Lecture 16

Introduction to Animation

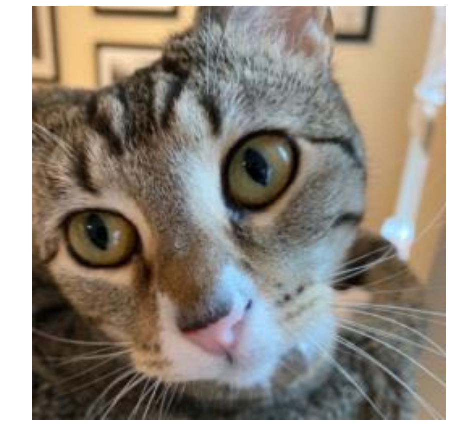
Interactive Computer Graphics
Stanford CS248, Winter 2021

Review from last time: Gaussian and Laplacian Pyramid Representations

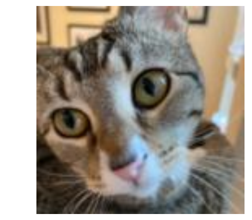
Gaussian pyramid

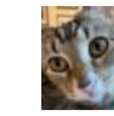
G_0 = original image

$G_1 = \text{down}(G_0)$



$G_2 = \text{down}(G_1)$



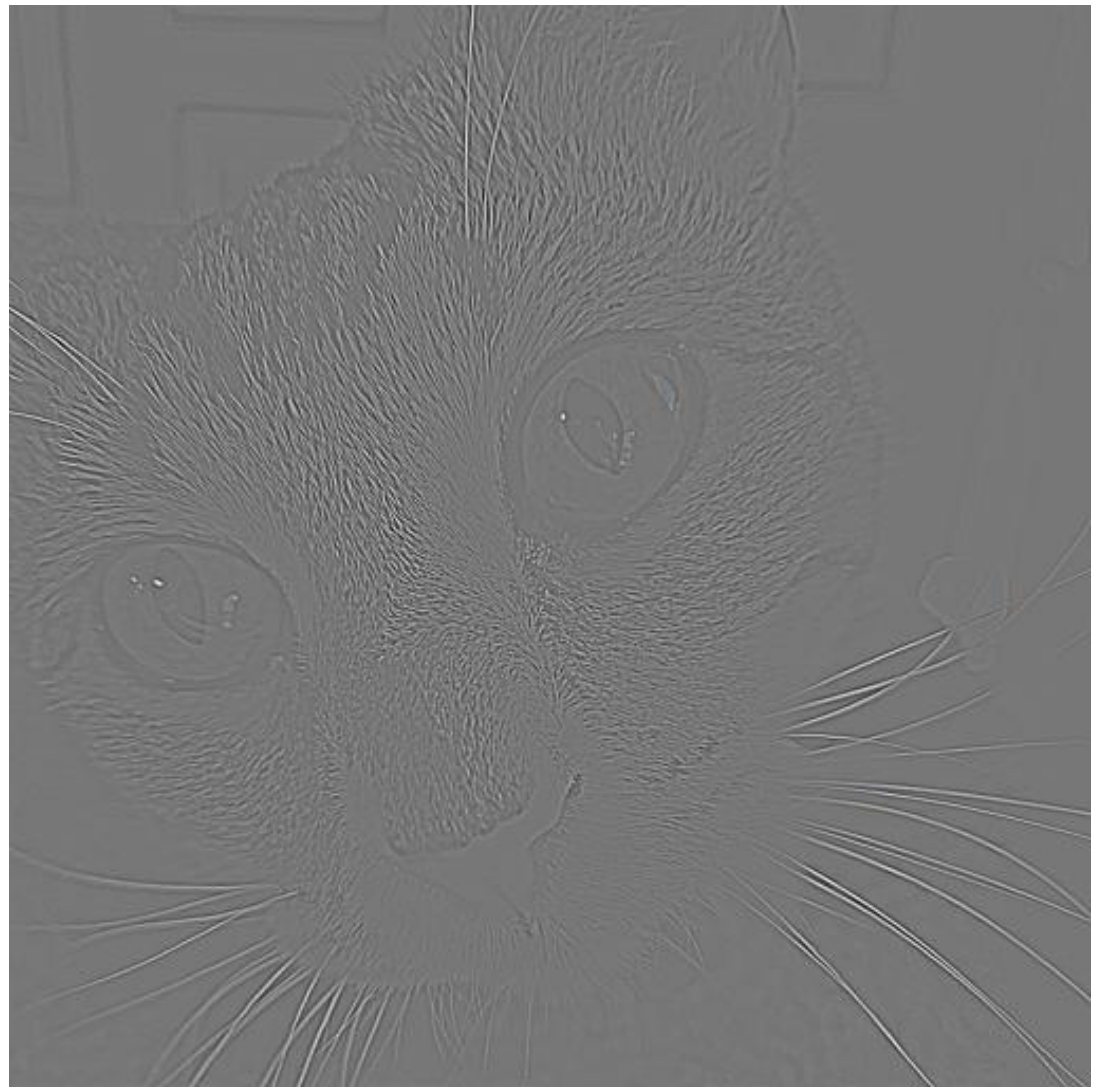


Each image in pyramid contains increasingly low-pass filtered signal

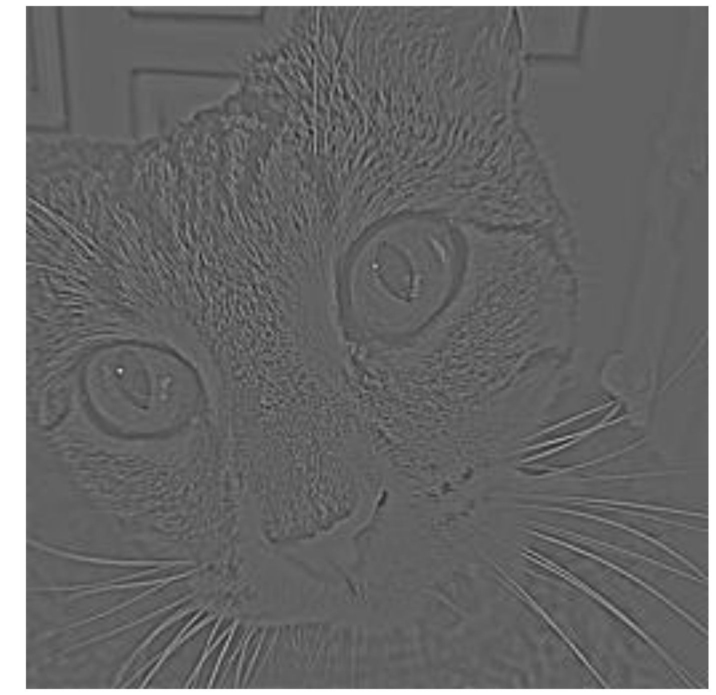
down() = Gaussian blur, then downsample by factor of 2 in both X and Y dimensions

Laplacian pyramid

$L_0 = G_0 - up(G_1)$



$L_1 = G_1 - up(G_2)$

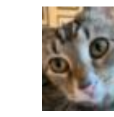


$L_2 = G_2 - up(G_3)$

$L_3 = G_3 - up(G_4)$

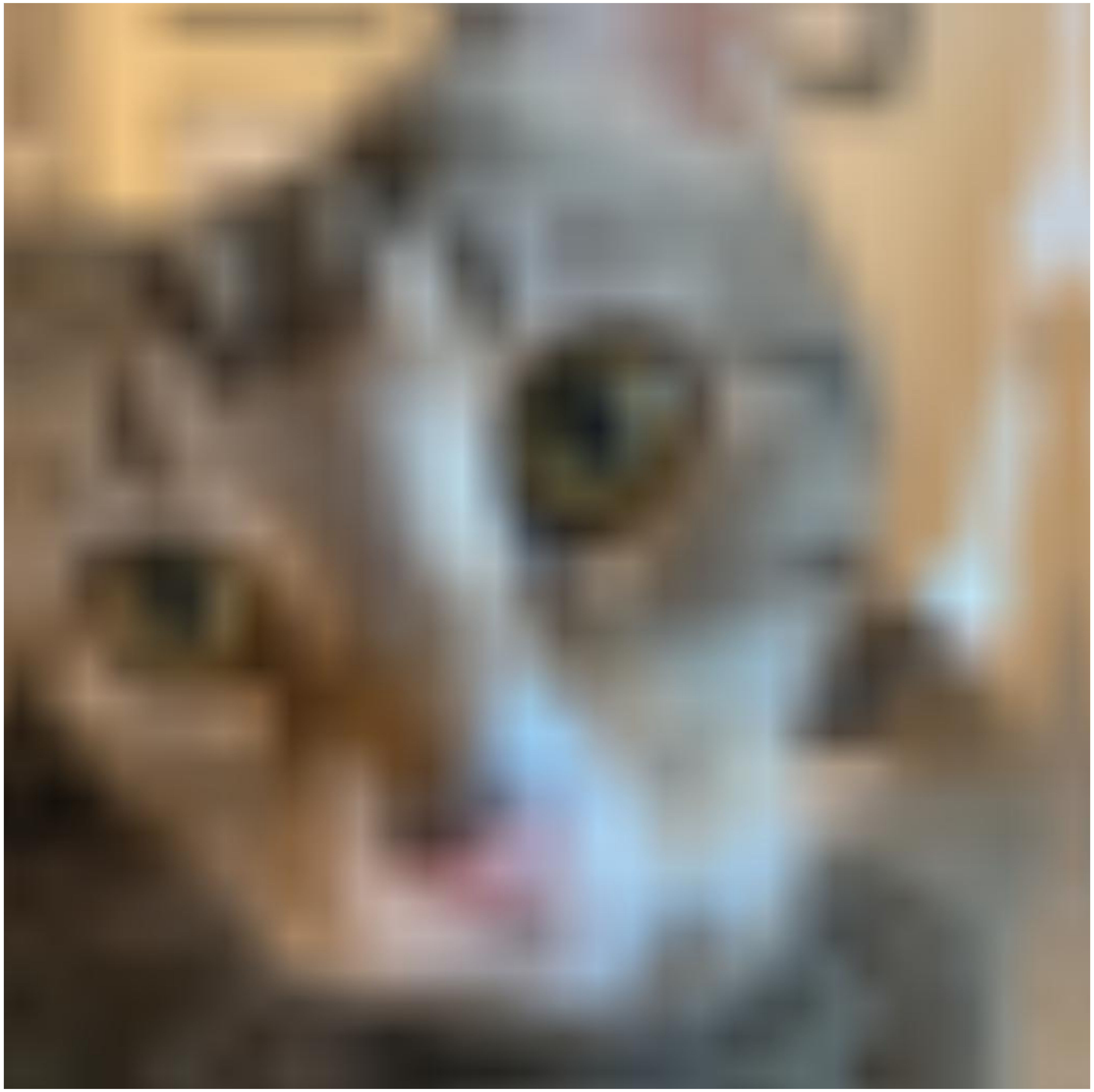
$L_4 = G_4 - up(G_5)$

$L_5 = G_5$



Question: how do you reconstruct original image from its Laplacian pyramid?

Laplacian pyramid



$L_5 = G_5$

Laplacian pyramid

$L_4 = G_4 - \text{up}(G_5)$

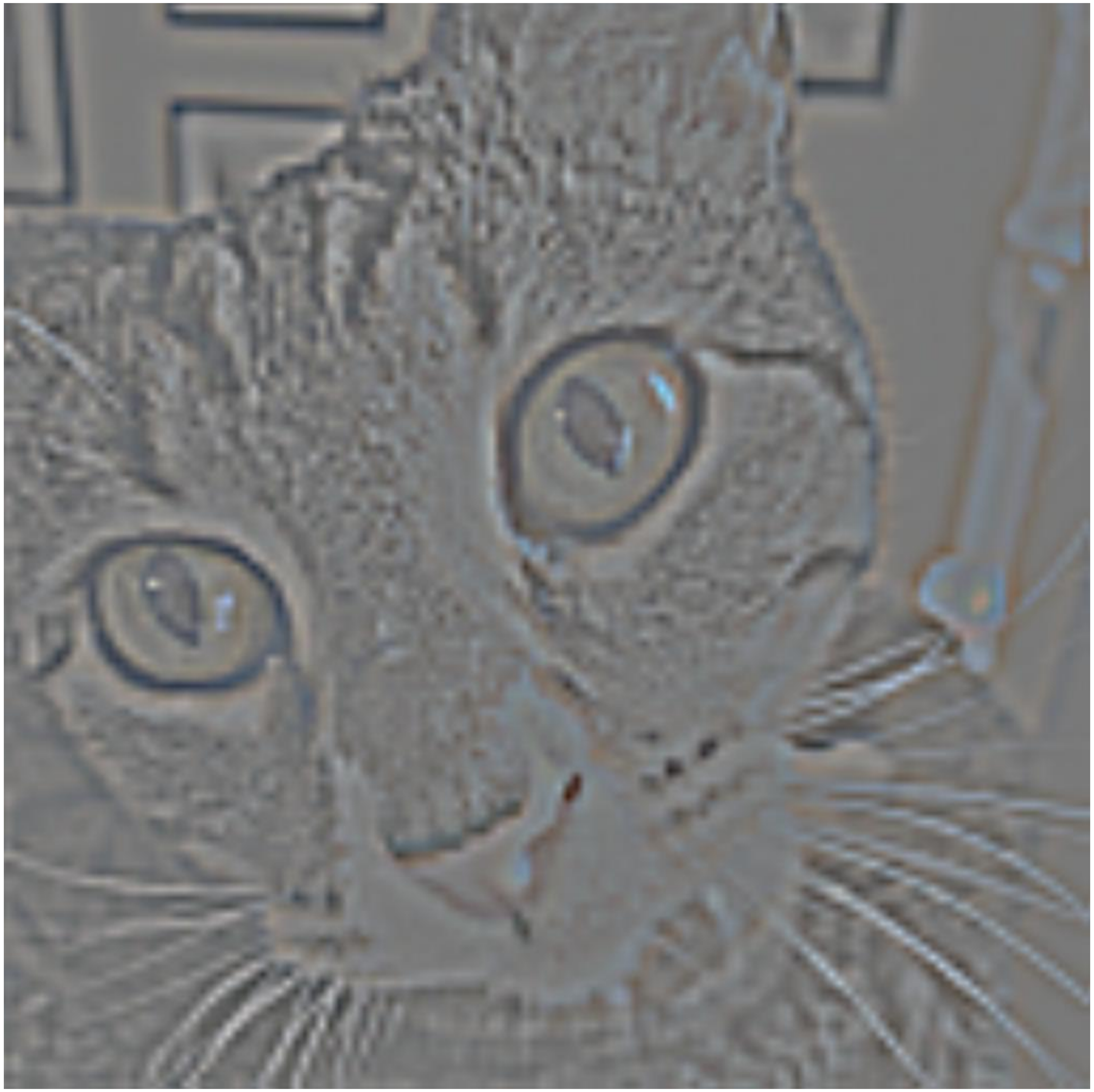
(upsampled back to full res for visualization)

Gaussian pyramid

G_4

(upsampled back to full res for visualization)

Laplacian pyramid



$L_3 = G_3 - \mathrm{up}(G_4)$

(upsampled back to full res for visualization)

Gaussian pyramid

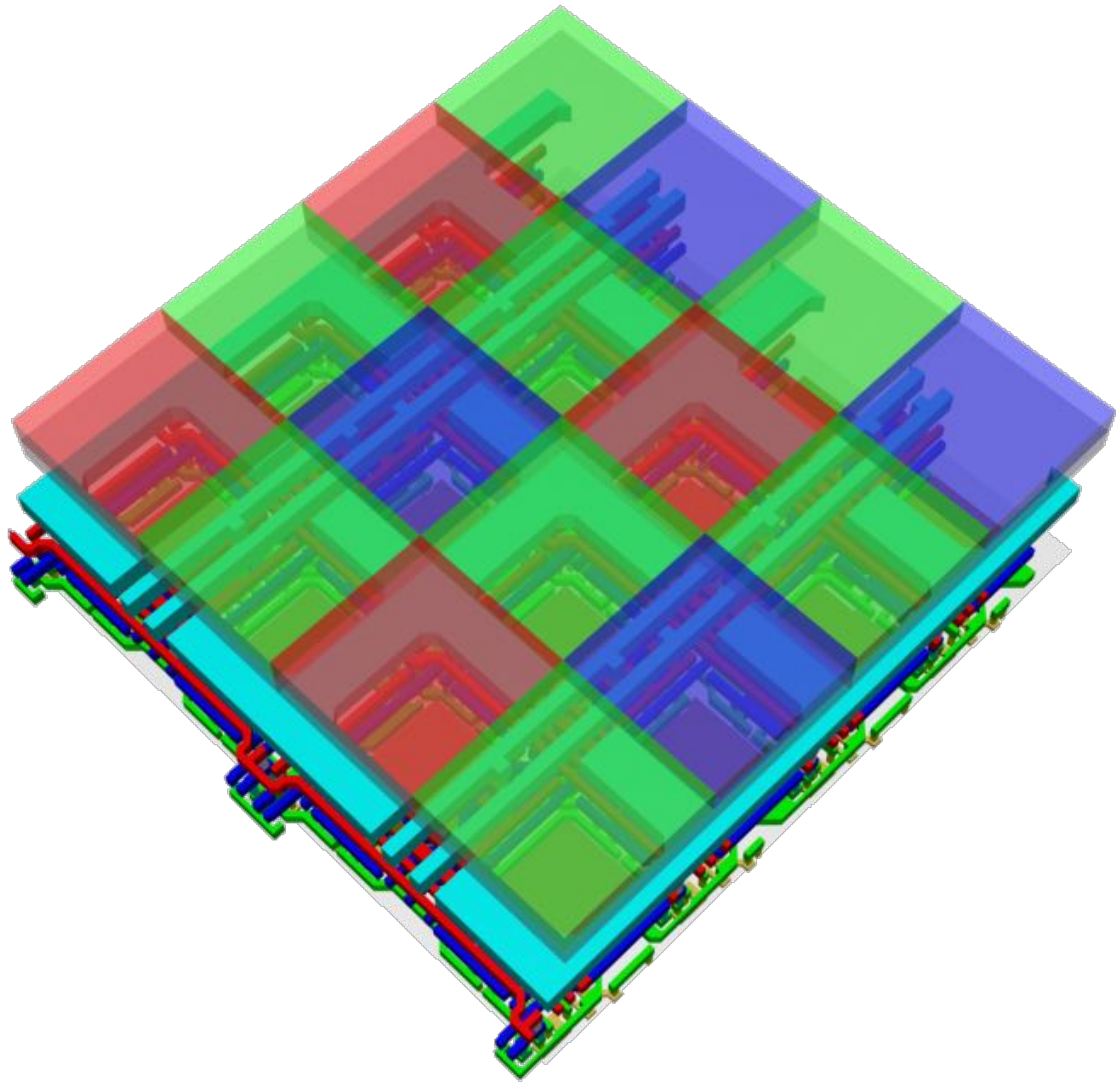
G_3

(upsampled back to full res for visualization)

One more topic related to cameras:
Exposure

Sensor with color filter array

(Different pixels have different photon frequency response curves)



Courtesy R. Motta, Pixim

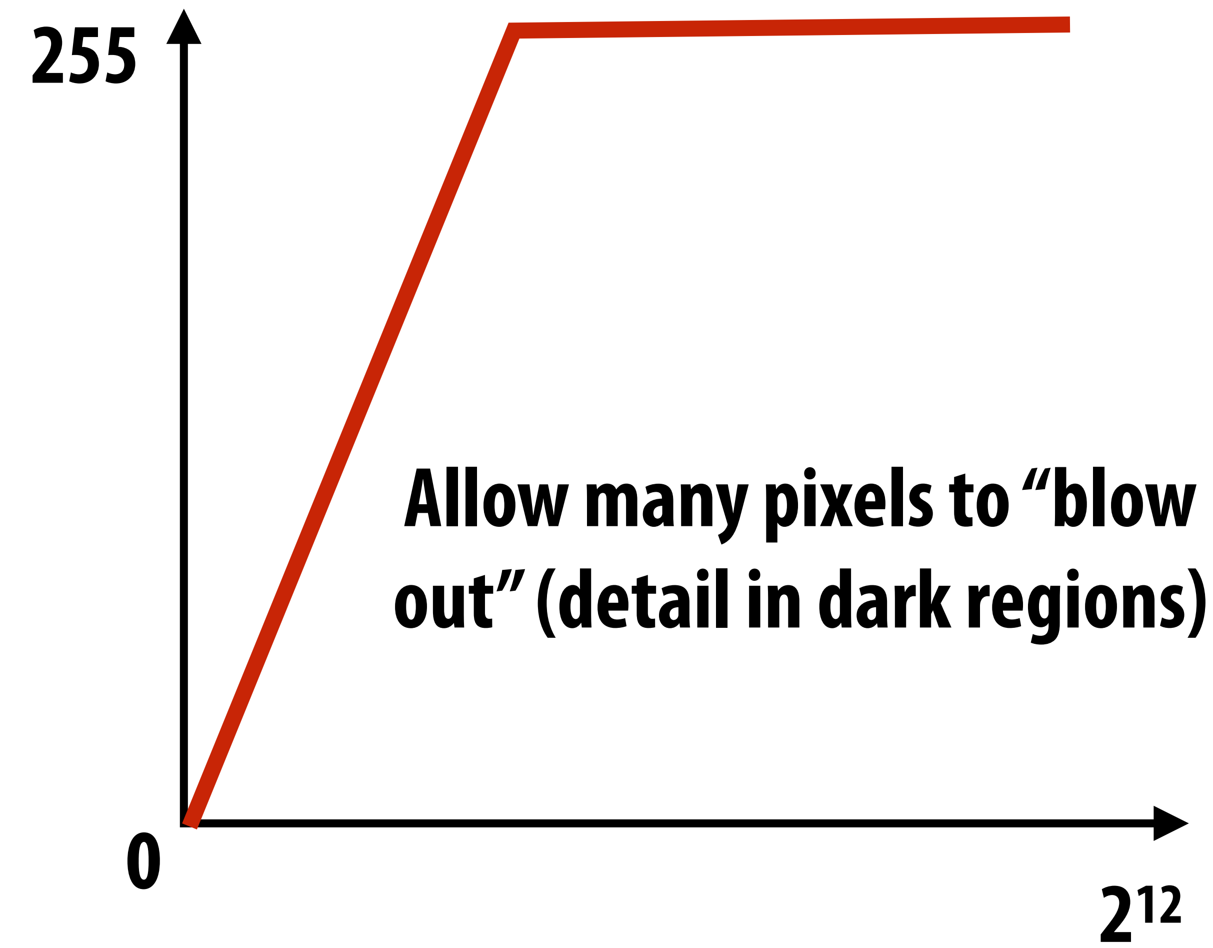
Saturated pixels

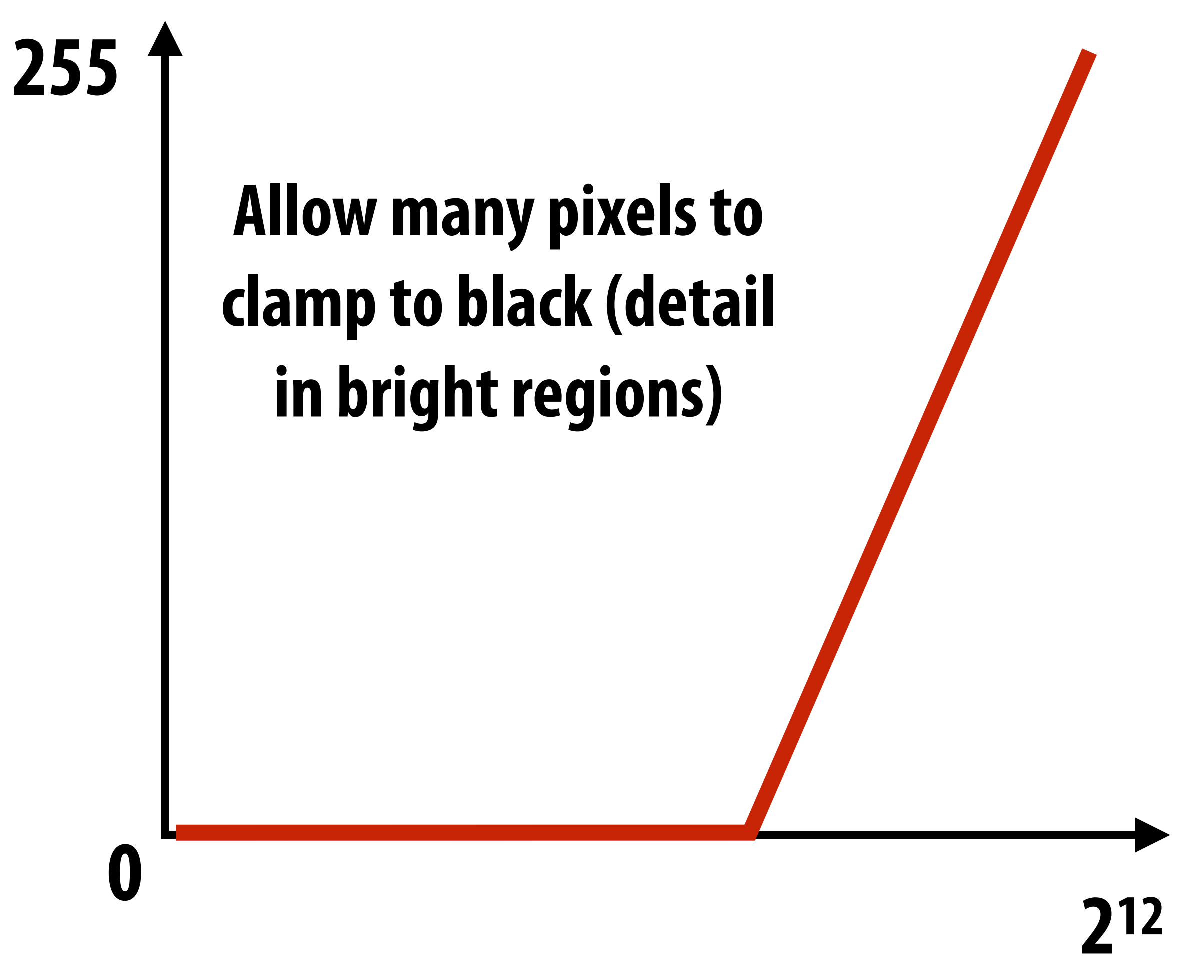
Pixels have saturated (no detail in image)

