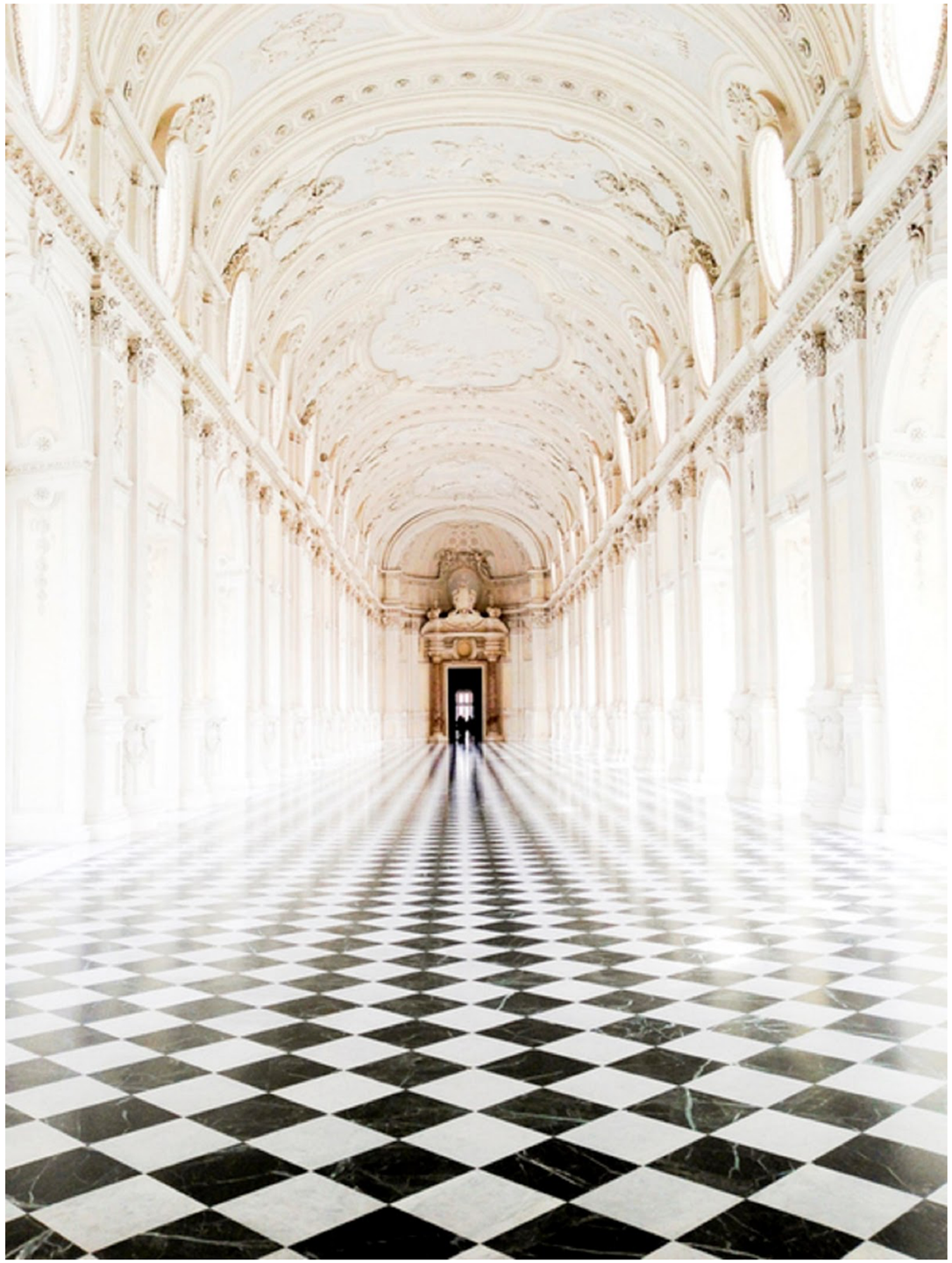
Lecture 4:

Perspective Projection and Texture Mapping

Interactive Computer Graphics
Stanford CS248, Winter 2021

Perspective and texture

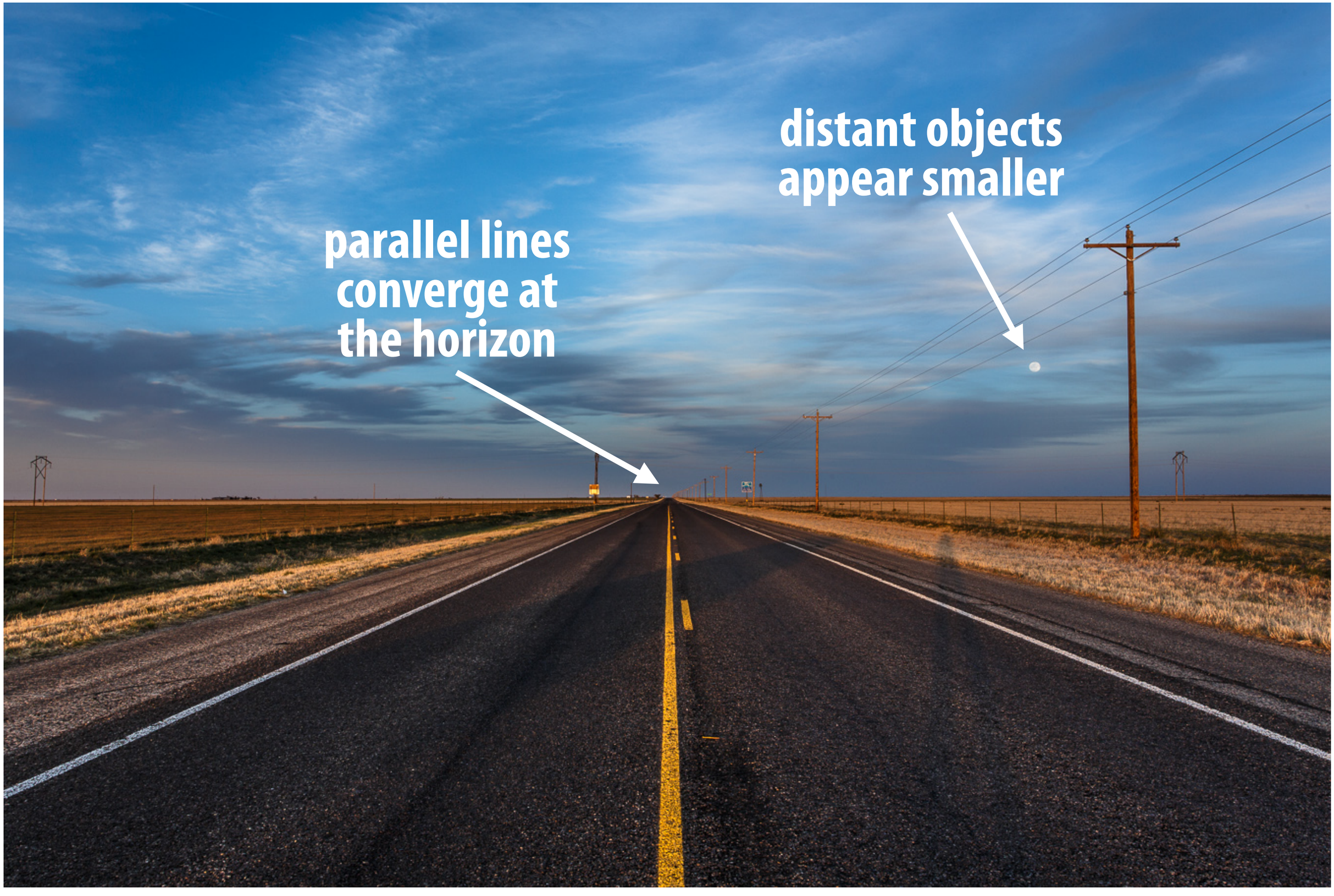
- **PREVIOUSLY:**
  - ***transformation*** **(how to manipulate primitives in space)**
  - ***rasterization*** **(how to turn primitives into colored pixels)**
- **TODAY:**
  - **see where these two ideas come crashing together!**
  - ***perspective*** **transformations**
  - **talk about how to map *texture* onto a primitive to get more detail**
  - **…and how perspective creates challenges for texture mapping!**



Why is it hard to render an image like this?

Perspective Projection

Perspective projection



Early painting: incorrect perspective

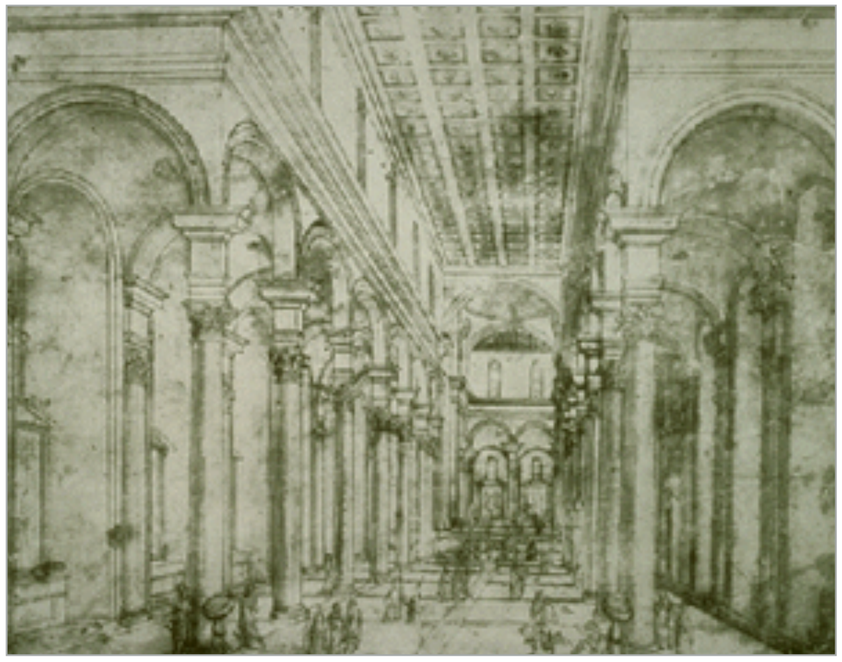
Carolingian painting from the 8-9th century

Perspective in art

Giotto 1290

Evolution toward correct perspective

Ambrogio Lorenzetti
Annunciation, 1344



Brunelleschi, elevation of Santo Spirito,
1434-83, Florence

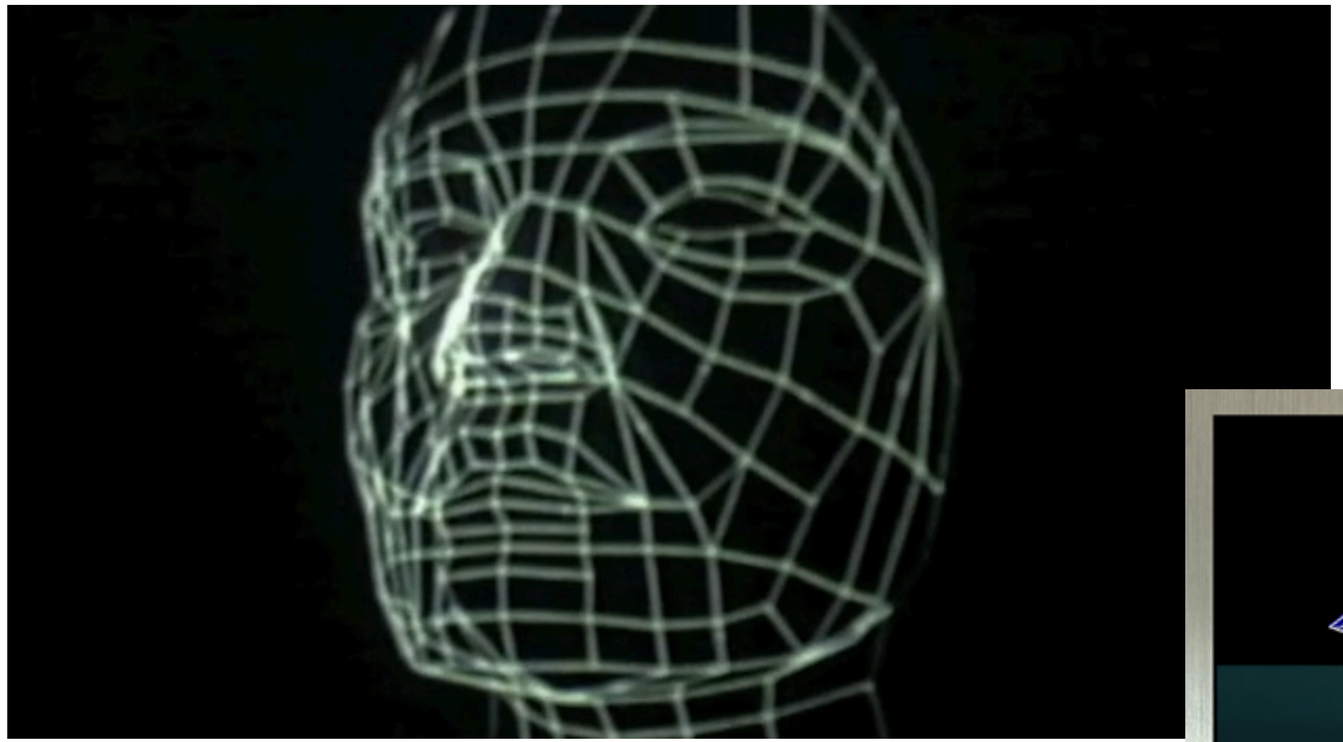
Masaccio – The Tribute Money c.1426-27
Fresco, The Brancacci Chapel, Florence

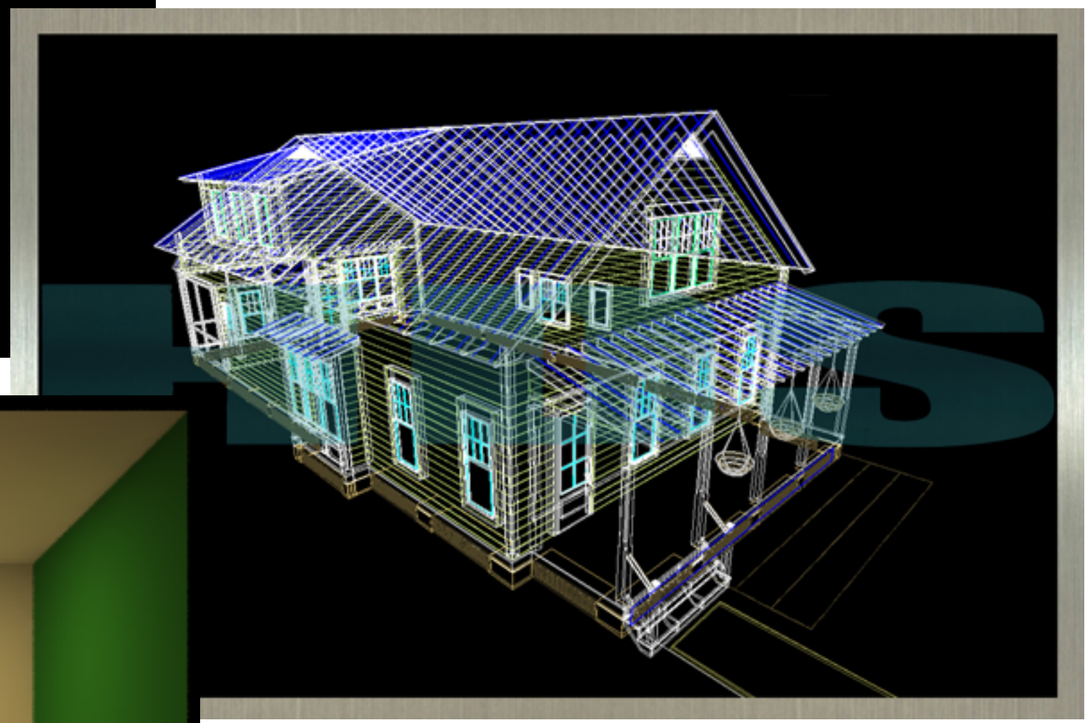
Perspective in art

Delivery of the Keys (Sistine Chapel), Perugino, 1482

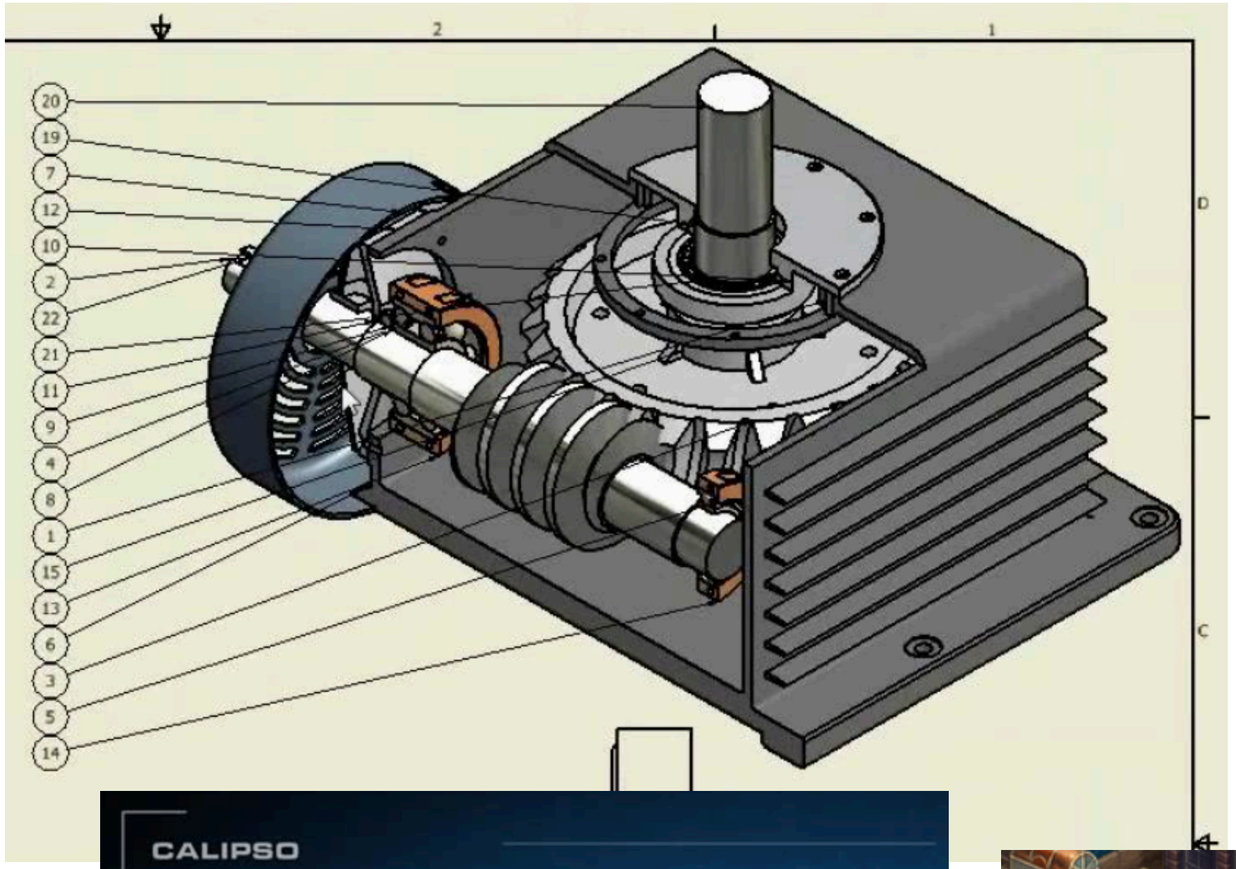
Later… rejection of proper perspective projection

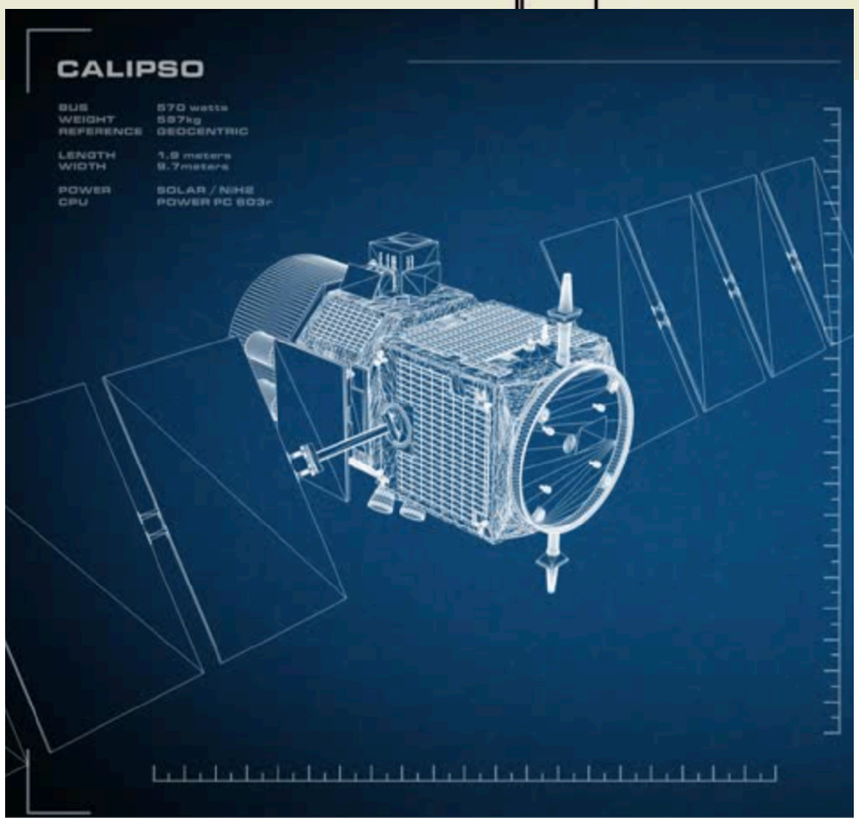
Correct perspective in computer graphics



