Lecture 2:

Drawing a Triangle

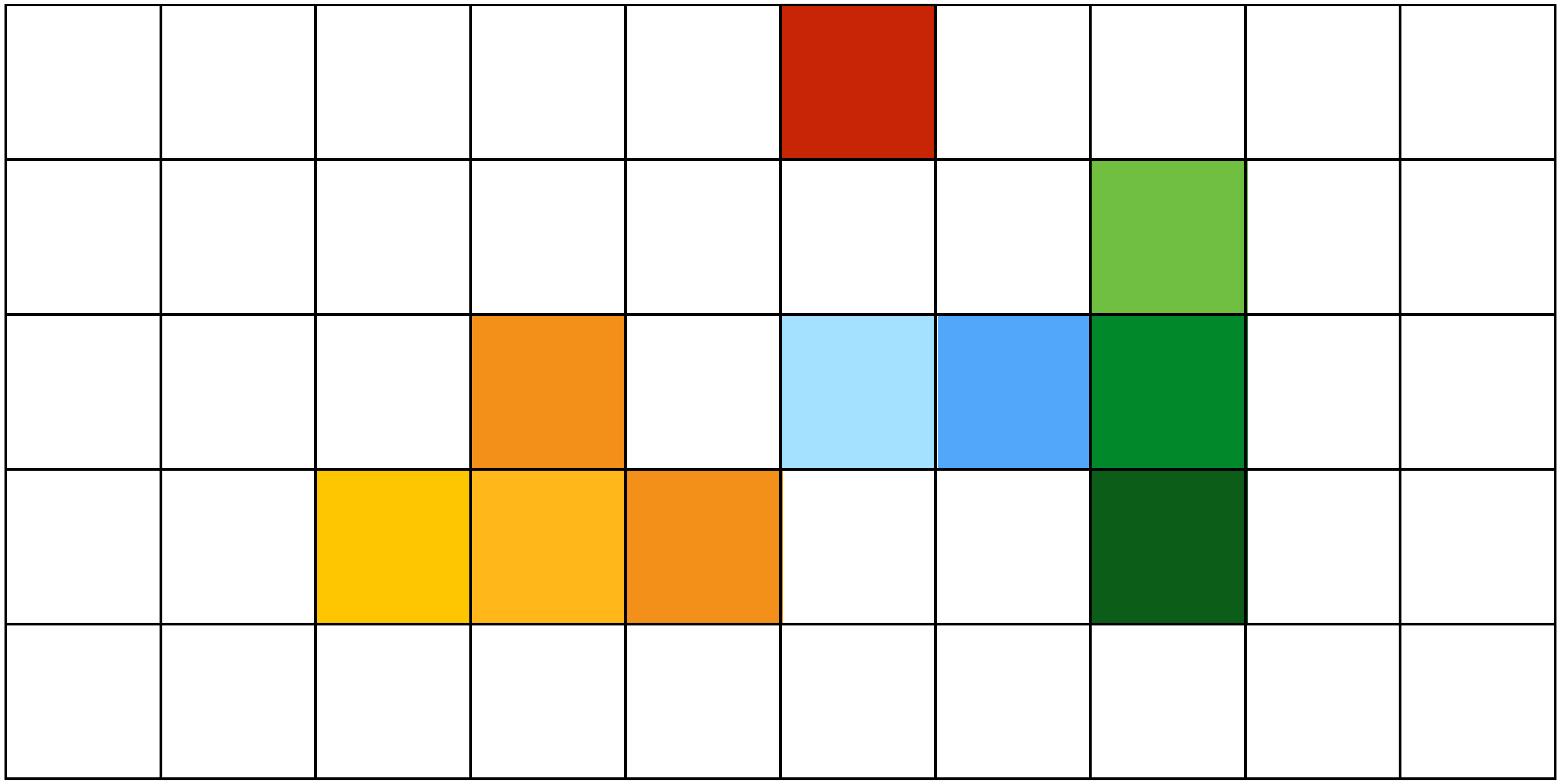
(+ the basics of sampling and anti-aliasing)

Interactive Computer Graphics

Stanford CS248, Winter 2021

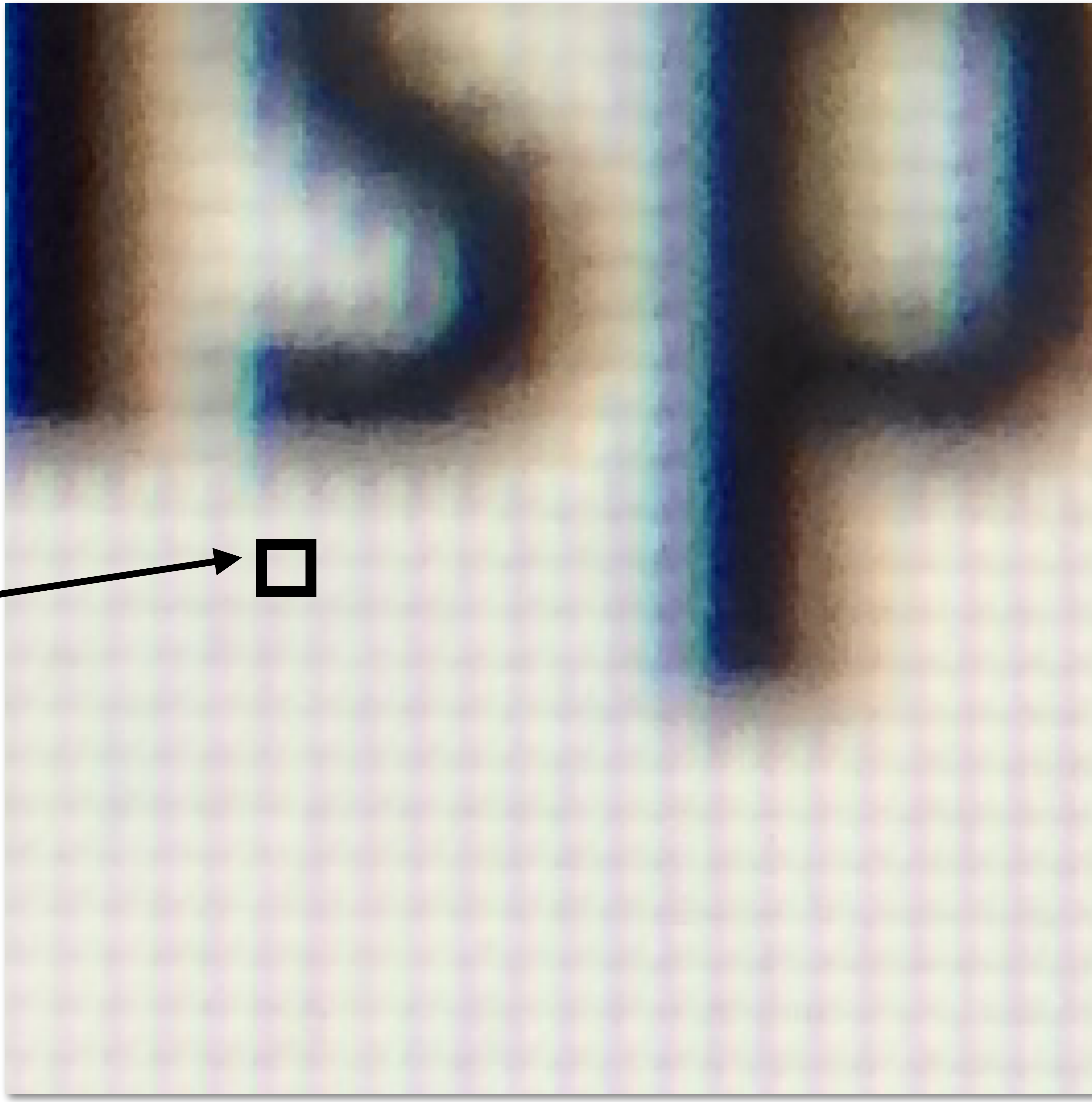
Last time

- A very simple notion of digital image representation
  - that we are about to challenge!
- An image: a 2D array of color values



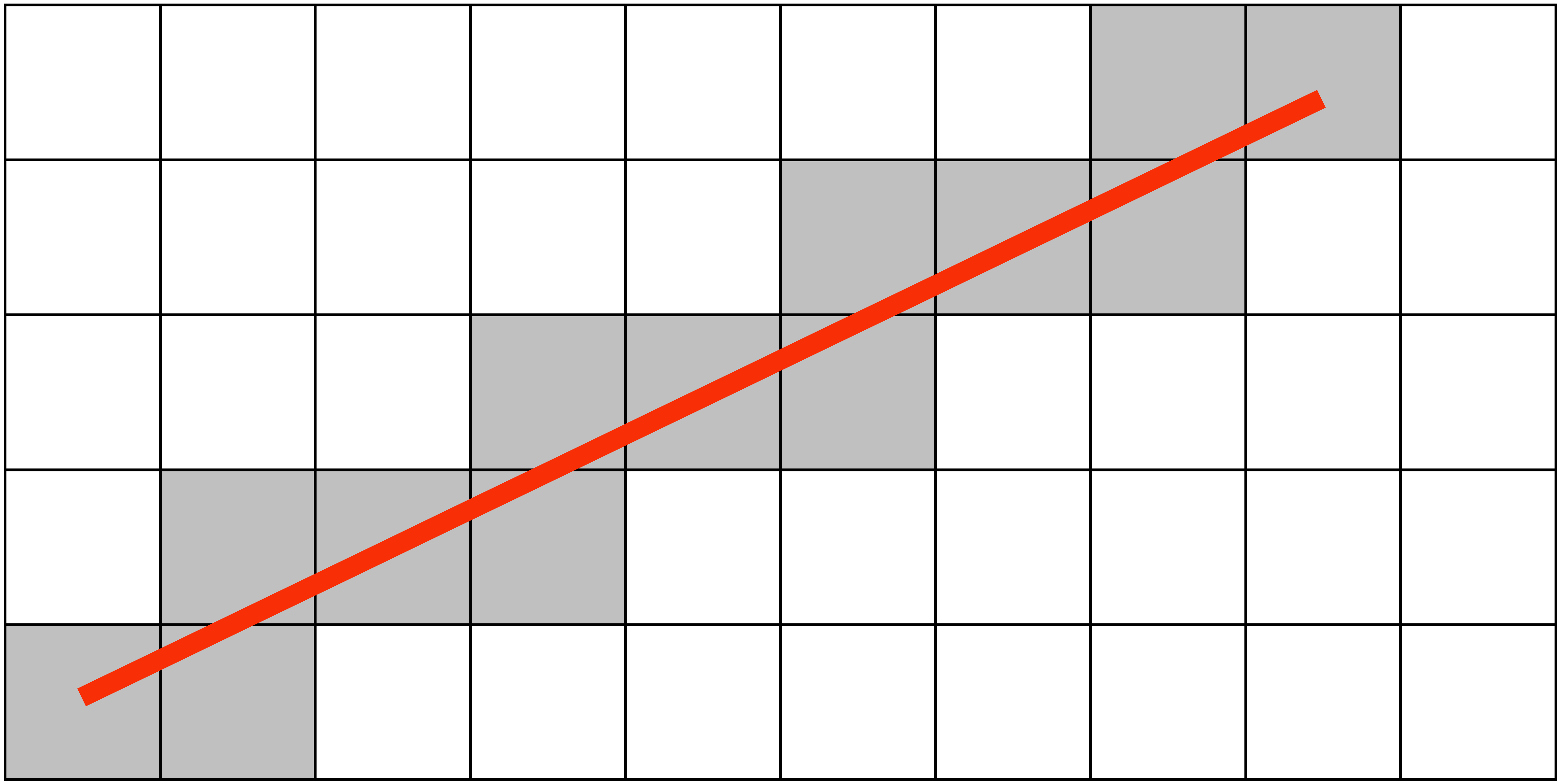
Last time

A display converts a color value at each pixel in an image to emitted light



**Display pixel
on my laptop**

Last time:
What pixels should we color in to draw a line?

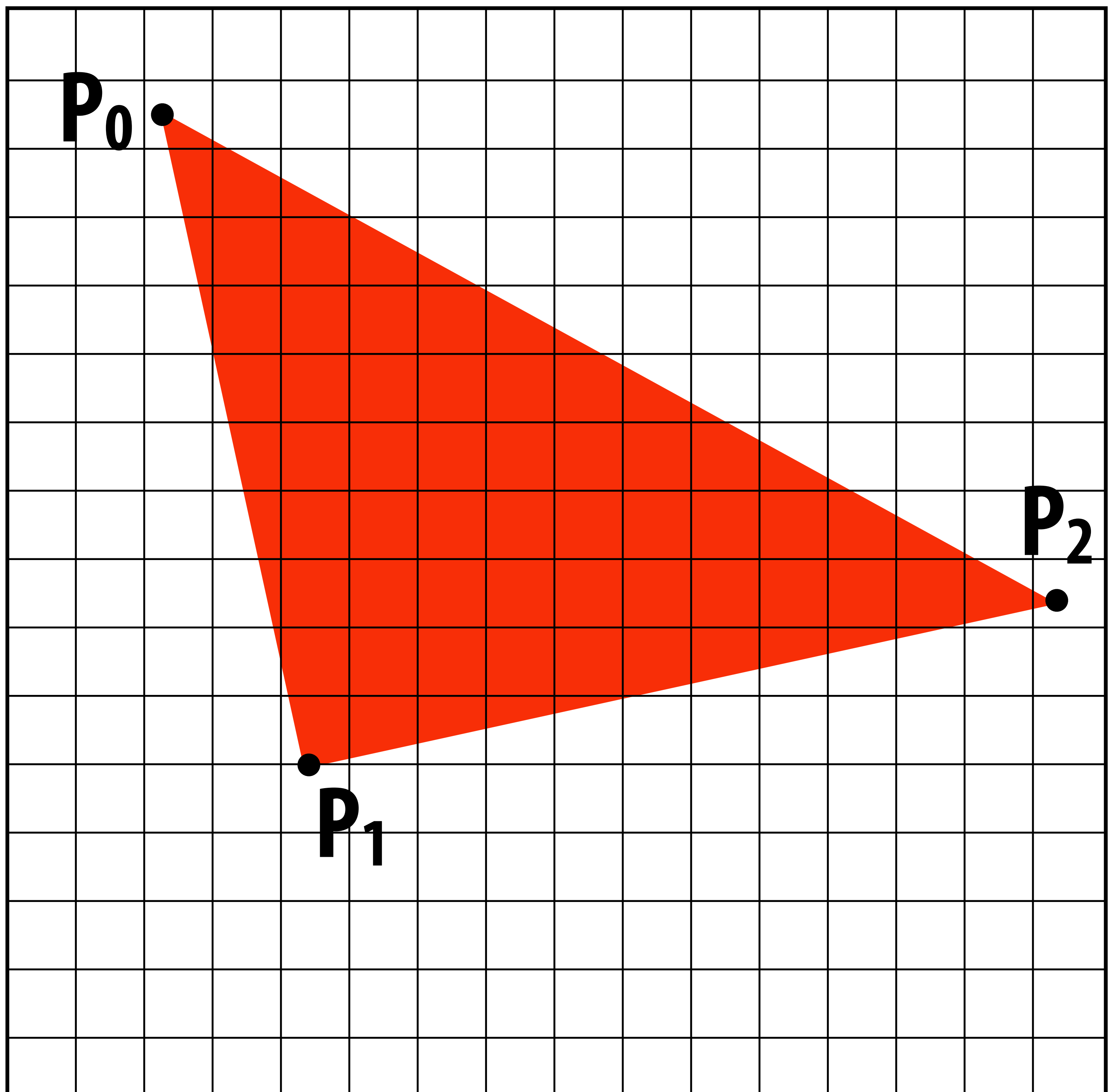


Today: drawing a triangle

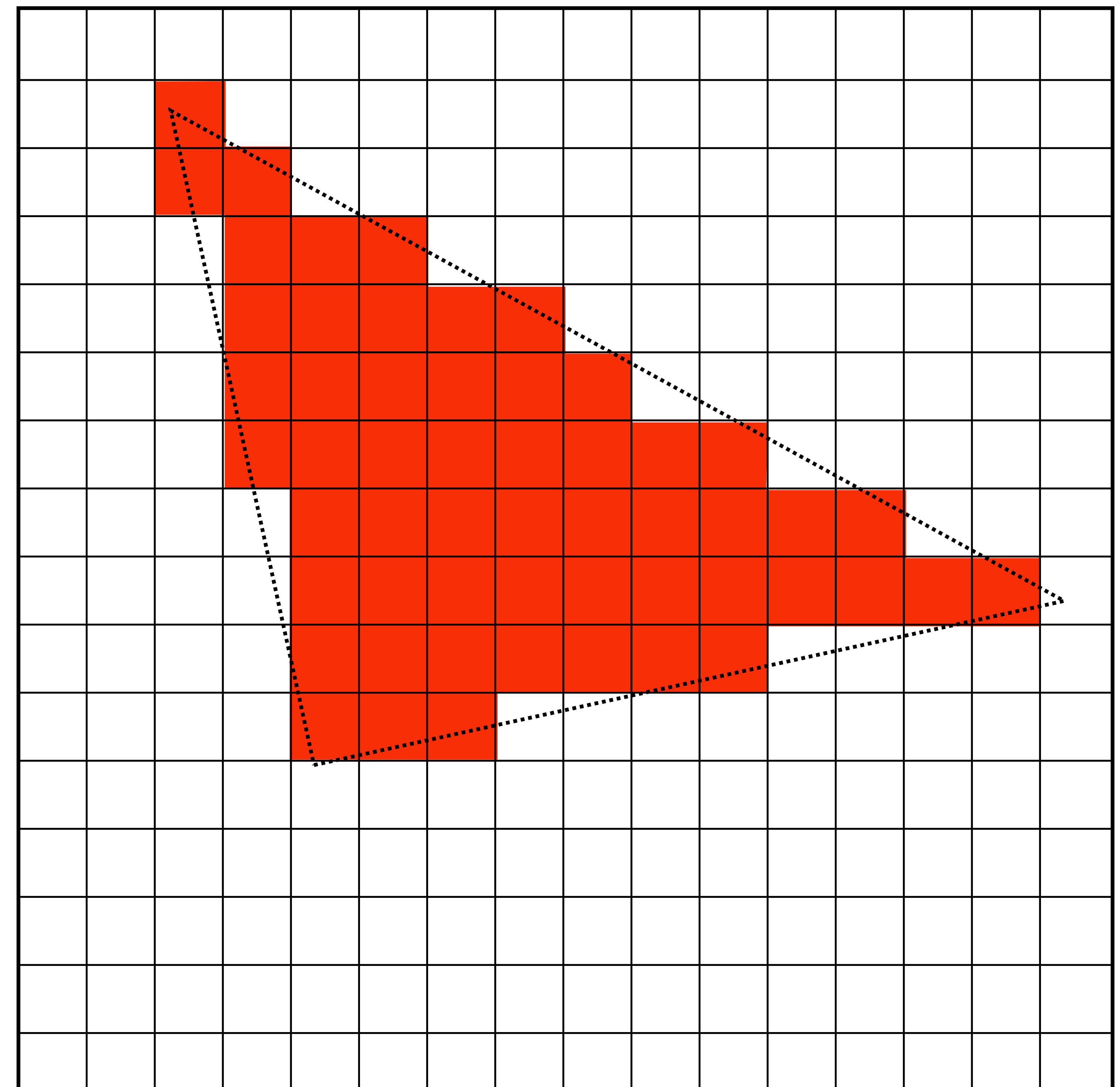
(Converting a representation of a triangle into an image)

"Triangle rasterization"

Input:
2D position of triangle vertices: P_0, P_1, P_2

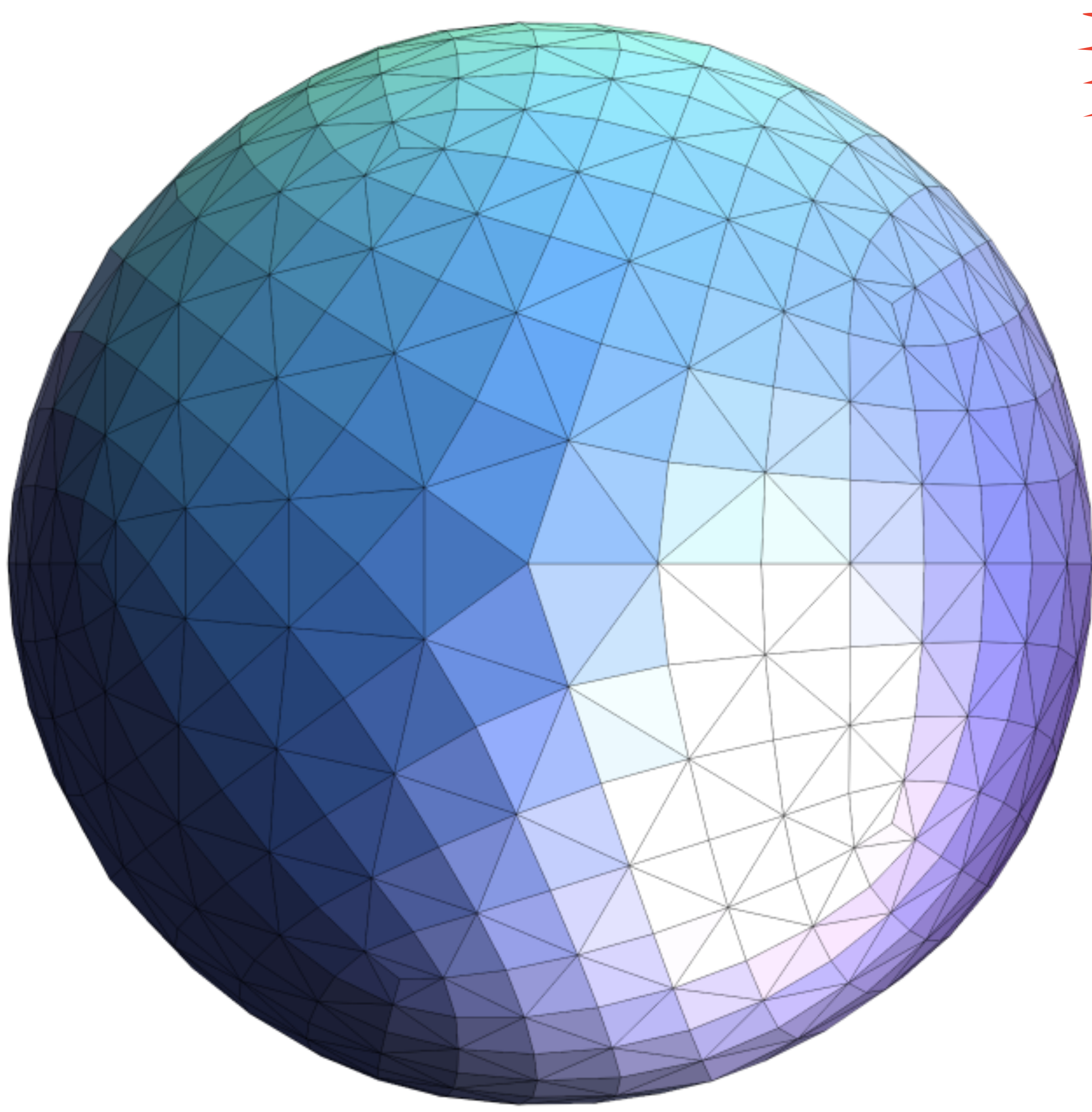


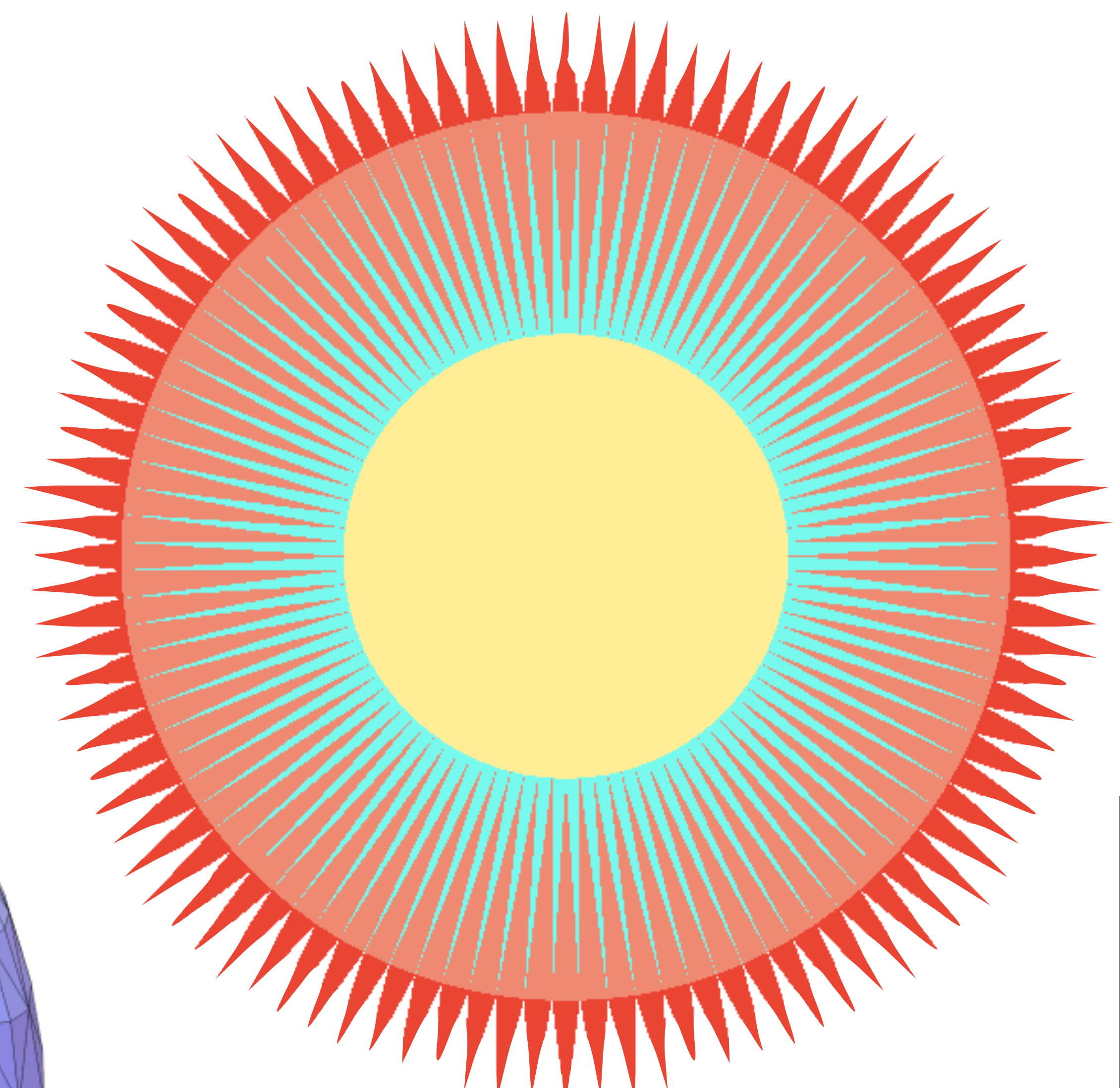
Output:
set of pixels "covered" by the triangle

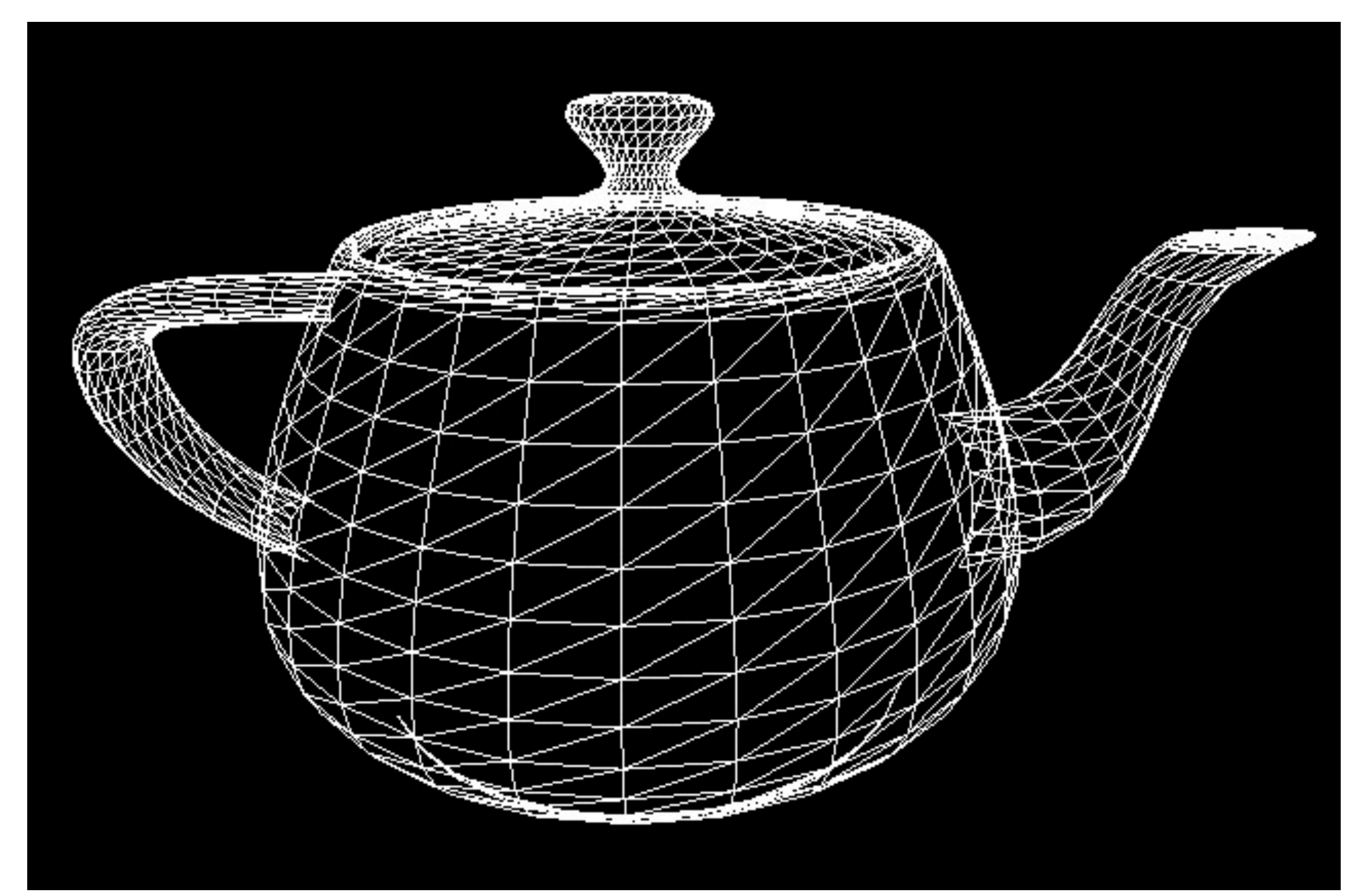


Why triangles?

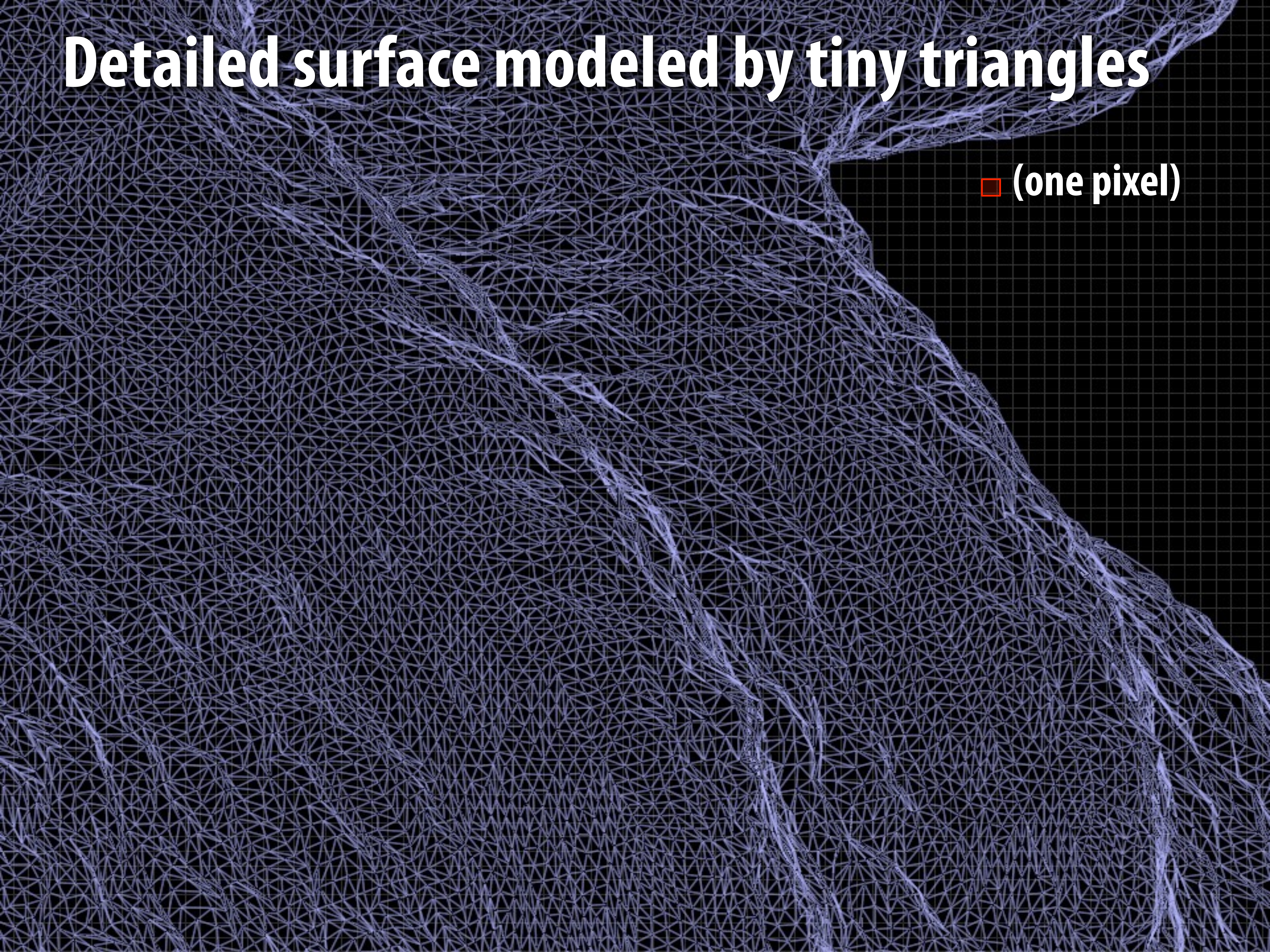
Triangles are a basic block for creating more complex shapes and surfaces



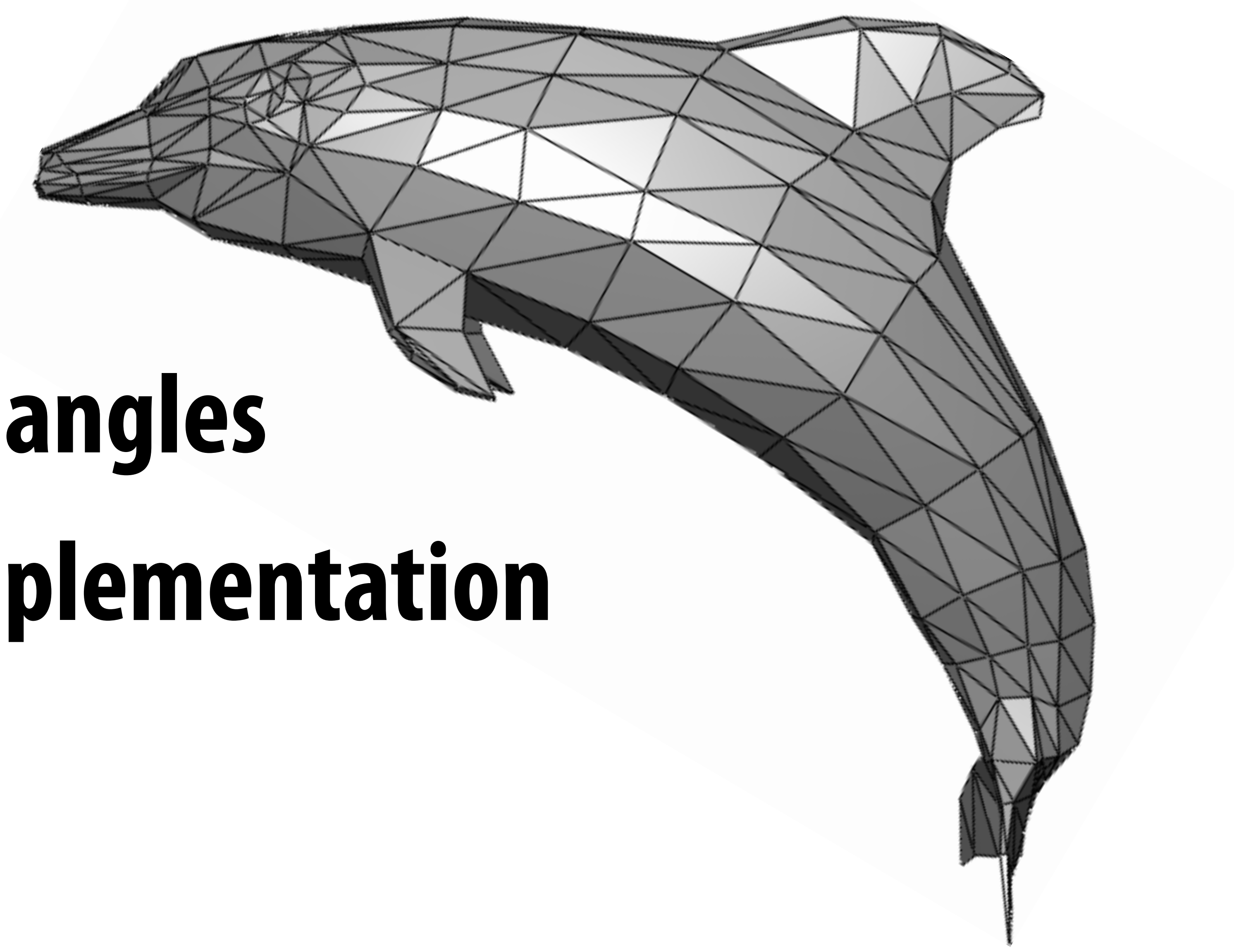




Detailed surface modeled by tiny triangles



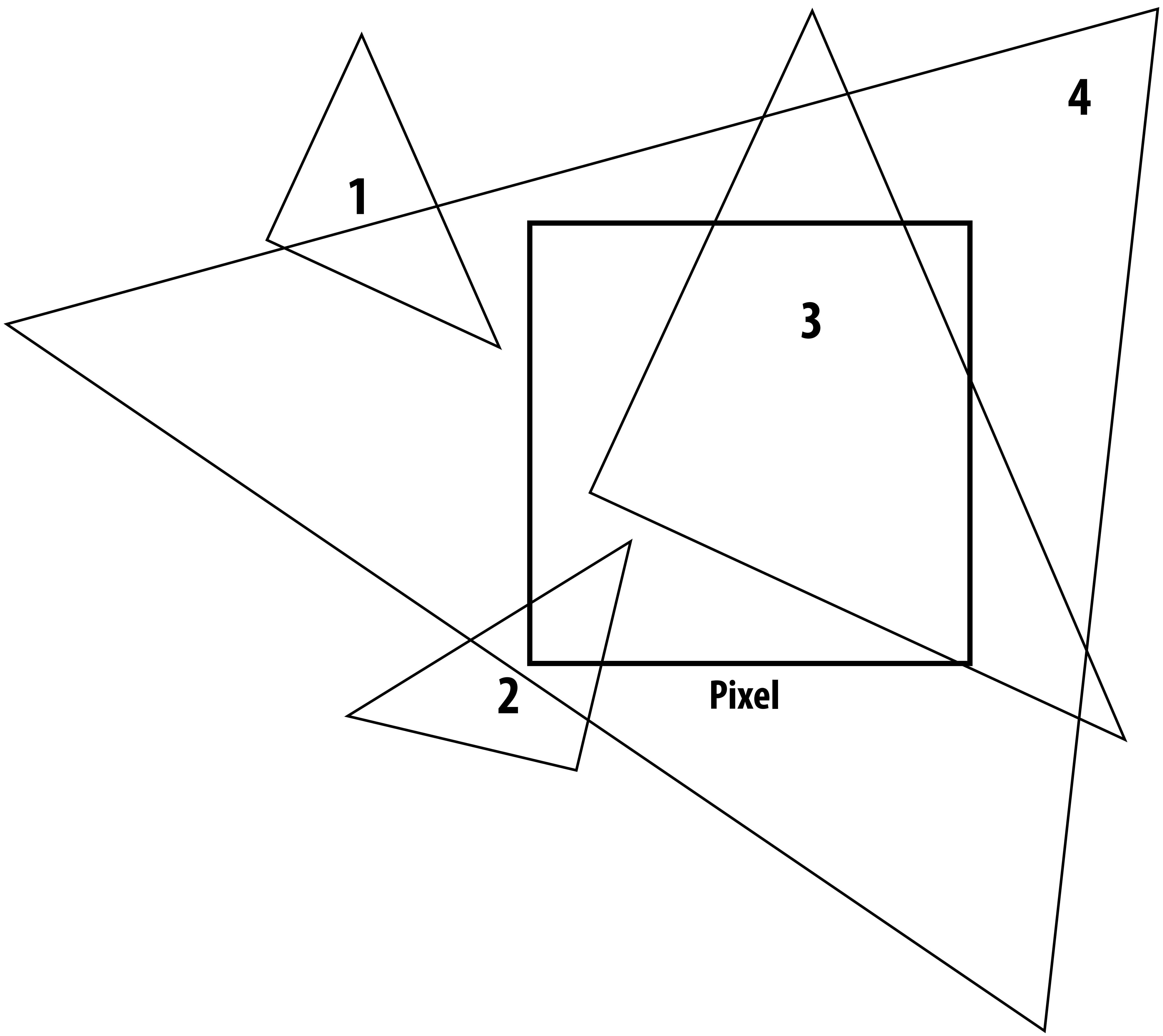
Triangles - a fundamental primitive



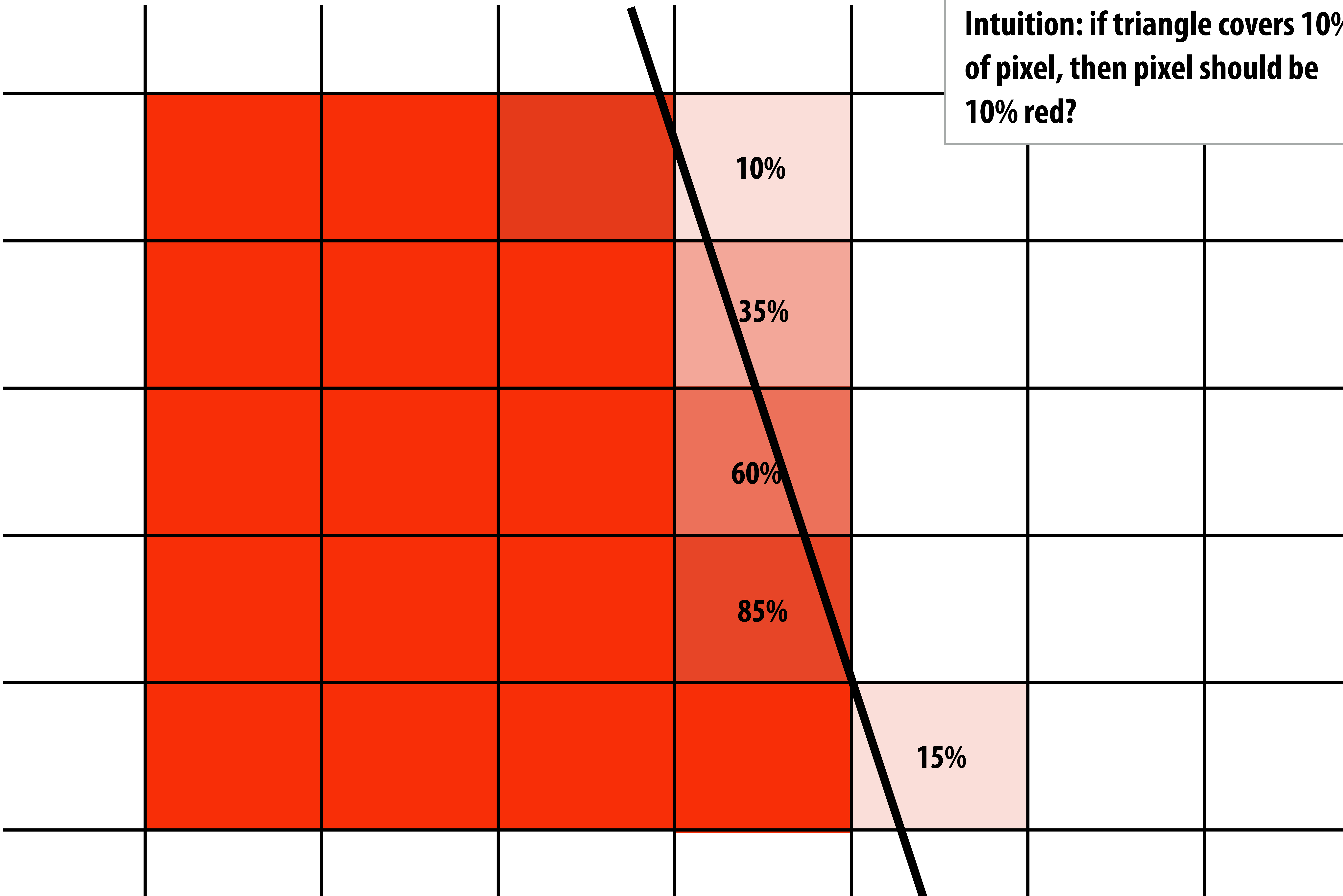
- **Why triangles?**
    - **Most basic polygon**
        - **Can break up other polygons into triangles**
        - **Allows programs to optimize one implementation**
    - **Triangles have unique properties**
        - **Guaranteed to be planar**
        - **Well-defined interior**
        - **Well-defined method for interpolating values at vertices over triangle (a topic of a future lecture)**

What does it mean for a pixel to be covered by a triangle?

Question: which triangles "cover" this pixel?

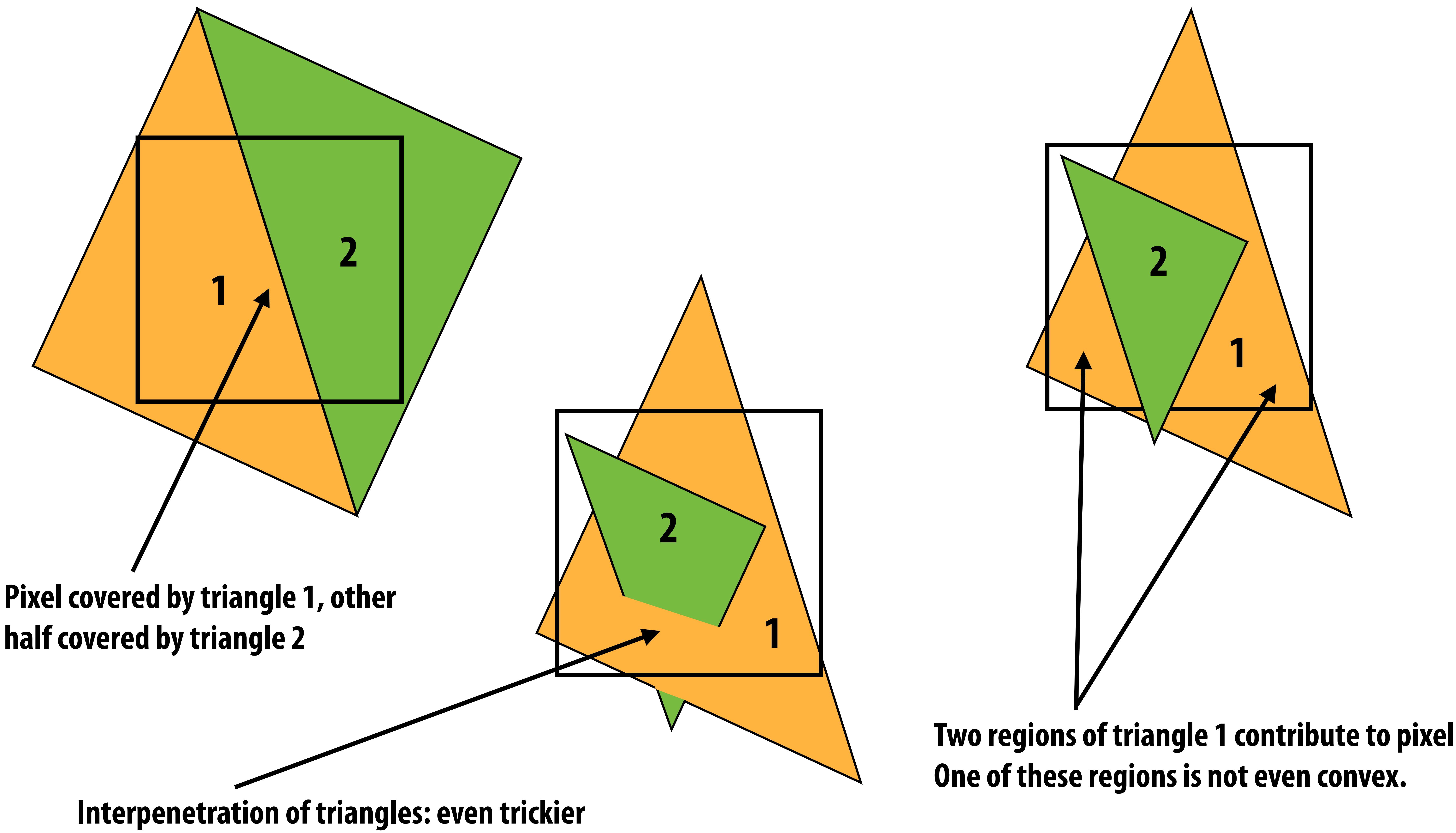


One option: compute fraction of pixel area covered by triangle, then color pixel according to this fraction.



Intuition: if triangle covers 10% of pixel, then pixel should be 10% red?

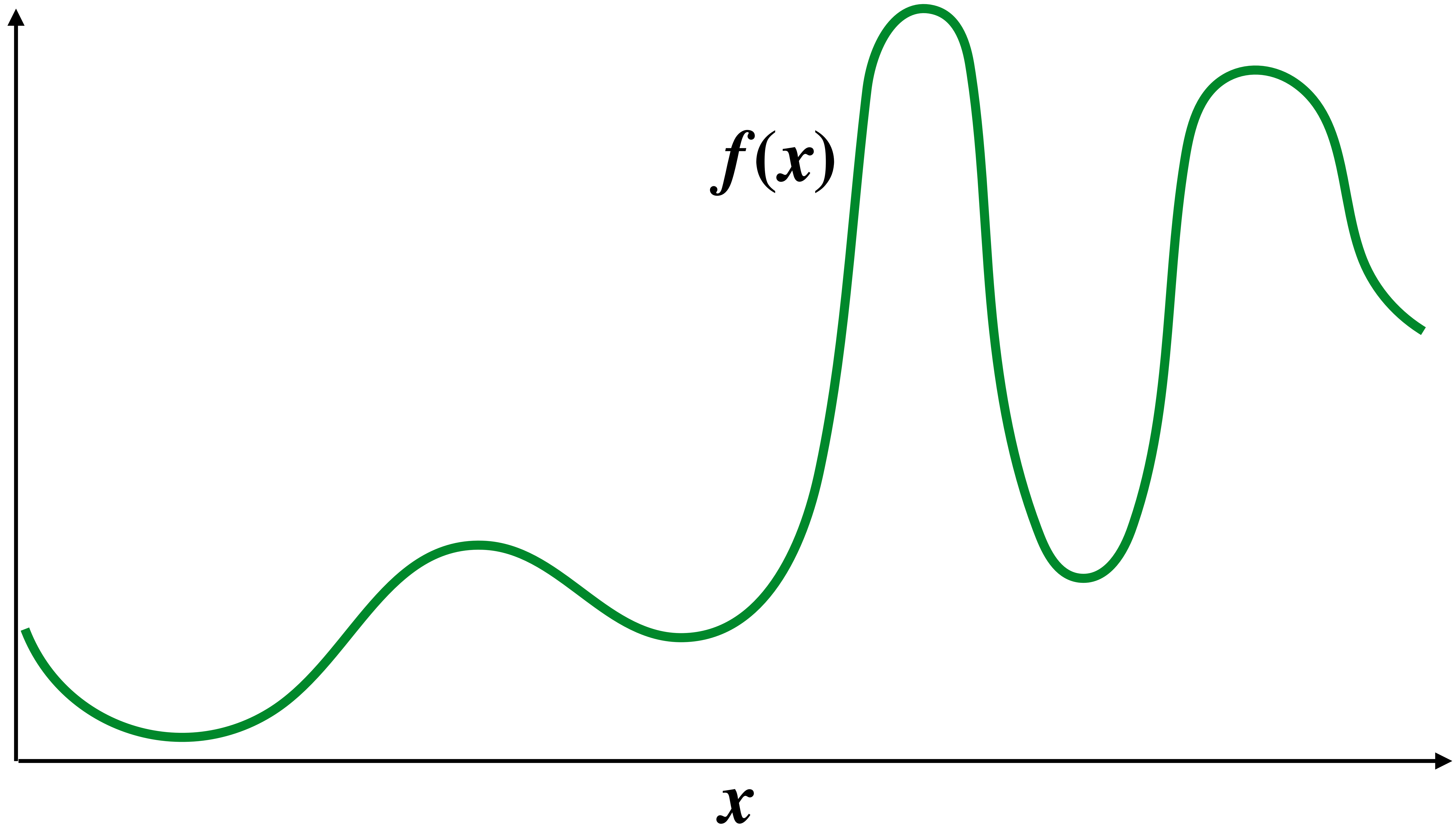
Analytical coverage schemes get tricky when considering occlusion of one triangle by another



Today we will draw triangles using a simple method: point sampling

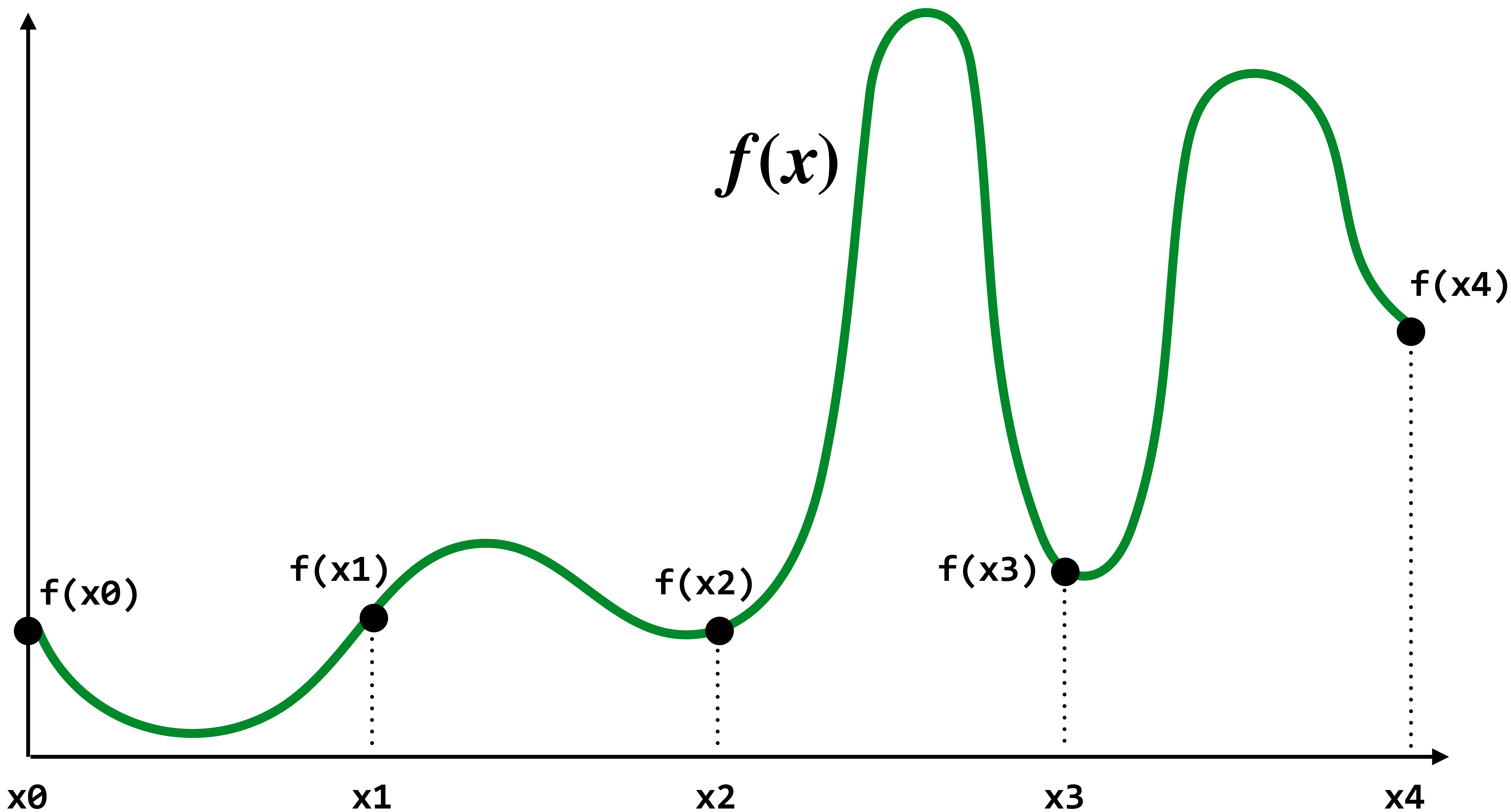
(let's consider sampling in 1D first)

Consider a 1D signal: f(x)



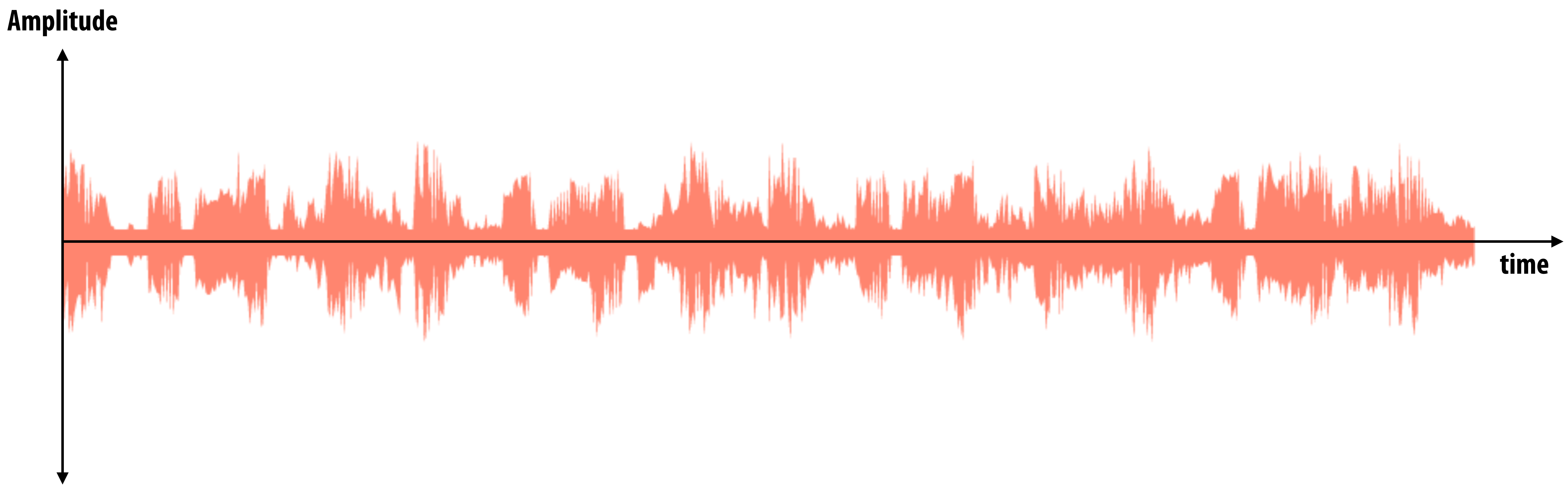
Sampling: taking measurements of a signal

Below: five measurements ("samples") of $f(x)$



Audio file: stores samples of a 1D signal

Audio is often sampled at 44.1 KHz



Sampling a function

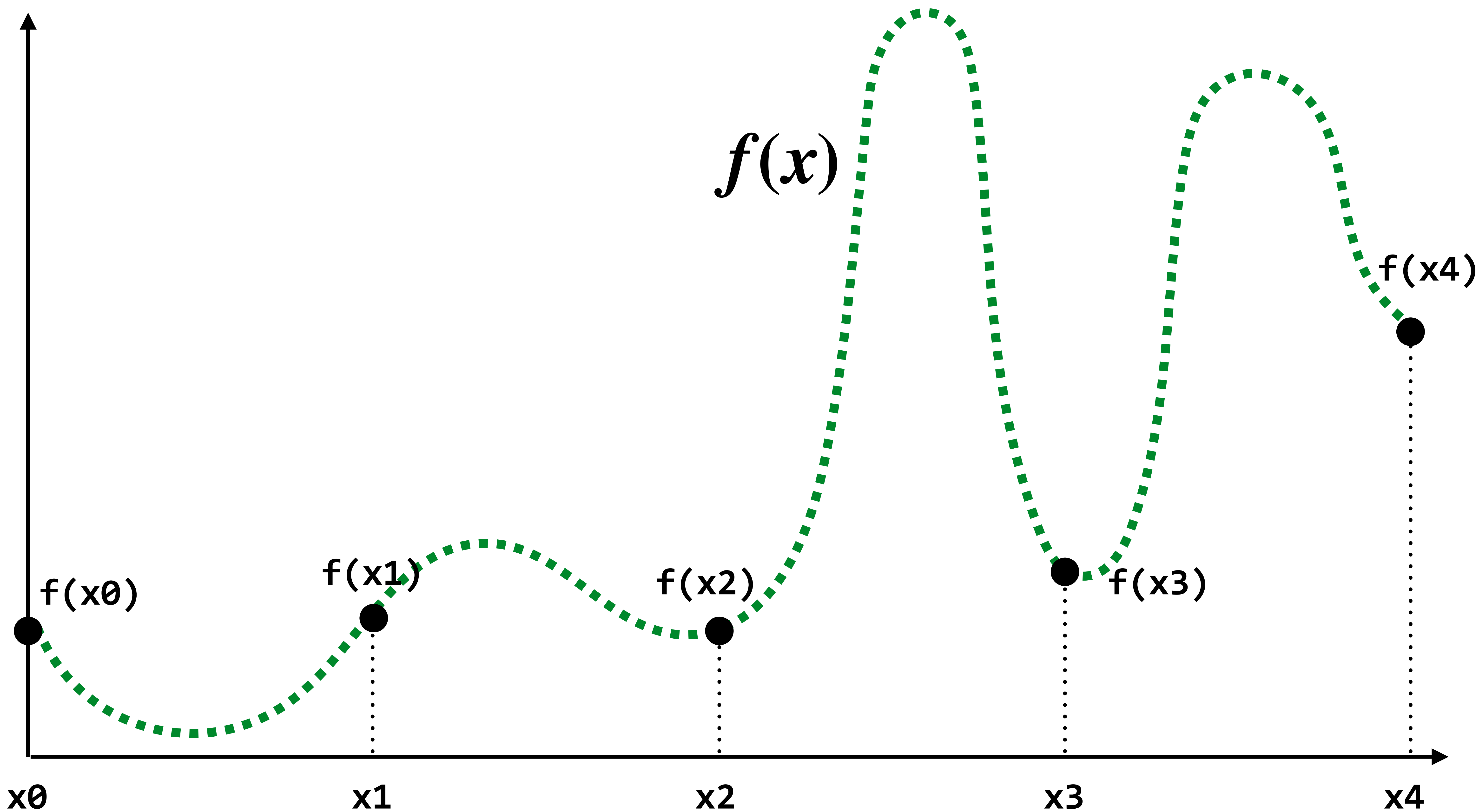
- **Evaluating a function at a point is sampling the function's value**

- **We can discretize a function by periodic sampling**

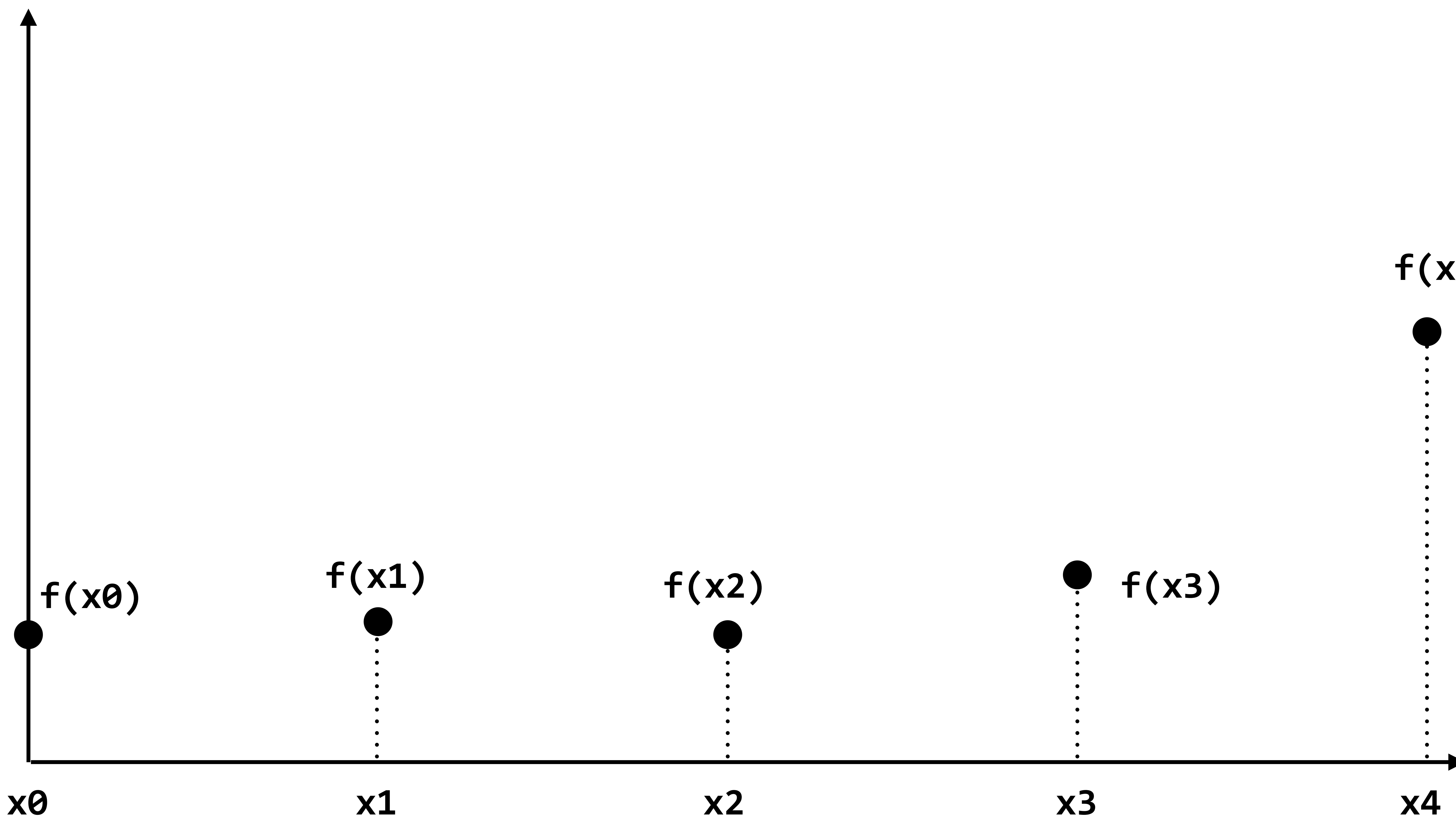
```
for(int x = 0; x < xmax; x++)
    output[x] = f(x);
```

- **Sampling is a core idea in graphics. In this class we'll sample time (1D), area (2D), angle (2D), volume (3D), etc ...**

Reconstruction: given a set of samples, how might we attempt to reconstruct the original signal $f(x)$?