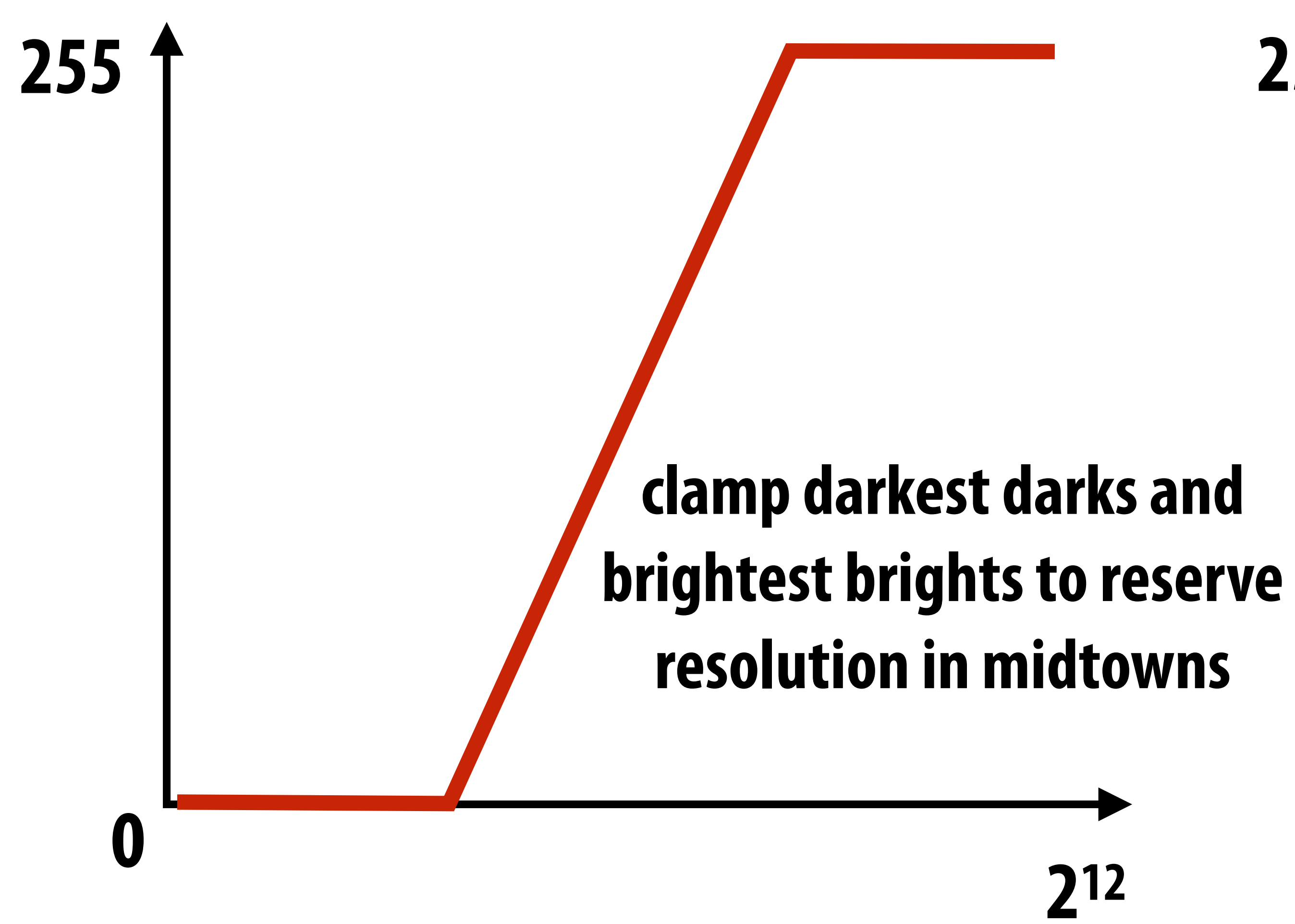
Global tone mapping

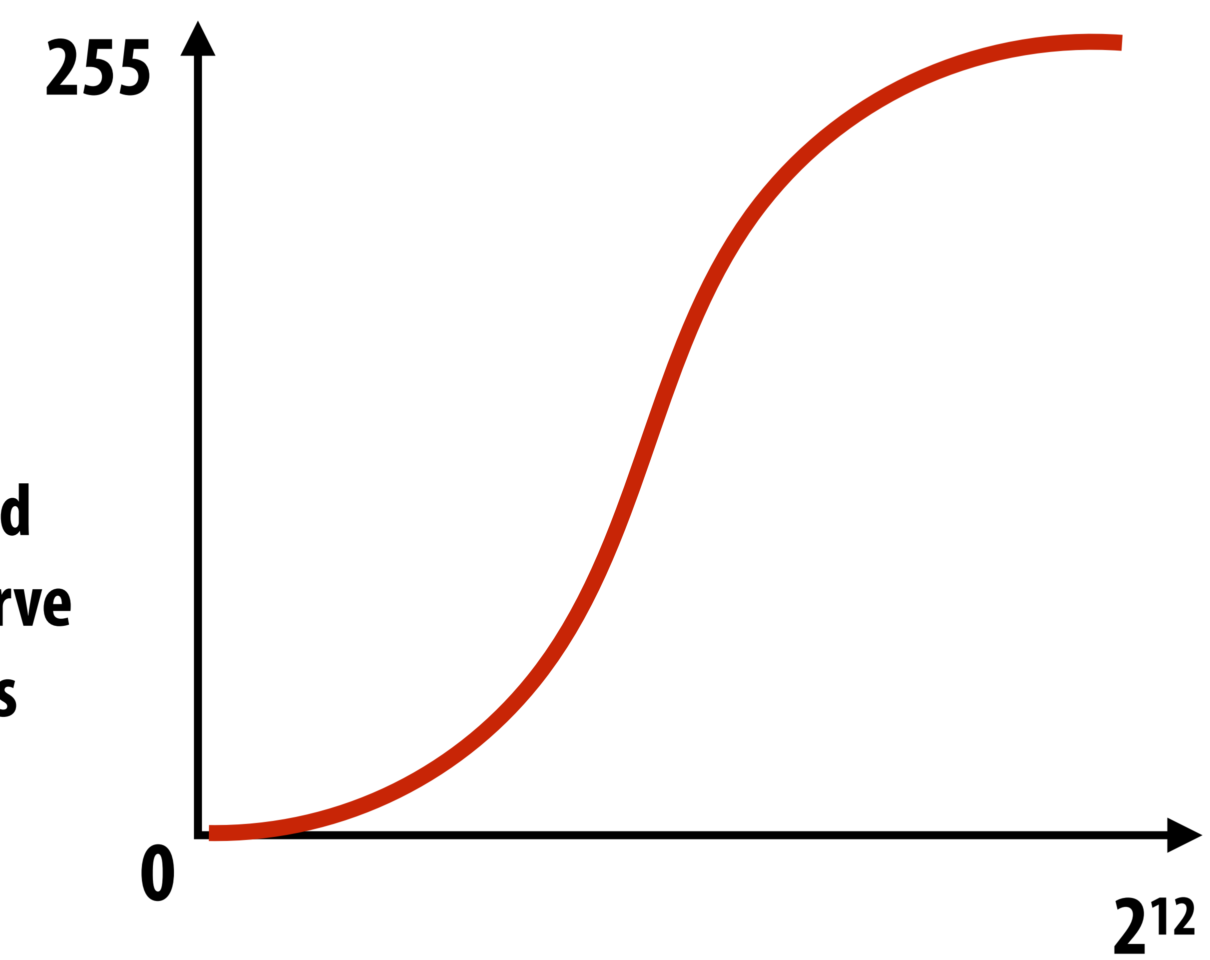
- Measured image values: 10-12 bits/pixel, but common image formats (8-bits/pixel)
- How to convert 12 bit number to 8 bit number?

$$out(x,y) = f(in(x,y))$$

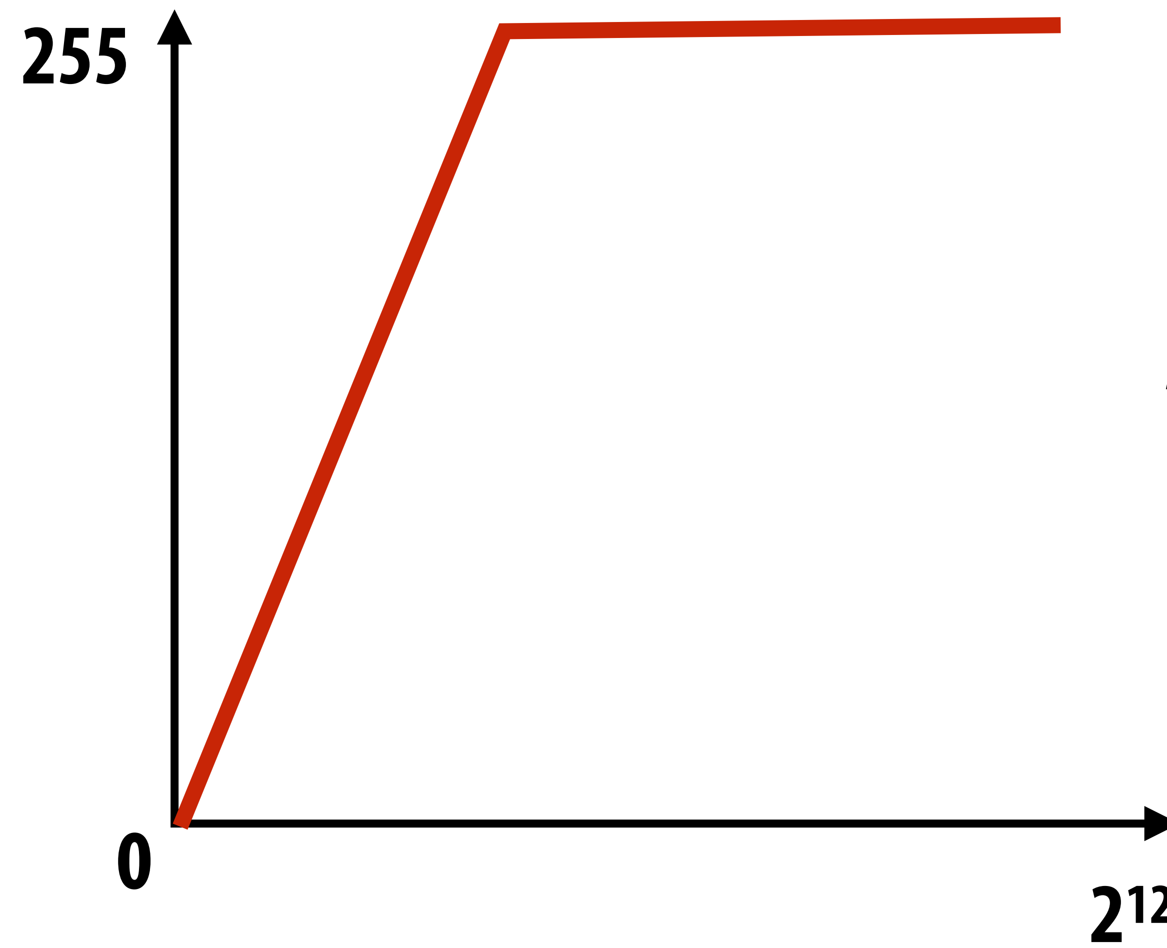




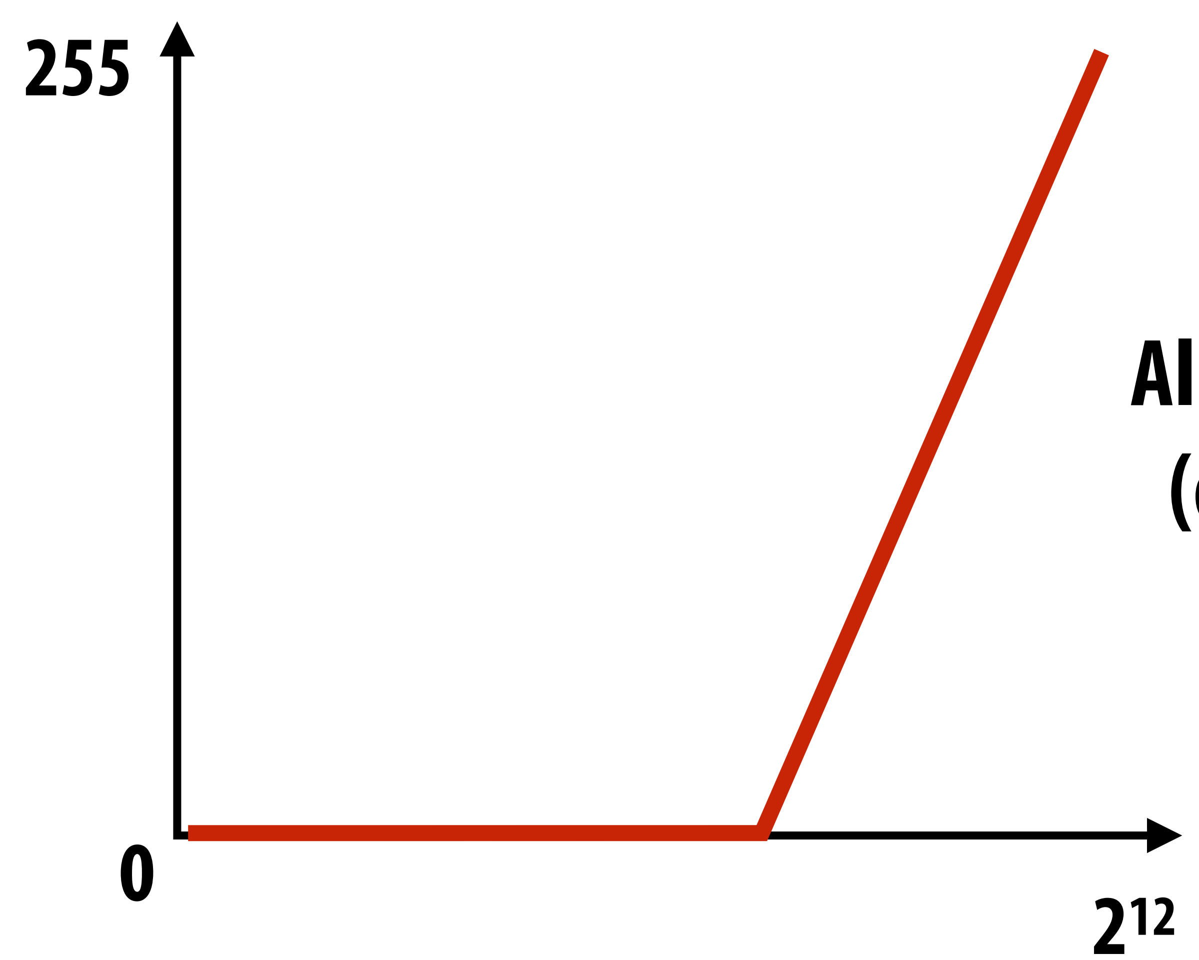




Global tone mapping



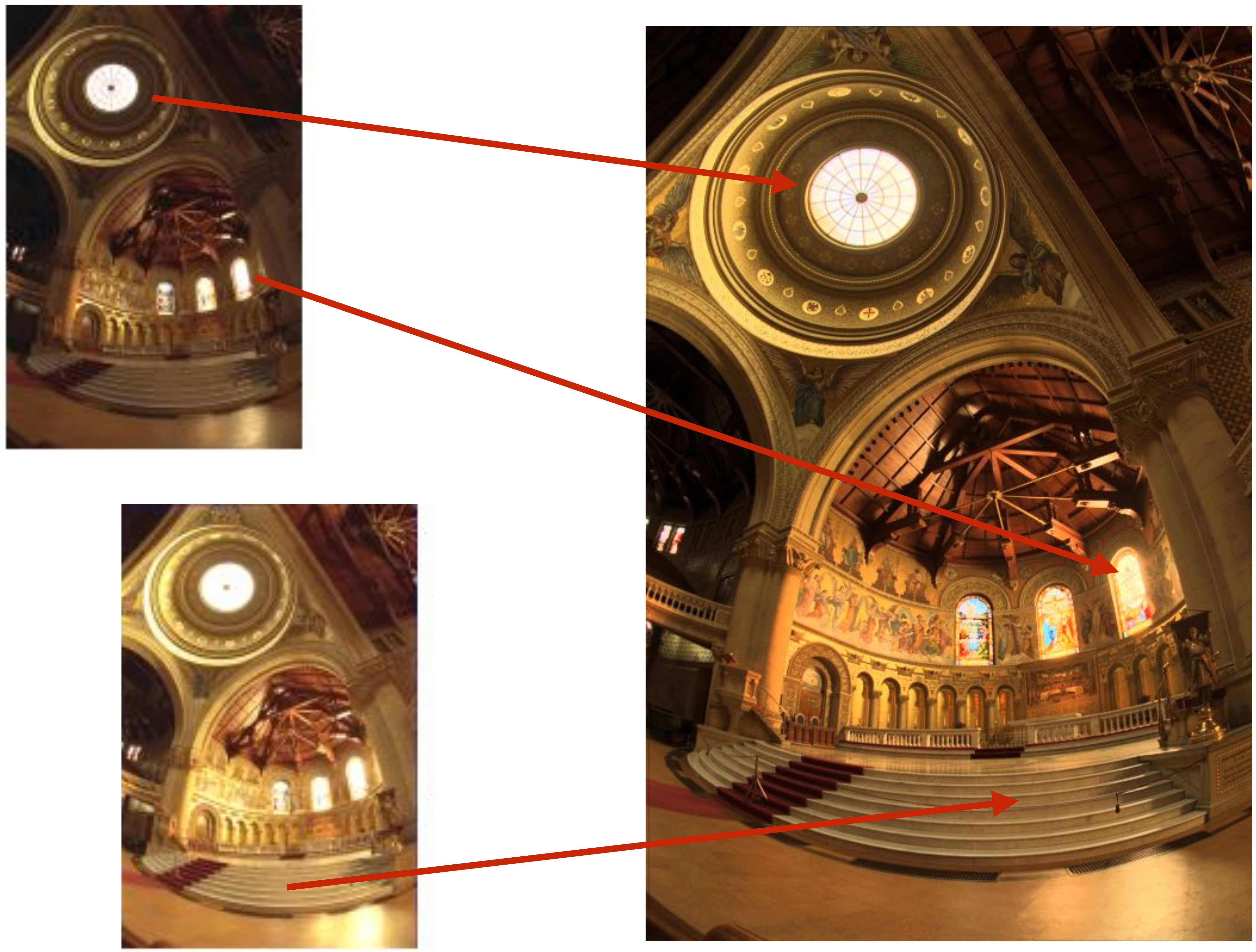
Allow many pixels to "blow out"
(detail remains in dark regions)



Allow many pixels to clamp to black
(detail remains in bright regions)

Local tone mapping

- **Different regions of the image undergo different tone mapping curves (preserve detail in both dark and bright regions)**



Local tone adjustment

Pixel values

Weight Masks

Improve picture's aesthetics by locally adjusting contrast, boosting dark regions, decreasing bright regions (no physical basis at all!)

Combined image
(unique weights per pixel)

[Image credit: Mertens 2007]

Challenge of merging images

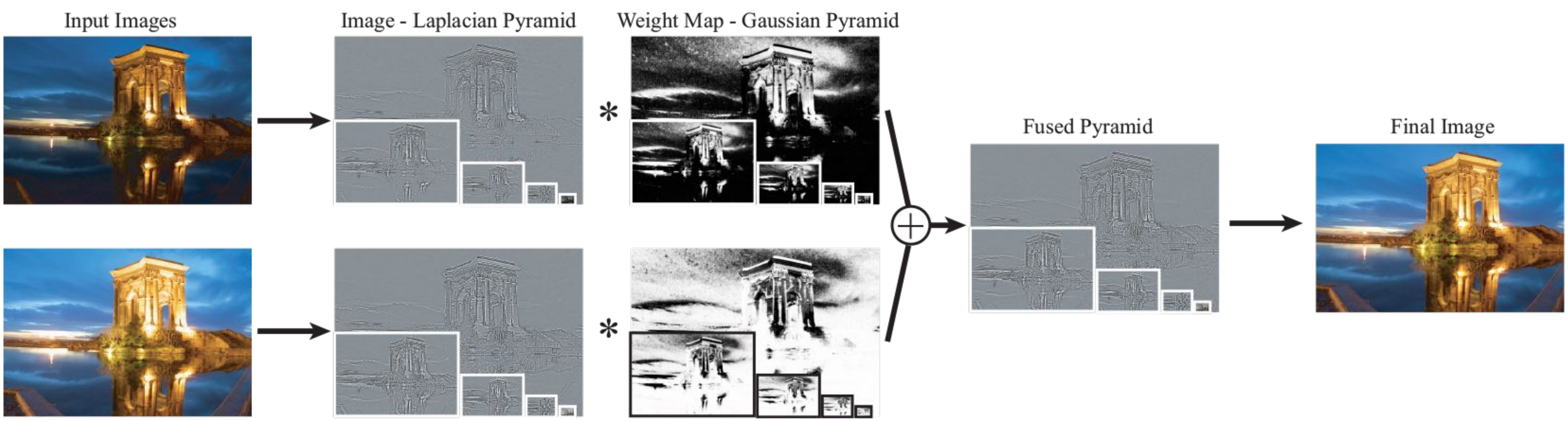
Four different exposures (corresponding weight masks not shown)

Merged result
(based on weight masks)
Notice "banding" since absolute intensity of
different exposures is different

Merged result
(after blurring weight mask)
Notice "halos" near edges

Use of Laplacian pyramid in tone mapping

- Compute weights for all Laplacian pyramid levels
- Merge pyramids (merge image features), not image pixels
- Then "flatten" merged pyramid to get final image



Challenges of merging images

Four exposures (weights not shown)

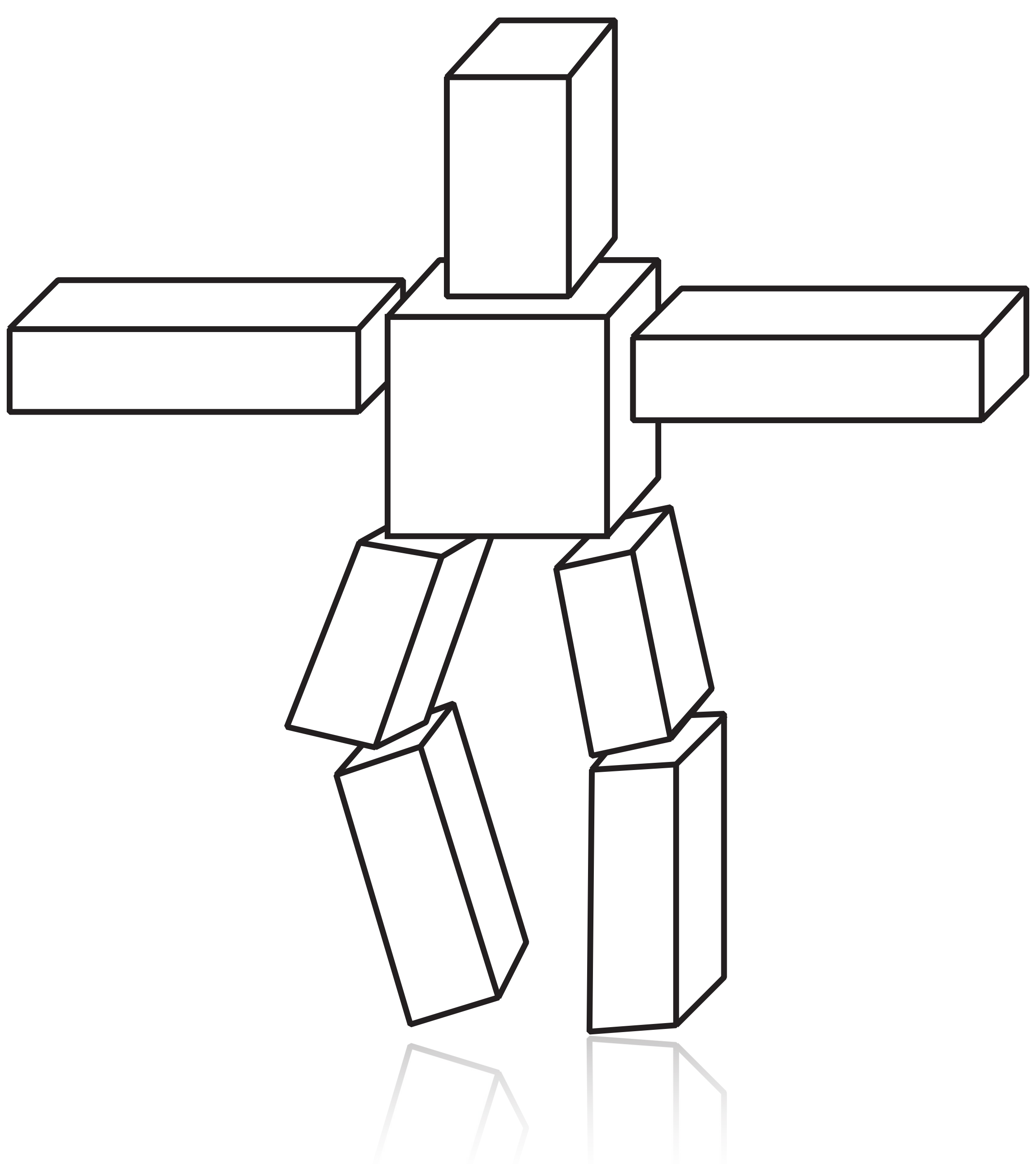
Merged result
(after blurring weight mask)
Notice “halos” near edges

Merged result
(based on multi-resolution pyramid merge)

Today: intro to animation

Increasing the complexity of our model of the world

Transformations



Geometry

Materials, lighting, ...