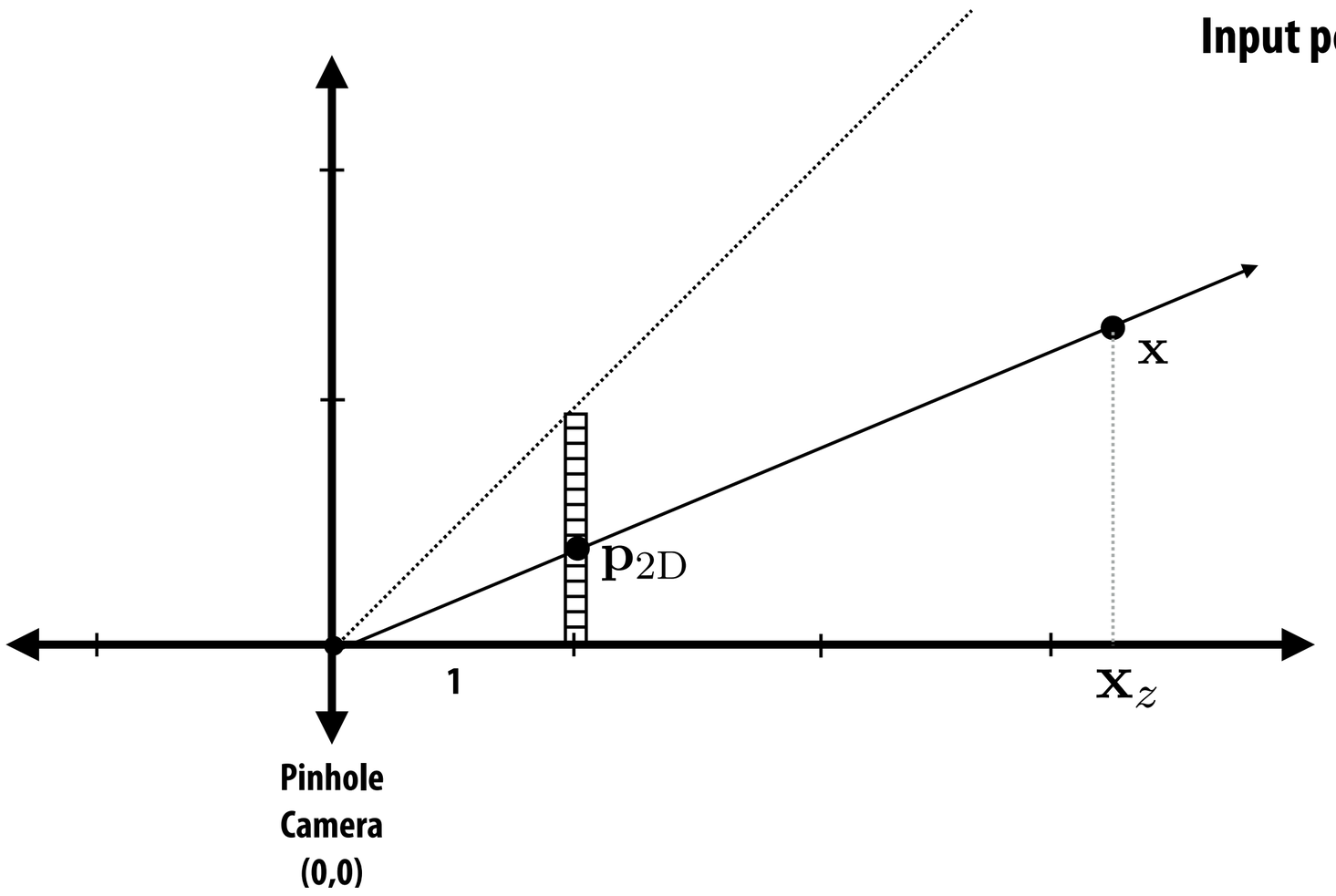


Rejection of perspective in computer graphics





Basic perspective projection



Input point in 3D-H: $\mathbf{x} = \begin{bmatrix} \mathbf{x}_x & \mathbf{x}_y & \mathbf{x}_z & 1 \end{bmatrix}^T$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Assumption:

Pinhole camera at (0,0) looking down z

Perspective vs. orthographic projection

- **Most basic version of perspective projection matrix:**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z \end{bmatrix} \longmapsto \begin{bmatrix} x/z \\ y/z \\ 1 \\ 1 \end{bmatrix}$$

objects shrink in distance

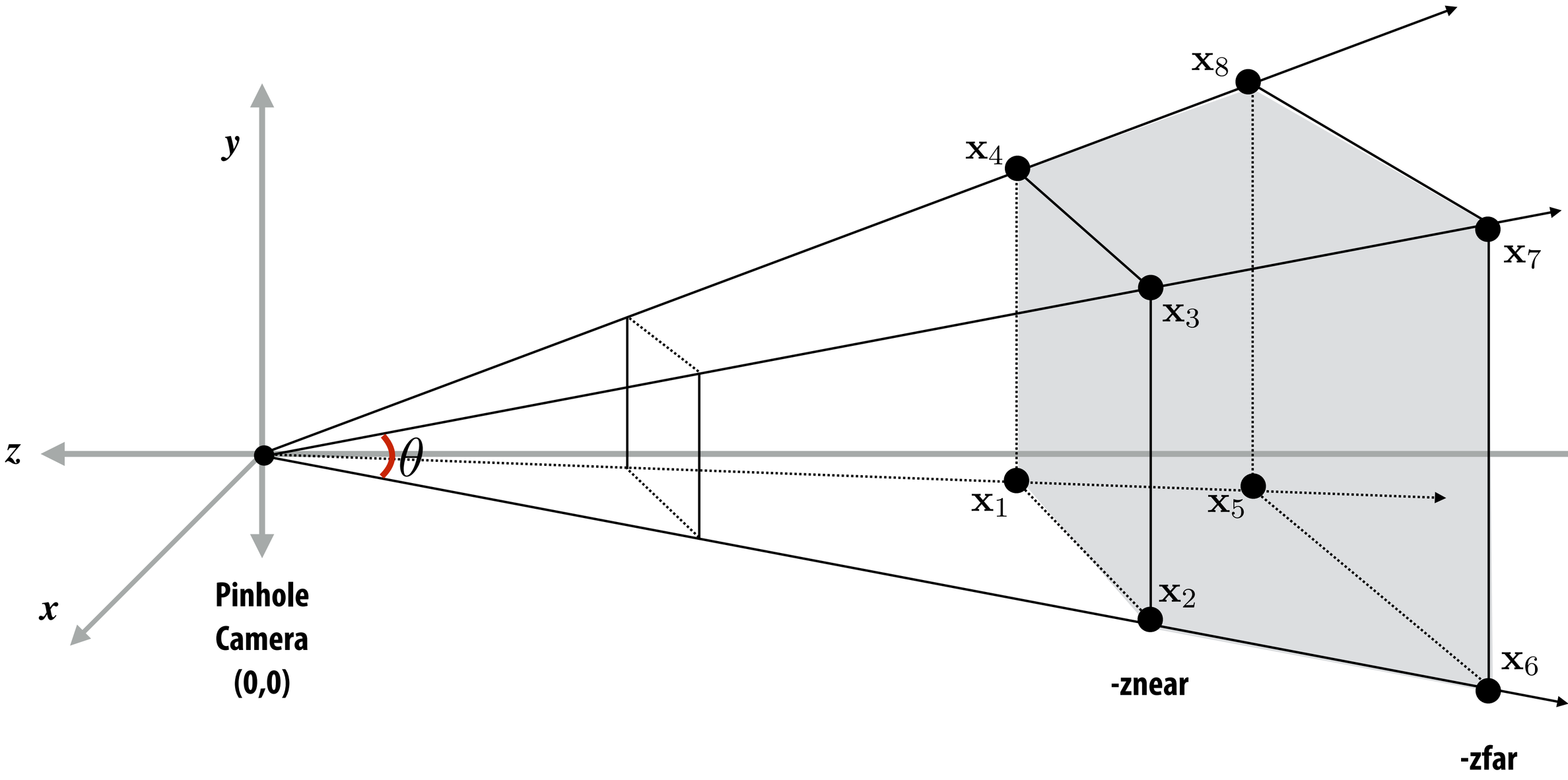
- **Most basic version of orthographic projection matrix:**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longmapsto \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

objects stay the same size

View frustum

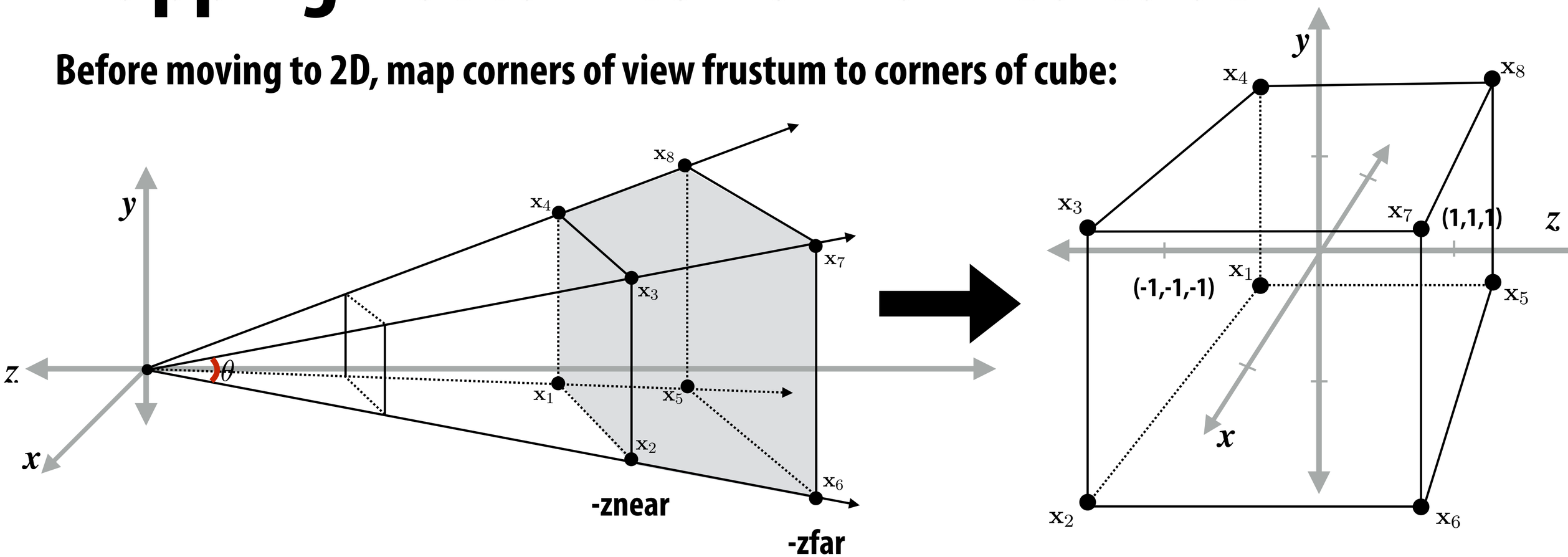
View frustum is the region of space the camera can see:



- Top/bottom/left/right planes correspond to sides of screen
- Near/far planes correspond to closest/furthest thing we want to draw

Mapping frustum to normalized cube

Before moving to 2D, map corners of view frustum to corners of cube:



View frustum corresponding to pinhole camera
(perspective projection transform transforms this volume to normalized cube)

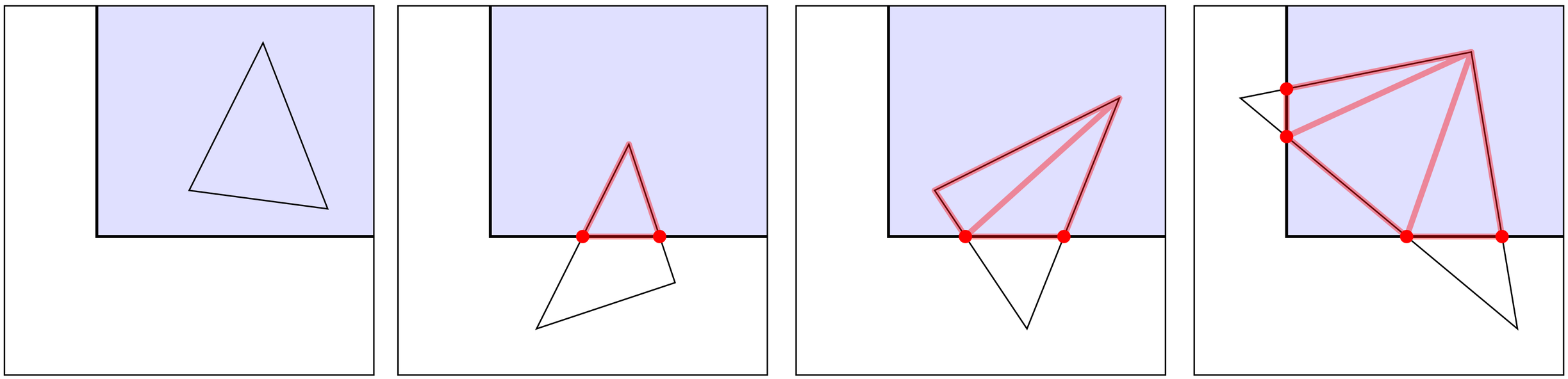
Why do we map frustum to unit cube?

1. Makes *clipping* much easier! (see next slide)
   - Can quickly discard geometry outside range [-1,1]
2. Represent all vertices in normalized cube in fixed point math

*** Question: what does the frustum of an orthographic camera look like?**

Clipping

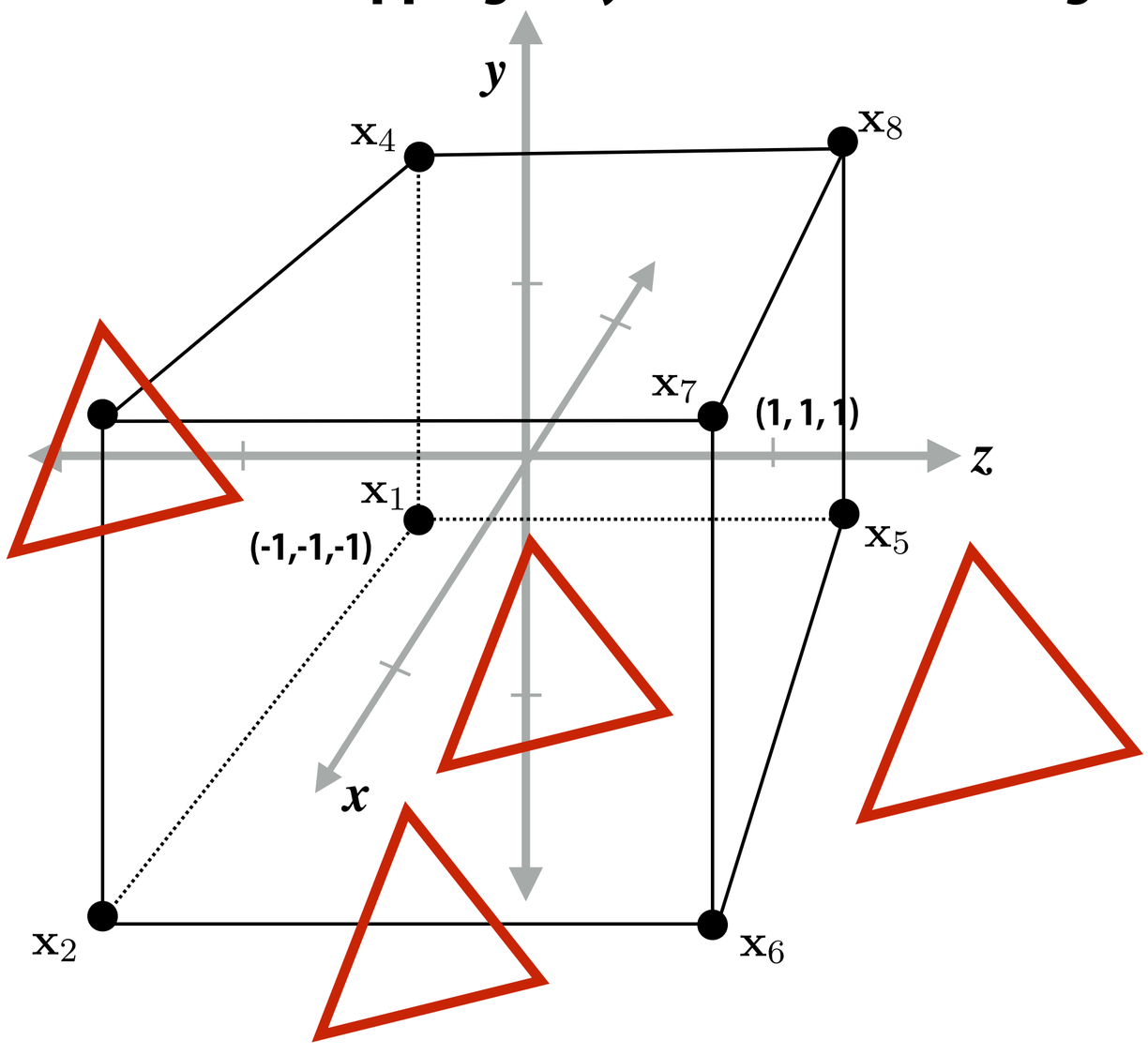
- "Clipping" is the process of eliminating triangles that aren't visible from the camera (because they outside the view frustum)
  - Don't waste time computing appearance of primitives the camera can't see!
  - Sample-in-triangle tests are expensive ("fine granularity" visibility)
  - Makes more sense to toss out entire primitives ("coarse granularity")
  - Must deal with primitives that are partially clipped…



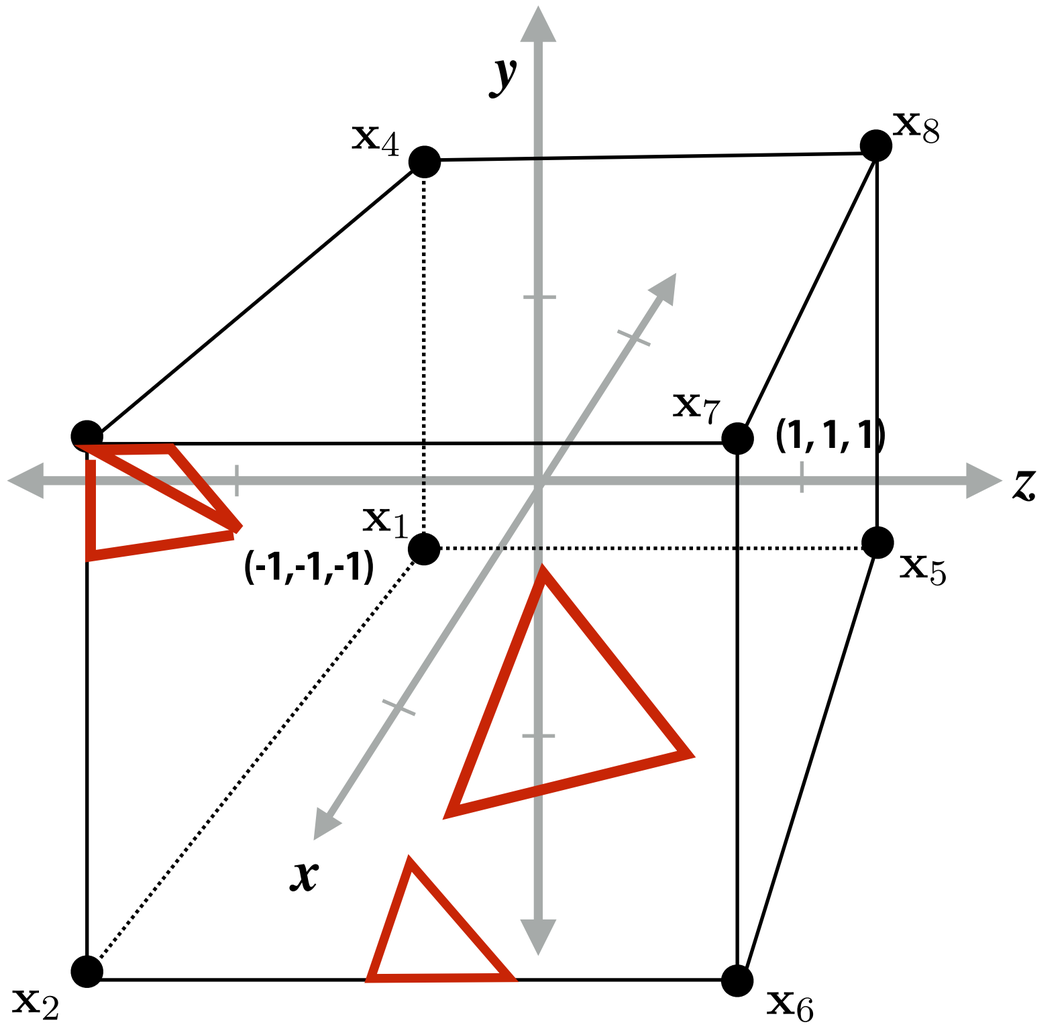
from: https://paroj.github.io/gltut/

Clipping in normalized device coordinates (NDC)