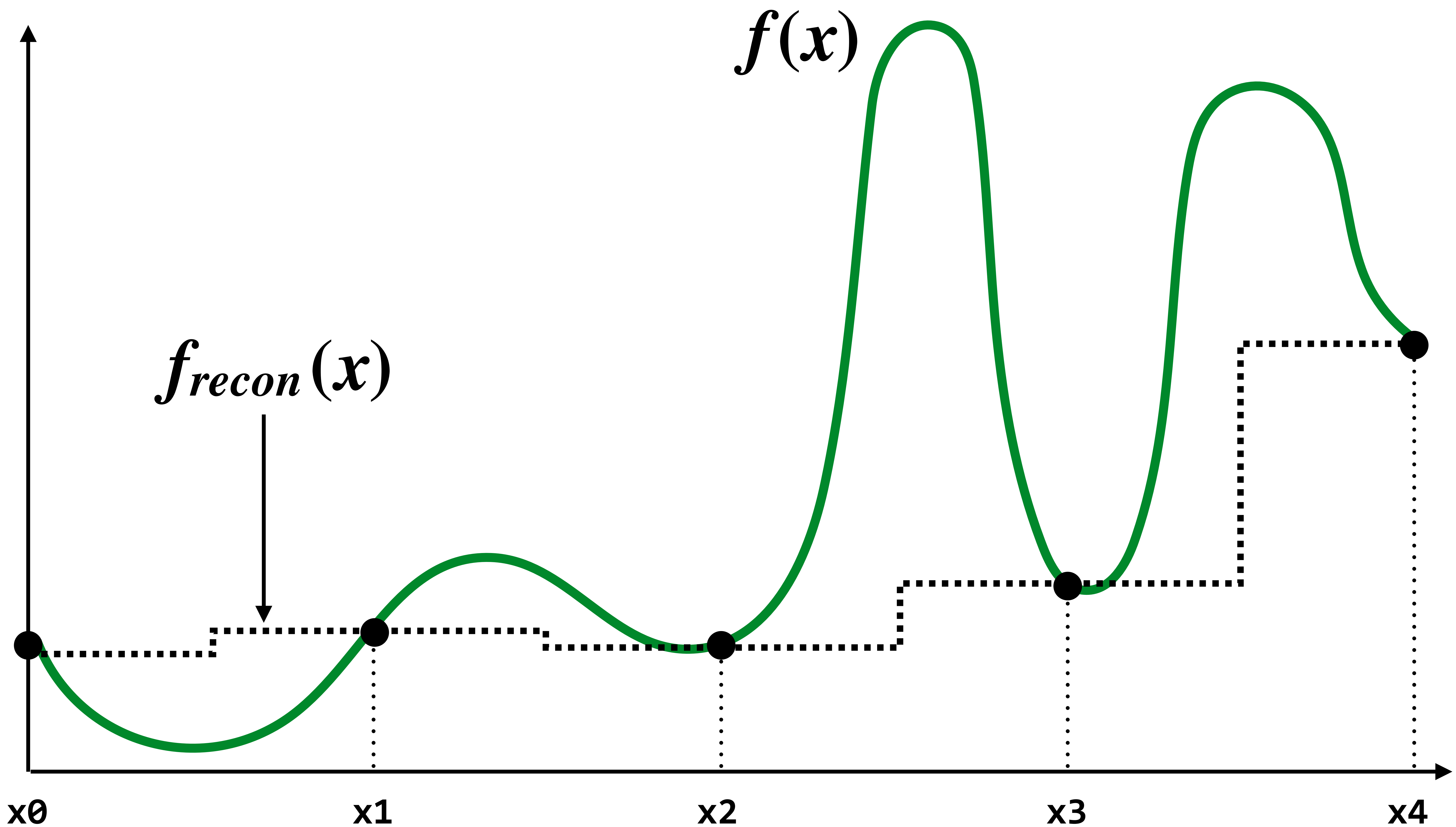
Reconstruction: given a set of samples, how might we attempt to reconstruct the original signal $f(x)$?



Piecewise constant approximation

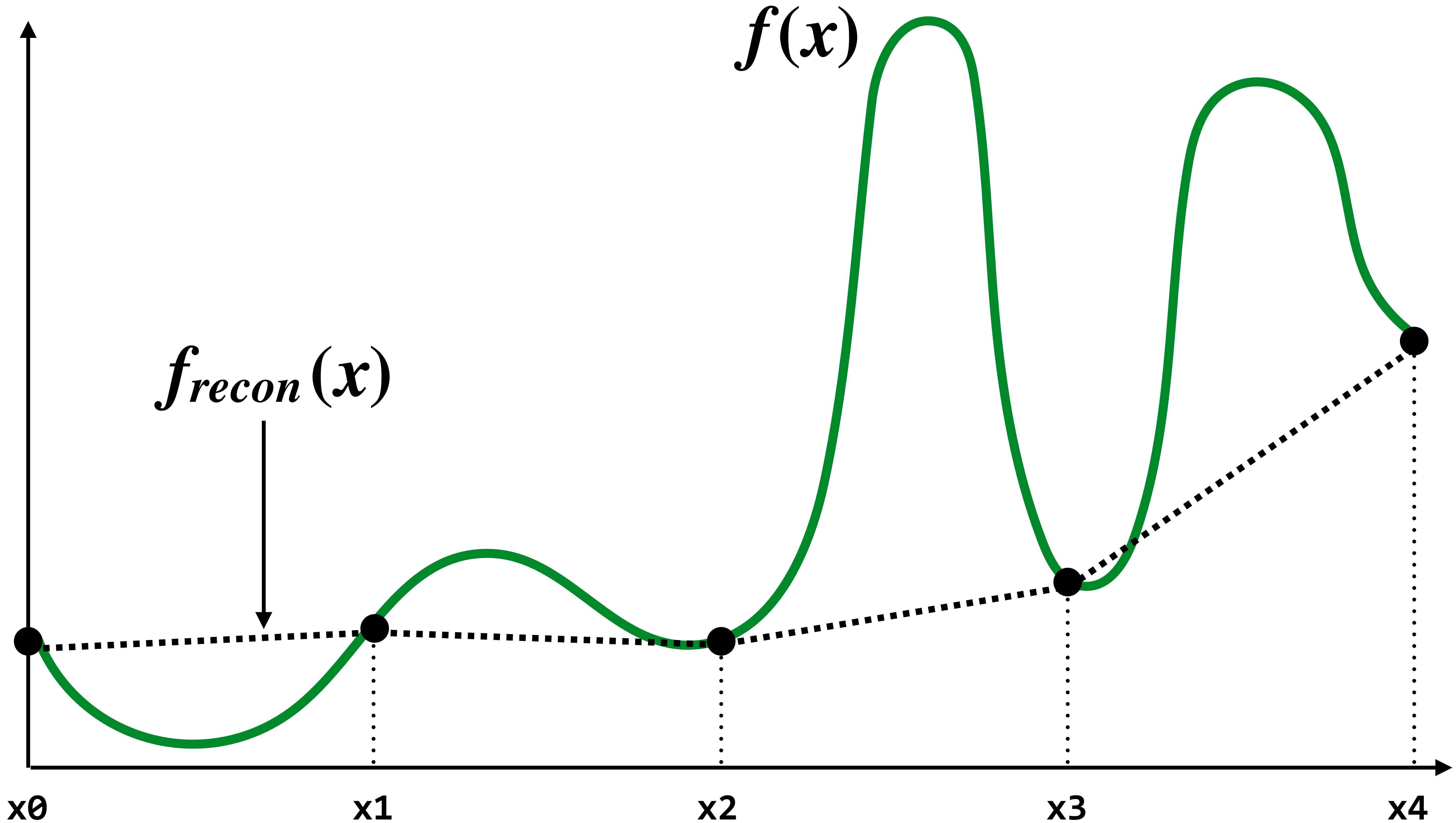
$f_{recon}(x)$ = value of sample closest to x

$f_{recon}(x)$ approximates $f(x)$

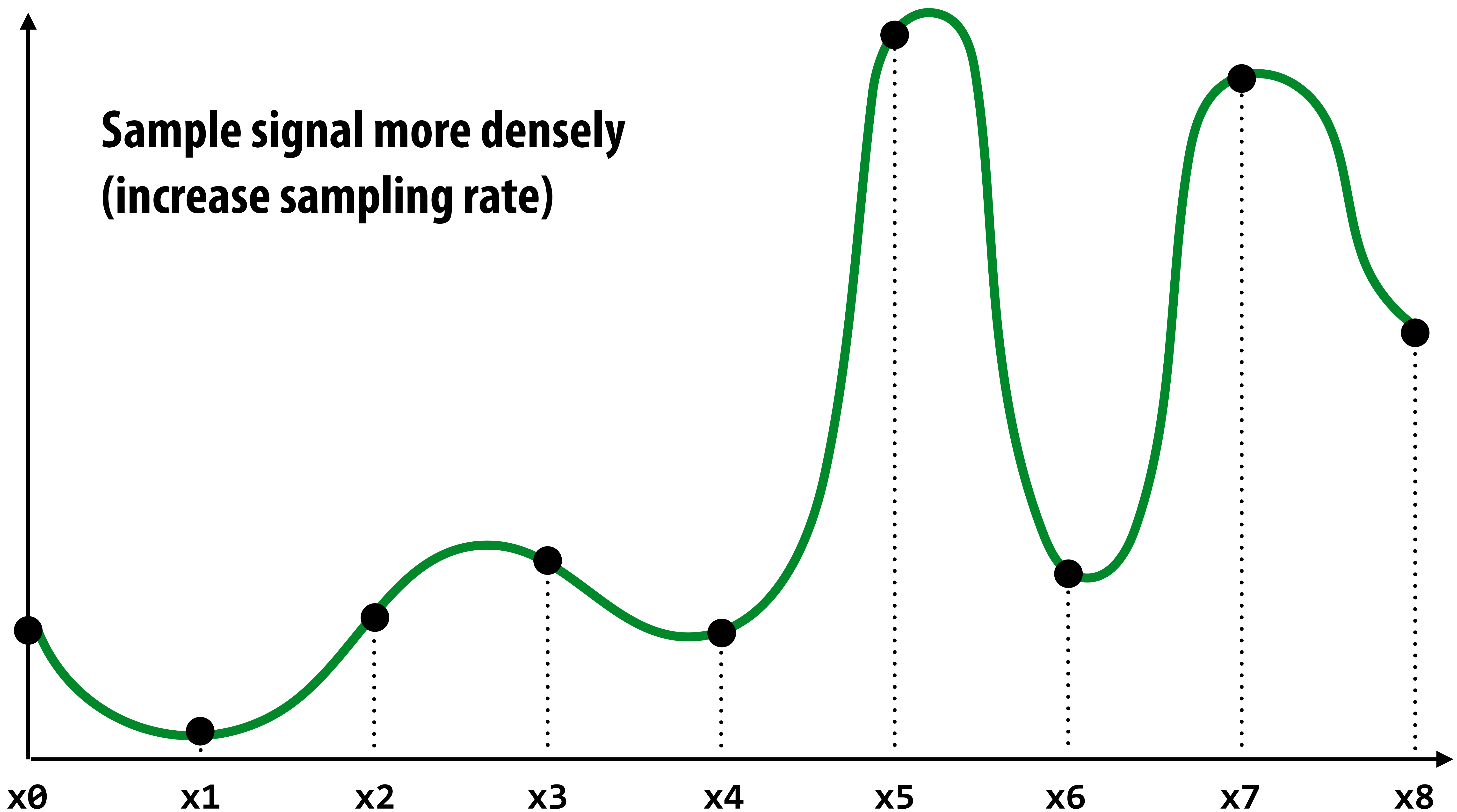


Piecewise linear approximation

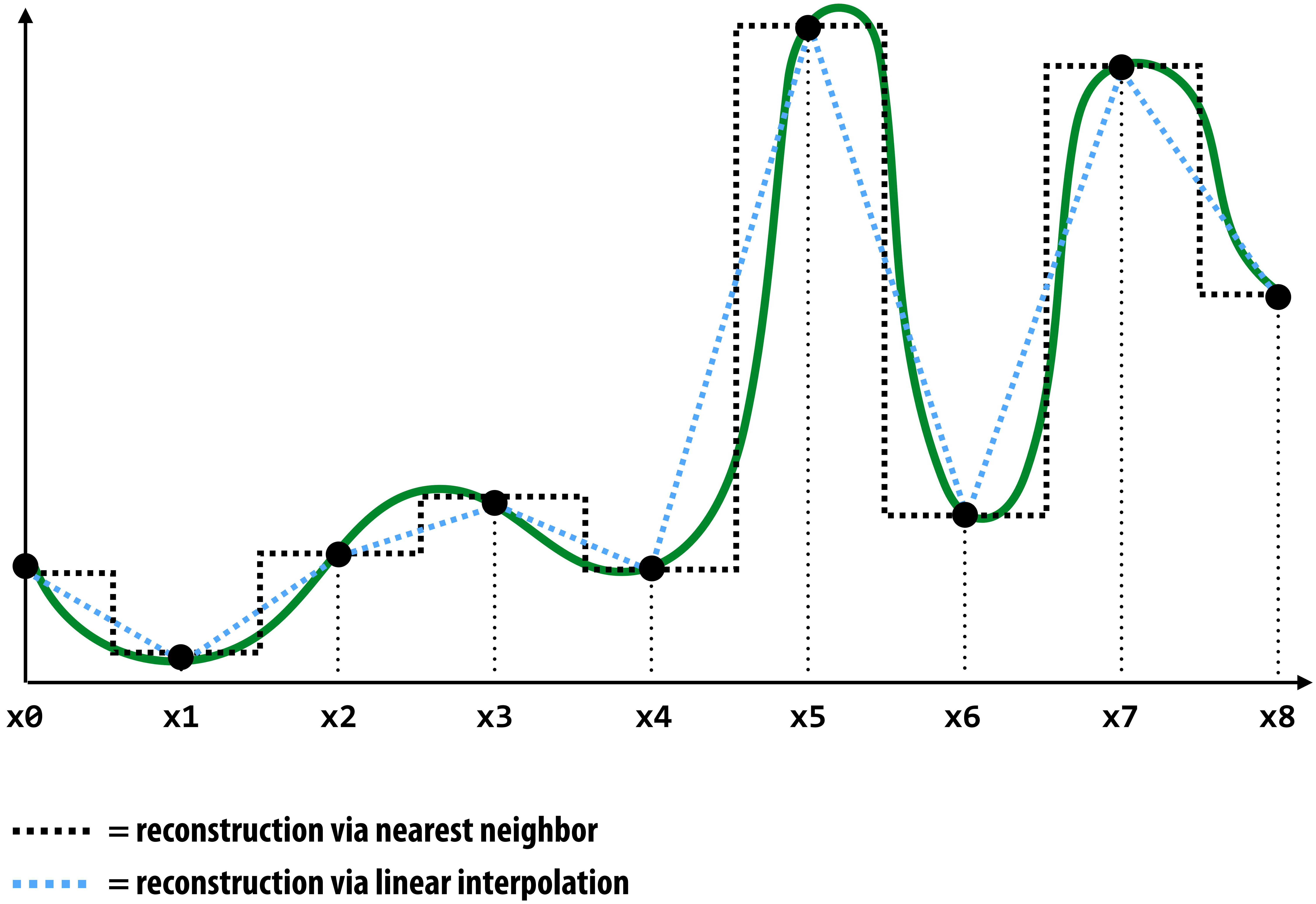
$f_{recon}(x)$ = linear interpolation between values of two closest samples to x



How can we represent the signal more accurately?



More accurate reconstructions result from denser sampling



Drawing a triangle by 2D sampling

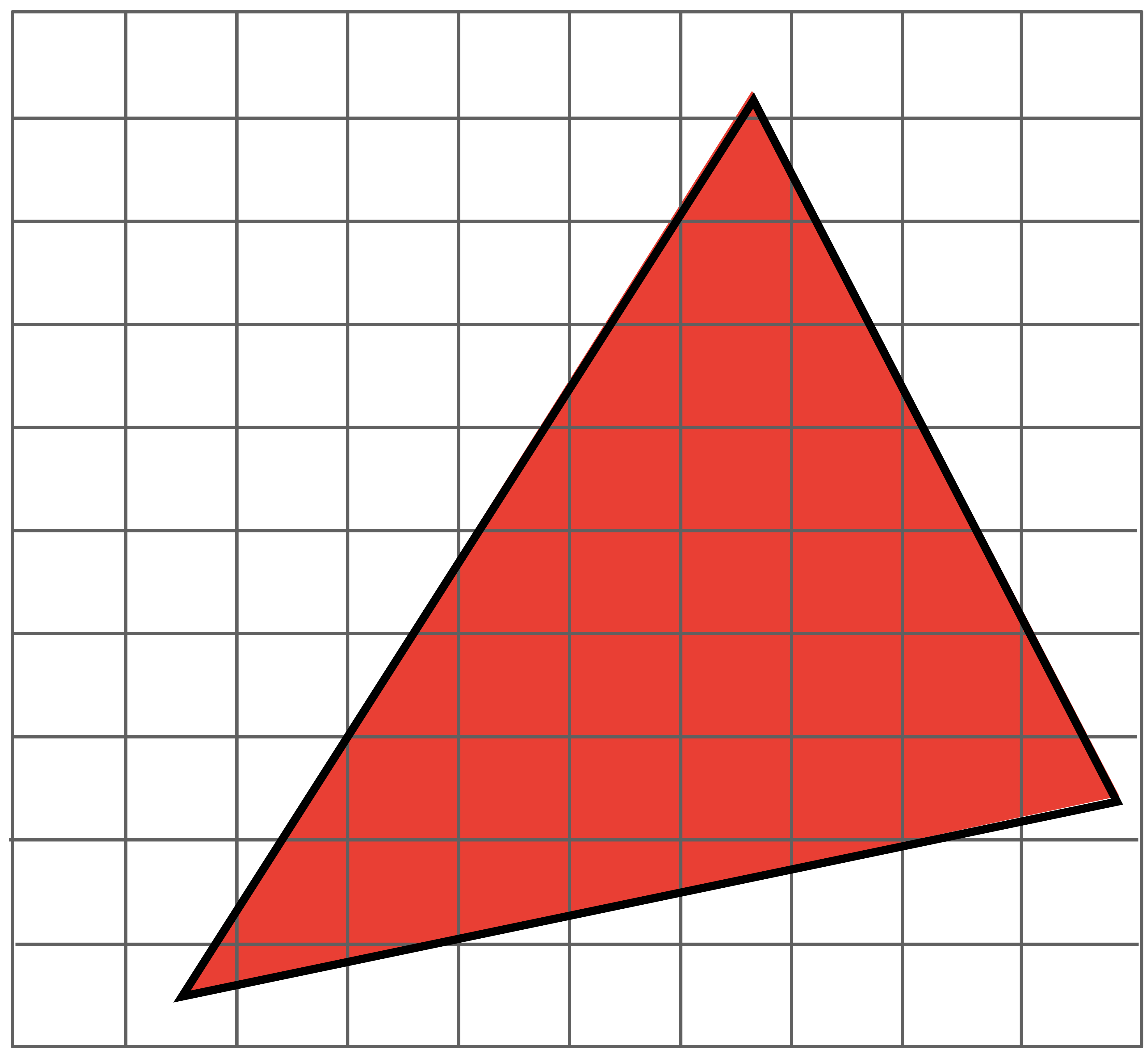
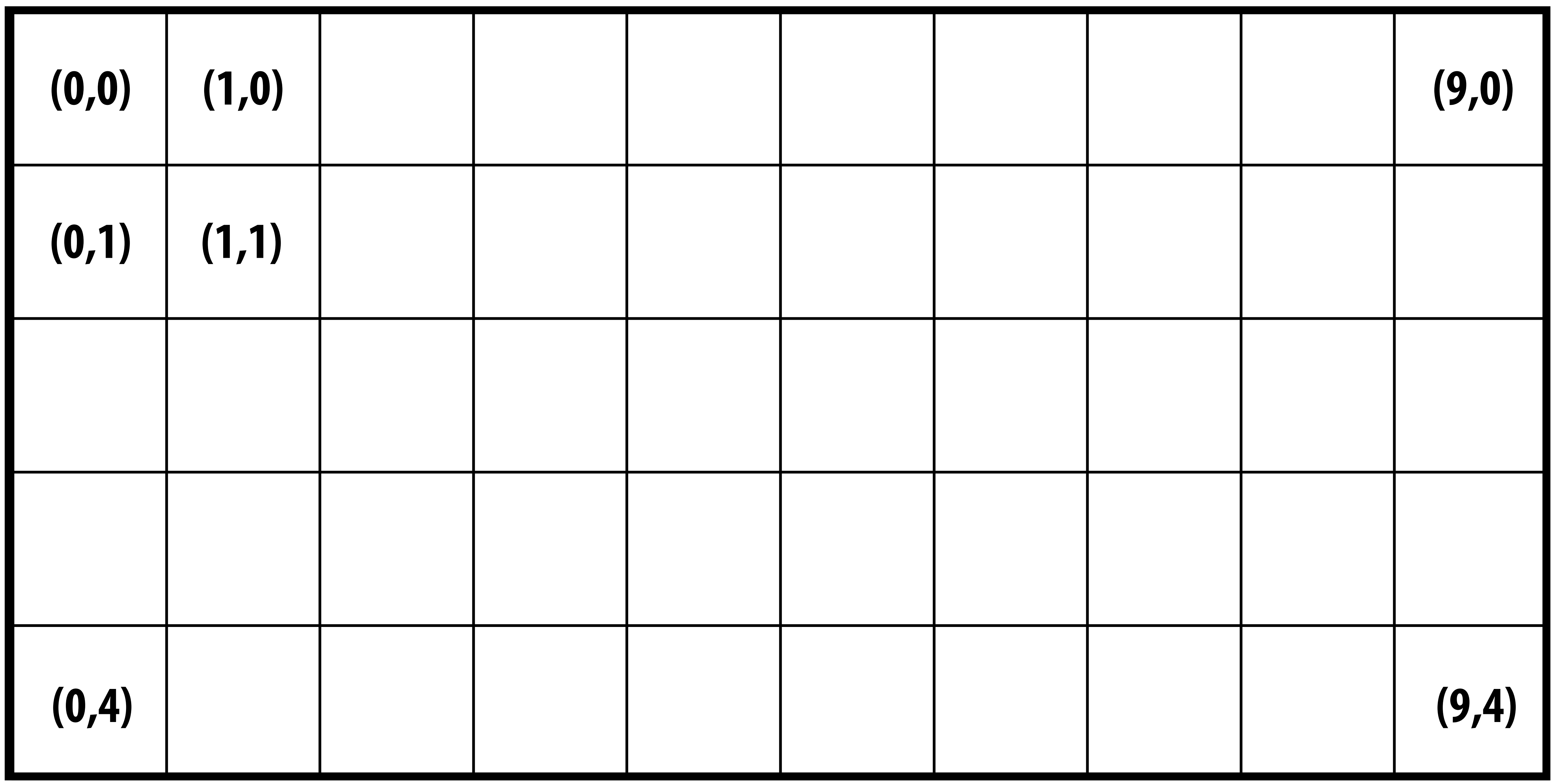


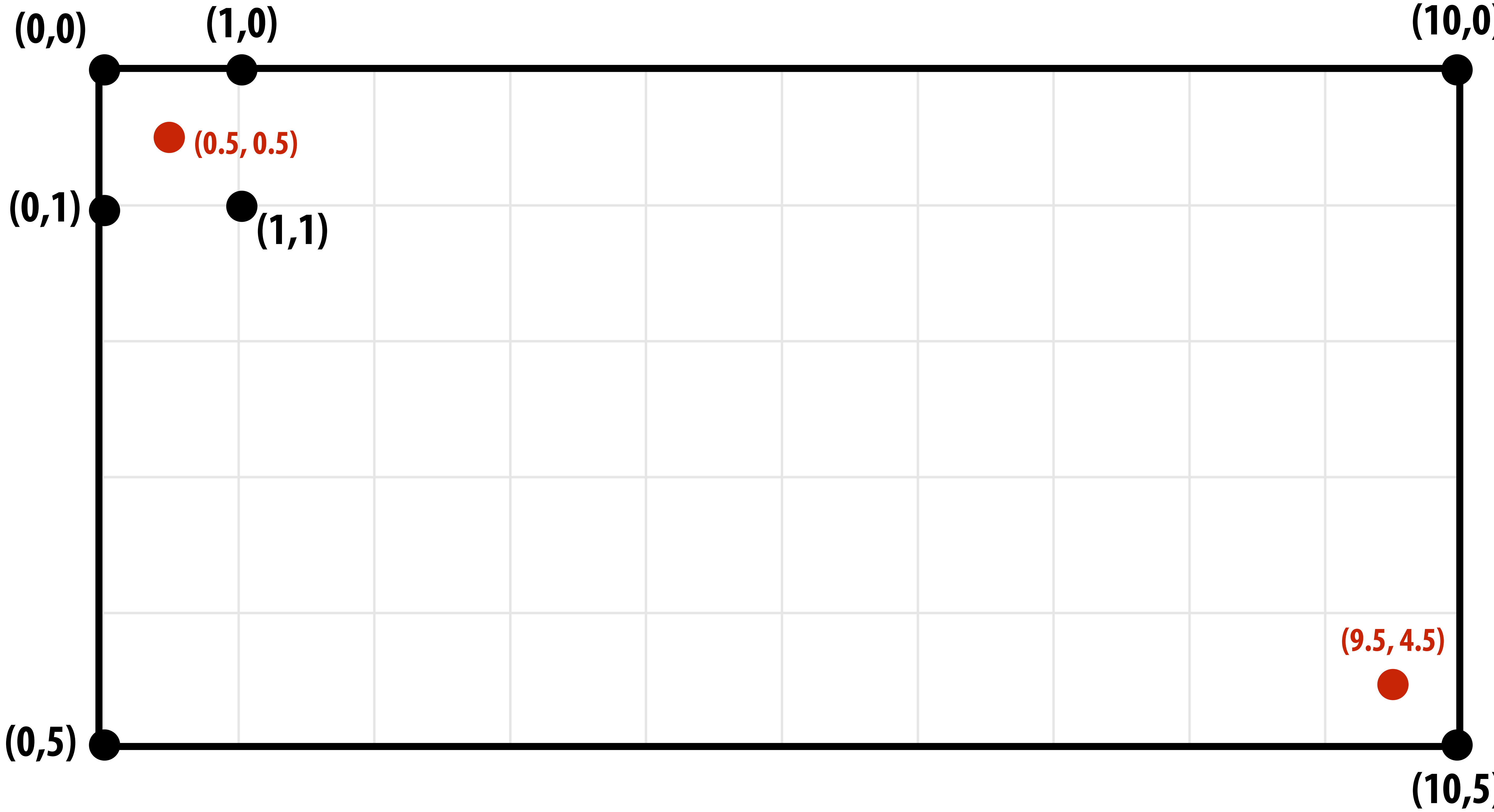
Image as a 2D matrix of pixels

Here I'm showing a 10 x 5 image

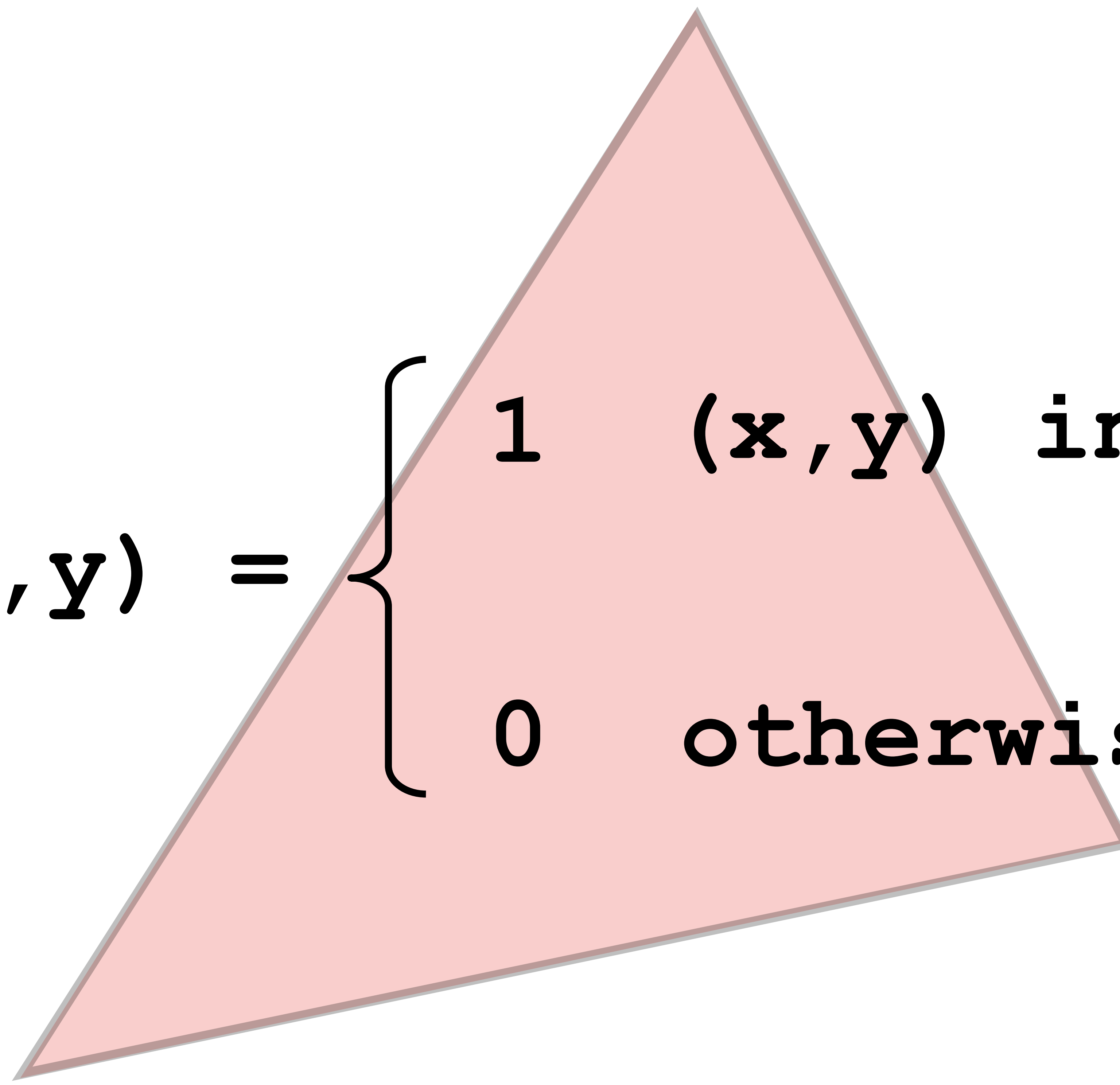
Identify pixel by its integer (x,y) coordinates



Continuous coordinate space over image

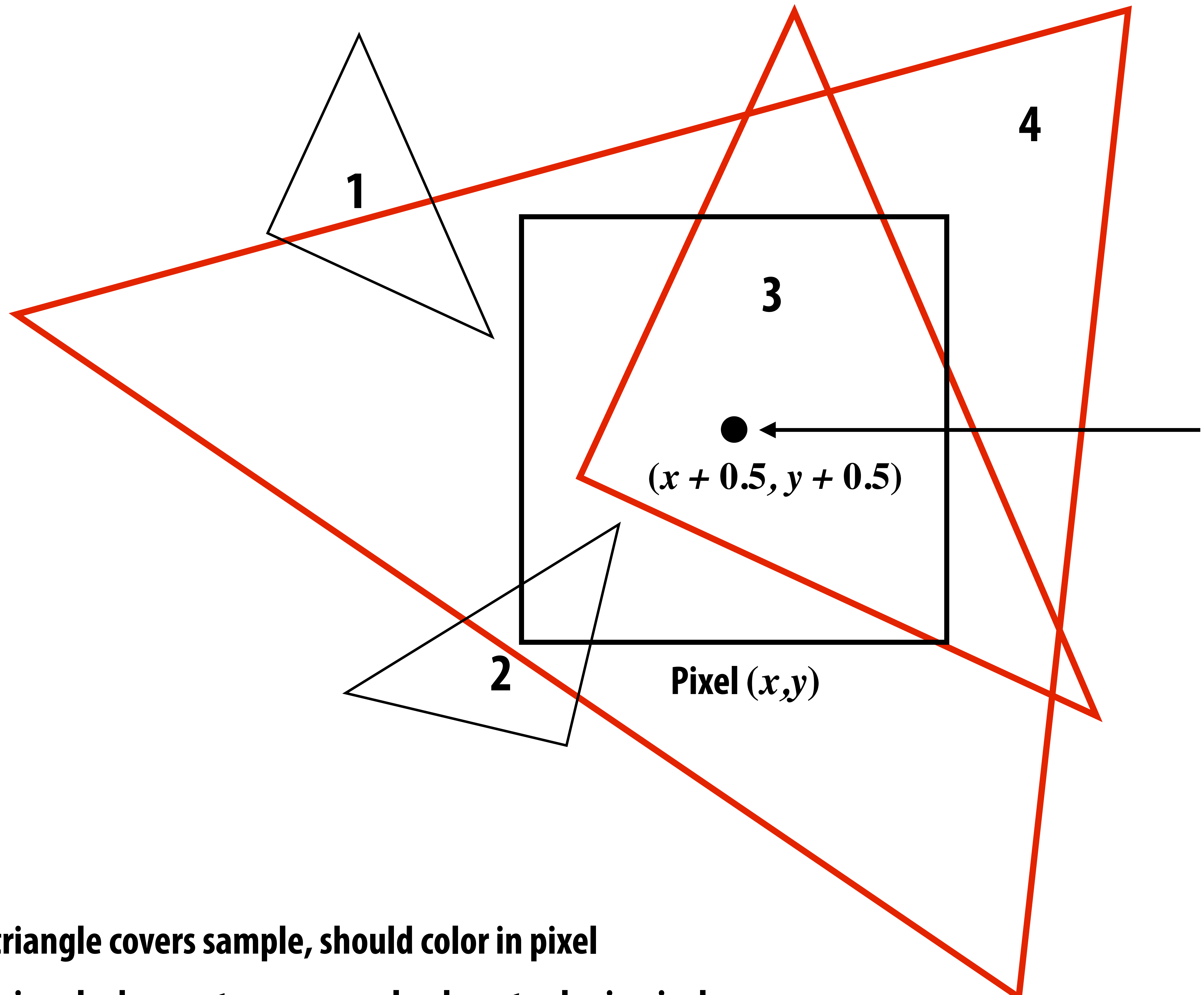


Define binary function: `inside(tri,x,y)`



$$\texttt{inside(t,x,y)} = \begin{cases} \texttt{1} & \texttt{(x,y) in triangle t} \\ \texttt{0} & \texttt{otherwise} \end{cases}$$

Sampling the binary function: `inside(tri,x,y)`

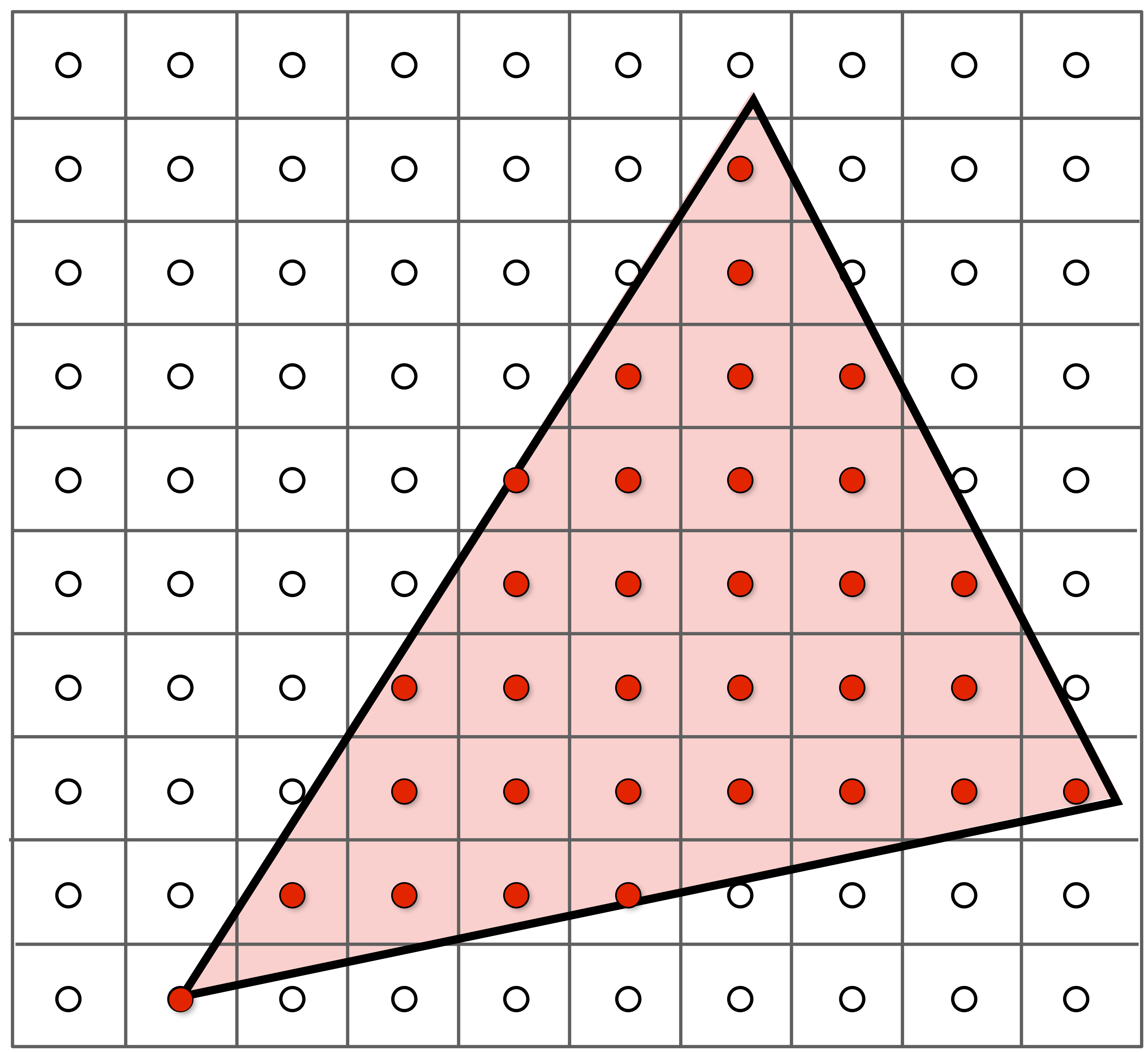


Example:
Here I chose the sample position to be at the pixel center.

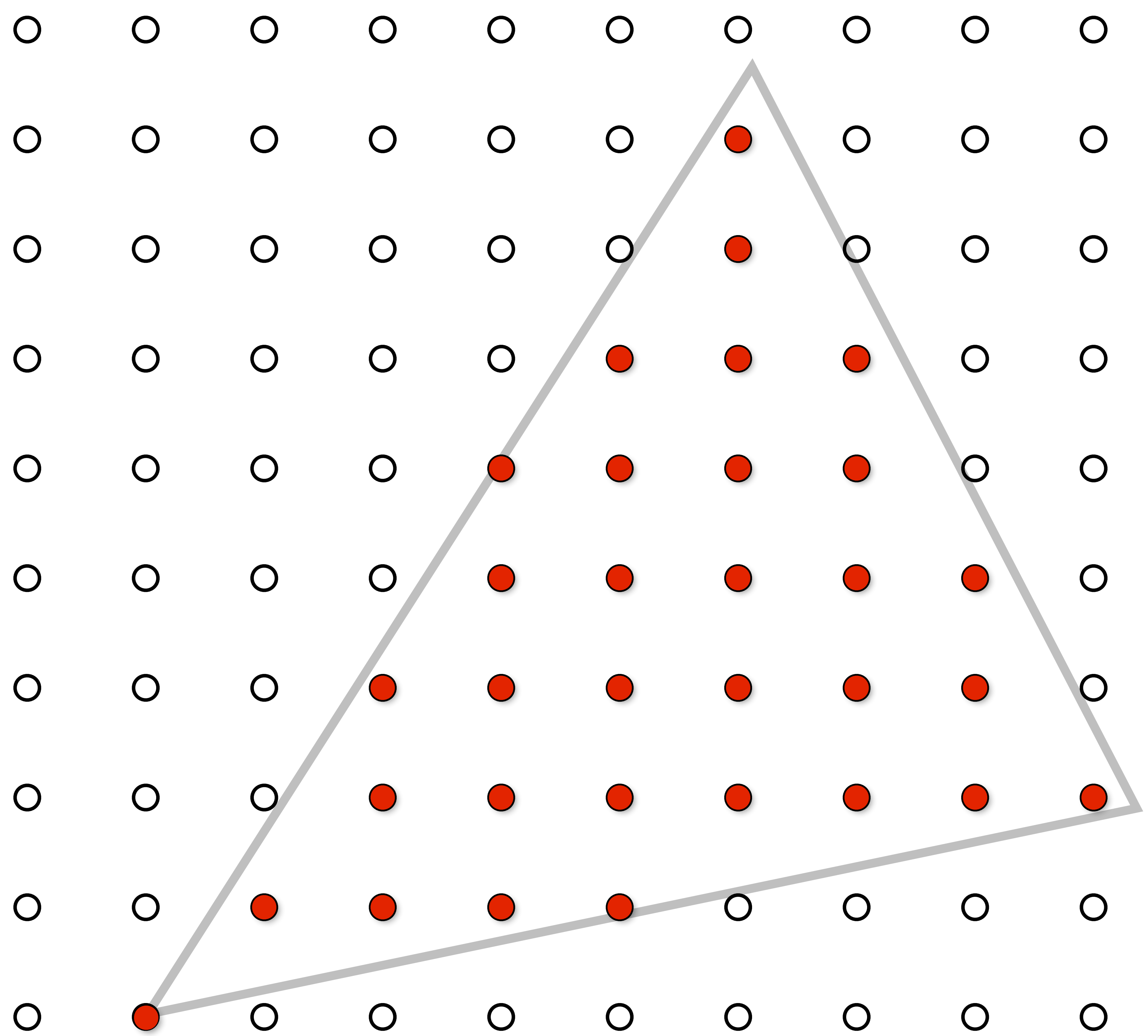
= triangle covers sample, should color in pixel

= triangle does not cover sample, do not color in pixel

Sample coverage at pixel centers



Sample coverage at pixel centers



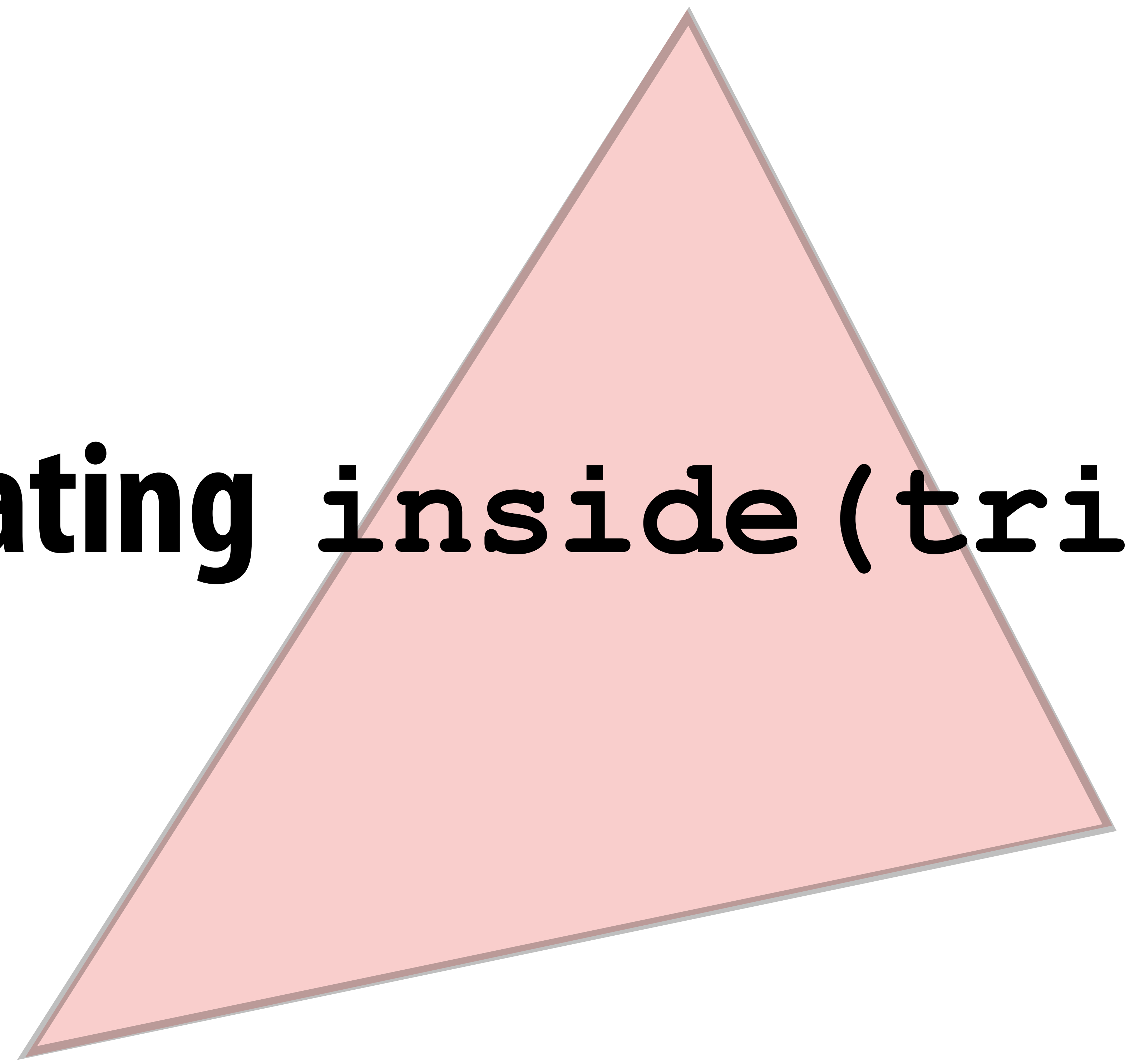
Rasterization = sampling a 2D binary function

```
for (int x = 0; x < xmax; x++)
   for (int y = 0; y < ymax; y++)
      image[x][y] = f(x + 0.5, y + 0.5);
```

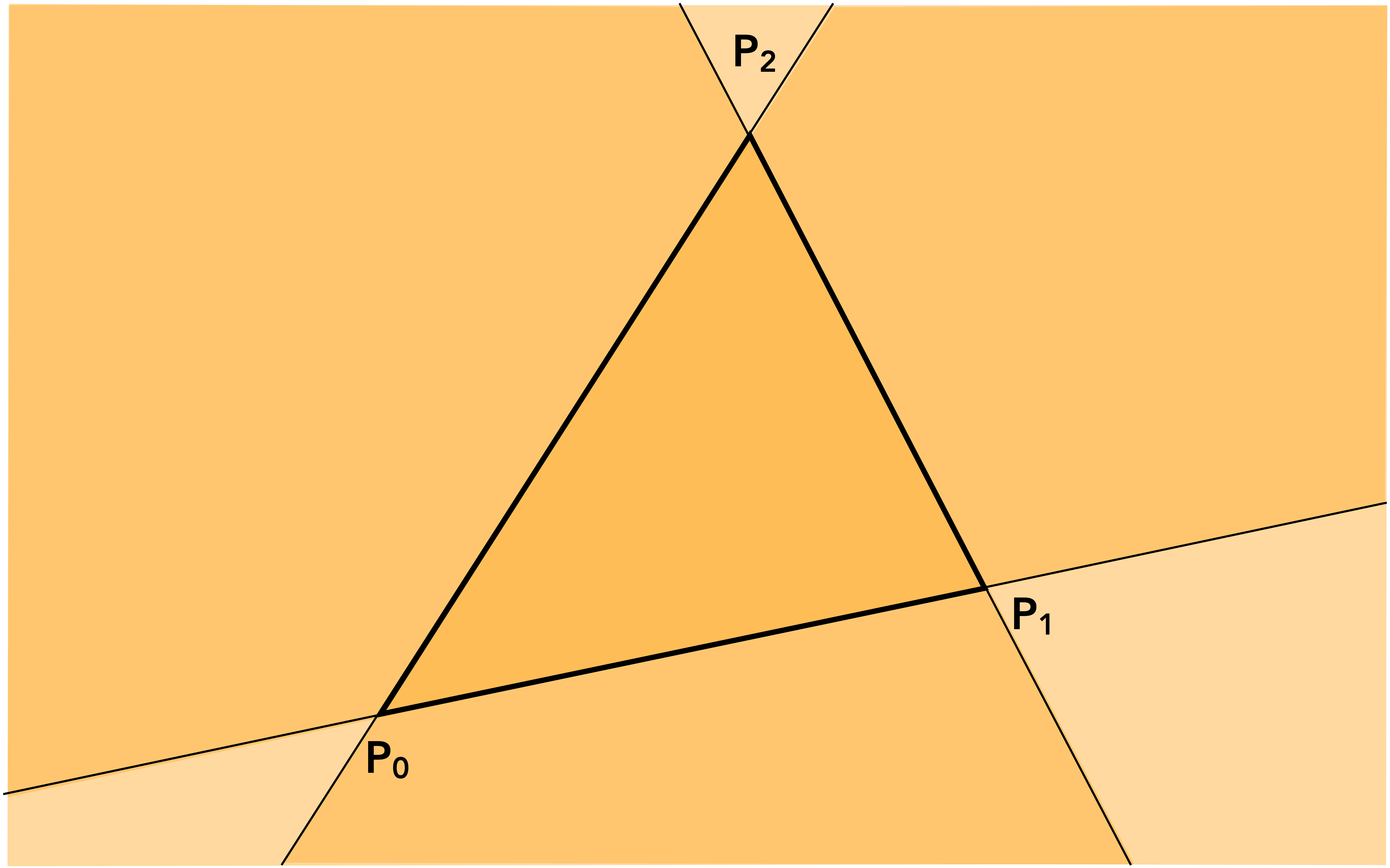
- **Rasterize triangle `tri` by sampling the function**

  `f(x,y) = inside(tri,x,y)`

Evaluating `inside(tri,x,y)`



Triangle = intersection of three half planes

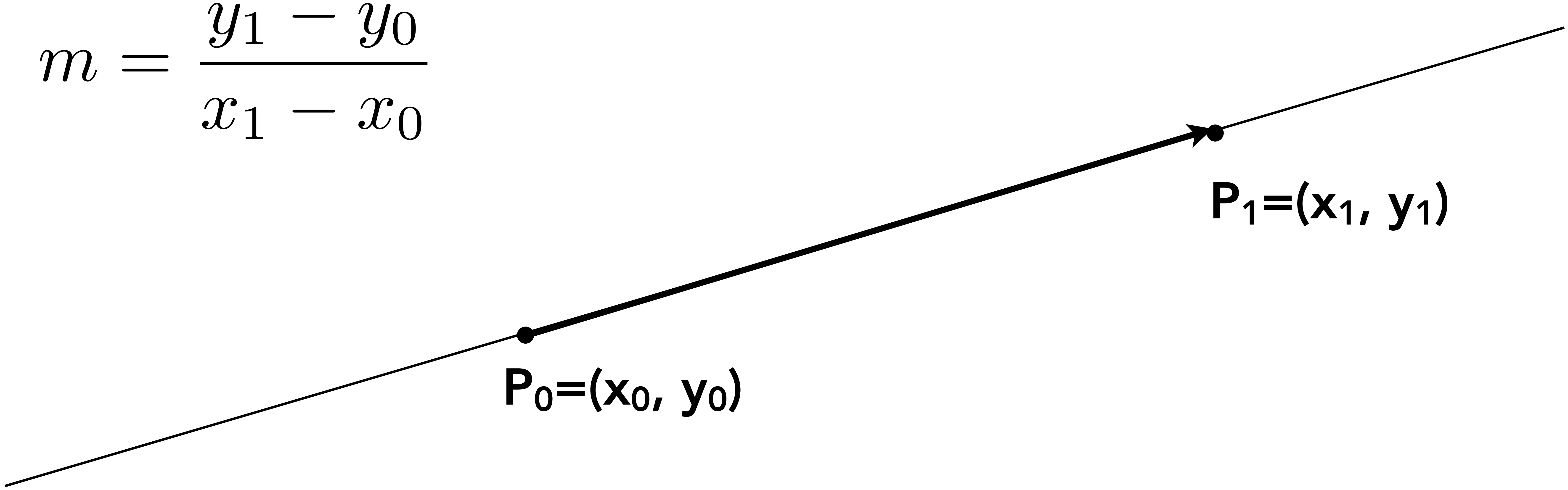


Point slope form of a line

(You might have seen this in high school)

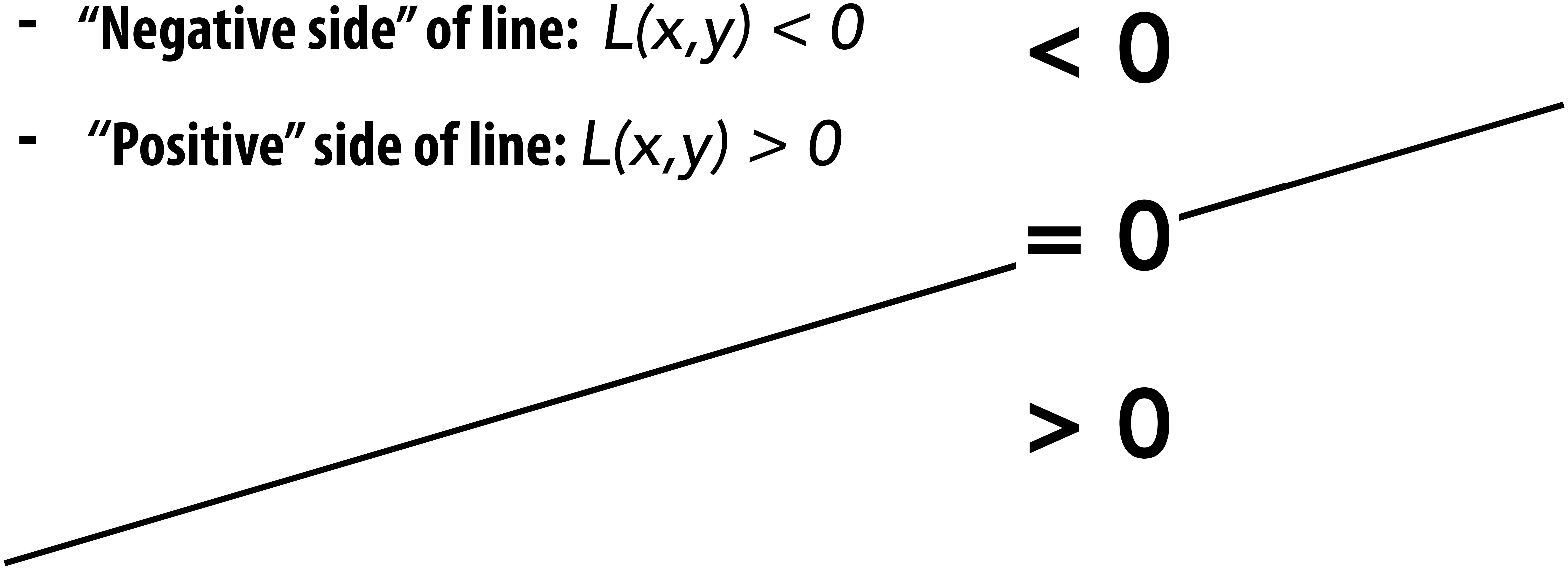
$$y - y_0 = m(x - x_0)$$

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

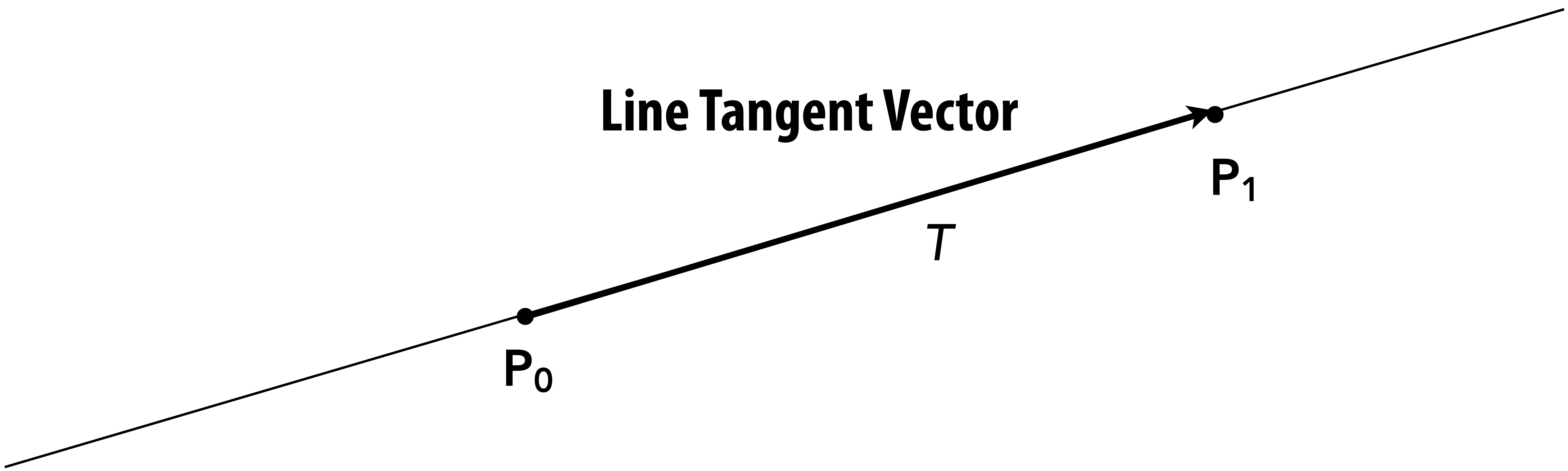


Each line defines two half-planes

- **Implicit line equation**
  - *L(x,y) = Ax + By + C*
  - **On the line:** *L(x,y) = 0*
  - **"Negative side" of line:** *L(x,y) < 0*
  - **"Positive" side of line:** *L(x,y) > 0*