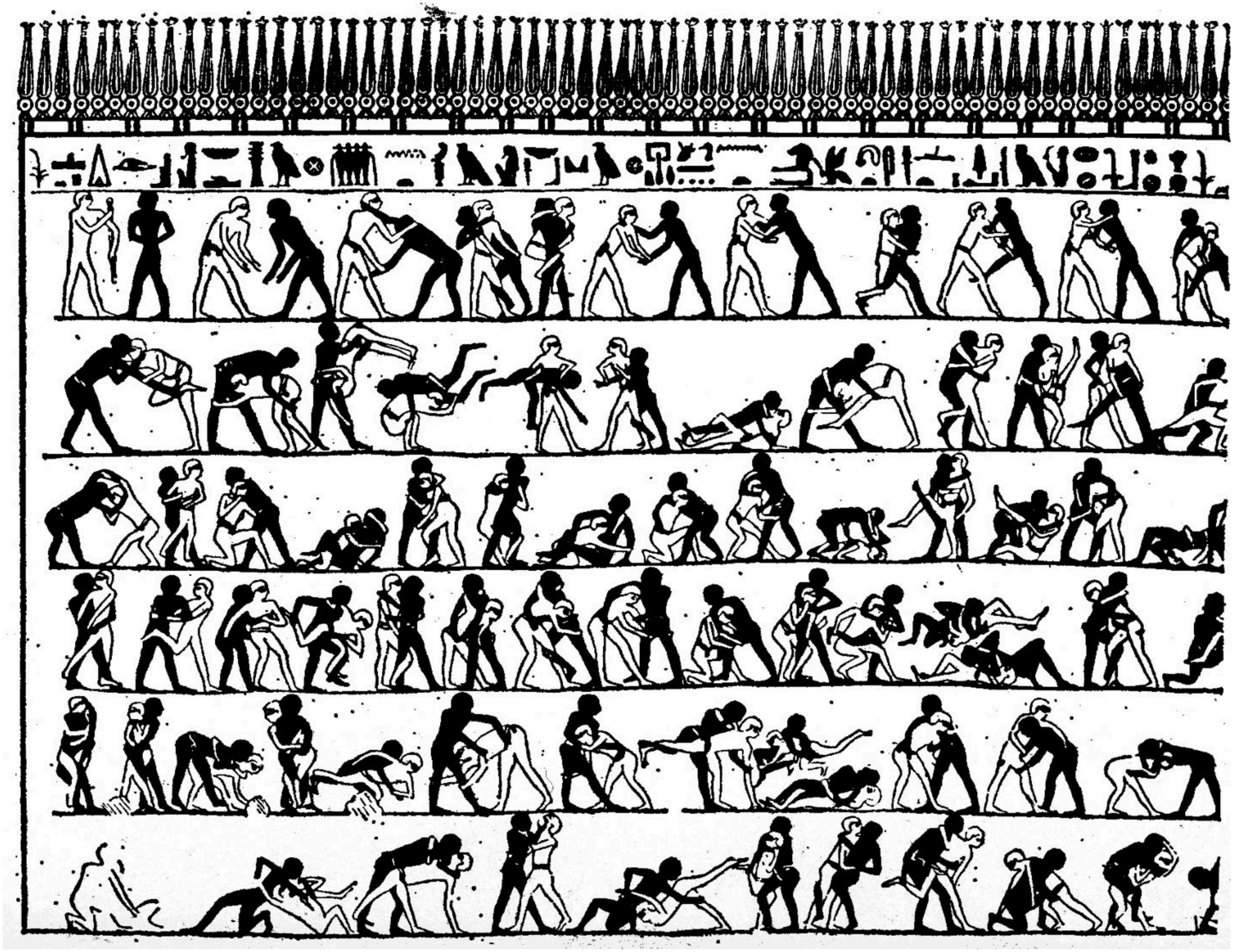
Increasing the complexity of our model of the world

...but what about *motion?*

First animation

(Shahr-e Sukhteh, Iran 3200 BCE)

History of animation



(tomb of Khnumhotep, Egypt 2400 BCE)

History of animation

(Phenakistoscope, 1831)

First film

- Originally used as scientific tool rather than for entertainment
- Critical *technology* that accelerated development of animation

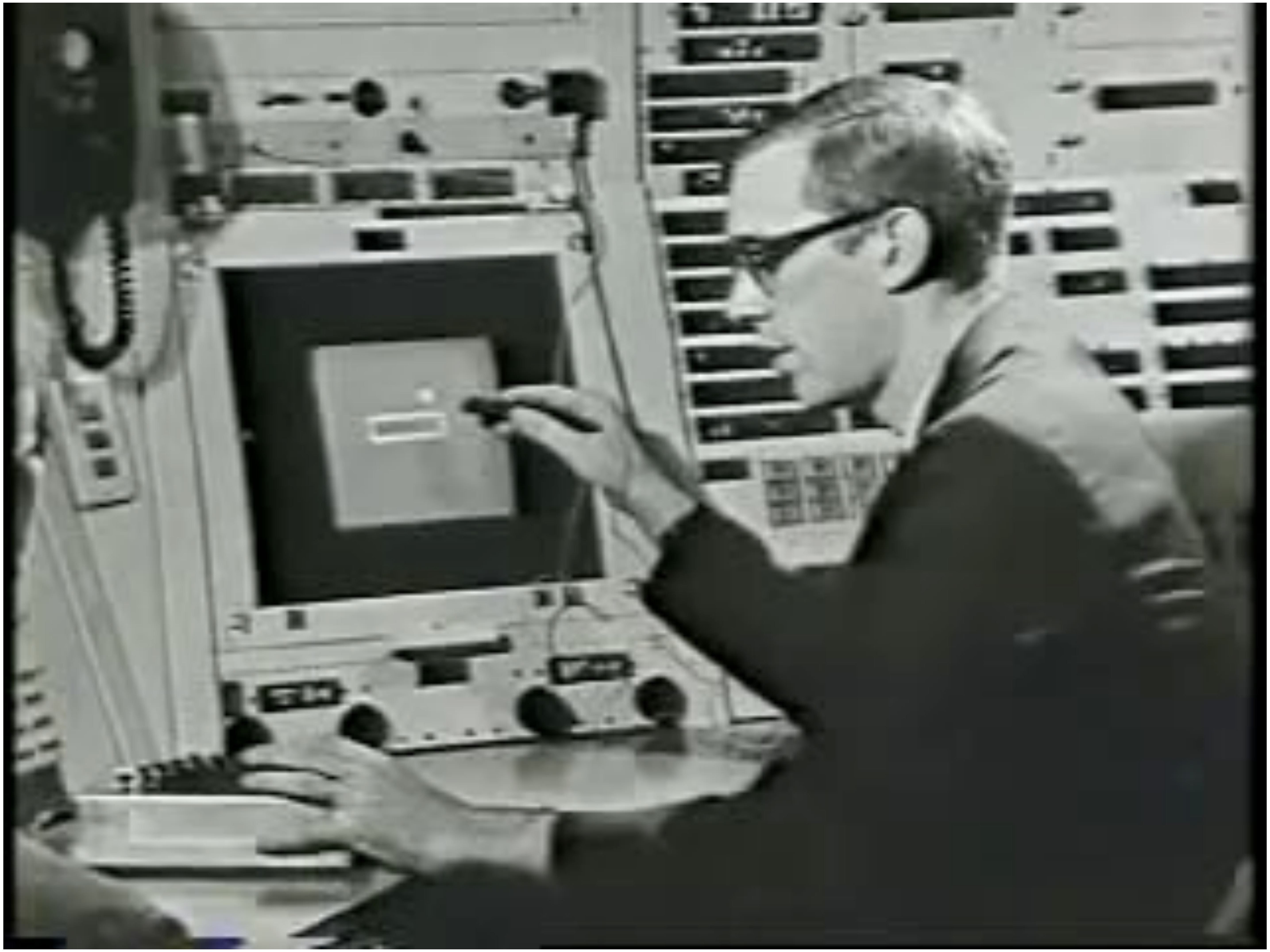
Eadweard Muybridge, *"Sallie Gardner"* (1878)

Interesting note: study commissioned by Leland Stanford (to determine if horse's feet ever off the ground)

First hand-drawn feature-length animation

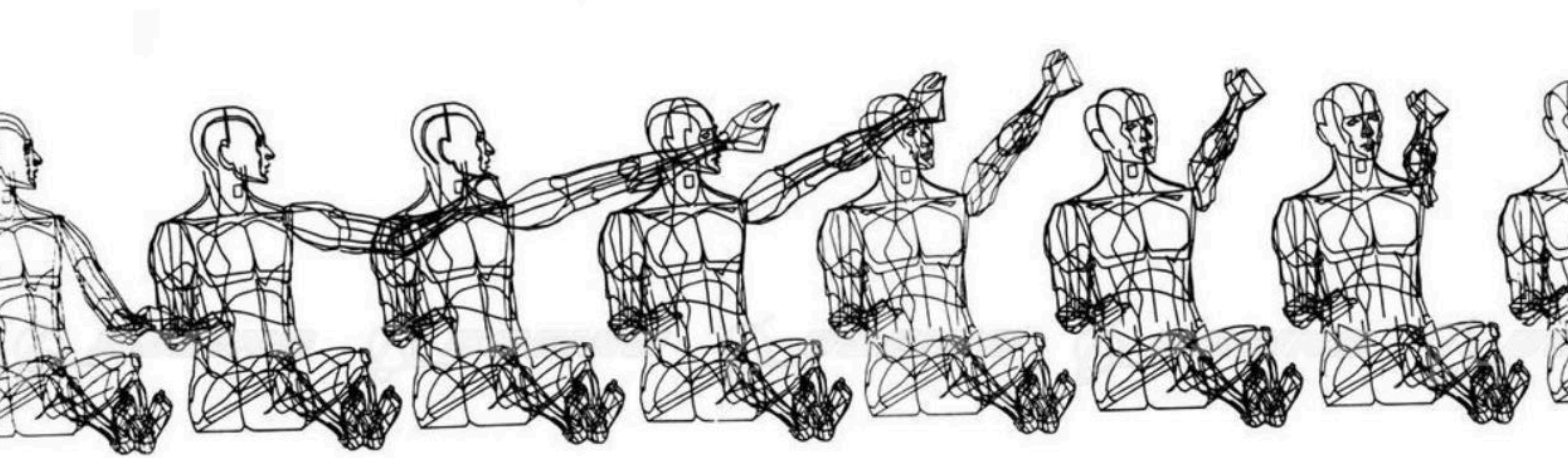
Disney, “Snow White and the Seven Dwarfs” (1937)

First digital-computer-generated animation



Ivan Sutherland, “Sketchpad” (1963)

First 3D computer animation

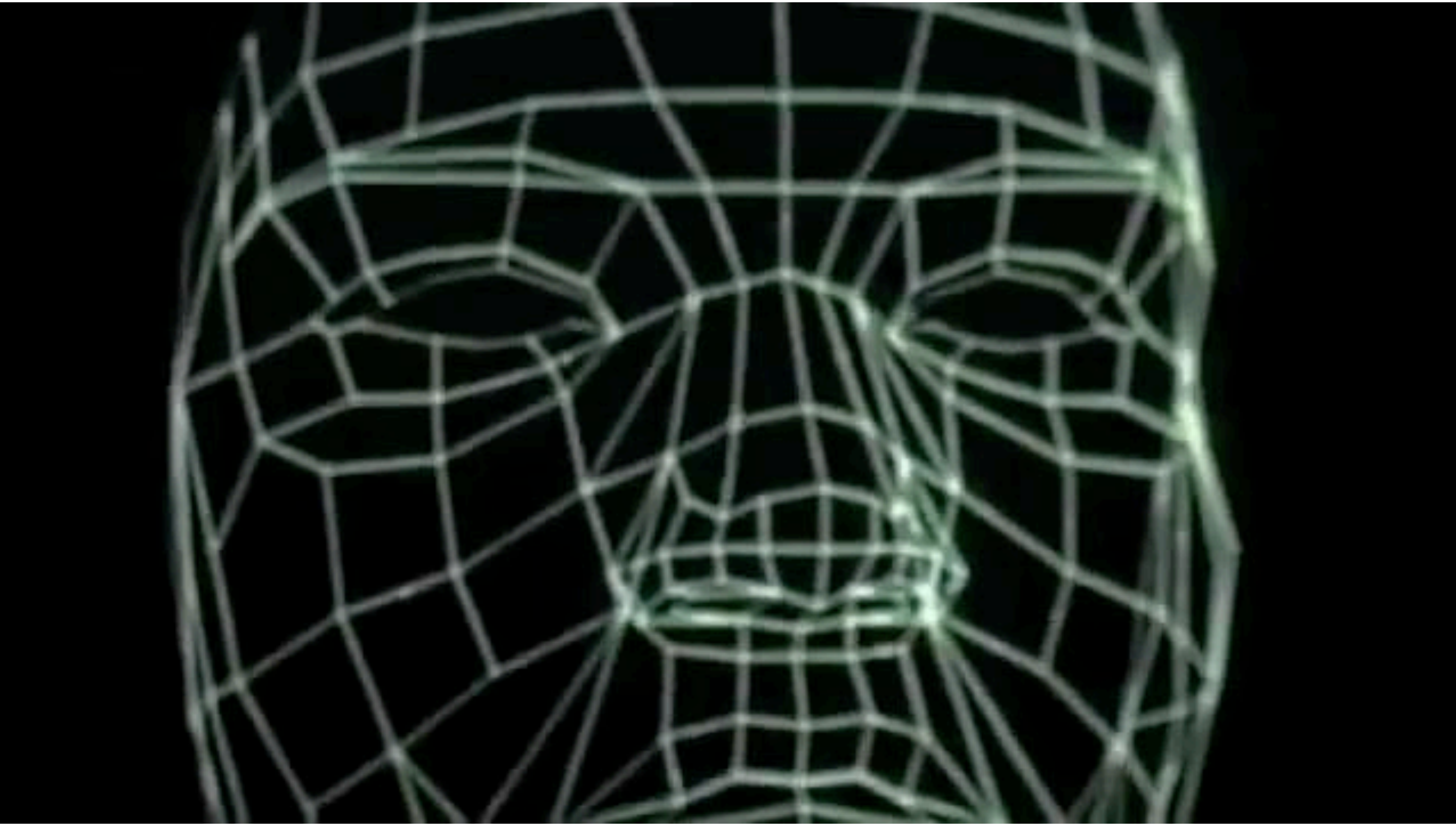


William Fetter, "Boeing Man" (1964)

Early computer animation

Nikolay Konstantinov, “Kitty” (1968)

Early computer animation



Ed Catmull & Fred Park, “Computer Animated Faces” (1972)

First a*ttempted* CG feature film

NYIT [Williams, Heckbert, Catmull, ...], "The Works" (1984)

First CG feature film

Pixar, “Toy Story” (1995)

Computer animation - present day

Notice combination of character animation, camera animation, and physical simulation in this clip.

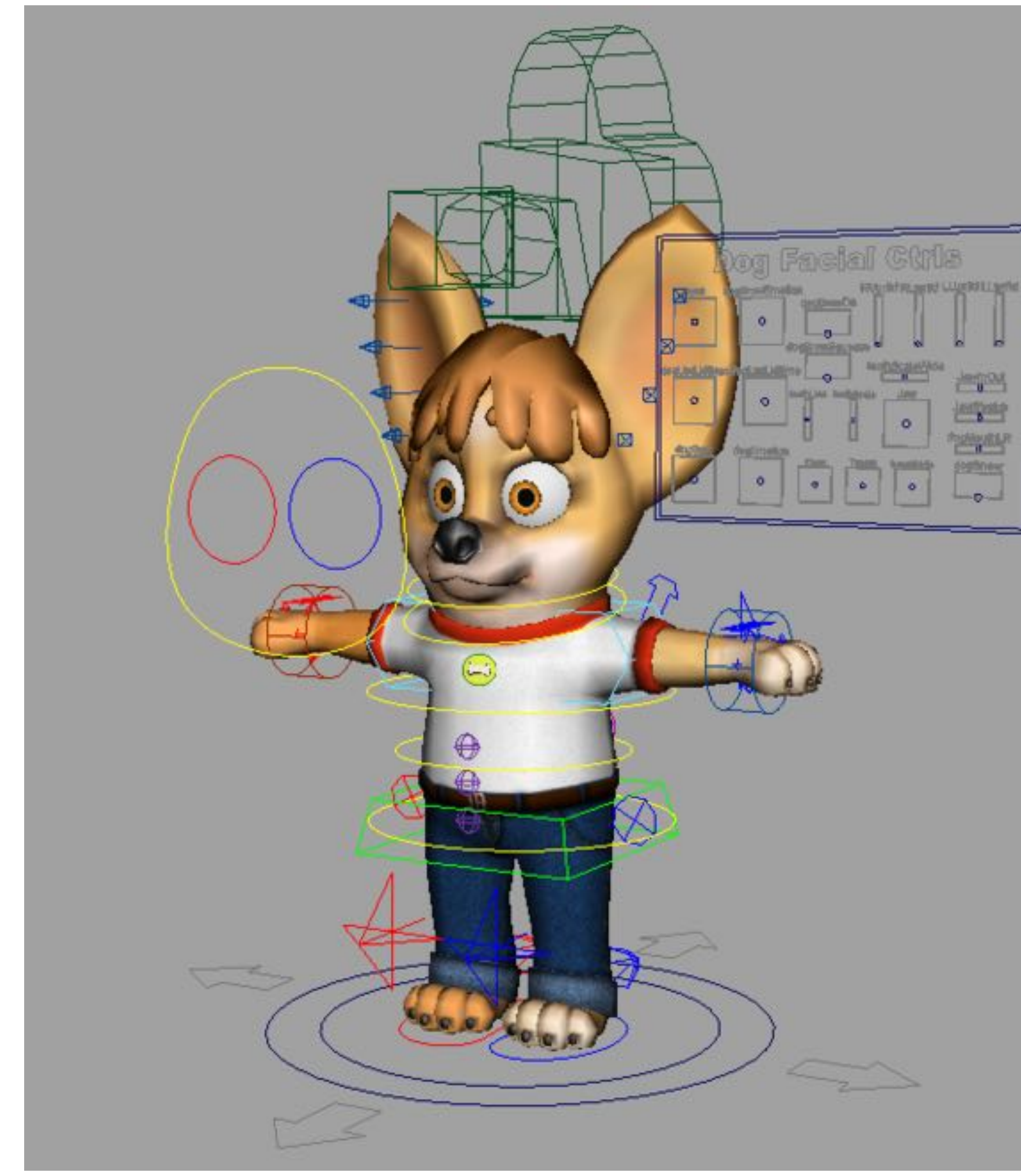
Pixar's Coco (2017)

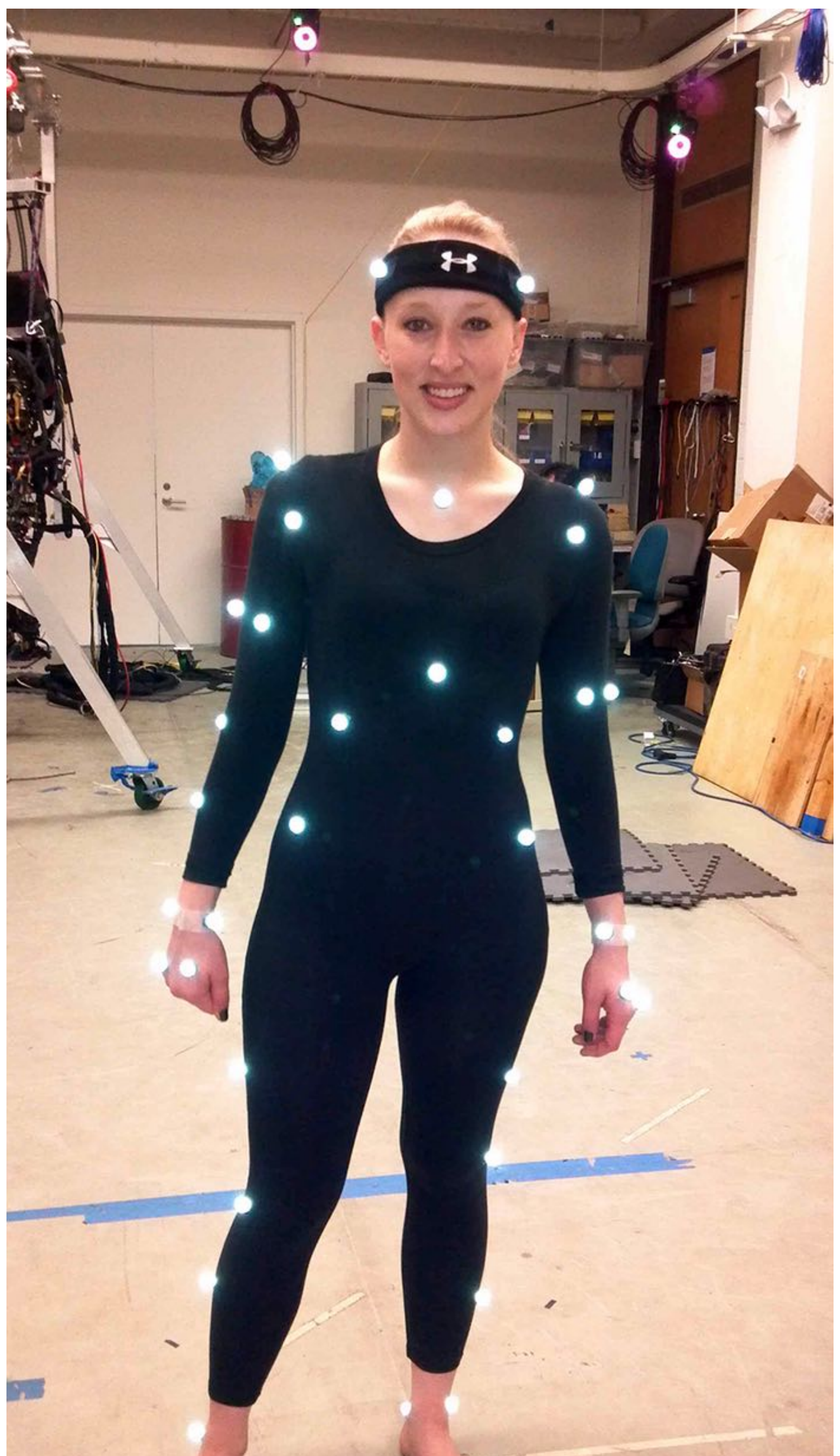
https://www.youtube.com/watch?v=GvicFasn_yM&t=4m5s

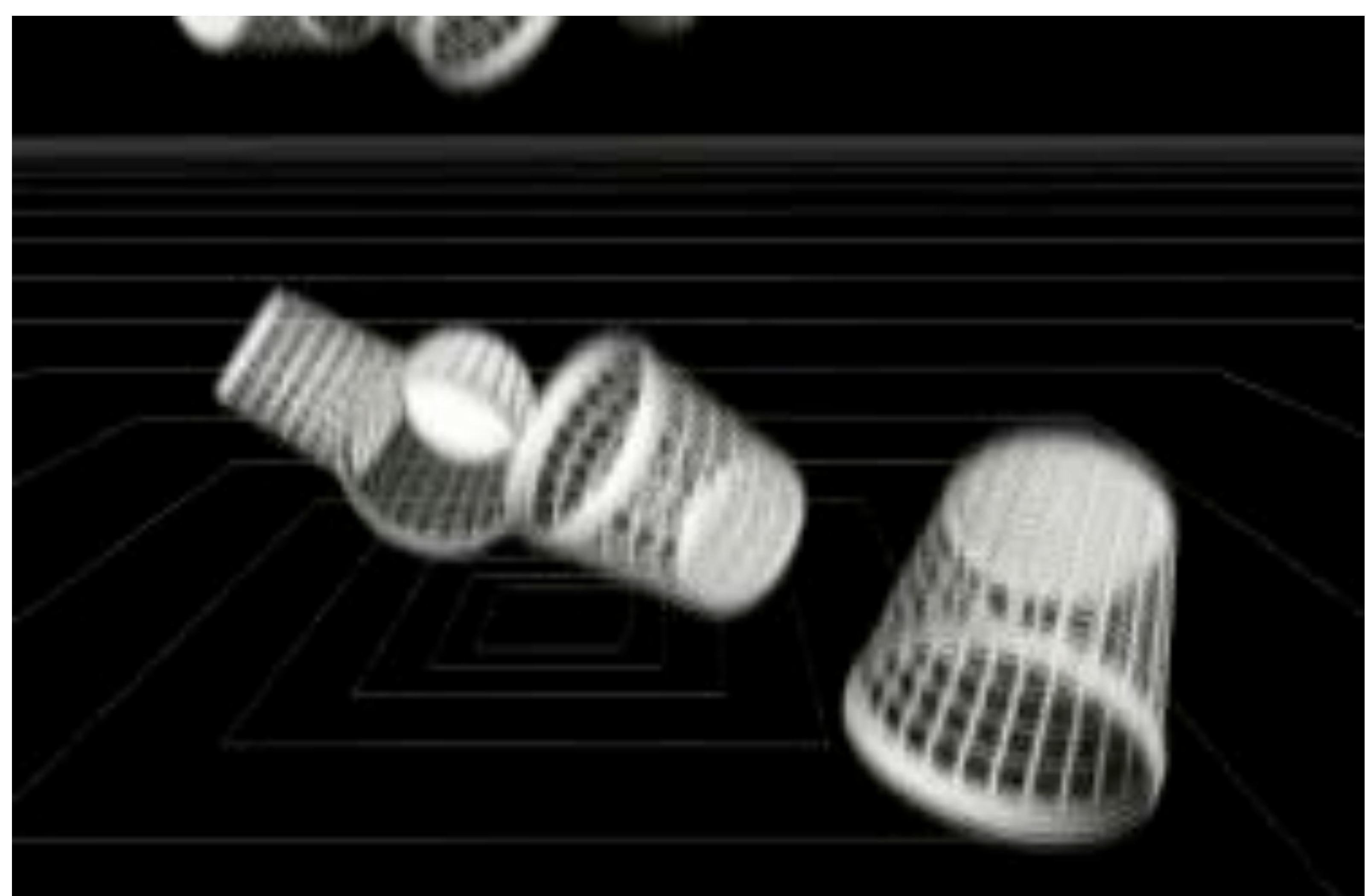
How do we describe motion on a computer?

