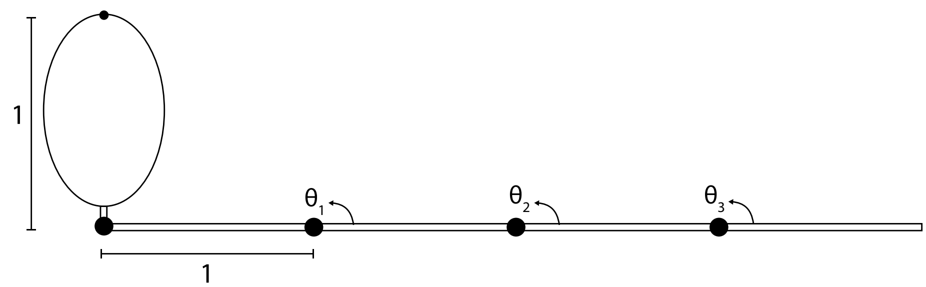
Stanford CS248: Interactive Computer Graphics
Exercise 6

Miscellaneous Problems

A. Imagine the human visual system could directly measure and interpret the full spectrum of incident light. (That is, your brain received and used full spectral information $L(\lambda)$ rather than just the response of S,M,L-cones). Why would this change to human perception make recording and displaying digital images and rendering pictures far more challenging? (Hint: consider reproducing the appearance of a real world scene on a display. The word metamer might be useful.)

B. Now that you've handed in programming assignment 3, you decide to go out clubbing to celebrate. You check the club's website and learn that tonight is "yellow light night", where the entire dance floor is illuminated in yellow looking light emitted from light sources that have red, green, and blue primaries. Your friend, who is in a glum mood, says, "I find it hard to party because I'm so sad that the CS248 programming assignments are over! I wish I could wear black tonight to show off my feelings, but I only have red shirts and blue shirts to choose from." You tell your friend, "Oh you can still look like you are wearing black!" Which shirt to you advise your friend to wear, and why?

C. Amazon rolls out a new three-elbowed autonomous delivery robot, “Zon”. The robot has arm segments of length 1, and joints θ_i that are constrained to rotate from 0 to 120 degrees. (In the direction shown in the figure.)



In a demonstration of its capabilities, Jeff Bezos asks the robot to touch the point on the top of it’s head (the top of the ellipse). Please give values of the θ_i’s that meet this goal (end of arm touching top of head).

Another Problem on Splines

In class we talked in detail about cubic Hermite splines. For a curve made up of $N-1$ cubic spline segments, the input to the ith spline segment was four values, the position of the segment's endpoints p_{i0} and p_{i1}, and the position derivatives u_{i0} and u_{i1} at these points. We derived the matrix form of the spline at segment i as follows:

$$p_i(t) = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{i0} \\ p_{i1} \\ u_{i0} \\ u_{i1} \end{bmatrix}$$

A. Consider animating the angle of rotation θ of a character's elbow using a Hermite spline. Specifically, for integers x ($0 \leq x < N$) and real-valued t ($0 \leq t \leq 1$), $\theta(x+t) = p_x(t)$. Describe the necessary constraints on the input points or derivatives needed to ensure the animation is C0 continuous (the elbow never jumps from one position to another) and C1 continuous (the elbow's velocity never makes a jump – which would suggest the character has infinitely strong muscles!)

B. Now imagine that instead of specifying points and velocities, you choose to define all $N-1$ spline segments using a sequence of N points p_i ($0 \leq i < N$), where segment i interpolates points p_i and p_{i+1}.

Unfortunately, the piecewise cubic curve just described is under-specified. (it doesn't have enough constraints!) So let's assume the derivative at each point p_i is set to $\frac{p_{i+1}-p_{i-1}}{2}$. **In this question, let's ignore the boundary cases for segments 0, and N-1, just answer for segments 1...N-2.** Given this definition you should be able to specify a 4×4 matrix that converts the new control points $p_{i-1}, p_i, p_{i+1}, p_{i+2}$ into the inputs for a Hermite spline. Fill in that matrix below.

$$f_i(t) = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$

C. Now multiply the two 4×4 matrices from part B to arrive at the matrix form of this new type of spline. (yes, this part both depends on your work on part B, and is a little mechanical, but we've given you three elements of the matrix to sanity check your work.

$$f_i(t) = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} & & \\ & & & \\ & & & \\ & & & 0 \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$

D. Finally, write the four basis functions $B_i(t)$ $(0 \leq i < 4)$ for this new spline. In other words what are the functions $B_i(t)$ such that:

$$f_i(t) = p_{i-1}B_0(t) + p_i B_1(t) + p_{i+1}B_2(t) + p_{i+2}B_3(t)$$

Note: If you were unable to derive a matrix in part C, to obtain credit you can answer this question in terms of symbolic matrix elements M_{ij} (i selects a row, j selects a column). However, if you did get an answer in part C, please answer this question in terms of that answer.

E. Finally, consider the following linear basis functions:

$$B_0(t) = 1 - t$$
$$B_1(t) = t$$

Consider an animation curve made of $N - 1$ segments, as defined by points P_i, just like in part B. Now each segment is defined by:

$$f_i(t) = p_i B_0(t) + p_{i+1} B_1(t)$$

Does the resulting animation have smooth motion (continuous velocity)? Why or why not?