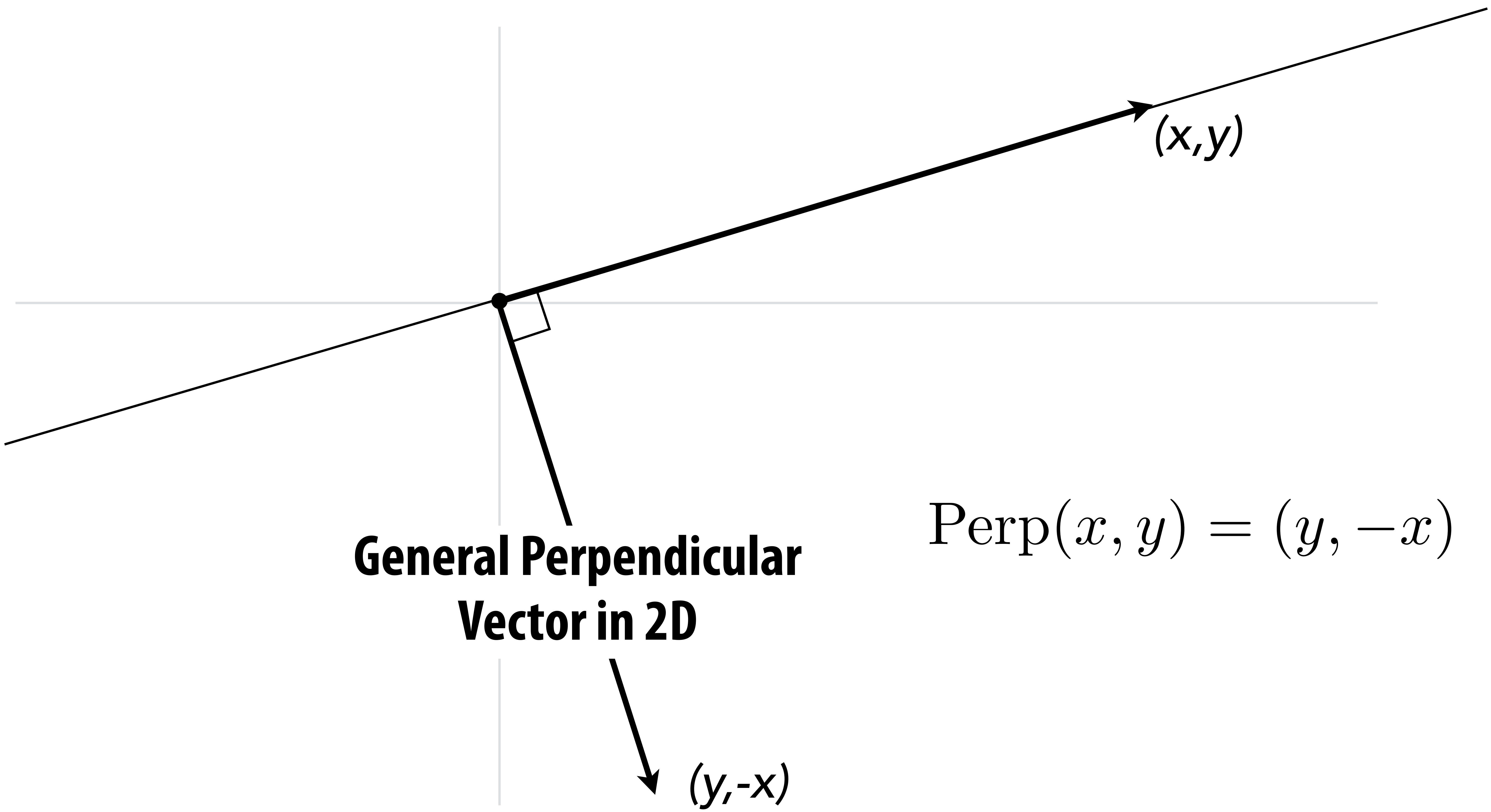


Line equation derivation



$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

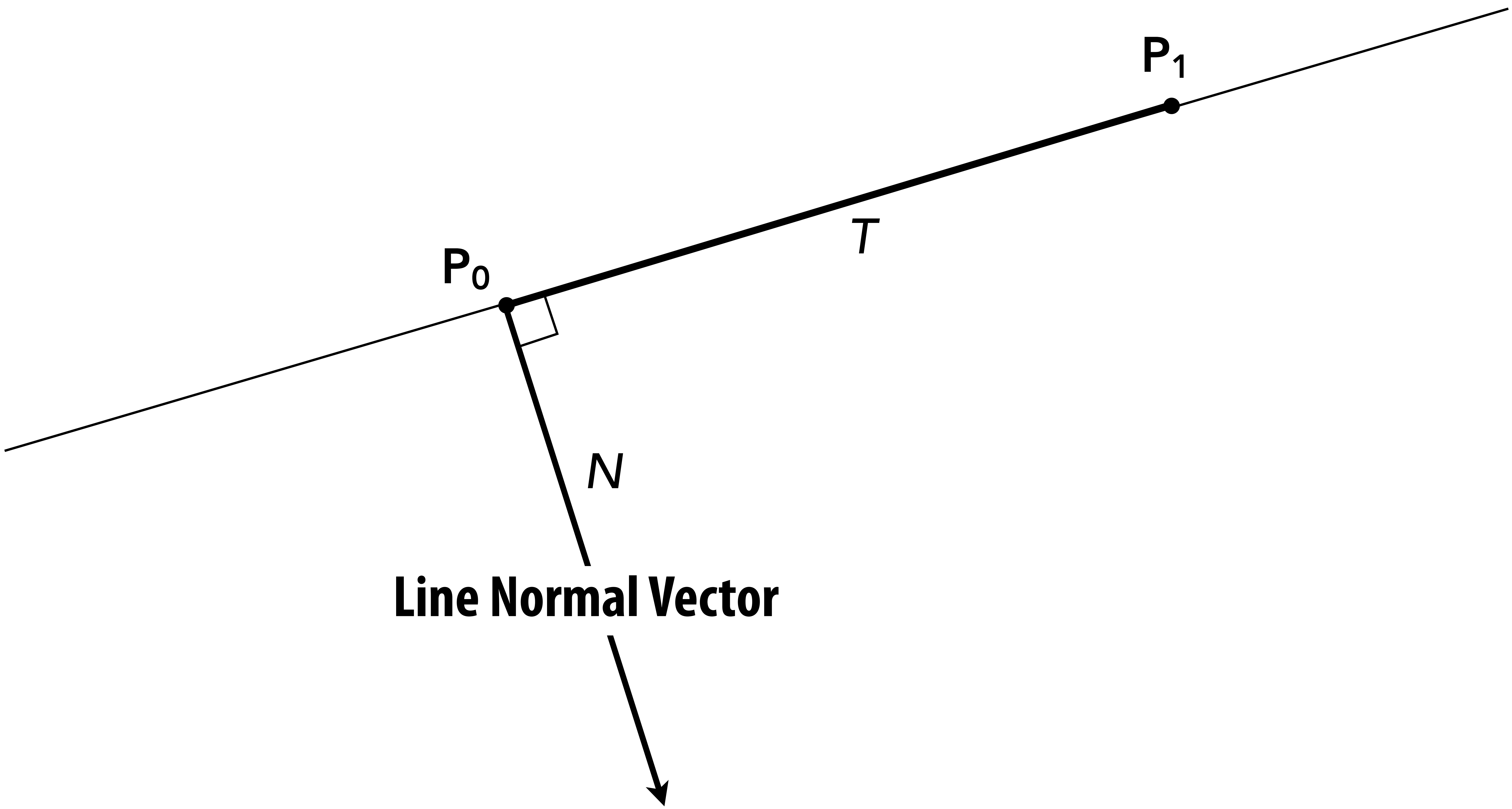
Line equation derivation



$$\mathrm{Perp}(x, y) = (y, -x)$$

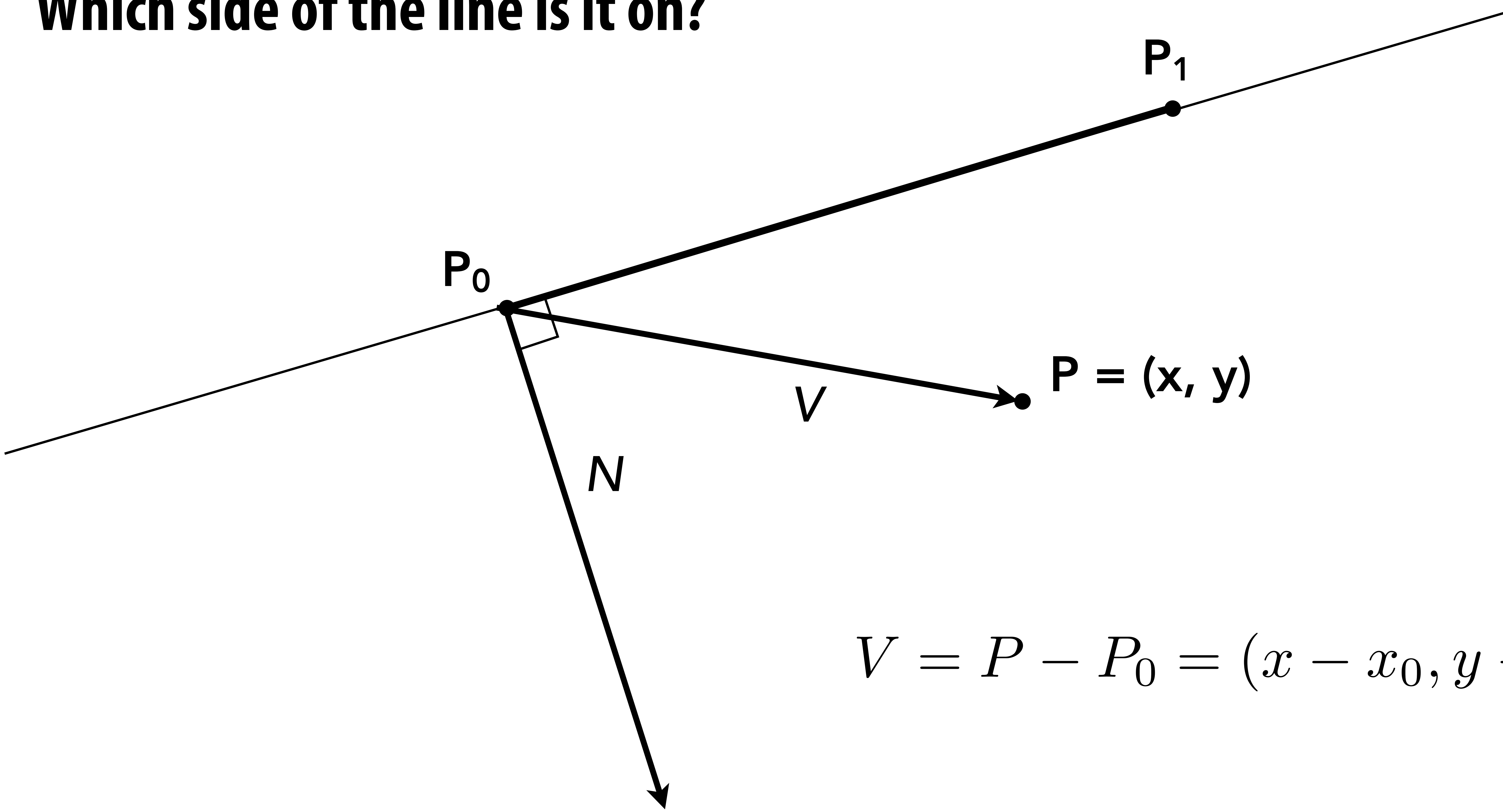
Line equation derivation

$$N = \mathrm{Perp}(T) = (y_1 - y_0, -(x_1 - x_0))$$



Line equation derivation

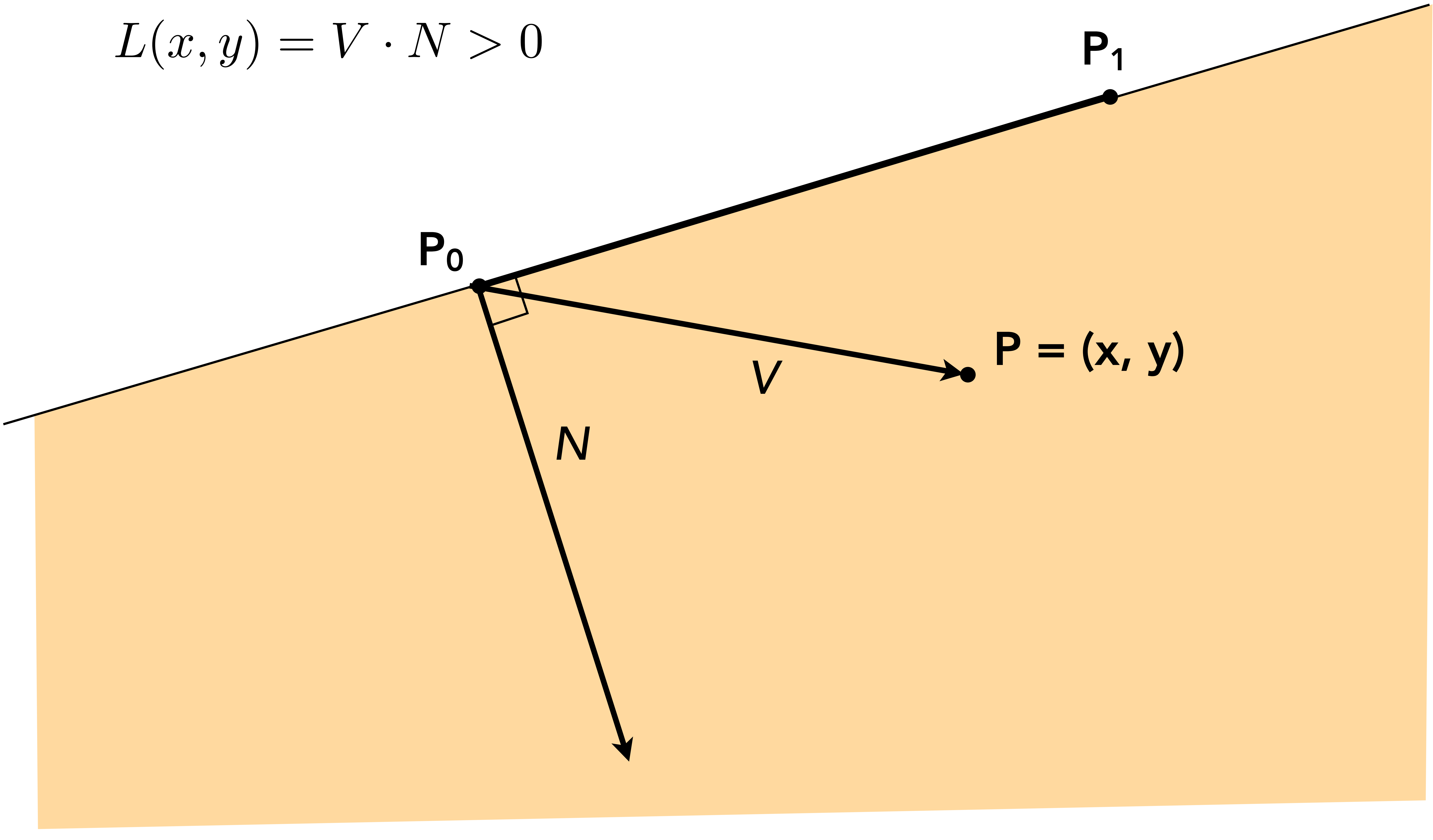
Now consider a point *P*=(x,y).
Which side of the line is it on?



$$V = P - P_0 = (x - x_0, y - y_0)$$

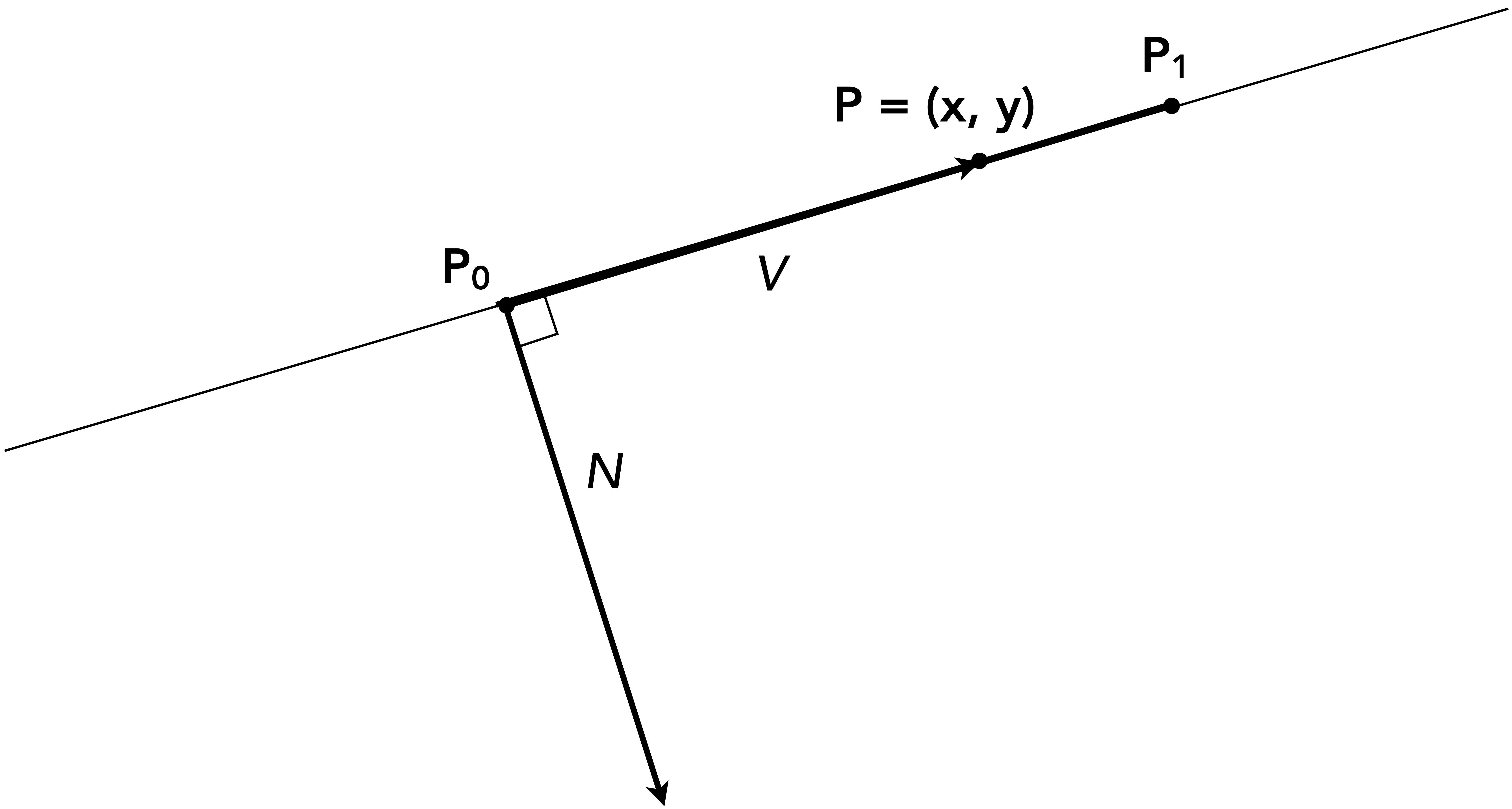
Line equation tests

$$L(x, y) = V \cdot N > 0$$

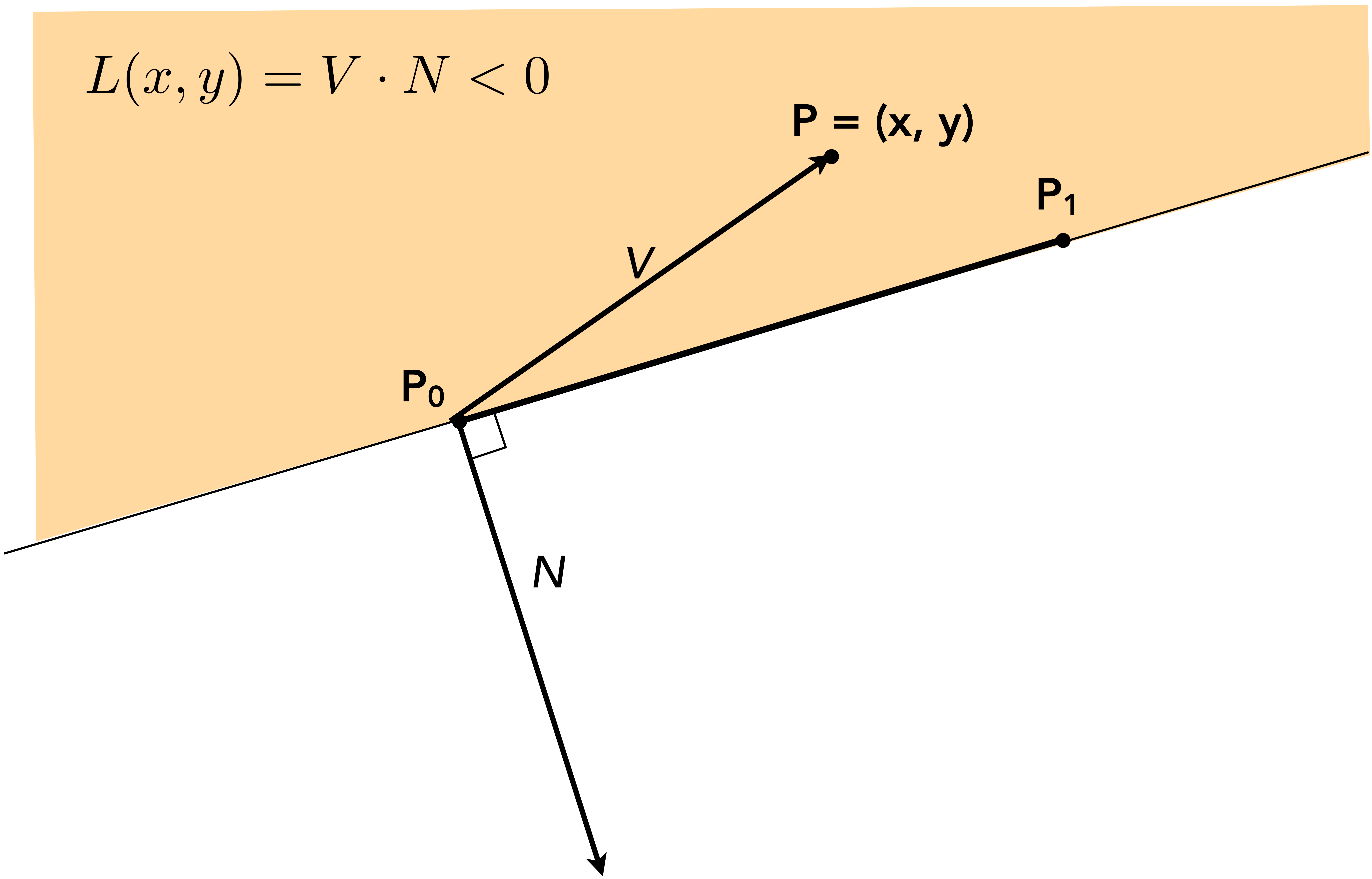


Line equation tests

$$L(x, y) = V \cdot N = 0$$

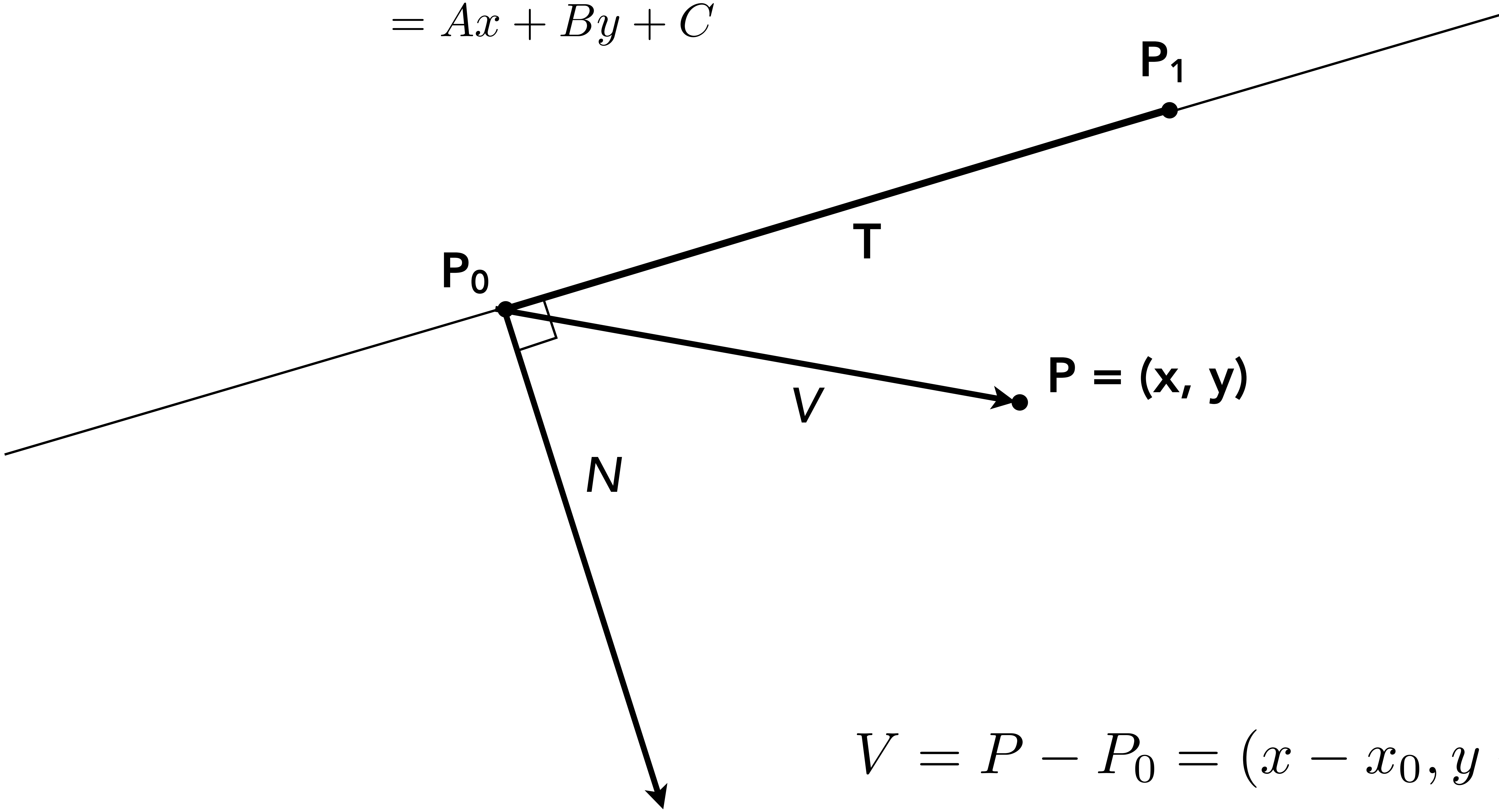


Line equation tests



Line equation derivation

$$\begin{aligned} L(x,y) = V \cdot N &= -(y - y_0)(x_1 - x_0) + (x - x_0)(y_1 - y_0) \\ &= (y_1 - y_0)x - (x_1 - x_0)y + y_0(x_1 - x_0) - x_0(y_1 - y_0) \\ &= Ax + By + C \end{aligned}$$



$$V = P - P_0 = (x - x_0, y - y_0)$$

$$N = \mathrm{Perp}(T) = (y_1 - y_0, -(x_1 - x_0))$$

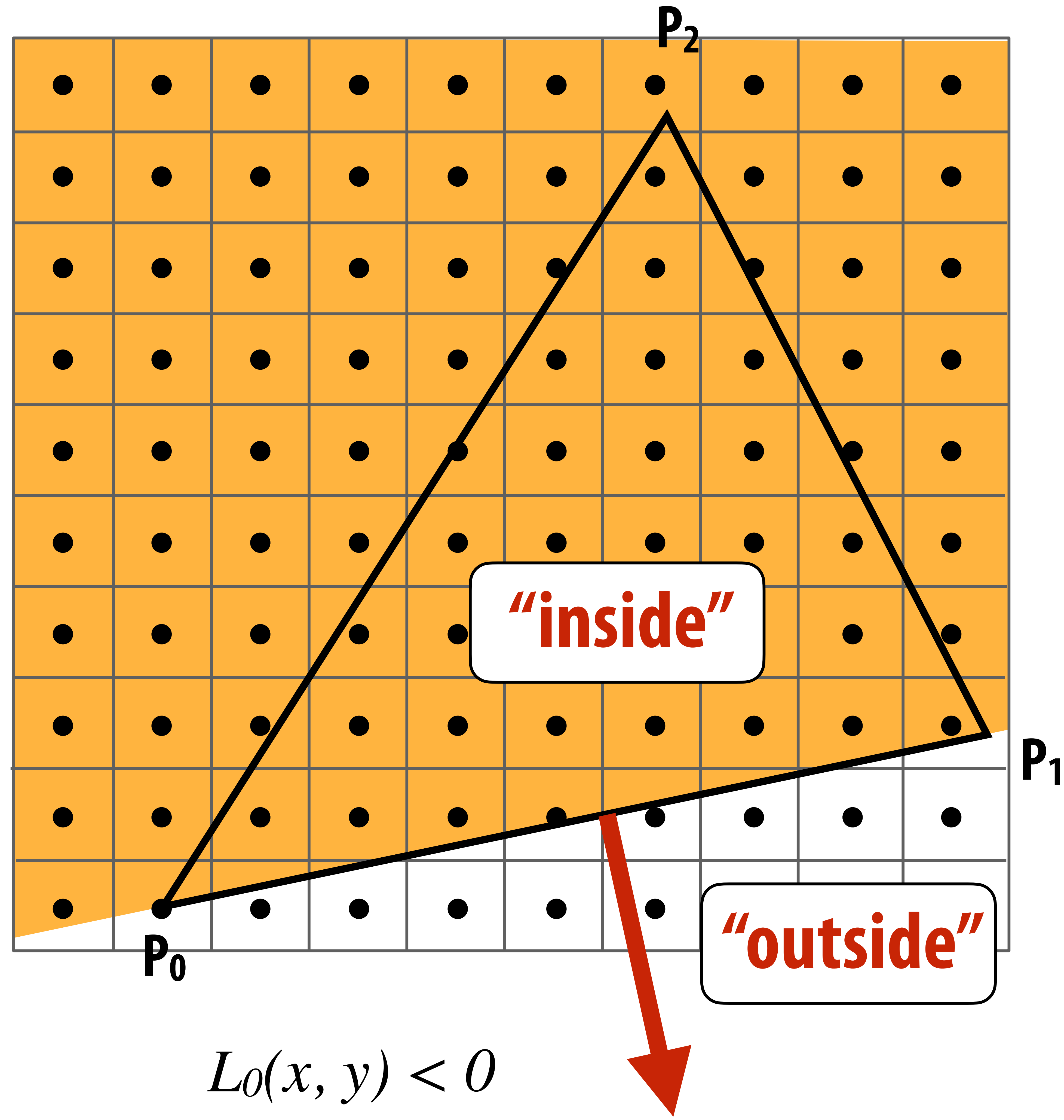
Point-in-triangle test

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = dX_i = X_{i+1} - X_i$
$C_i = Y_i (X_{i+1} - X_i) - X_i (Y_{i+1} - Y_i)$

$L_i(x, y) = dY_i x - dX_i y + C_i$

$L_i(x, y) = 0$: point on edge
> 0 : outside edge
< 0 : inside edge



$L_0(x, y) < 0$

Point-in-triangle test

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = dX_i = X_{i+1} - X_i$
$C_i = Y_i\,(X_{i+1} - X_i) - X_i\,(Y_{i+1} - Y_i)$

$L_i\,(x, y) = dY_i\,x - dX_i\,y + C_i$

$L_i\,(x, y) = 0$: point on edge
> 0 : outside edge
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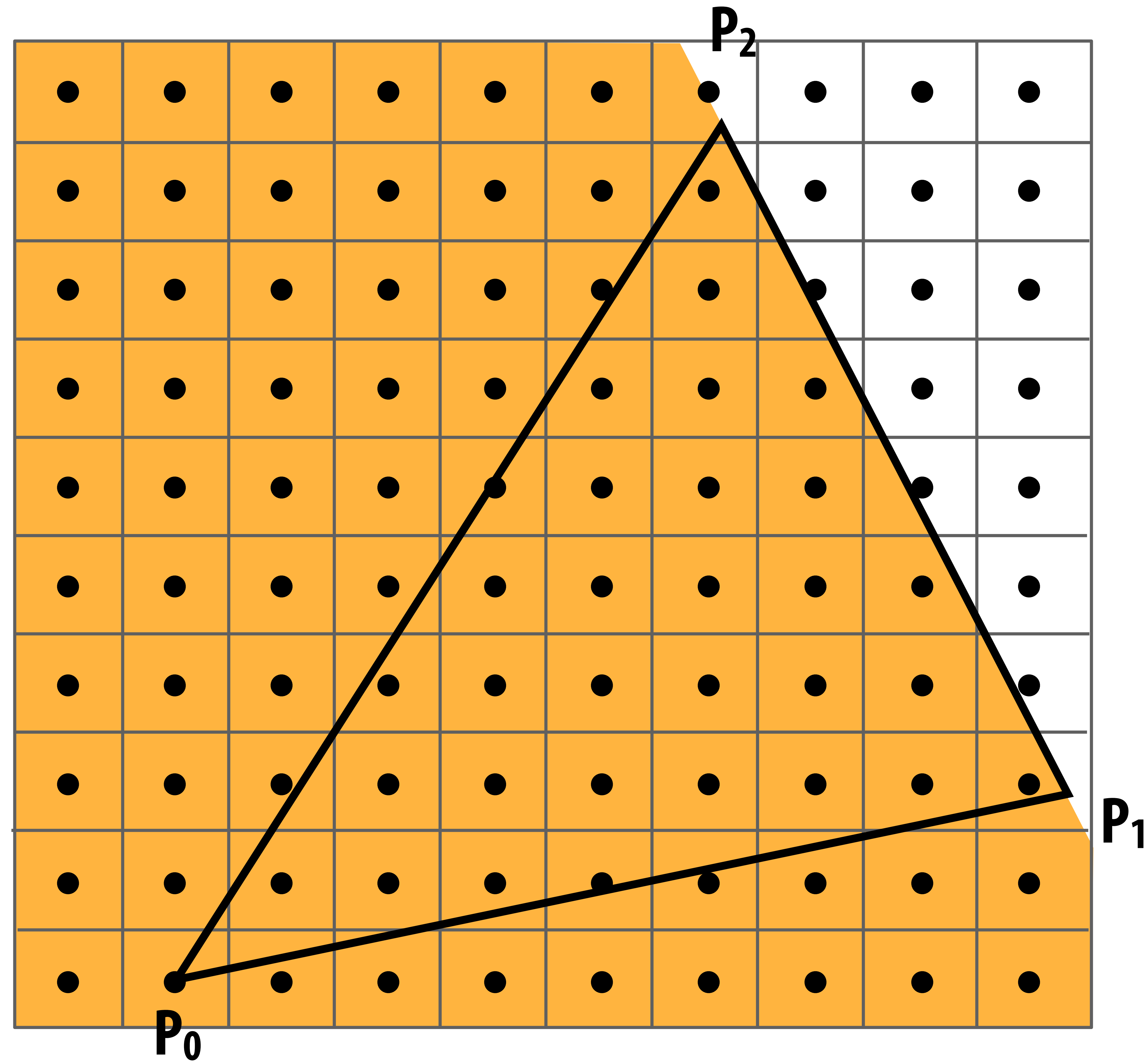


$L_1(x, y) < 0$

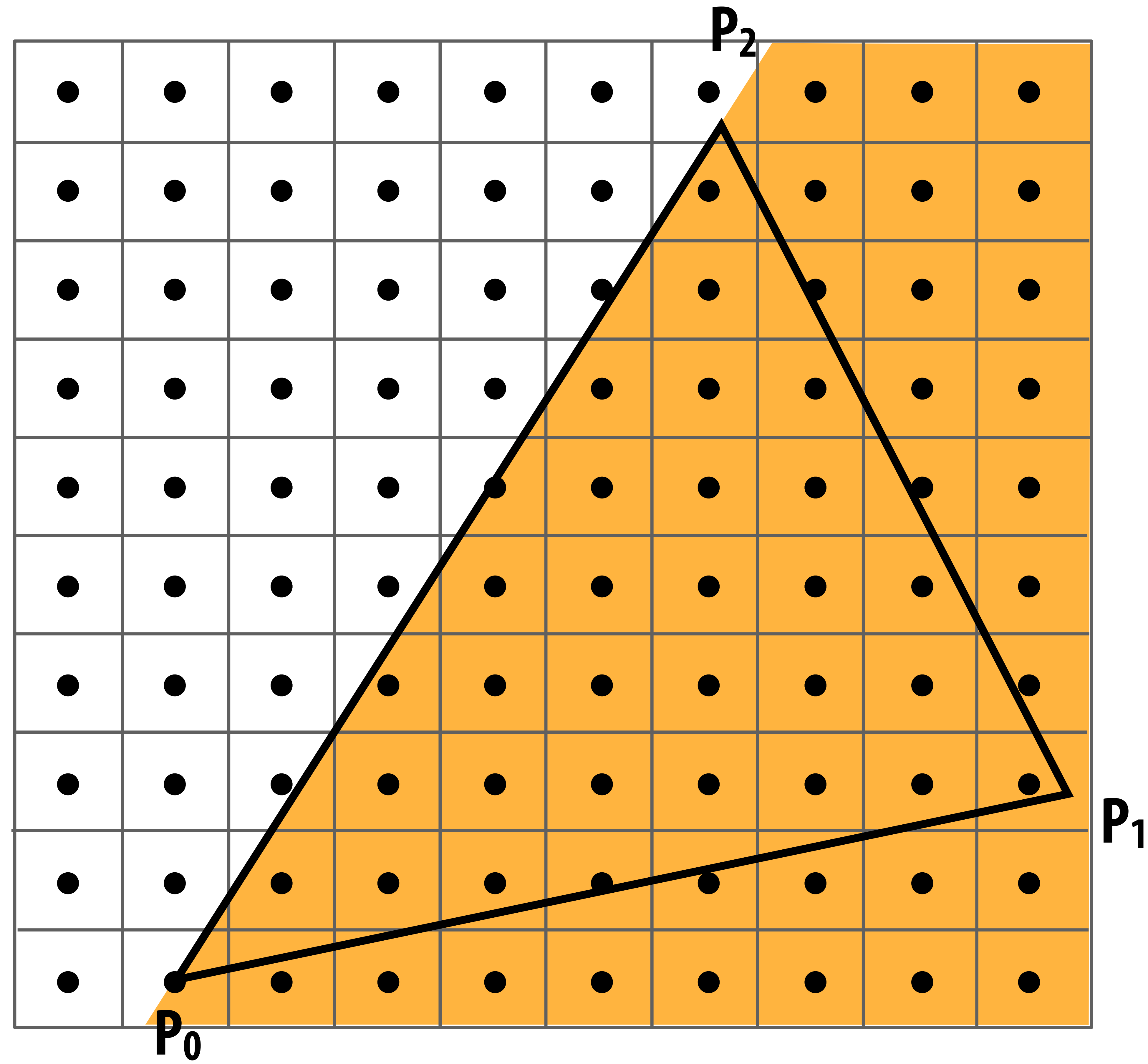
Point-in-triangle test

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = dX_i = X_{i+1} - X_i$
$C_i = Y_i (X_{i+1} - X_i) - X_i (Y_{i+1} - Y_i)$

$L_i(x, y) = dY_i\, x - dX_i\, y + C_i$

$L_i(x, y) = 0$: point on edge
> 0 : outside edge
< 0 : inside edge



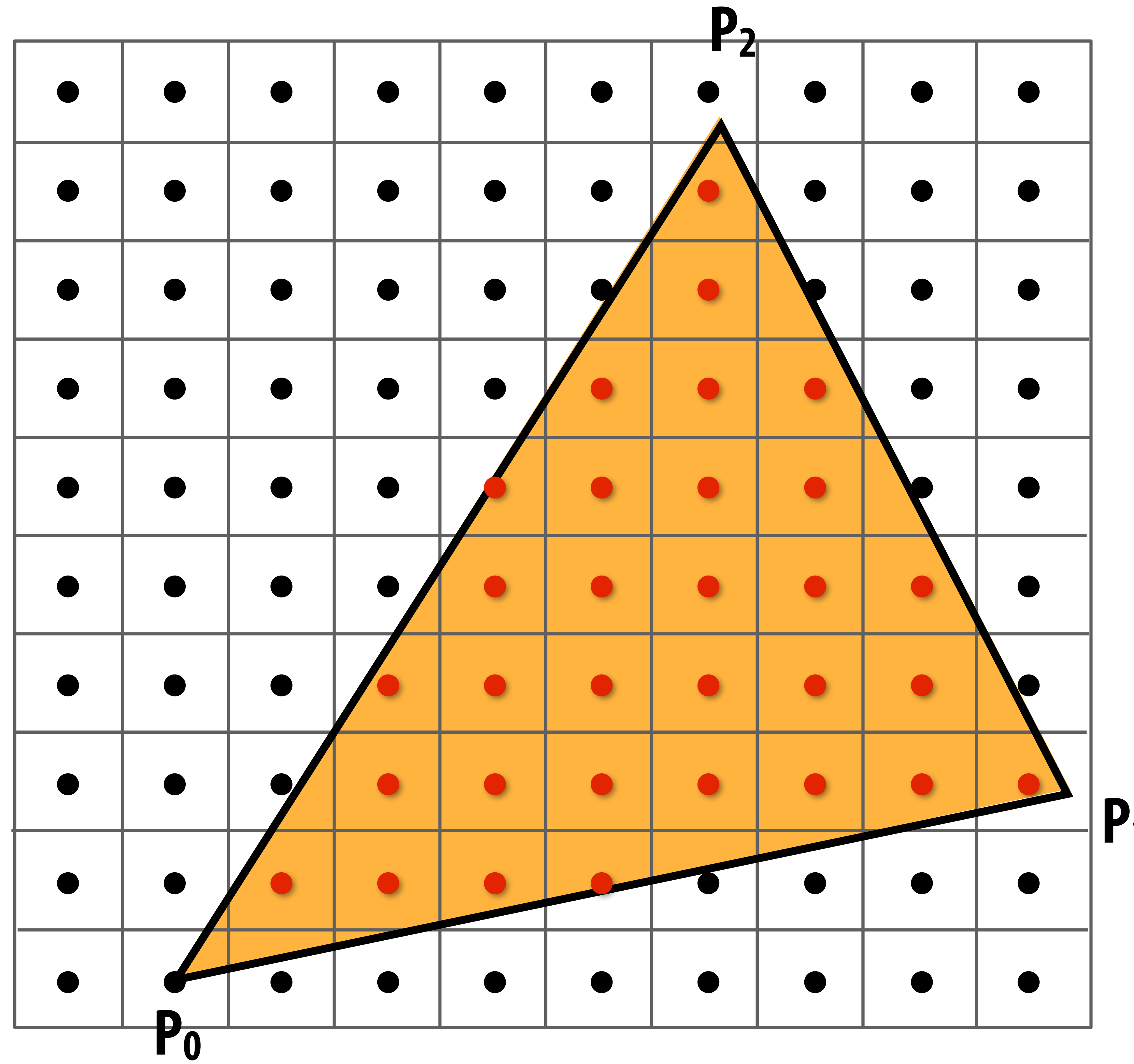
$L_2(x, y) < 0$

Point-in-triangle test

Sample point $s = (sx, sy)$ **is inside the triangle if it is inside all three edges.**

$$
\begin{aligned}
&inside(sx, sy) = \\
&\quad L_0(sx, sy) < 0 \;\&\& \\
&\quad L_1(sx, sy) < 0 \;\&\& \\
&\quad L_2(sx, sy) < 0;
\end{aligned}
$$

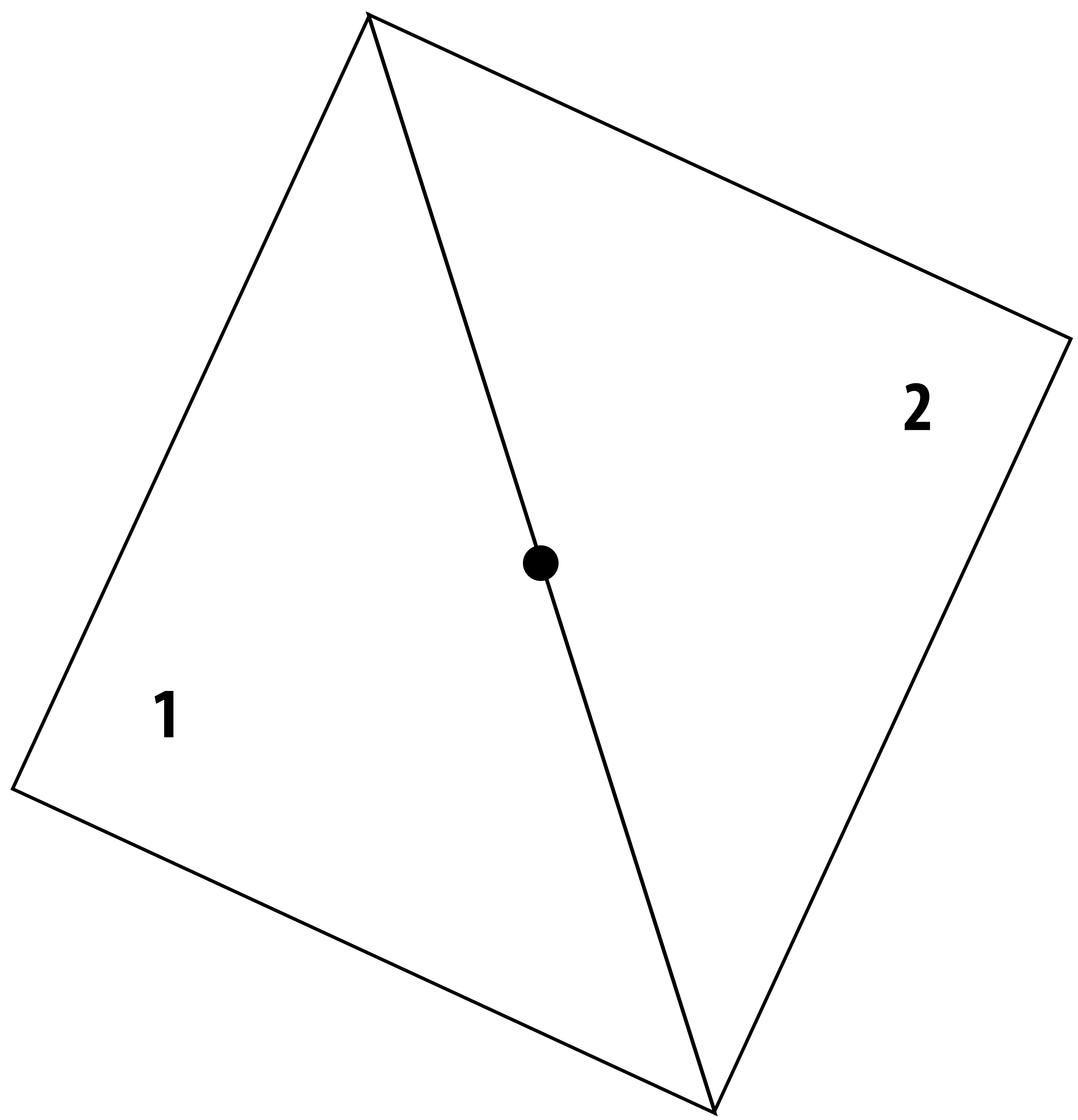
Note: actual implementation of $inside(sx,sy)$ **involves ≤ checks based on the triangle coverage edge rules (see next slides)**



Sample points inside triangle are highlighted red.

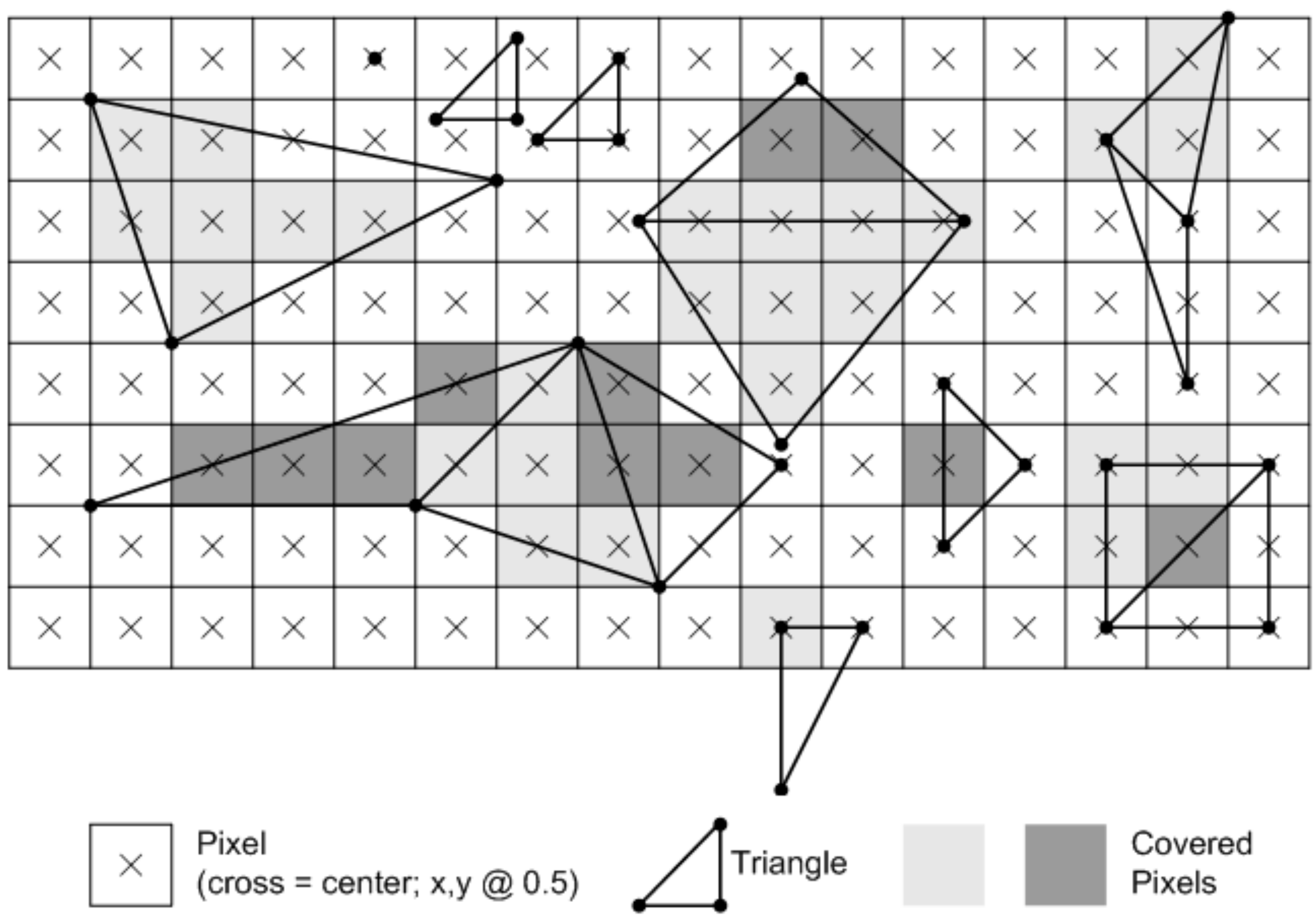
Edge cases (literally)

Is this sample point covered by triangle 1? or triangle 2? or both?



OpenGL/Direct3D edge rules

- When edge falls directly on a screen sample point, the sample is classified as within triangle if the edge is a "top edge" or "left edge"
  - Top edge: horizontal edge that is above all other edges
  - Left edge: an edge that is not exactly horizontal and is on the left side of the triangle. (triangle can have one or two left edges)



Source: Direct3D Programming Guide, Microsoft

Finding covered samples: incremental triangle traversal

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$

$B_i = dX_i = X_{i+1} - X_i$

$C_i = Y_i (X_{i+1} - X_i) - X_i (Y_{i+1} - Y_i)$

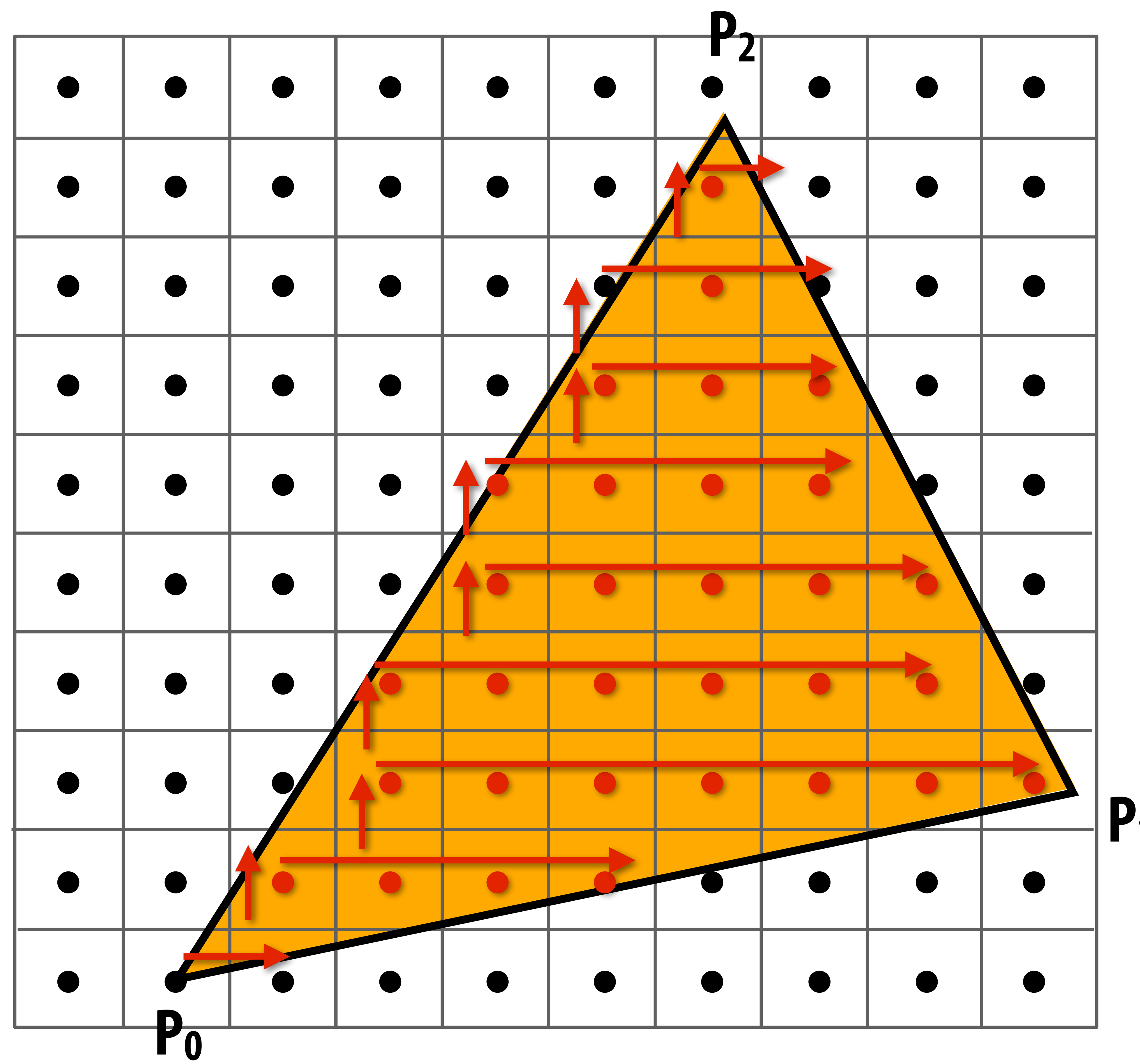
$L_i (x, y) = dY_i\, x - dX_i\, y + C_i$

$$L_i (x, y) = \begin{cases} 0 & : \text{point on edge} \\ > 0 & : \text{outside edge} \\ < 0 & : \text{inside edge} \end{cases}$$

Efficient incremental update:

$L_i (x+1,y) = L_i (x,y) + dY_i = L_i (x,y) + A_i$

$L_i (x,y+1) = L_i (x,y) - dX_i = L_i (x,y) + B_i$



Incremental update saves computation:
Only one addition per edge, per sample test

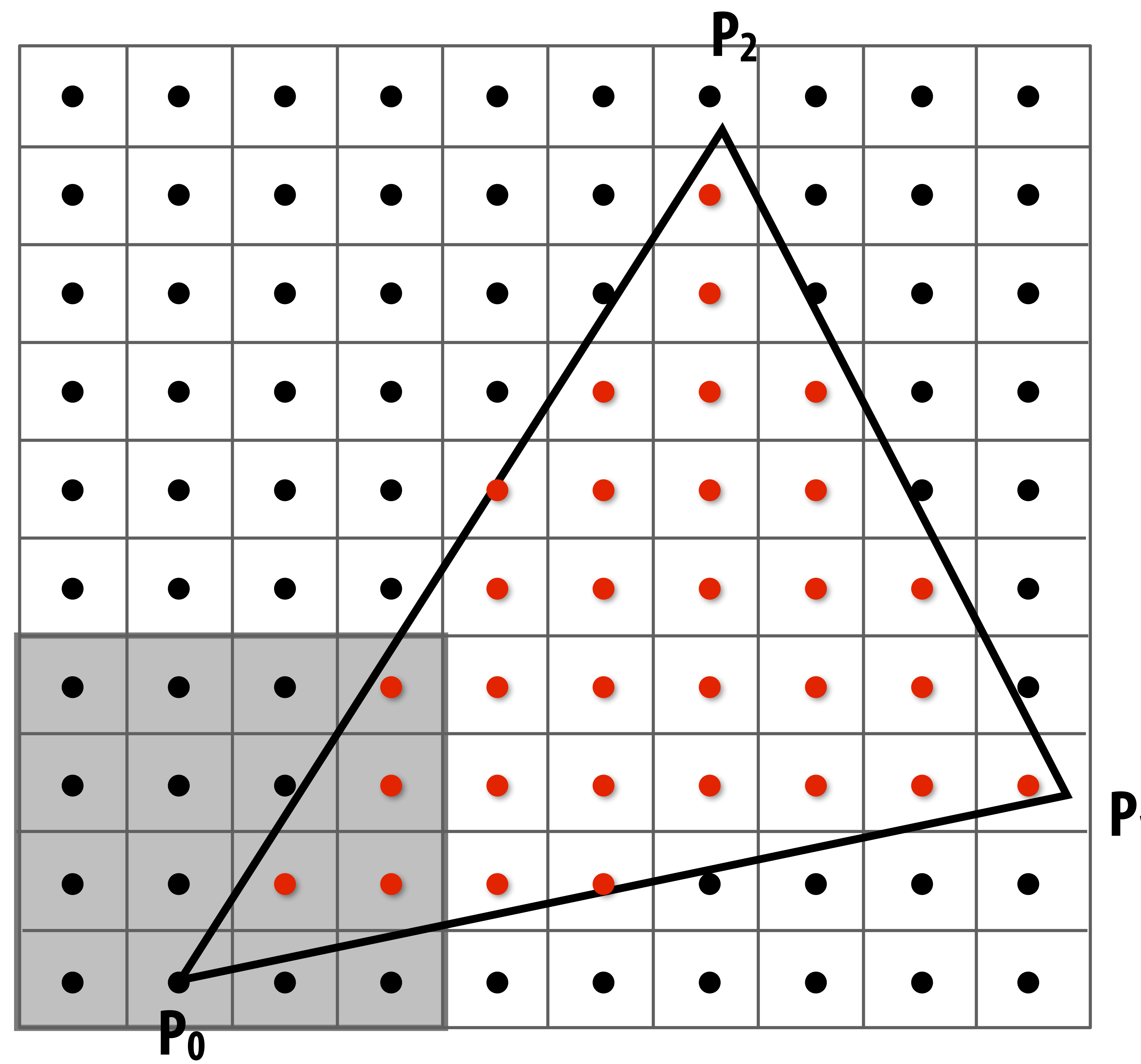
Many traversal orders are possible: backtrack, zig-zag, Hilbert/Morton curves

Modern approach: tiled triangle traversal

Traverse triangle in blocks

Test all samples in block against triangle in parallel

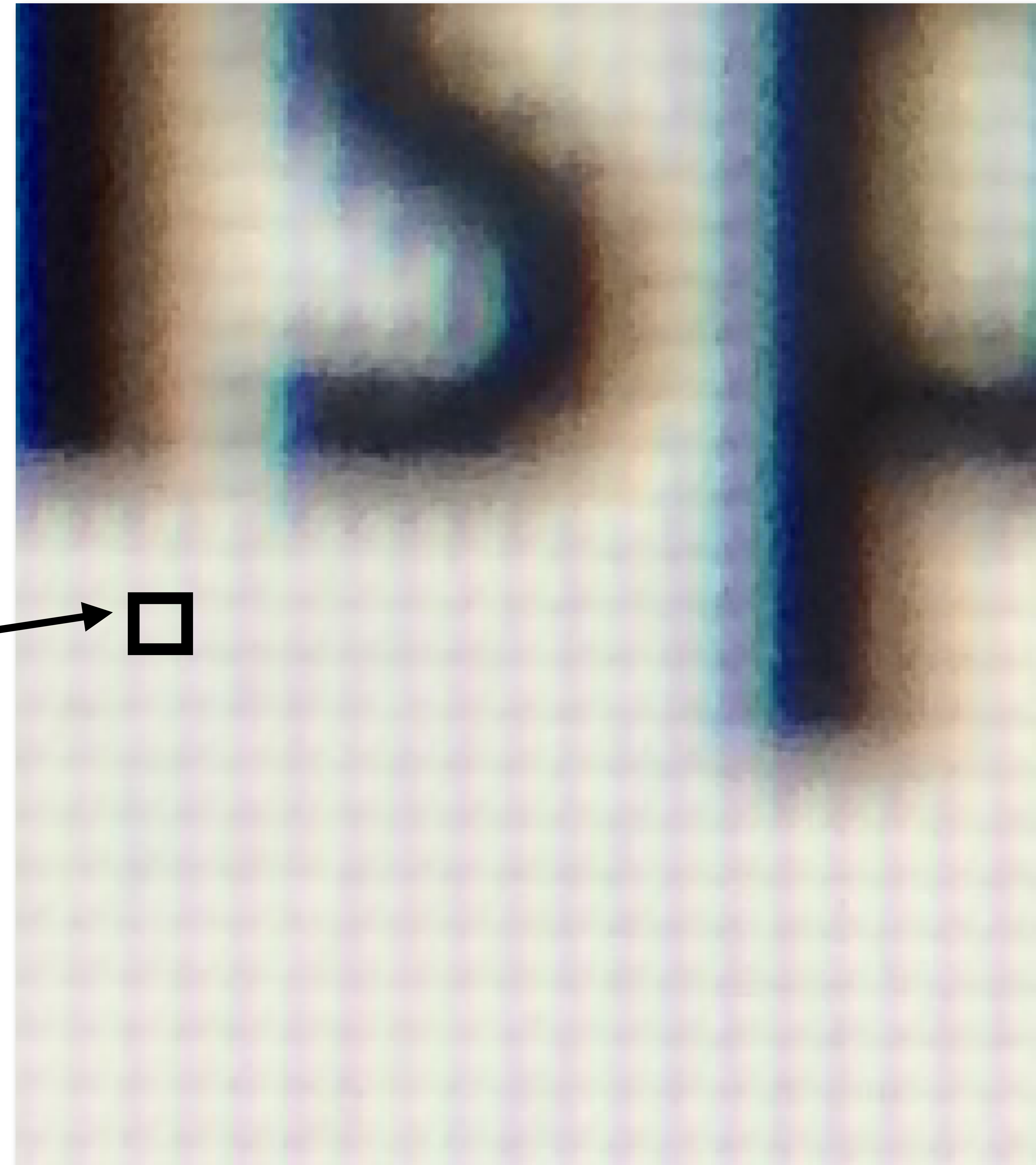
Advantages:
- Simplicity of parallel execution overcomes cost of extra point-in-triangle tests (most triangles are big enough to cover many samples)
- Can skip sample testing work: entire block not in triangle ("early out"), entire block entirely within triangle ("early in")
- Additional advantages related to accelerating occlusion computations (not discussed today)



All modern graphics processors (GPUs) have special-purpose hardware for efficiently performing point-in-triangle tests