Basic techniques in computer animation

- **Artist-directed (e.g., keyframing)**
- **Data-driven (e.g., motion capture)**
- **Procedural (e.g., simulation)**

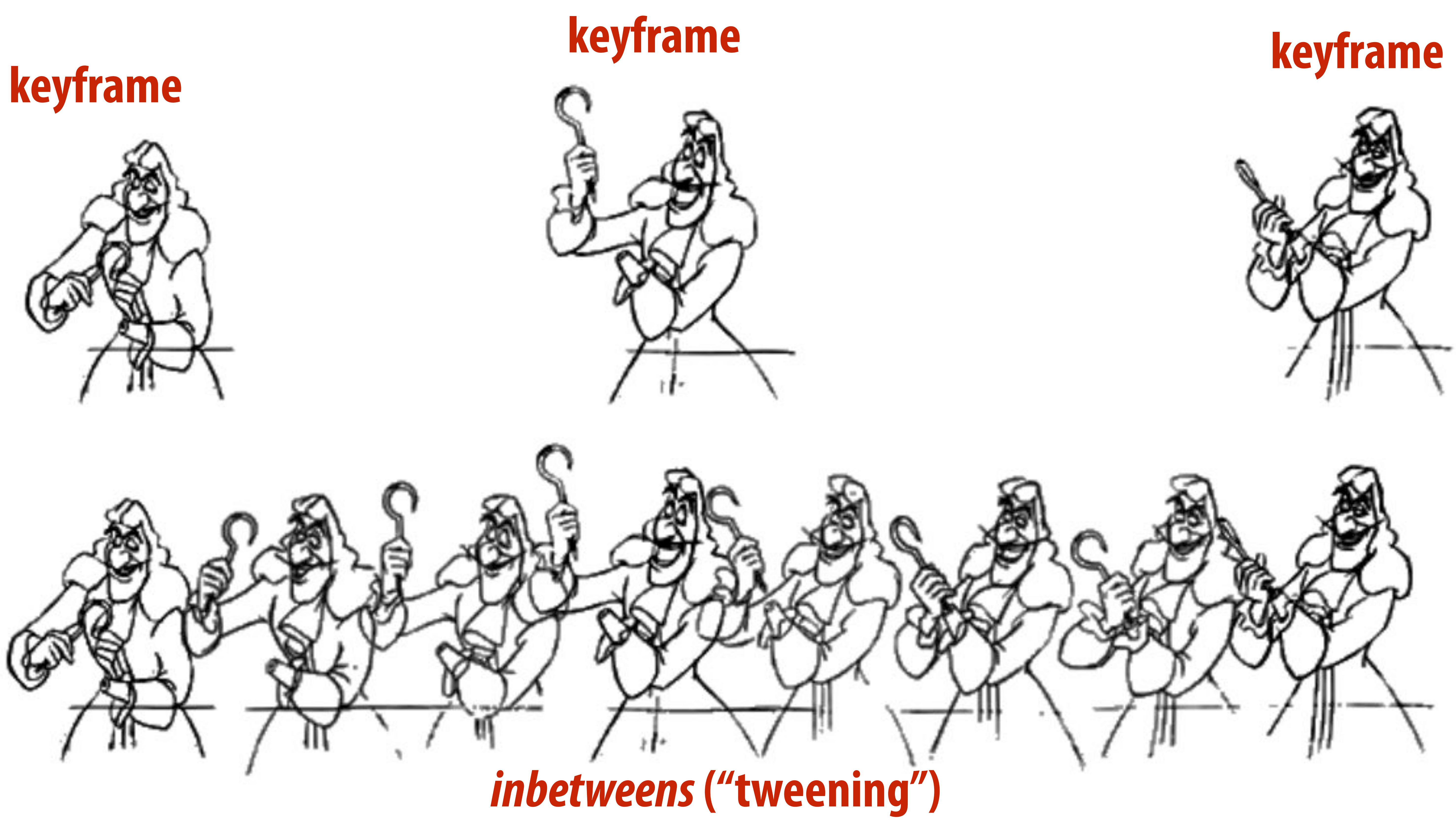






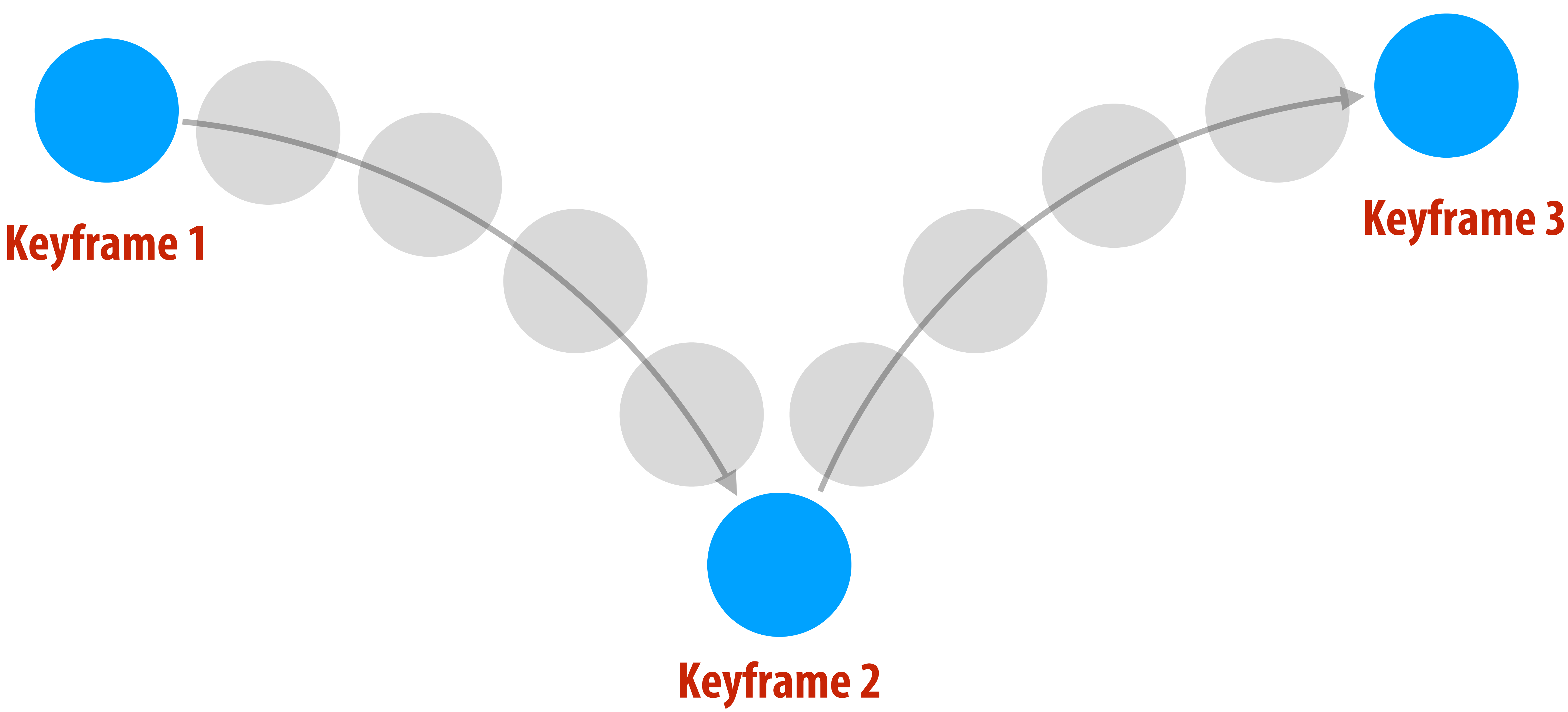
Generating motion (hand-drawn)

- Senior artist draws *keyframes*
- Assistant draws *inbetweens*
- Tedious / labor intensive (opportunity for technology!)



Keyframing

- **Basic idea:**
  - **Animator specifies important events only**
  - **Computer fills in the rest via interpolation/approximation**
- **"Events" don't have to be position**
- **Could be color, light intensity, camera zoom, ...**



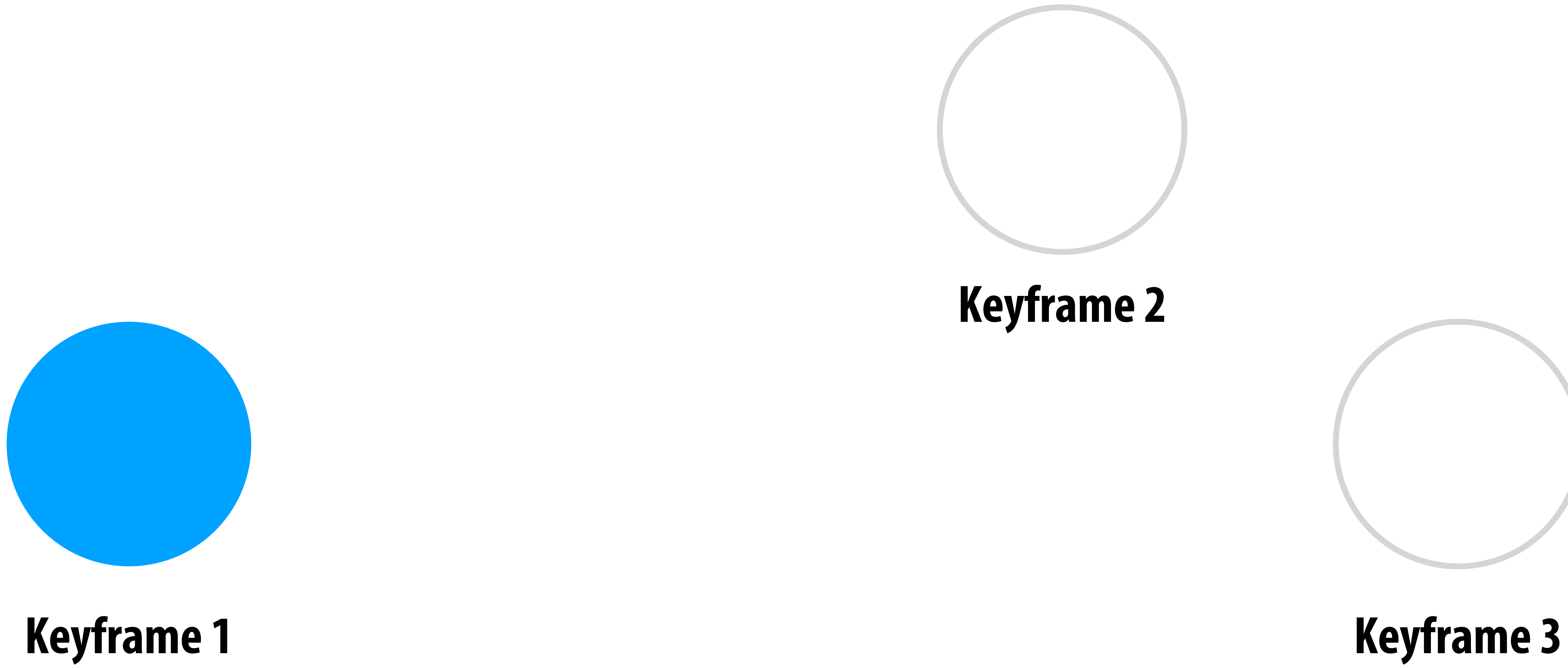
Keyframing example

Keyframe 1

Keyframe 2

Keyframing example

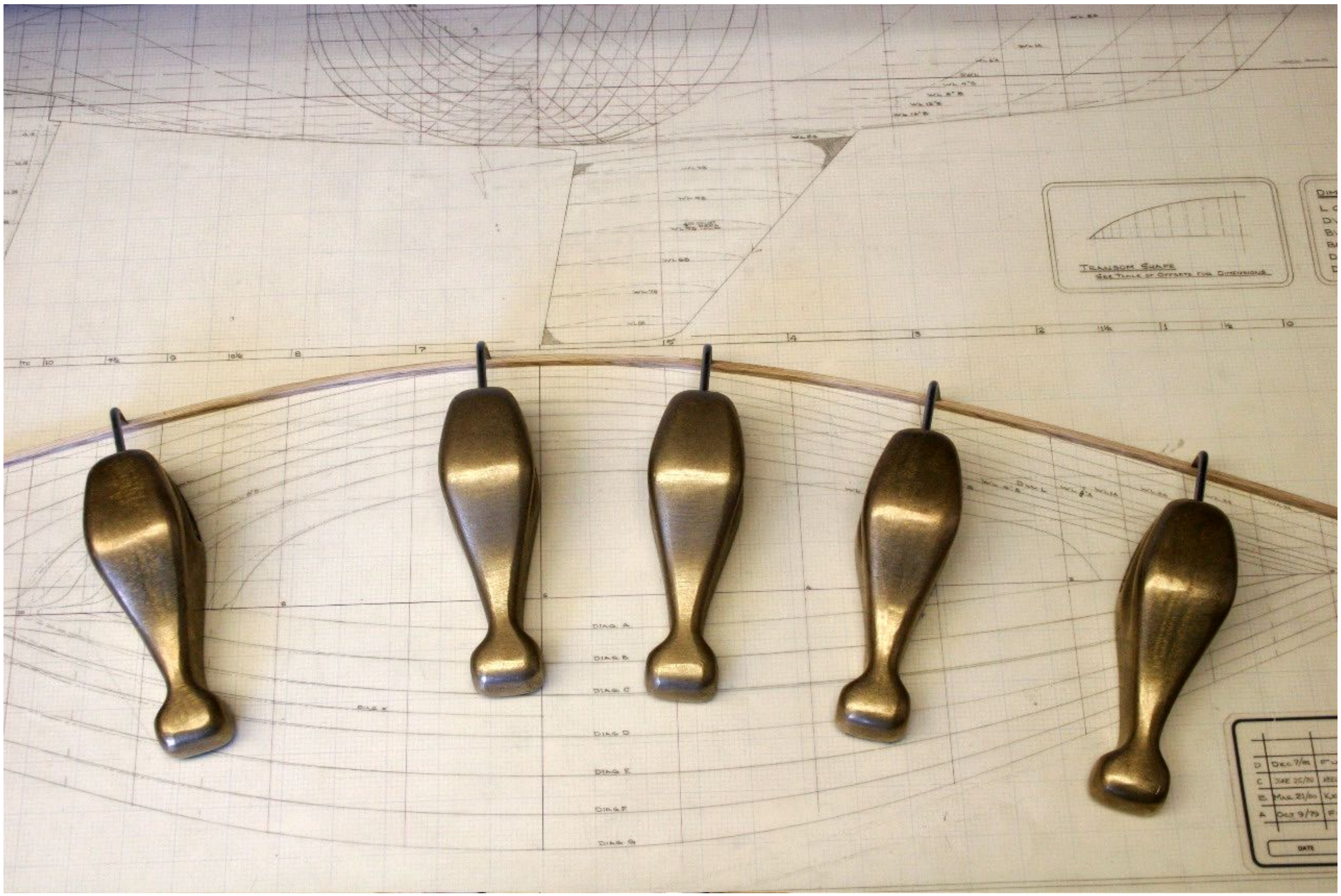
Keyframing example



How do we interpolate data?

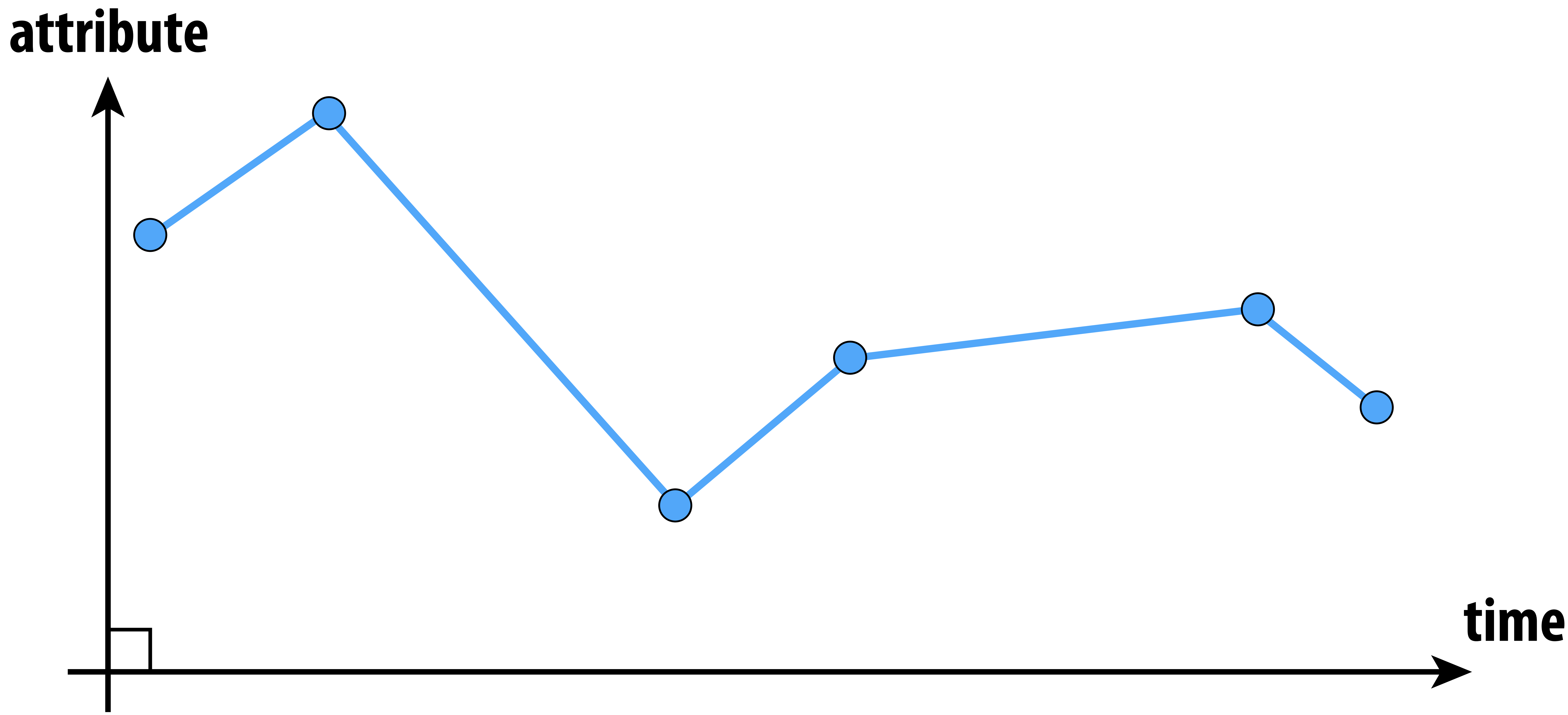
Spline interpolation

- **Mathematical theory of interpolation arose from study of thin strips of wood or metal ("splines") under various forces**



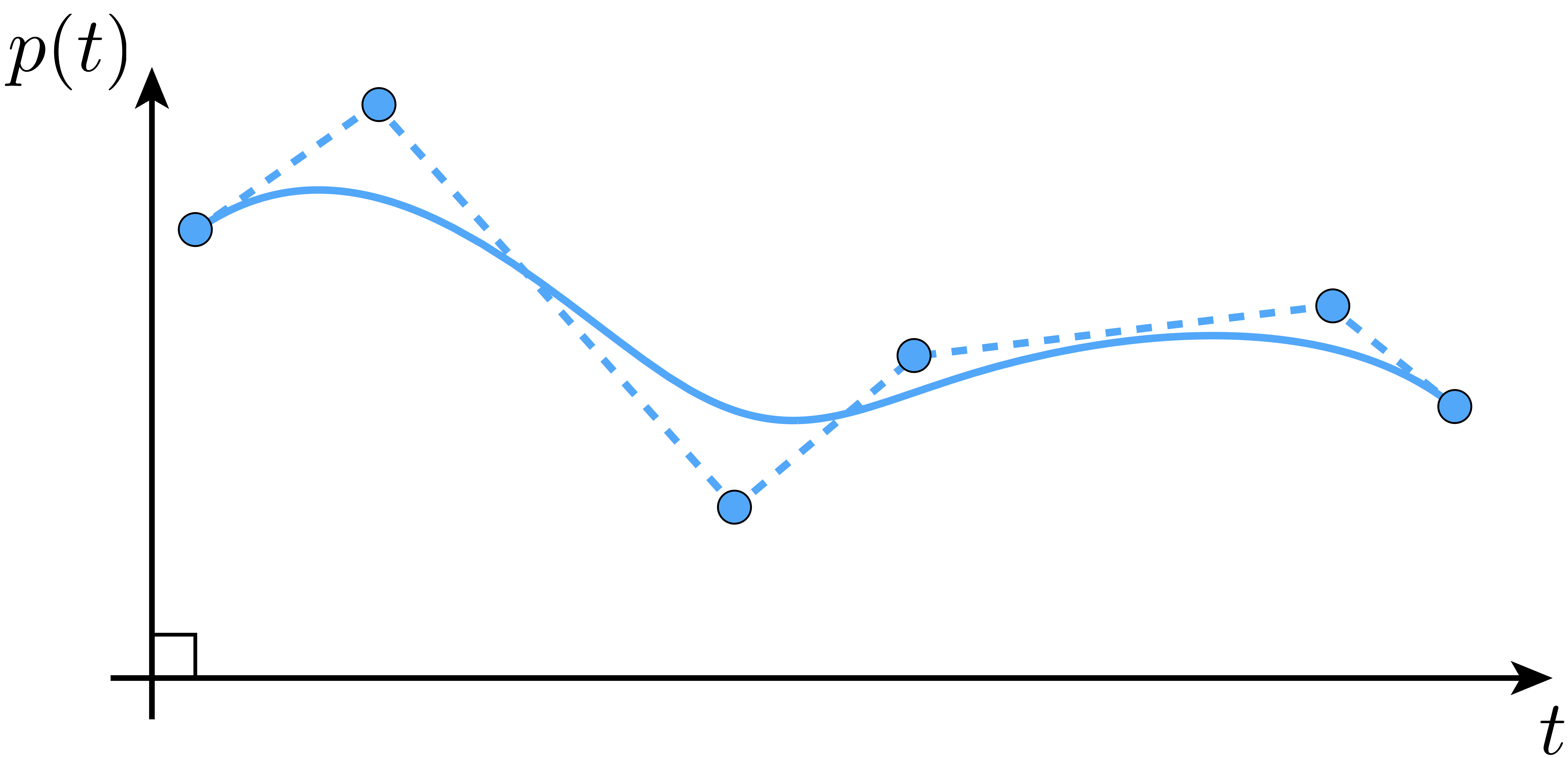
Interpolation

- Basic idea: "connect the dots"
- E.g., *piecewise linear interpolation*
- Simple, but yields "rough" motion (infinite acceleration at keyframes)



Piecewise polynomial interpolation

- **Common interpolant: piecewise polynomial "spline"**



Basic motivation: get better continuity than piecewise linear!