- Discard triangles that lie complete outside the normalized cube (culling)
  - They are off screen, don't bother processing them further
- Clip triangles that extend beyond the cube… to the sides of the cube
  - Note: clipping may create more triangles



Triangles before clipping

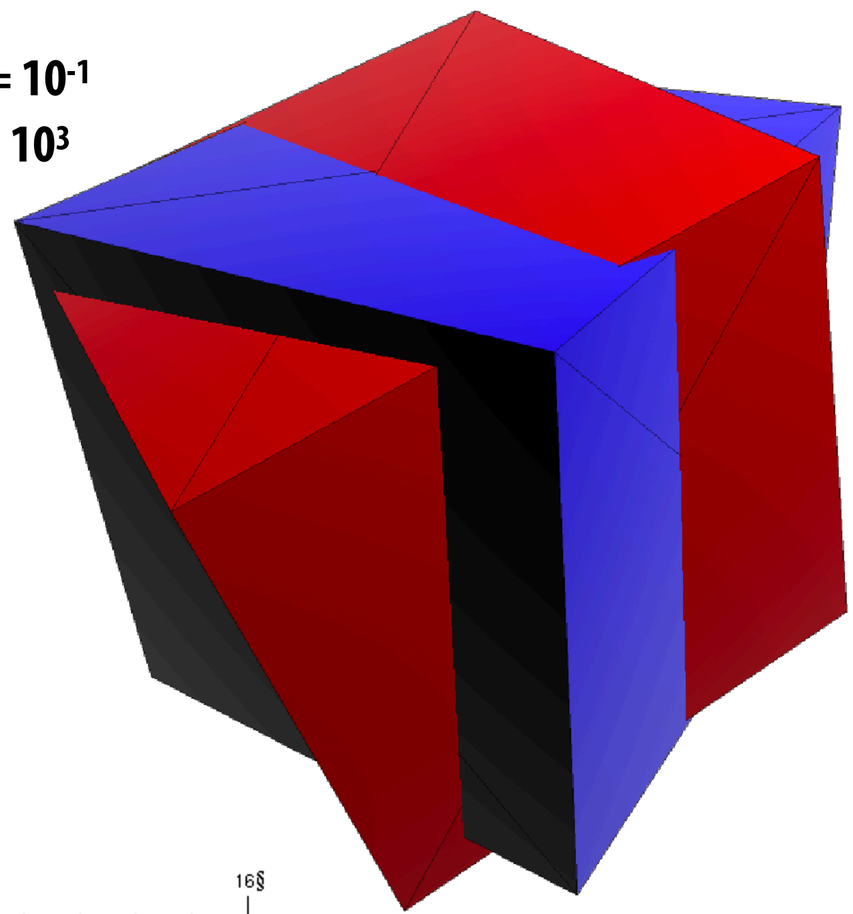


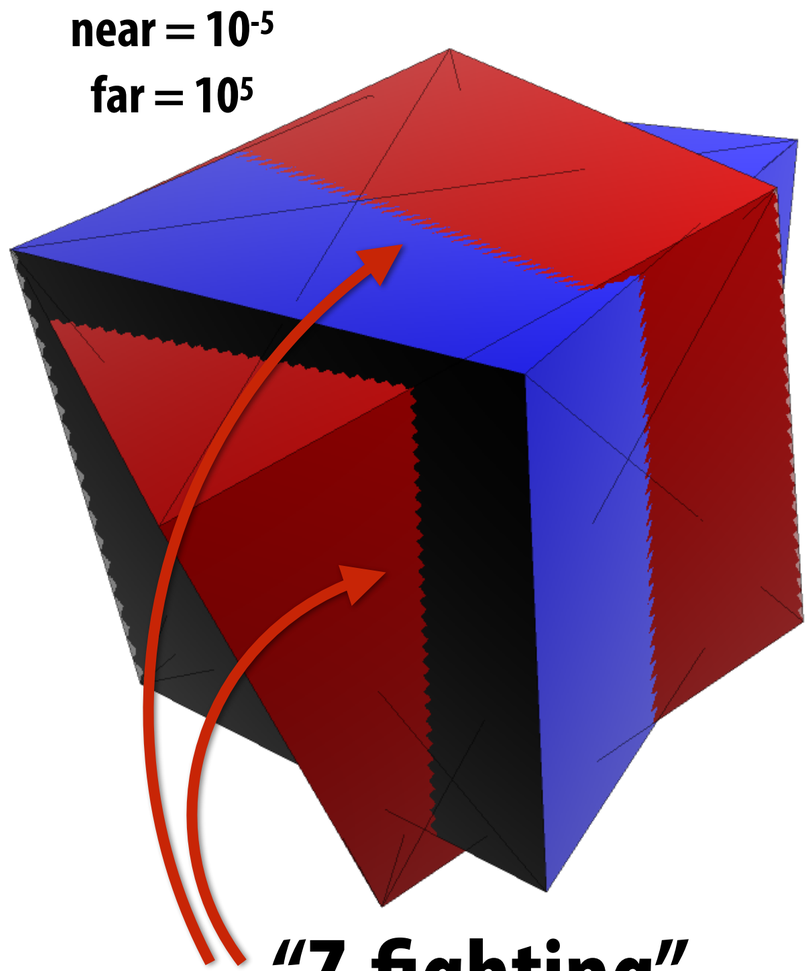
Triangles after clipping

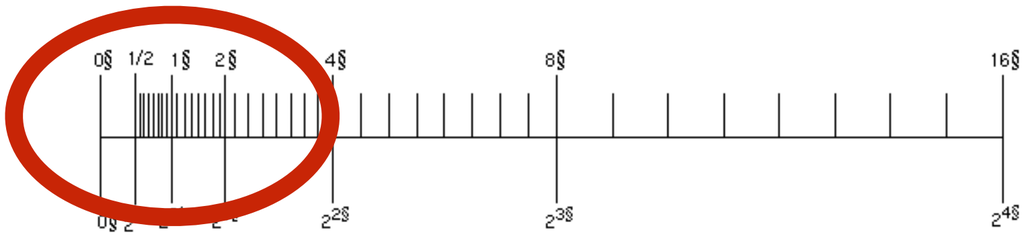
* These figures are correct: OpenGL normalized device coordinates is left-handed coordinate space

Detailed aside: why near/far plane clipping?

- Primitives (e.g., triangles) may have vertices both in front and behind camera! (Causes headaches for rasterization, e.g., checking if fragments are behind camera
- Avoid divide by zero in perspective divide (near plane clipping)
- Also important for dealing with finite precision of depth buffer



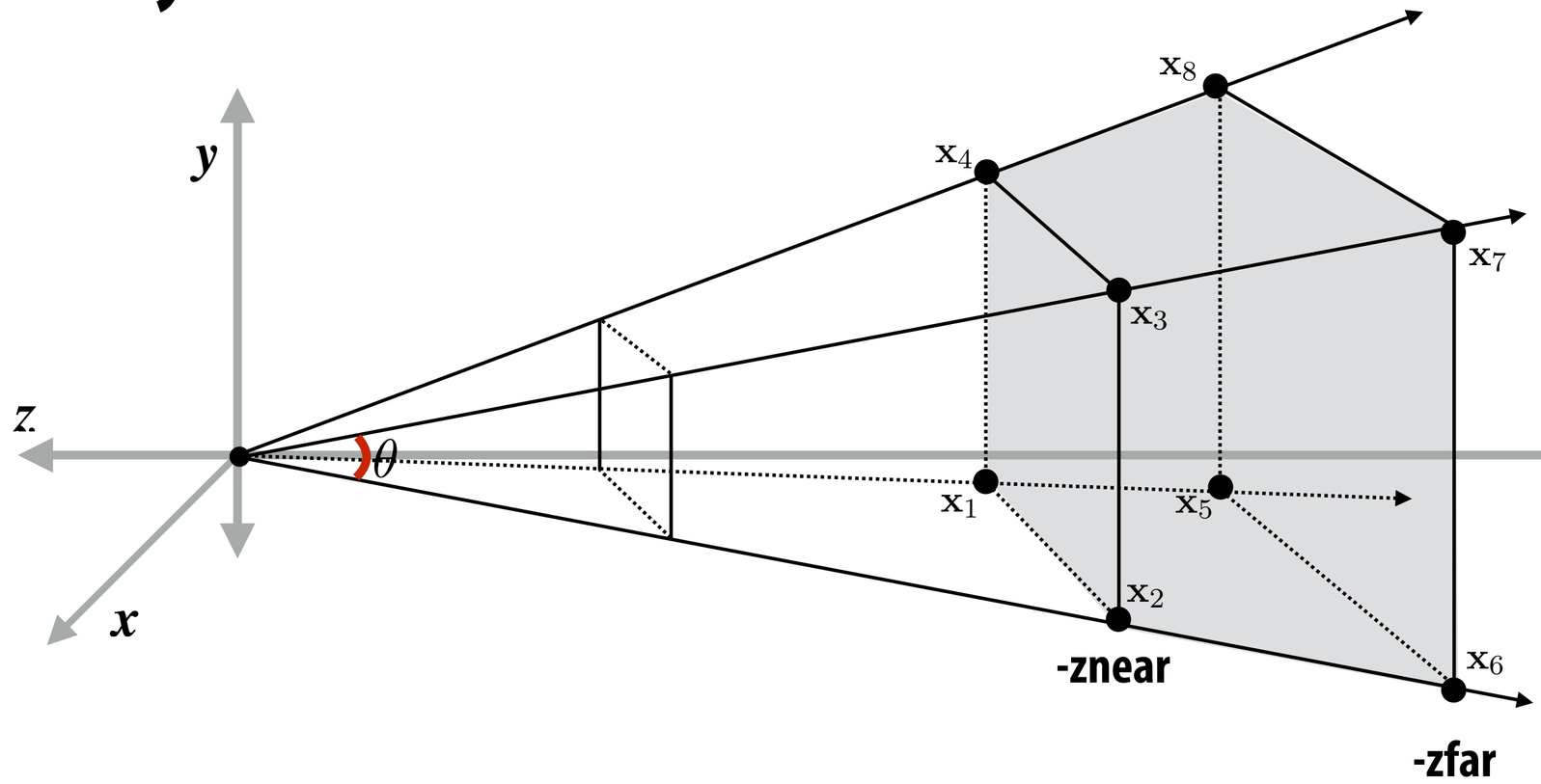




floating point has more "resolution" near zero—hence more precise resolution of primitive-primitive intersection

Matrix for perspective transform

Takes into account geometry of view frustum:



$$\begin{pmatrix} \frac{n}{r} & 0 & 0 & 0 \\ 0 & \frac{n}{t} & 0 & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

left (l), right (r), top (t), bottom (b), near (n), far (f)

(matrix at left is perspective projection for frustum that is symmetric about x,y axes: l=-r, t=-b)

For a derivation: http://www.songho.ca/opengl/gl_projectionmatrix.html

Recall: screen transform

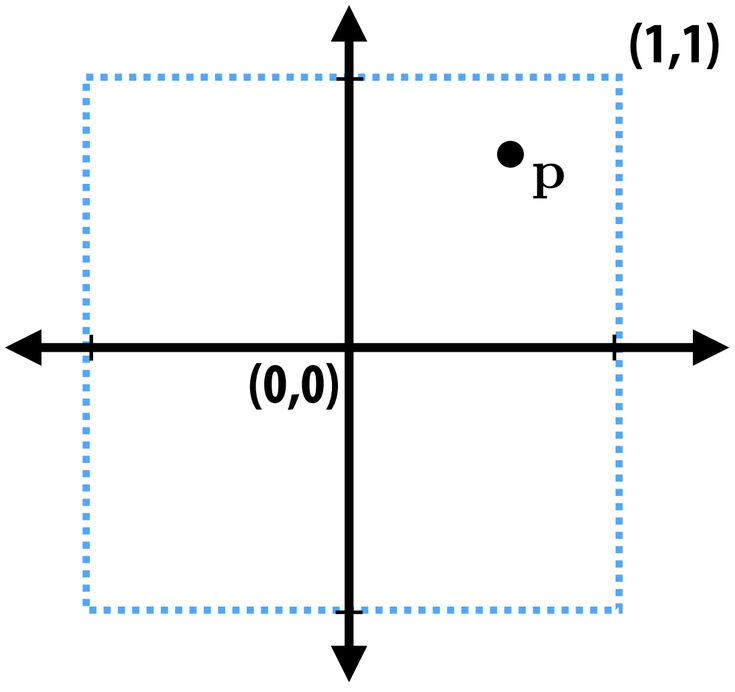
After divide, coordinates in [-1,1] have to be "stretched" to fit the screen

Example:

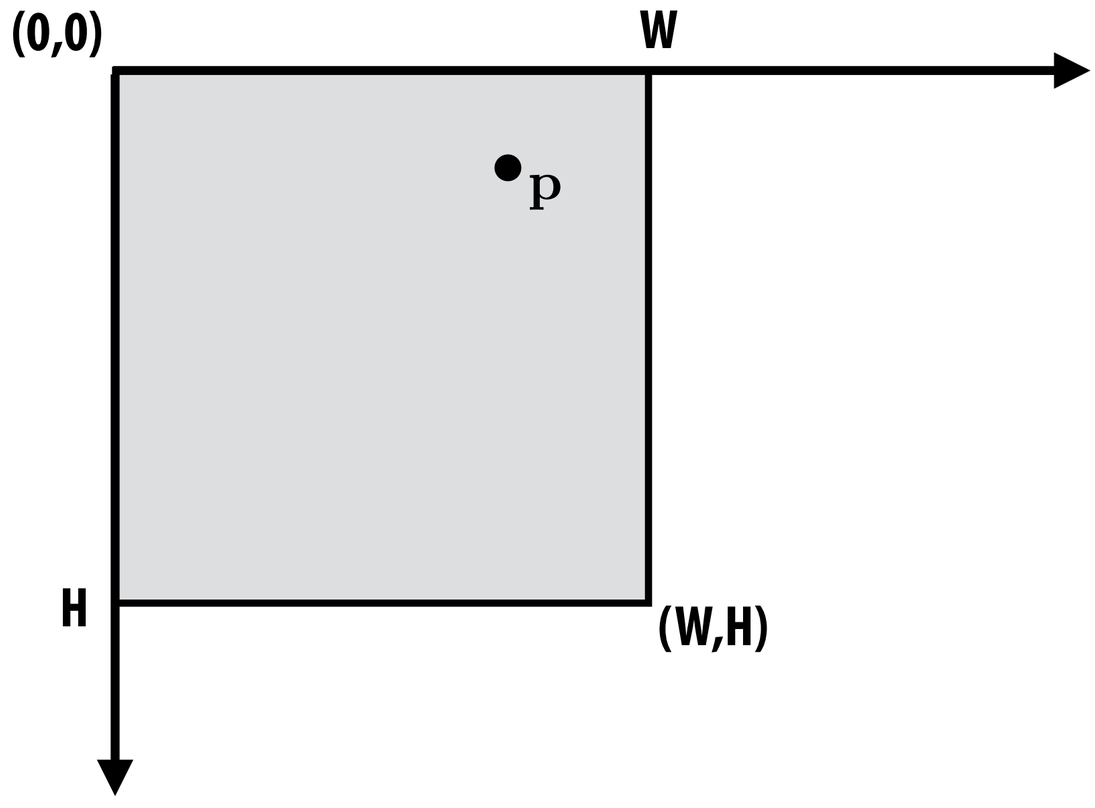
All points within (-1,1) to (1,1) region are on screen

(1,1) in normalized space maps to (W,0) in screen

Normalized coordinate space:



Screen (W x H output image) coordinate space:



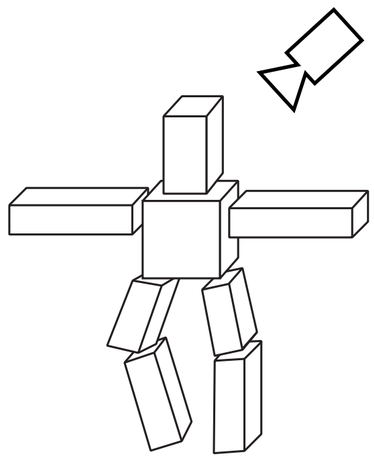
Step 1: reflect about x-axis

Step 2: translate by (1,1)

Step 3: scale by (W/2,H/2)

Transformations: from objects to the screen

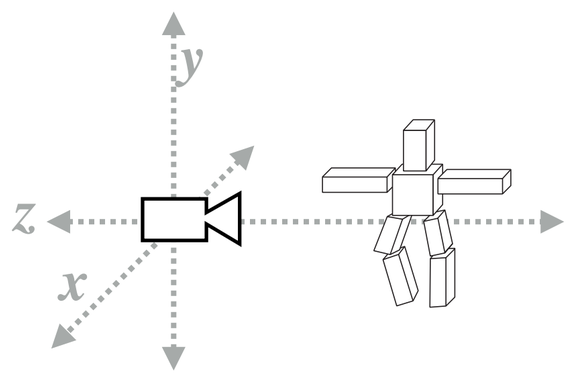
[WORLD COORDINATES]



original description of objects

view transform

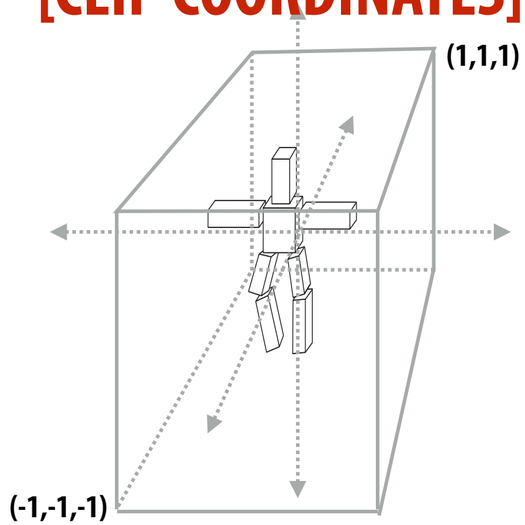
[VIEW COORDINATES]



vertex positions now expressed relative to camera; camera is sitting at origin looking down -z direction (can canonicalize projection matrix)

projection transform

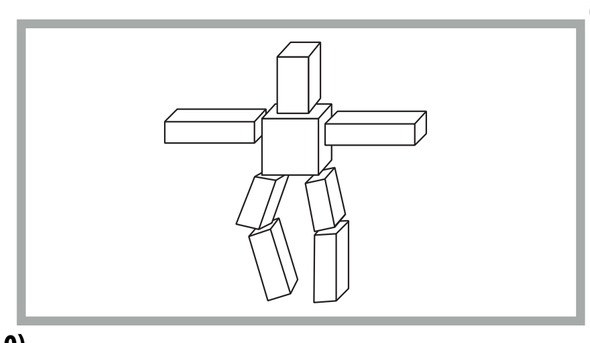
[CLIP COORDINATES]



everything visible to the camera is mapped to unit cube for easy "clipping"

screen transform

[WINDOW COORDINATES]