Recall: pixels on a screen

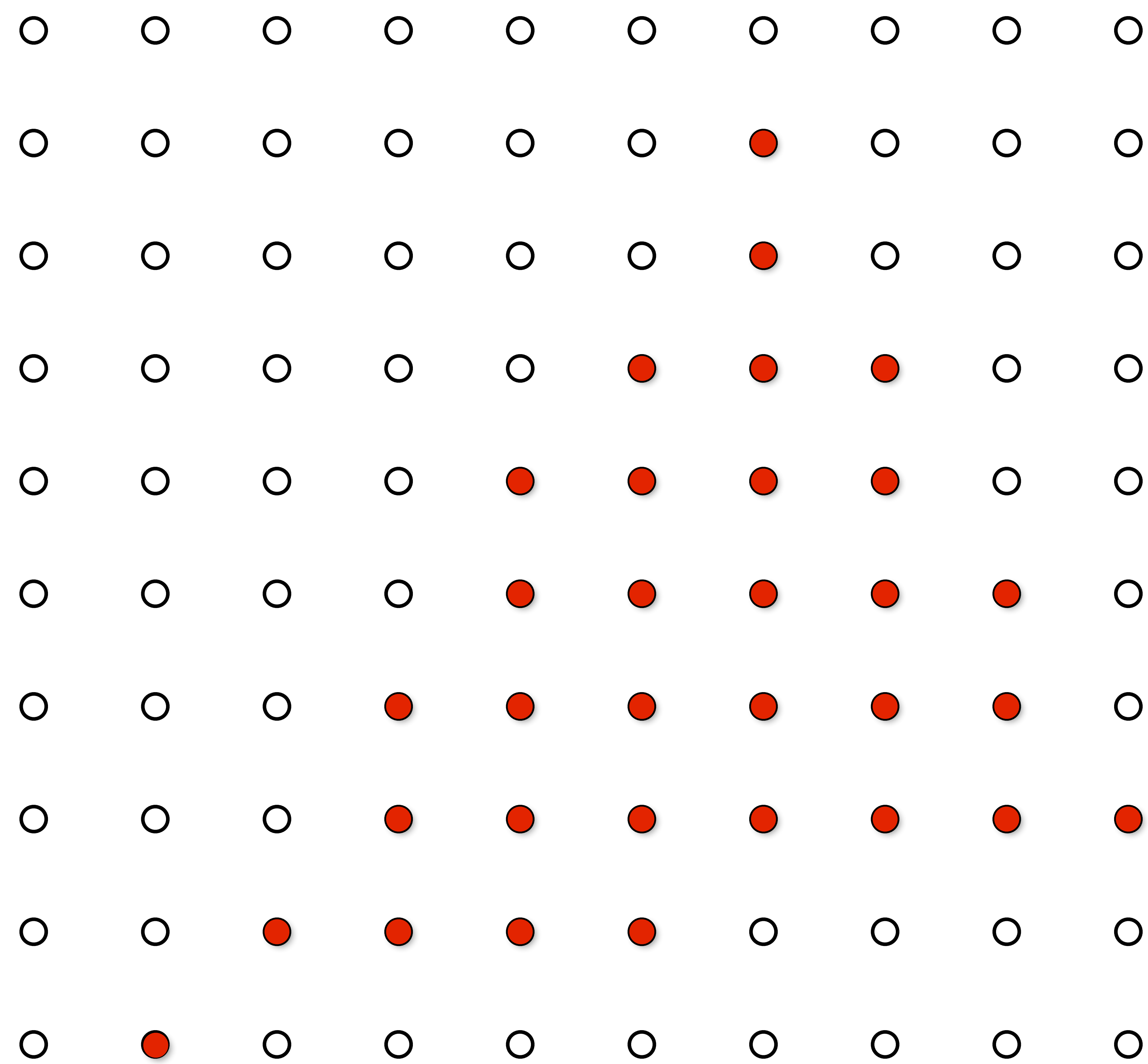
Each image sample sent to the display is converted into a little square of light of the appropriate color:
(a pixel = picture element)



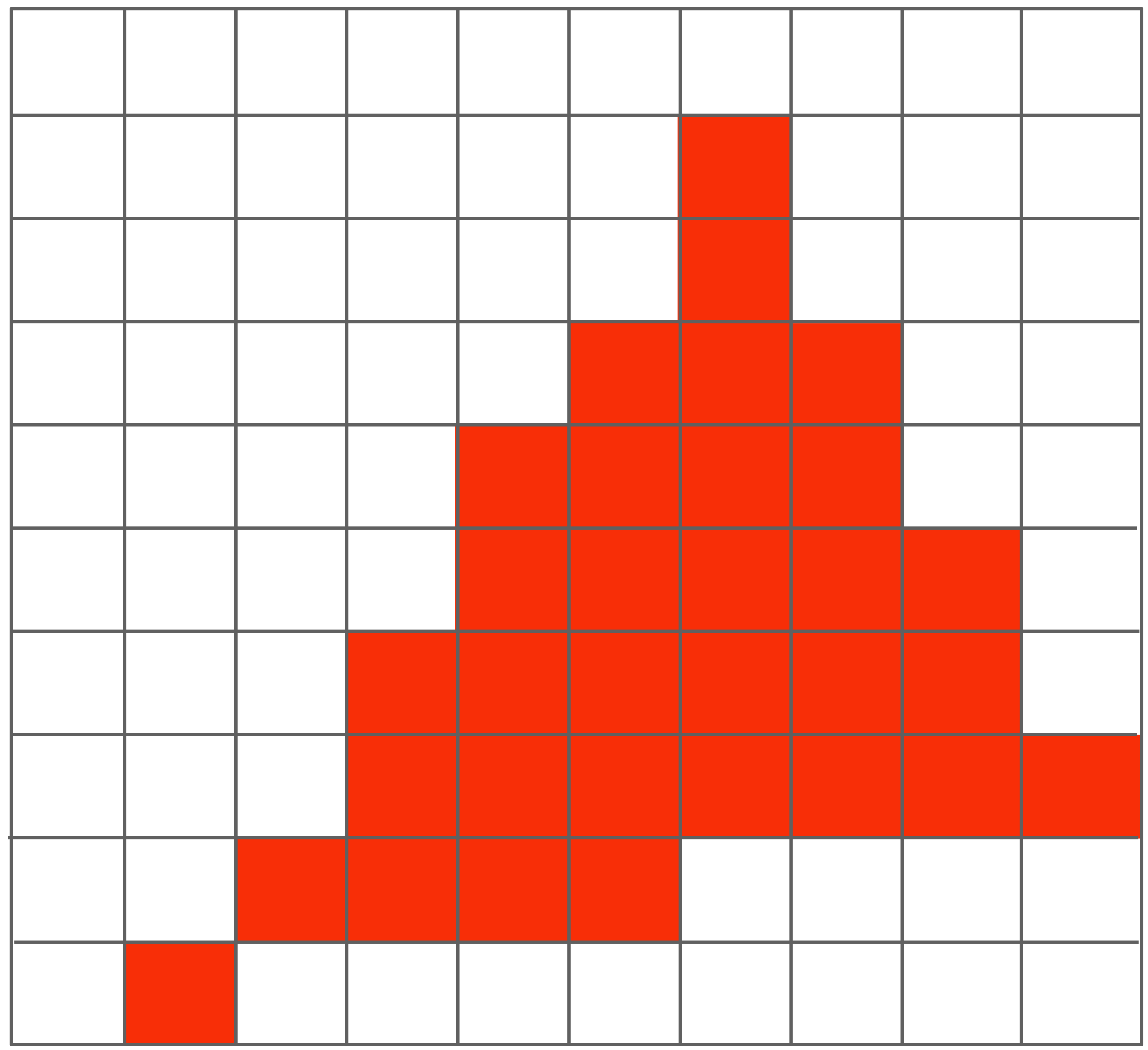
LCD display pixel on my laptop

*** Thinking of each LCD pixel as emitting a square of uniform intensity light of a single color is a bit of an approximation to how real displays work, but it will do for now.**

So, if we send the display this sampled signal

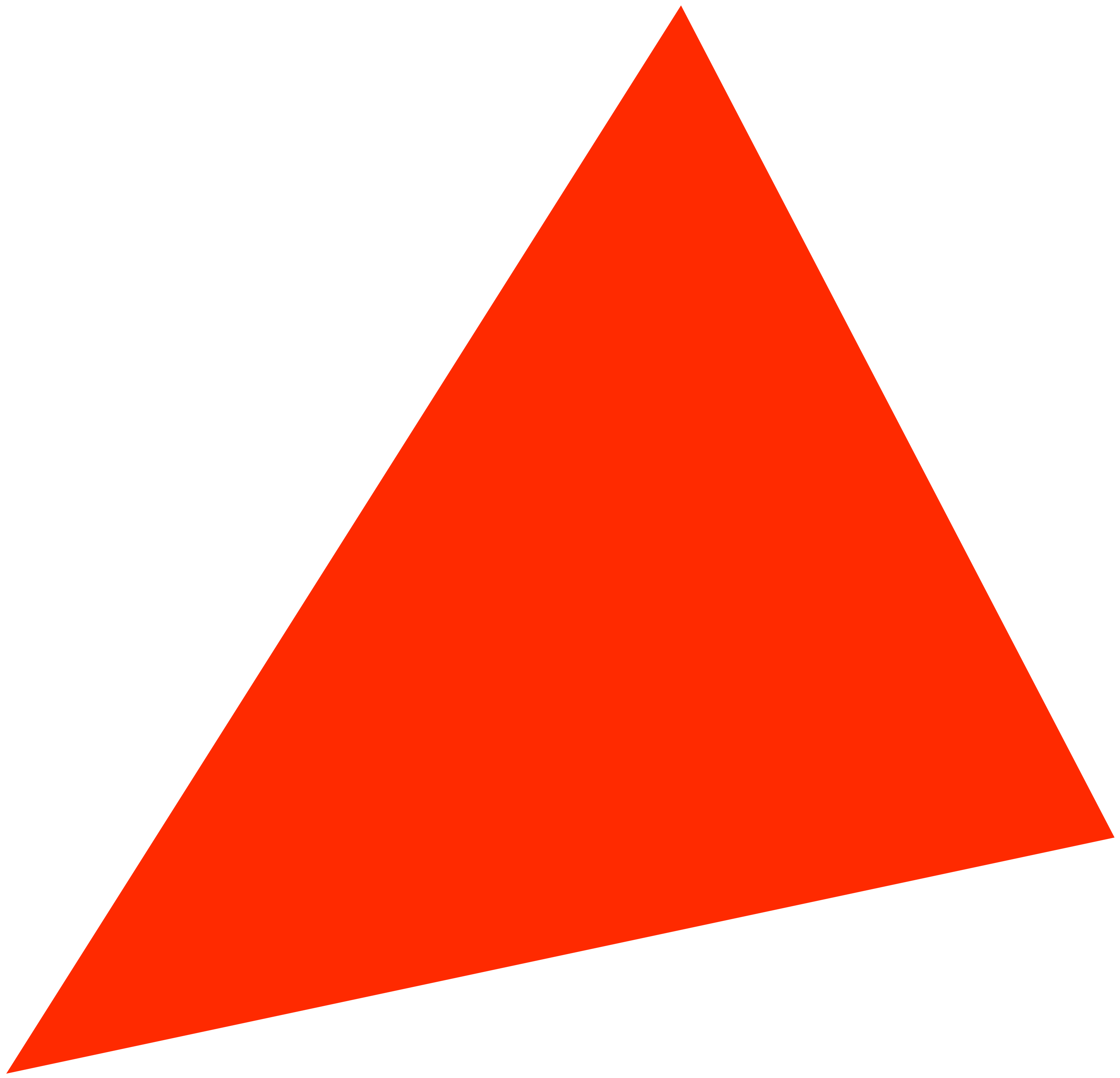


The display physically emits this signal

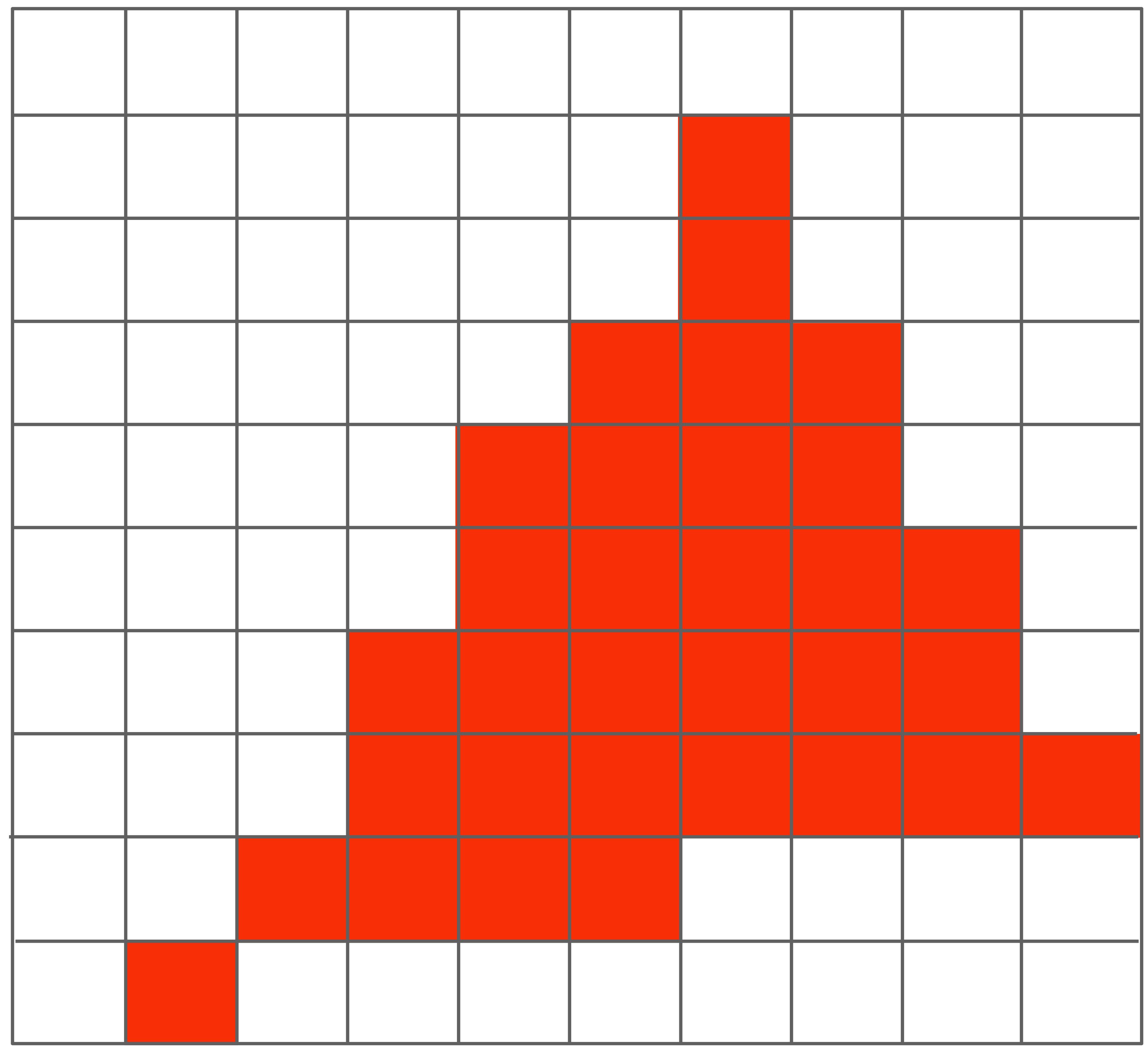


Given our simplified "square pixel" display assumption, we've effectively performed a piecewise constant reconstruction

Compare: the continuous triangle function

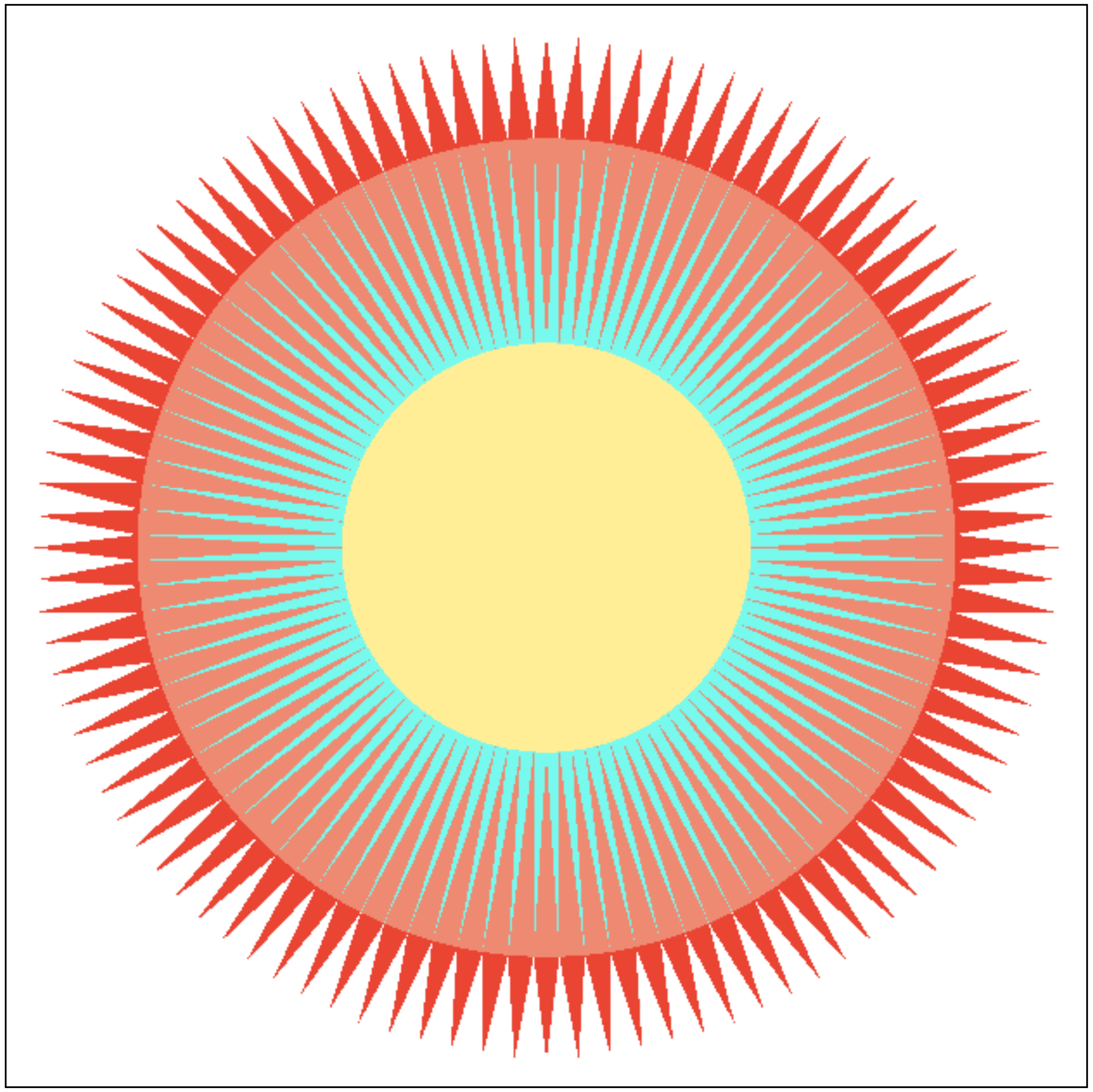


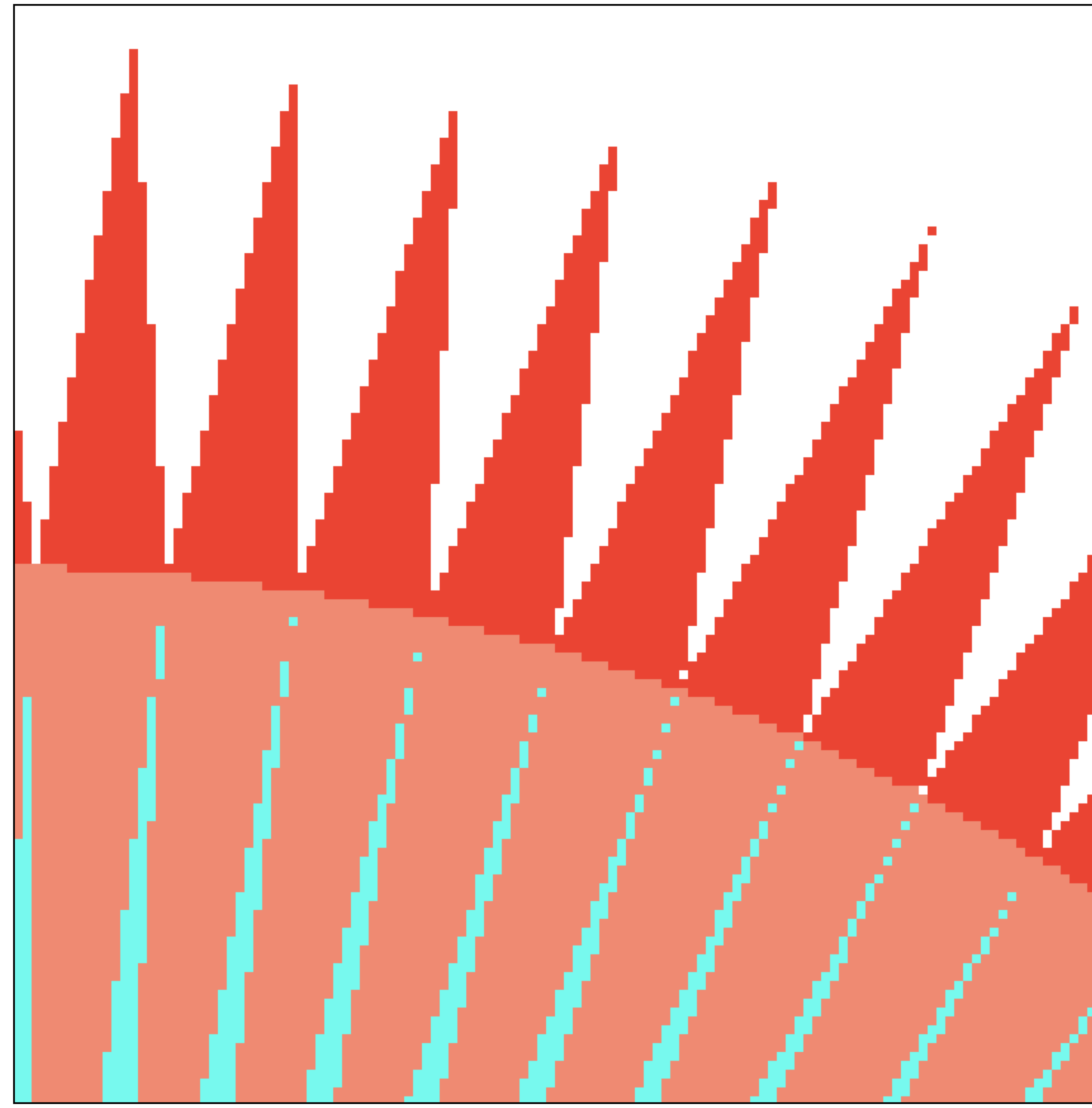
What's wrong with this picture?



Jaggies!

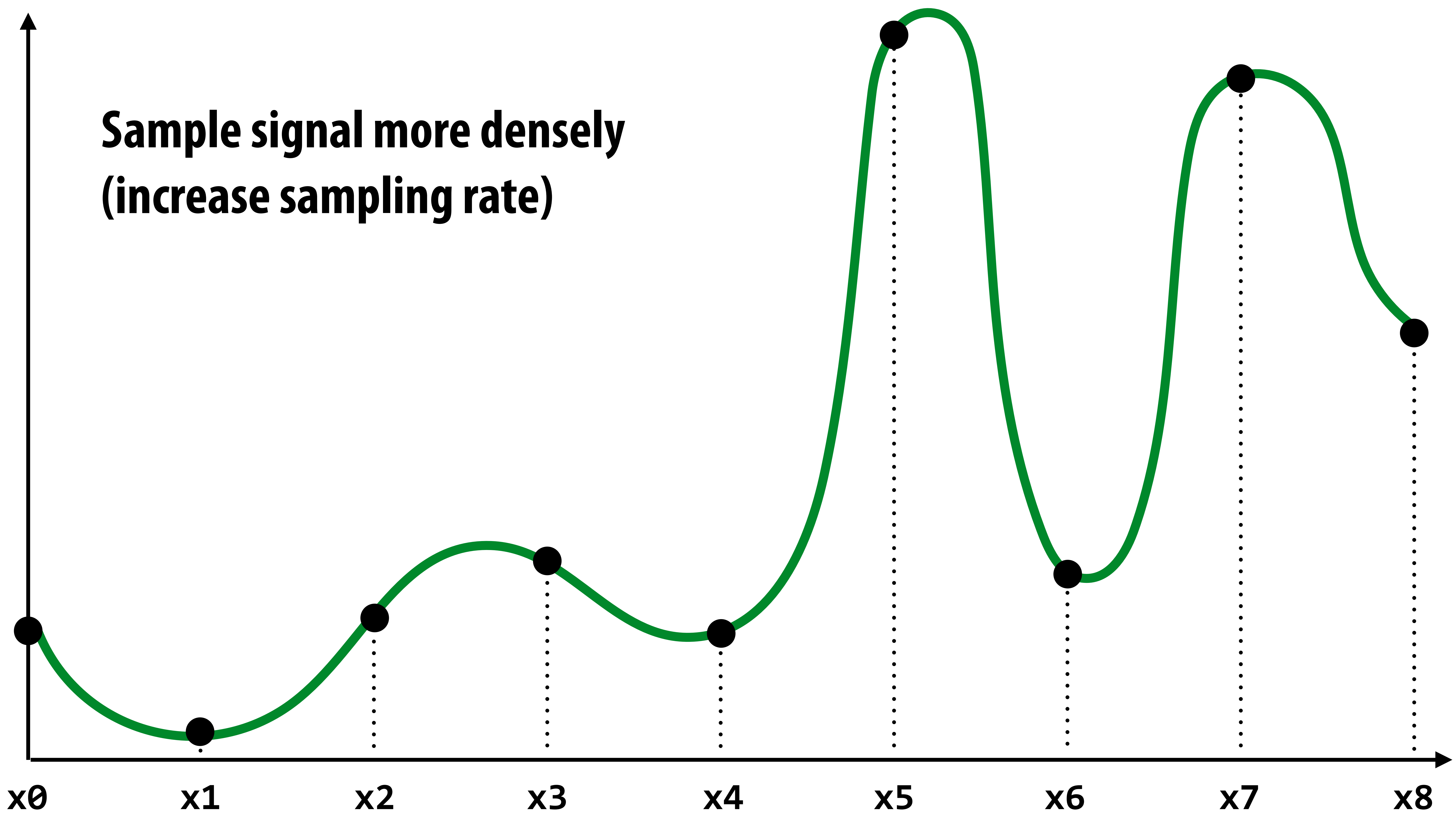
Jaggies (staircase pattern)



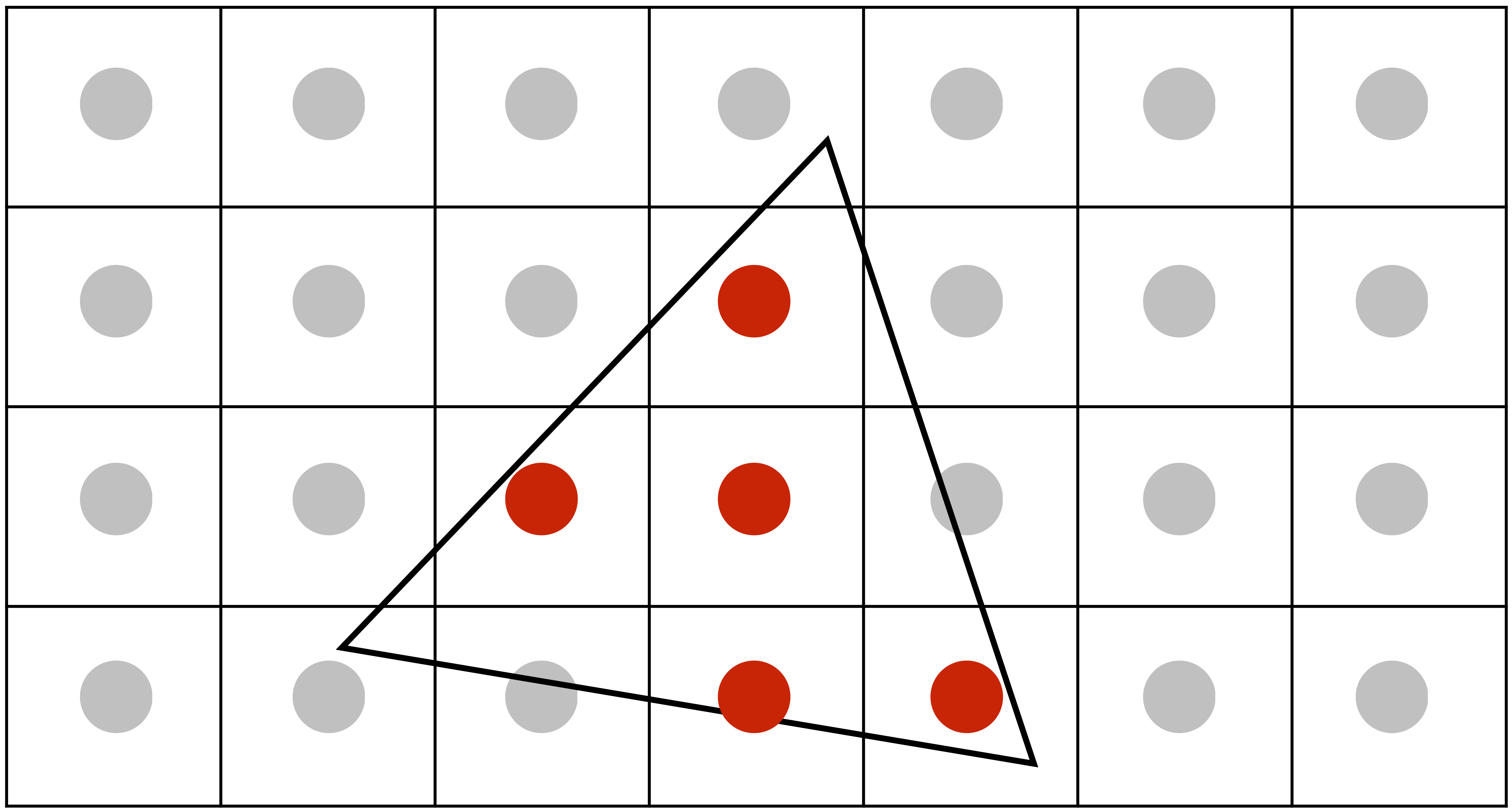


Is this the best we can do?

Reminder: how can we represent a sampled signal more accurately?



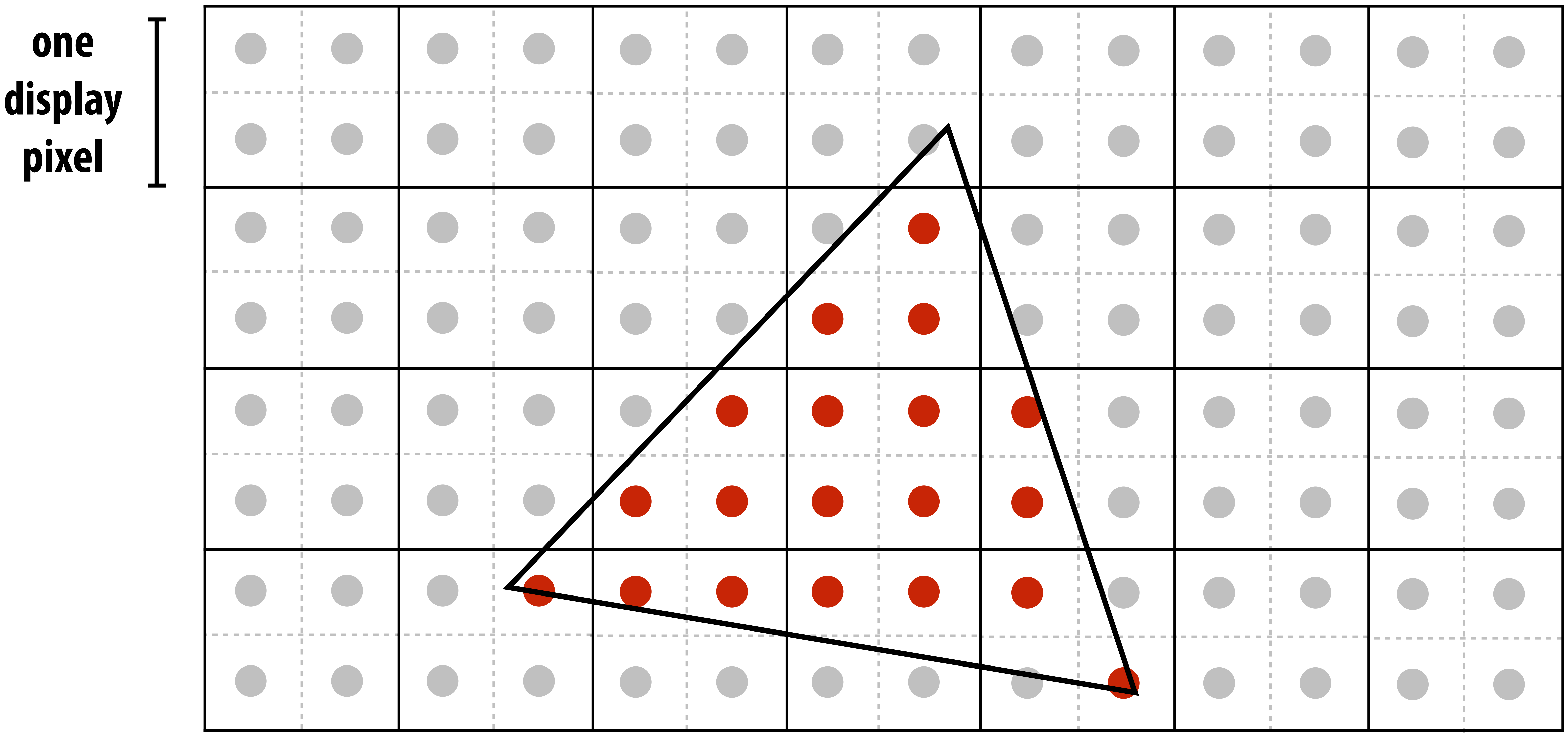
Point sampling: one sample per pixel



Supersampling: step 1

Take N x N samples in each pixel

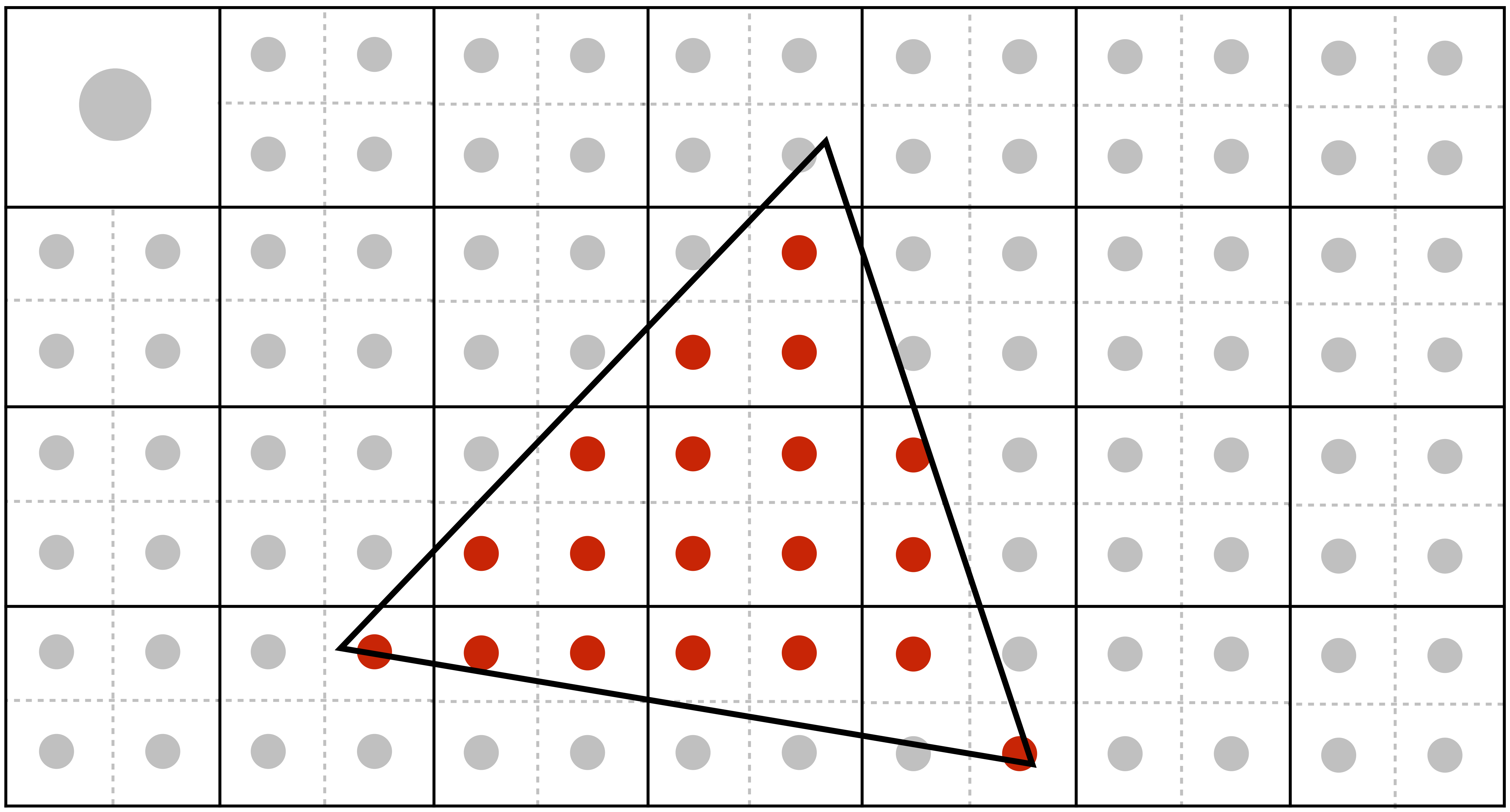
(but… how do we use these samples to drive a display, since there are four times more samples than display pixels!)



2x2 supersampling

Supersampling: step 2

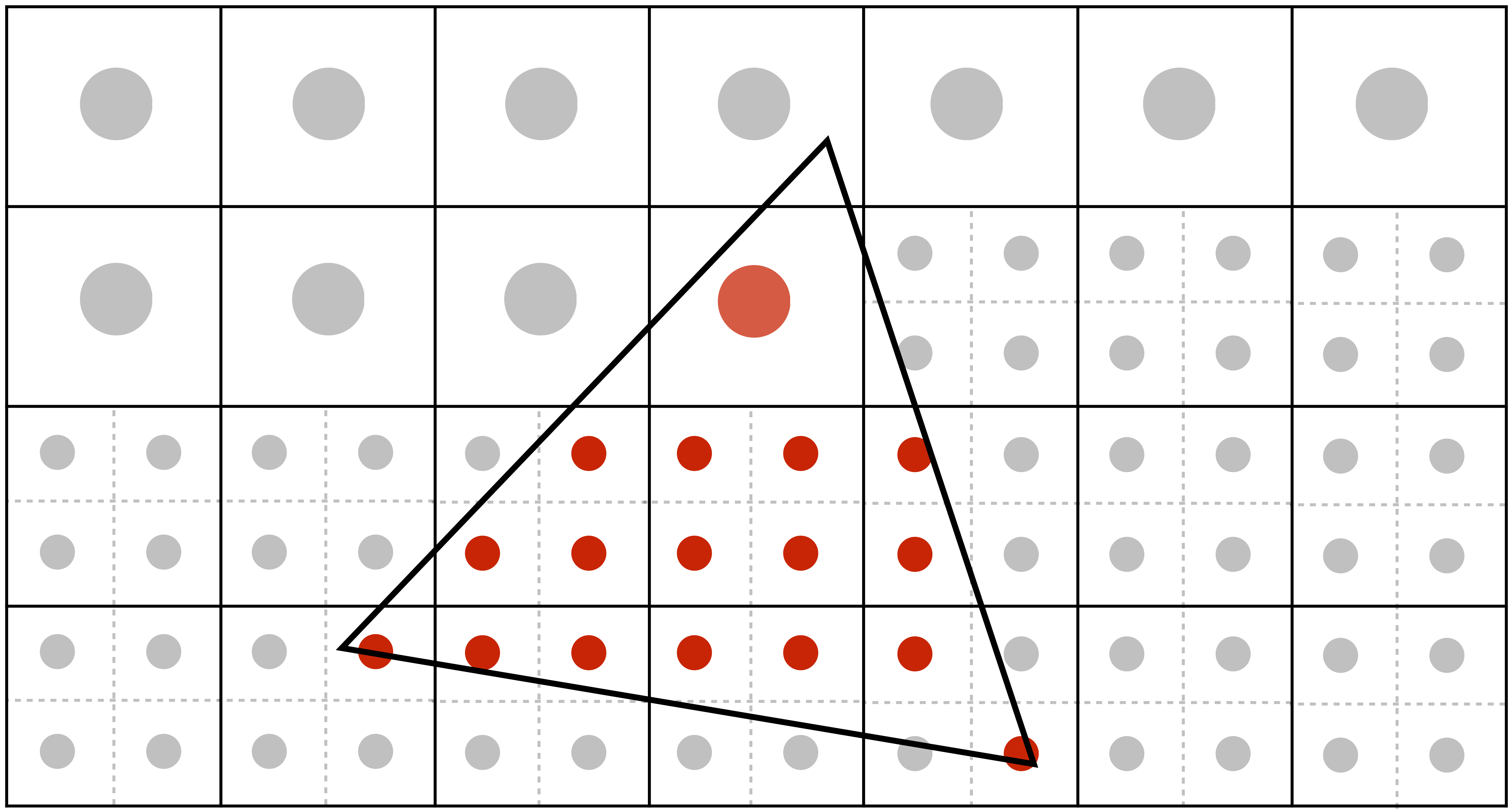
Average the N x N samples “inside” each pixel



Averaging down

Supersampling: step 2

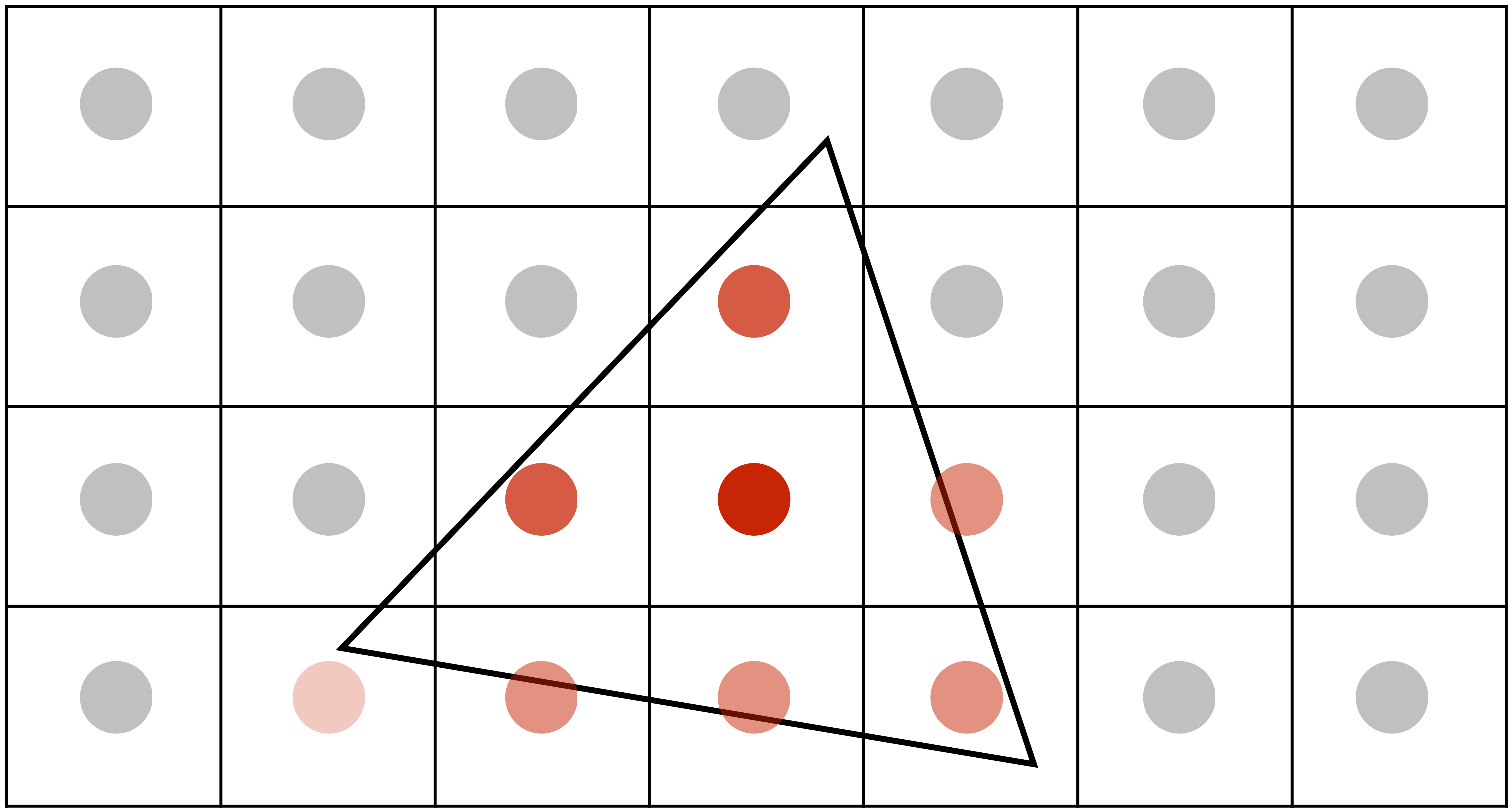
Average the N x N samples “inside” each pixel



Averaging down

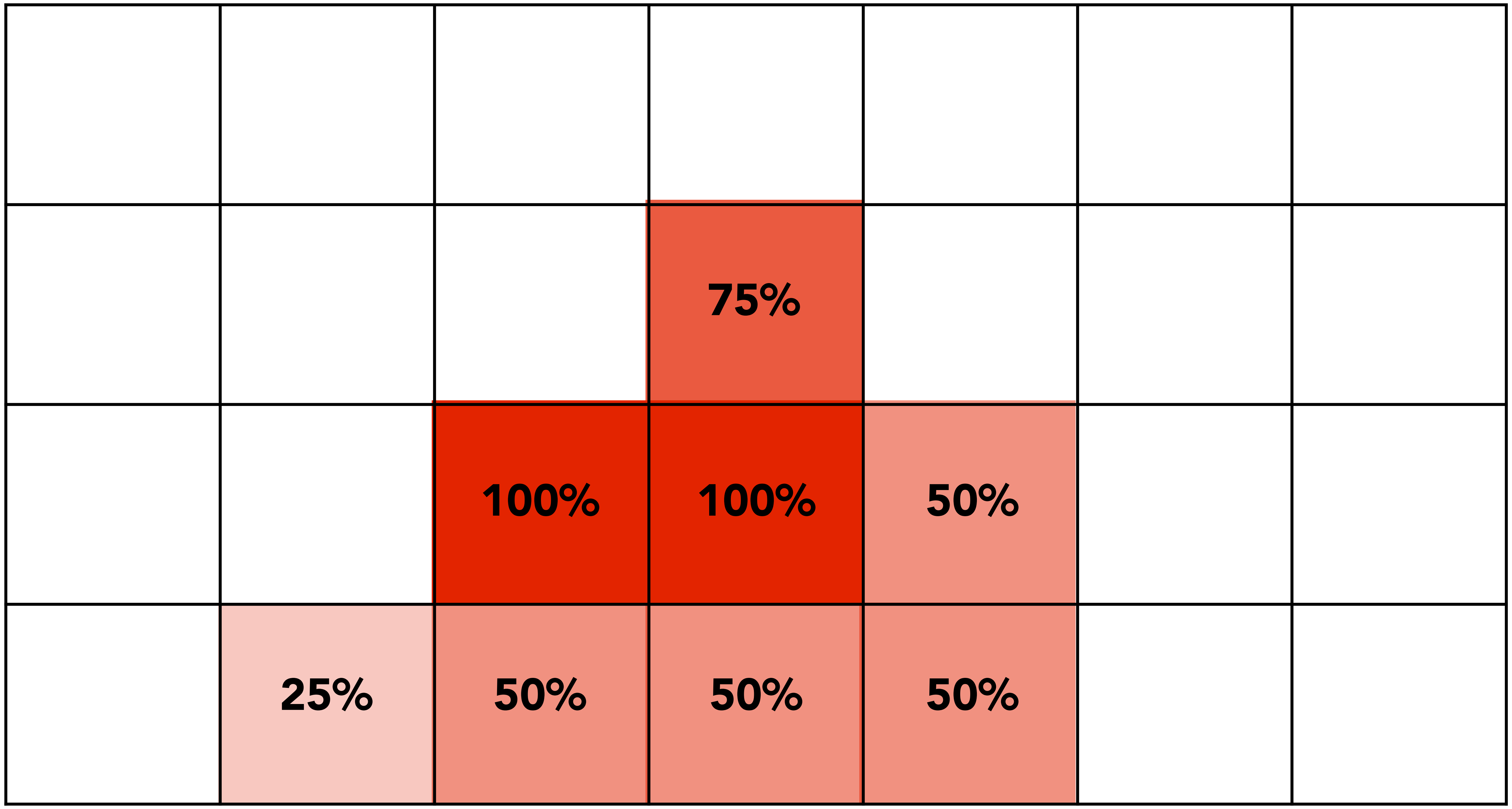
Supersampling: step 2

Average the N x N samples "inside" each pixel

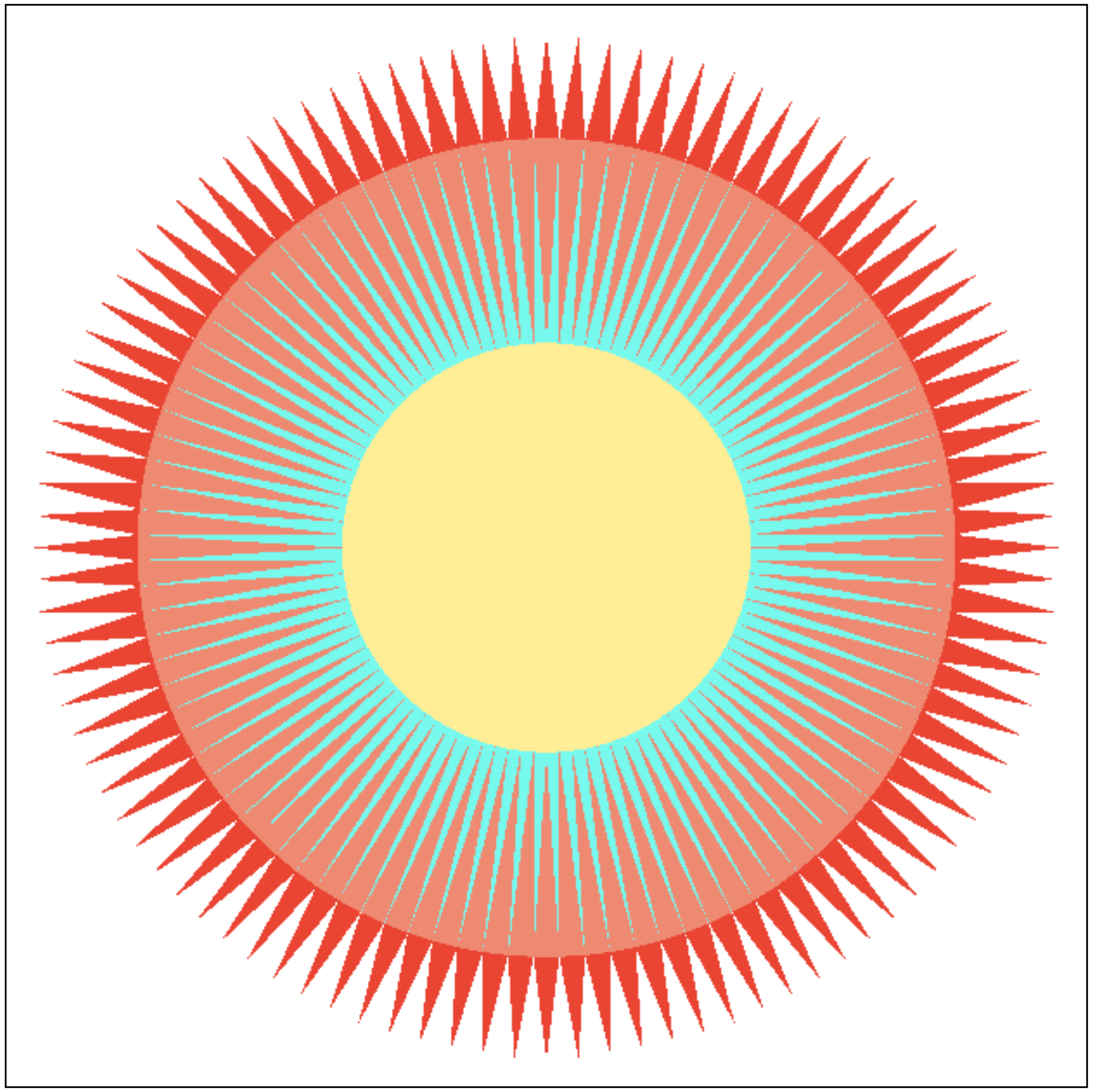


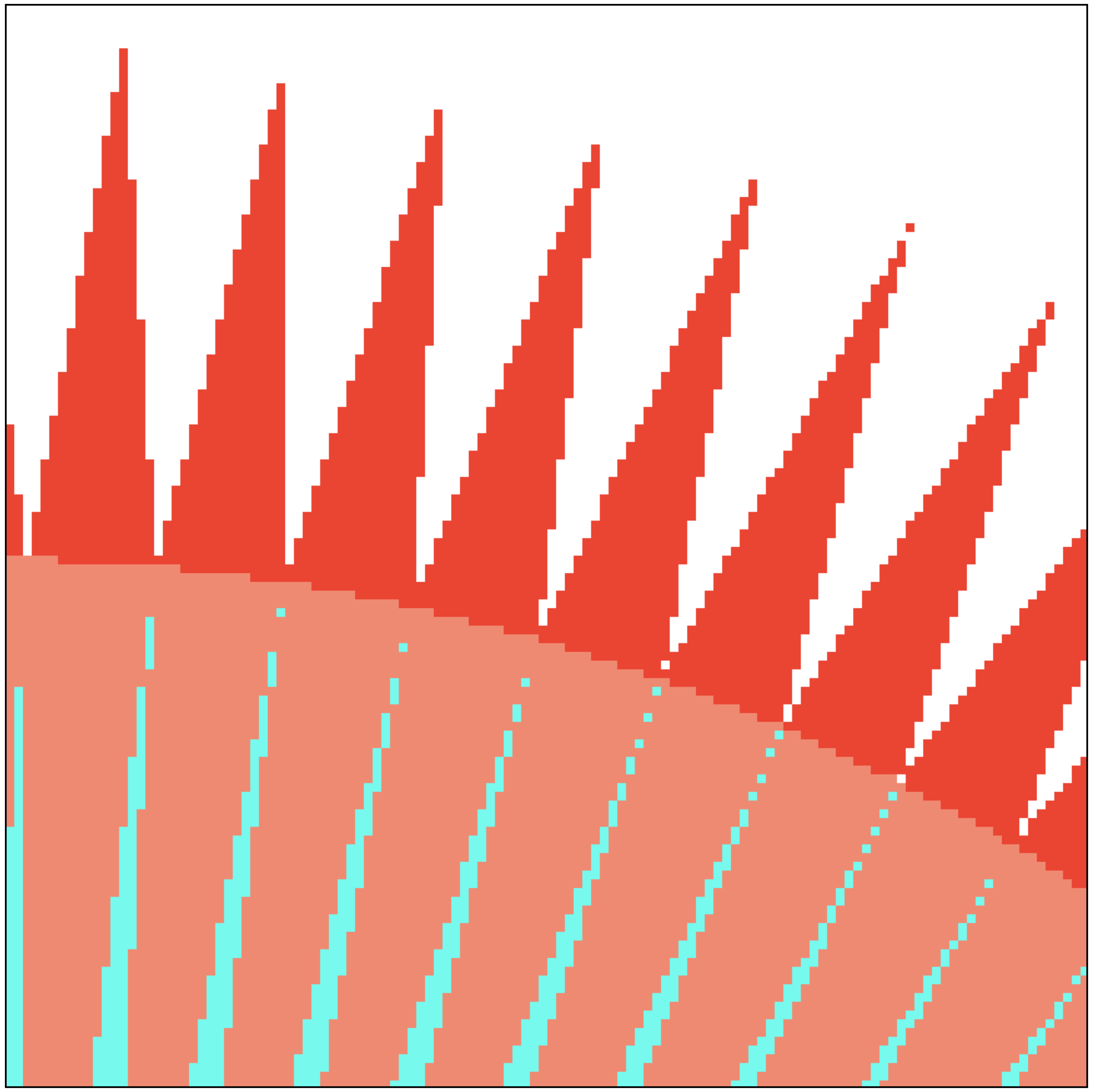
Supersampling: result

This is the corresponding signal emitted by the display

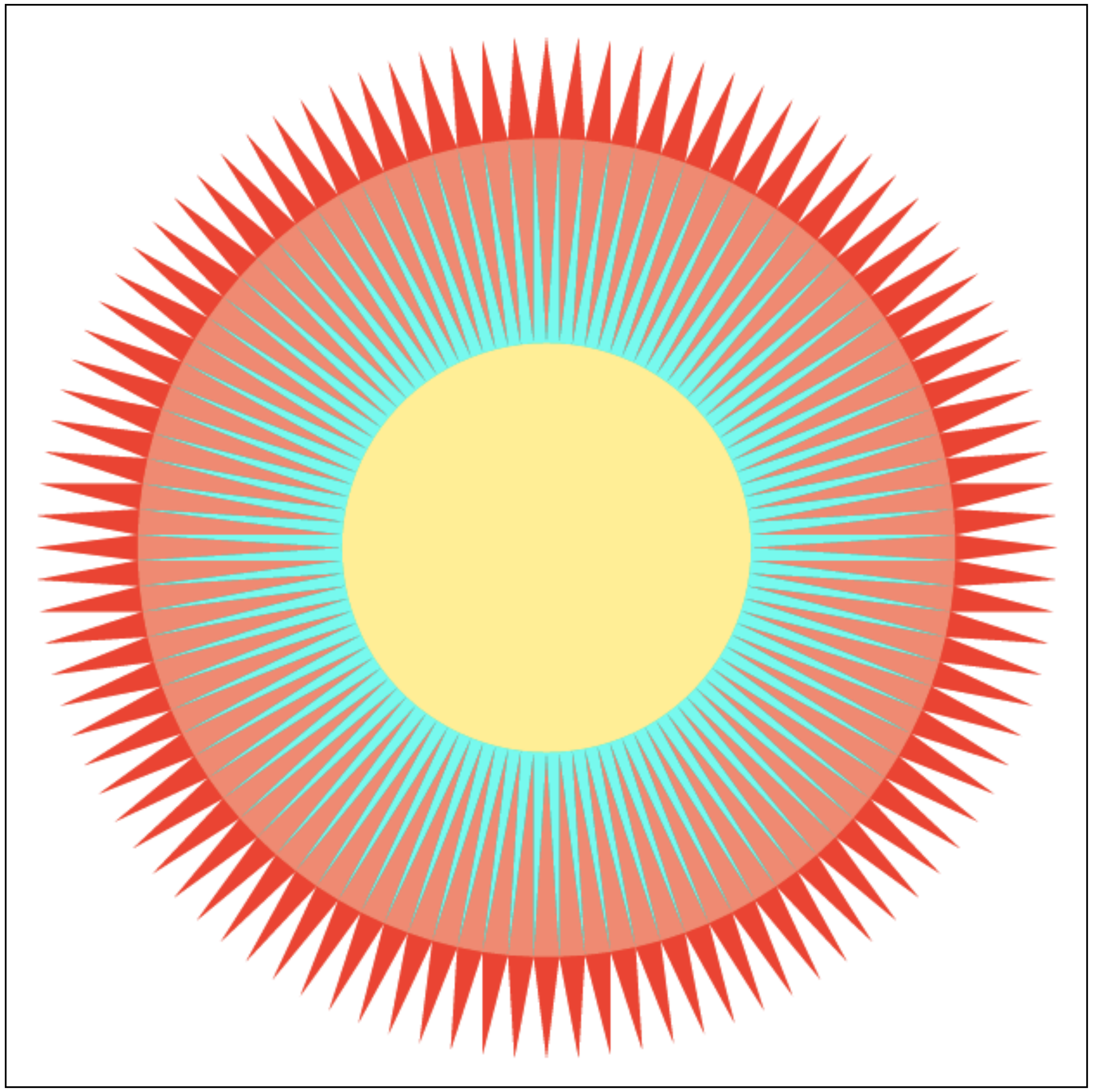


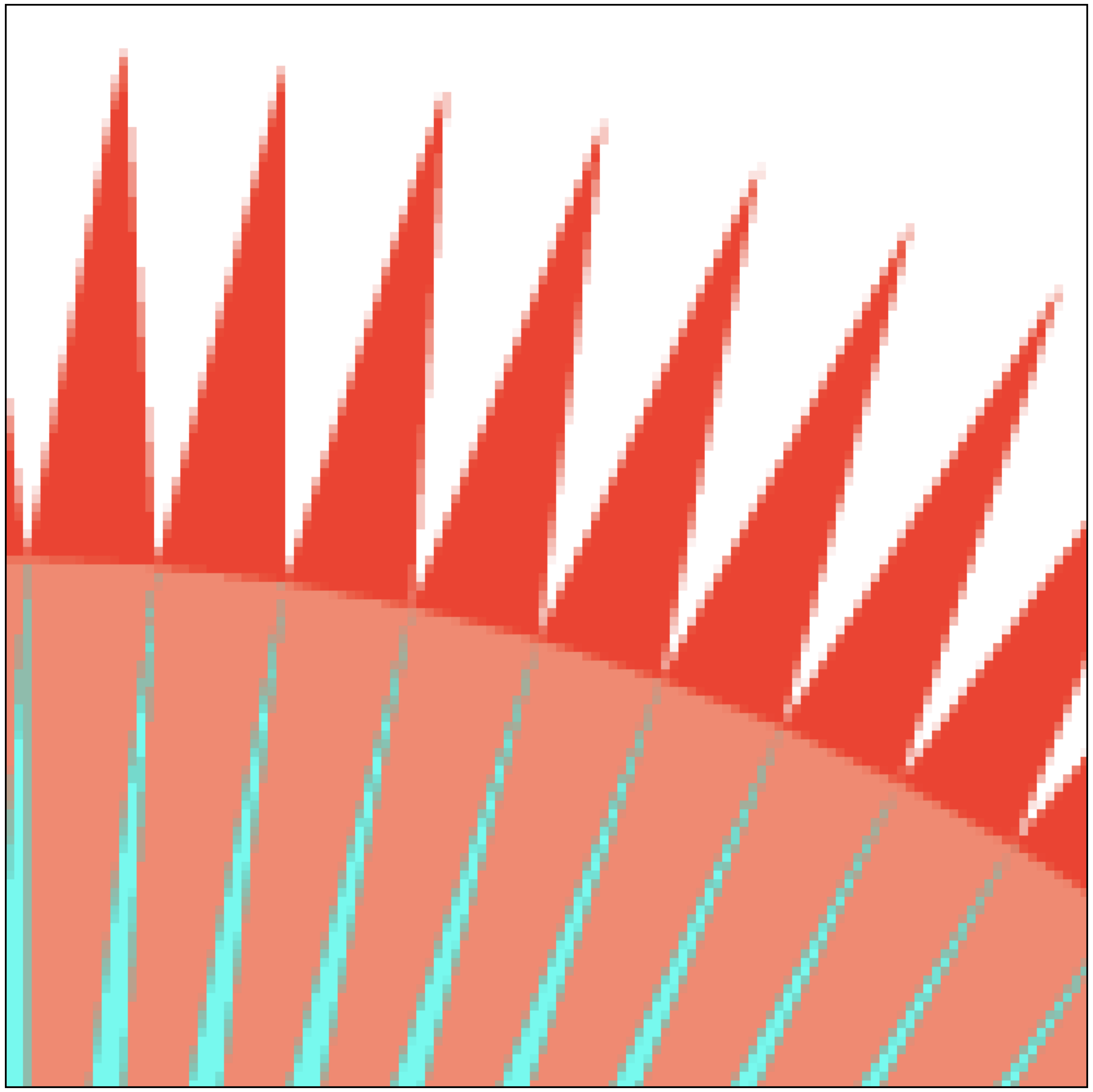
Images rendered using one sample per pixel