Splines

- **In general, a *spline* is any piecewise polynomial function**
- **In 1D, spline interpolates data over the real line:**

$$(t_i, f_i), \quad i = 0, \ldots, n$$

“knots” values

$$t_i < t_{i+1}$$

- **“Interpolates” means that the function *exactly* passes through those values:**

$$f(t_i) = f_i \quad \forall i$$

- **The only other condition is that the function is a *polynomial* when restricted to any interval between knots:**

polynomial

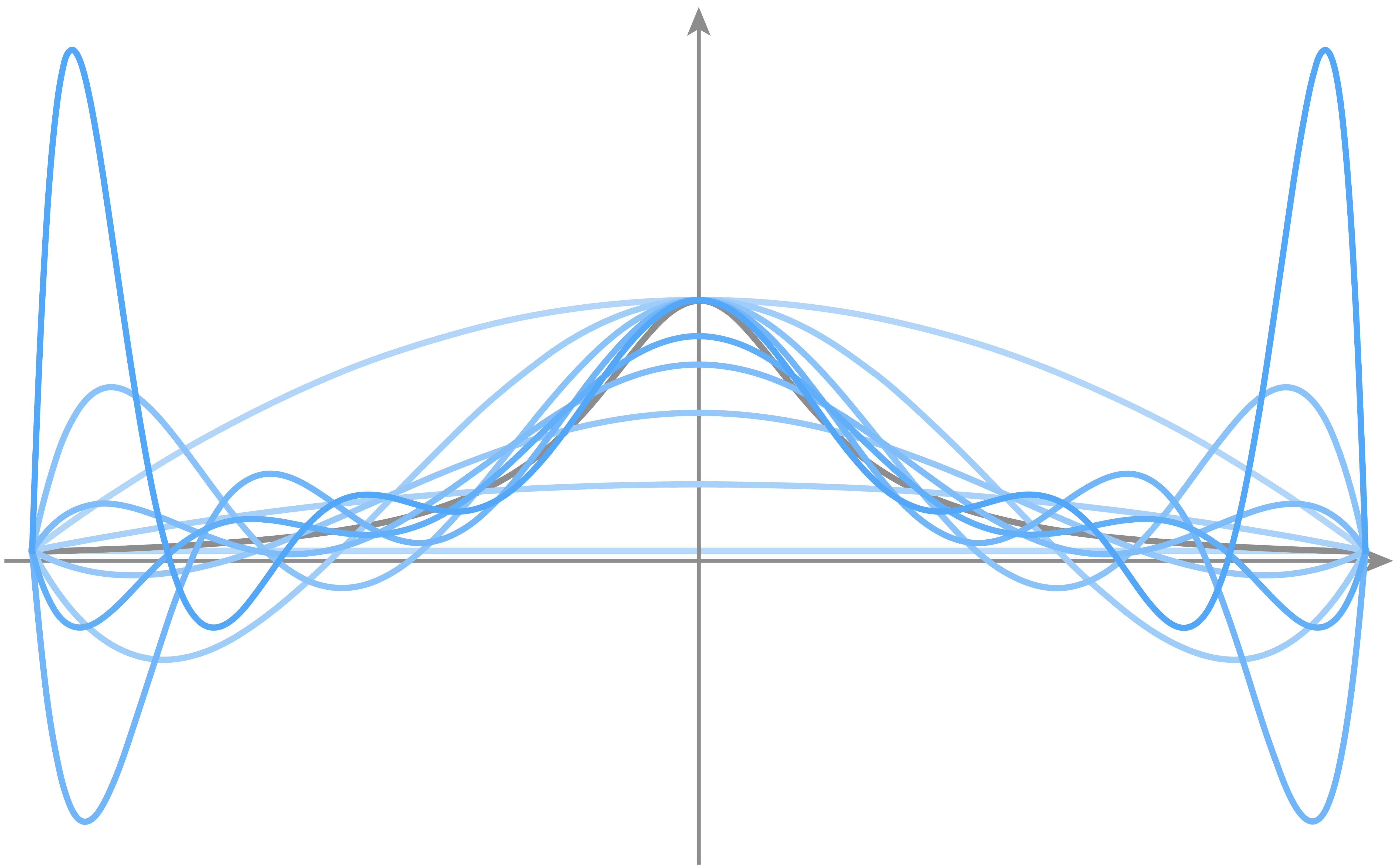
degree

$$\text{for } t_i \leq t \leq t_{i+1}, f(t) = \sum_{j=1}^{d} c_j t^j =: p_i(t)$$

coefficients

What's so special about *cubic* polynomials?

- Splines most commonly used for interpolation are *cubic* (d=3)
- Can provide "reasonable" continuity
- Tempting to use higher-degree polynomials to get higher-order continuity
- But high degree can lead to oscillation, ultimately *worse* approximation:

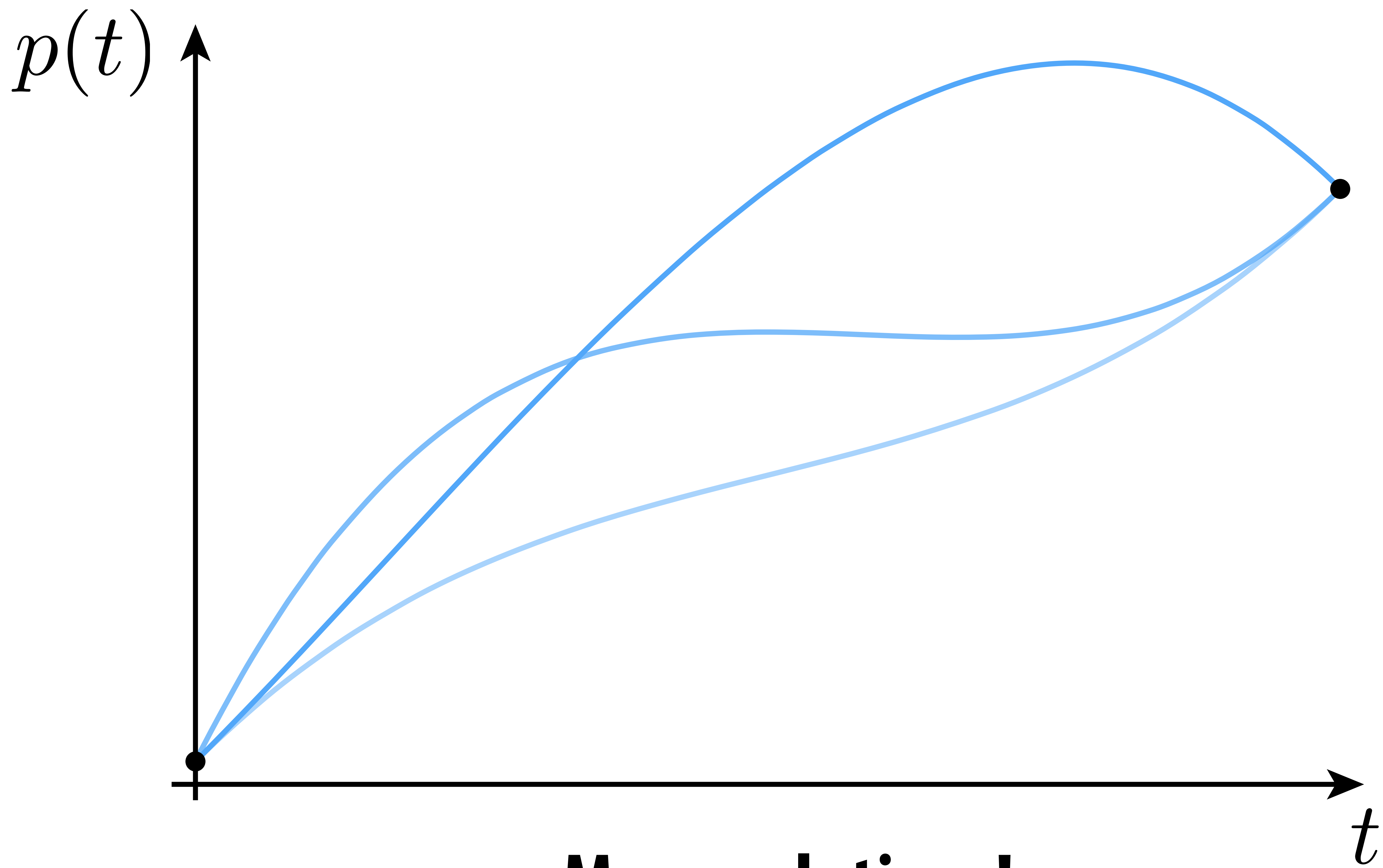


Fitting a cubic polynomial to endpoints

- **Consider a *single* cubic polynomial**

$$p(t) = at^3 + bt^2 + ct + d$$

- **Suppose we want it to match two given endpoints:**



Many solutions!

Cubic polynomial - degrees of freedom

- **Why are there so many different solutions?**
- **Cubic polynomial has four *degrees of freedom (DOFs)*, namely four coefficients (a,b,c,d) that we can manipulate/control**
- **Only need *two* degrees of freedom to specify endpoints:**

$$p(t) = at^3 + bt^2 + ct + d$$

$$p(0) = p_0 \qquad \Rightarrow d = p_0$$

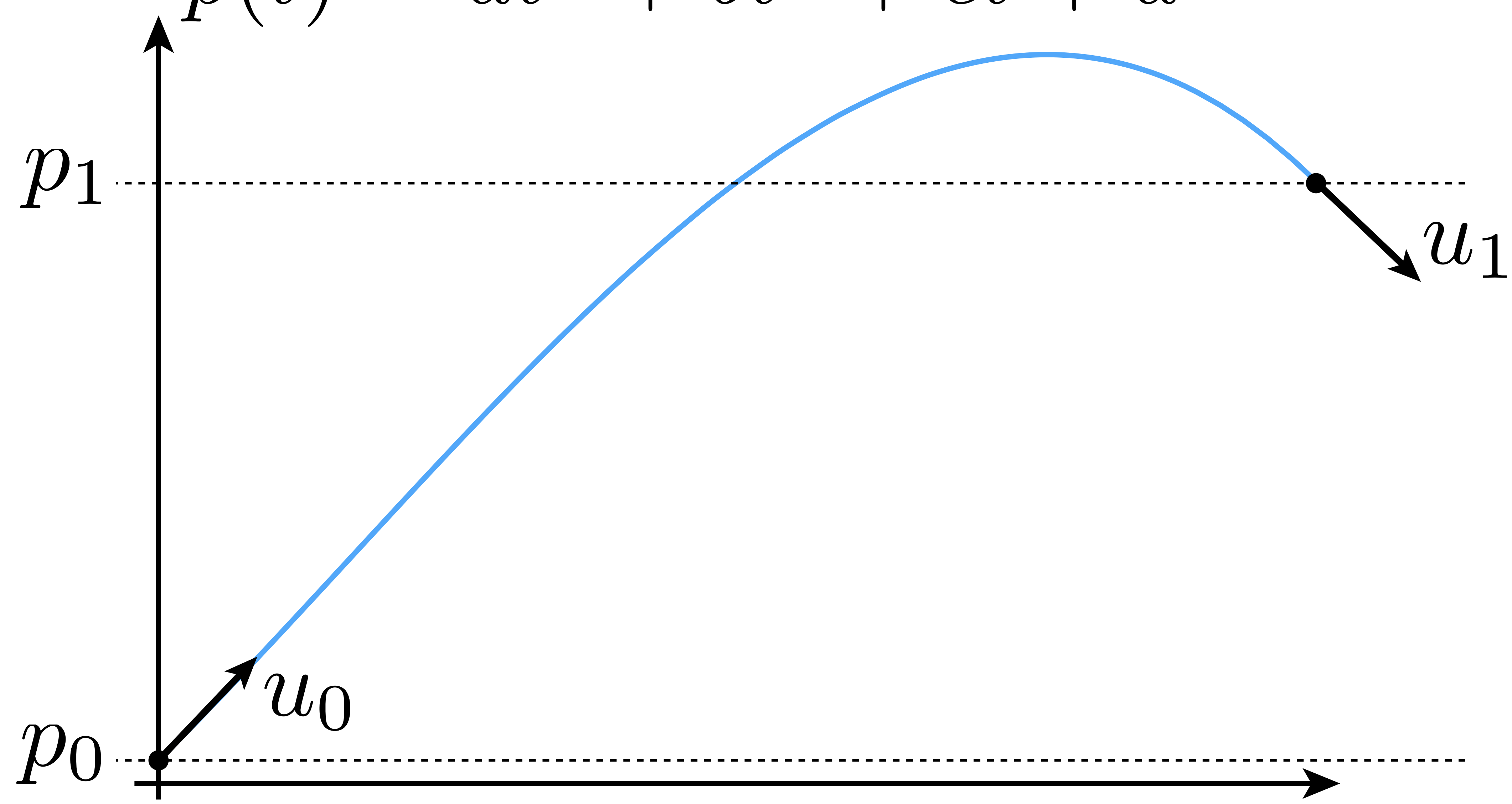
$$p(1) = p_1 \qquad \Rightarrow a + b + c + d = p_1$$

- **Overall, four unknowns but only *two* equations**
- **Not enough to uniquely determine the curve!**

Fitting cubic to endpoints and derivatives

- **What if we also match specified *derivatives* at endpoints?**

$$p(t) = at^3 + bt^2 + ct + d$$



$$p(0) = p_0 \quad \Rightarrow d = p_0$$

$$p(1) = p_1 \quad \Rightarrow a + b + c + d = p_1$$

$$p'(0) = u_0 \quad \Rightarrow c = u_0$$

$$p'(1) = u_1 \quad \Rightarrow 3a + 2b + c = u_1$$

Splines as linear systems

- **Now we have four equations and four unknowns**
- **Could also express as a matrix equation:**

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

- **This is a common way to define a spline**
  - **Each condition on spline leads to a linear equality**
  - **Hence, if we have m degrees of freedom, we need m (linearly independent!) conditions to determine spline**

Solve for polynomial coefficients

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

Matrix form

- **Interpolates endpoints, matches derivatives**

$$p(t) = at^3 + bt^2 + ct + d$$

$$p(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

Interpretation 1: matrix rows = coefficient formulas

$$p(t) = at^3 + bt^2 + ct + d$$

$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

Interpretation 2: matrix cols = ???

$$p(t) = at^3 + bt^2 + ct + d$$

$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

$$= \begin{bmatrix} 2t^3 - 3t^2 + 1 \\ -2t^3 + 3t^2 \\ t^3 - 2t^2 + t \\ t^3 - t^2 \end{bmatrix}^T \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

Hermite basis functions

$$p(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} H_0(t) & H_1(t) & H_2(t) & H_3(t) \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

One common basis for cubic polynomials

$$f_0(t) = t^3$$
$$f_1(t) = t^2$$
$$f_2(t) = t$$
$$f_3(t) = 1$$

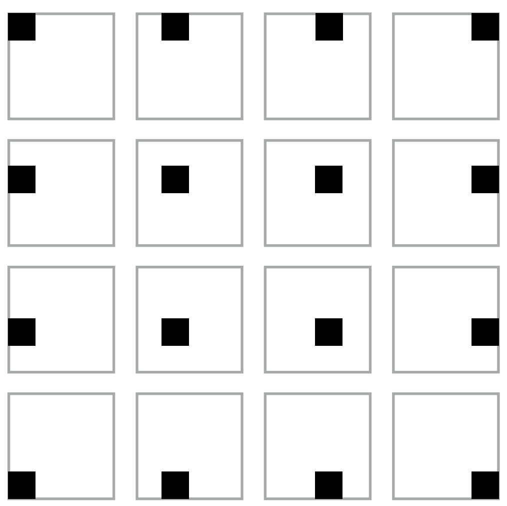
Hermite Basis for cubic polynomials

$$H_0(t) = 2t^3 - 3t^2 + 1$$
$$H_1(t) = -2t^3 + 3t^2$$
$$H_2(t) = t^3 - 2t^2 + t$$
$$H_3(t) = t^3 - t^2$$

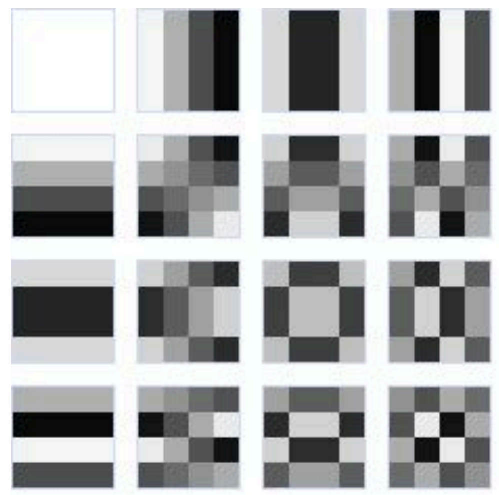
Either basis can represent a cubic polynomial through linear combination!

Recall other examples of representing signals in different bases!

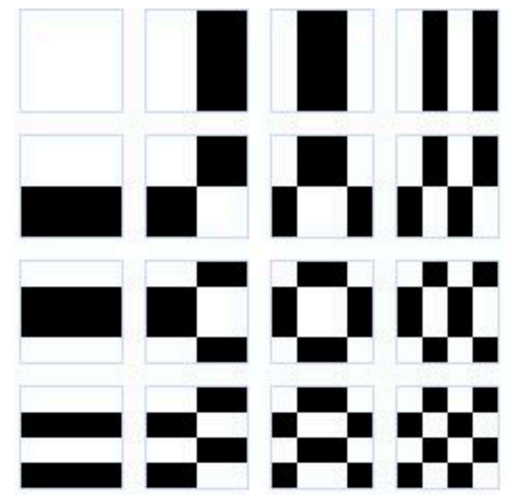
Representing Images



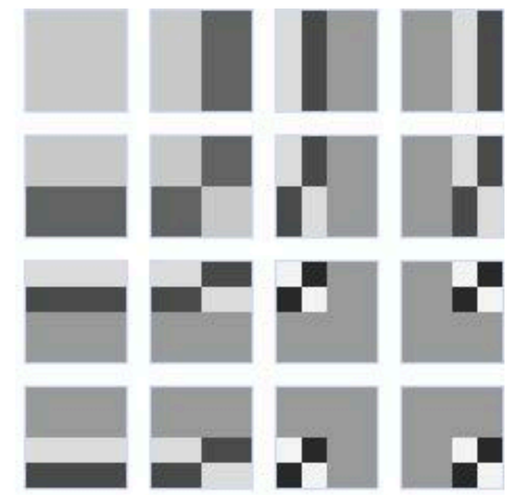
Pixel Basis



DCT

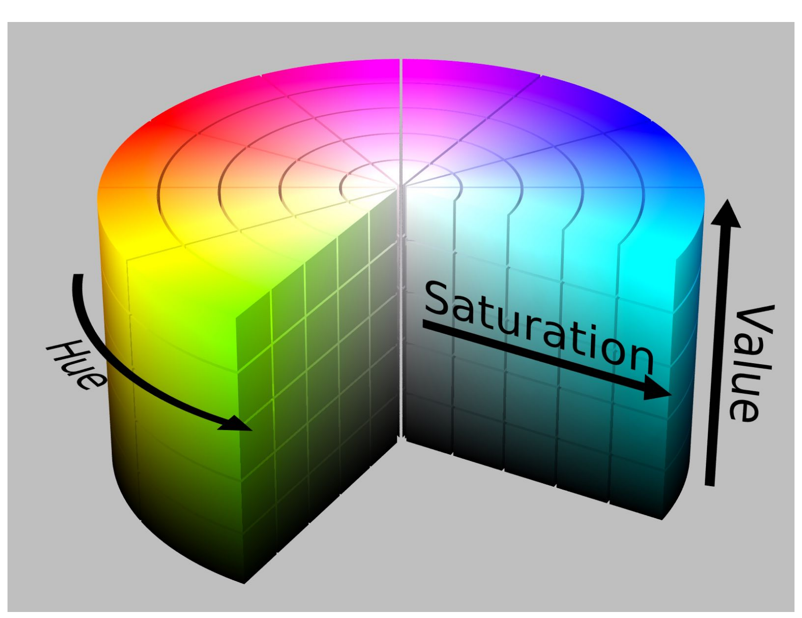


Walsh-Hadamard

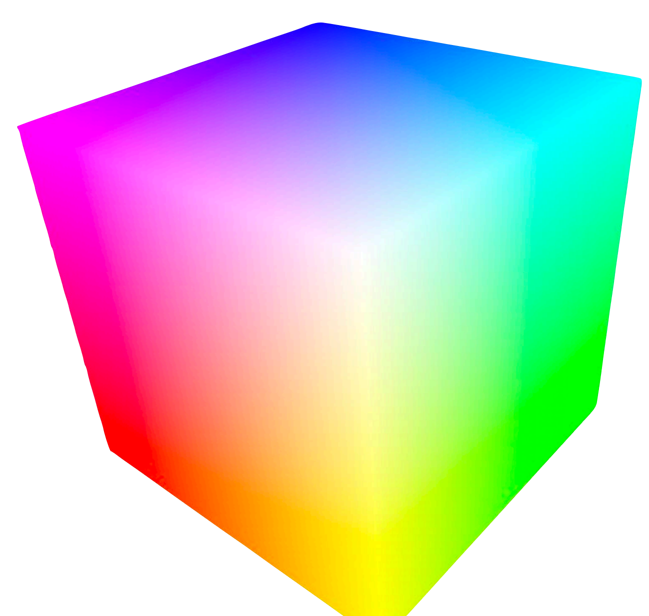


Haar Wavelet

Representing Color

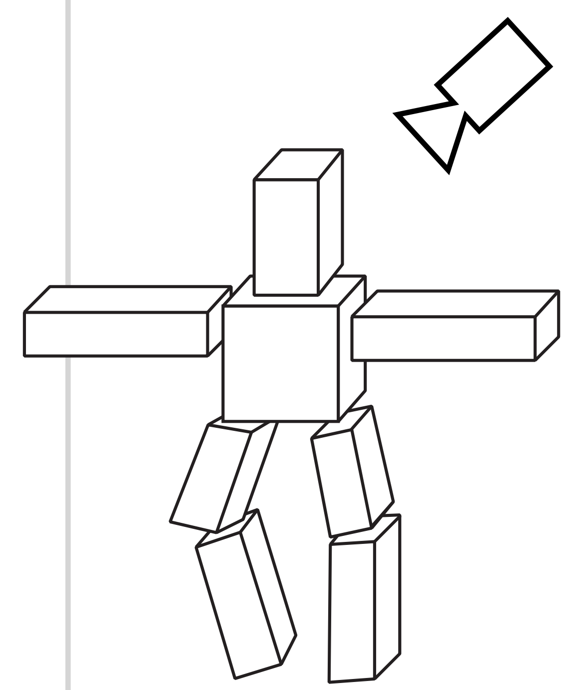


HSV

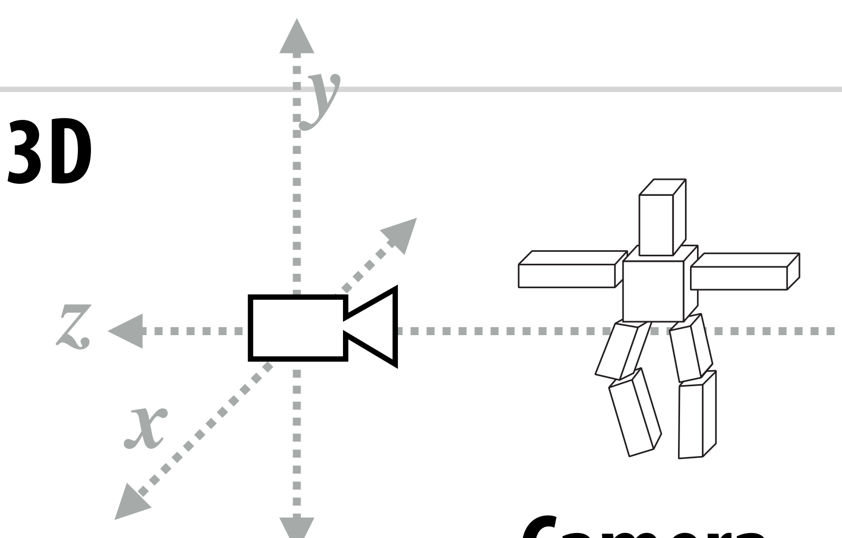


RGB

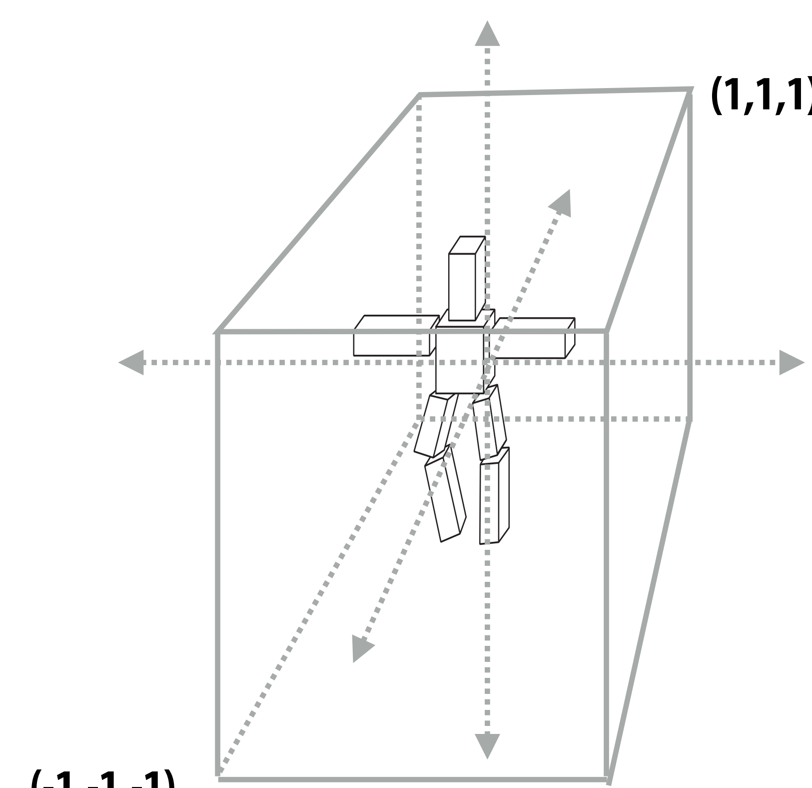
Points in 3D



World coordinates

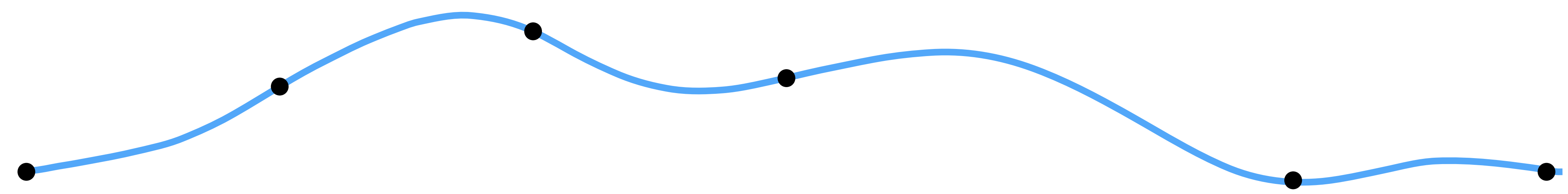


Camera coordinates



NDC

Natural splines



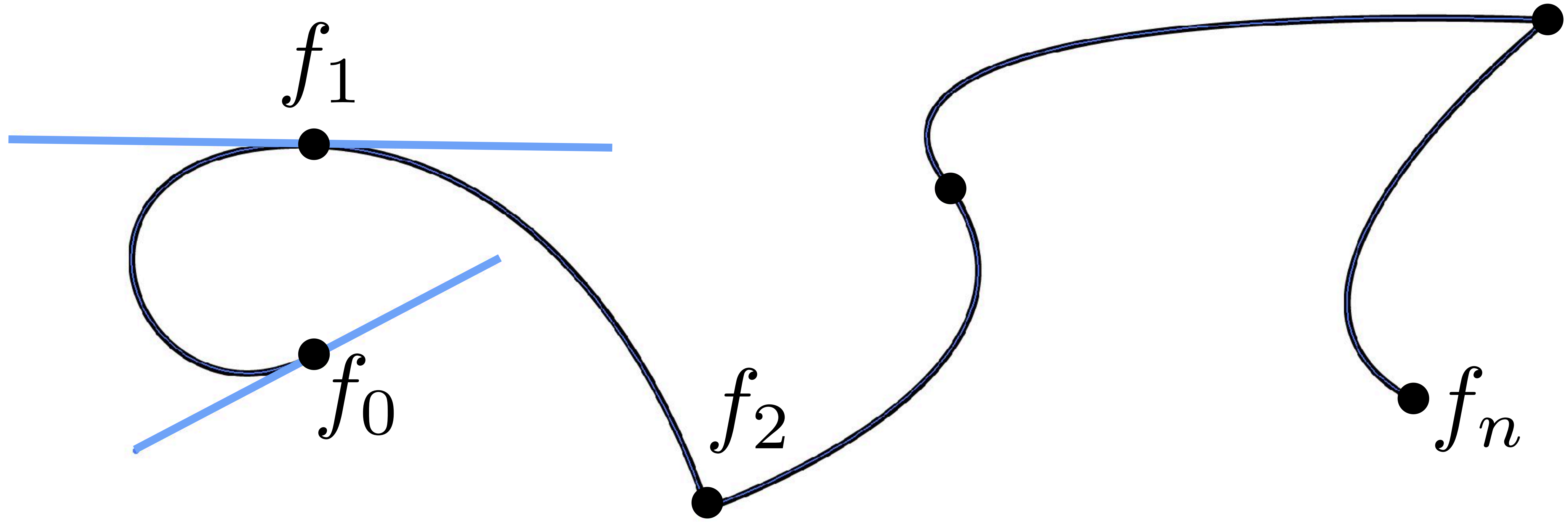
- Now consider *piecewise* spline made of *n* cubic polynomials p_i
- For each interval, want polynomial "piece" p_i to interpolate data (e.g., keyframes) at both endpoints:
  $$p_i(t_i) = f_i, \;\; p_i(t_{i+1}) = f_{i+1}, \;\; i = 0, \ldots, n-1$$
- Want tangents to agree at endpoints ("C^1 continuity"):
  $$p_i'(t_{i+1}) = p_{i+1}'(t_{i+1}), \;\; i = 0, \ldots, n-2$$
- Also want curvature to agree at endpoints ("C^2 continuity"):
  $$p_i''(t_{i+1}) = p_{i+1}''(t_{i+1}), \;\; i = 0, \ldots, n-2$$
- How many equations do we have at this point?
  - 2n+(n-1)+(n-1) = 4n-2
- Pin down remaining DOFs by setting 2nd derivative (curvature) to zero at endpoints