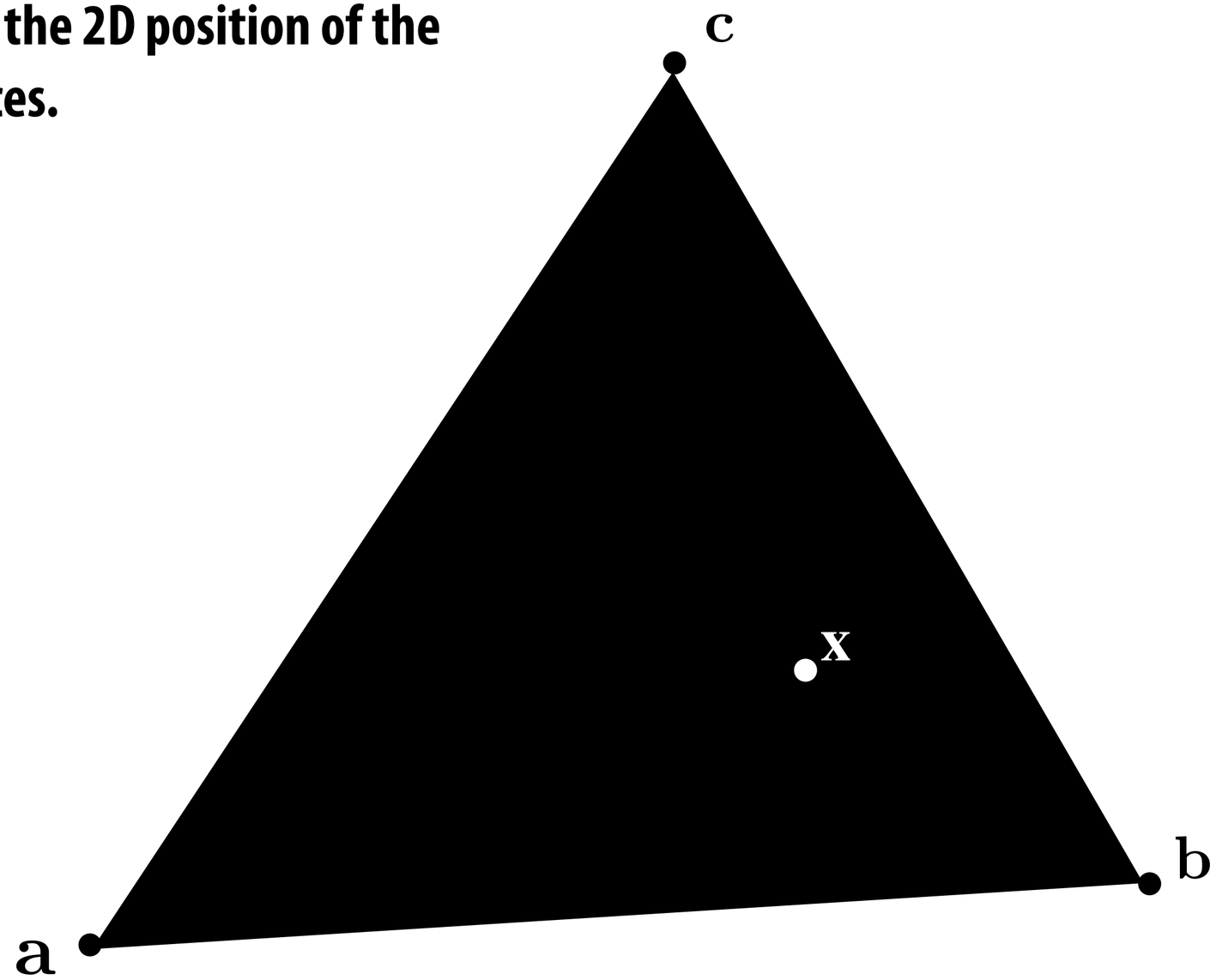
objects now in 2D screen coordinates

primitives are now 2D and can be drawn via rasterization

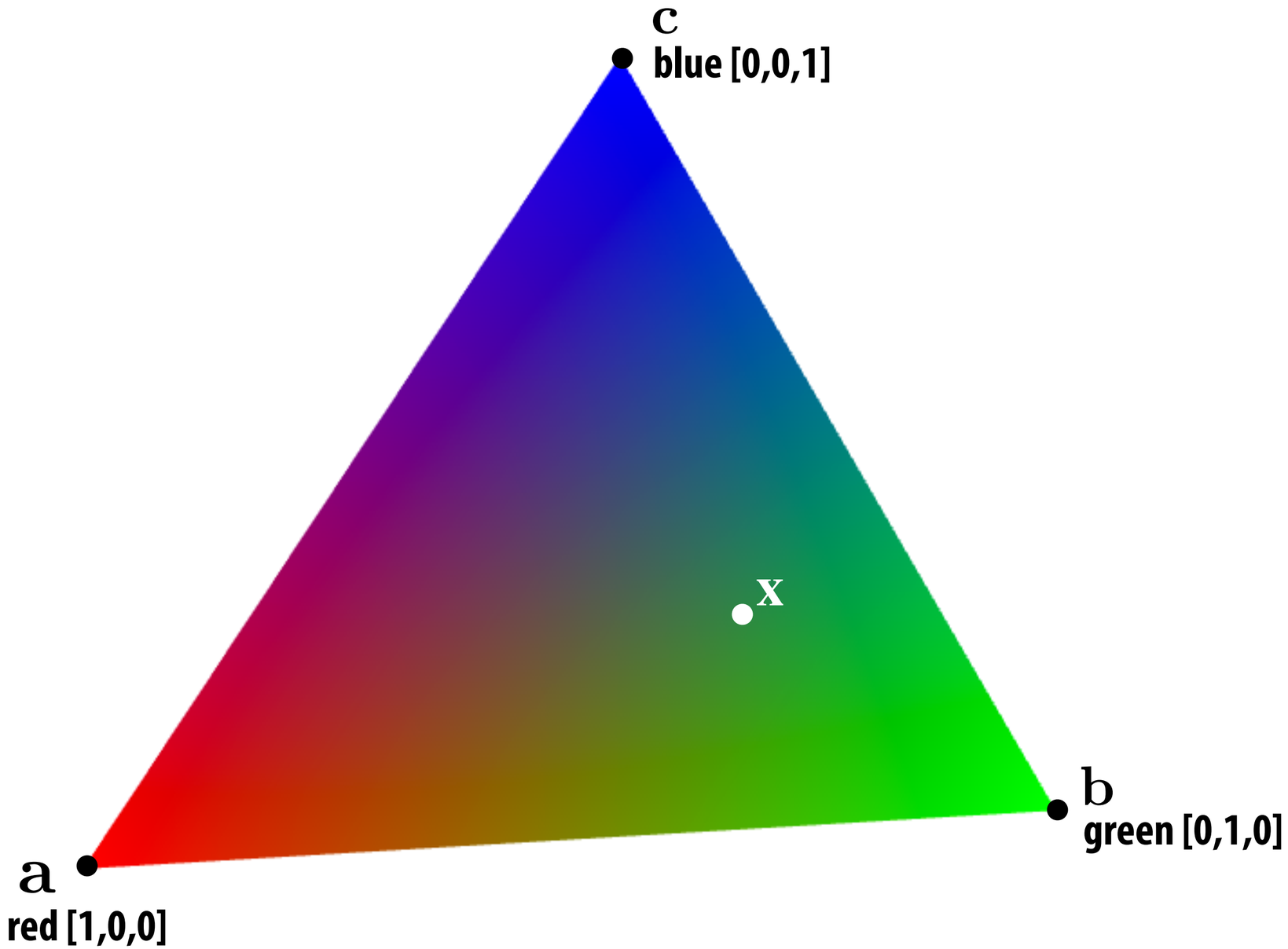
Texture mapping

Recall the function coverage(x,y) from lecture 2

In lecture 2 we discussed how to sample coverage given the 2D position of the triangle's vertices.



Consider sampling color(x,y)



What is the triangle's color at the point x ?

Review: interpolation in 1D

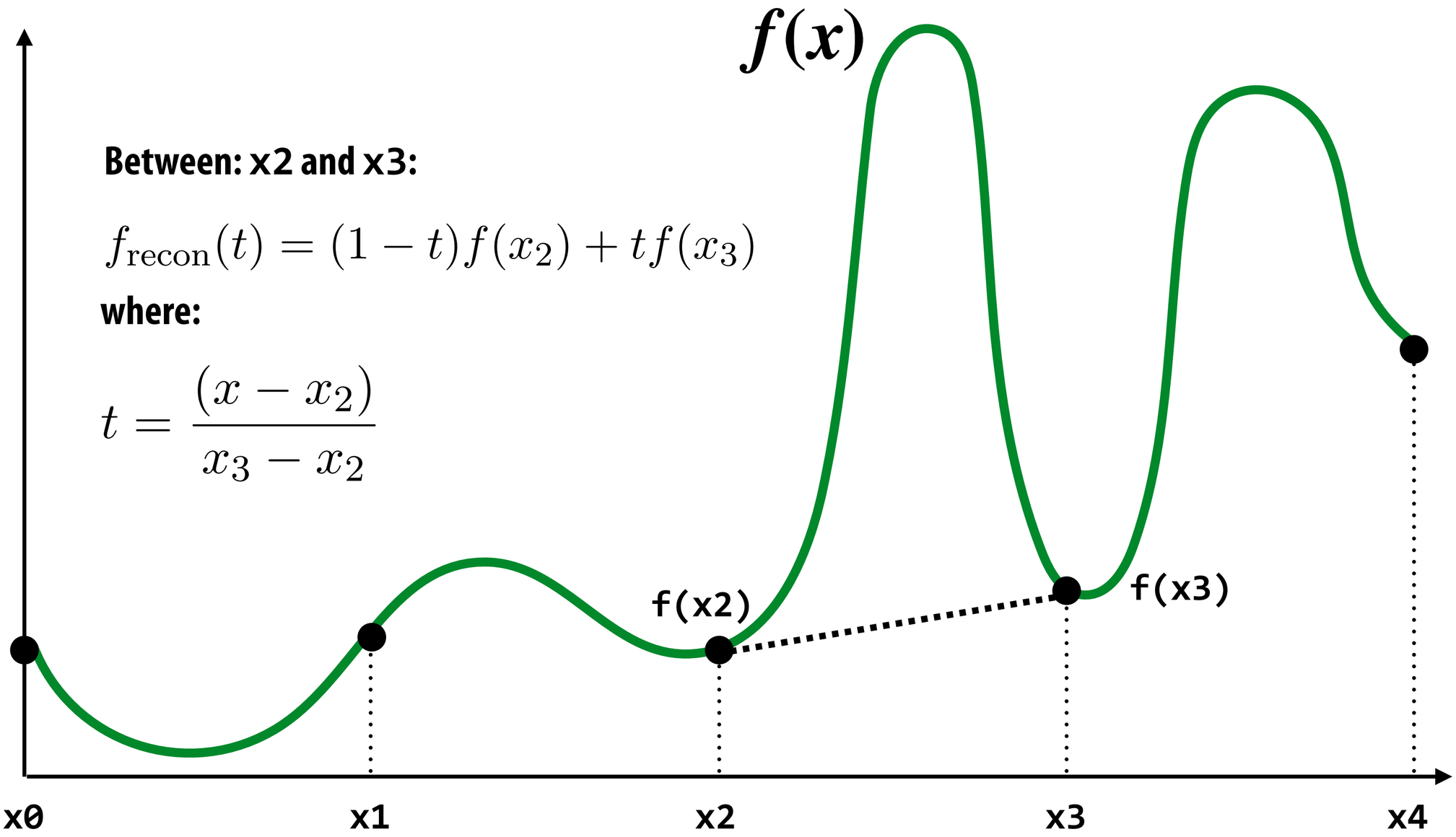
$f_{recon}(x)$ = linear interpolation between values of two closest samples to x

Between: x2 and x3:

$$f_{\text{recon}}(t) = (1-t)f(x_2) + tf(x_3)$$

where:

$$t = \frac{(x - x_2)}{x_3 - x_2}$$

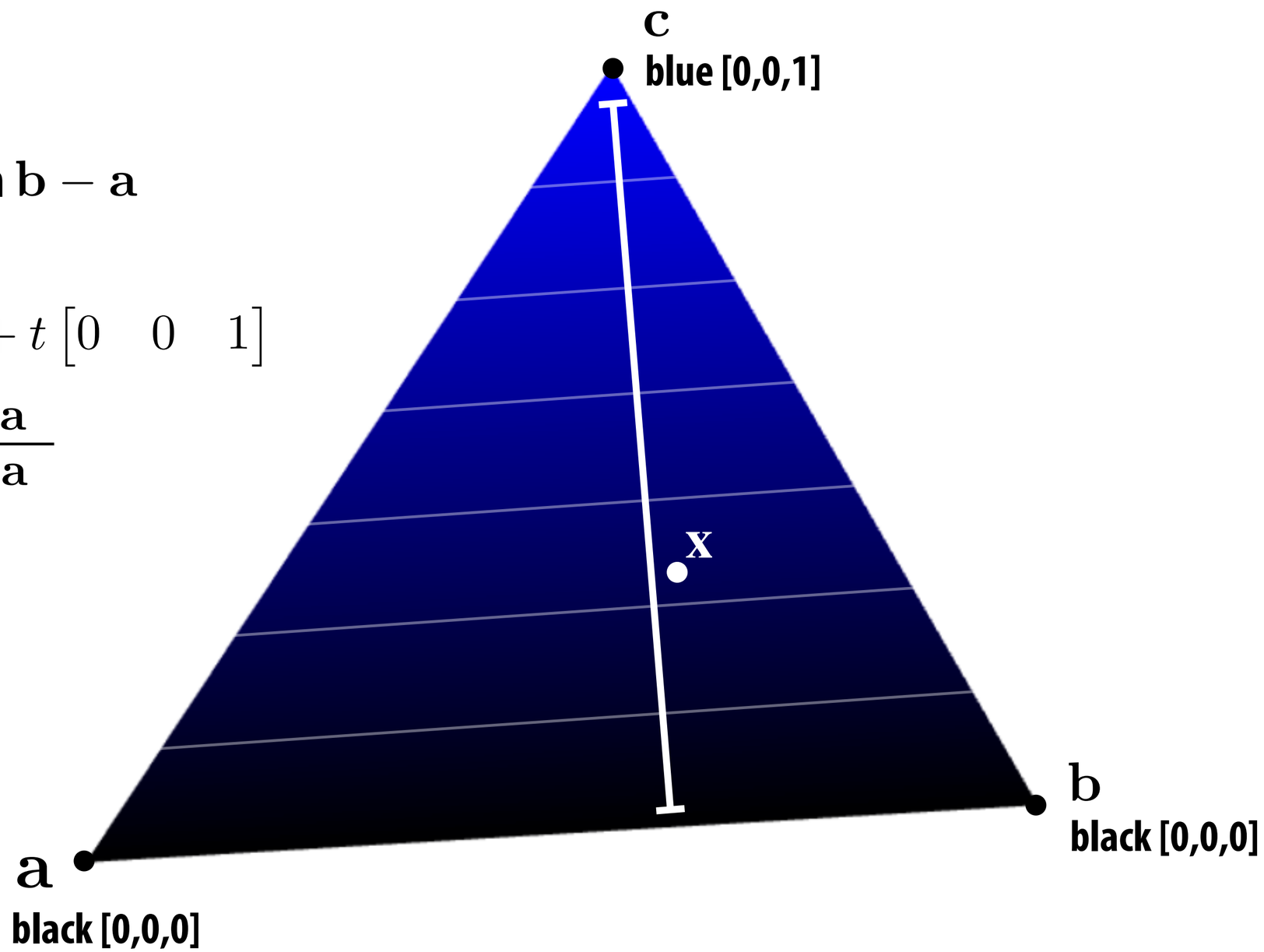


Consider similar behavior on triangle

Color depends on distance from $\mathbf{b} - \mathbf{a}$

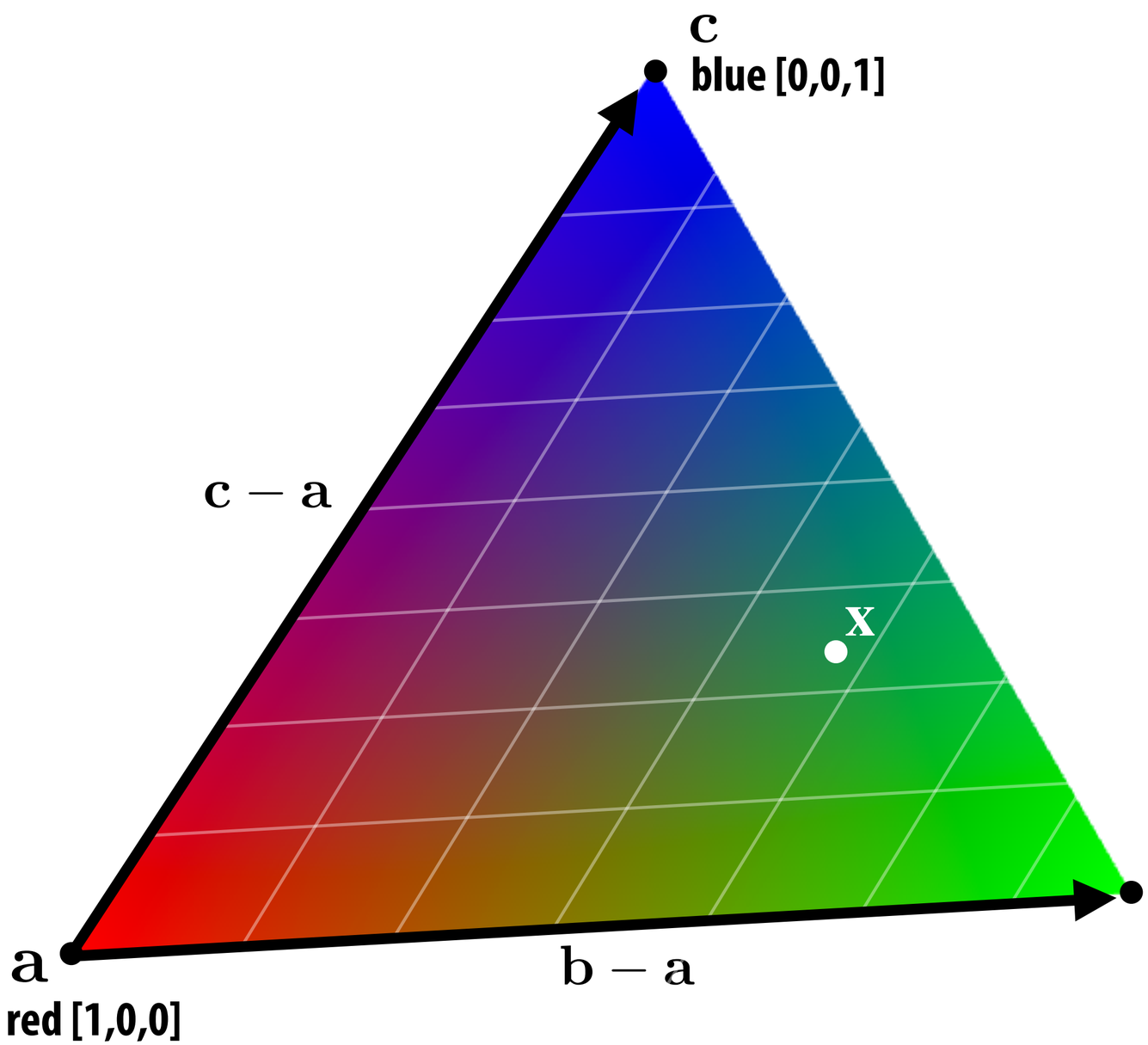
color at $(1-t)\begin{bmatrix}0 & 0 & 0\end{bmatrix} + t\begin{bmatrix}0 & 0 & 1\end{bmatrix}$

$$t = \frac{\textbf{distance from } \mathbf{x} \textbf{ to } \mathbf{b}-\mathbf{a}}{\textbf{distance from } \mathbf{c} \textbf{ to } \mathbf{b}-\mathbf{a}}$$



How can we interpolate in 2D between three values?