4x4 supersampling + downsampling



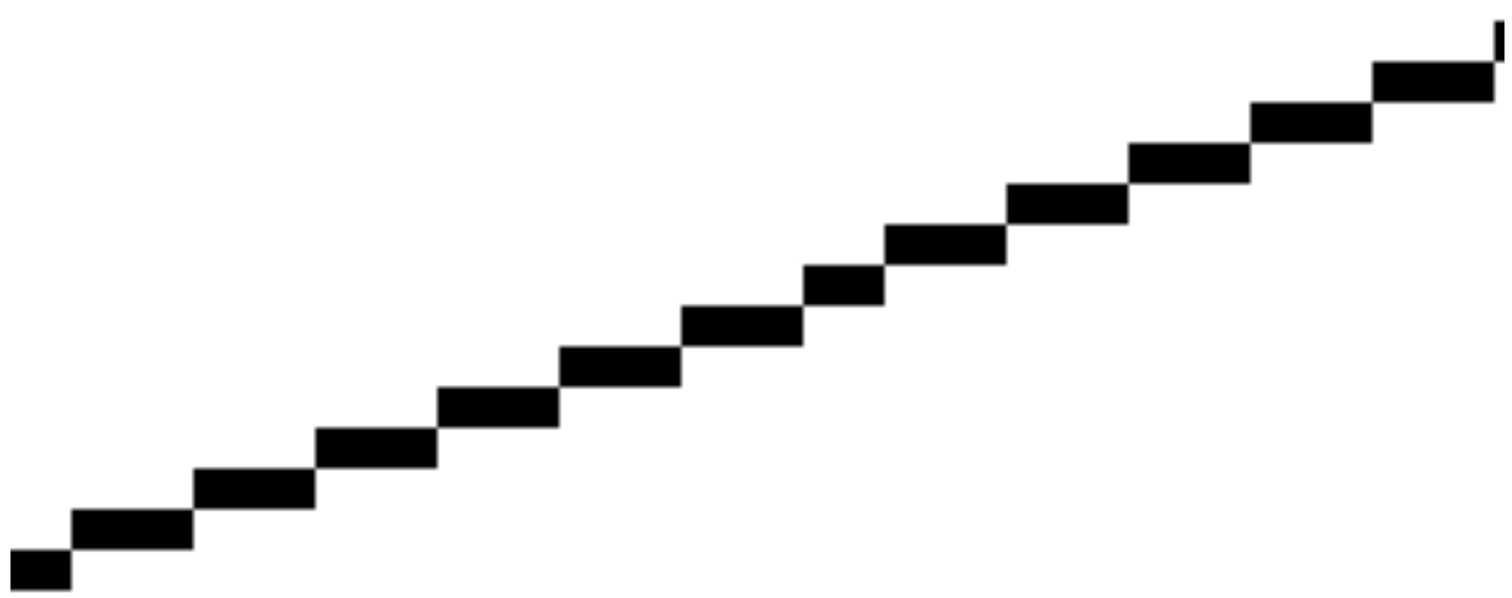


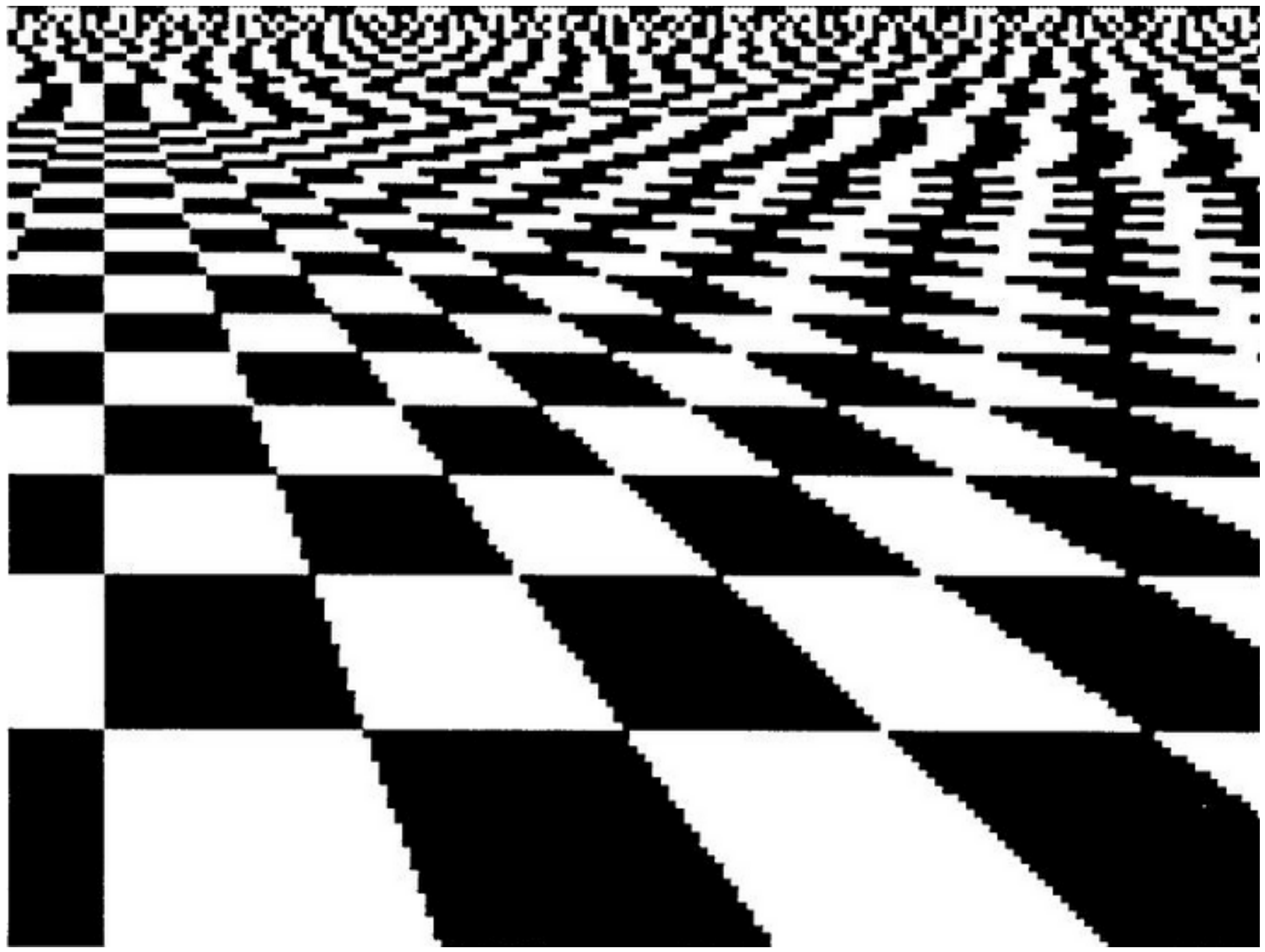
Pixel value is average of 4x4 samples per pixel

Let's understand what just happened in a more principled way

More examples of sampling artifacts in computer graphics

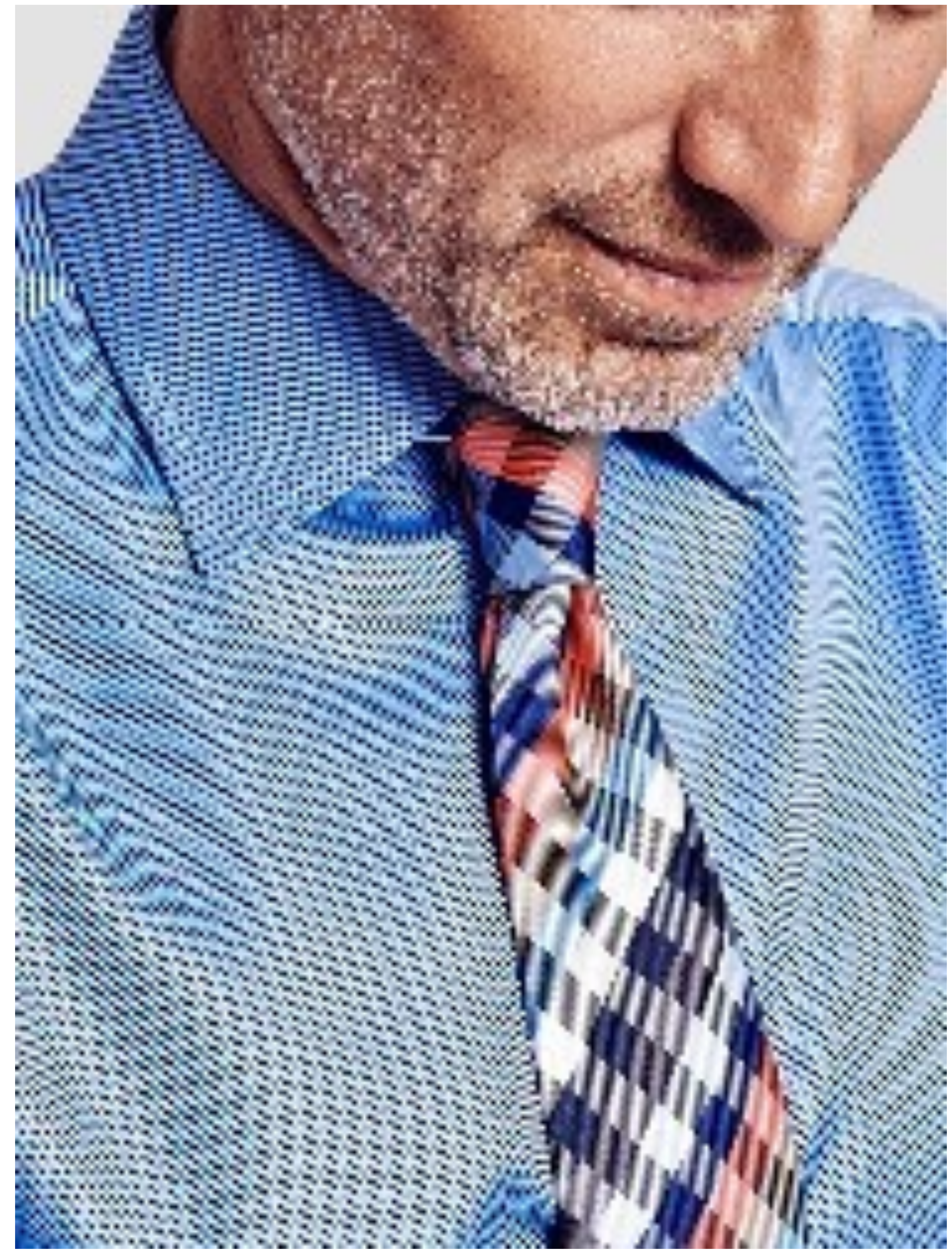
Jaggies (staircase pattern)





Moiré patterns in imaging

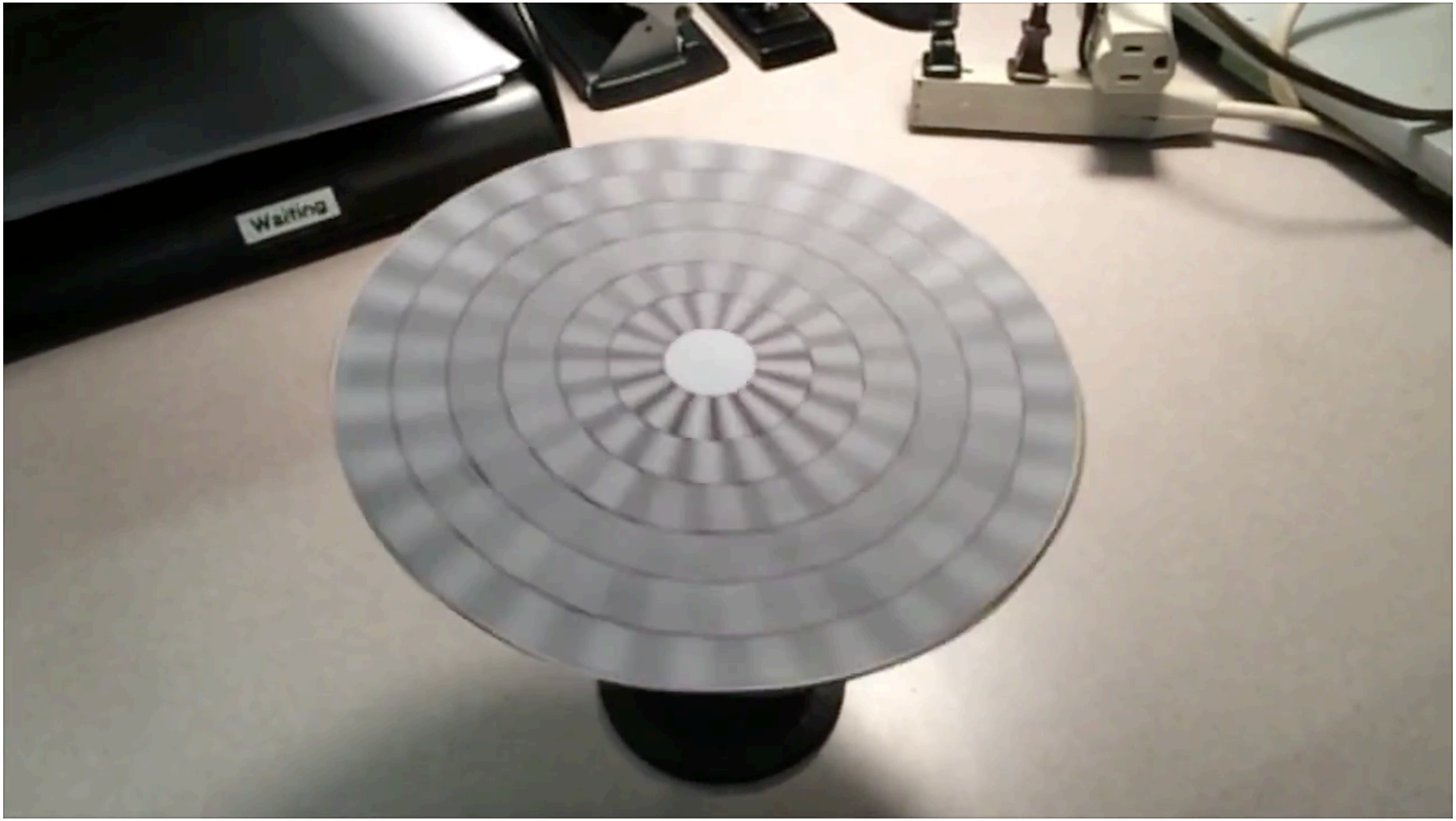
Full resolution image



1/2 resolution image:
skip pixel odd rows and columns

lystit.com

Wagon wheel illusion (false motion)



Camera's frame rate (temporal sampling rate) is too low for rapidly spinning wheel.

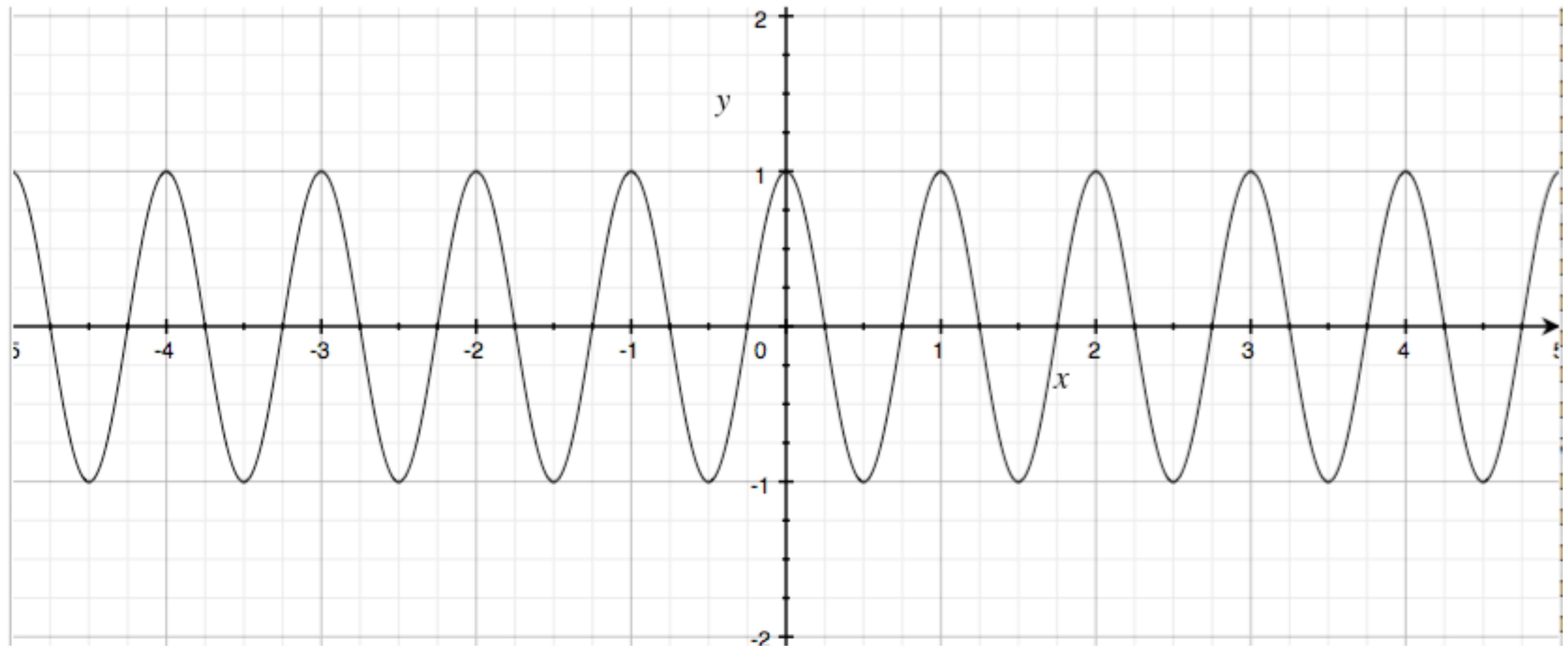
Created by Jesse Mason, https://www.youtube.com/watch?v=QOwzkND_ooU

Sampling artifacts in computer graphics

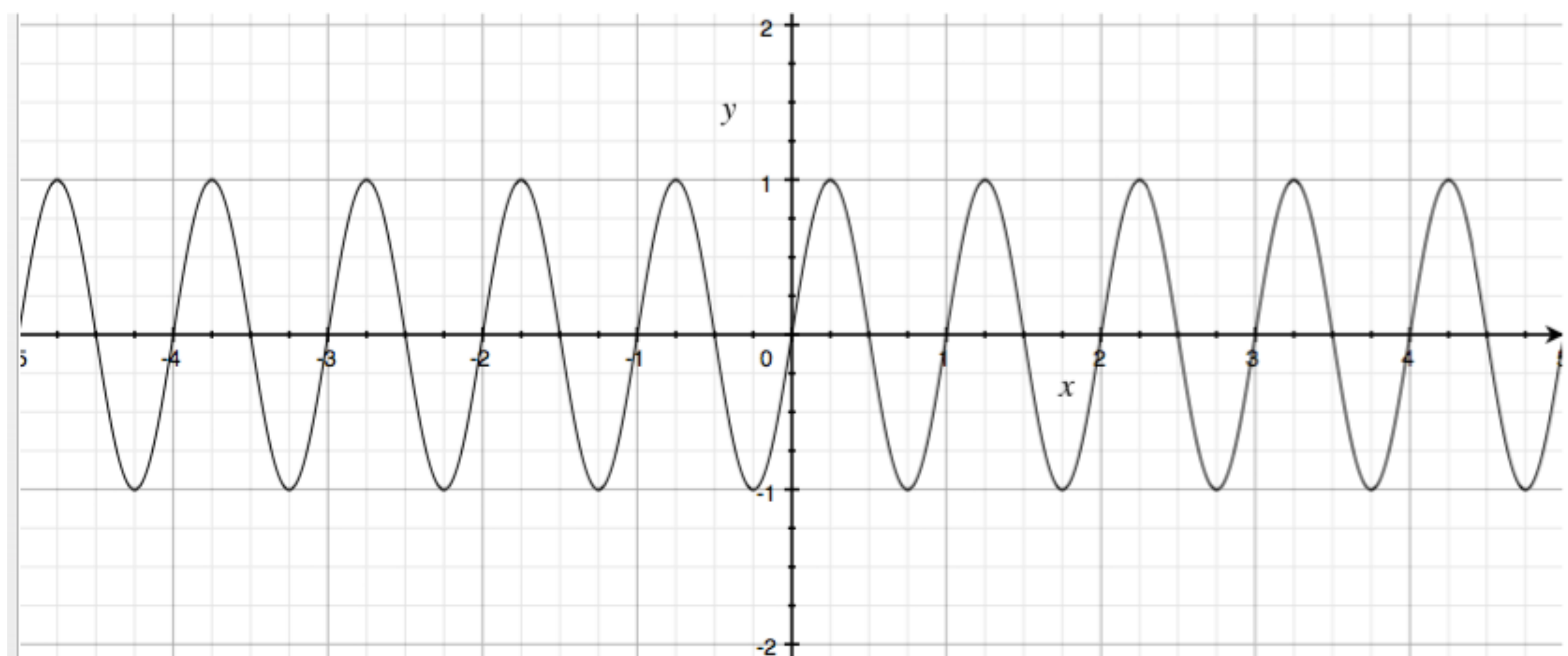
- **Artifacts due to sampling - "Aliasing"**
  - **Jaggies – sampling in space**
  - **Wagon wheel effect – sampling in time**
  - **Moire – undersampling images (and texture maps)**
  - **[Many more] …**

- **We notice this in fast-changing signals, when we sample the signal too sparsely**

Sines and cosines



$$\cos 2\pi x$$

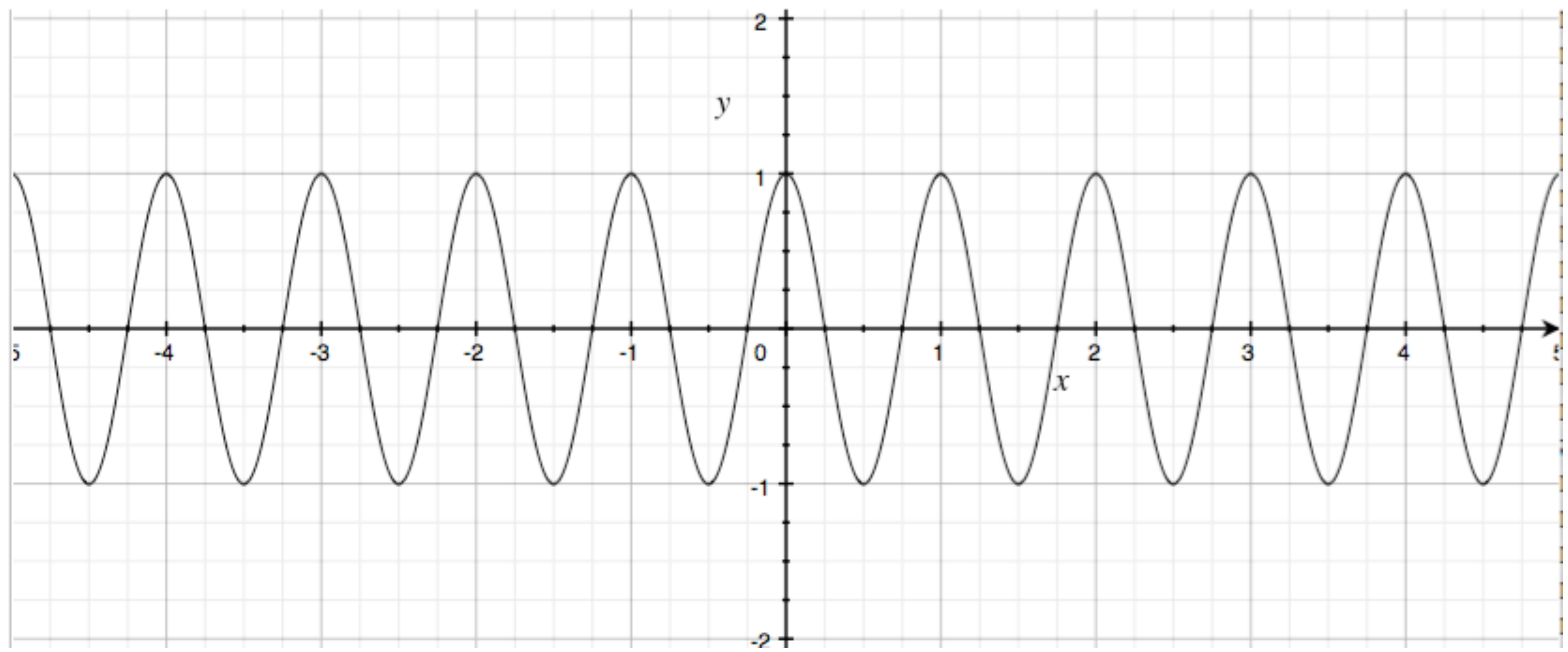


$$\sin 2\pi x$$

Frequencies

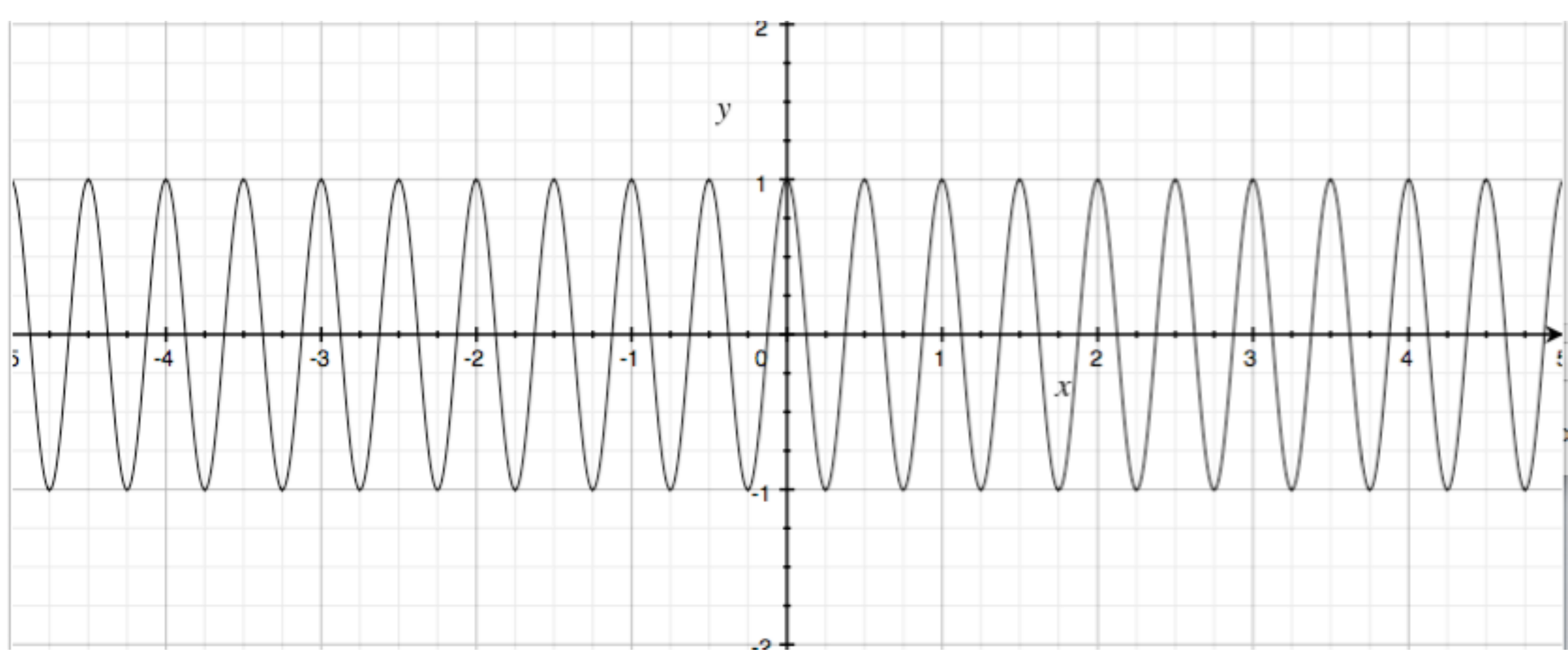
$$\cos 2\pi f x$$

$$f = \frac{1}{T}$$



$$f = 1$$

$$\cos 2\pi x$$

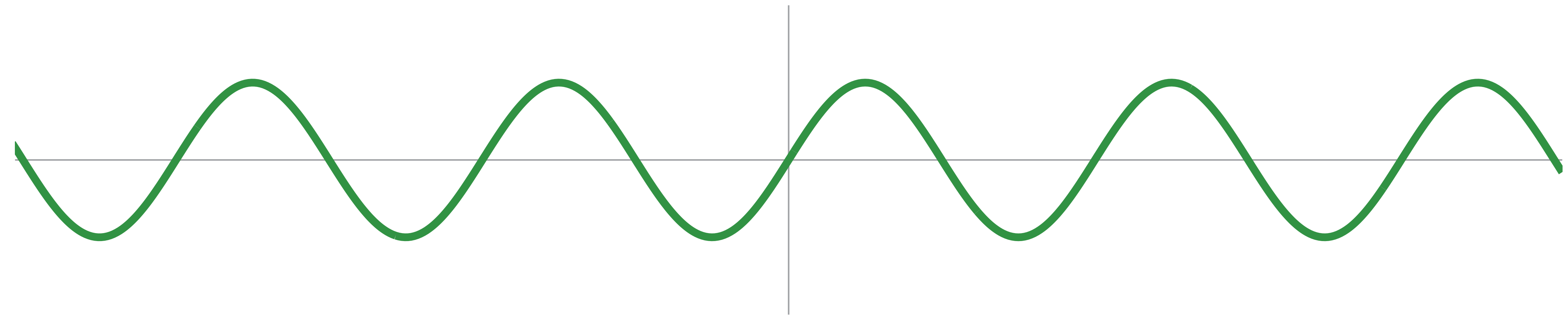


$$f = 2$$

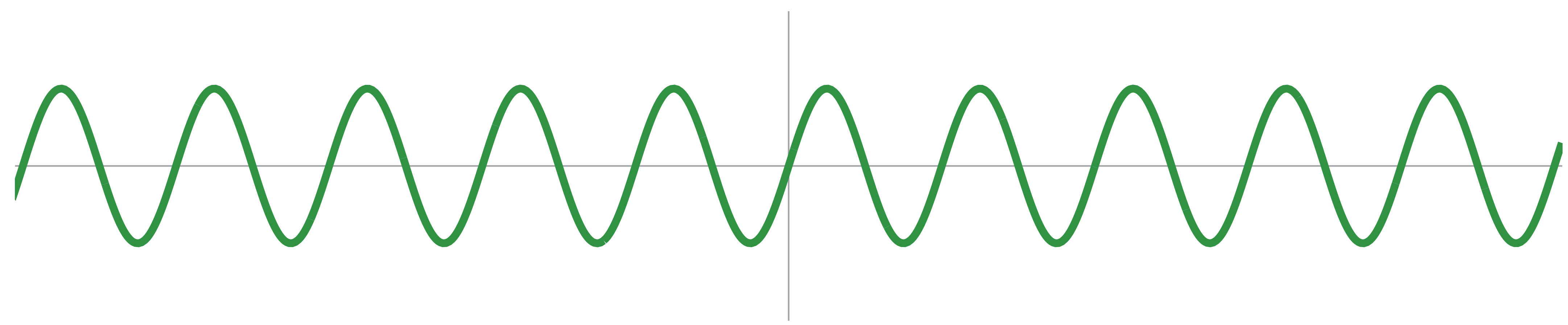
$$\cos 4\pi x$$

Representing sound wave as a superposition of frequencies

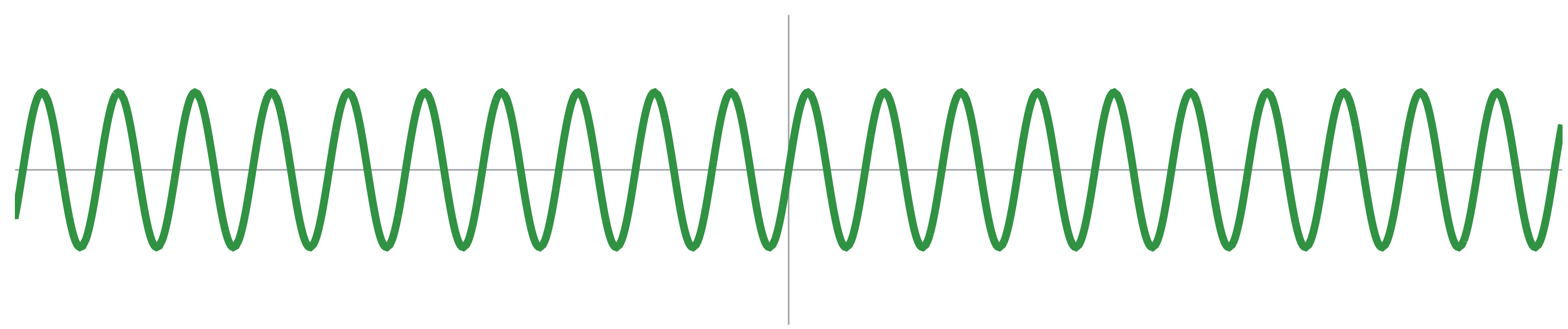
$f_1(x) = sin(\pi x)$



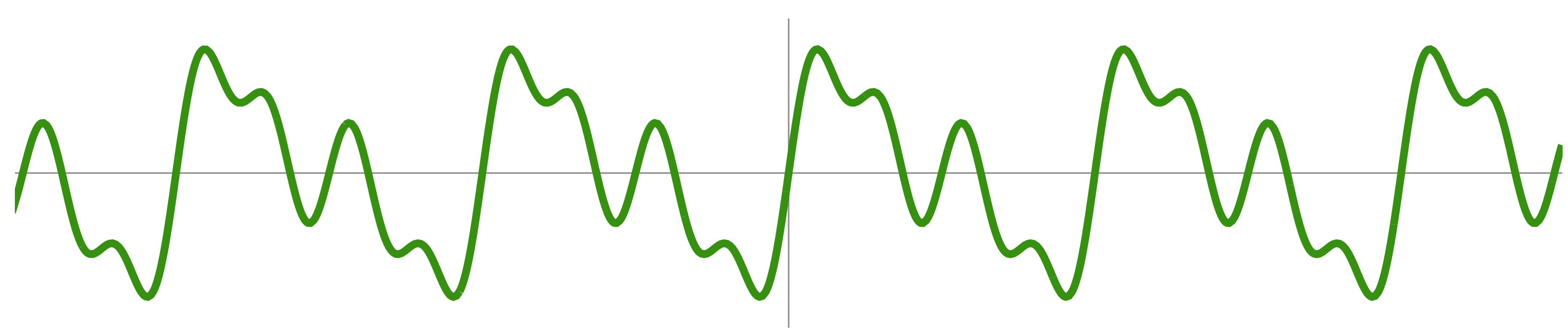
$f_2(x) = sin(2\pi x)$



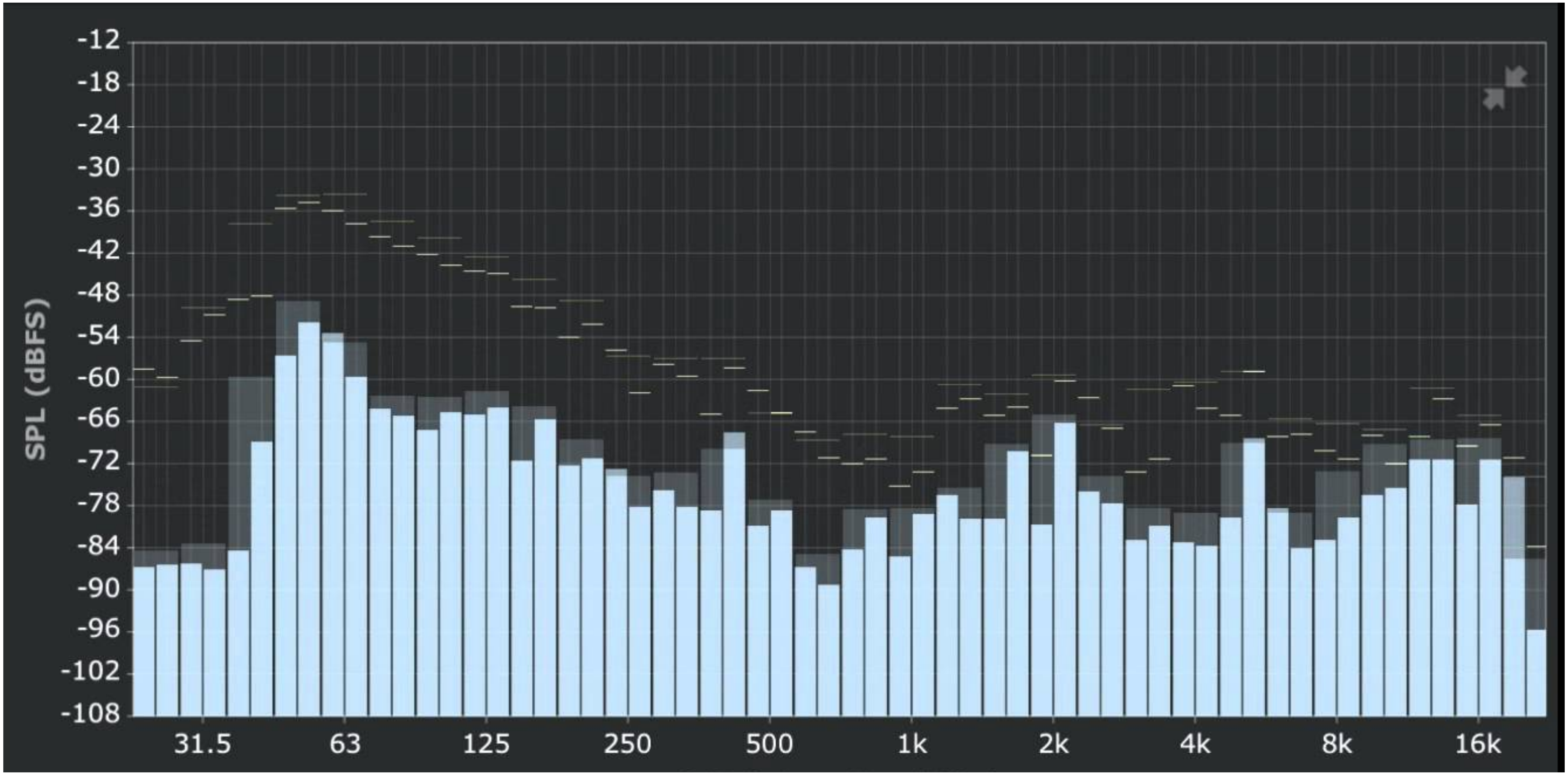
$f_4(x) = sin(4\pi x)$



$f(x) = 1.0\, f_1(x) + 0.75\, f_2(x) + 0.5\, f_4(x)$



Audio spectrum analyzer: representing sound as a sum of its constituent frequencies



Intensity of low-frequencies (bass)

Intensity of high frequencies

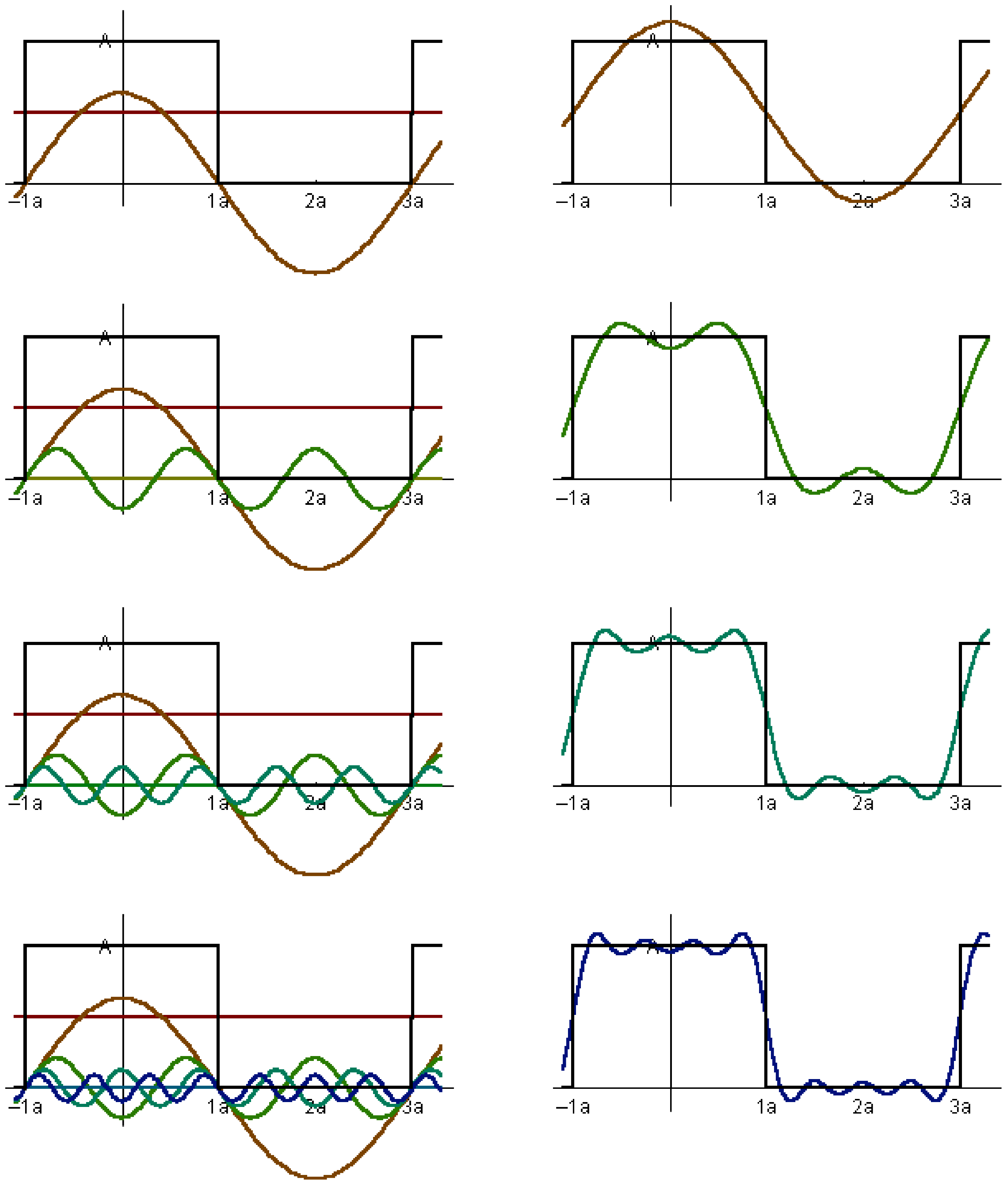
Image credit: ONYX Apps

How to compute frequency-domain representation of a signal?

Fourier transform

Represent a function as a weighted sum of sines and cosines

Joseph Fourier 1768 - 1830



$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

Fourier transform

- **Convert representation of signal from primal domain (spatial/temporal) to frequency domain by projecting signal into its component frequencies**

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \omega} dx$$

$$= \int_{-\infty}^{\infty} f(x)(\cos(2\pi\omega x) - i\sin(2\pi\omega x)) dx$$

Recall:

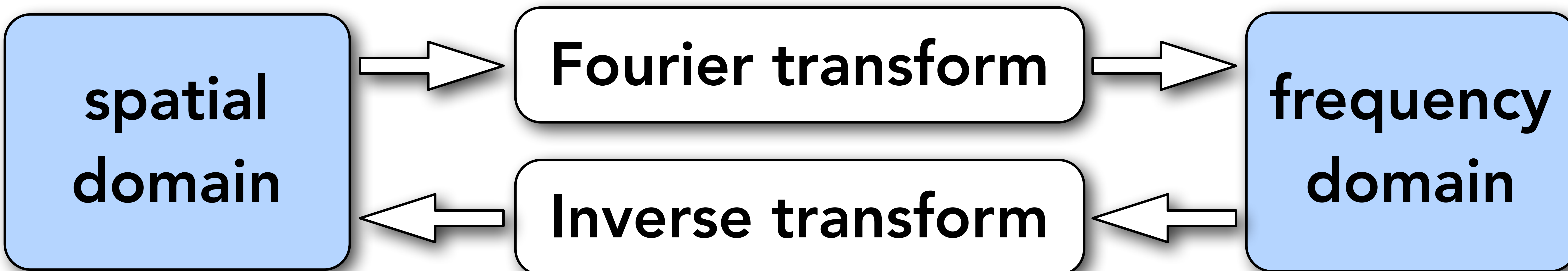
$$e^{ix} = \cos x + i \sin x$$

- **2D form:**

$$F(u,v) = \int\int f(x,y) e^{-2\pi i (ux+vy)} dx dy$$

Fourier transform decomposes a signal into its constituent frequencies

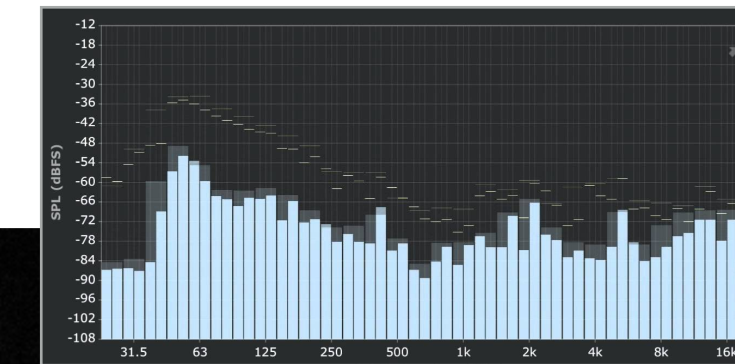
$$f(x) \qquad F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx \qquad F(\omega)$$



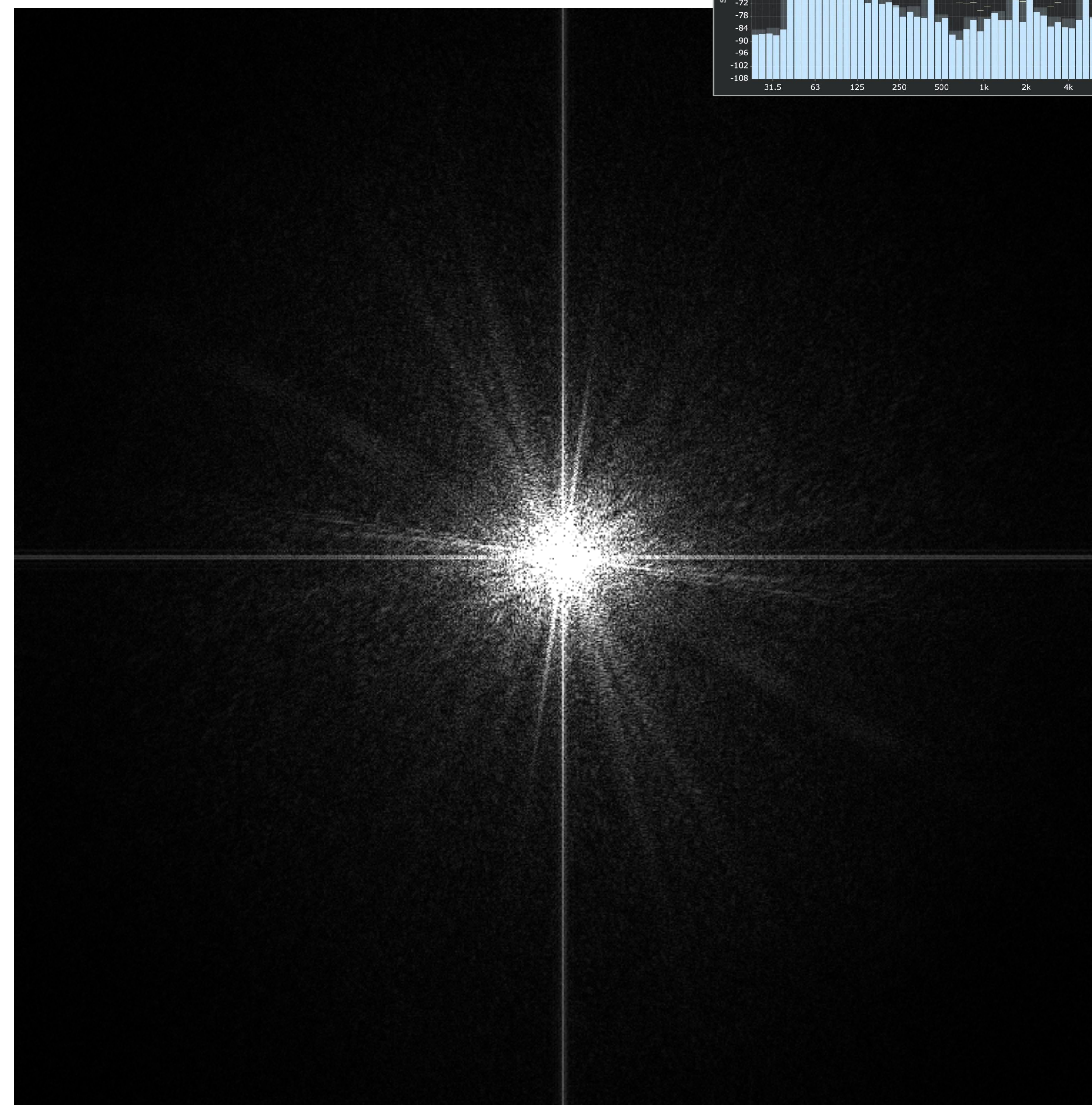
$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

Visualizing the frequency content of images

Visualization below is the 2D frequency domain equivalent of the 1D audio spectrum I showed you earlier *

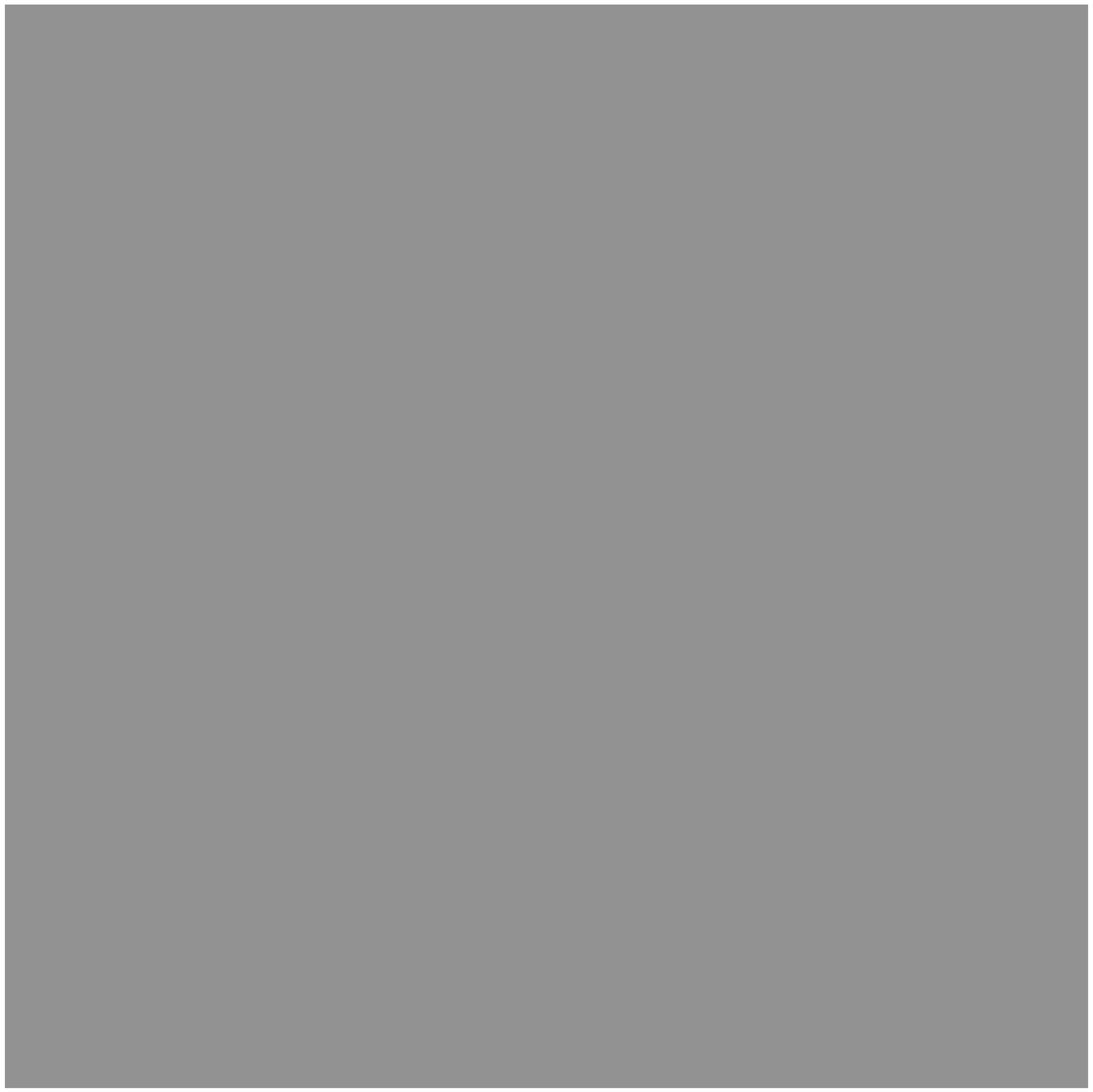


Spatial domain result

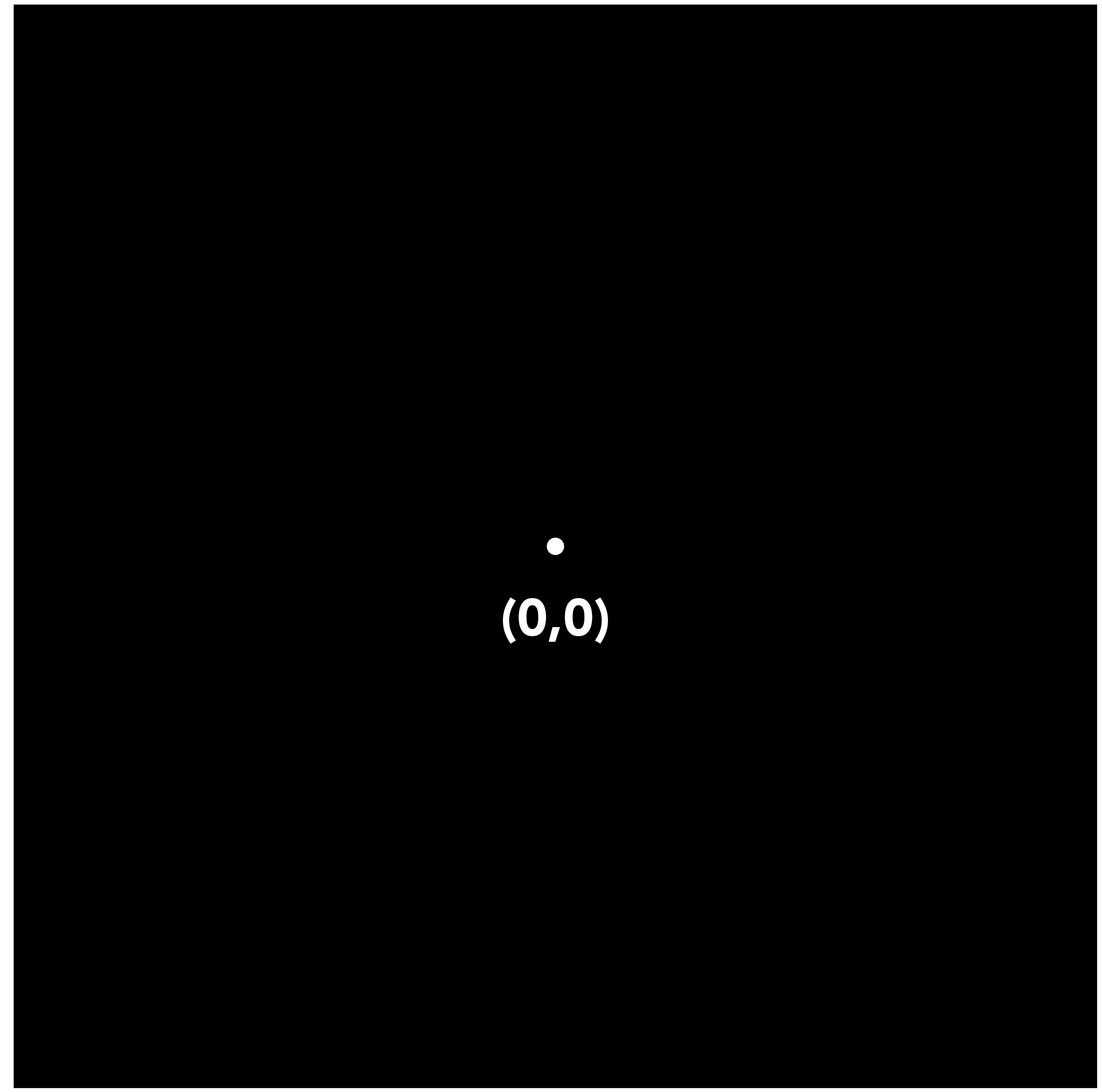


Spectrum

Constant signal (in primal domain)