Spline desiderata

- In general, what are some properties of a "good" spline?
  - INTERPOLATION: spline passes *exactly* through data points
  - CONTINUITY: at least *twice* differentiable everywhere (for animation = constant "acceleration")
  - LOCALITY: moving one control point doesn't affect whole curve
- How does our natural spline do?
  - INTERPOLATION: yes, by construction
  - CONTINUITY: C^2 everywhere, by construction
  - LOCALITY: no, coefficients depend on global linear system
- Many other types of splines we can consider
- Spoiler: there is "no free lunch" with cubic splines (can't simultaneously get all three properties)

Back to Hermite splines from earlier in lecture

- **Hermite: each cubic "piece" specified by endpoints and tangents:**



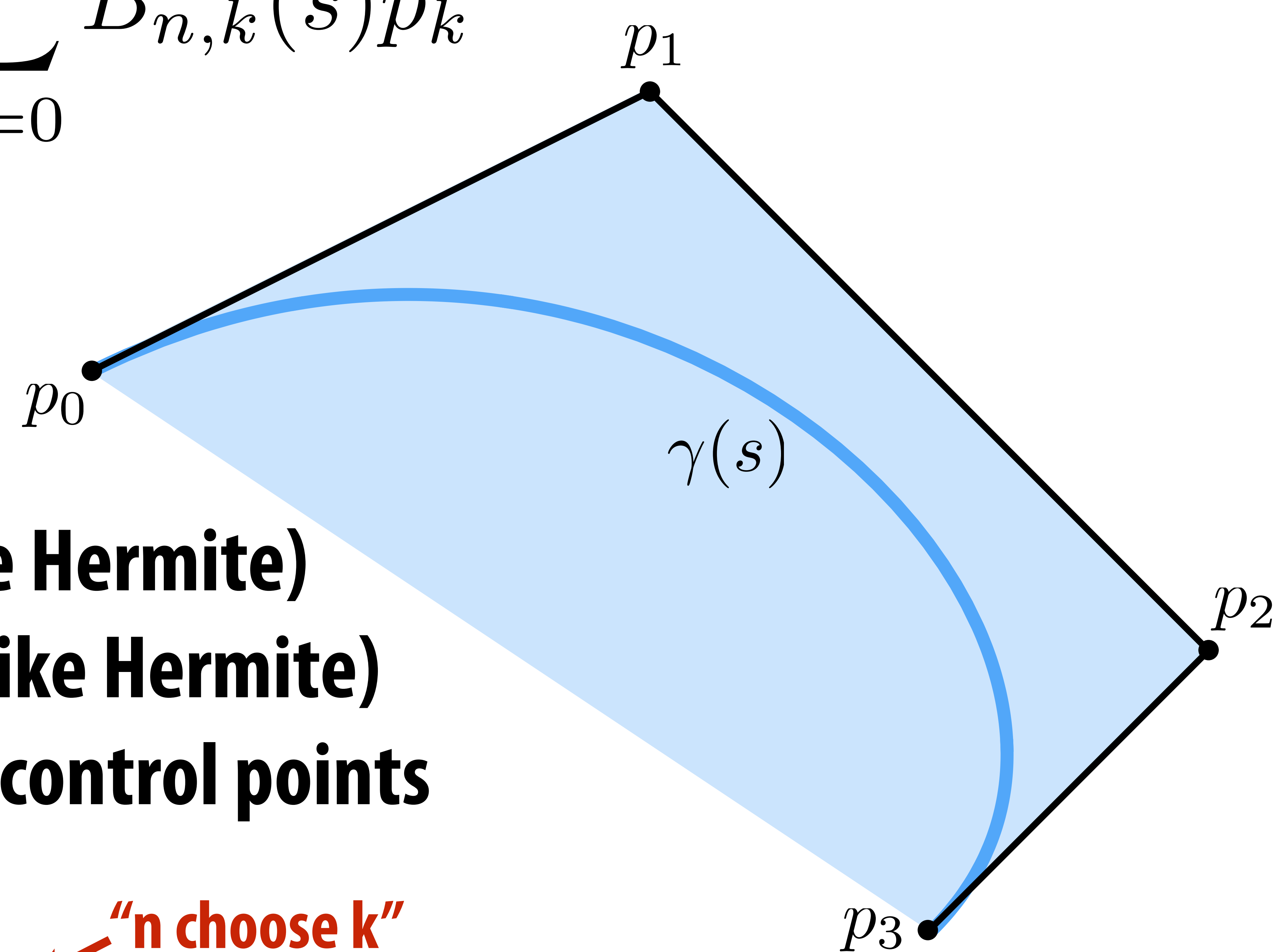
- **Commonly used for 2D vector art (Illustrator, Inkscape, SVG, ...)**
- **Can we get tangent (C1) continuity?**
- **Sure: set both tangents to same value on both sides of knot!**
  - **E.g., f_1 above, but not f_2**

Bézier curves

- A Bézier curve is a curve expressed in the Bernstein basis:

$$\gamma(s) := \sum_{k=0}^{n} B_{n,k}(s) p_k$$

control points



- For n=3, get "cubic Bézier":
- Properties:
  1. interpolates endpoints (like Hermite)
  2. tangent to end segments (like Hermite)
  3. contained in convex hull of control points

degree

0≤x≤1

"n choose k"

$$B_k^n(x) := \binom{n}{k} x^k (1-x)^{n-k}$$

k=0,…,n

Properties of Hermite/Bézier spline

- More precisely, want endpoints to interpolate data:

$$p_i(t_i) = f_i,\;\; p_i(t_{i+1}) = f_{i+1},\;\; i = 0, \ldots, n-1$$

- Also want tangents to interpolate some given data:

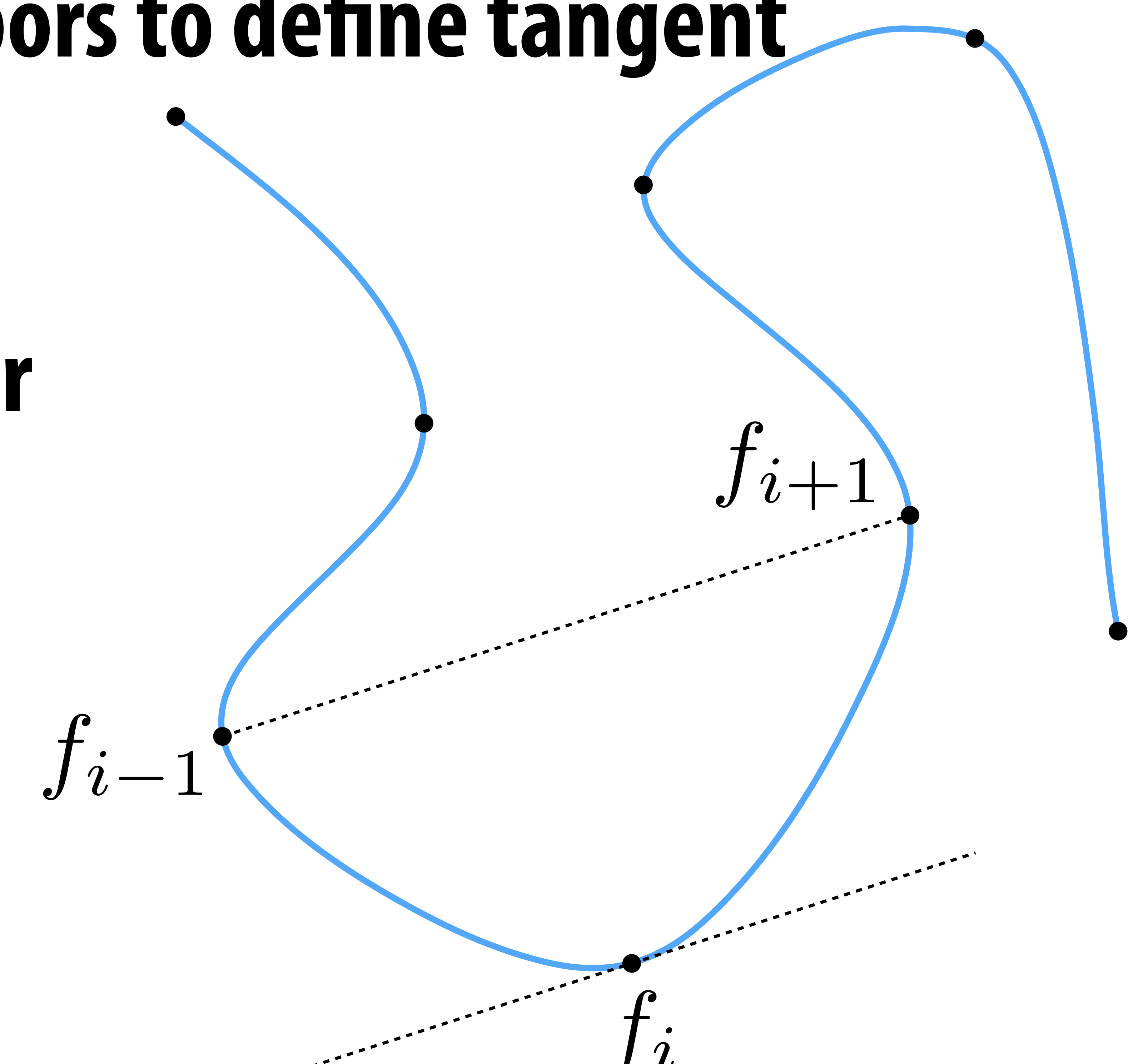
$$p_i'(t_i) = u_i,\;\; p_i'(t_{i+1}) = u_{i+1},\;\; i = 0, i, \ldots, n-1$$

- How is this *different* from our natural spline's tangent condition?
- There, tangents didn't have to match any prescribed value—they merely had to be the same. Here, they are given.
- How many conditions overall?
  - 2n + 2n = 4n
- What properties does this curve have?
  - INTERPOLATION and LOCALITY, but not C^2 CONTINUITY

Catmull-Rom splines

- Sometimes makes sense to specify *tangents* (e.g., illustration)
- Often more convenient to just specify *values*
- Catmull-Rom: specialization of Hermite spline, determined by values alone
- Basic idea: use difference of neighbors to define tangent

$$u_i := \frac{f_{i+1} - f_{i-1}}{t_{i+1} - t_{i-1}}$$



- All the same properties as any other Hermite spline (locality, etc.)
- Commonly used to interpolate motion in computer animation.
- Many, many variants, but Catmull-Rom is usually good starting point

Spline desiderata, revisited

	INTERPOLATION	CONTINUITY	LOCALITY
natural	YES	YES	NO
Hermite	YES	NO	YES
???	NO	YES	YES

See B-Splines

But what quantities do we seek to interpolate?

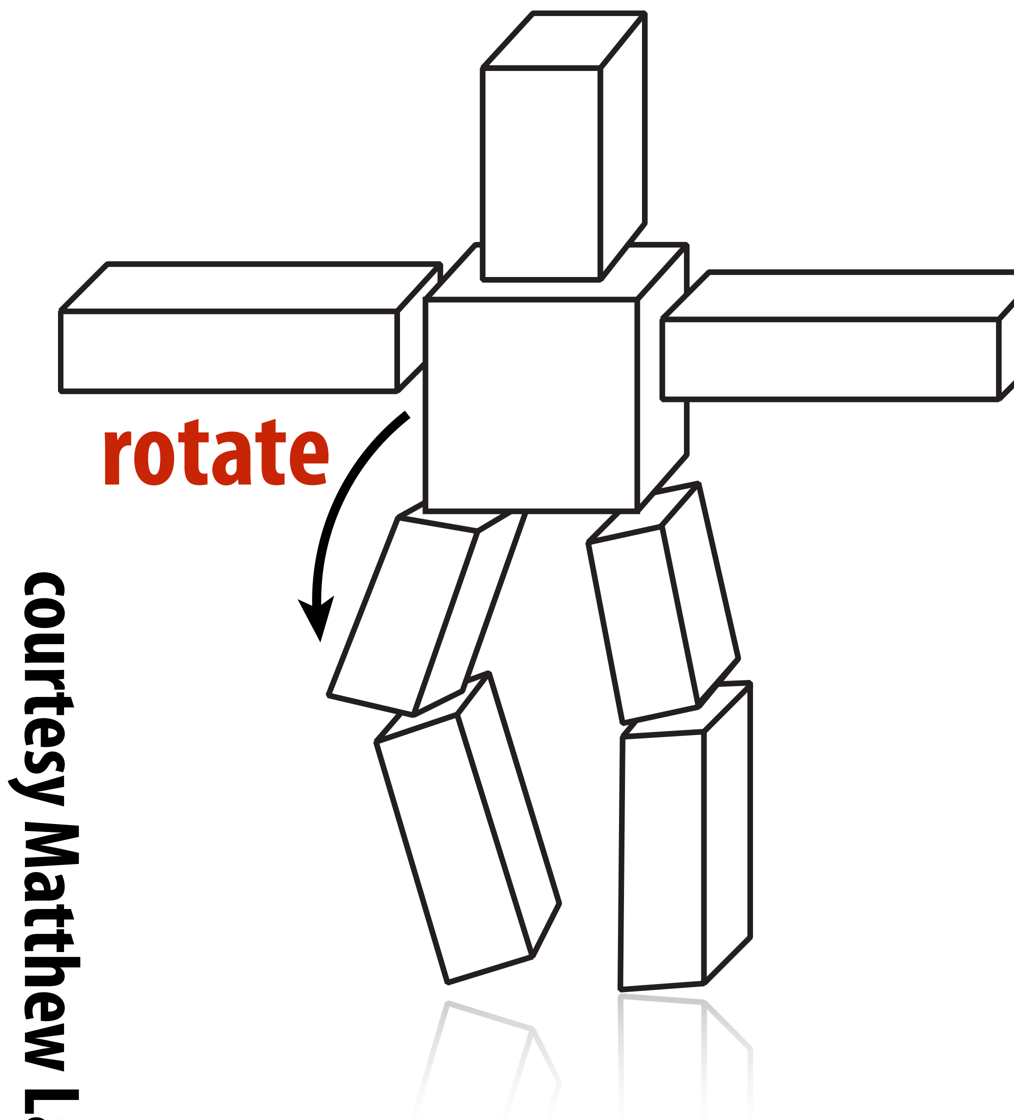
Simple example: camera path

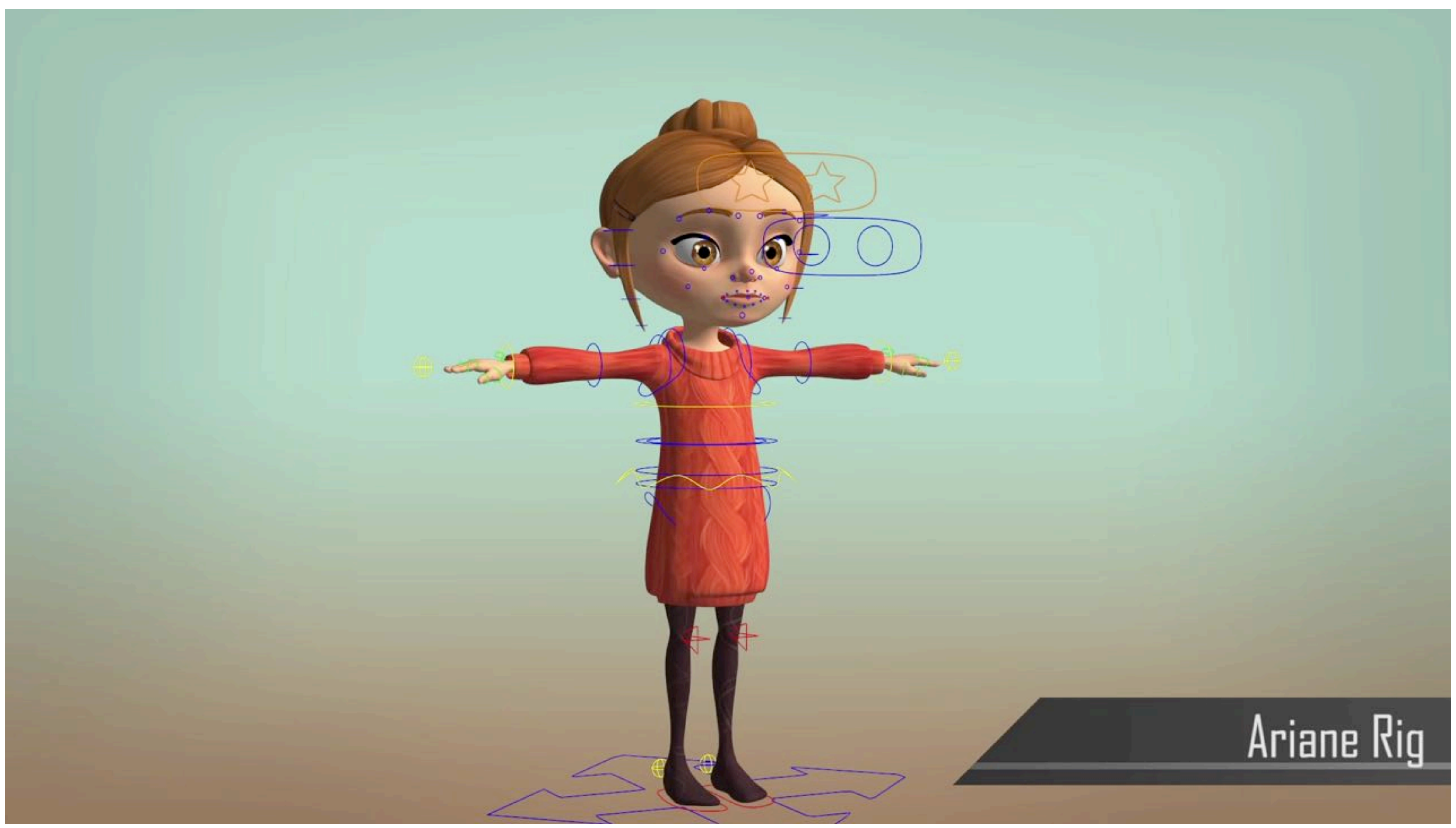
- Animate position, direction, "up" direction of camera
  - each path is a function $f(t) = (x(t), y(t), z(t))$
  - each component (x,y,z) is a spline

Zaha Hadid Architects—City of Dreams Hotel Tower

Character animation

- *Scene graph/kinematic chain*: scene as tree of transformations
- E.g. in our "cube person," configuration of a leg might be expressed as rotation relative to body
- Animate by interpolating transformations
- Often have sophisticated "rig":





courtesy Matthew Lailler

Even w/ computer "tweening," its a lot of work to animate!