Interpolation via barycentric coordinates



$\mathbf{b}-\mathbf{a}$ and $\mathbf{c}-\mathbf{a}$ form a non-orthogonal basis for points in triangle (origin at $\mathbf{a}$)

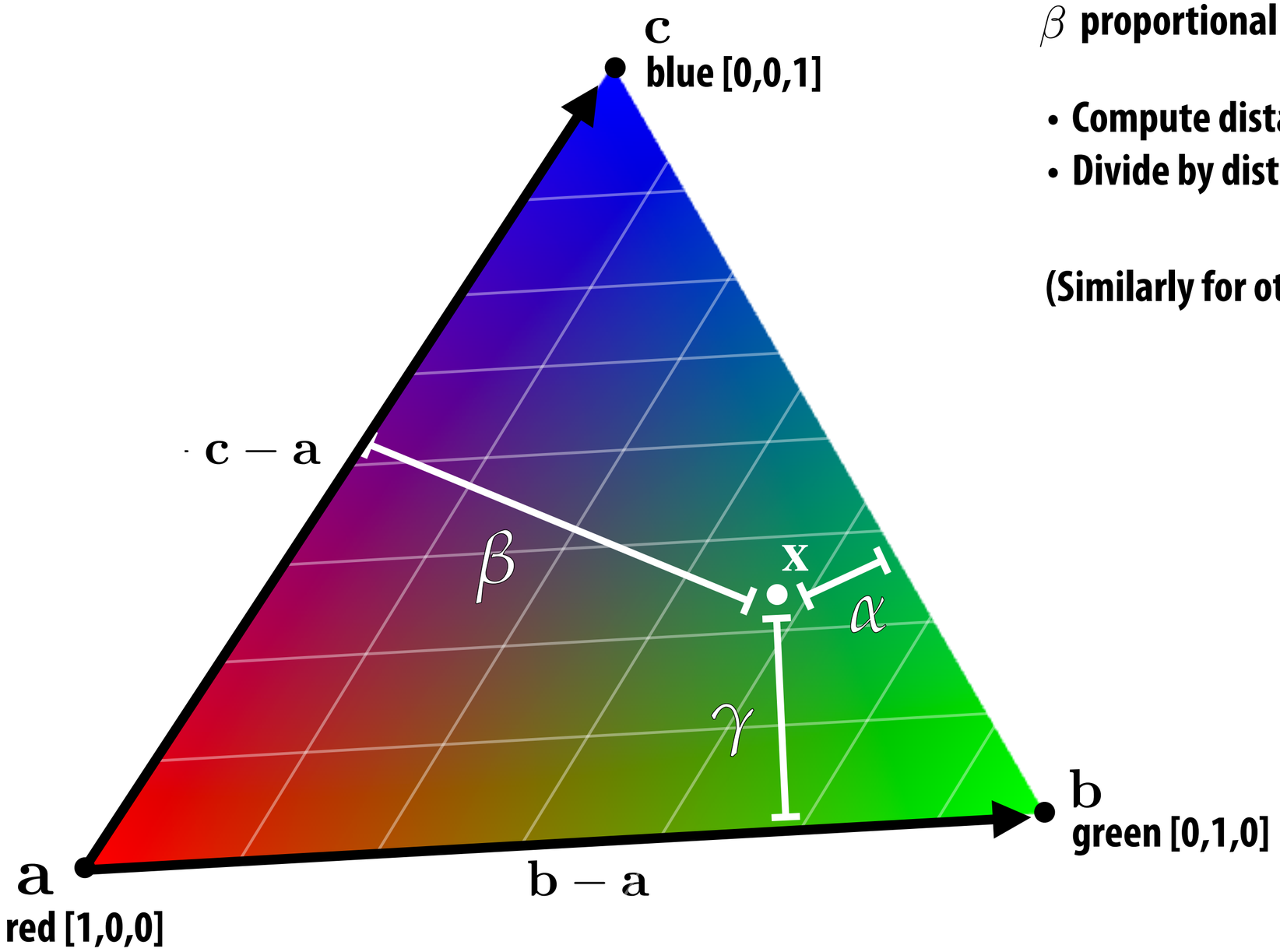
$$\begin{aligned}\mathbf{x} &= \mathbf{a} + \beta(\mathbf{b}-\mathbf{a}) + \gamma(\mathbf{c}-\mathbf{a}) \\ &= (1-\beta-\gamma)\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} \\ &= \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}\end{aligned}$$

$$\alpha + \beta + \gamma = 1$$

Color at $\mathbf{x}$ is linear combination of color at three triangle vertices.

$$\mathbf{x}_{\text{color}} = \alpha\mathbf{a}_{\text{color}} + \beta\mathbf{b}_{\text{color}} + \gamma\,\mathbf{c}_{\text{color}}$$

Barycentric coordinates as scaled distances

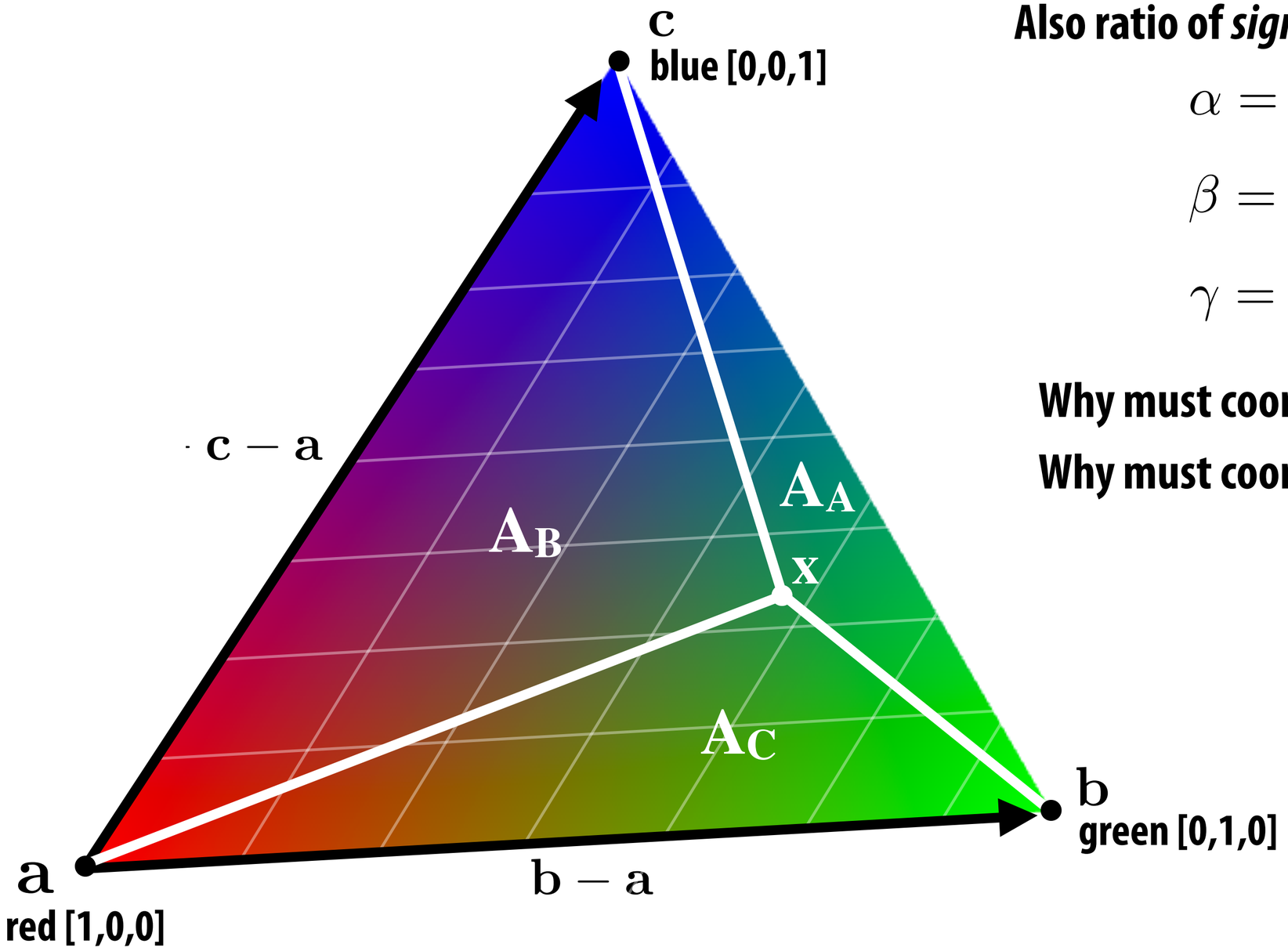


β proportional to distance from $\mathbf{x}$ to edge $\mathbf{c} - \mathbf{a}$

- Compute distance of x from line ca
- Divide by distance of b from line ca ("height")

(Similarly for other two barycentric coordinates)

Barycentric coordinates as ratio of areas



Also ratio of *signed* areas:

$$\alpha = A_A/A$$

$$\beta = A_B/A$$

$$\gamma = A_C/A$$

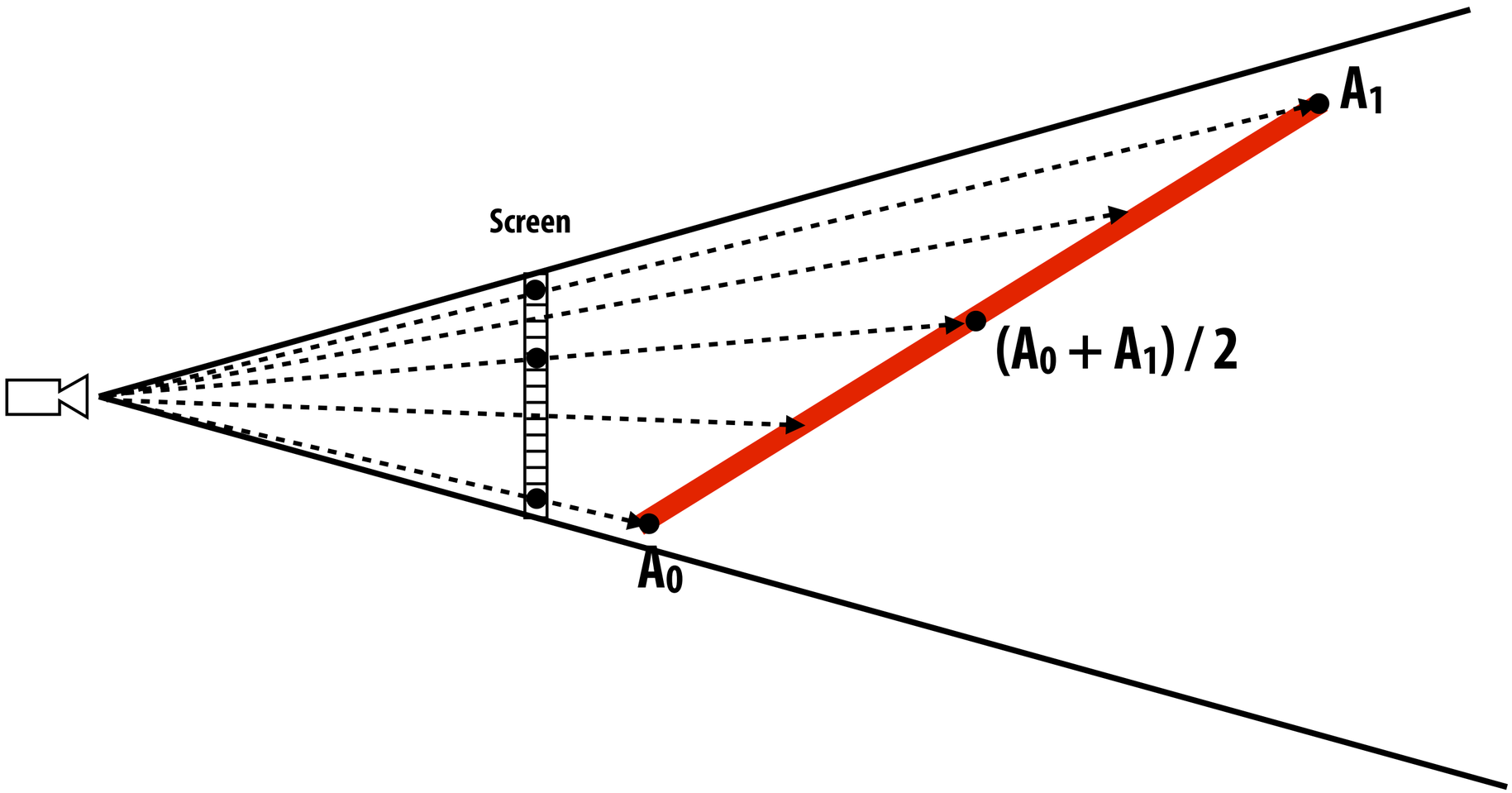
Why must coordinates sum to one?

Why must coordinates be between 0 and 1?

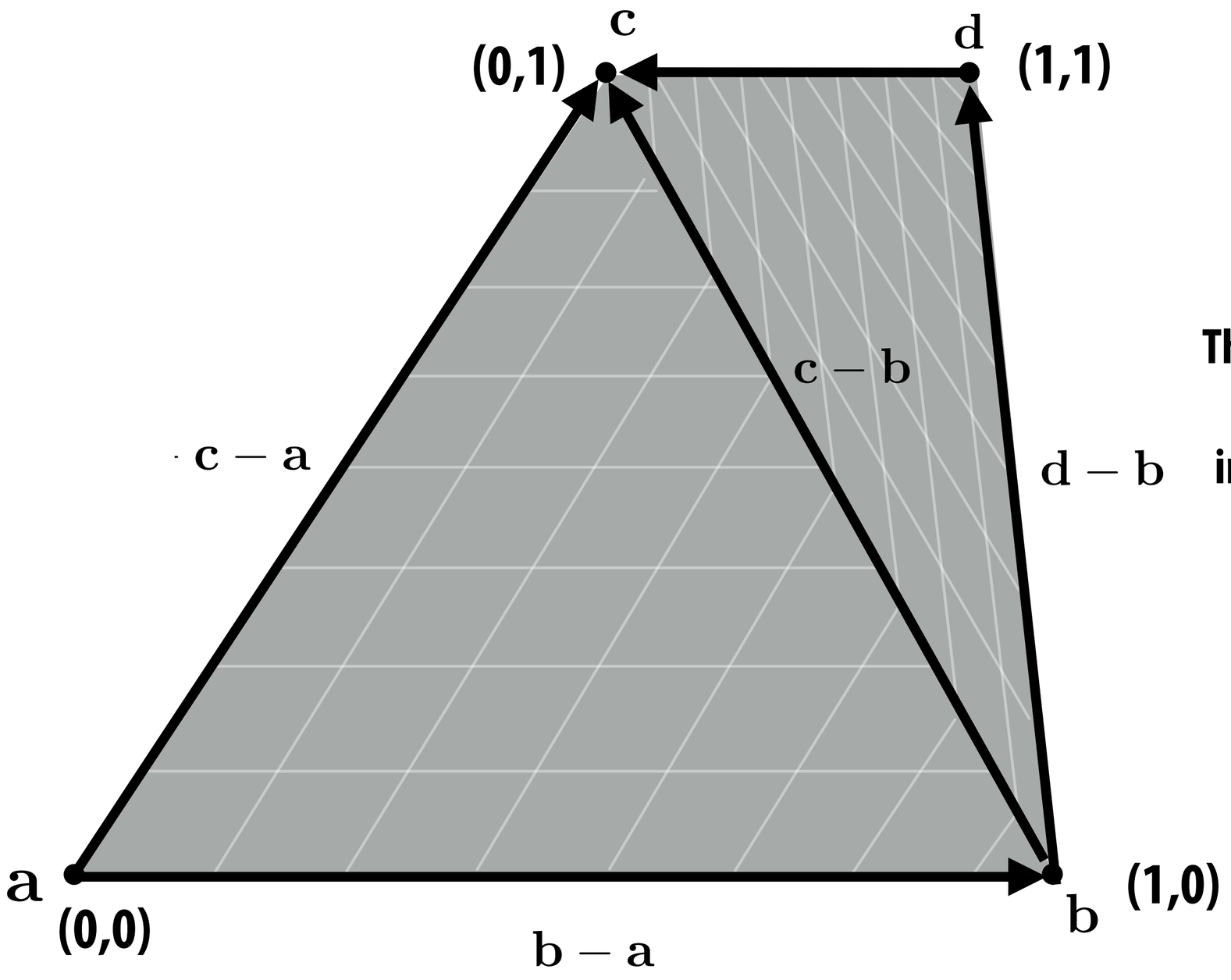
Incorrect interpolation under perspective

Interpolating attribute values linearly in screen space (using projected vertex positions to define the triangle) is not the same as interpolating linearly in 3D space, and then projecting.

Due to perspective projection, barycentric interpolation of values on a triangle with vertices of different depths is not an linear function of screen XY coordinates



Linear interpolation in screen space

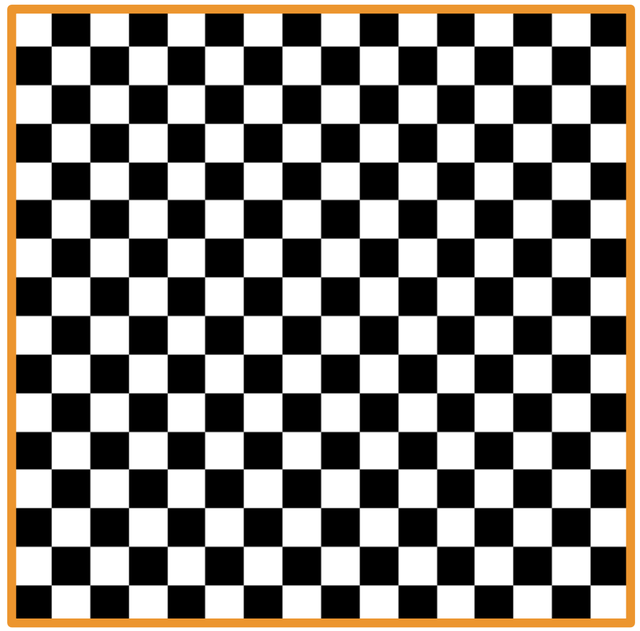


This is a plane (two triangles), tilted down and rendered under perspective.

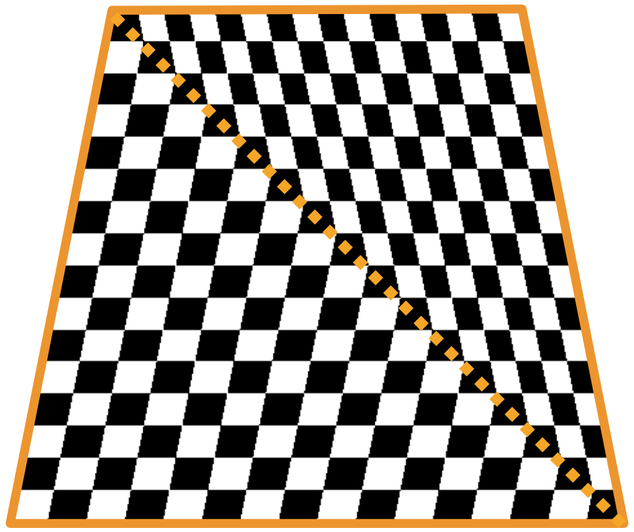
The white lines are isolines — showing where the x and y coordinate of the interpolated values is the same across the triangle

Perspective correct interpolation

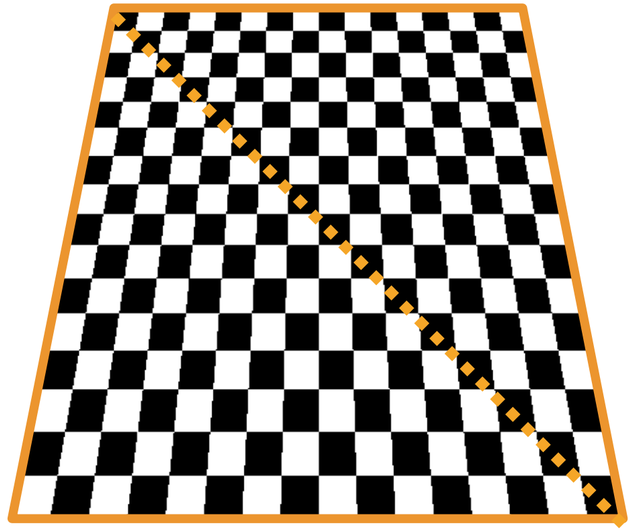
This is a plane (two triangles), tilted down and rendered under perspective.



Texture



Affine screen-space interpolation



World-space interpolation

Perspective correct interpolation (after projection to 2D)

- Basic recipe:
  - To interpolate some attribute A over a triangle…
  - Let z be the depth of the triangle at each vertex
  - Evaluate Z := 1/z and P := A/z at each vertex
  - Interpolate Z(x,y) and P(x,y) using standard (2D) barycentric coords
  - At each sample at 2D screen coord (x,y), divide P(x,y) by Z(x,y) to get A(x,y)
  - $P/Z = (A/z) / (1/z) = A$

In other words… A is not affine in 2D screen coordinates (X,Y), but A/z is!