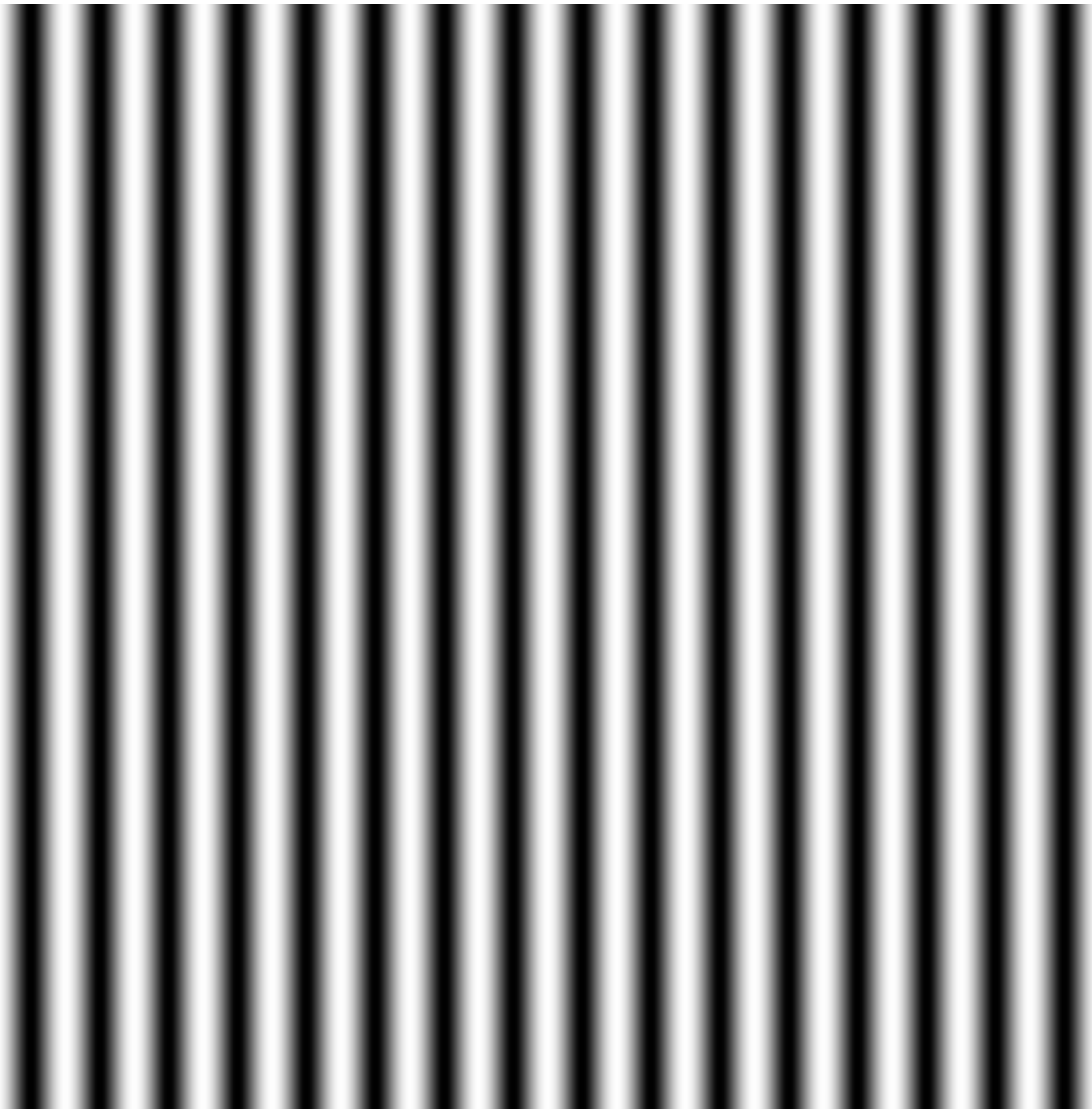


Spatial domain

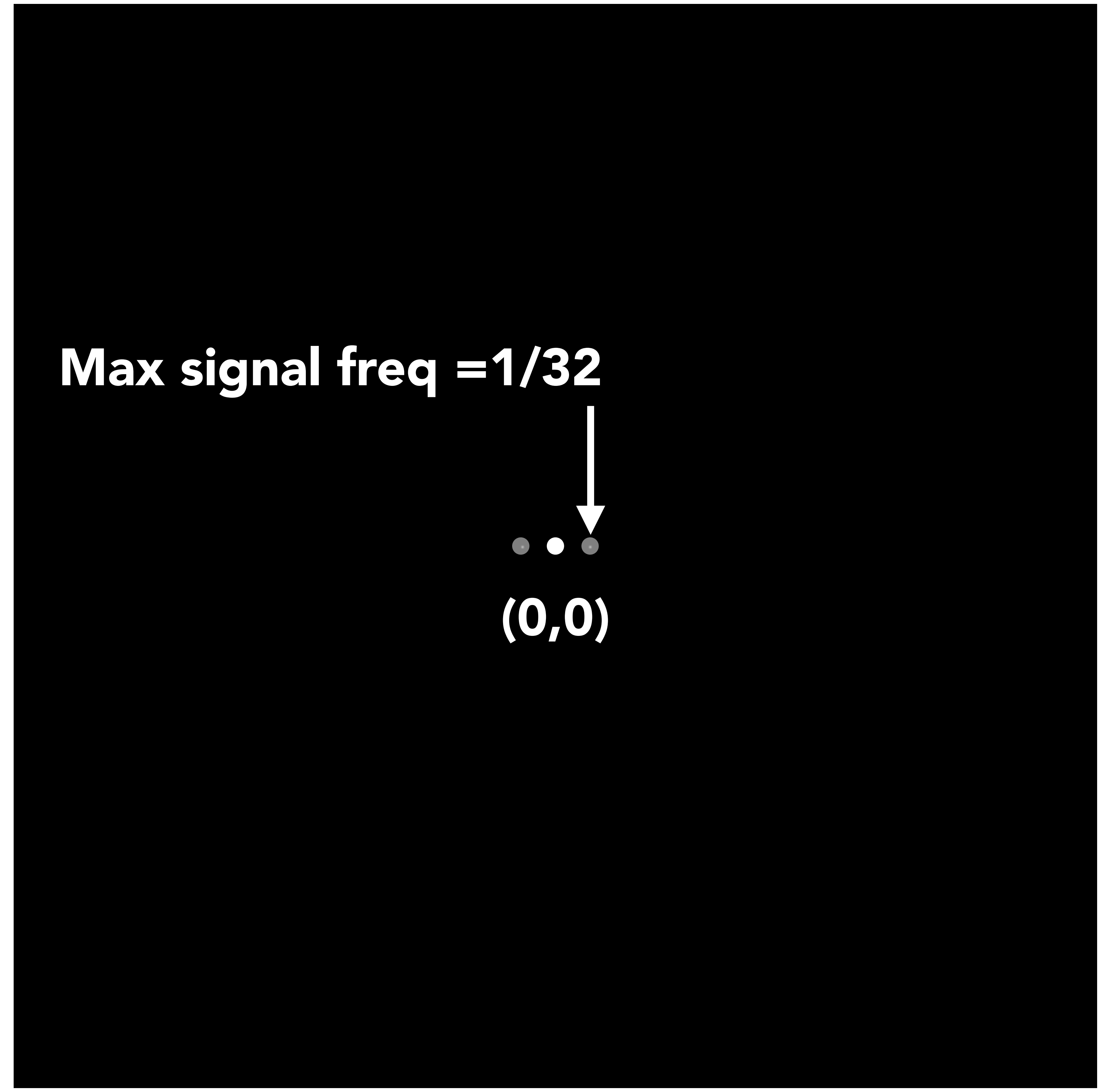


Frequency domain

$\sin(2\pi/32)x$ — frequency 1/32; 32 pixels per cycle

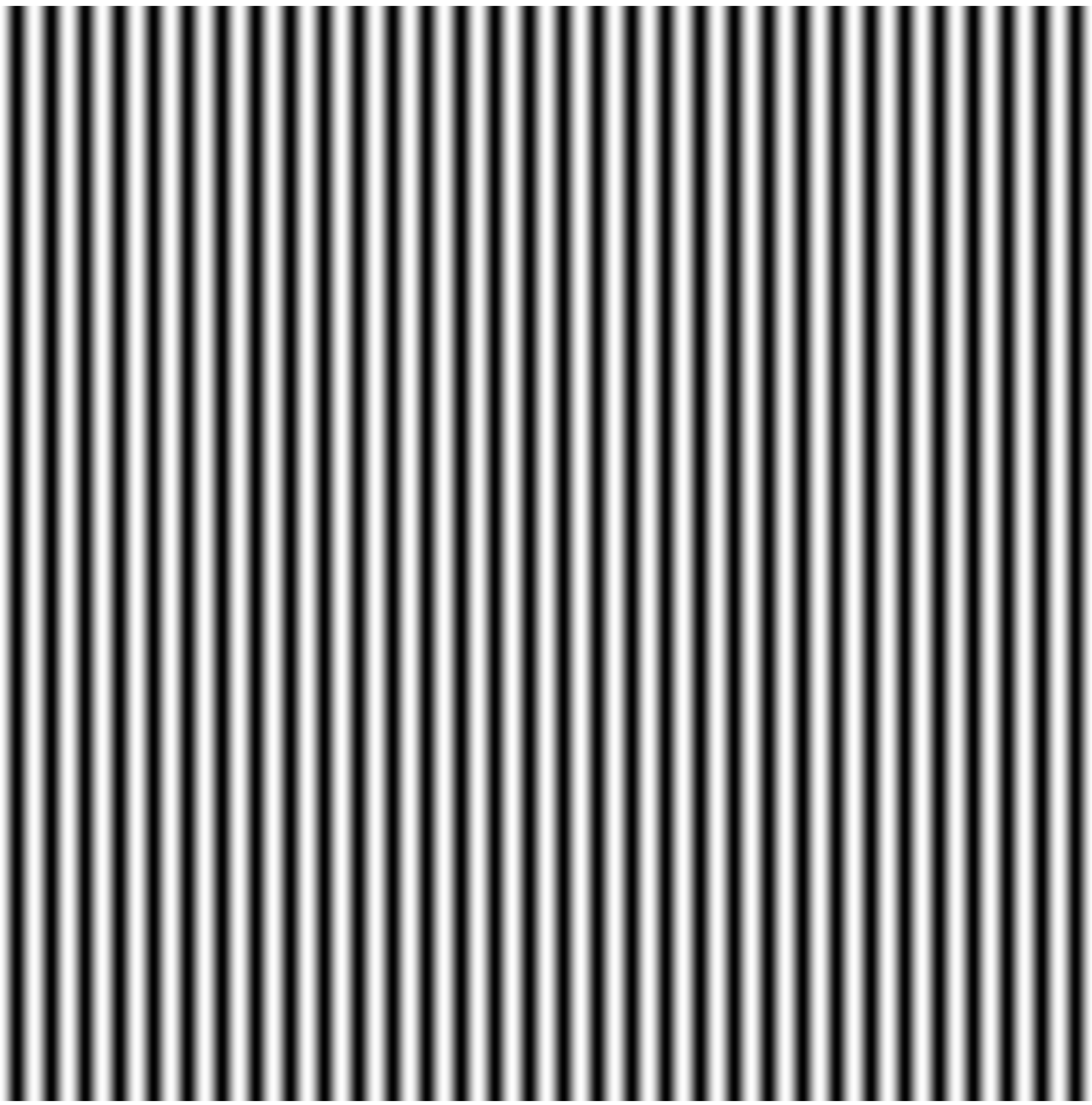


Spatial domain

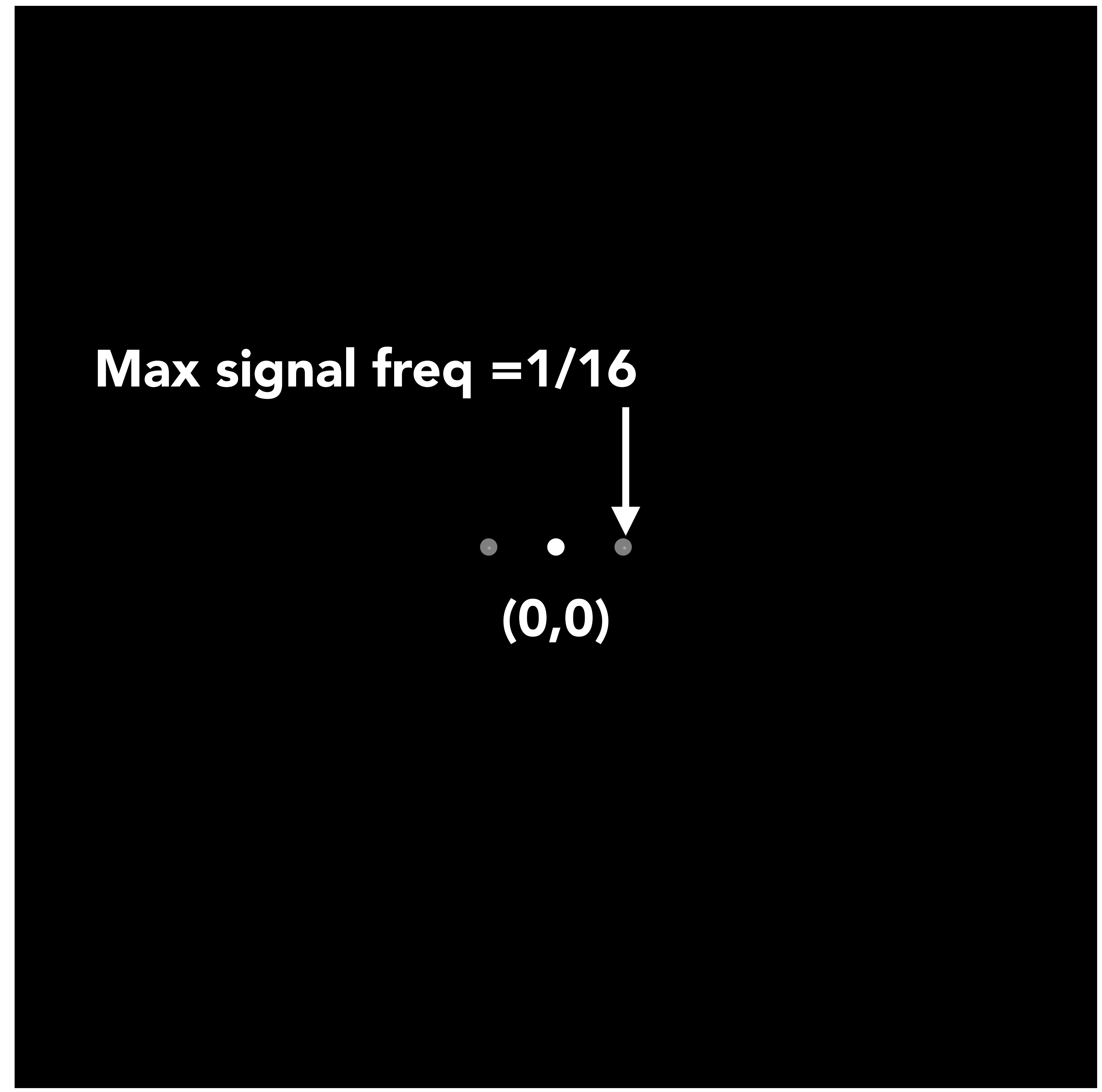


Frequency domain

$\sin(2\pi/16)x$ — frequency 1/16; 16 pixels per cycle

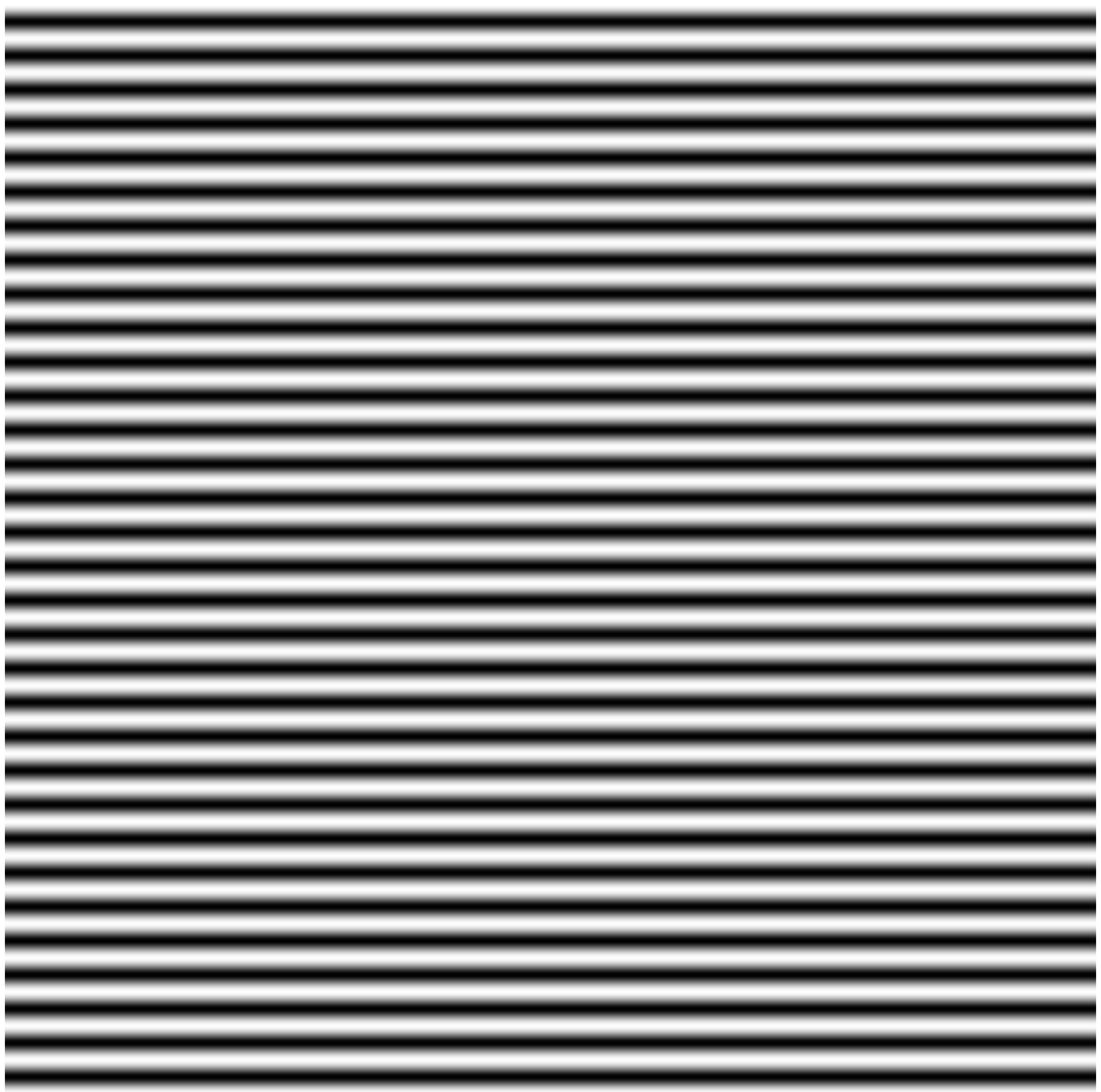


Spatial domain

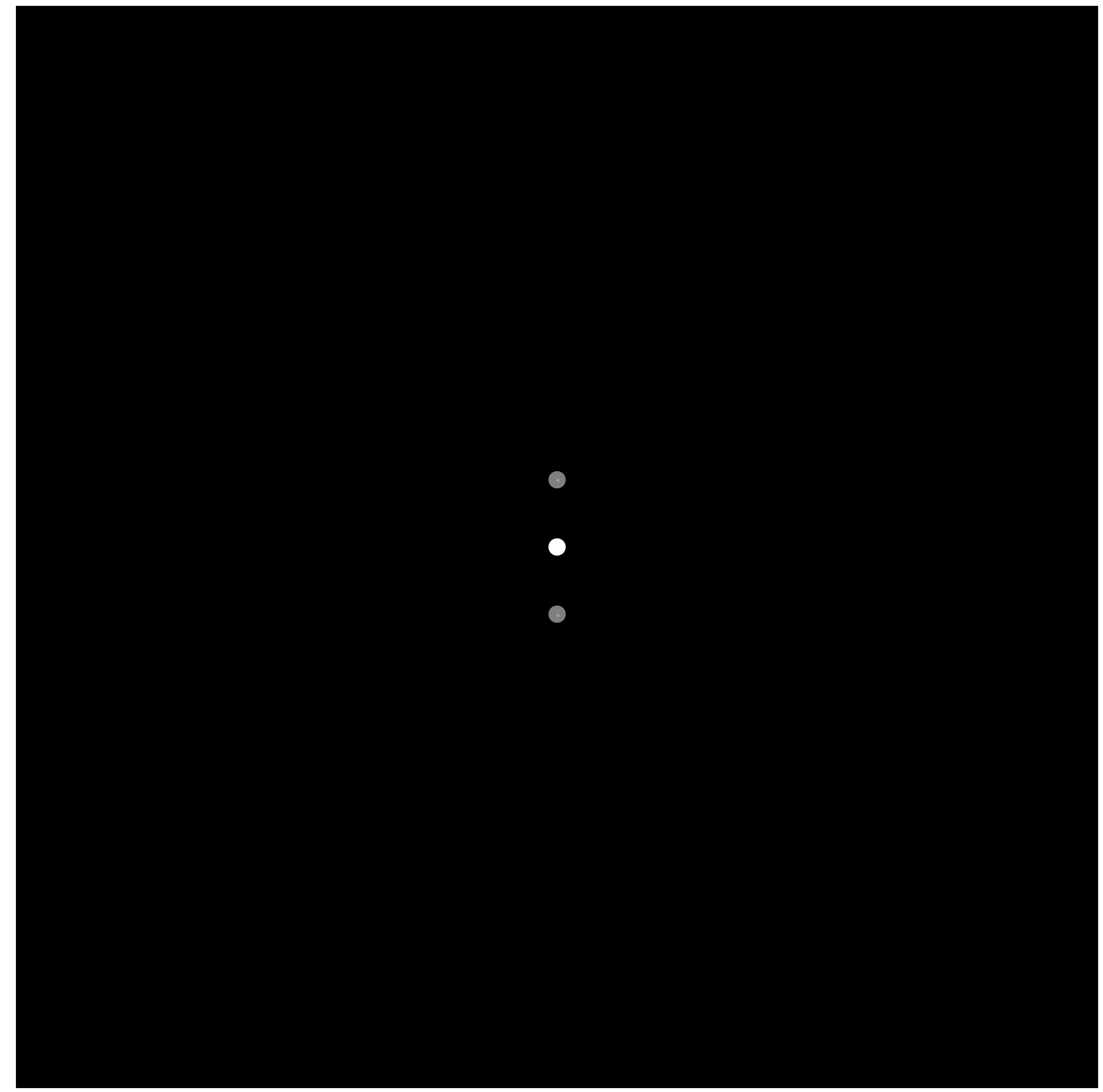


Frequency domain

$$\sin(2\pi/16)y$$

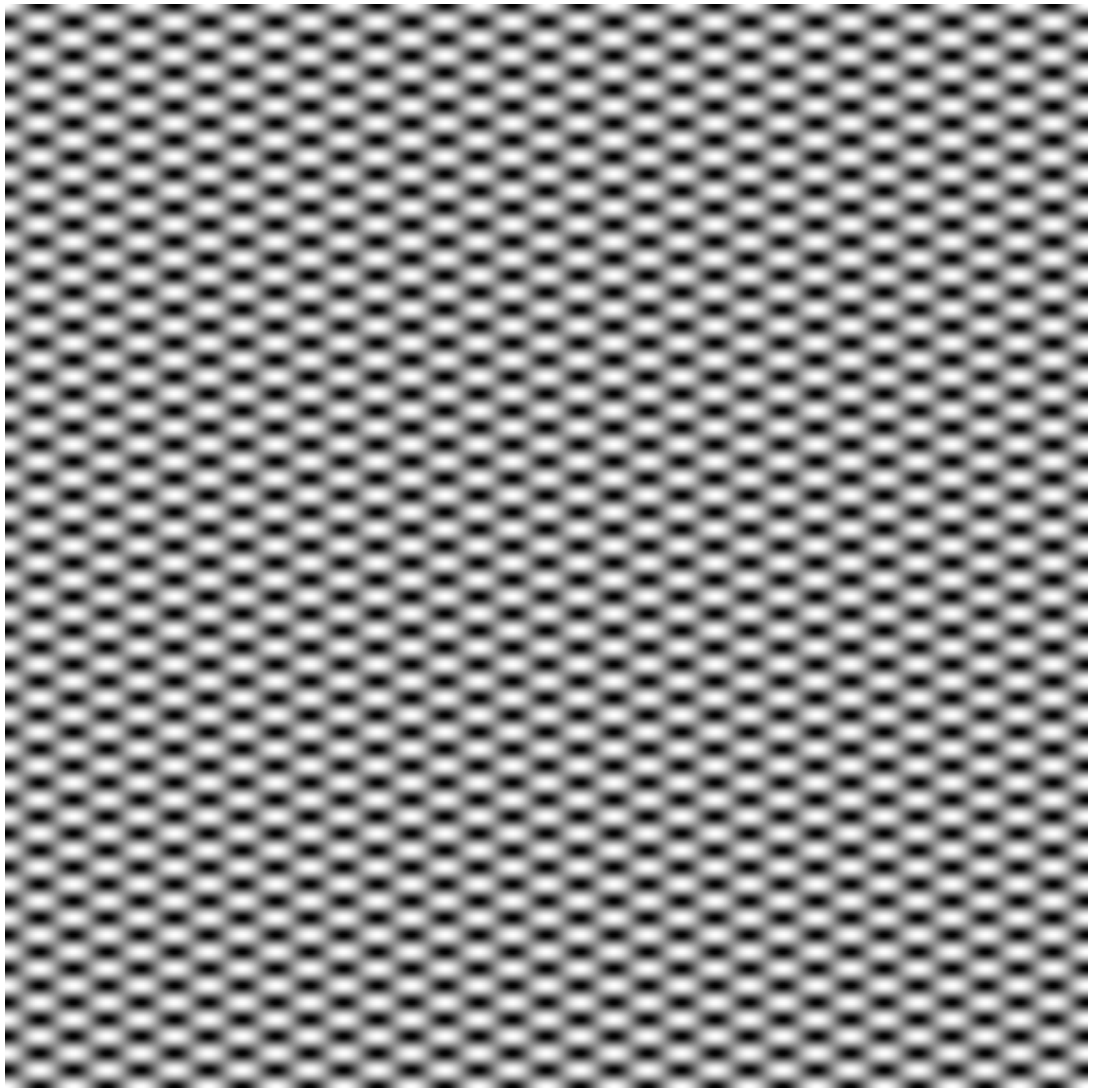


Spatial domain

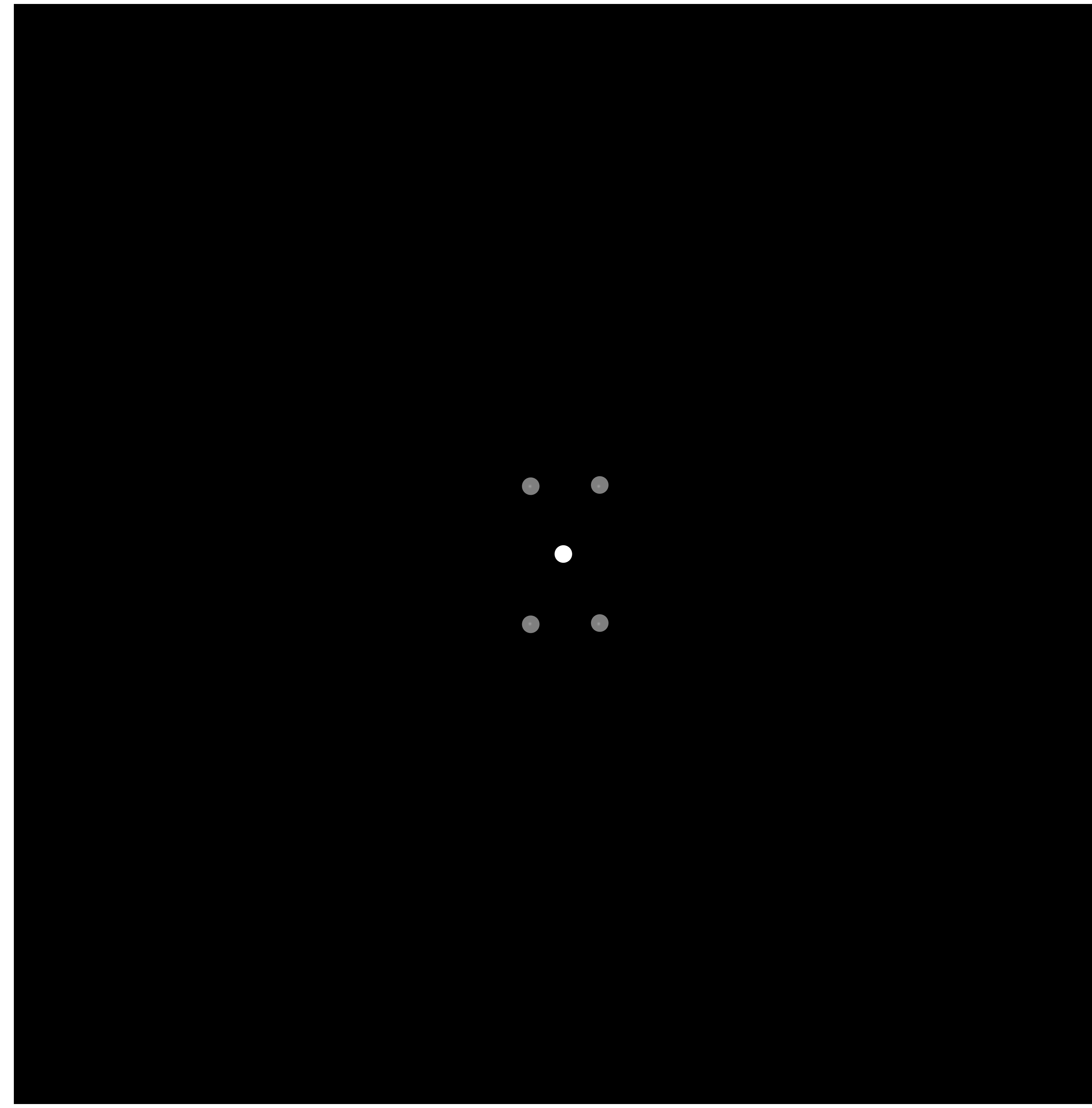


Frequency domain

$$\sin(2\pi/32)x \times \sin(2\pi/16)y$$

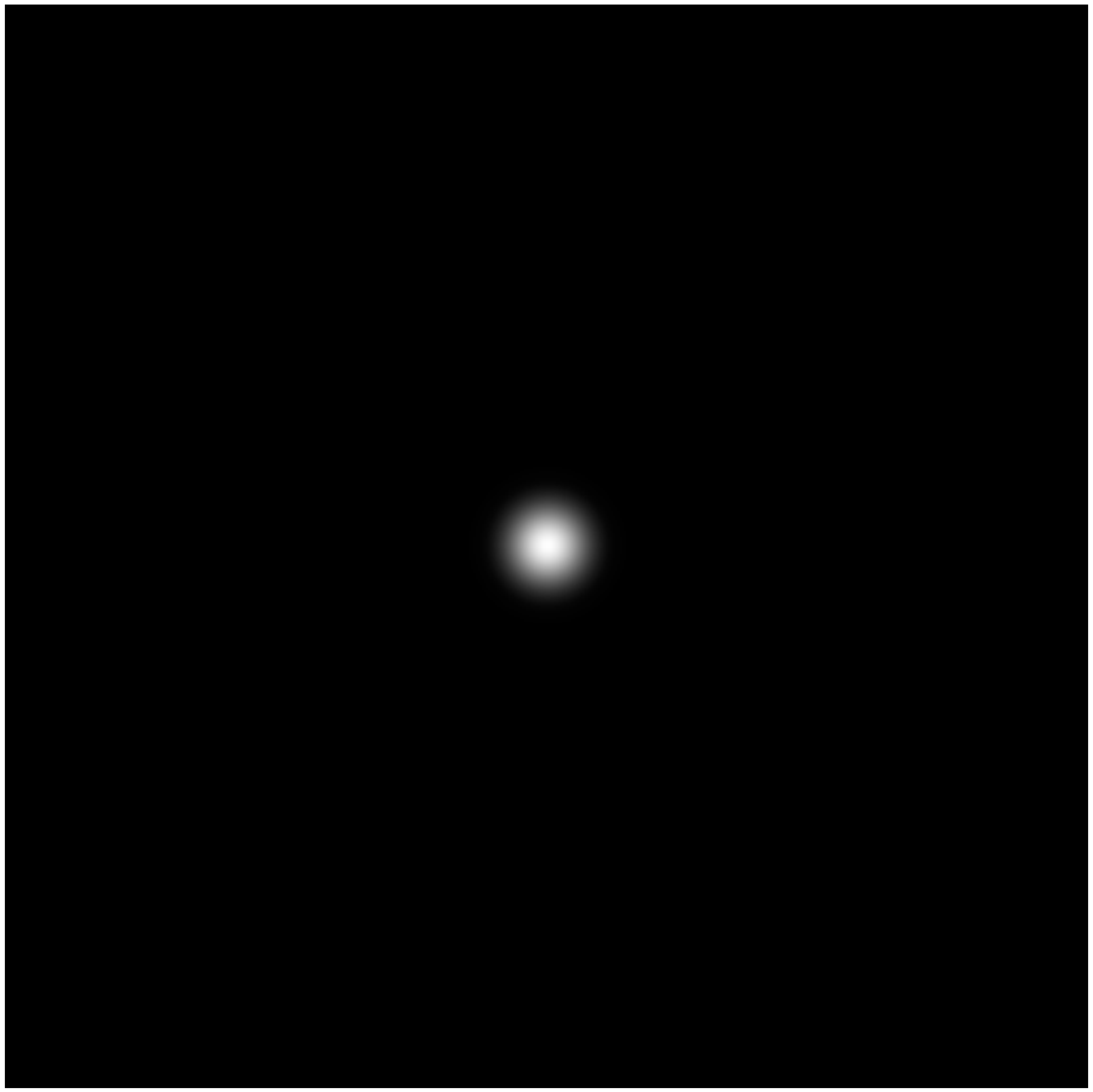


Spatial domain

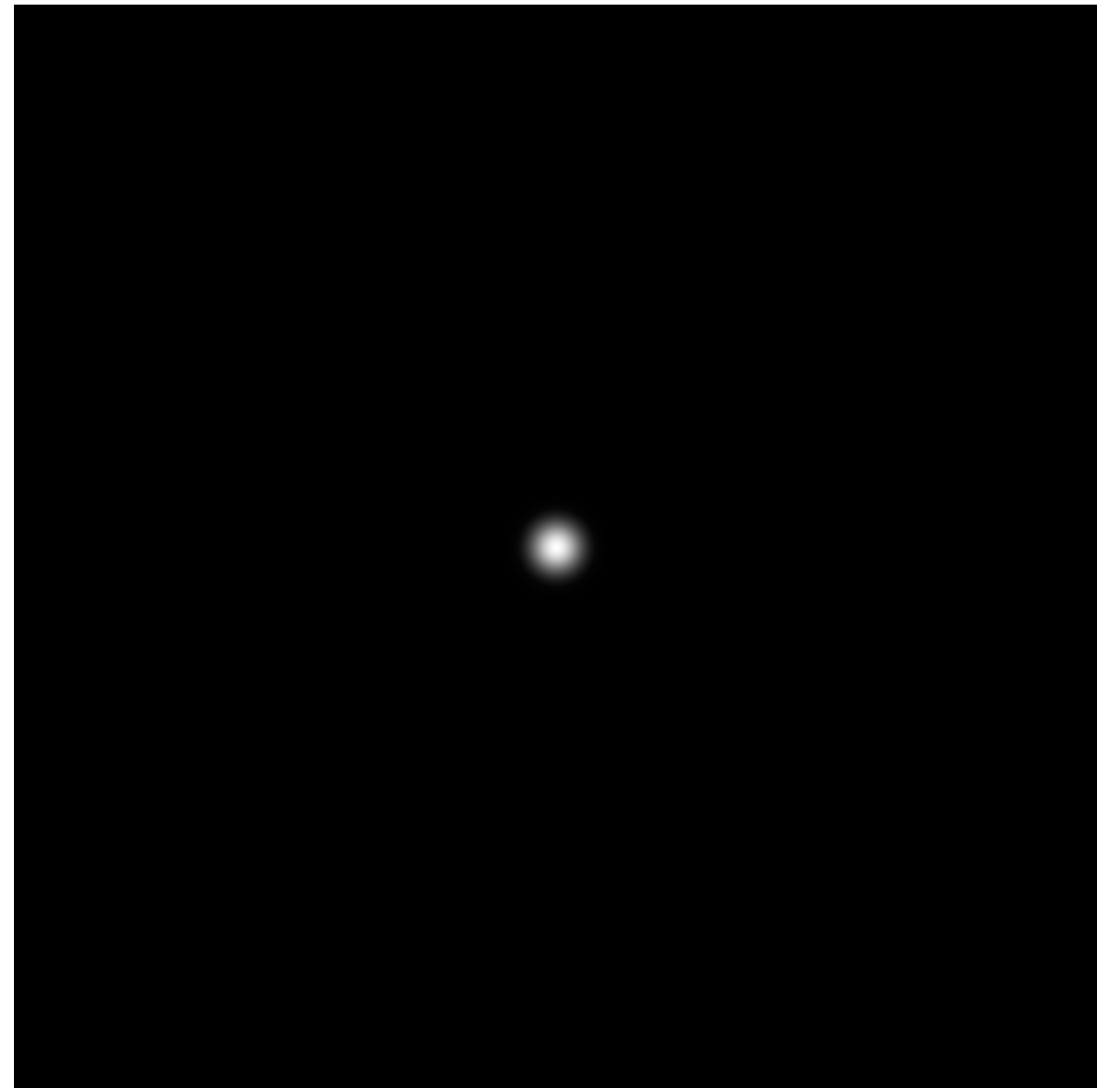


Frequency domain

$$\exp(-r^2/16^2)$$

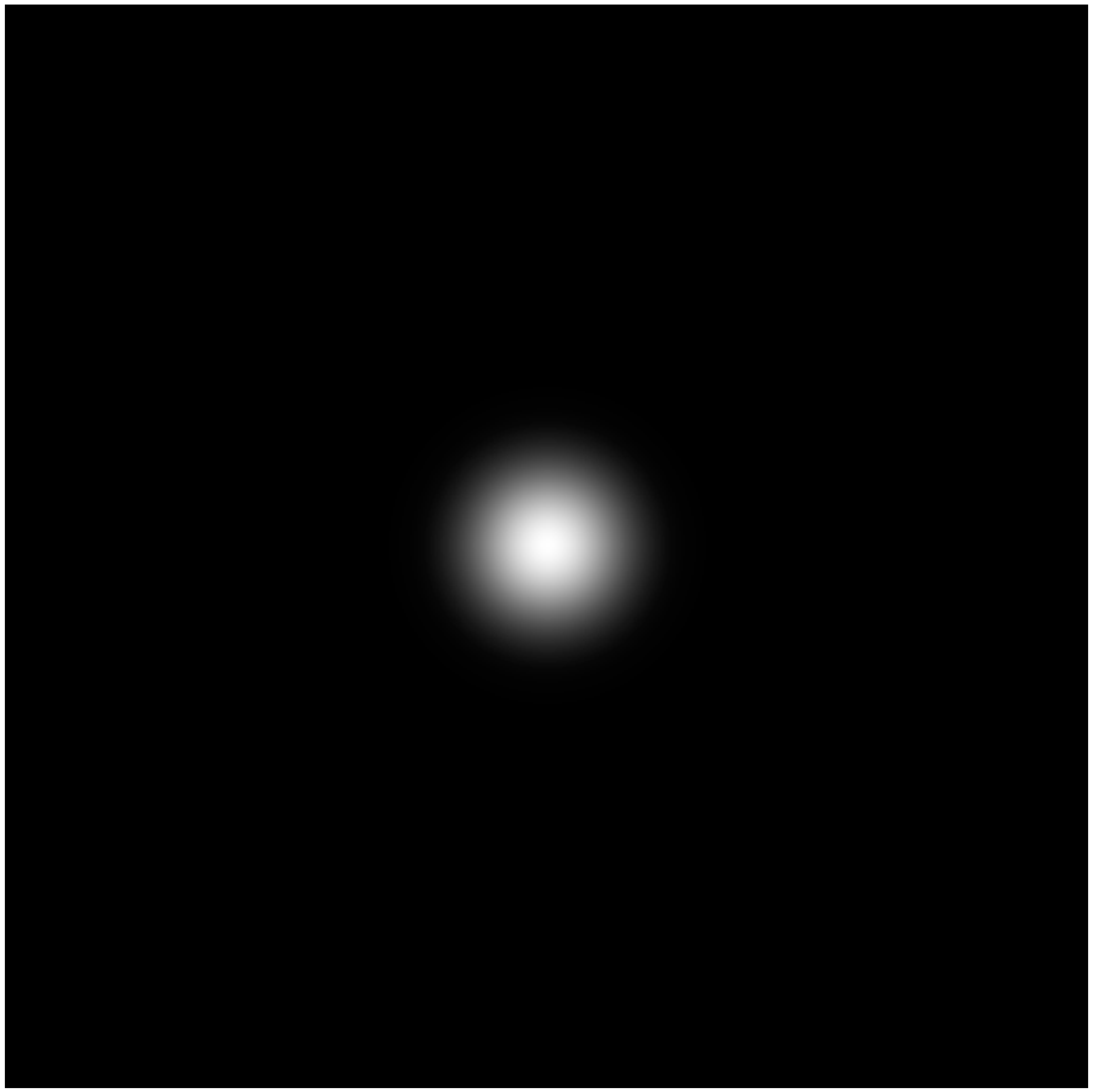


Spatial domain

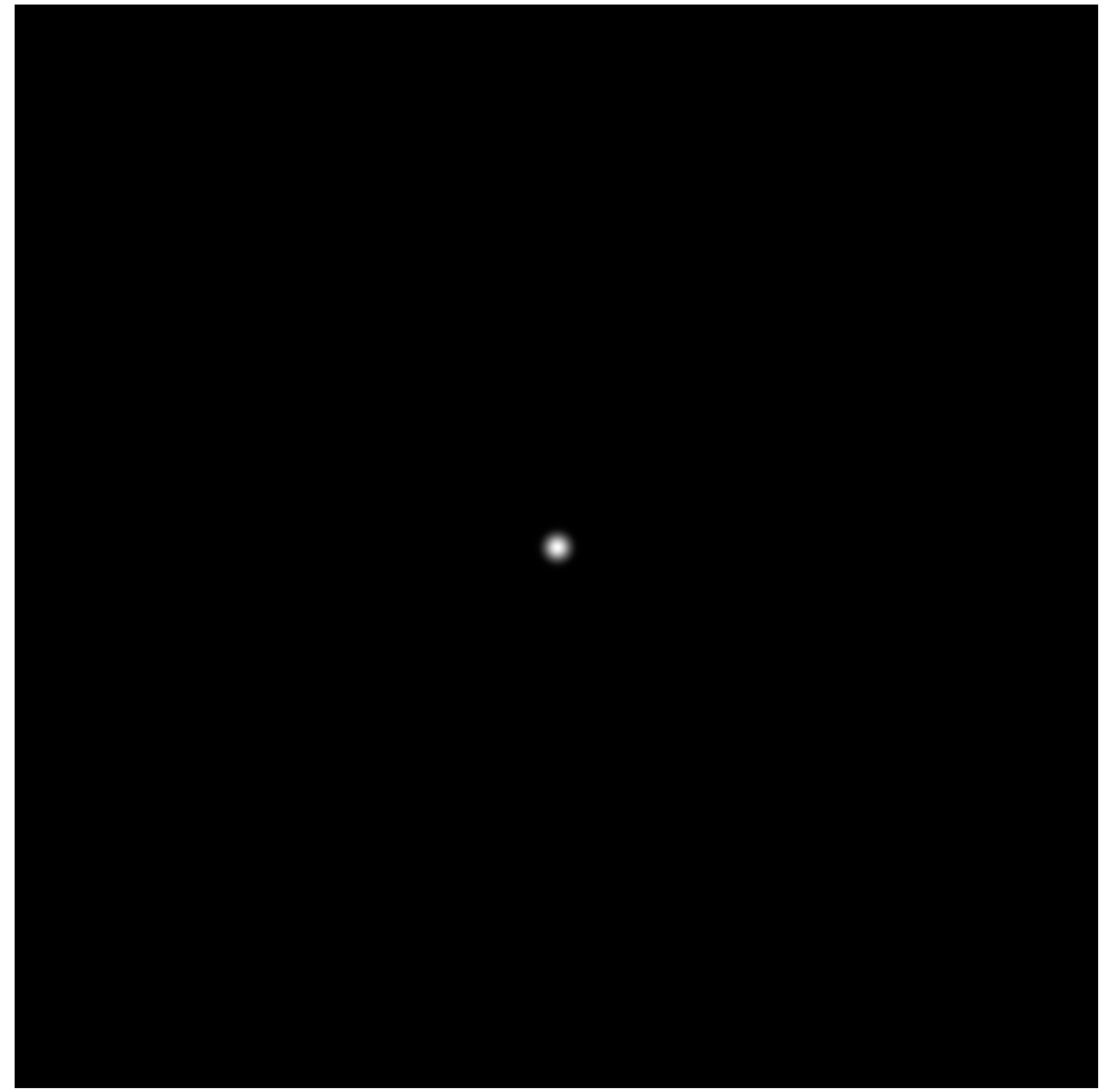


Frequency domain

$$\exp(-r^2/32^2)$$

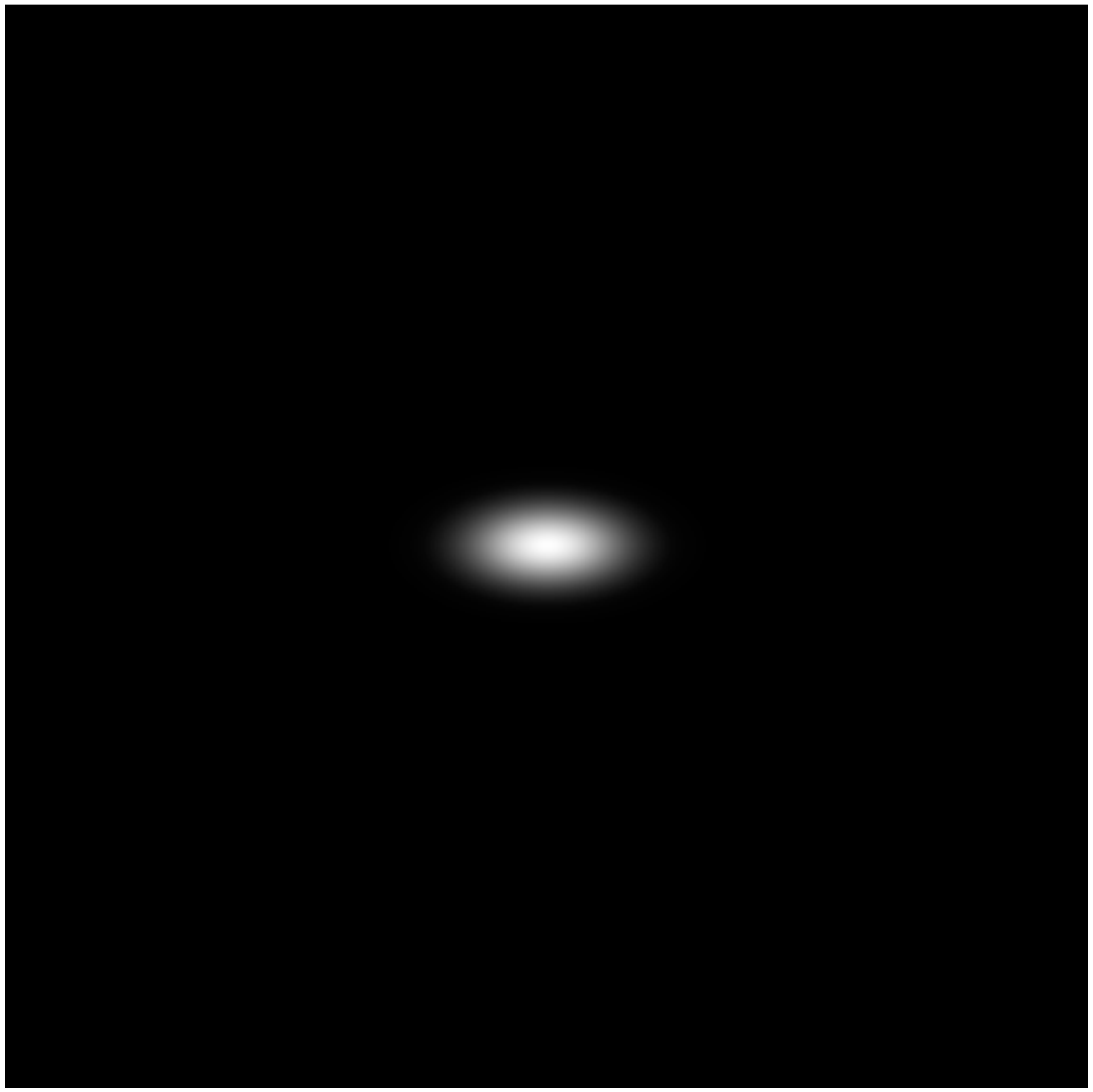


Spatial domain

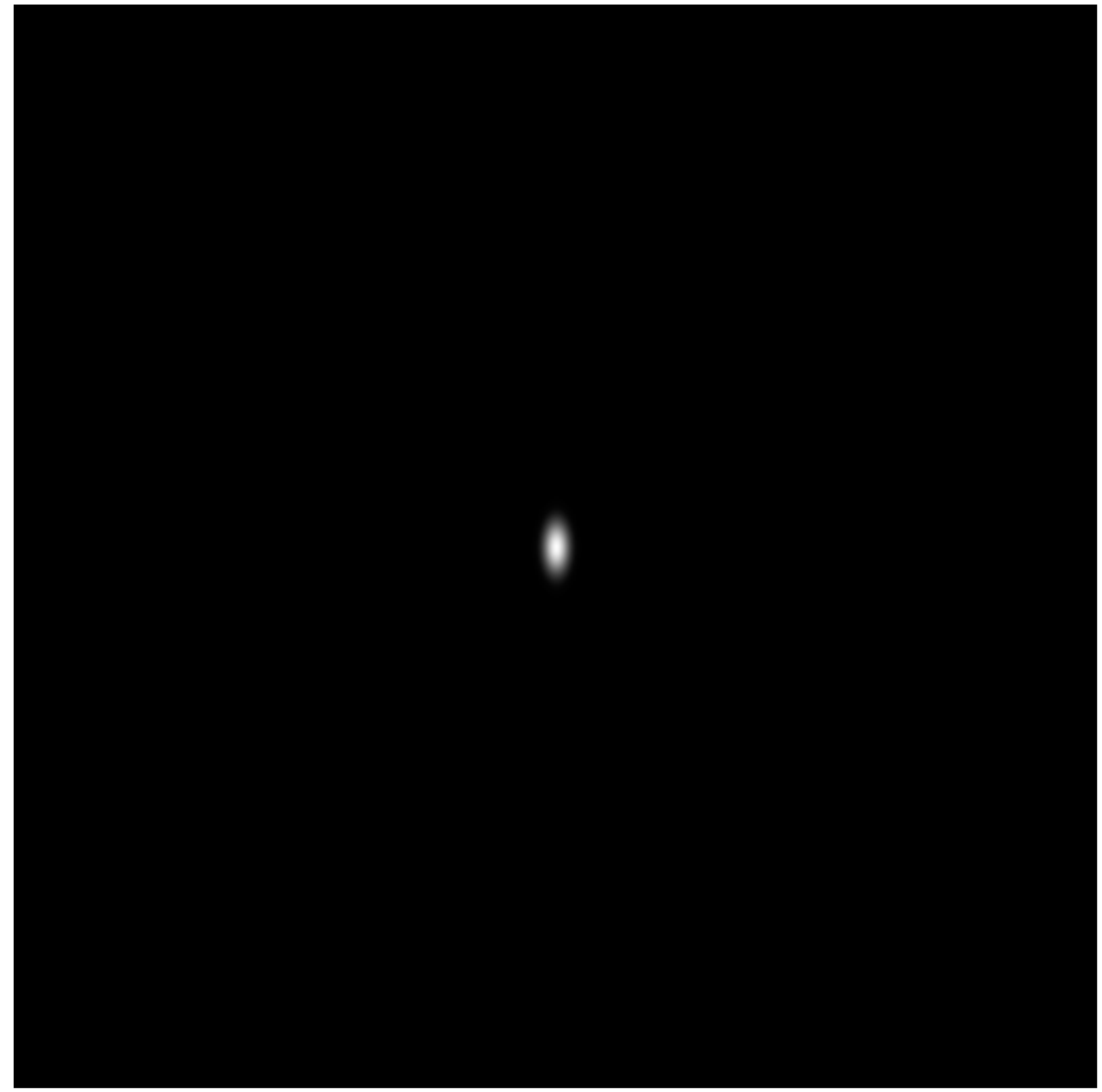


Frequency domain

$$\exp(-x^2/32^2) \times \exp(-y^2/16^2)$$



Spatial domain

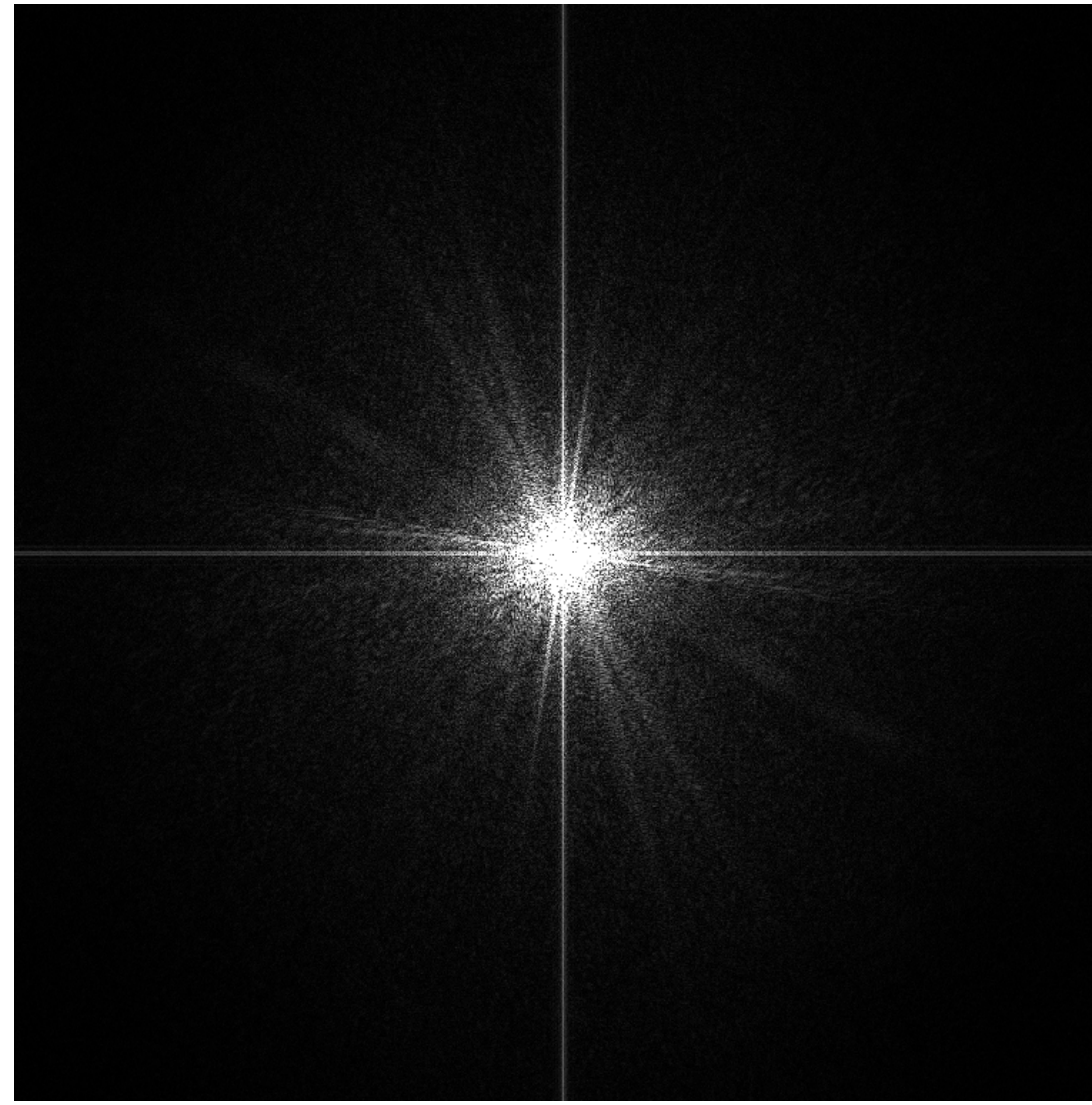


Frequency domain

Image filtering
(in the frequency domain)

Manipulating the frequency content of images

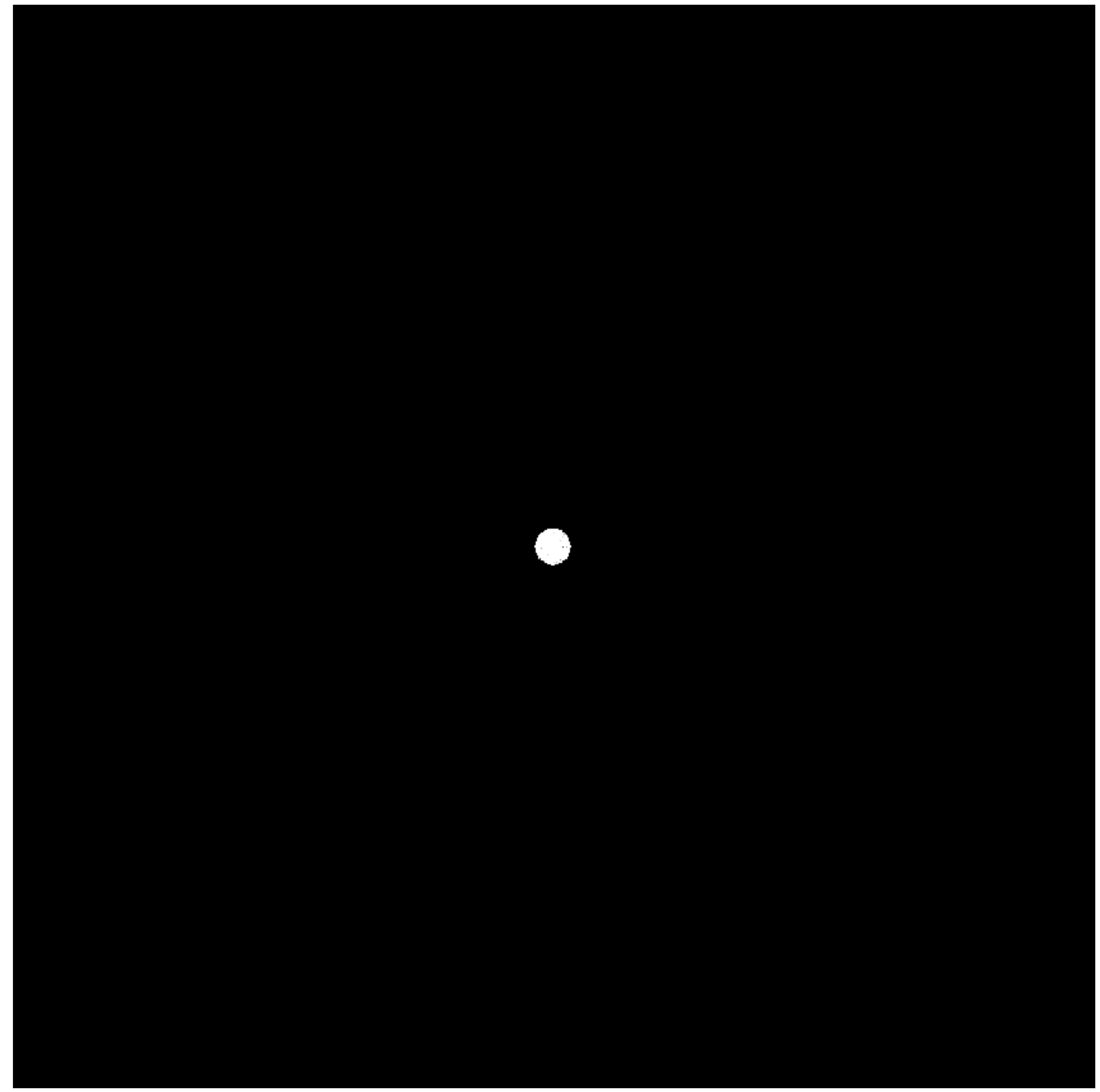
Spatial domain



Frequency domain

Low frequencies only (smooth gradients)

Spatial domain



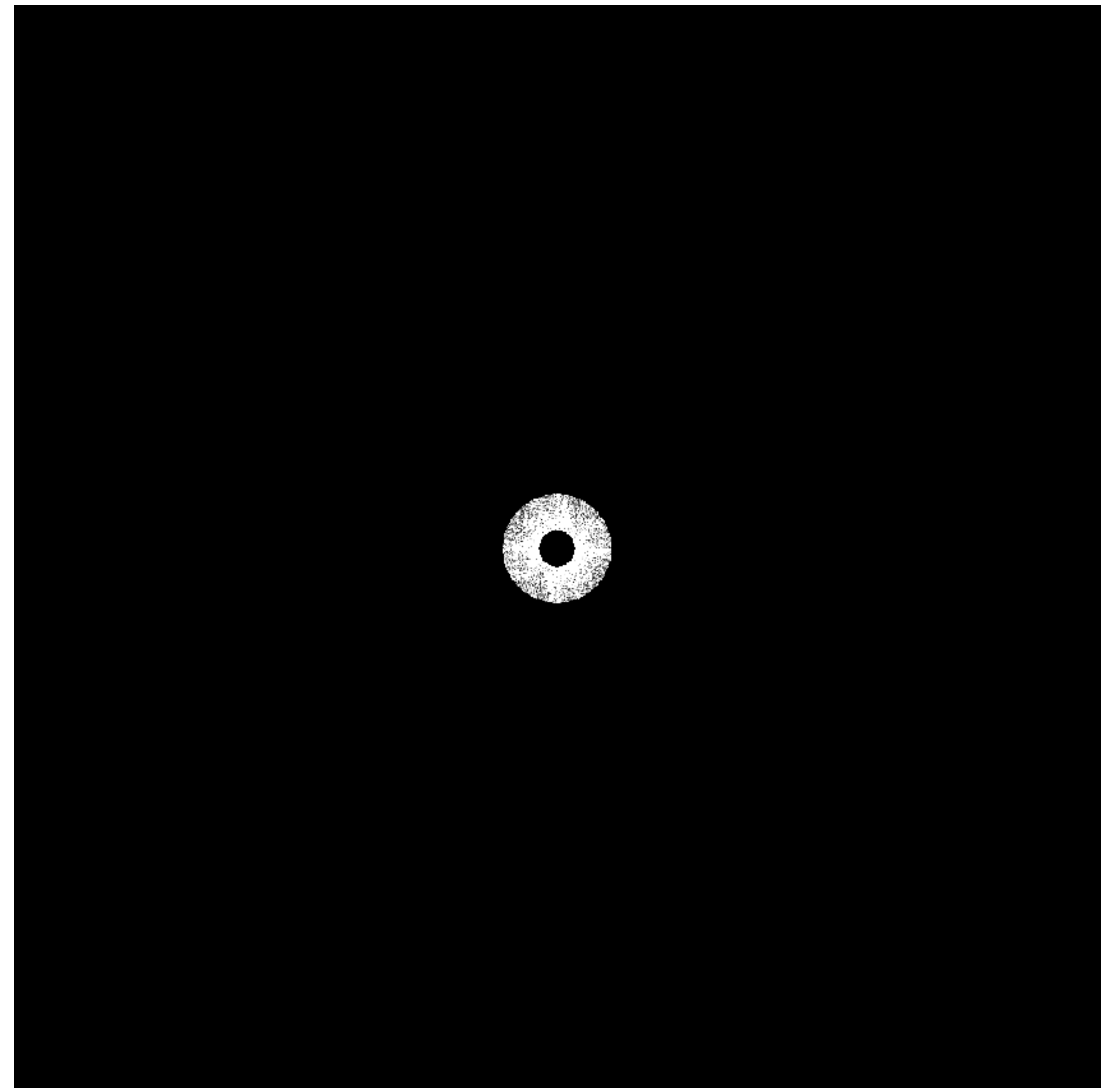
Frequency domain

(after low-pass filter)

All frequencies above cutoff have 0 magnitude

Mid-range frequencies

Spatial domain

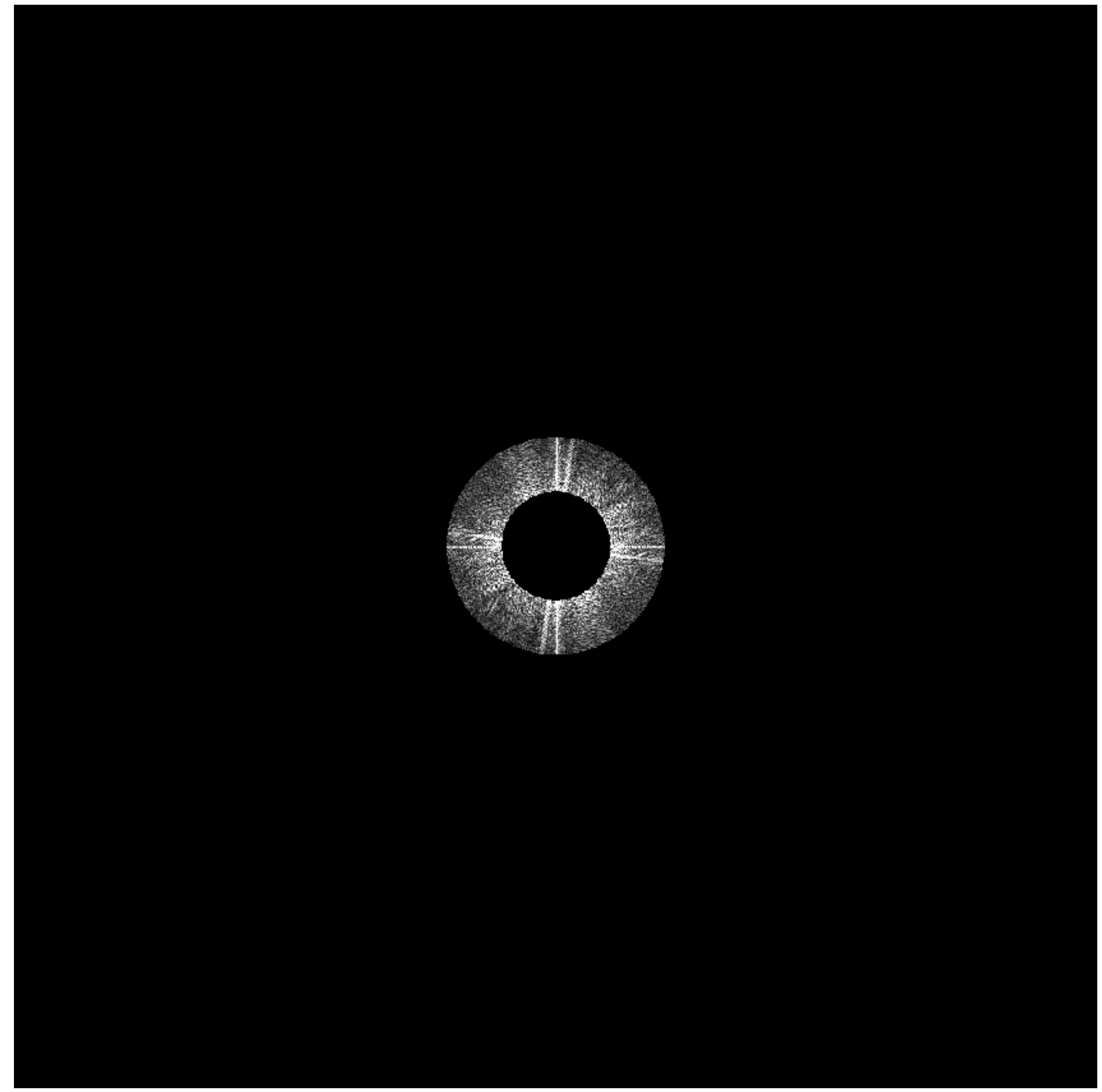


Frequency domain

(after band-pass filter)