Blend shapes

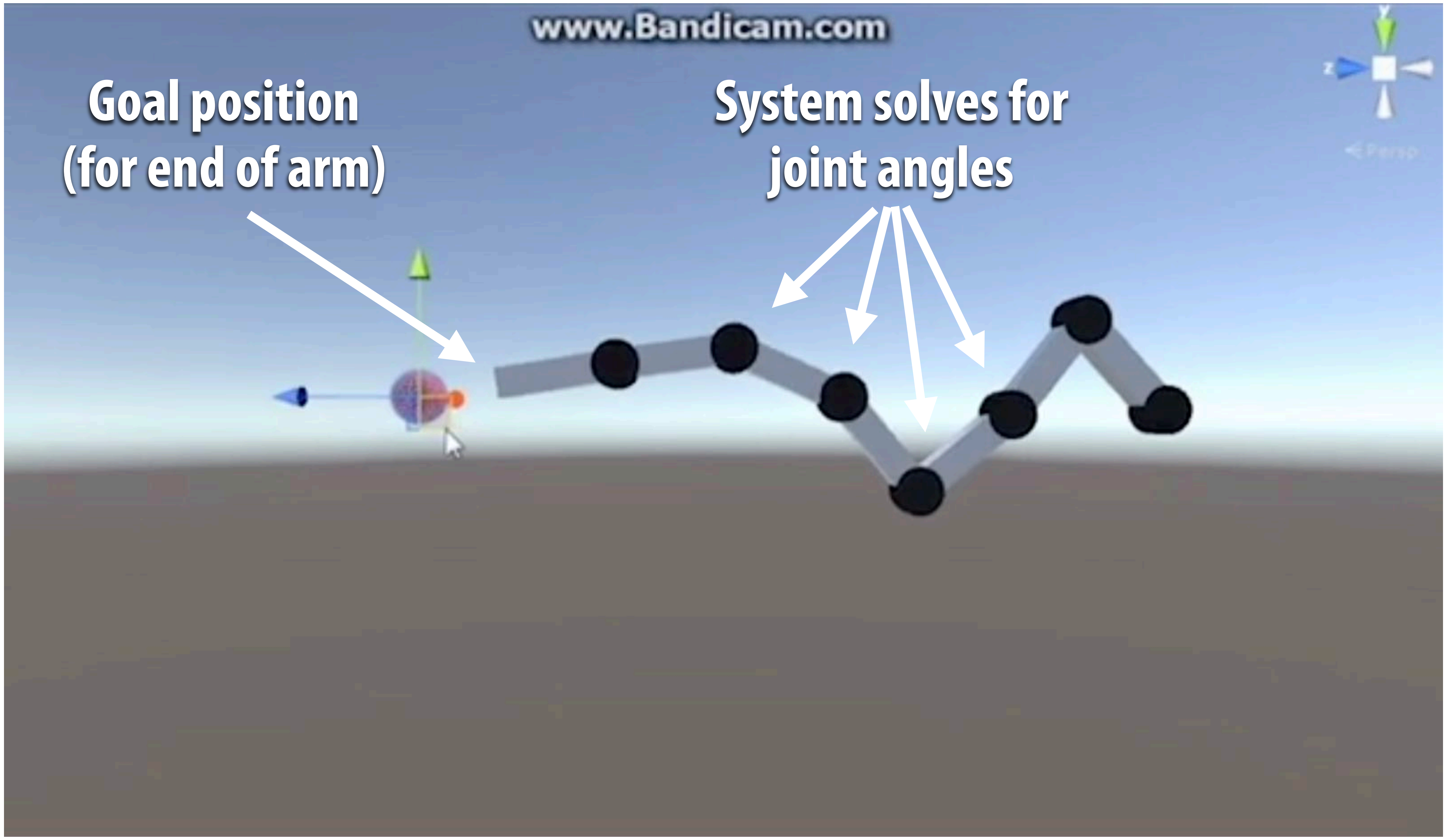
- Instead of skeleton, interpolate directly between surfaces
- E.g., model a collection of facial expressions:

courtesy Félix Ferrand

- Simplest scheme: take linear combination of vertex positions
- Spline used to control choice of weights over time

Inverse kinematics

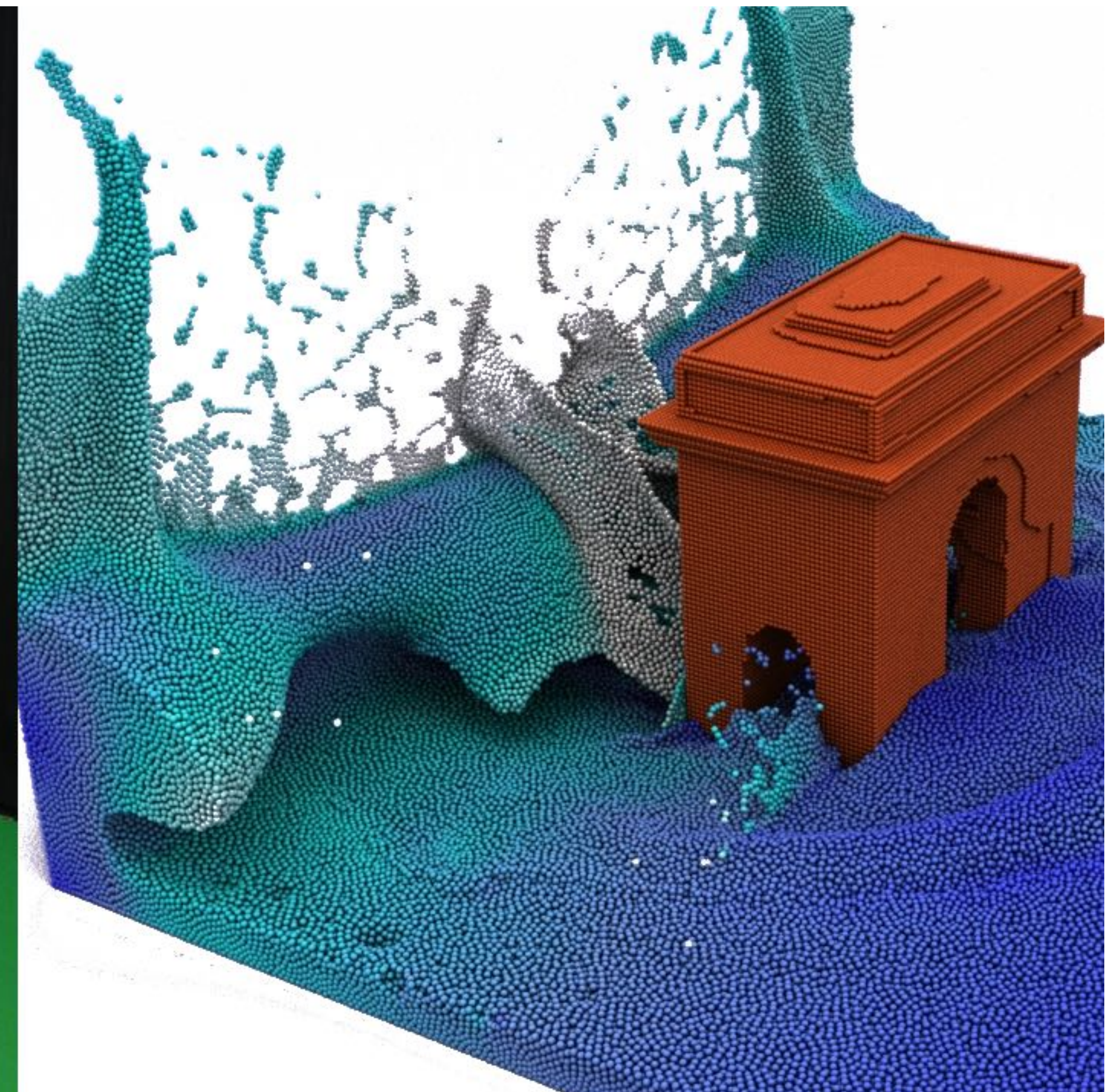
- Important technique in animation and robotics
- Rather than adjust individual transformations, set "goal" and use algorithm to come up with plausible motion:



Many algorithms—to be discussed in a future lecture

Coming up next...

- Even with "computer-aided tweening," animating a scene by hand takes a lot of work!
- Will see how data capture and physical simulation can help

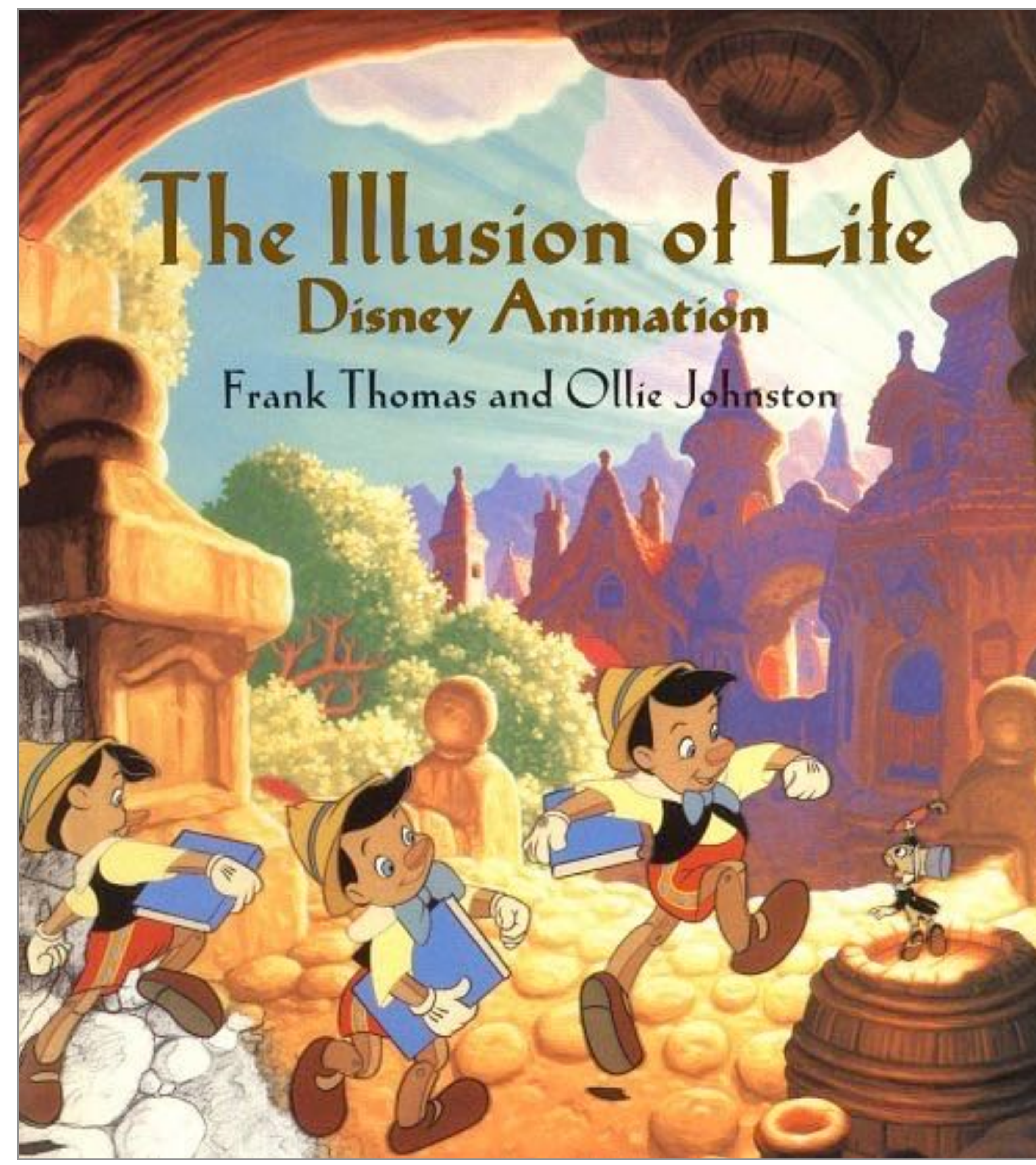


Principles of animation

Animation principles

- **From**
  - **"Principles of Traditional Animation Applied to 3D Computer Animation" - John Lasseter, ACM Computer Graphics, 21(4), 1987**

- **In turn from**
  - **"The Illusion of Life"
    Frank Thomas and Ollie Johnson**



http://www.siggraph.org/education/materials/HyperGraph/animation/character_animation/principles/prin_trad_anim.htm

12 animation principles

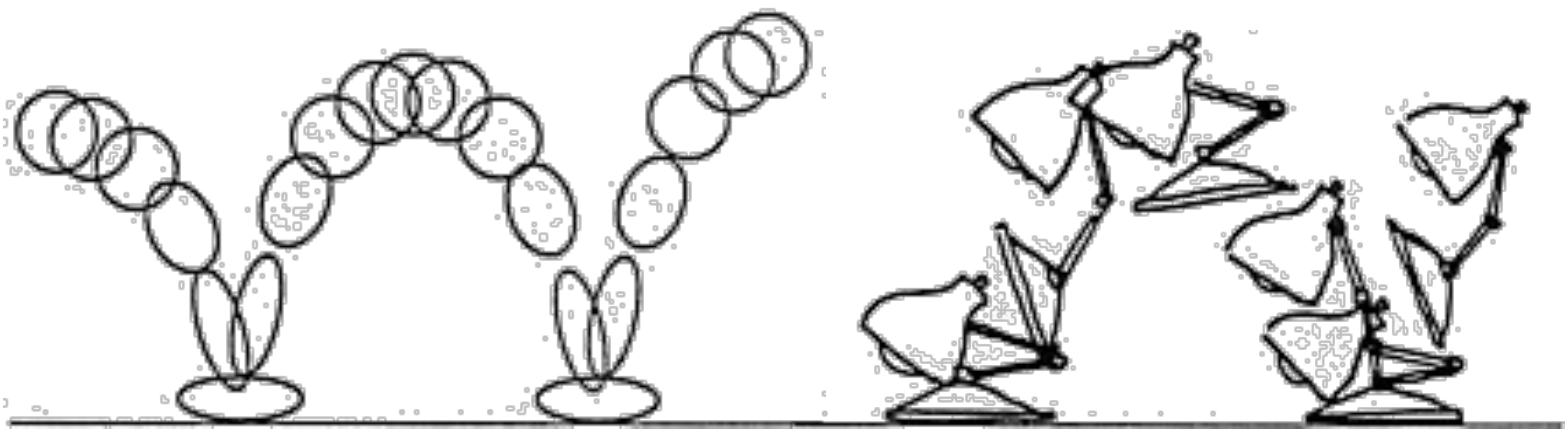
1. Squash and stretch
2. Anticipation
3. Staging
4. Straight ahead and pose-to-pose
5. Follow through
6. Ease-in and ease-out
7. Arcs
8. Secondary action
9. Timing
10. Exaggeration
11. Solid drawings
12. Appeal

12 animation principles

Cento Lodgiani, https://vimeo.com/93206523

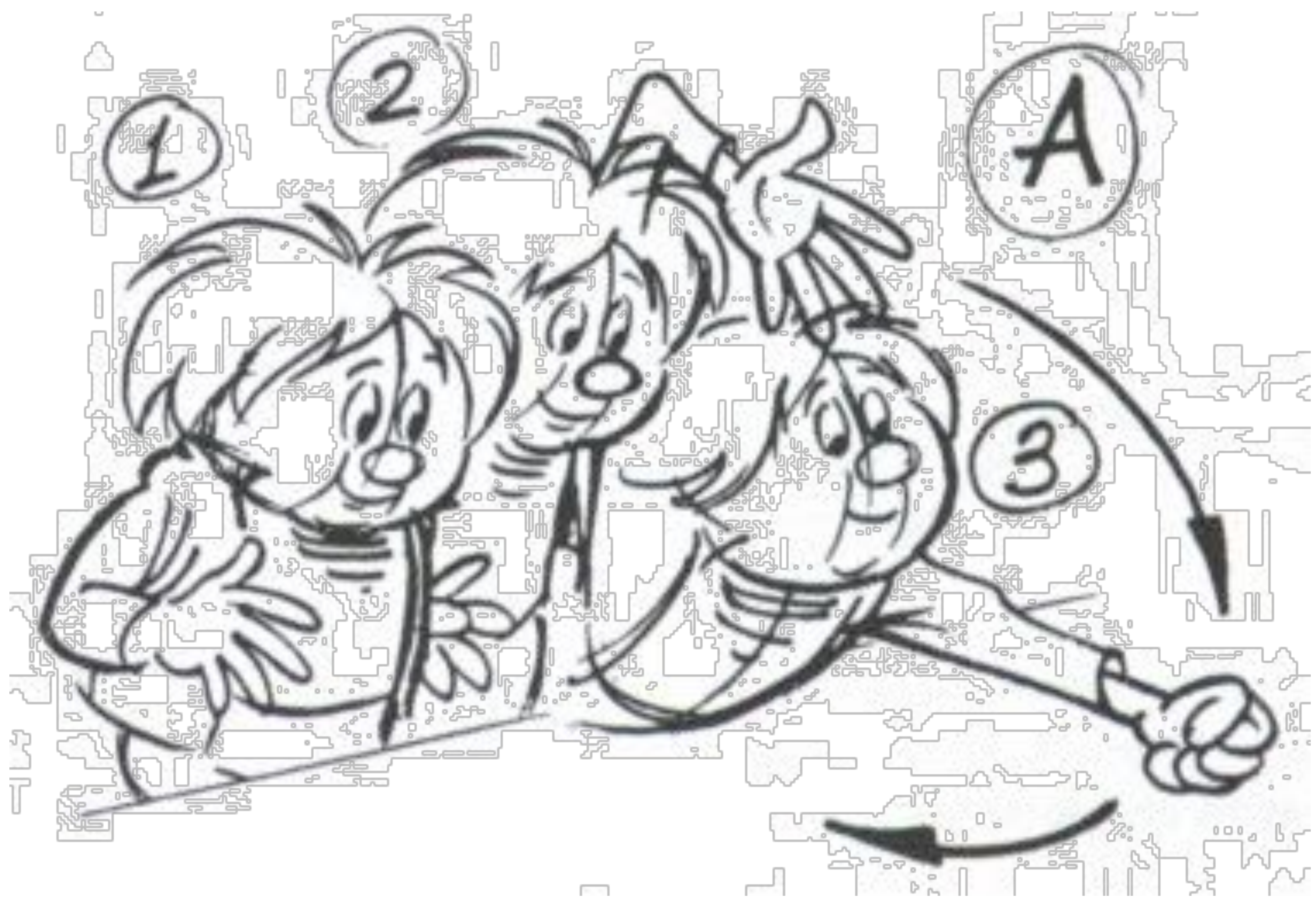
Squash and stretch

- Refers to defining the rigidity and mass of an object by distorting its shape during an action
- Shape of object changes during movement, but not its volume



Anticipation

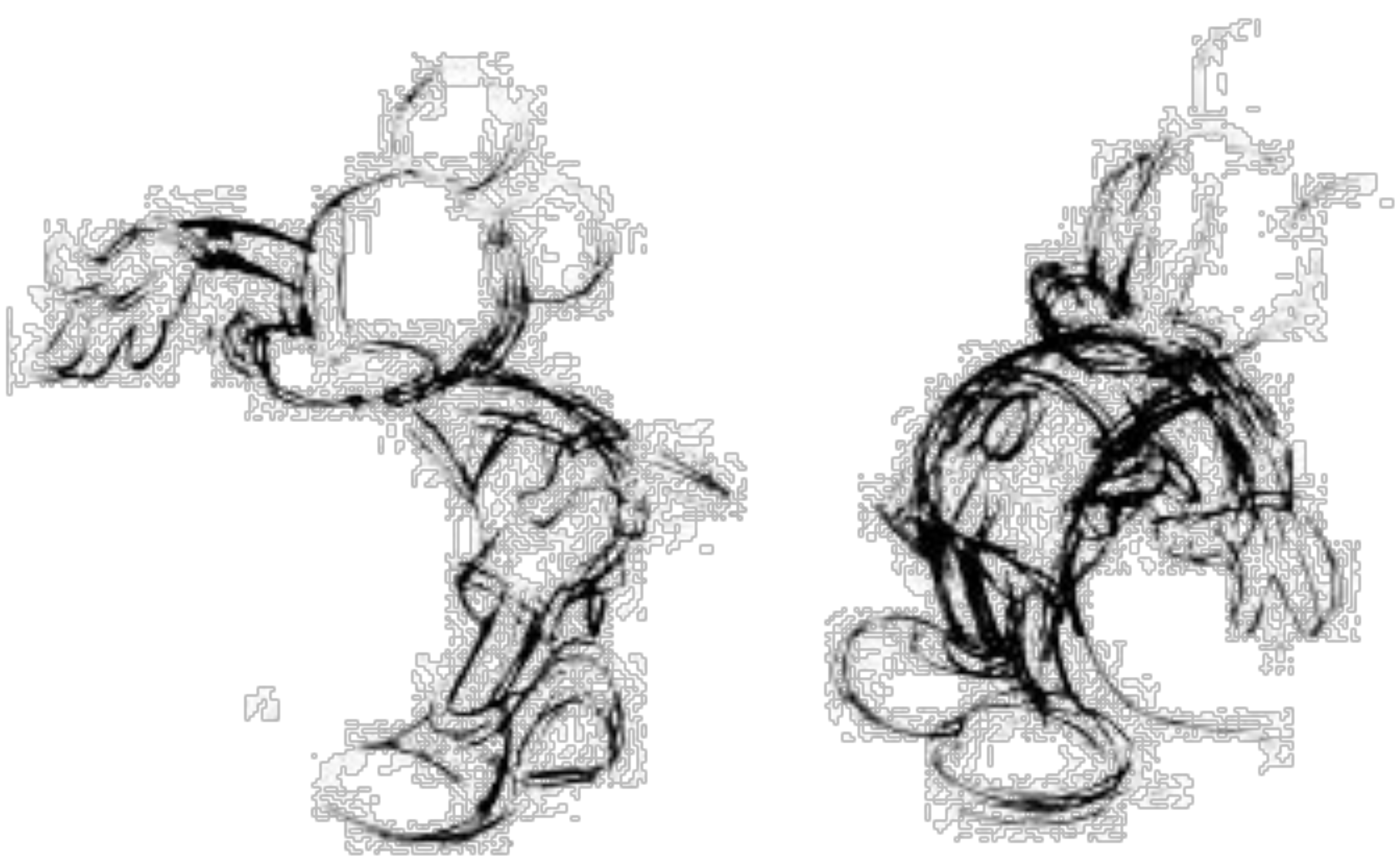
- Prepare for each movement
- For physical realism
- To direct audience's attention



Timing for Animation, Whitaker & Halas

Staging

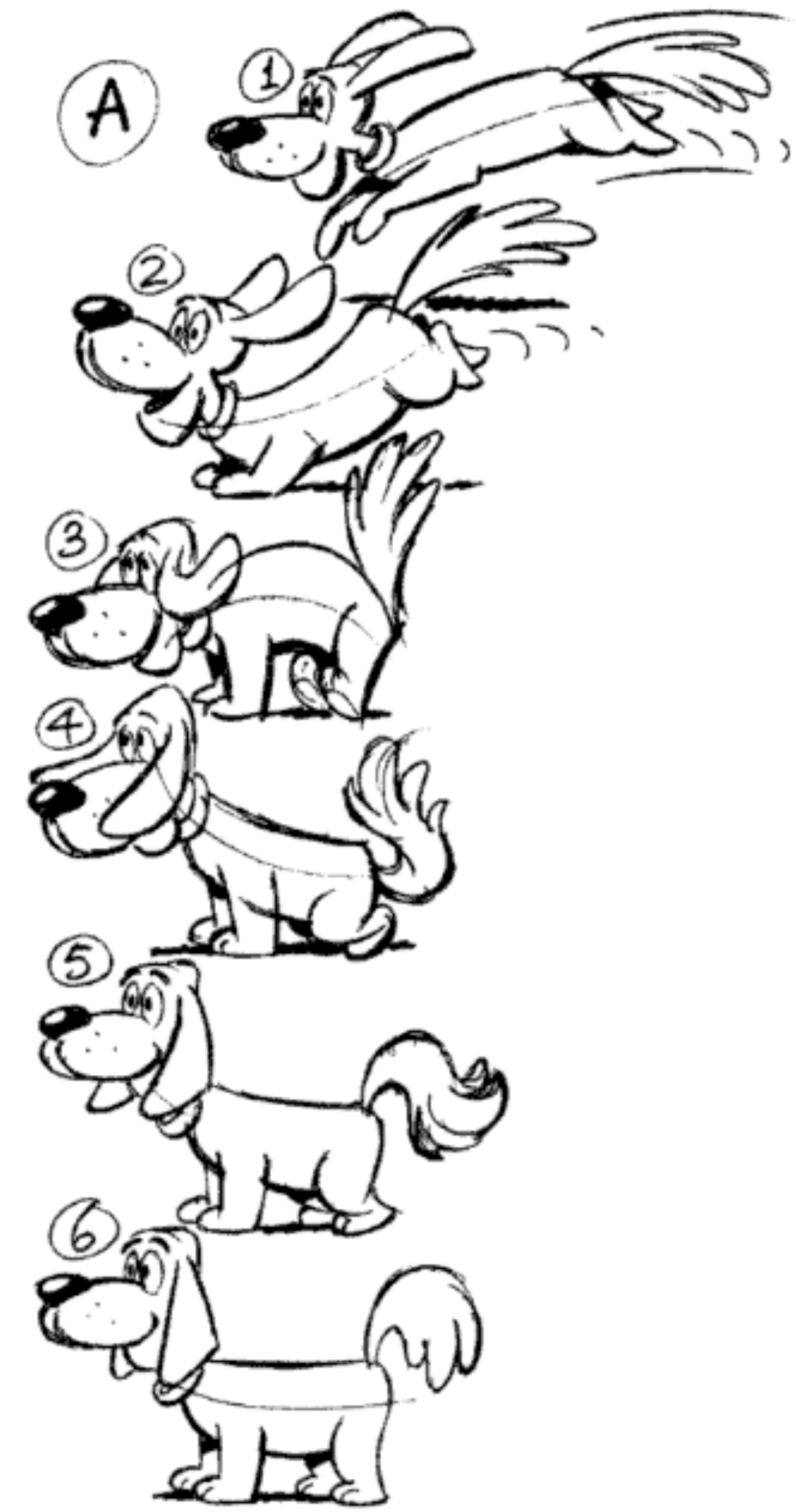
- Picture is 2D
- Make situation clear
- Audience looking in right place
- Action clear in silhouette



Disney Animation: The Illusion of Life

Follow through

- Overlapping motion
- Motion doesn't stop suddenly
- Pieces continue at different rates
- One motion starts while previous is finishing, keeps animation smooth