For a derivation, see Low, "Perspective-Correct Interpolation" (I'll also add some useful notes on the web site)

Texture mapping

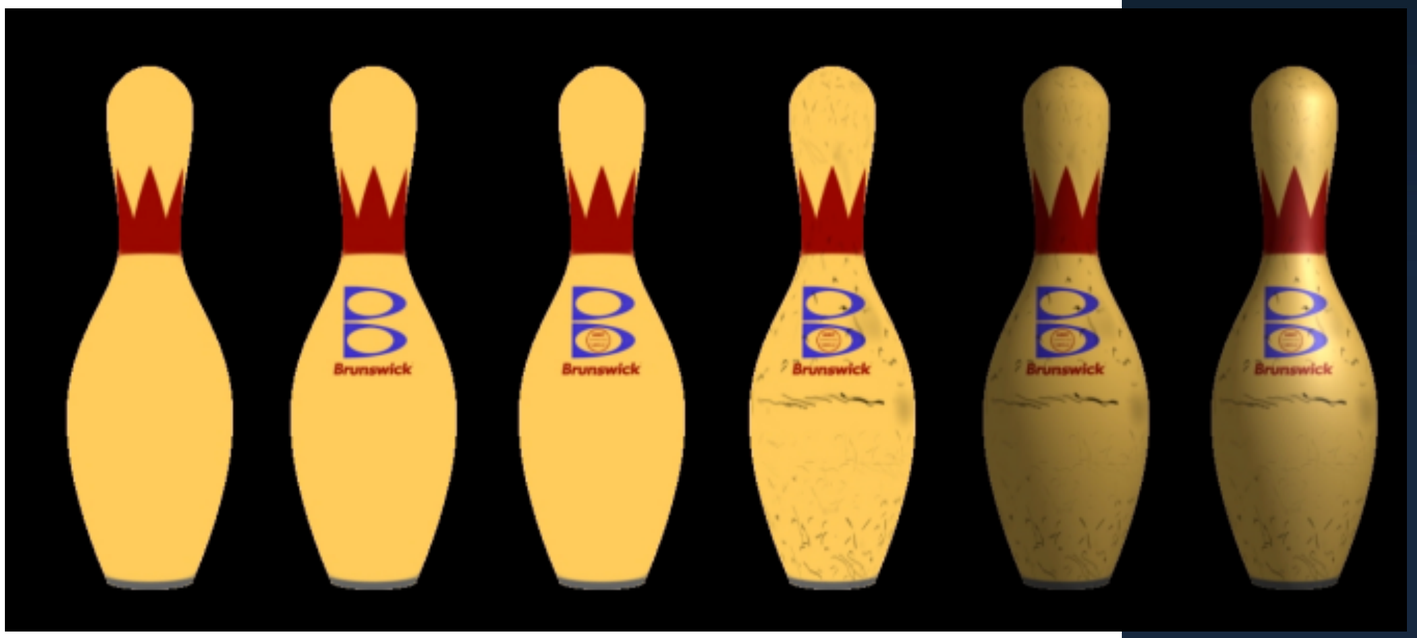
Many uses of texture mapping

Define variation in surface reflectance

Pattern on ball

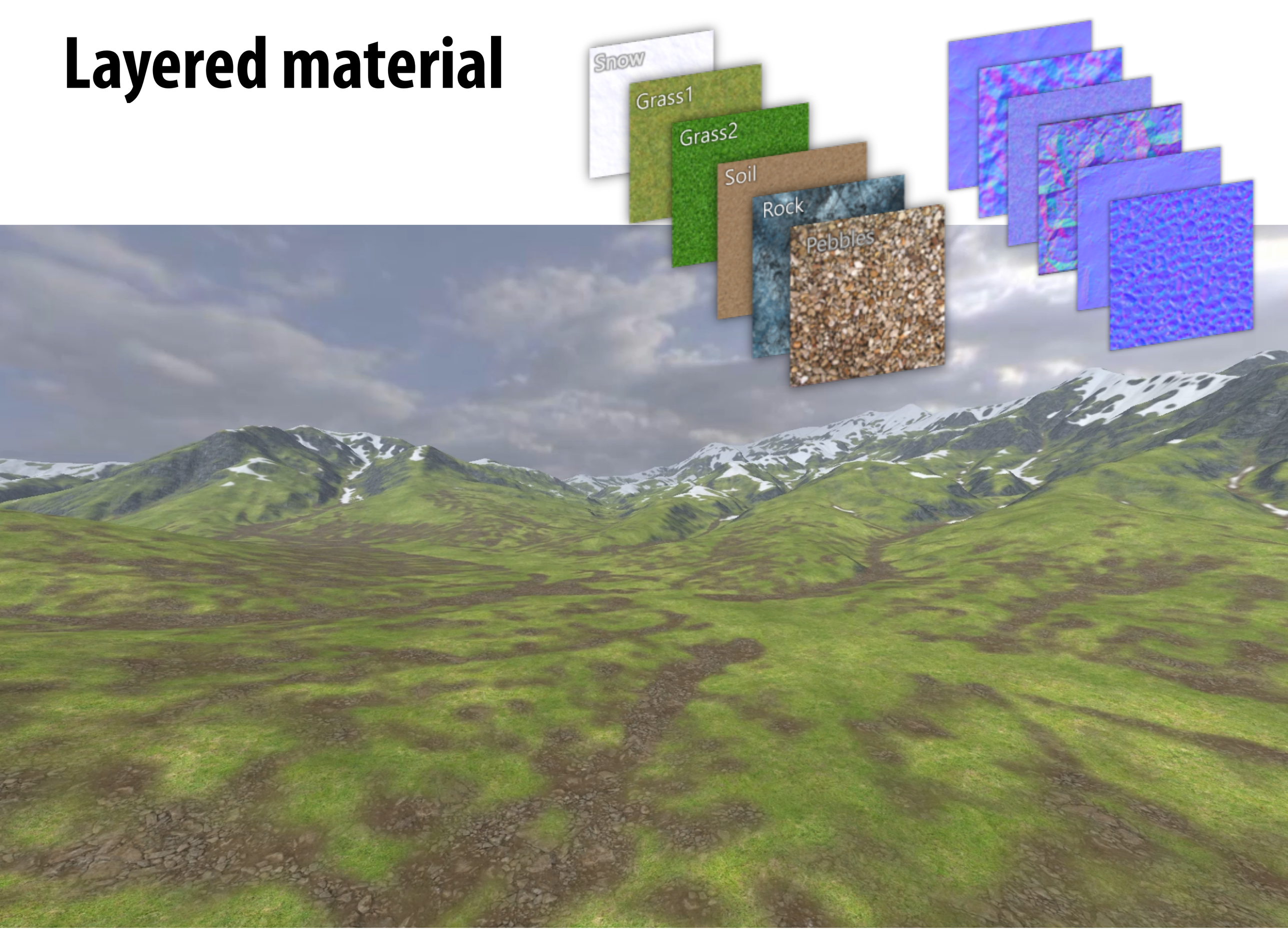
Wood grain on floor

Describe surface material properties



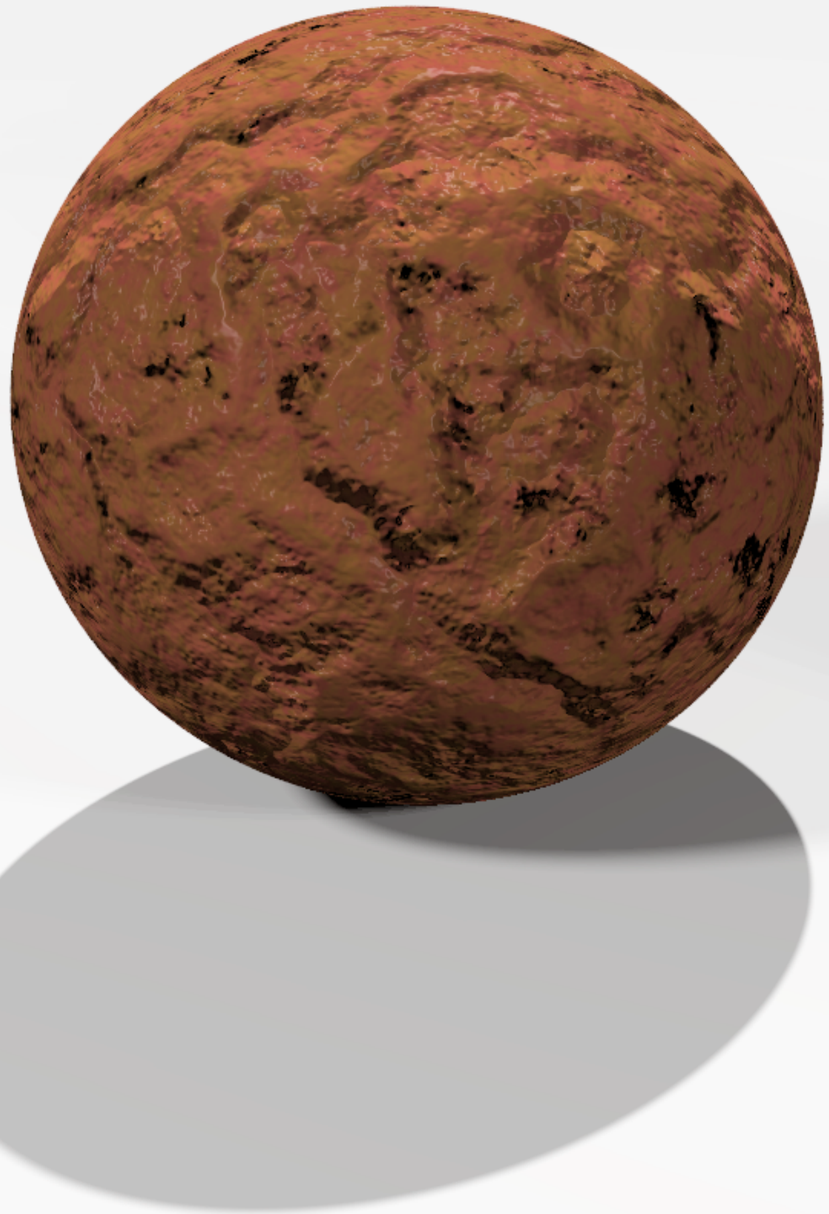
Multiple layers of texture maps for color, logos, scratches, etc.

Layered material



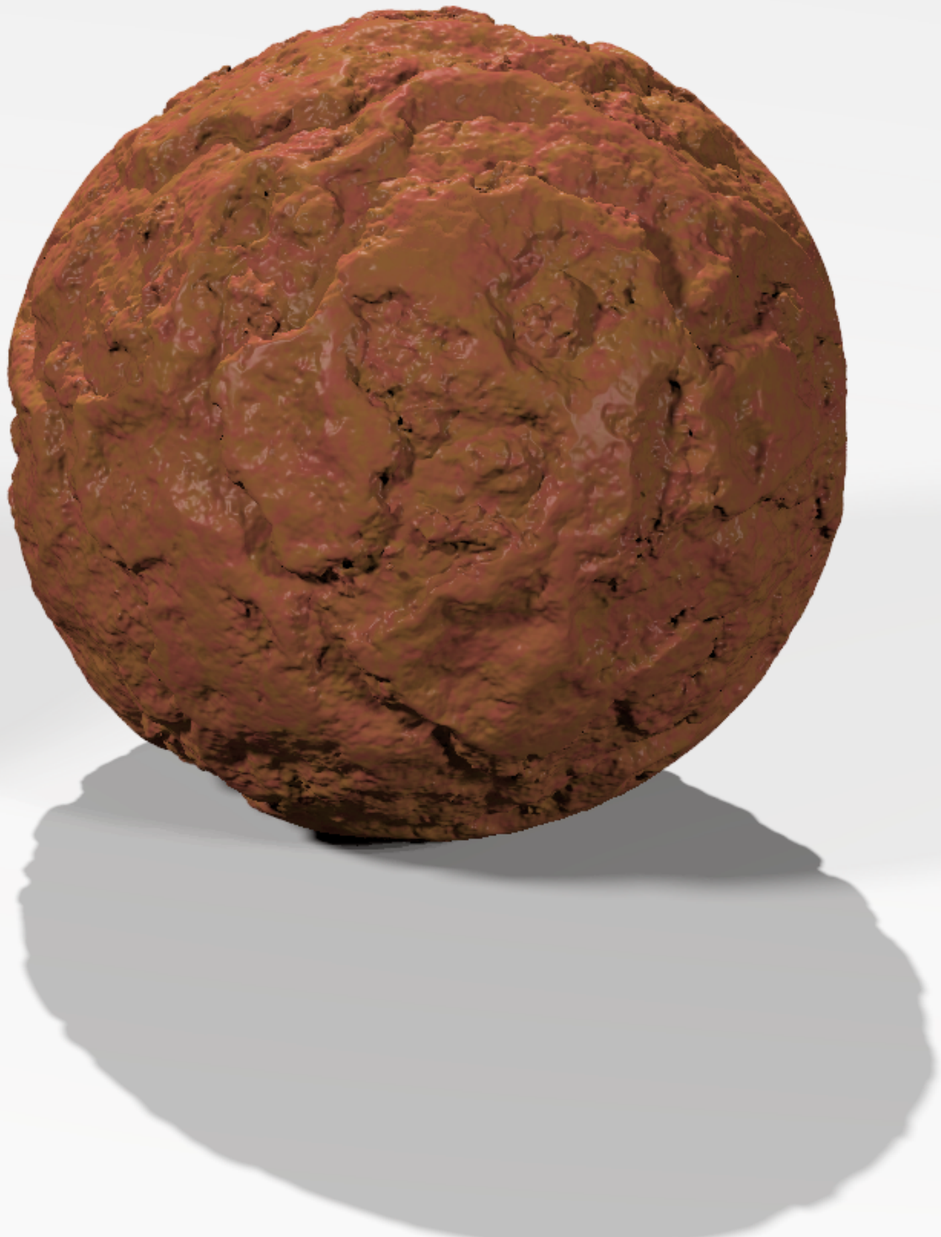
Normal and displacement mapping

normal mapping



Use texture value to perturb surface normal to "fake" appearance of a bumpy surface (note smooth silhouette/shadow reveals that surface geometry is not actually bumpy!)

displacement mapping



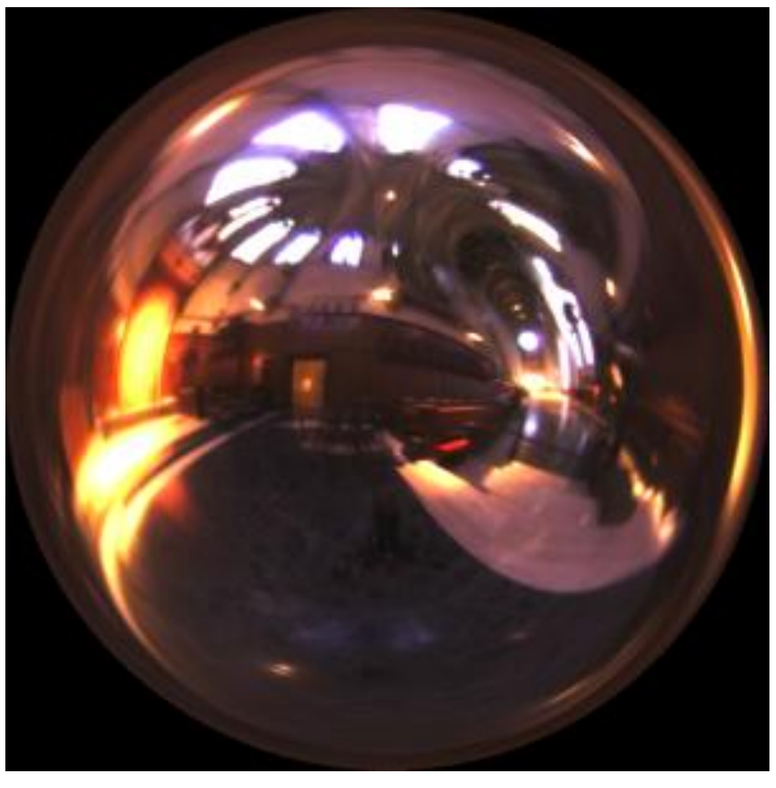
dice up surface geometry into tiny triangles & offset positions according to texture values (note bumpy silhouette and shadow boundary)

Represent precomputed lighting and shadows

Original model

With ambient occlusion

Extracted ambient occlusion map

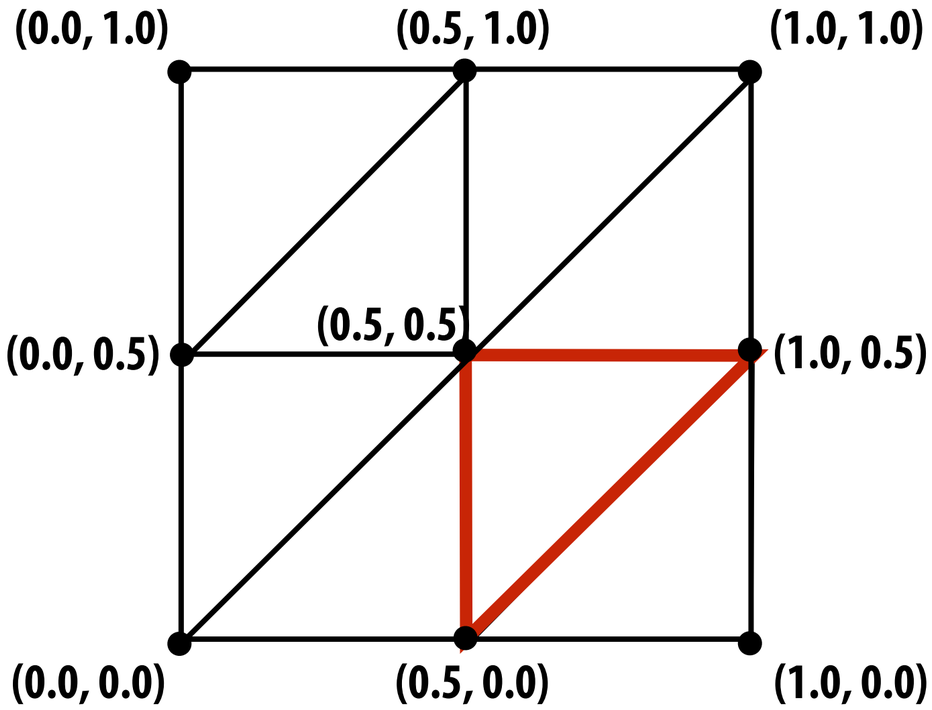


Grace Cathedral environment map

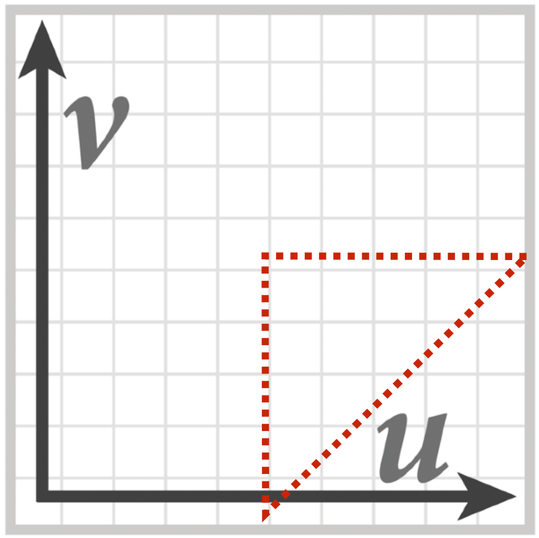
Environment map used in rendering

Texture coordinates

"Texture coordinates" define a mapping from surface coordinates (points on triangle) to points in texture domain.

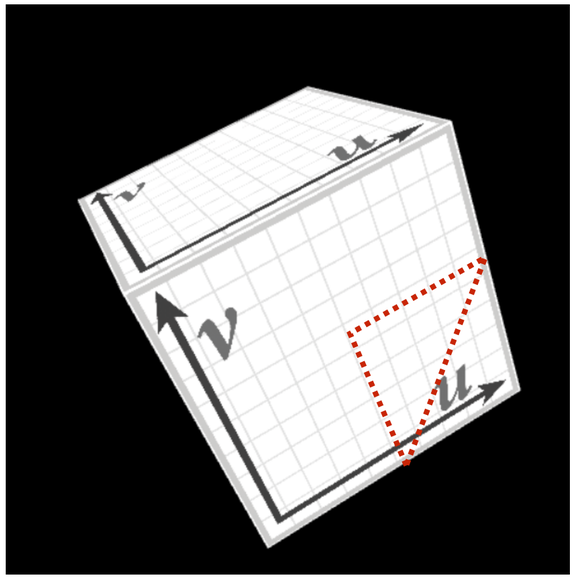


Eight triangles (one face of cube) with surface parameterization provided as per-vertex texture coordinates.



`myTex(u,v)` is a function defined on the $[0,1]^2$ domain (represented by 2048x2048 image)

Location of highlighted triangle in texture space shown in red.