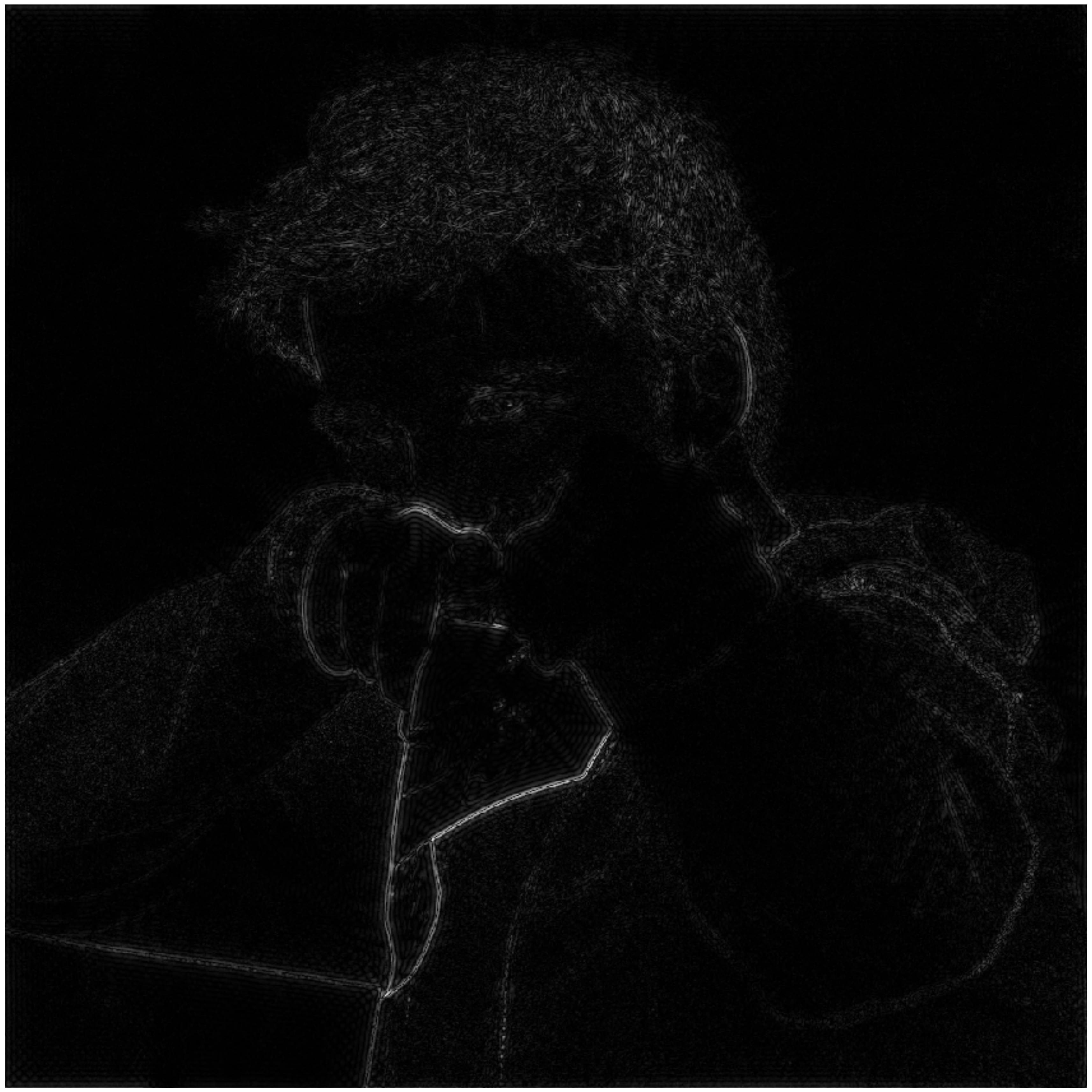
Mid-range frequencies

Spatial domain

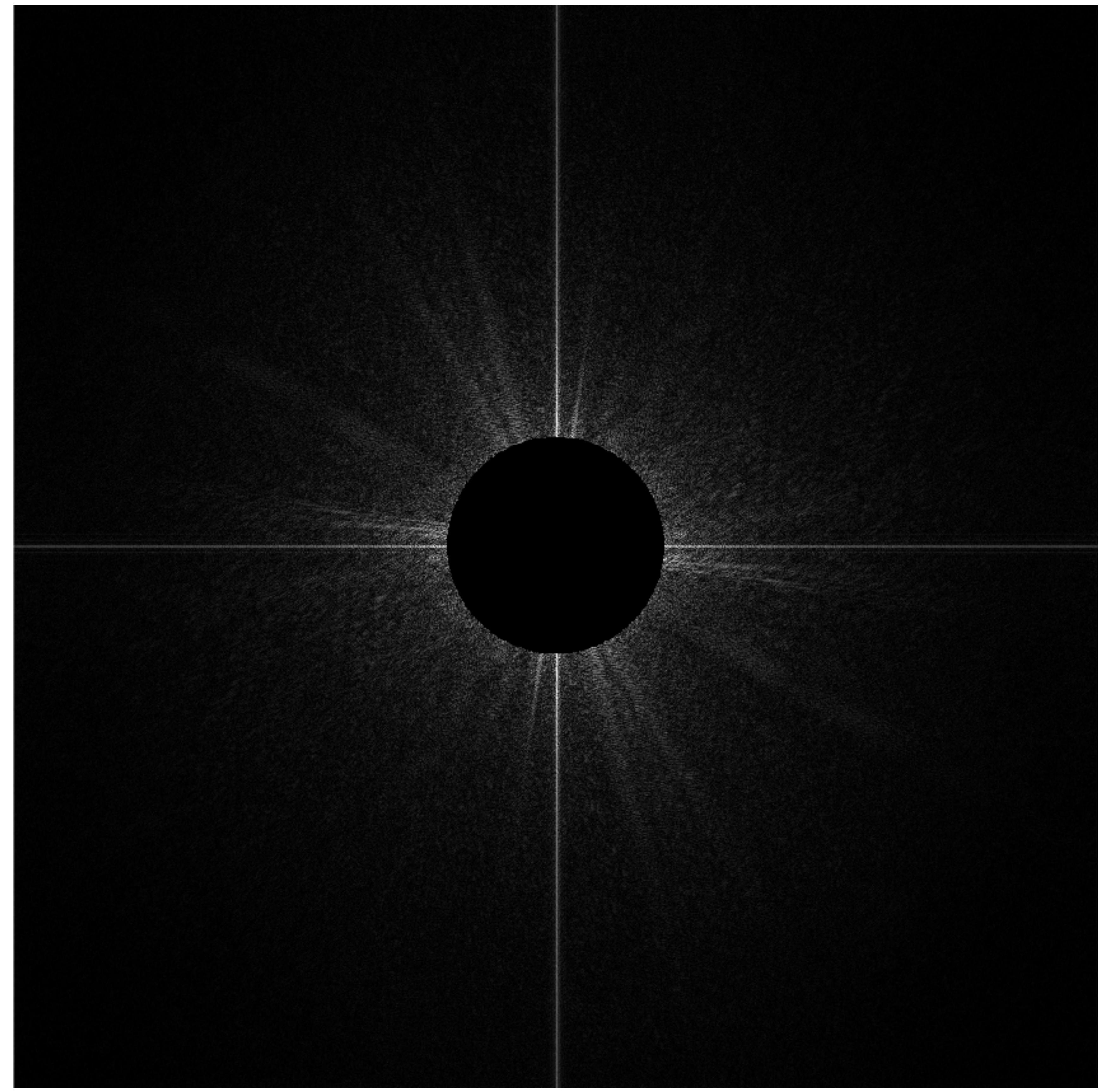


Frequency domain
(after band-pass filter)

High frequencies (edges)

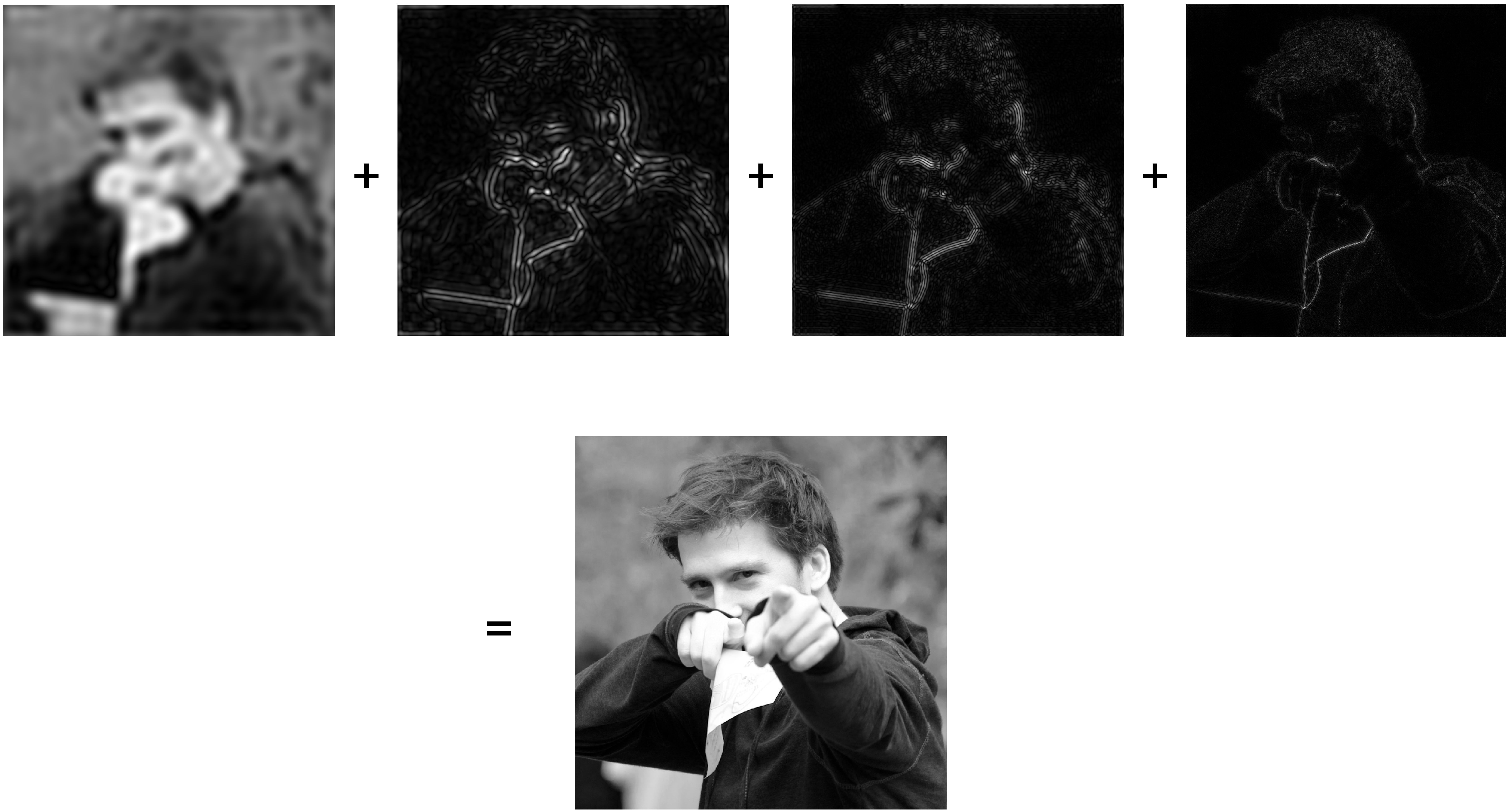


Spatial domain
(strongest edges)

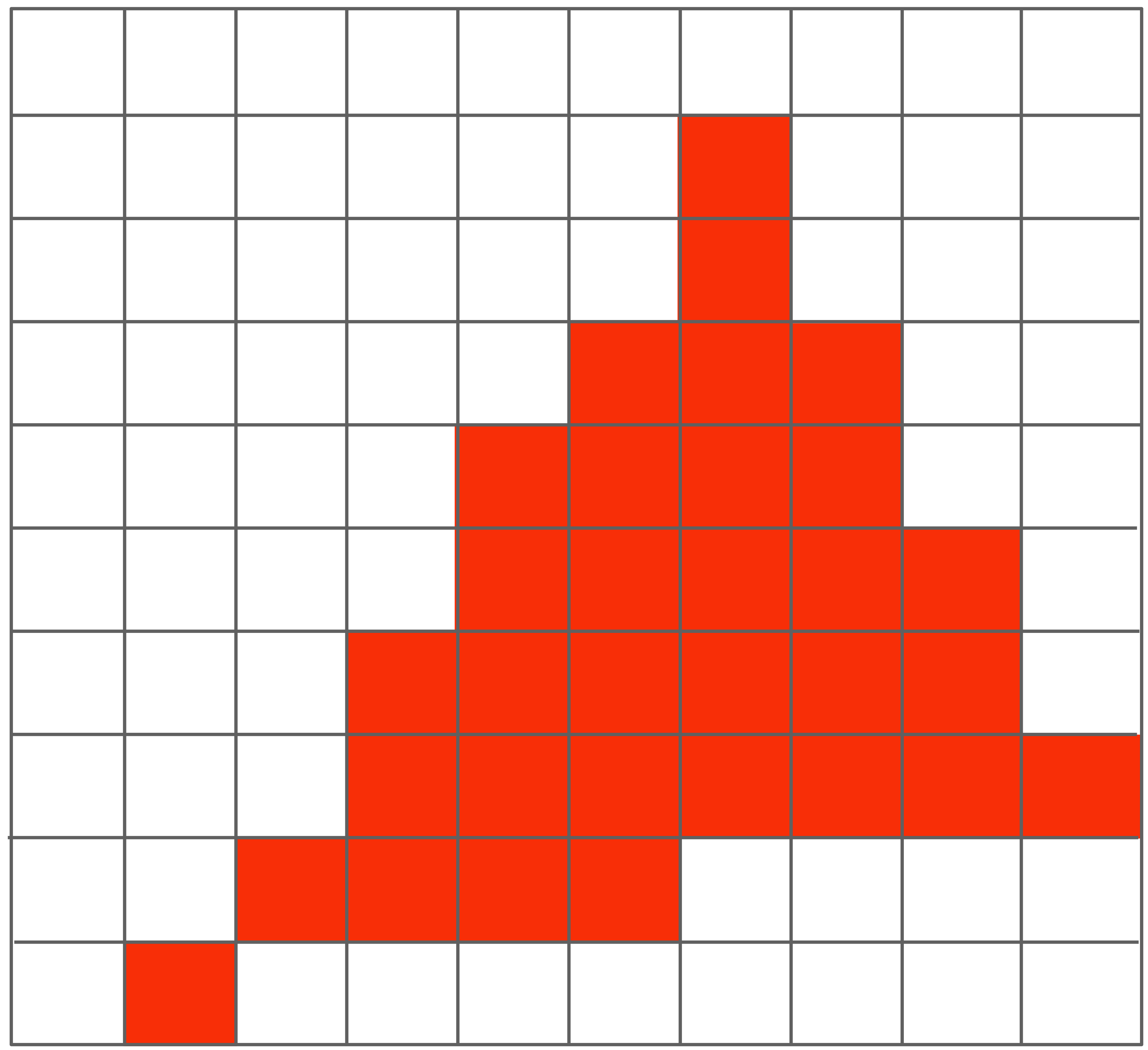


Frequency domain
(after high-pass filter)
All frequencies below threshold have 0 magnitude

An image as a sum of its frequency components

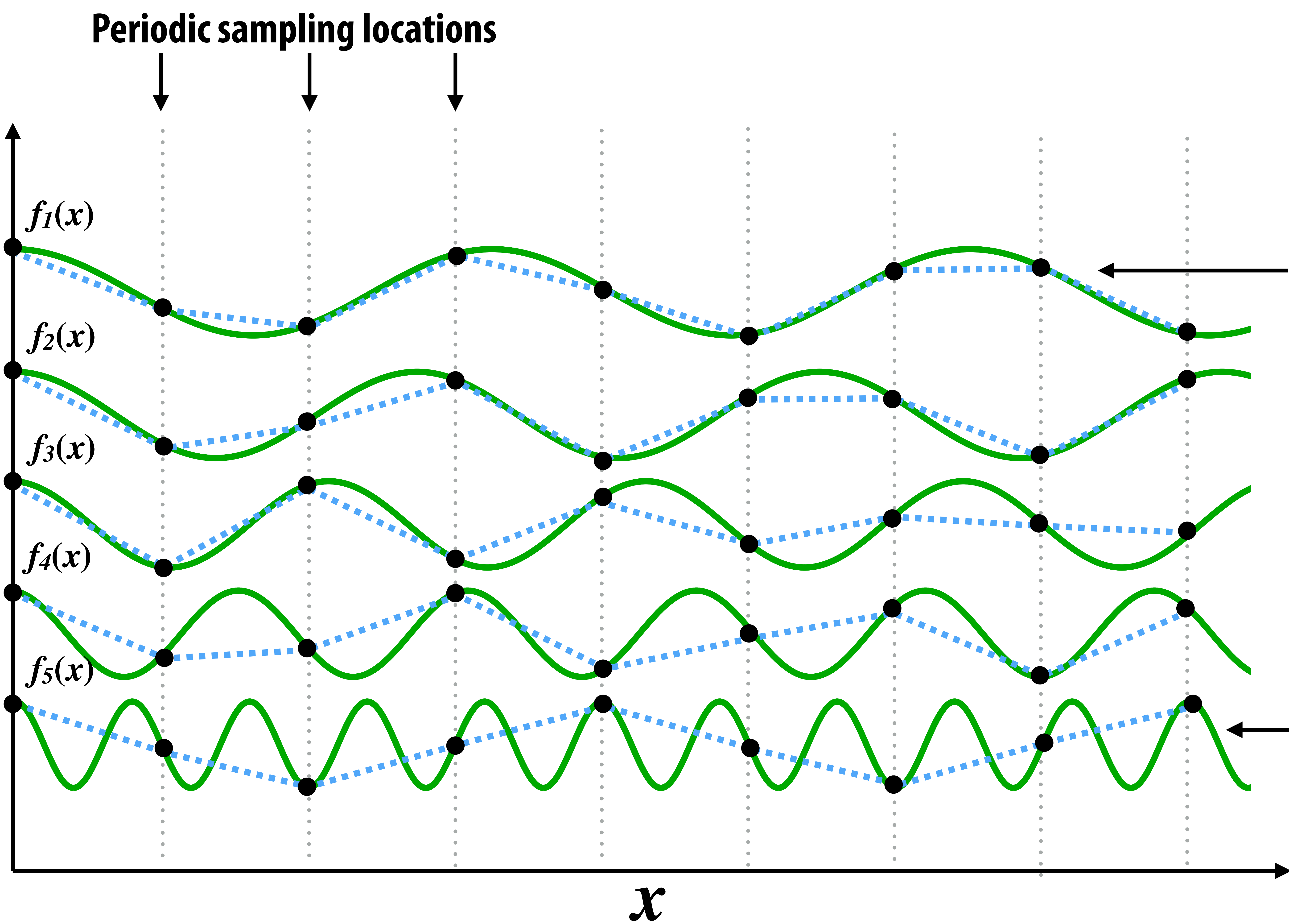


Back to our problem of artifacts in images



Jaggies!

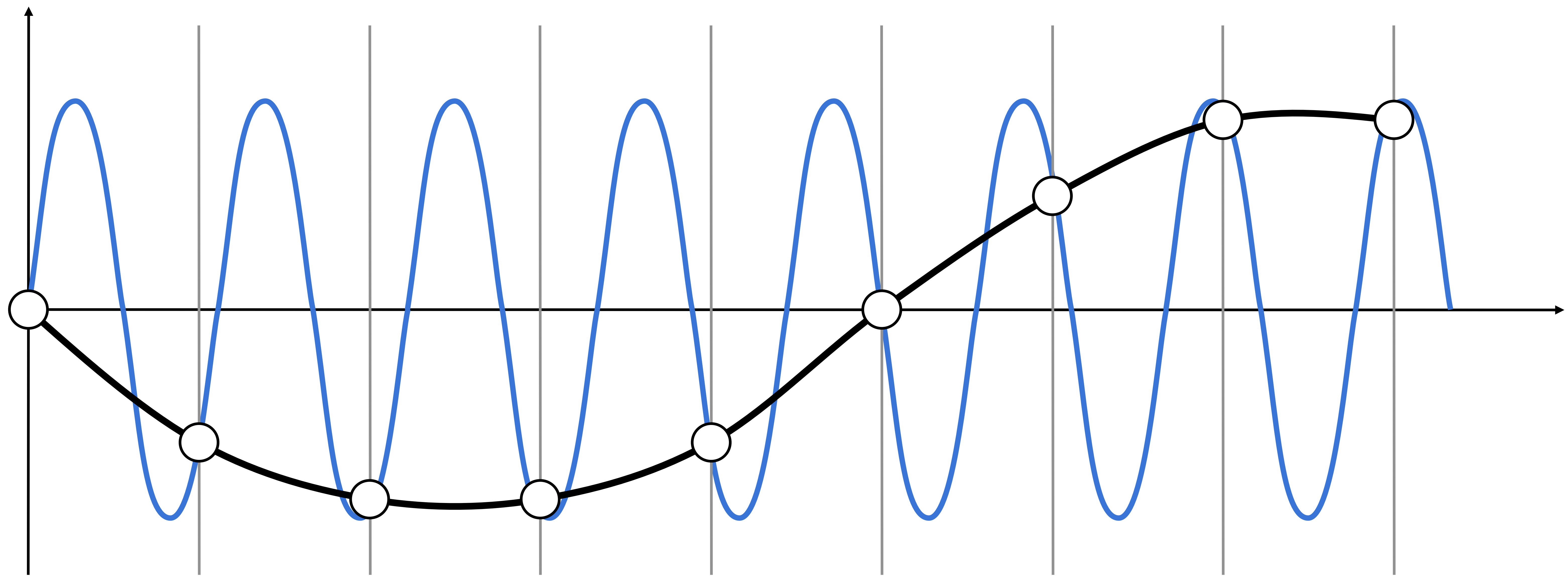
Higher frequencies need denser sampling



Low-frequency signal: sampled adequately for reasonable reconstruction

High-frequency signal is insufficiently sampled: reconstruction incorrectly appears to be from a low frequency signal

Undersampling creates frequency "aliases"



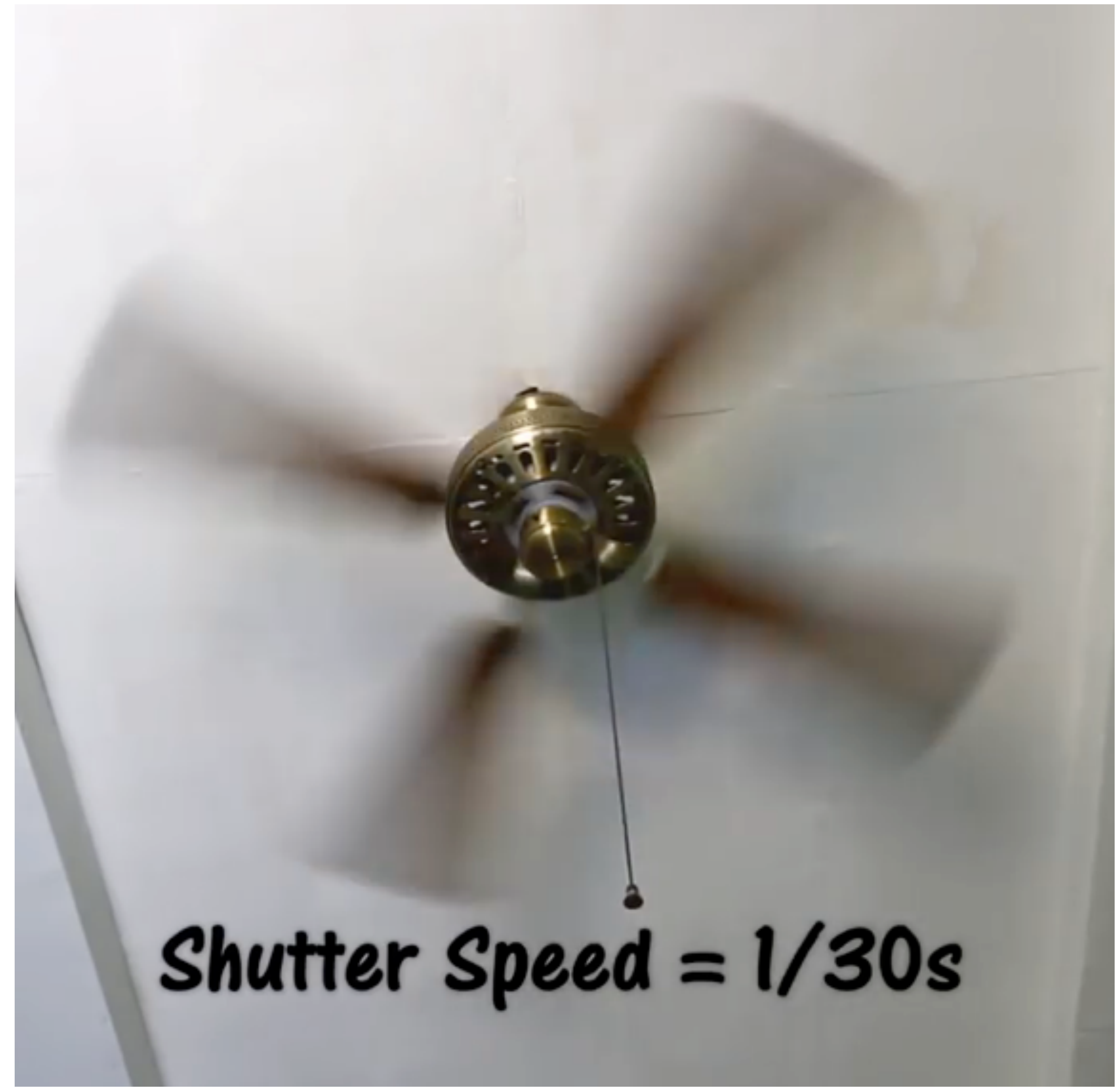
High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

Anti-aliasing idea: filter out high frequencies before sampling

Video: point vs antialiased sampling

Point in time



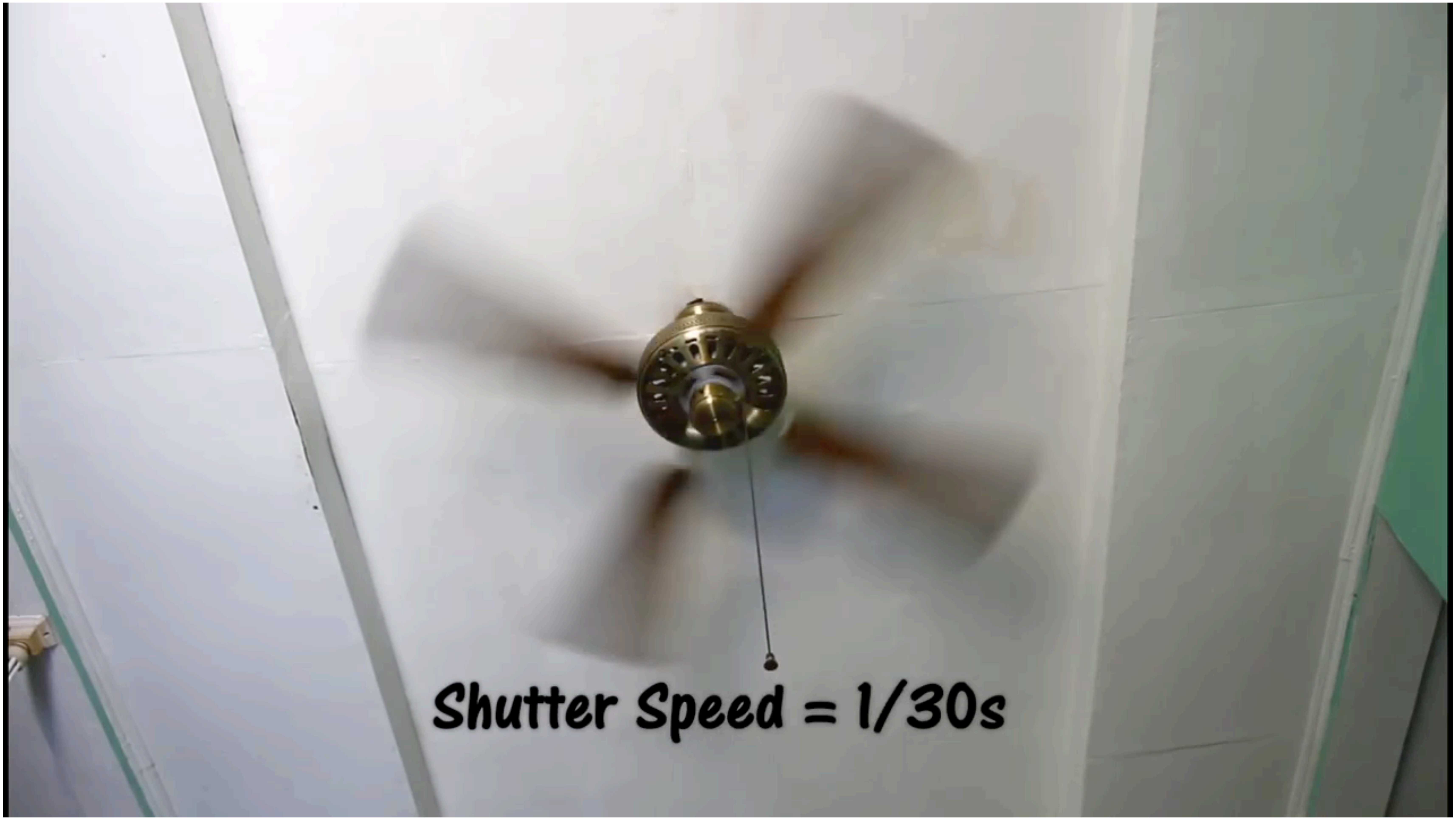
Motion blurred

Video: point sampling in time

Credit: Aris & cams youtube, https://youtu.be/NoWwxTktoFs

30 fps video. 1/800 second exposure is sharp in time, causes time aliasing.

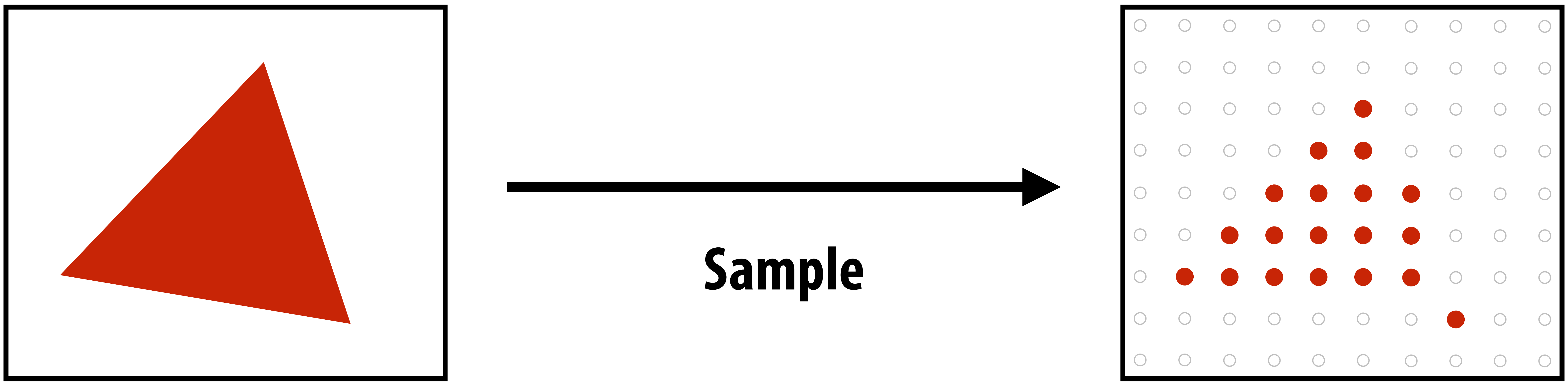
Video: motion-blurred sampling



Credit: Aris & cams youtube, https://youtu.be/NoWwxTktoFs

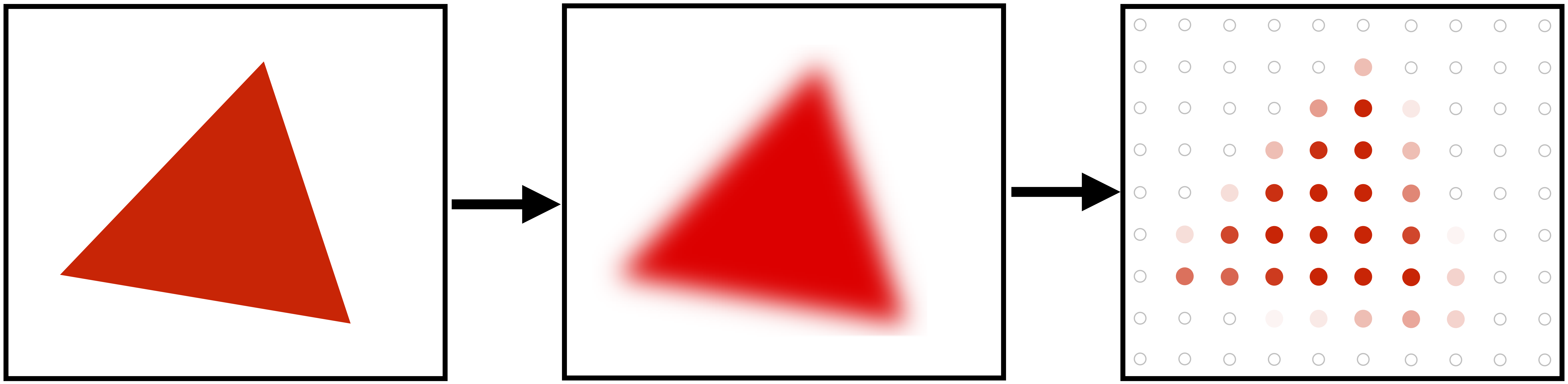
30 fps video. 1/30 second exposure is motion-blurred in time, reduces aliasing.

Rasterization is sampling in 2D space



Note jaggies in rasterized triangle
(pixel values are either red or white: sample is in or out of triangle)

Anti-aliasing by pre-filtering the signal



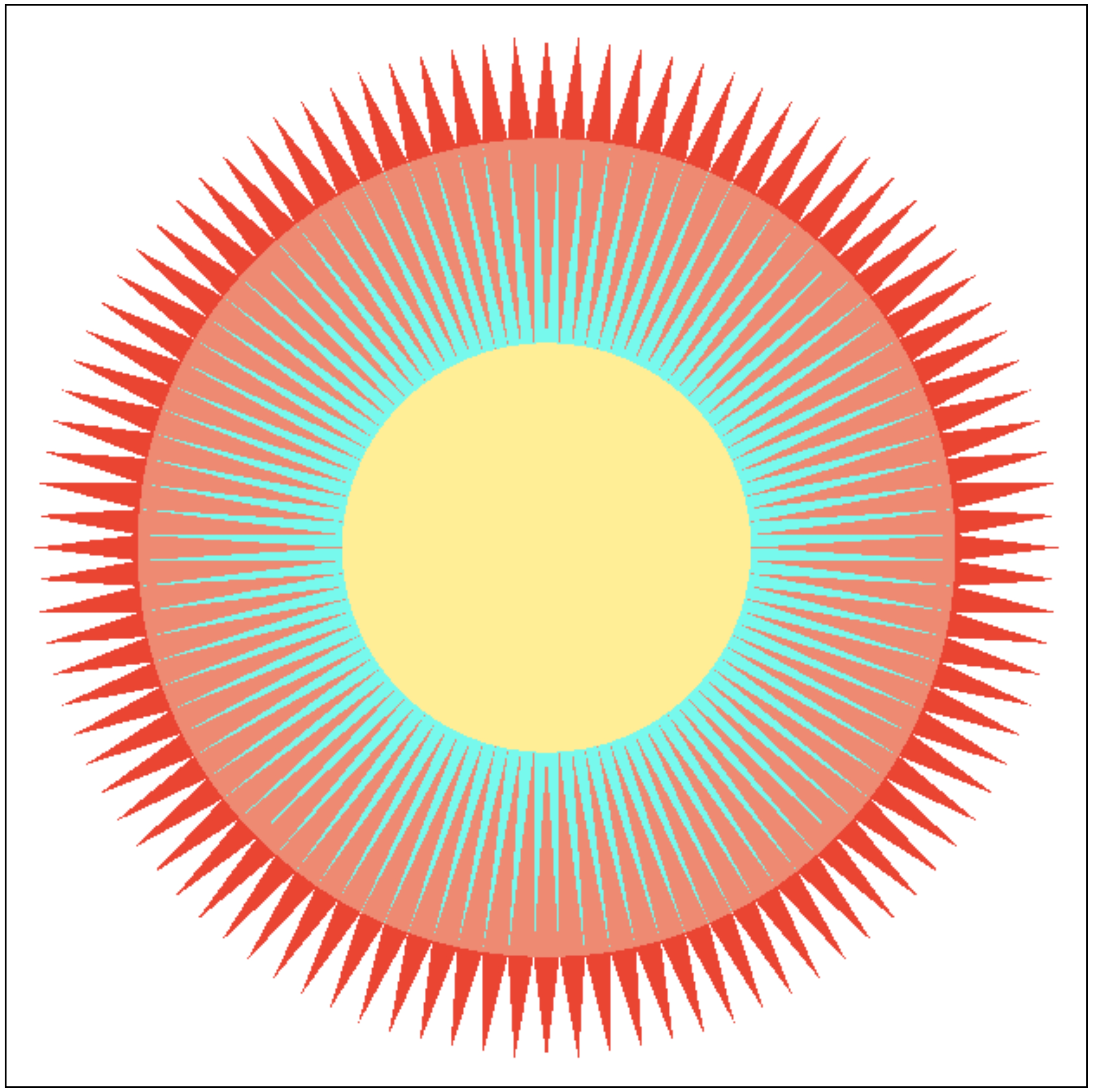
Pre-filter

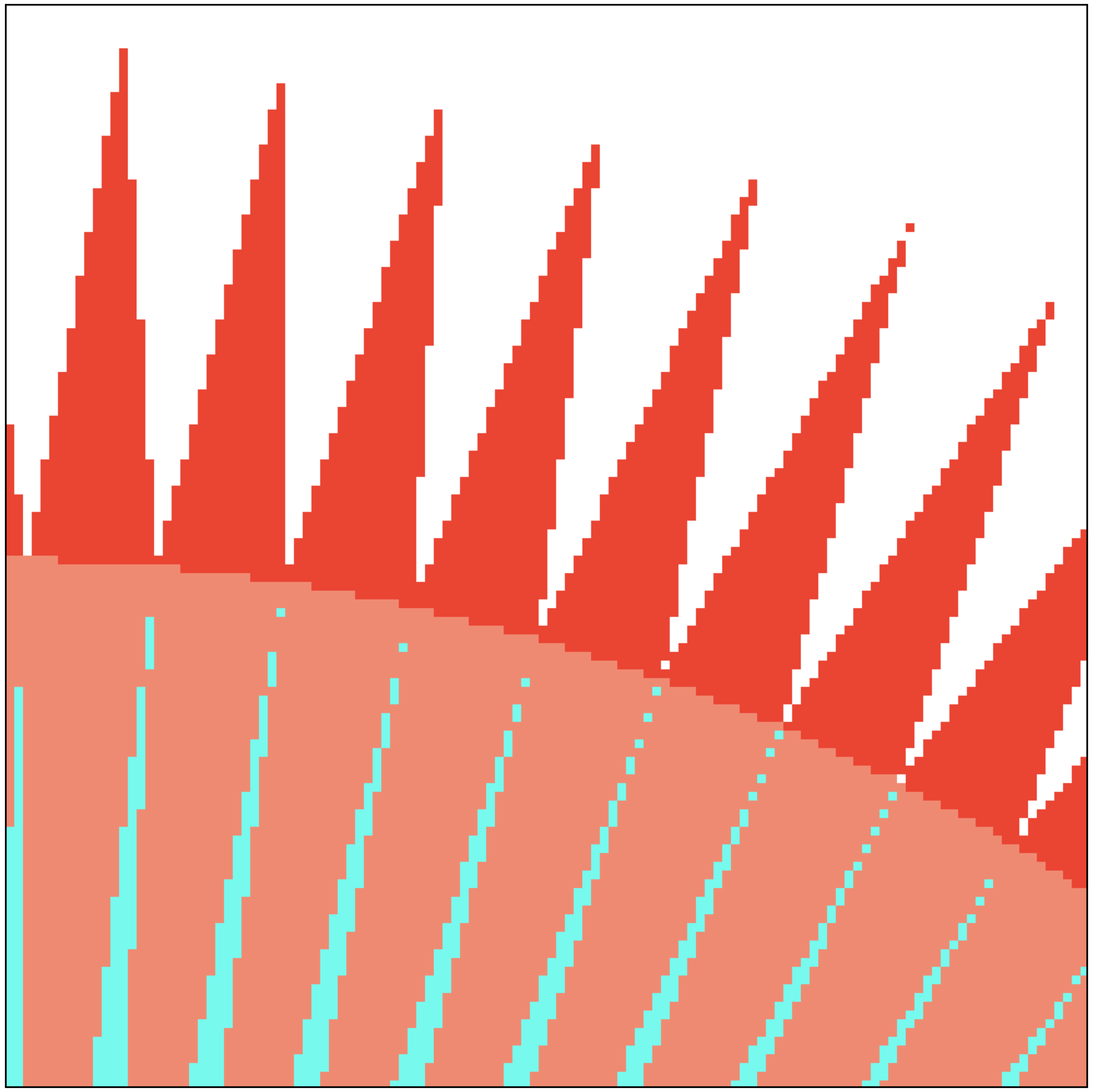
(remove frequencies above Nyquist limit)

Sample

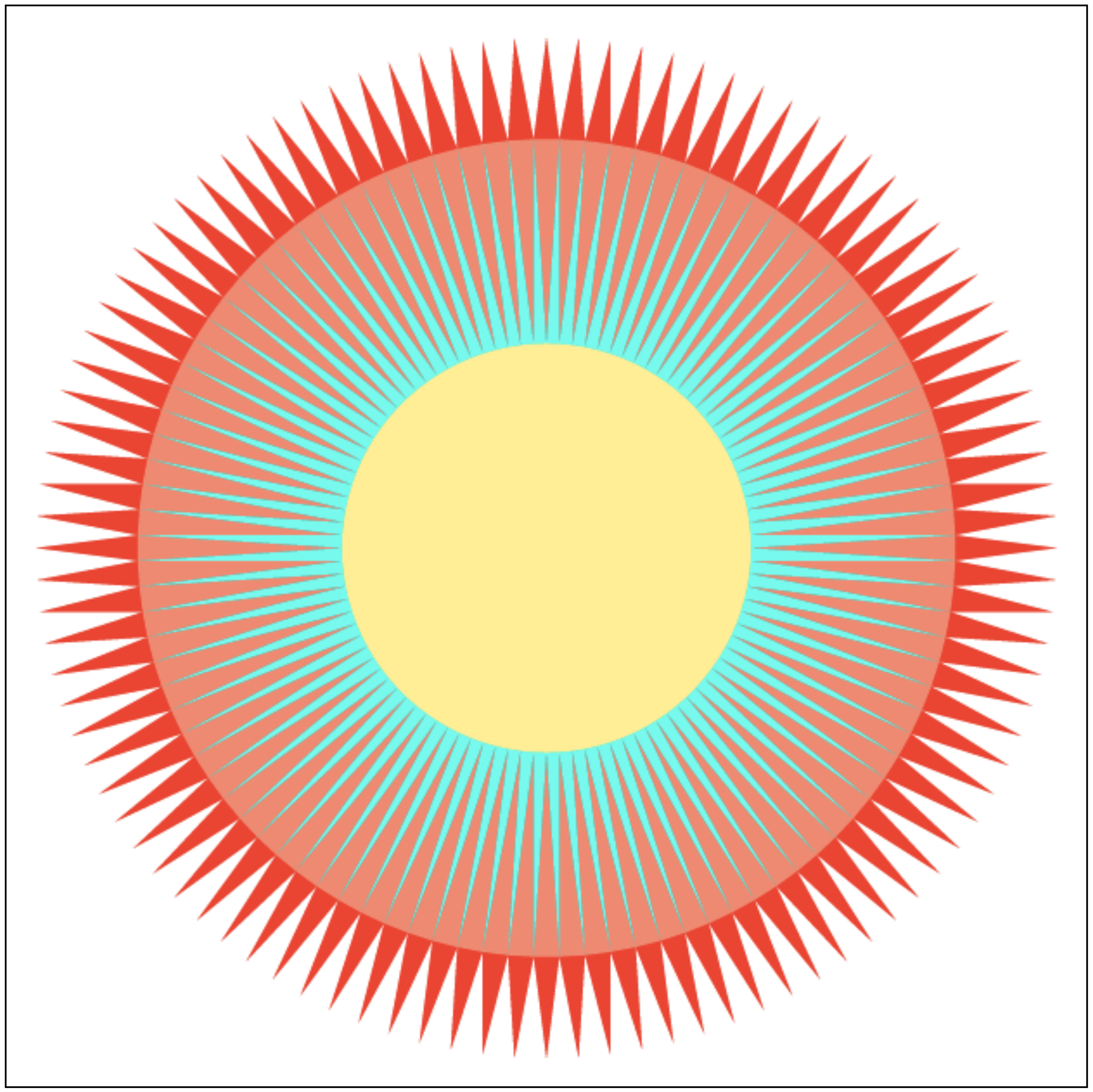
Note anti-aliased edges of rasterized triangle: pixel values take intermediate values

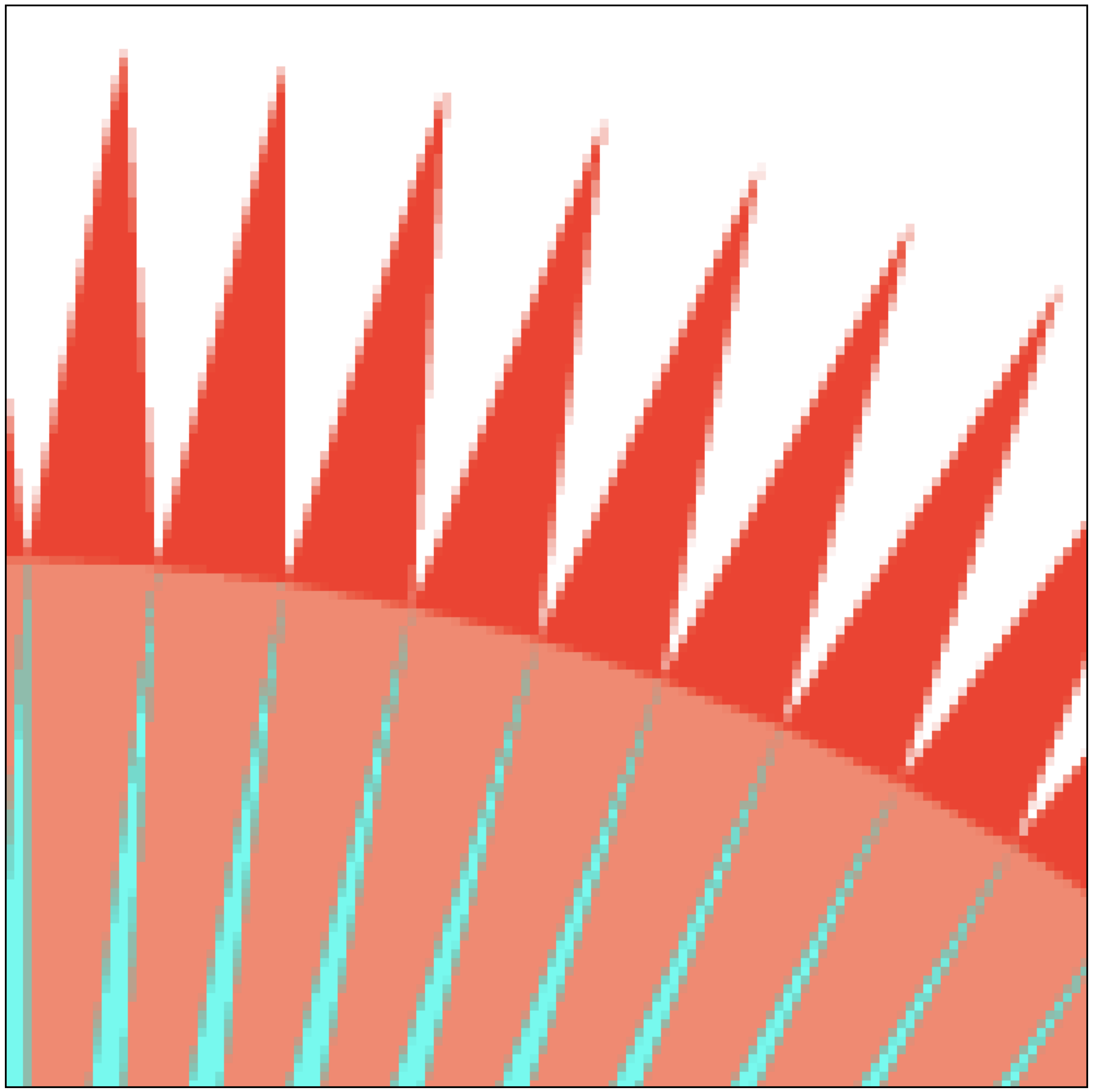
Images rendered using one sample per pixel



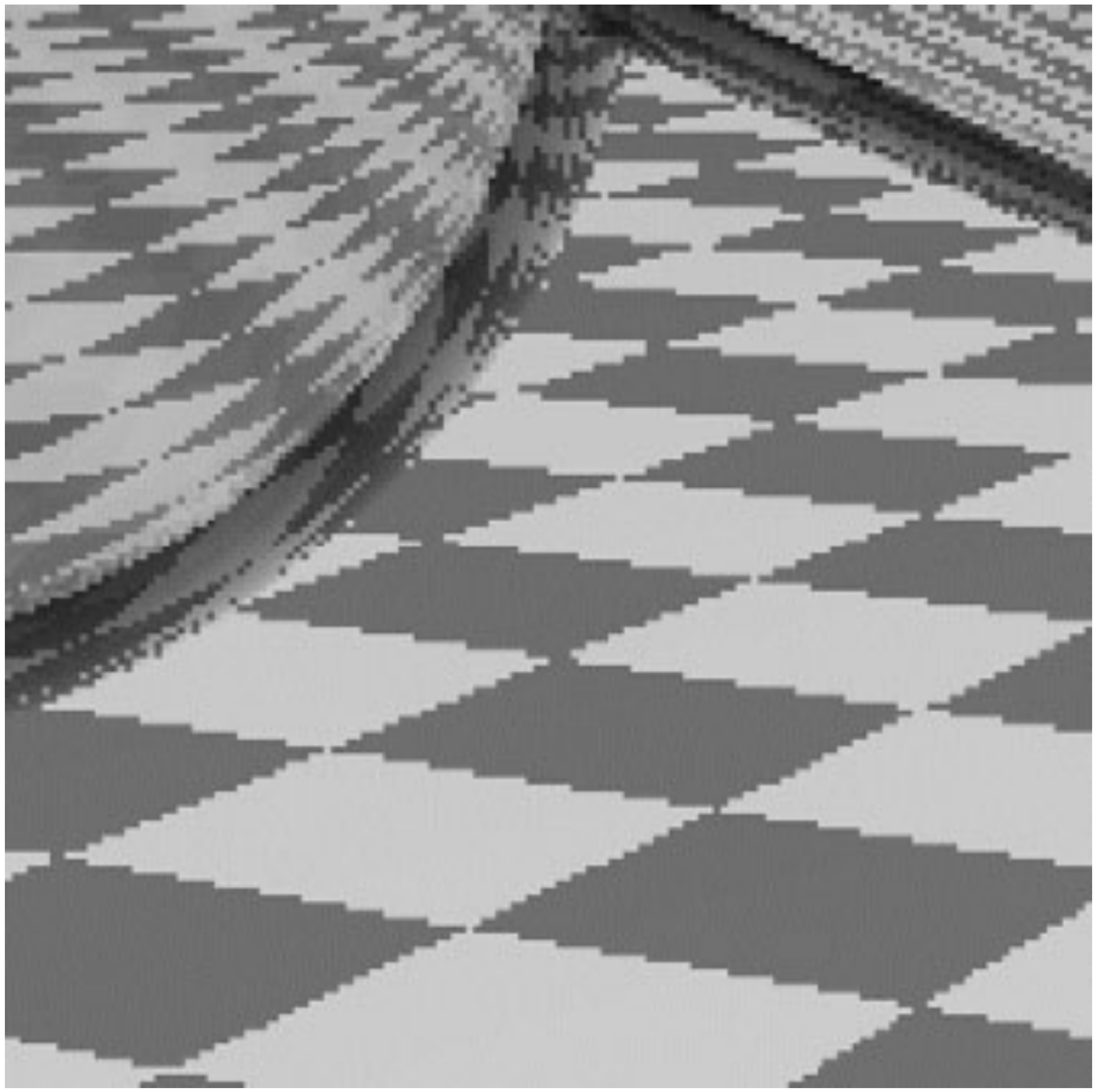


Anti-aliased results

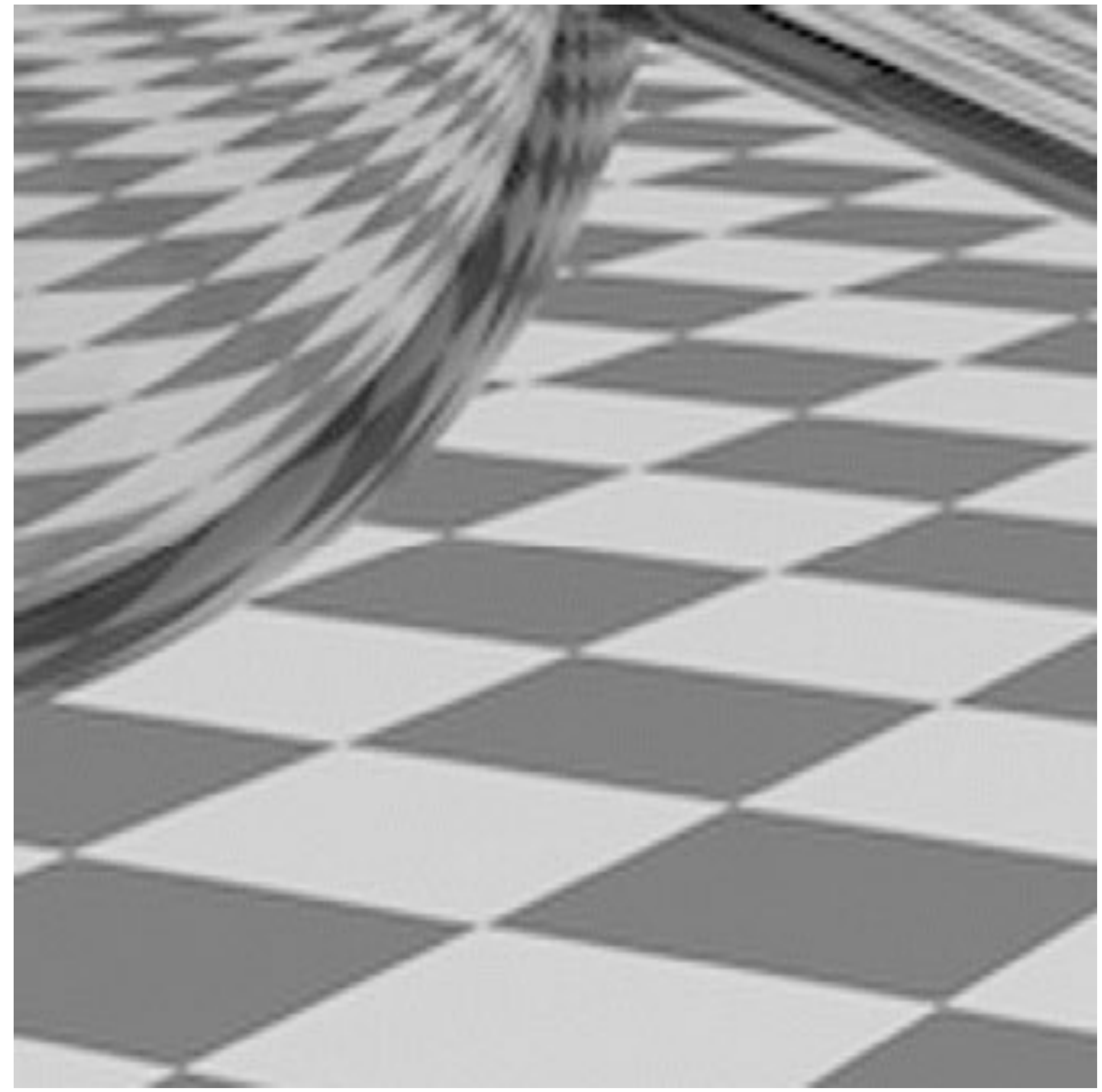




Benefits of anti-aliasing

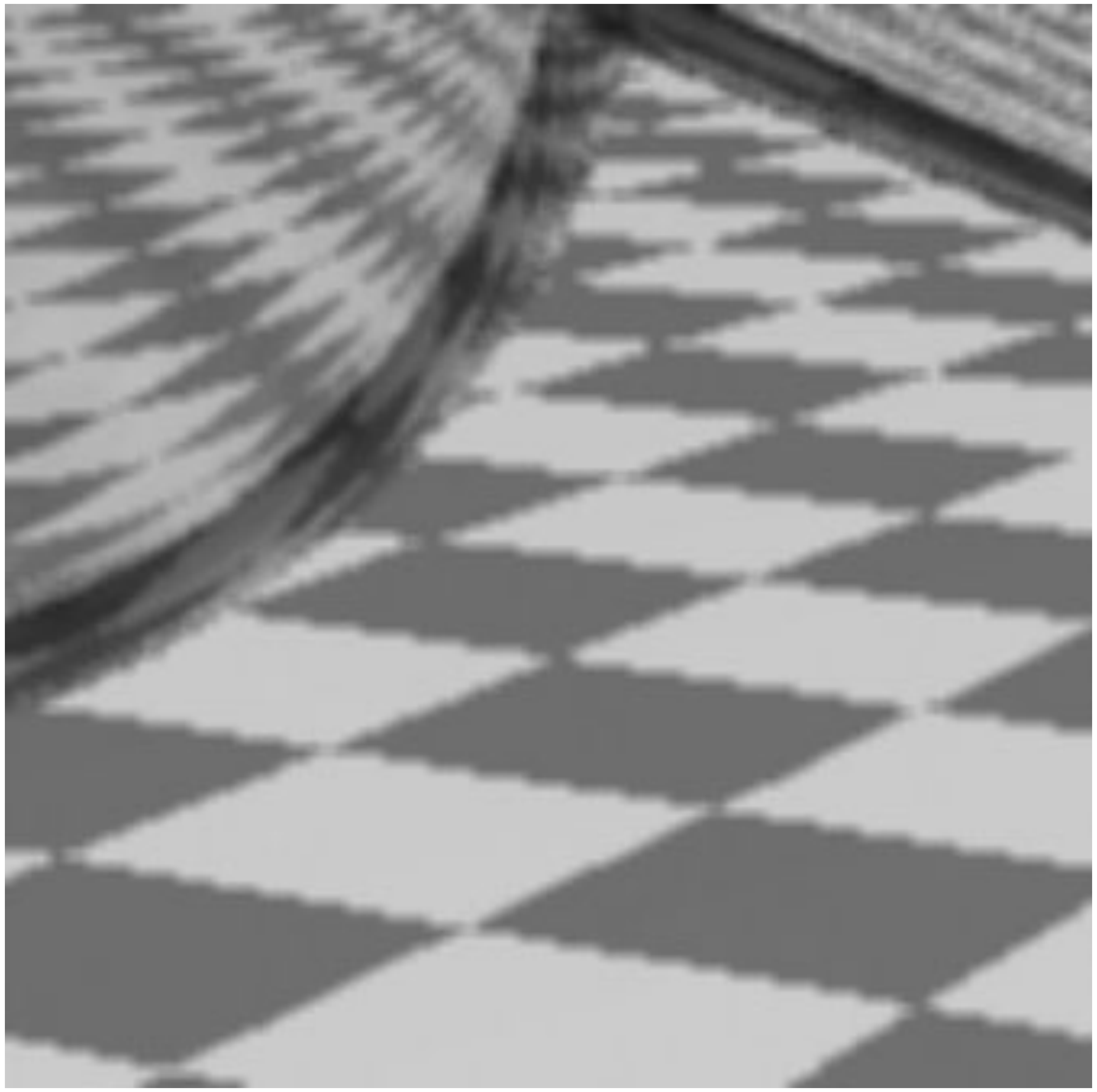


Jaggies

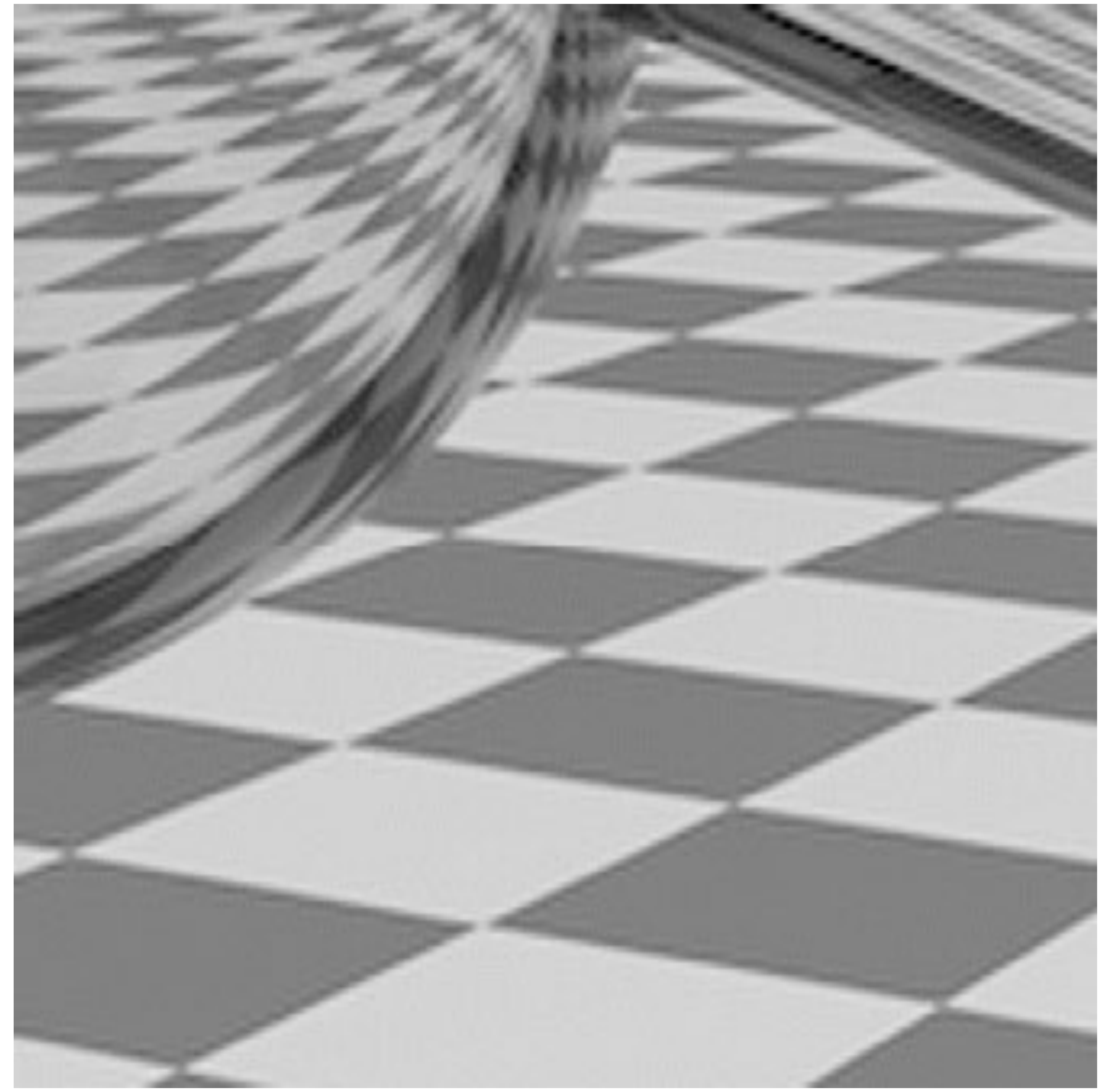


Pre-filtered

Anti-aliasing vs blurring an aliased result

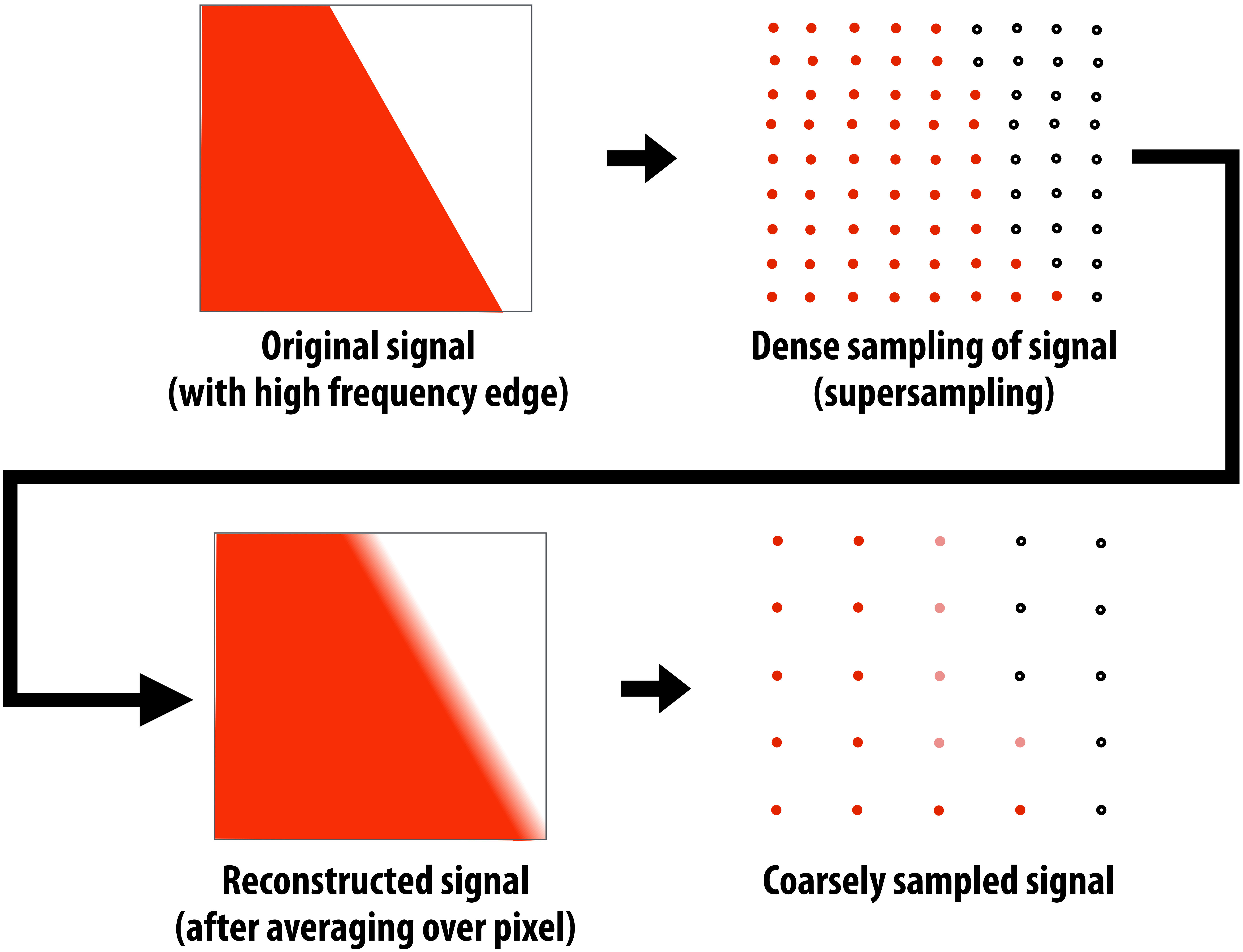


Blurred Jaggies
("Sample then blur jaggies")