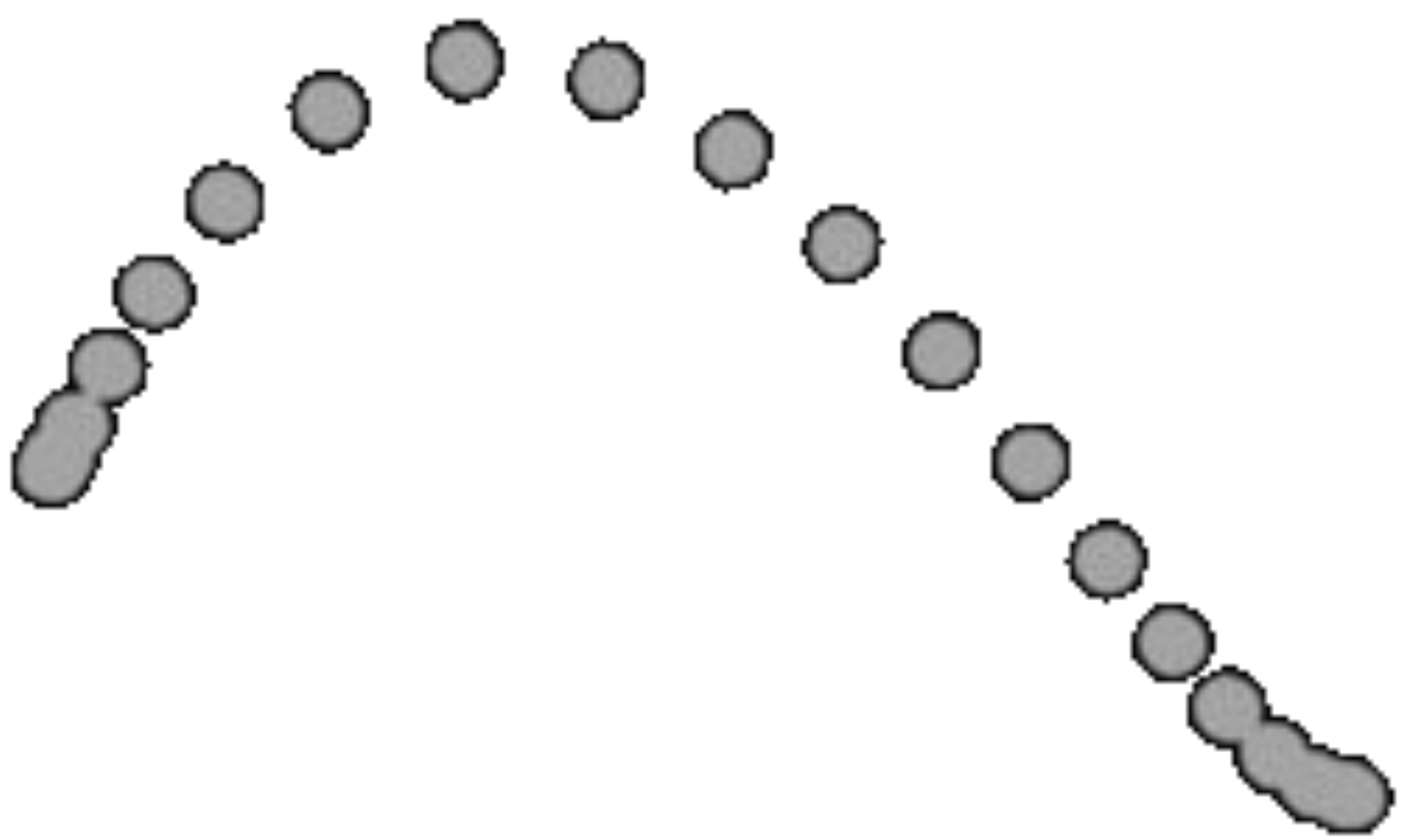


Timing for Animation, Whitaker & Halas

Ease-in and ease-out

Movement doesn't start and stop abruptly

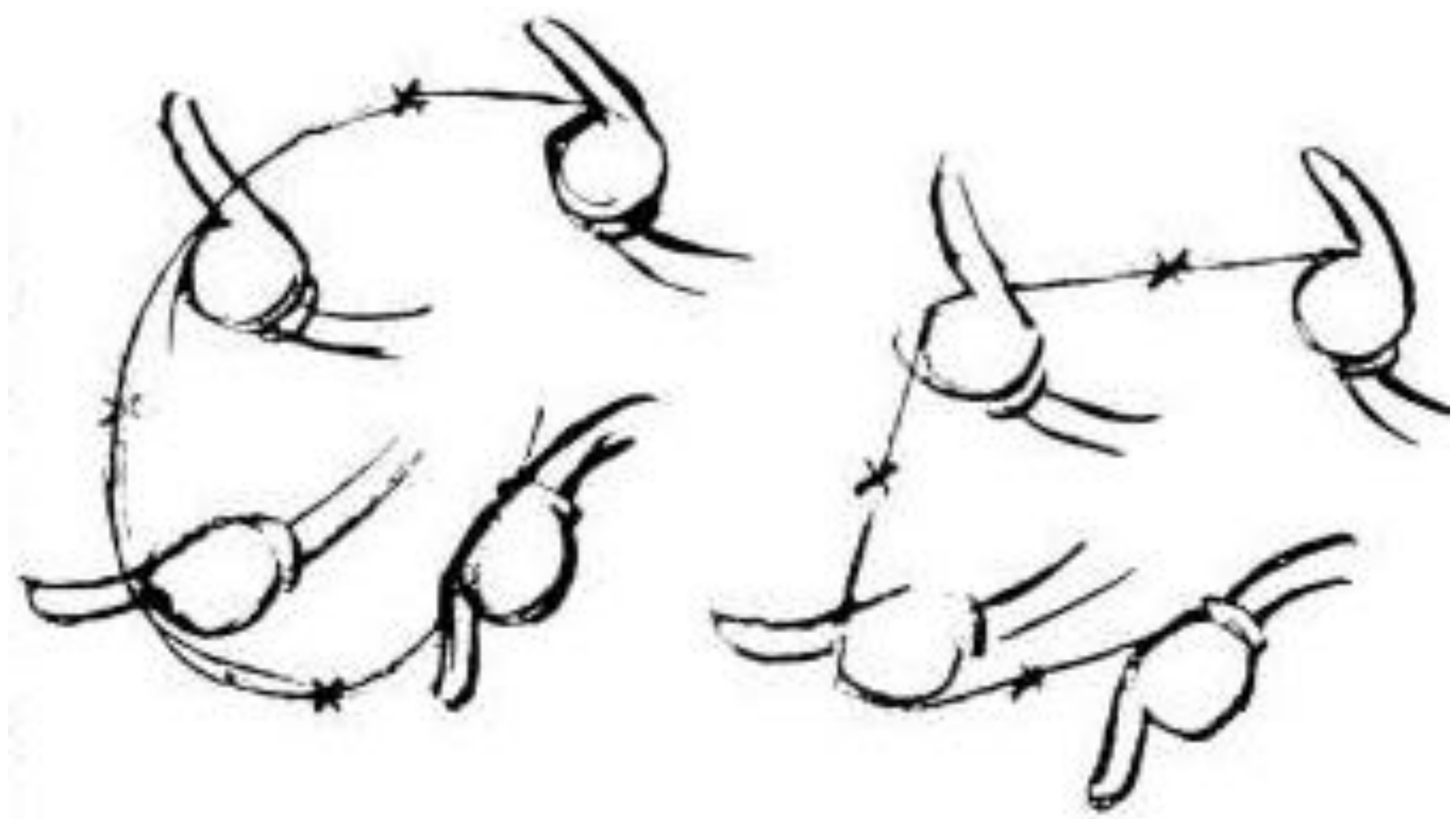
Also contributes to weight and emotion



Arcs

Move in curves, not in straight lines

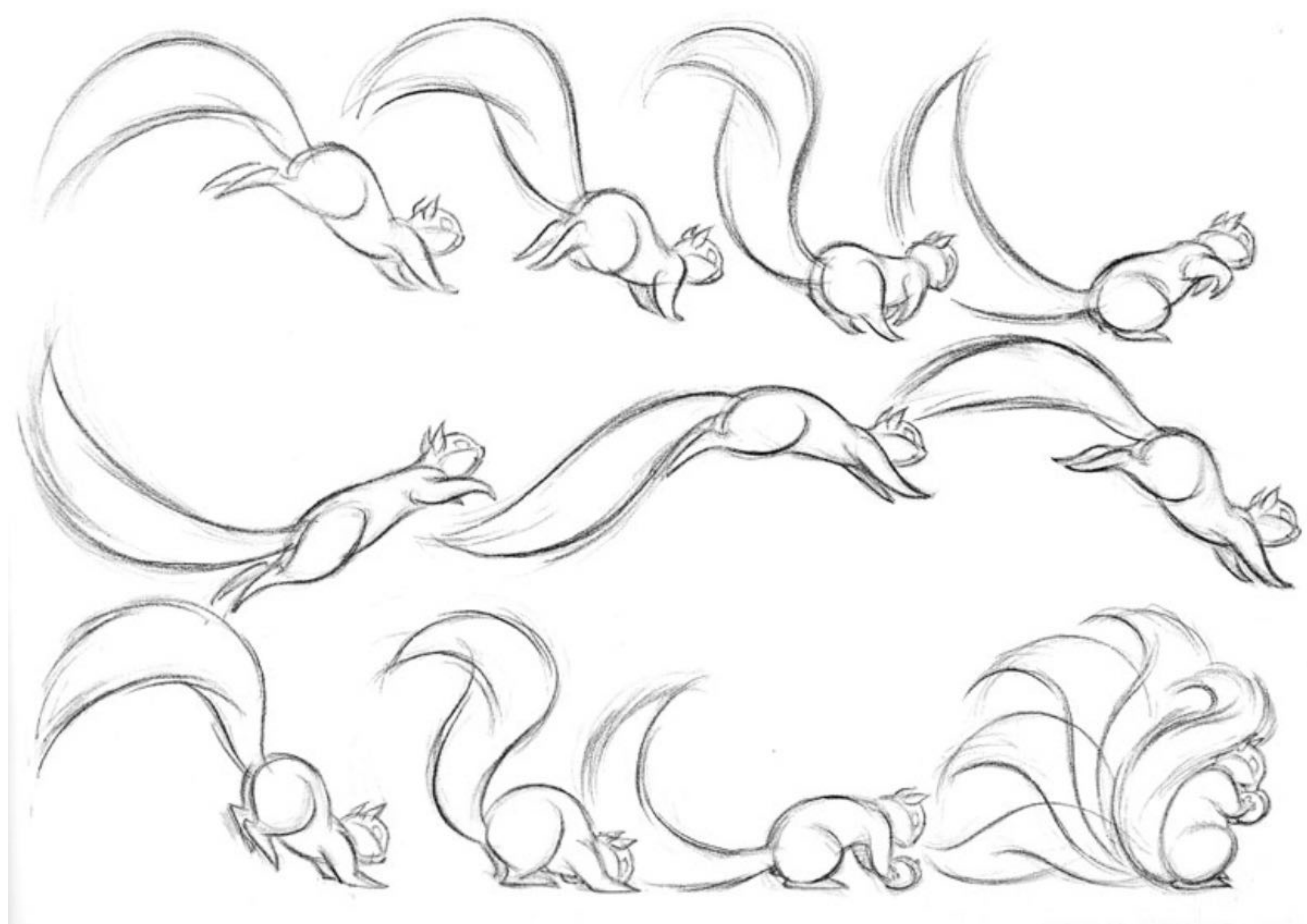
This is how living creatures move



Disney Animation: The Illusion of Life

Secondary action

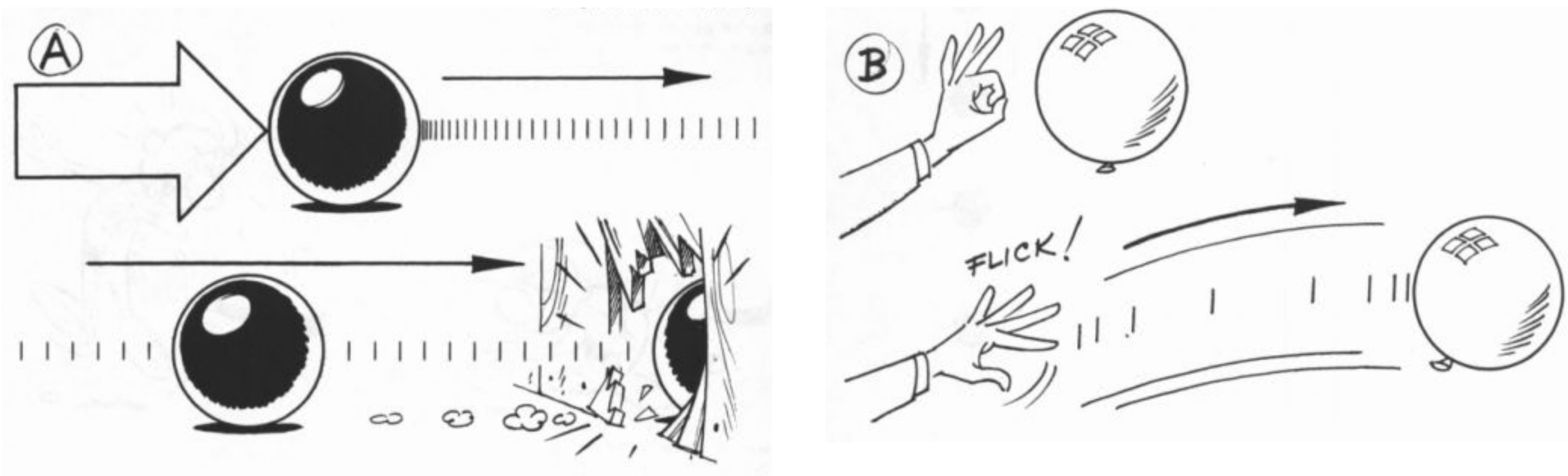
- Motion that results from some other action
- Needed for interest and realism
- Shouldn't distract from primary motion



Cartoon Animation, Preston Blair

Timing

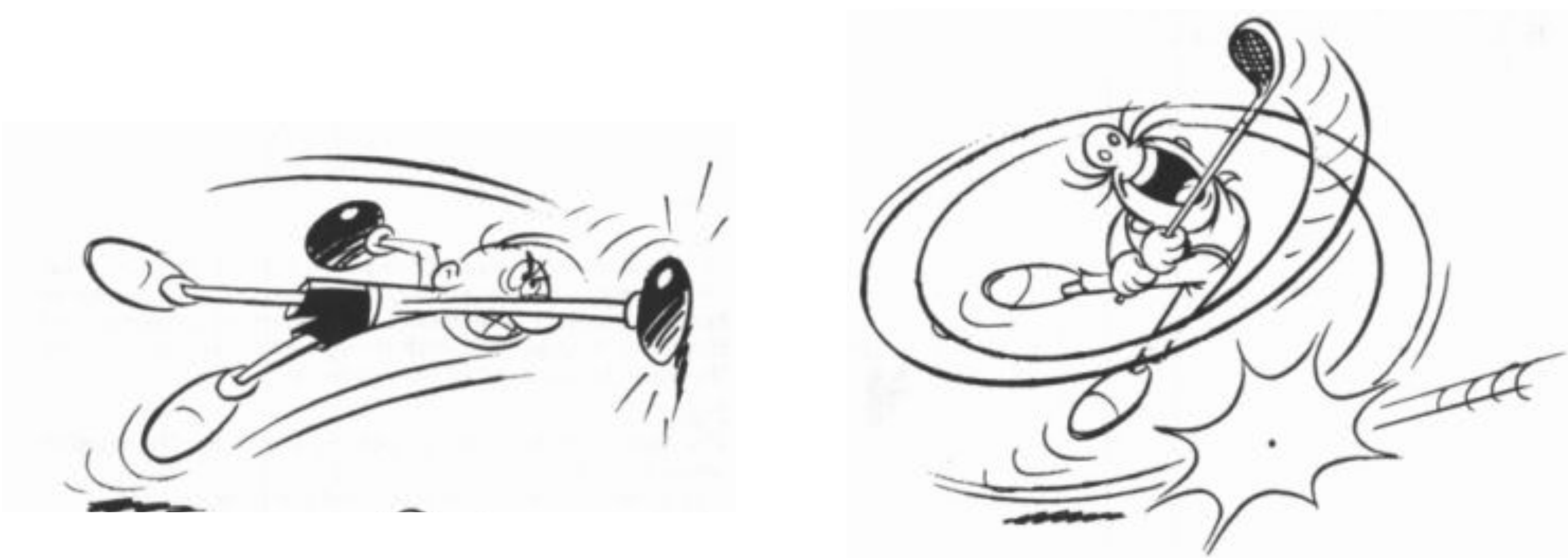
- **Rate of acceleration conveys weight**
- **Speed and acceleration of character's movements convey emotion**



Timing for Animation, Whitaker & Halas

Exaggeration

- Helps make actions clear
- Helps emphasize story points and emotion
- Must balance with non-exaggerated parts



Timing for Animation, Whitaker & Halas

Appeal

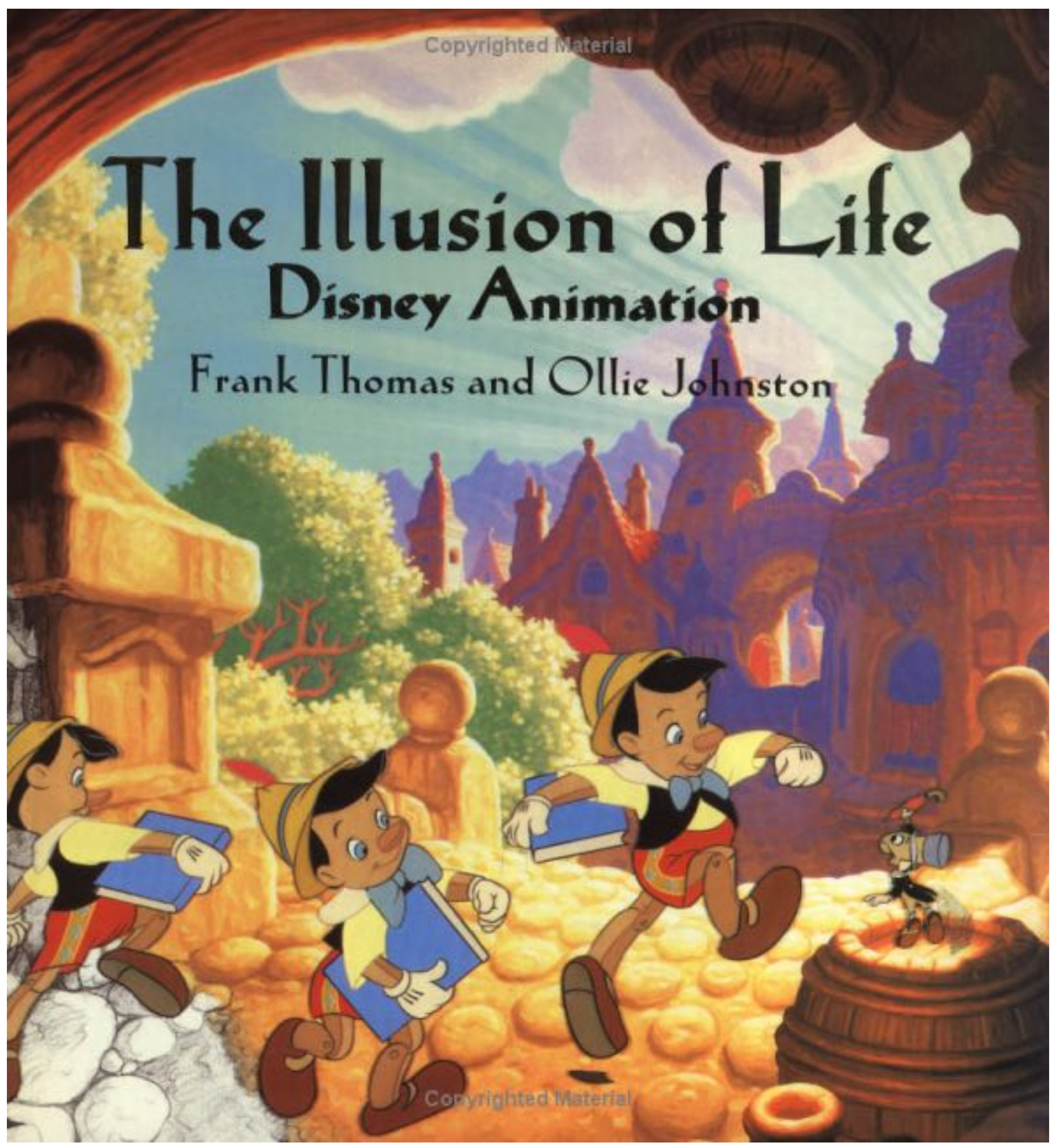
- Attractive to the eye, strong design
- Avoid symmetries

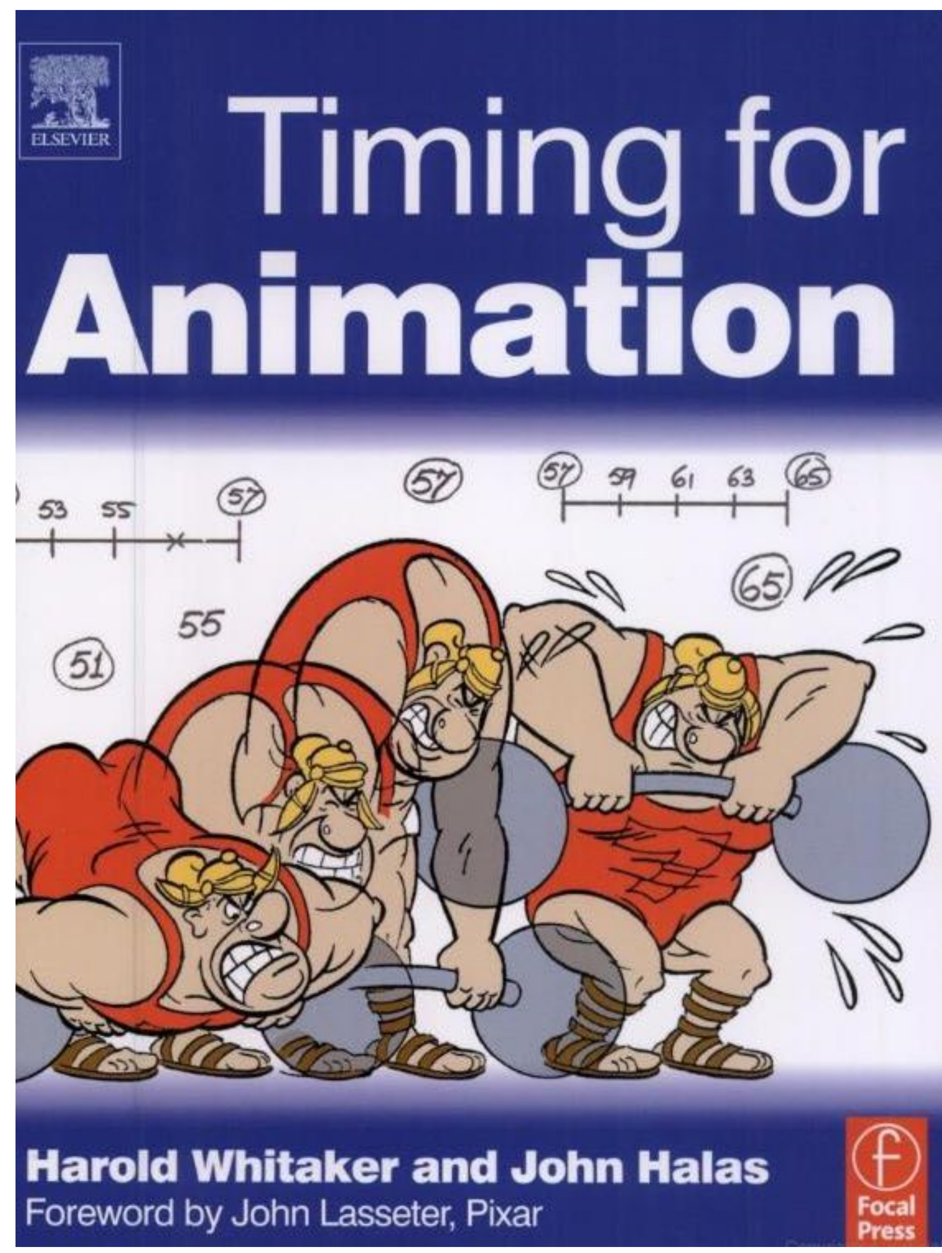
Disney Animation: The Illusion of Life

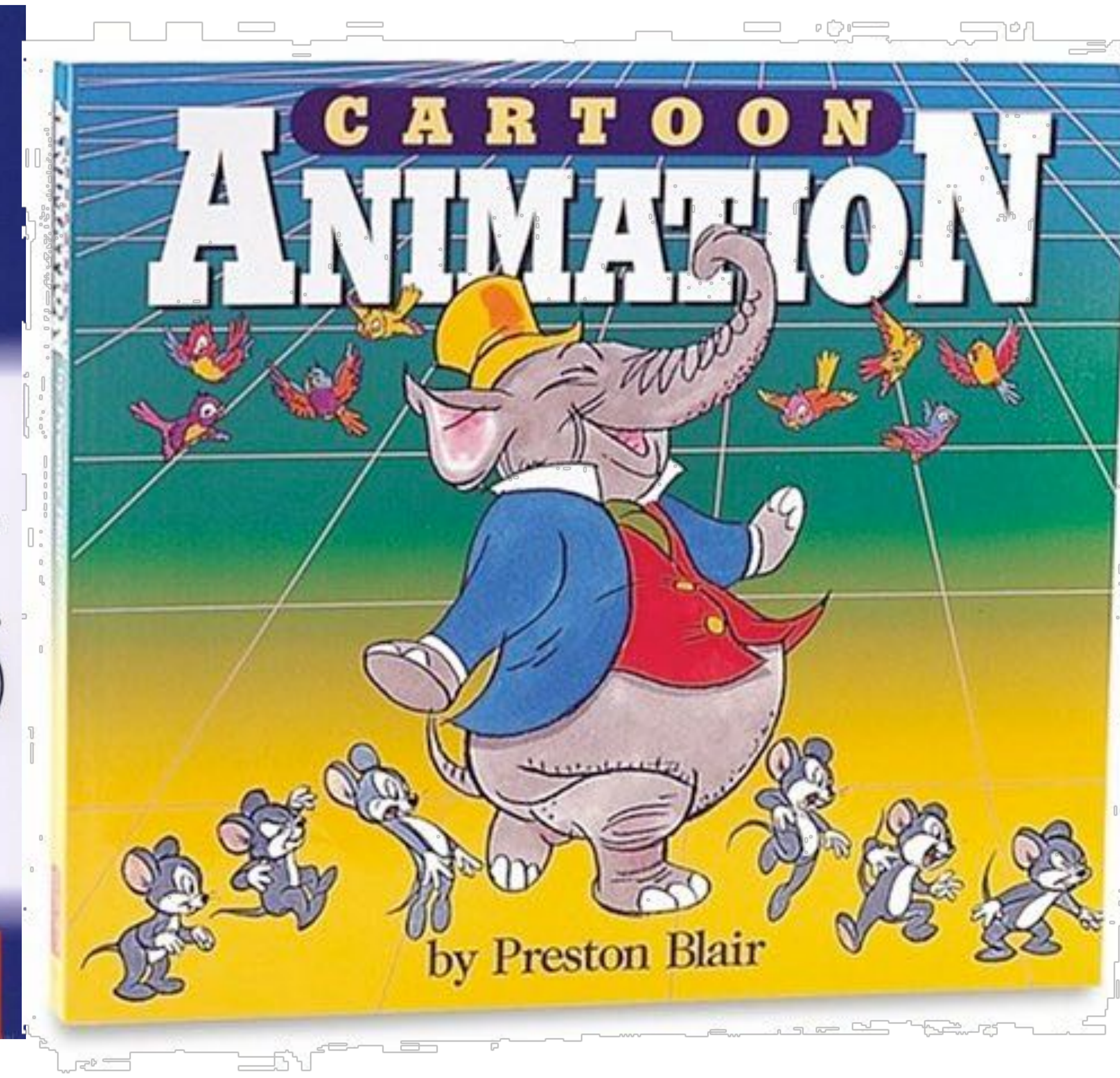
Personality

- Action of character is result of its thoughts
- Know purpose and mood before animating each action
- No two characters move the same way

Further reading







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