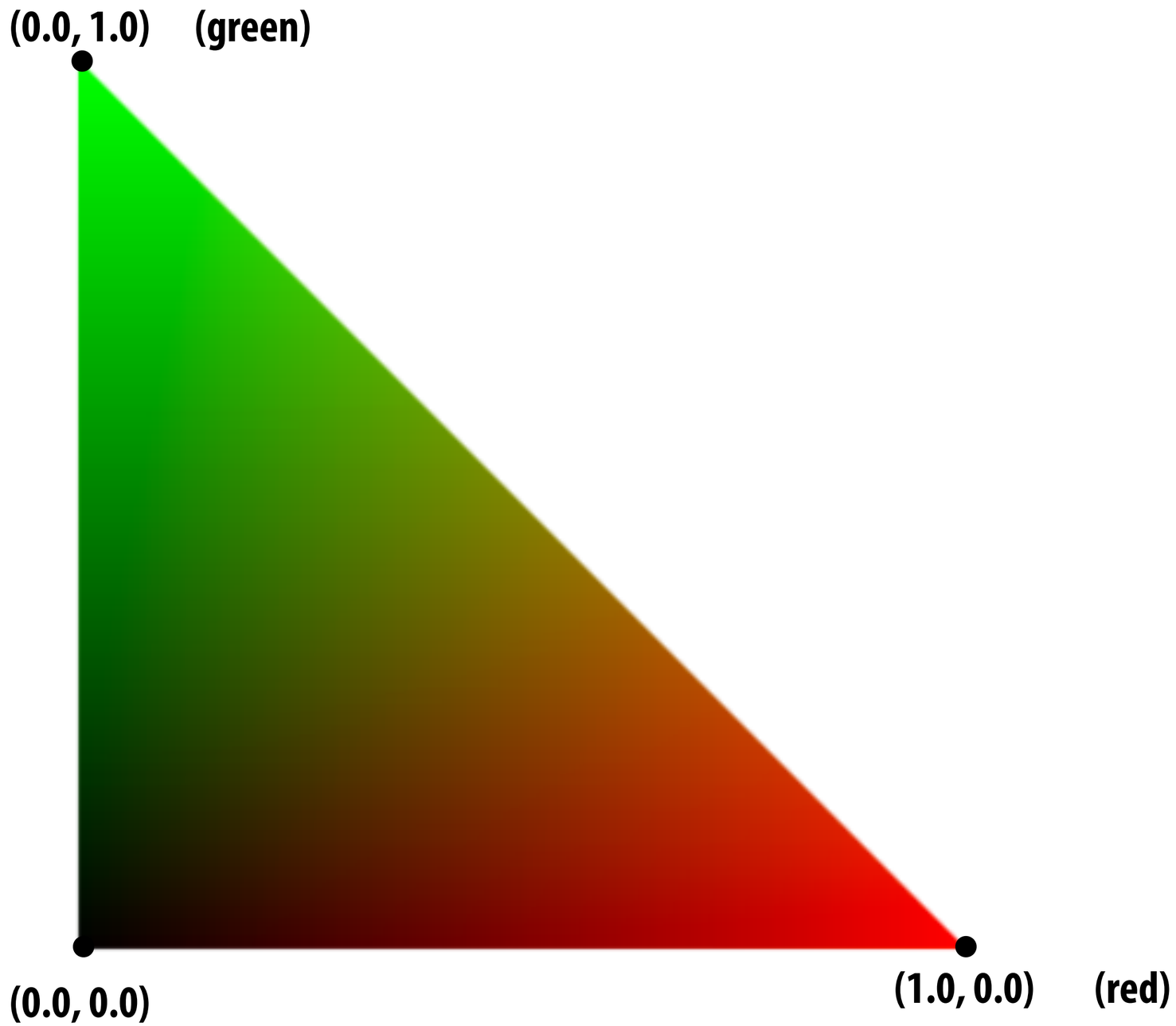
Final rendered result (entire cube shown).

Location of triangle after projection onto screen shown in red.

Today we'll assume surface-to-texture space mapping is provided as per vertex attribute
(Not discussing methods for generating surface texture parameterizations)

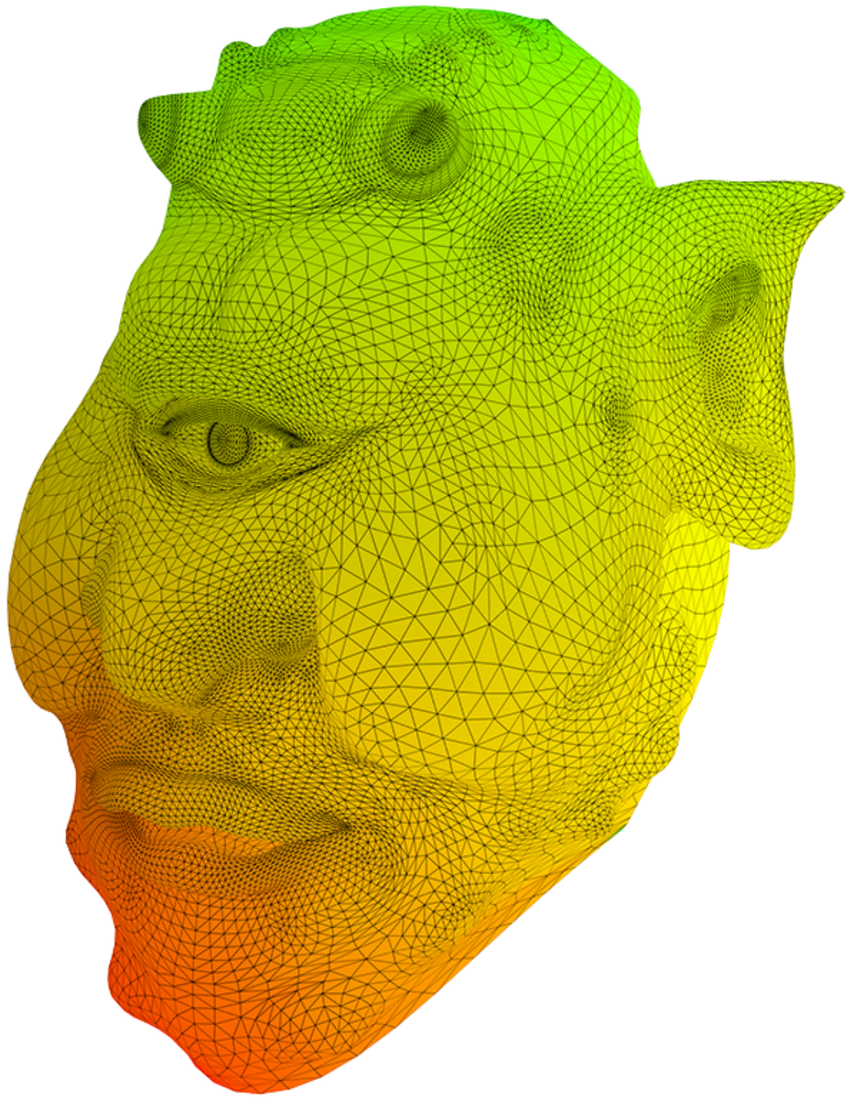
Visualization of texture coordinates

Texture coordinates linearly interpolated over triangle

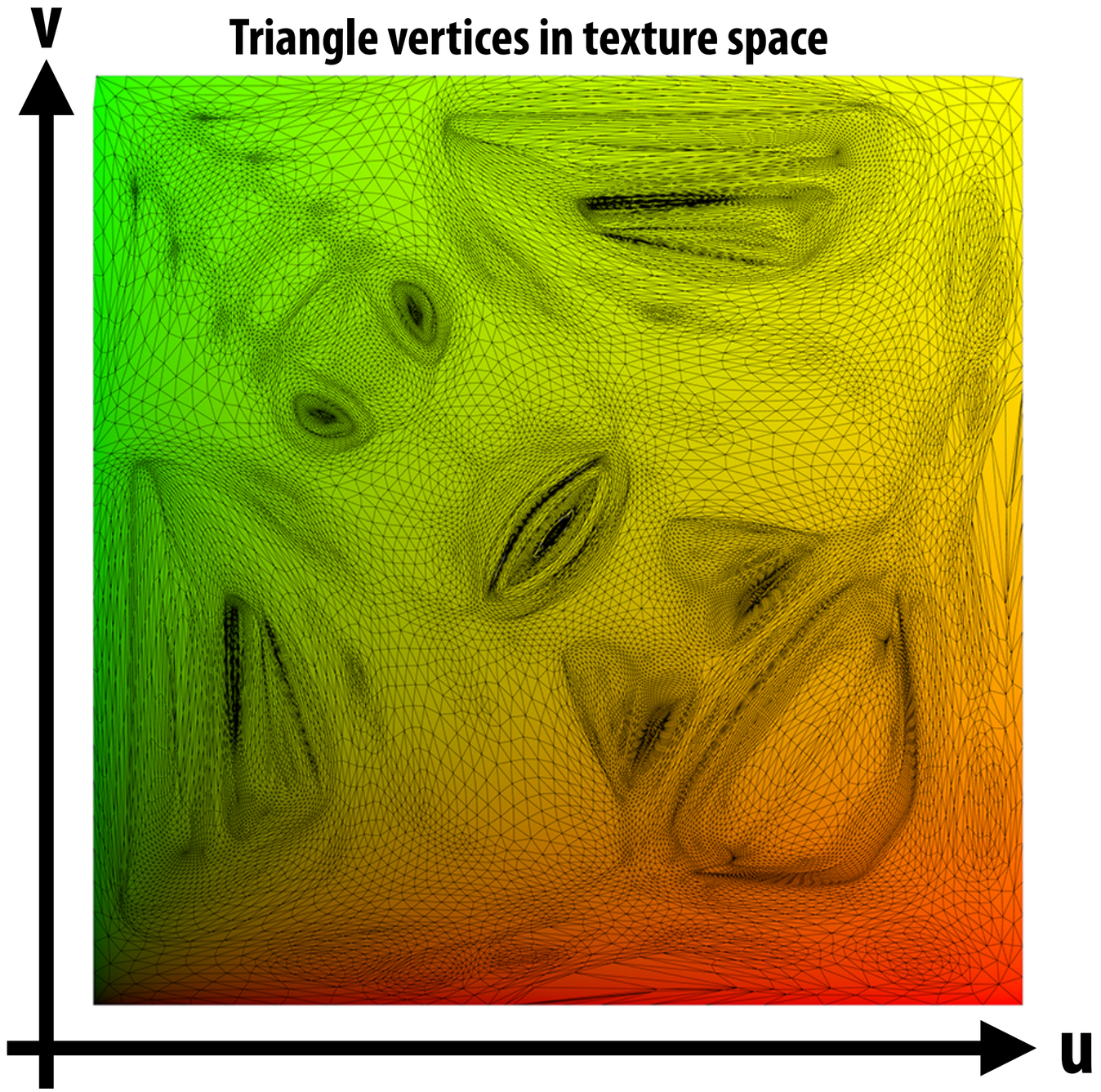


More complex mapping

Visualization of texture coordinates



Triangle vertices in texture space

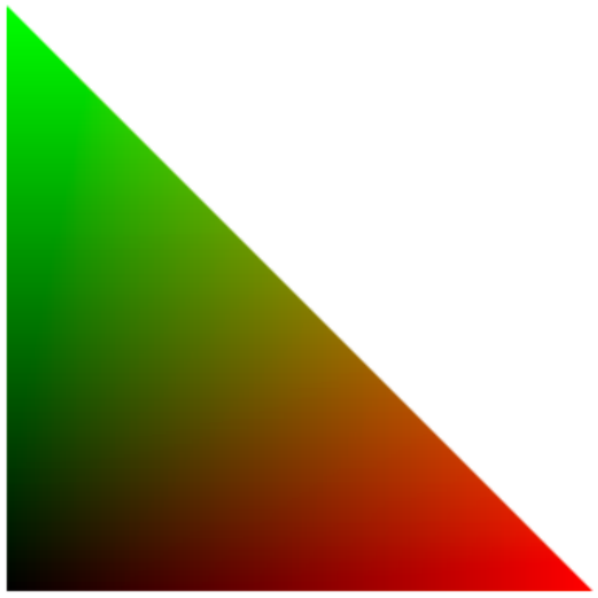


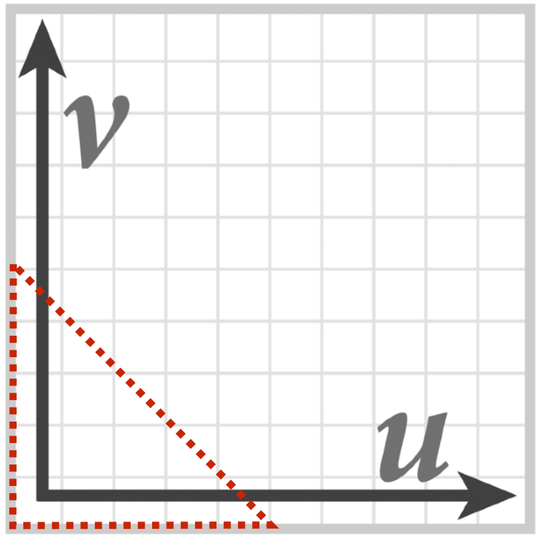
Each vertex has a coordinate (u,v) in texture space.

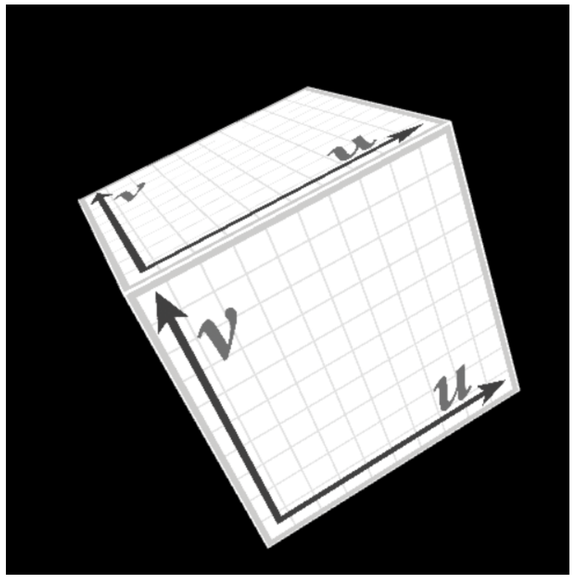
(Actually coming up with these coordinates is another story!)

Texture sampling 101

- Basic algorithm for mapping texture to surface:
  - For each color sample location (X,Y)
    - Interpolate U and V coordinates across triangle to get value at (X,Y)
    - Sample (evaluate) texture at (U,V)
    - Set color of fragment to sampled texture value