Pre-filtered
("blur then sample")

Recall our anti-aliasing technique from the first half of lecture



Filtering = convolution

1D convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1	2	1

1D convolution

Signal	1	3	5	3	7	1	3	8	6	4

Filter	1	2	1

1x1 + 3x2 + 5x1 = 12

Result	12									

1D convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1	2	1

3x1 + 5x2 + 3x1 = 16

Result

12	16								

1D convolution

Signal	1	3	5	3	7	1	3	8	6	4
Filter			1	2	1					

5x1 + 3x2 + 7x1 = 18

Result	12	16	18							

Box filter (used in a 2D convolution)

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Example: 3x3 box filter

2D convolution with box filter blurs the image

Original image

Blurred
(convolve with box filter)

Hmm… this reminds me of a low-pass filter…

Discrete 2D convolution

$$\underbrace{(f * g)(x, y)}_{\text{output image}} = \sum_{i,j=-\infty}^{\infty} \underbrace{f(i, j)}_{\text{filter}} \underbrace{I(x - i, y - j)}_{\text{input image}}$$

Consider $f(i, j)$ **that is nonzero only when:** $-1 \leq i, j \leq 1$

Then:

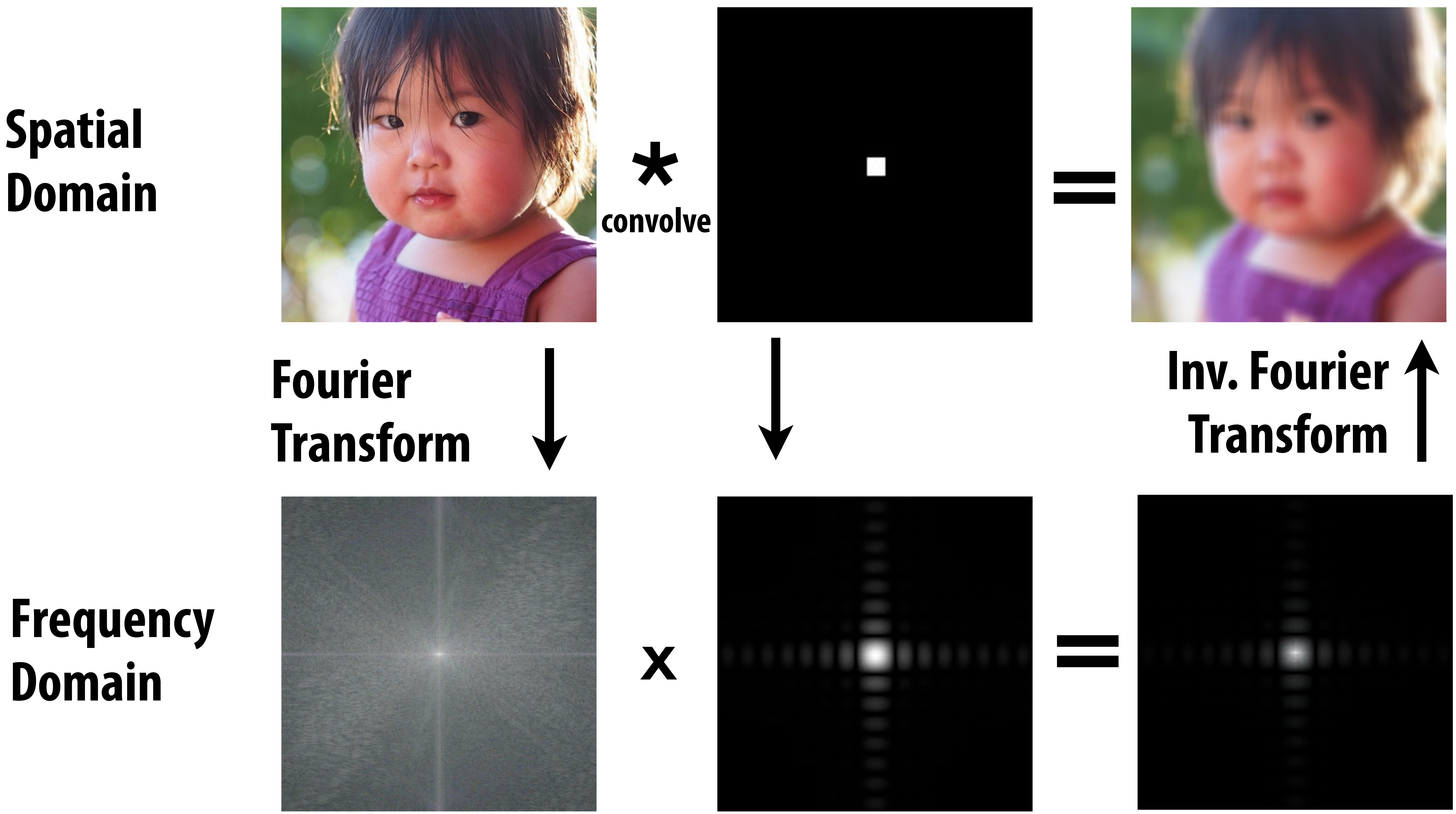
$$(f * g)(x, y) = \sum_{i,j=-1}^{1} f(i, j) I(x - i, y - j)$$

And we can represent f(i,j) as a 3x3 matrix of values where:

$$f(i, j) = \mathbf{F}_{i,j}$$ **(often called: "filter weights", "filter kernel")**

Convolution theorem

Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa



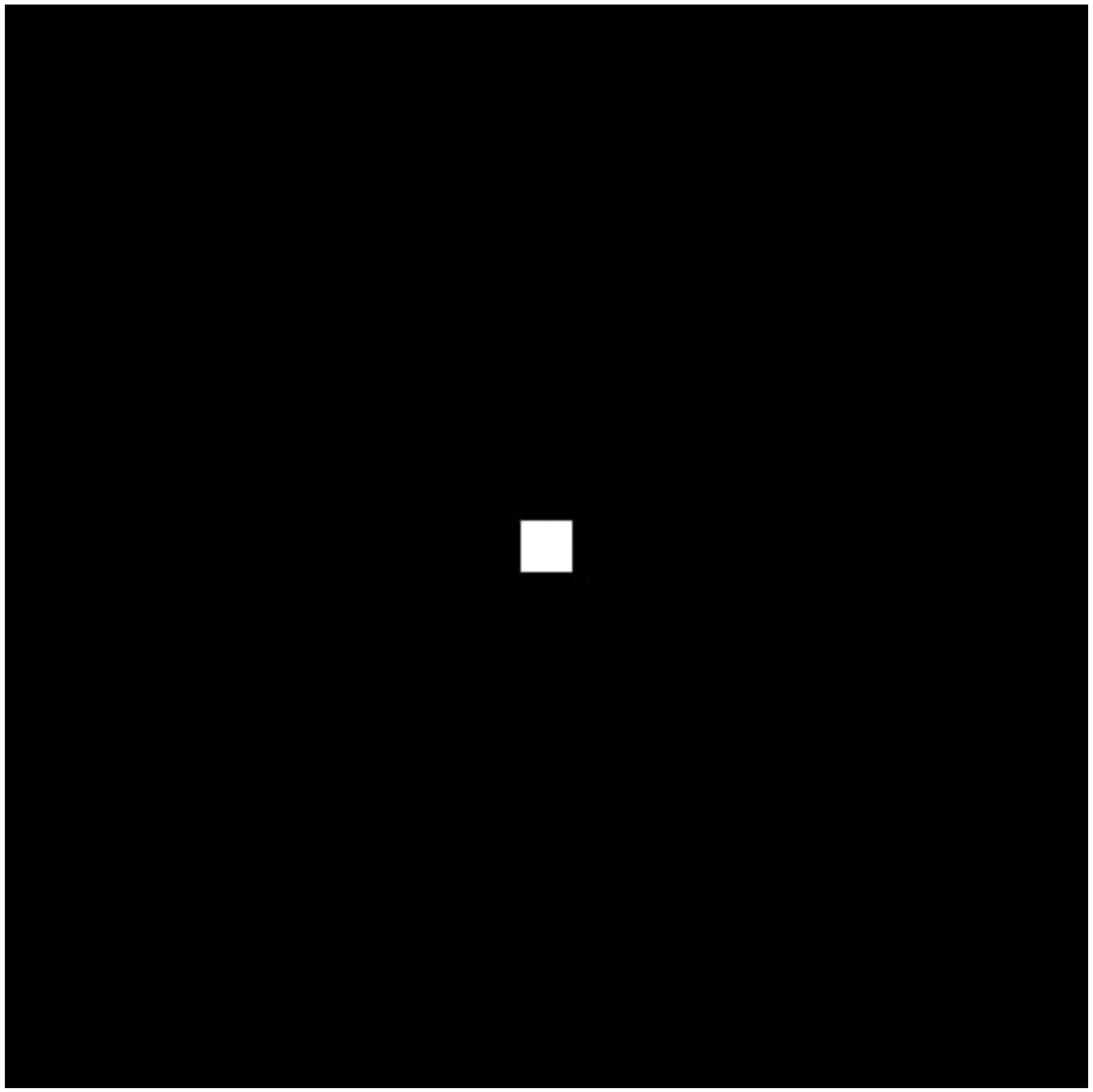
Convolution theorem

- Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa

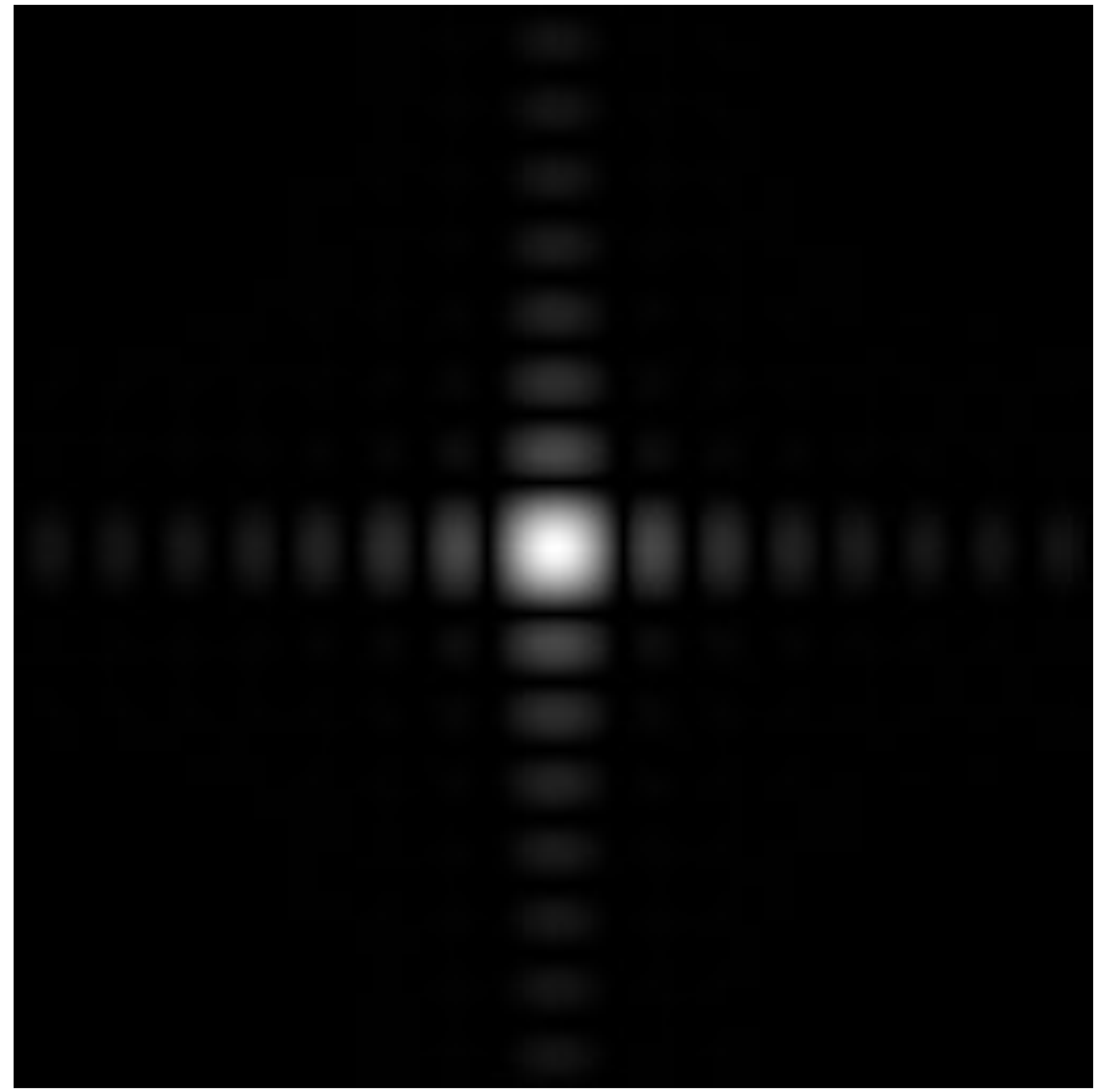
- Pre-filtering option 1:
    - Filter by convolution in the spatial domain

- Pre-filtering option 2:
    - Transform to frequency domain (Fourier transform)
    - Multiply by Fourier transform of convolution kernel
    - Transform back to spatial domain (inverse Fourier)

Box function = "low pass" filter

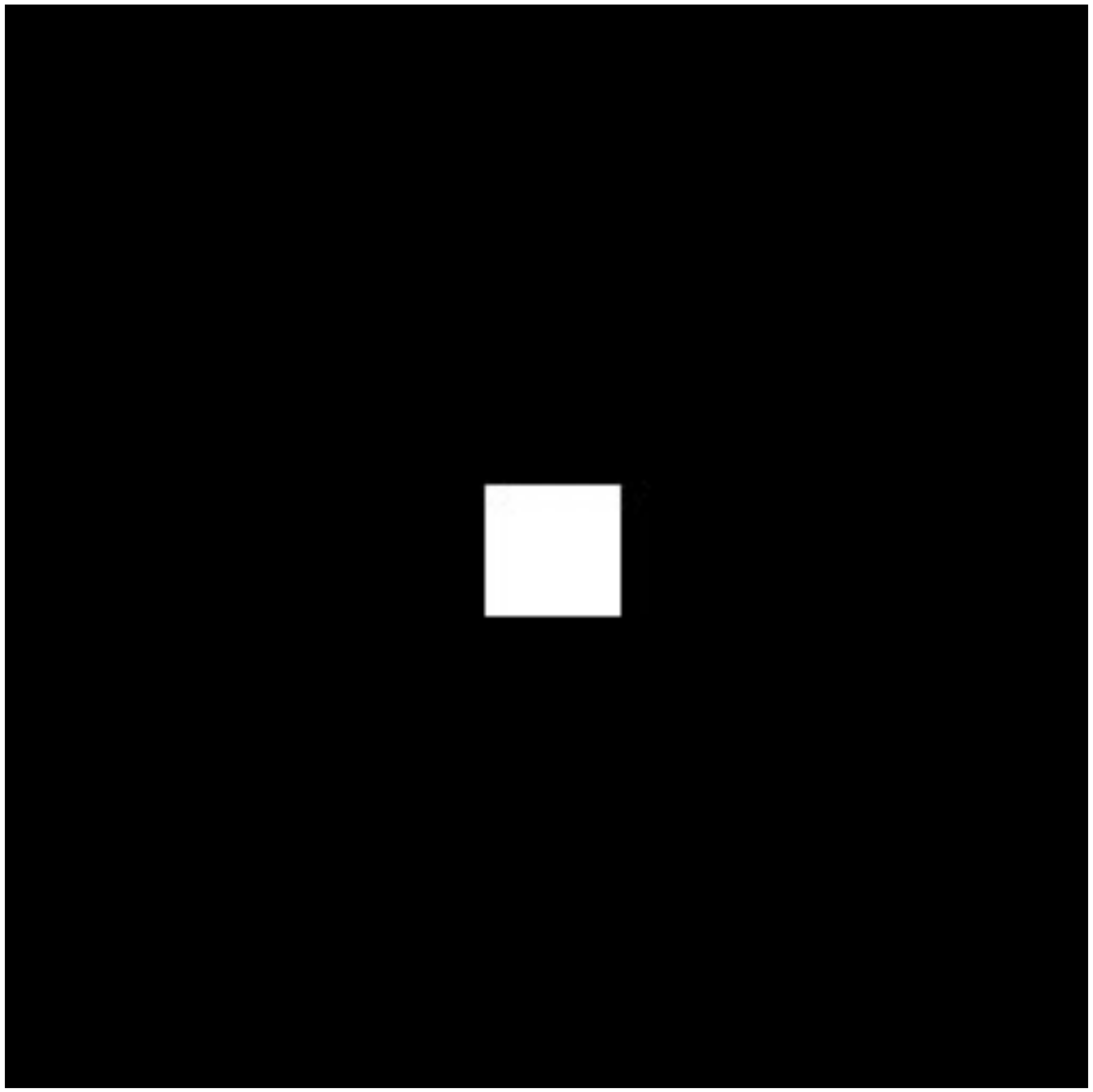


Spatial domain

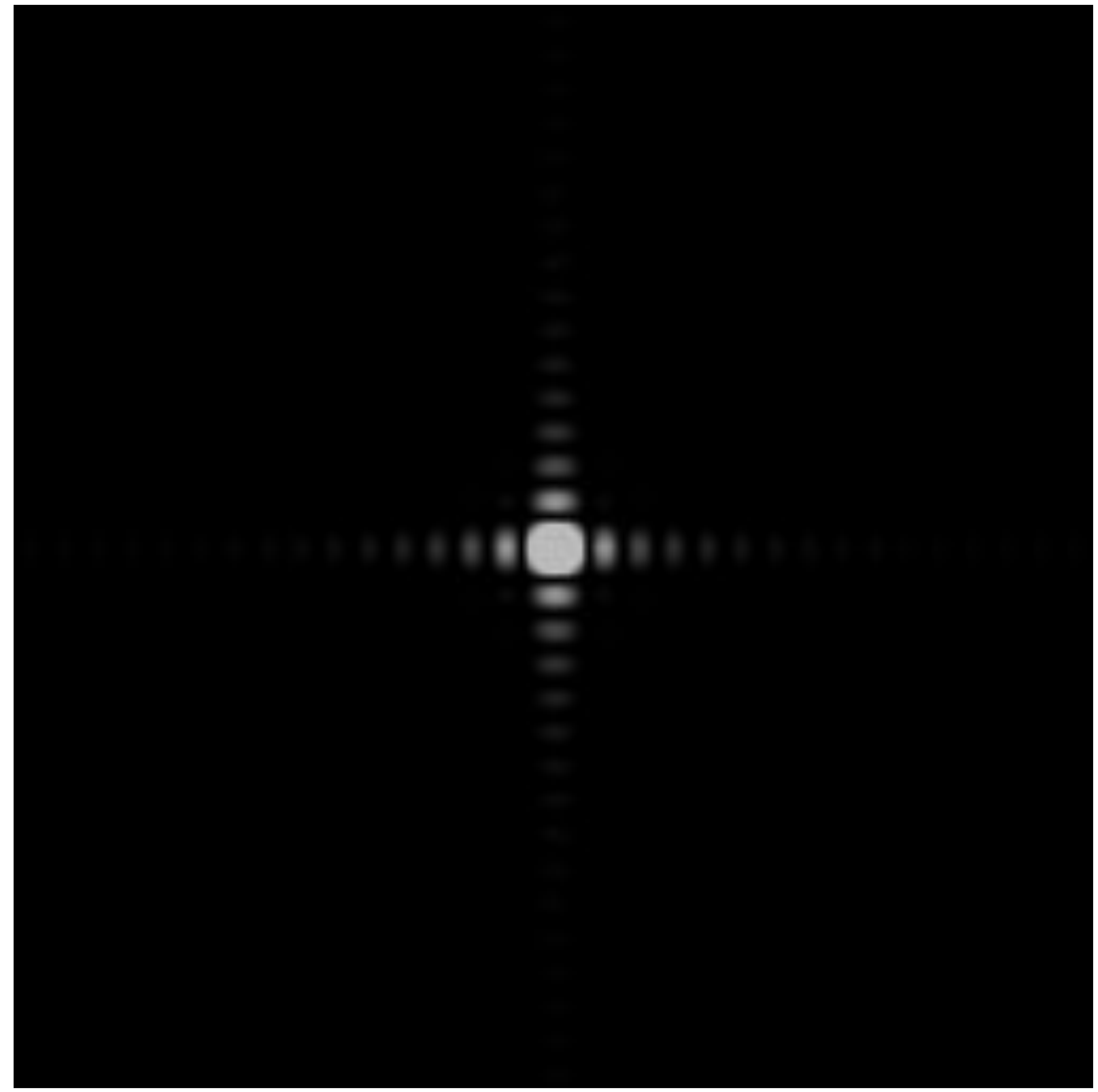


Frequency domain

Wider filter kernel = retain only lower frequencies



Spatial domain



Frequency domain

Wider filter kernel = lower frequencies

- As a filter is localized in the spatial domain,
  it spreads out in frequency domain

- Conversely, as a filter is localized in frequency domain, it spreads out in the spatial domain

How can we reduce aliasing error?

- Increase sampling rate
  - Higher resolution displays, sensors, framebuffers…
  - But: costly and may need very high resolution to sufficiently reduce aliasing
- Anti-aliasing
  - Simple idea: remove (or reduce) high frequencies before sampling
  - How to filter out high frequencies before sampling?

Anti-aliasing by averaging values in pixel area

- **Convince yourself the following are the same:**

- **Option 1:**
  - **Convolve f(x,y) by a 1-pixel box-blur**
  - **Then sample at every pixel**

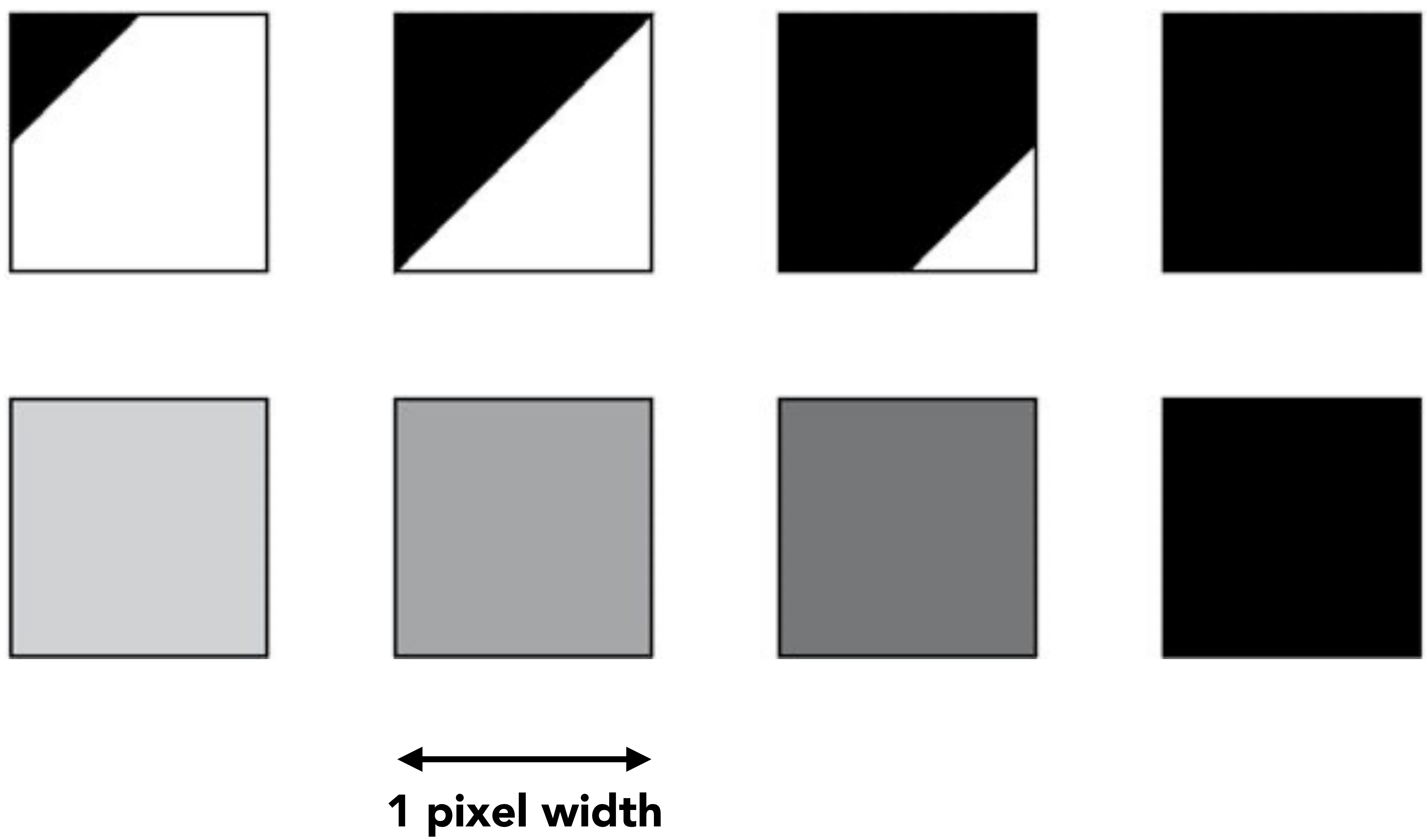
- **Option 2:**
  - **Compute the average value of f(x,y) in the pixel**

Anti-aliasing by computing average pixel value

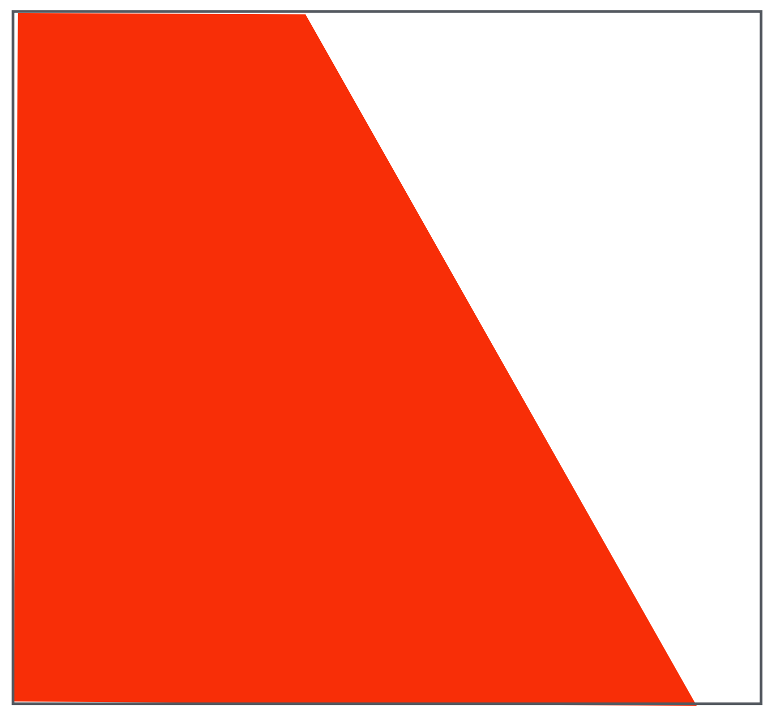
In rasterizing one triangle, the value of f(x,y) = inside(tri,x,y) averaged over the area of a pixel is equal to the amount of the pixel covered by the triangle.

Original

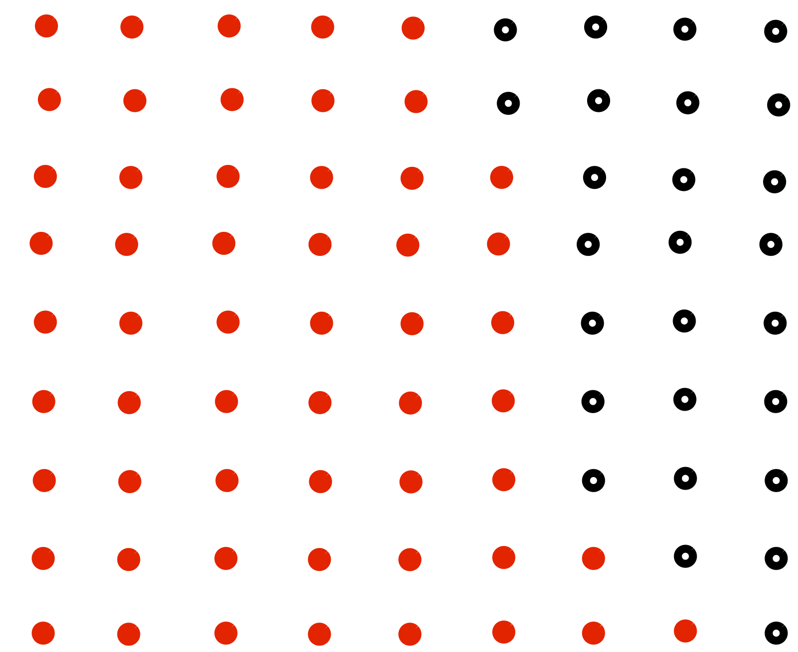
Filtered



Putting it all together: anti-aliasing via supersampling

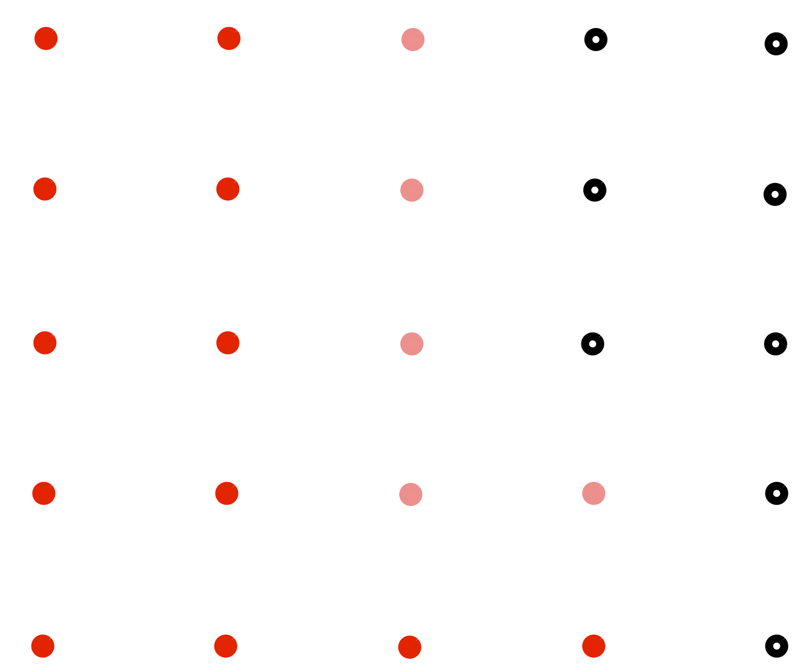


Original signal
(with high frequency edge)



Dense sampling of signal
(supersampling)

Reconstructed signal
(averaging over pixel (via convolution) yields new signal with high frequencies removed)



Coarse sampling of reconstructed signal exhibits less aliasing

Today's summary

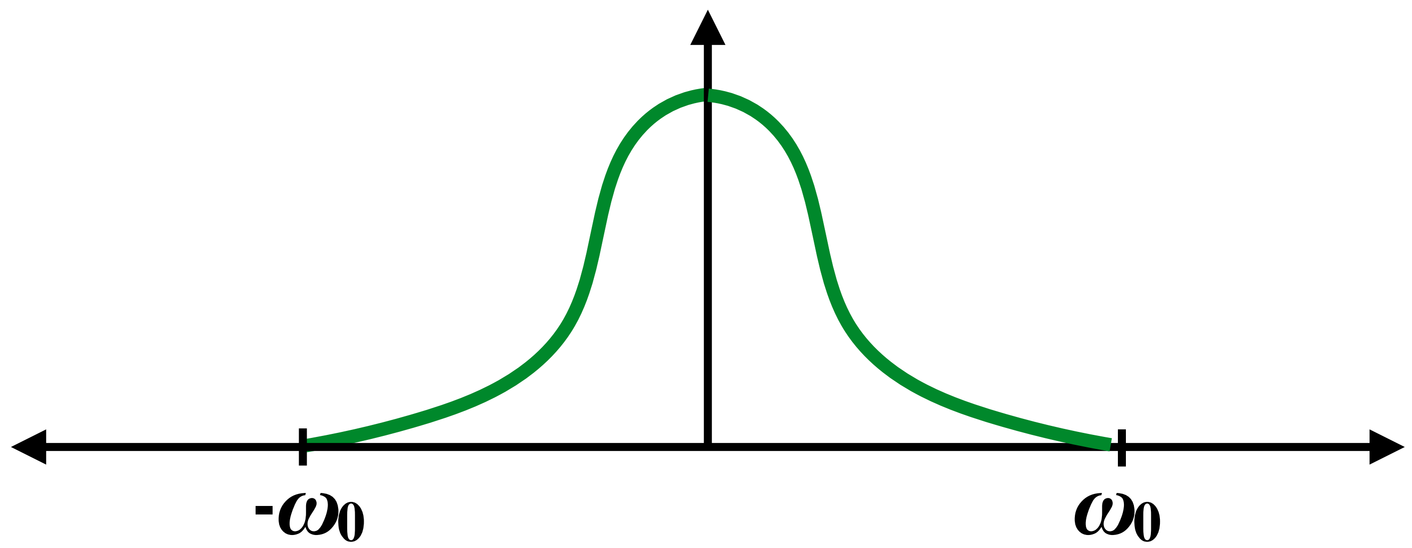
- Drawing a triangle = sampling triangle/screen coverage
- Pitfall of sampling: aliasing
- Reduce aliasing by prefiltering signal
  - Supersample
  - Reconstruct via convolution (average coverage over pixel)
    - Higher frequencies removed
  - Sample reconstructed signal once per pixel

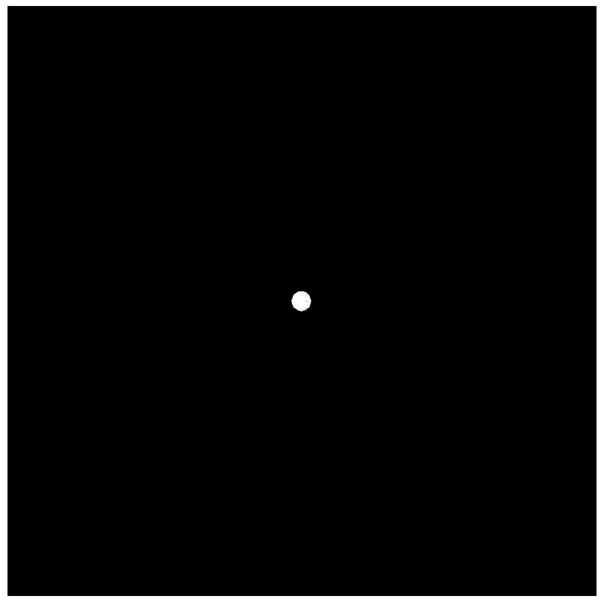
- There is much, much more to sampling theory and practice…

Bonus slides:
How much pre-filtering do we need to avoid aliasing?

Nyquist-Shannon theorem

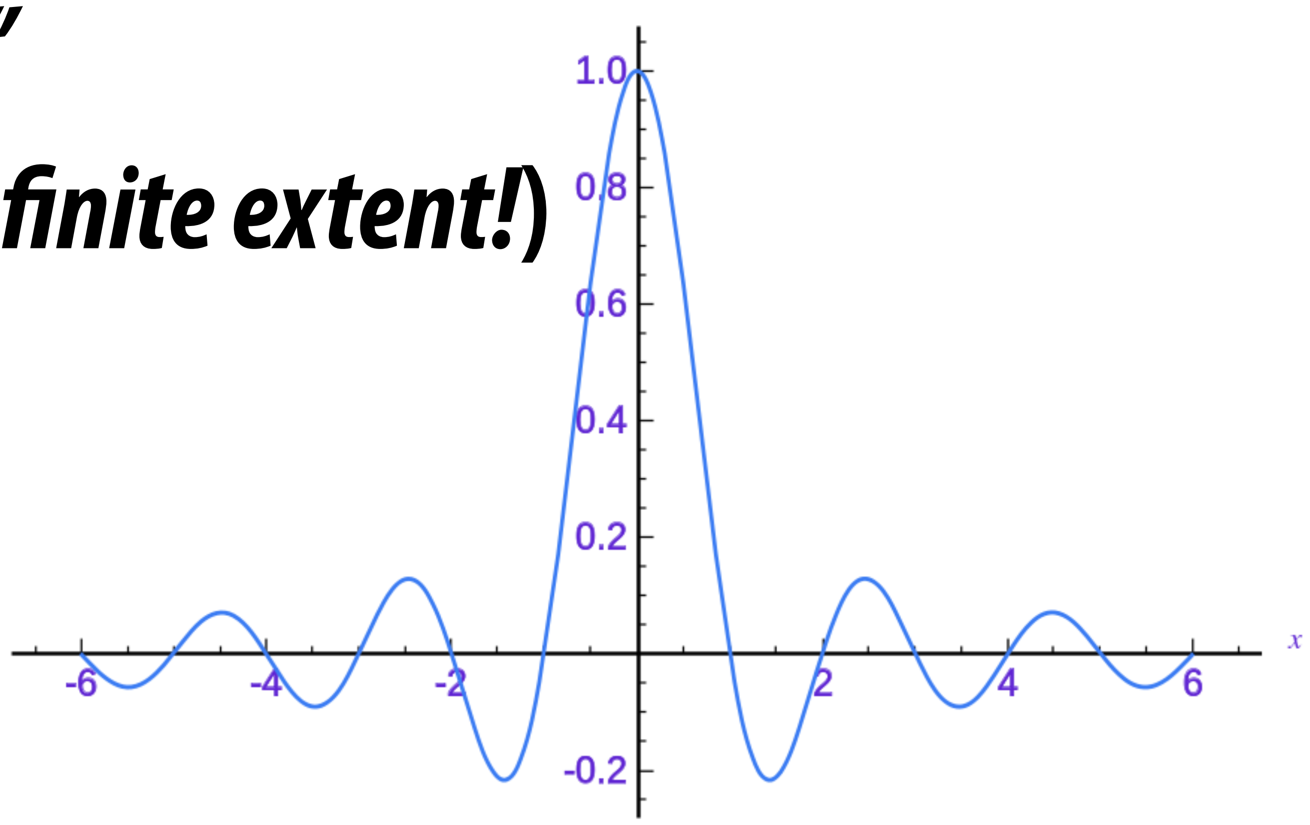
- Consider a band-limited signal: has no frequencies above ω_0
  - 1D: consider low-pass filtered audio signal
  - 2D: recall the blurred image example from a few slides ago





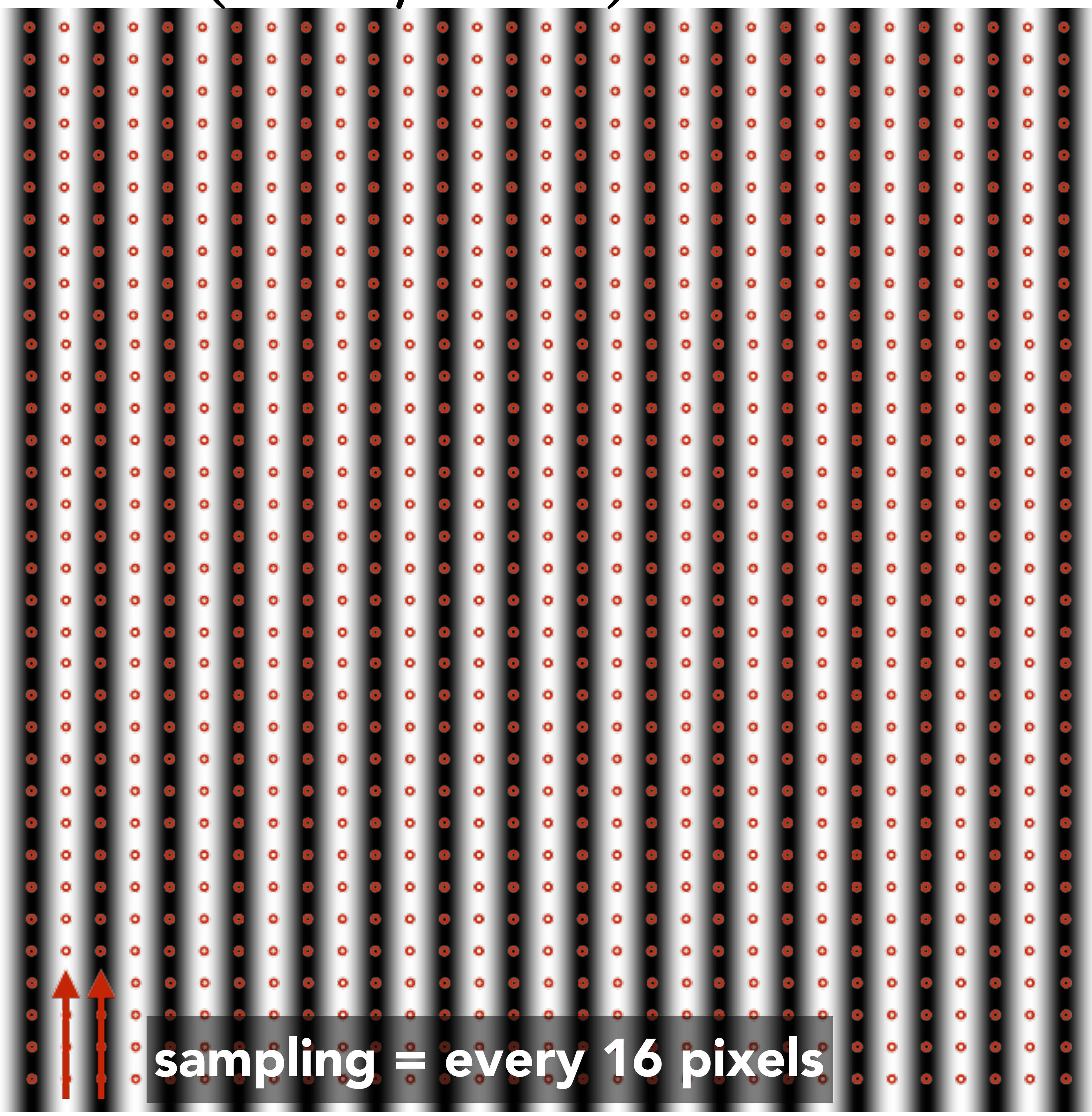
- The signal can be perfectly reconstructed if sampled with period $T = 1 / 2\omega_0$
- And reconstruction is performed using a *"sinc filter"*
  - Ideal filter with no frequencies above cutoff (*infinite extent!*)

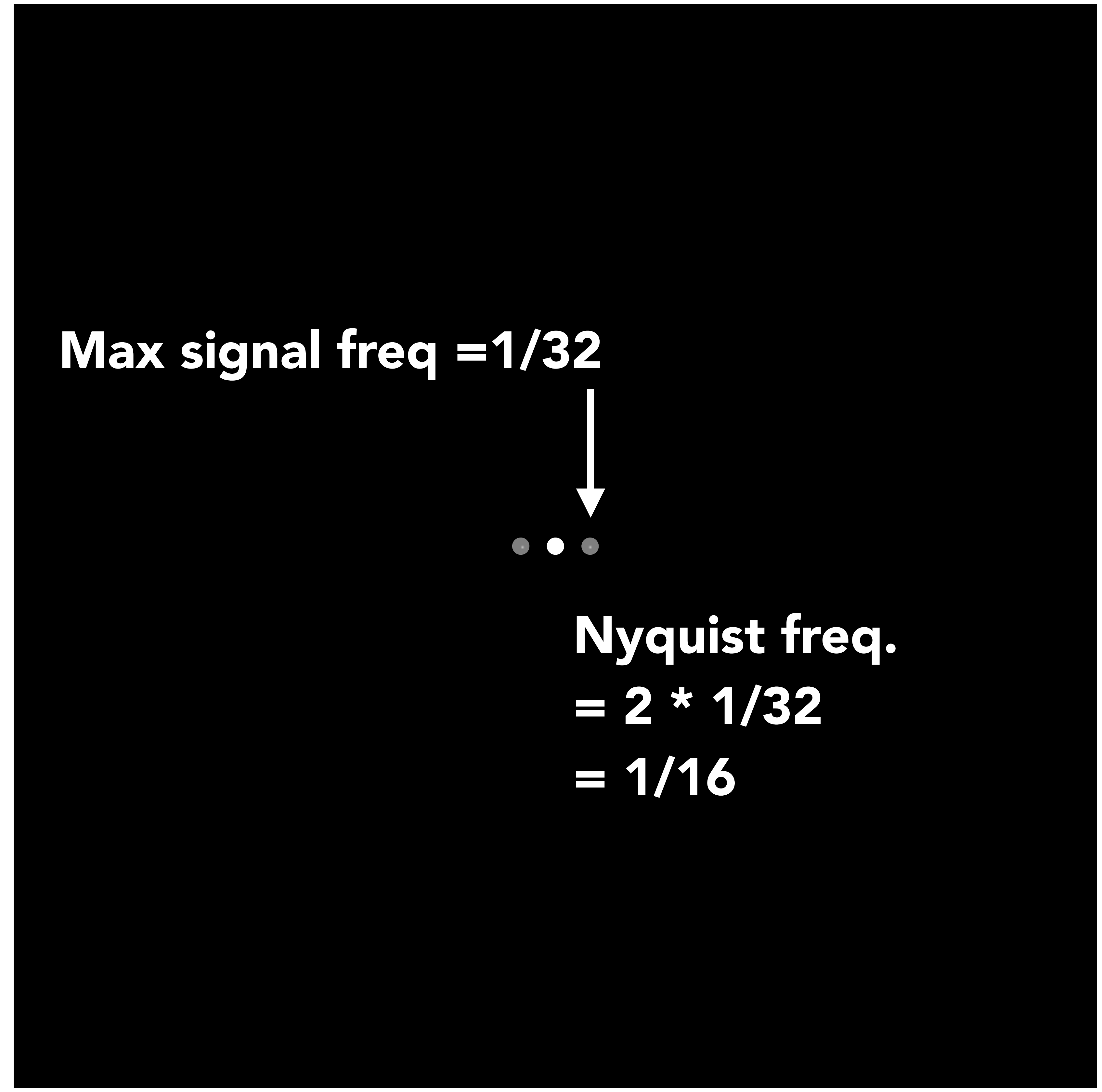
$$sinc(x) = \frac{sin(\pi x))}{\pi x}$$



Signal vs Nyquist frequency: example

$\sin(2\pi/32)x$ — **frequency 1/32; 32 pixels per cycle**



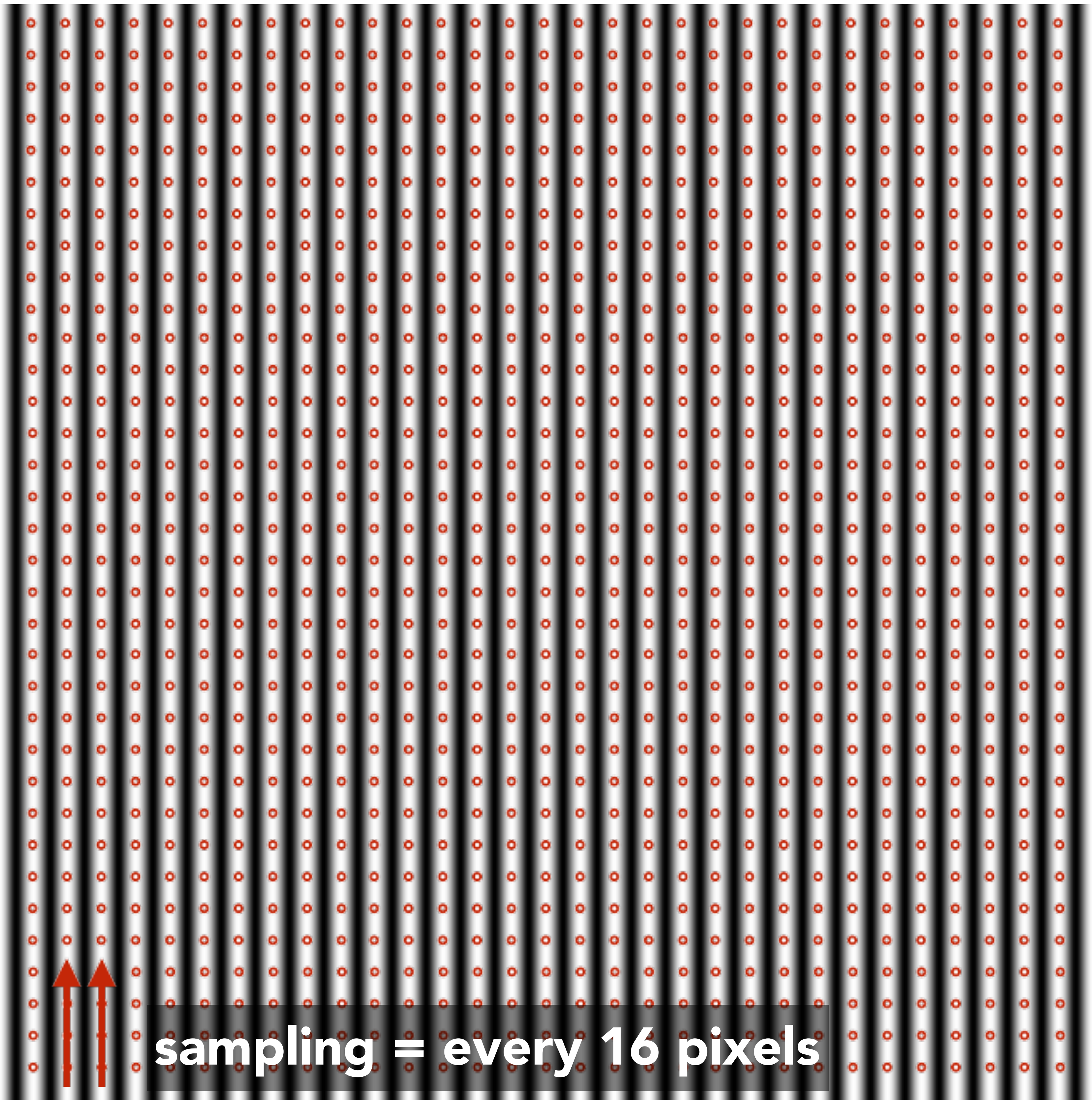


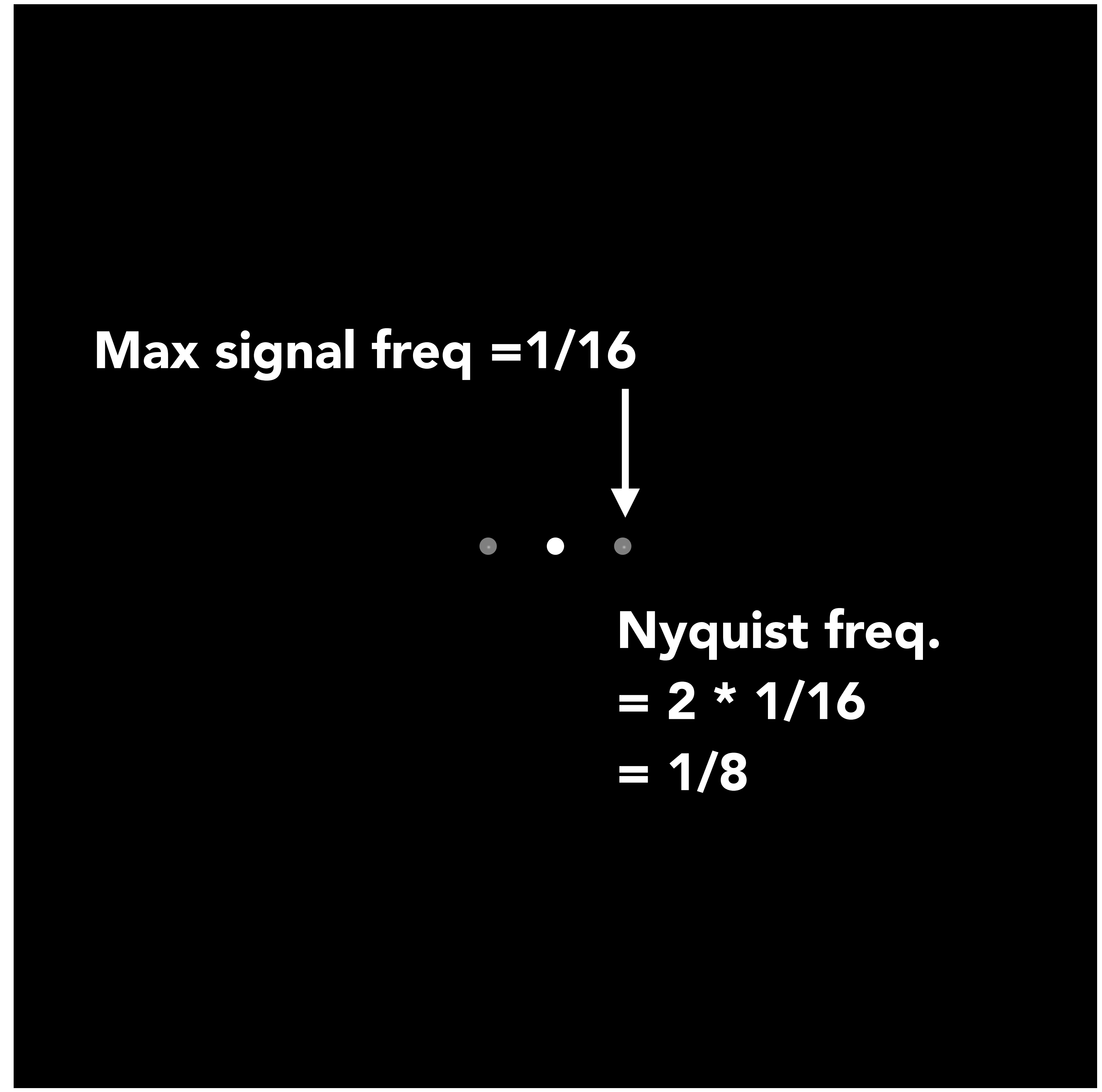
Spatial domain | **Frequency domain**

No Aliasing!

Signal vs Nyquist frequency: example

$\sin(2\pi/16)x$ — **frequency 1/16; 16 pixels per cycle**





Aliasing! (due to undersampling)

Reminder: Nyquist theorem

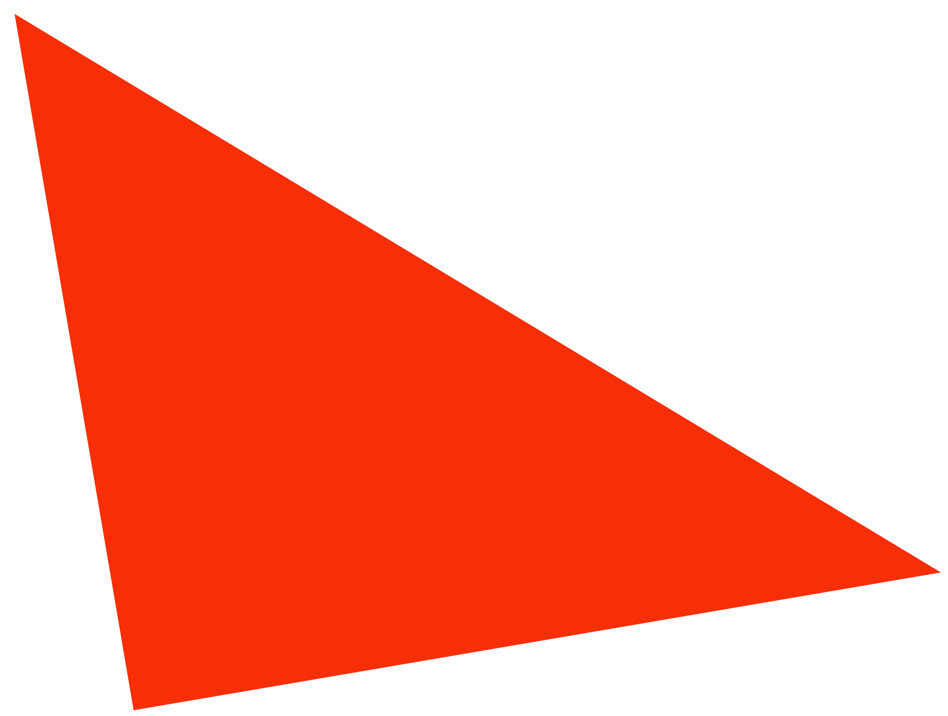
Theorem: We get no aliasing from frequencies in the signal that are less than the Nyquist frequency
(which is defined as half the sampling frequency)

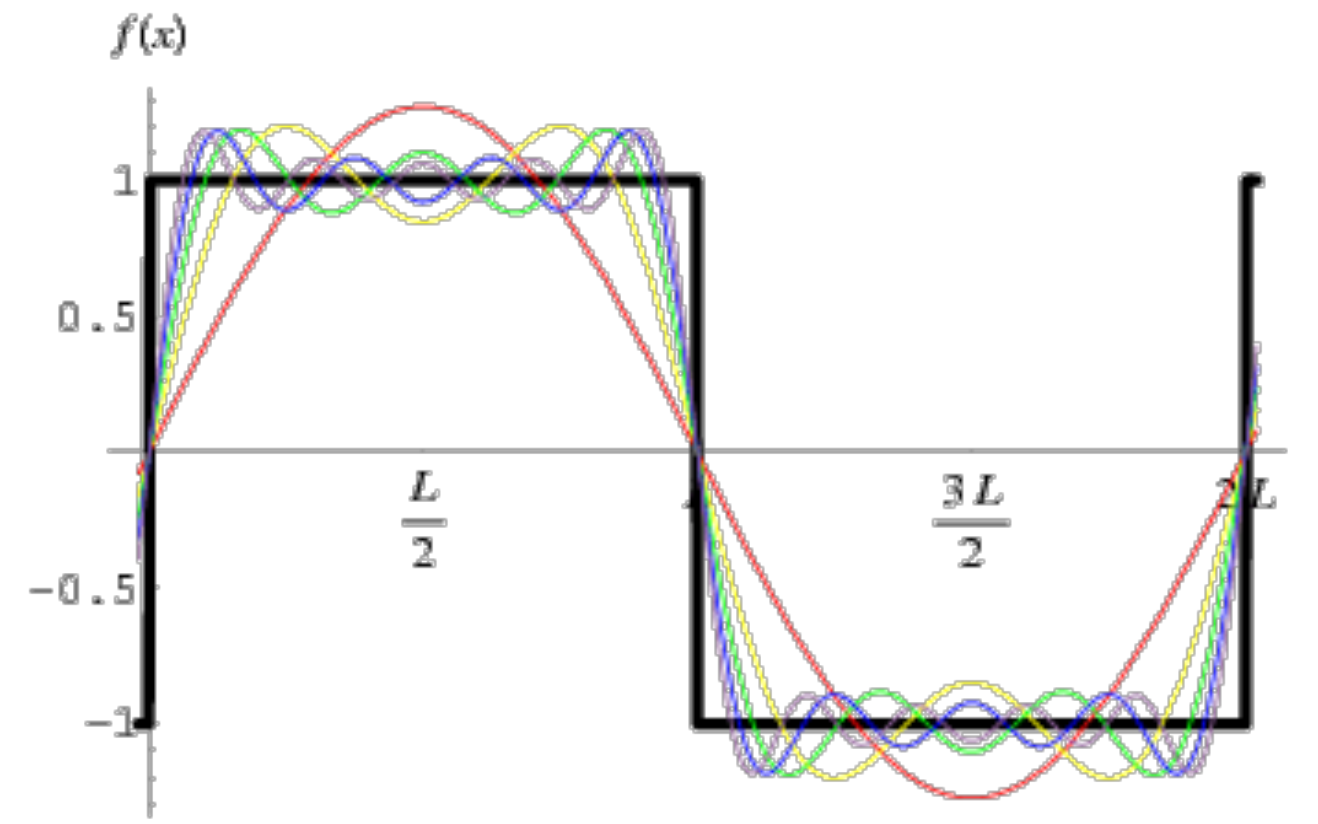
Consequence: sampling at twice the highest frequency in the signal will eliminate aliasing

Challenges of sampling-based approaches in graphics

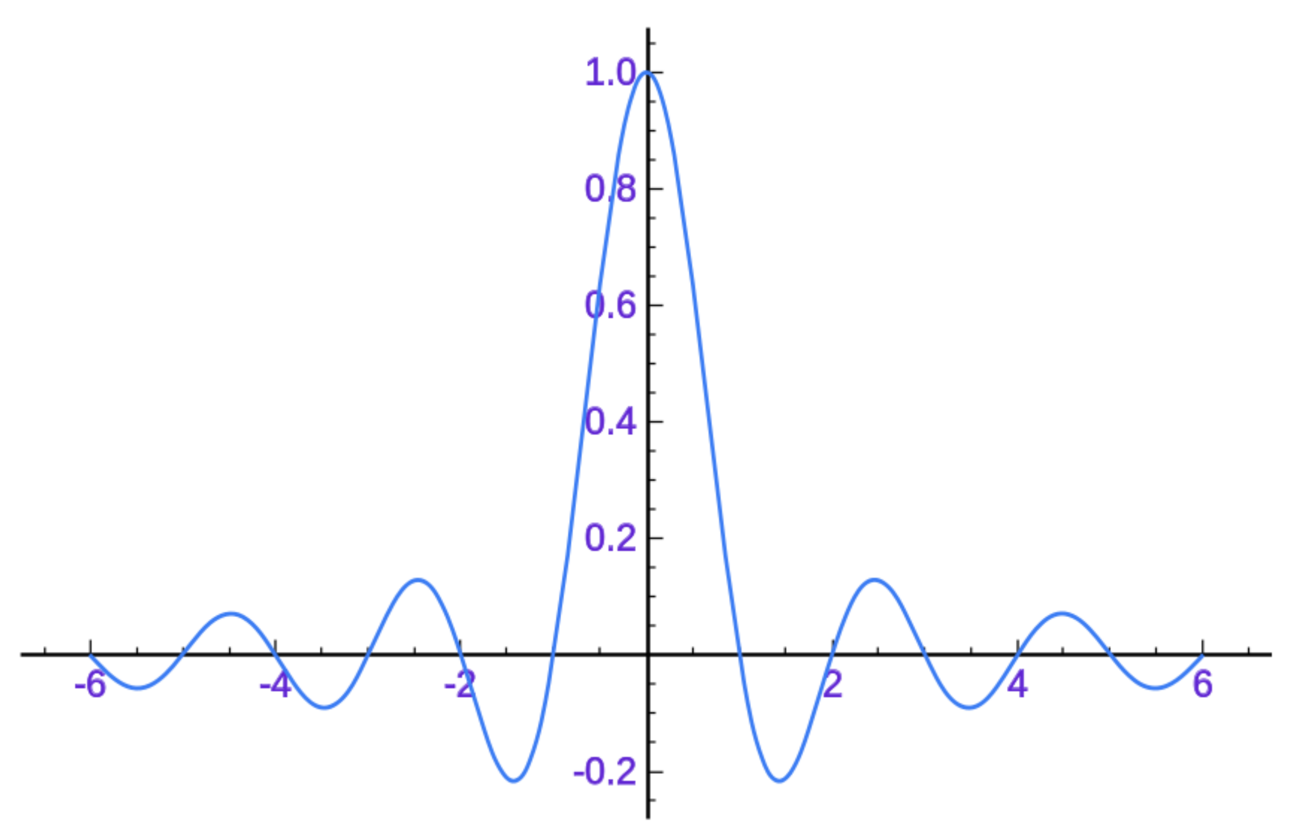
- **Our signals are not always band-limited in computer graphics. Why?**

Hint:





- **Also, infinite extent of "ideal" reconstruction filter (sinc) is impractical for efficient implementations. Why?**



Acknowledgements

- Thanks to Ren Ng, Pat Hanrahan, Keenan Crane for slide materials