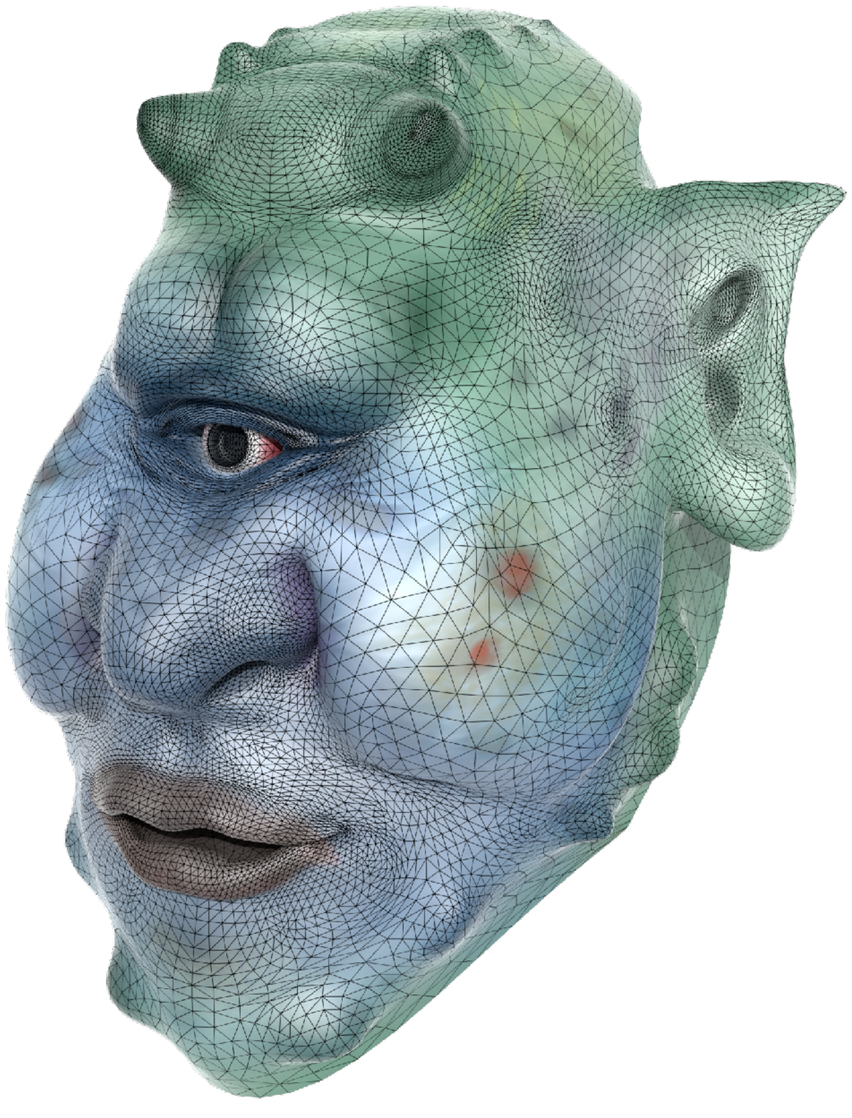




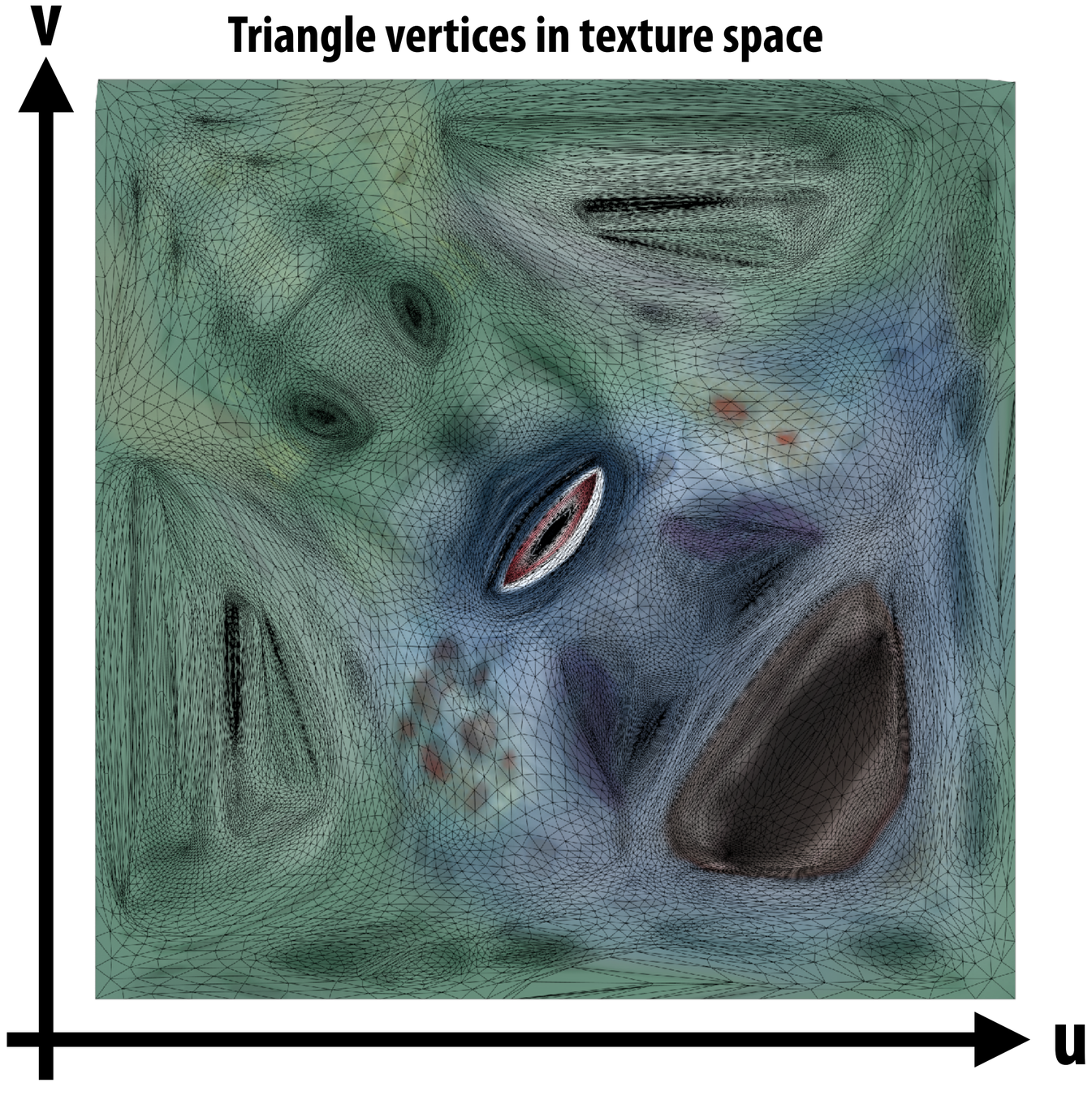


Texture mapping adds detail

Rendered result



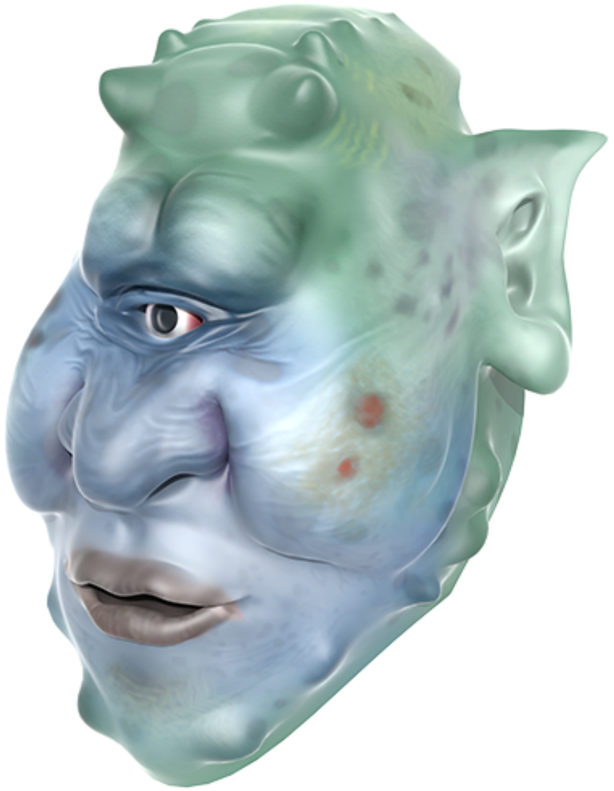
Triangle vertices in texture space



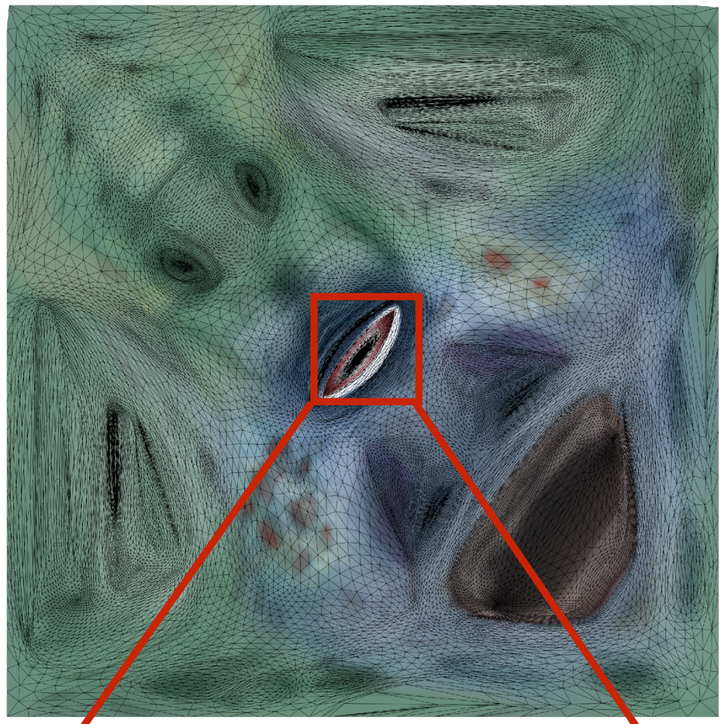
Texture mapping adds detail

rendering without texture

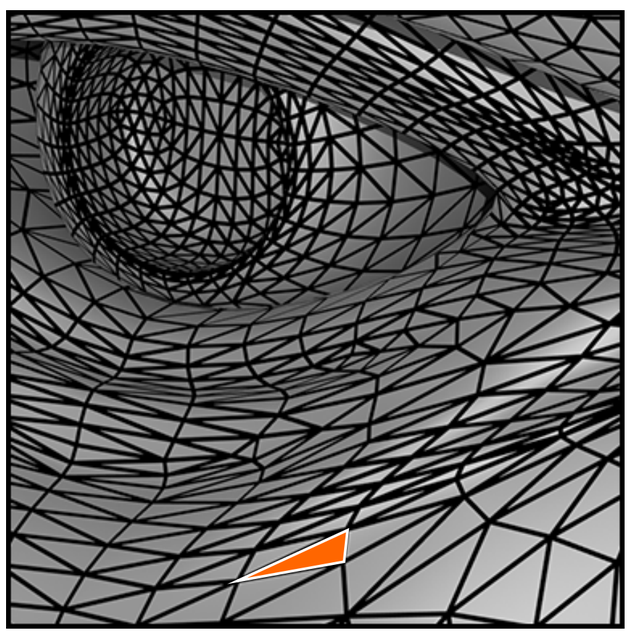
rendering with texture

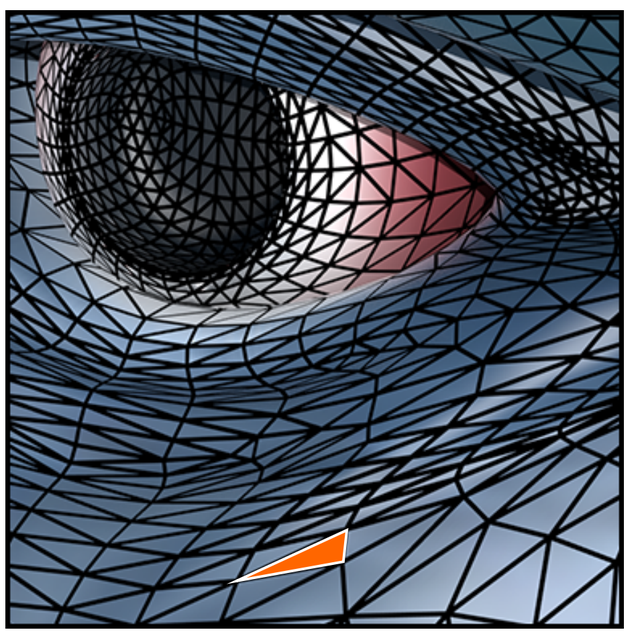


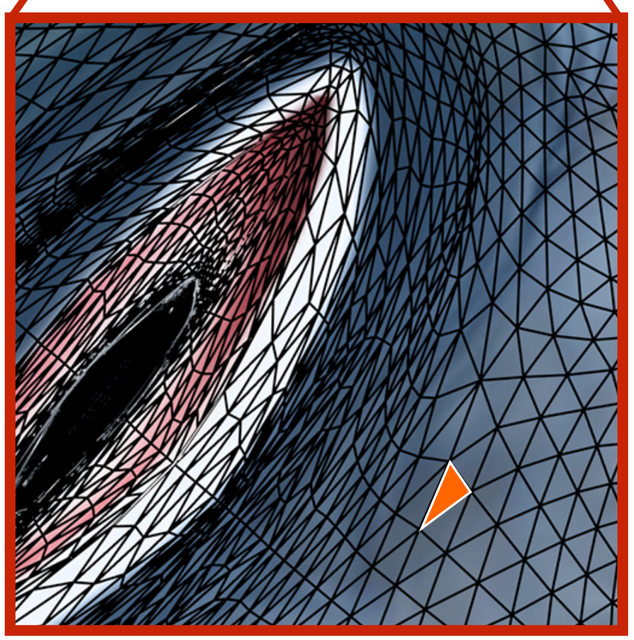
texture image



zoom

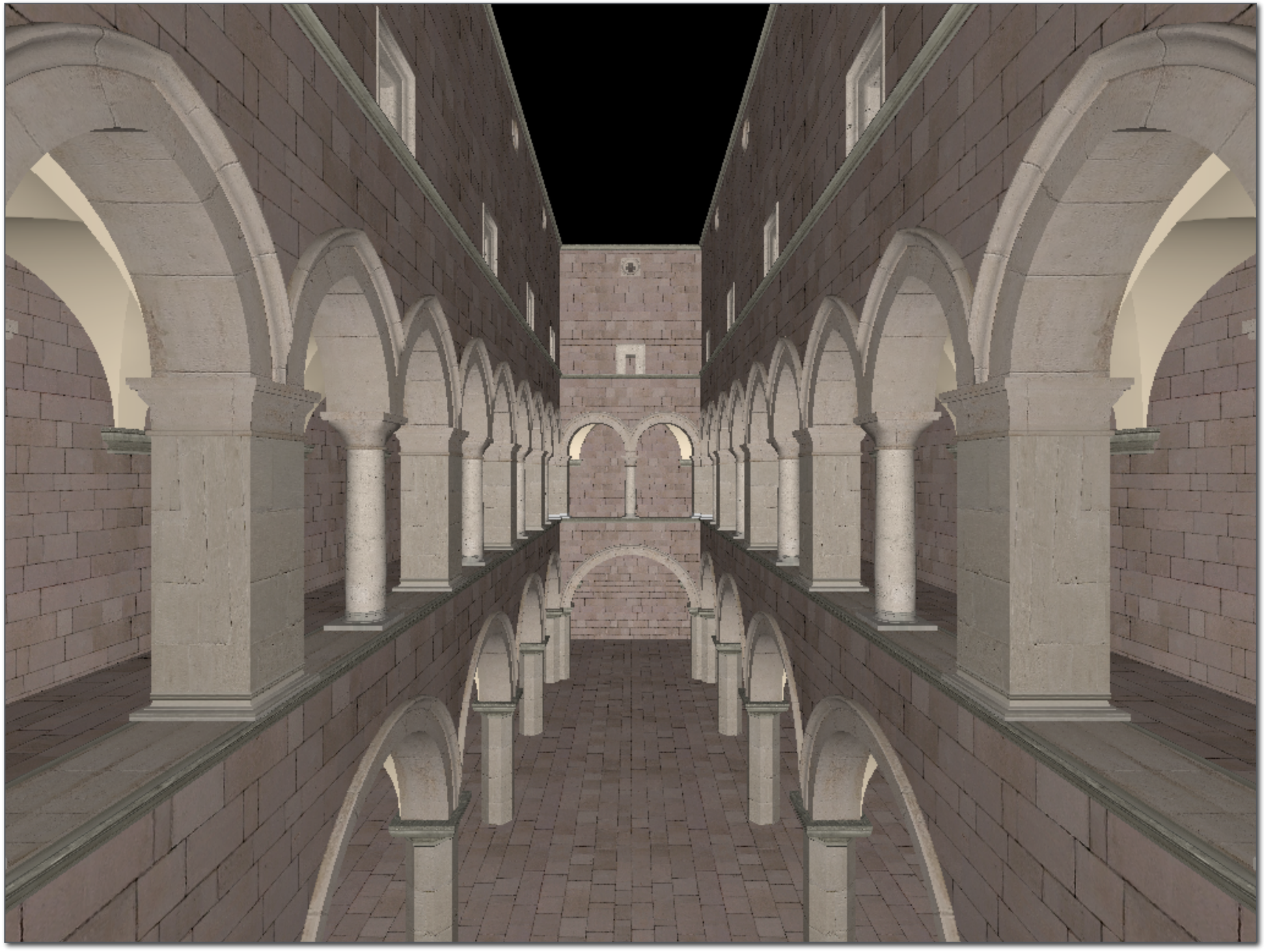




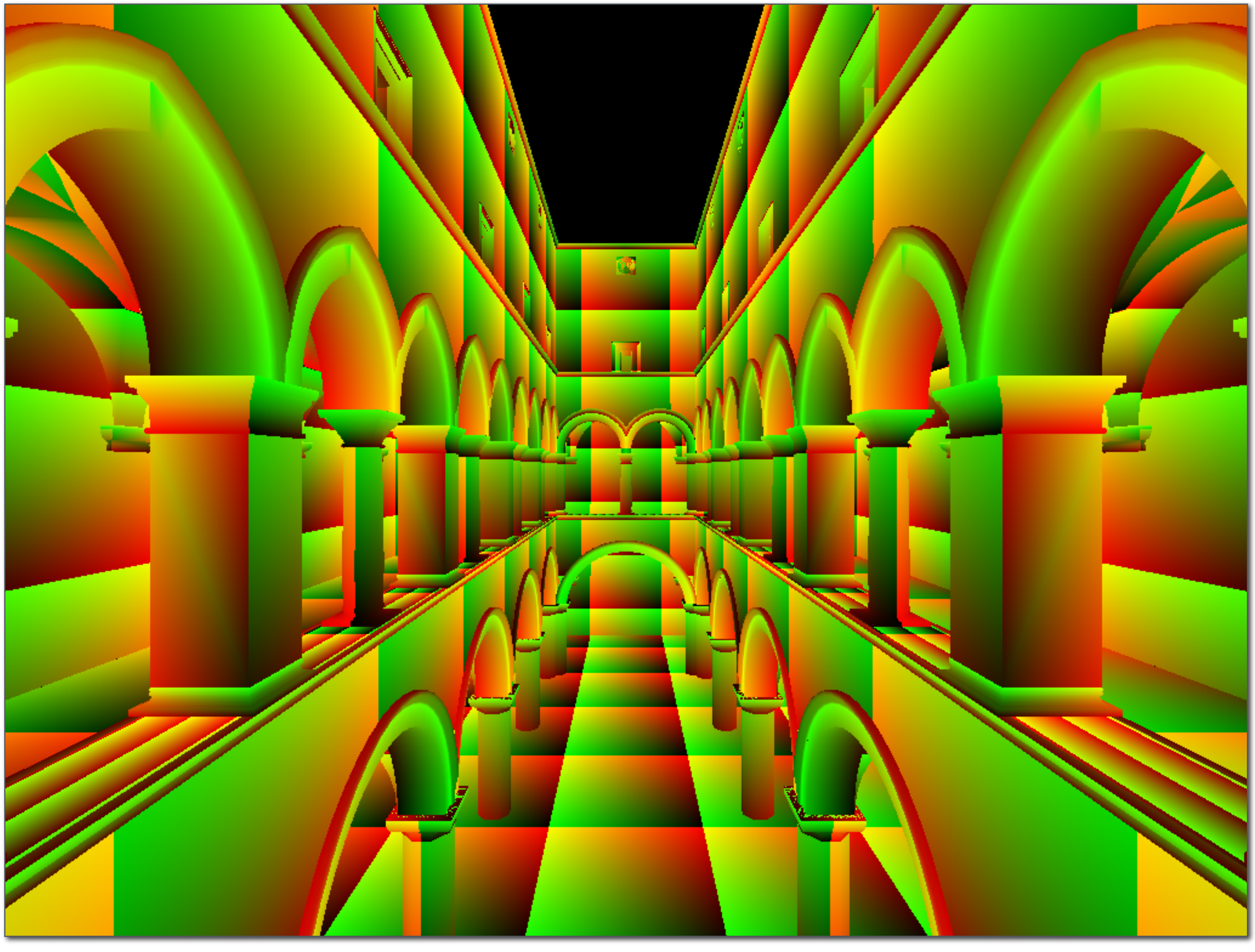


Each triangle "copies" a piece of the image back to the surface.

Example textured scene

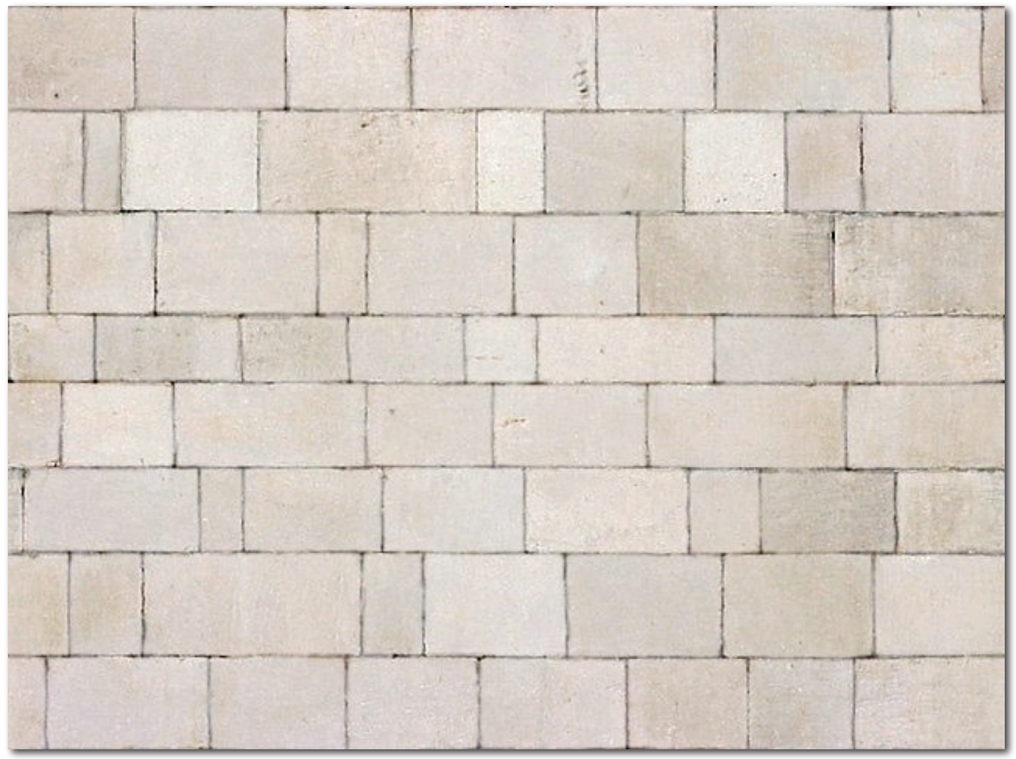


Visualization of texture coordinates



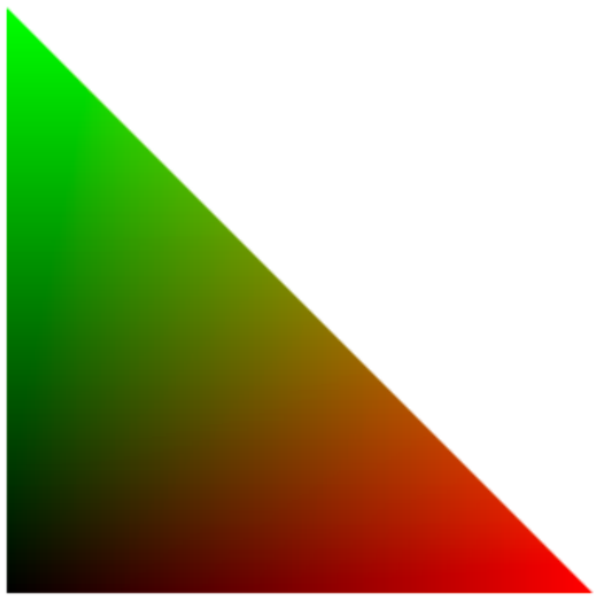
Notice texture coordinates repeat over surface.

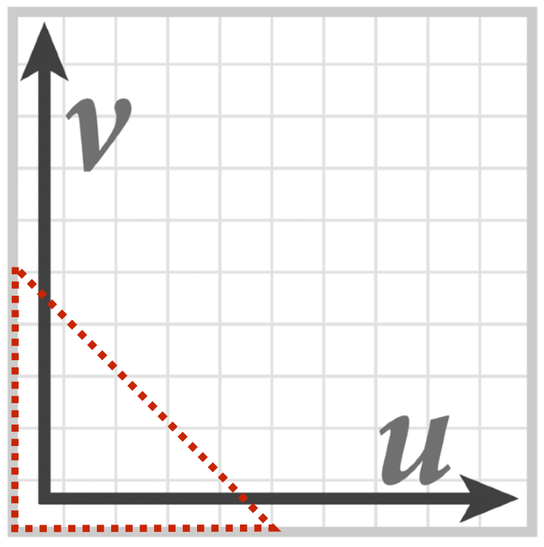
Example textures used in previous scene

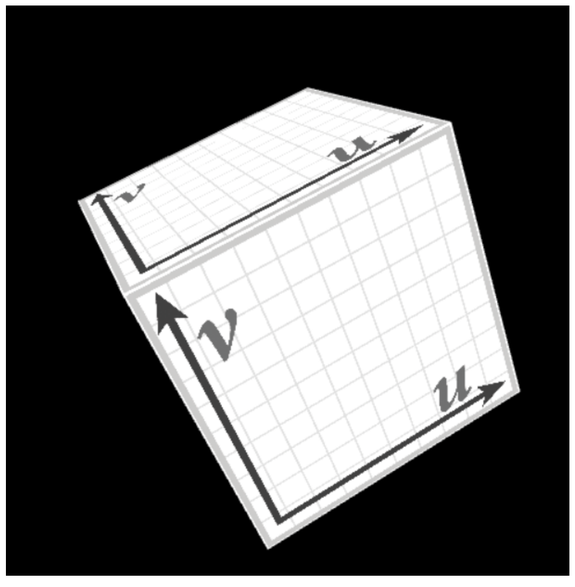


Texture mapping: basic algorithm

- Basic algorithm for mapping texture image onto a surface:
  - For each color sample location (X,Y) in the image
    - Interpolate U and V texture coordinates across triangle to get texture coordinate value at (X,Y)
    - Sample texture map at location (U,V)
    - Set output image sample color to sampled texture value

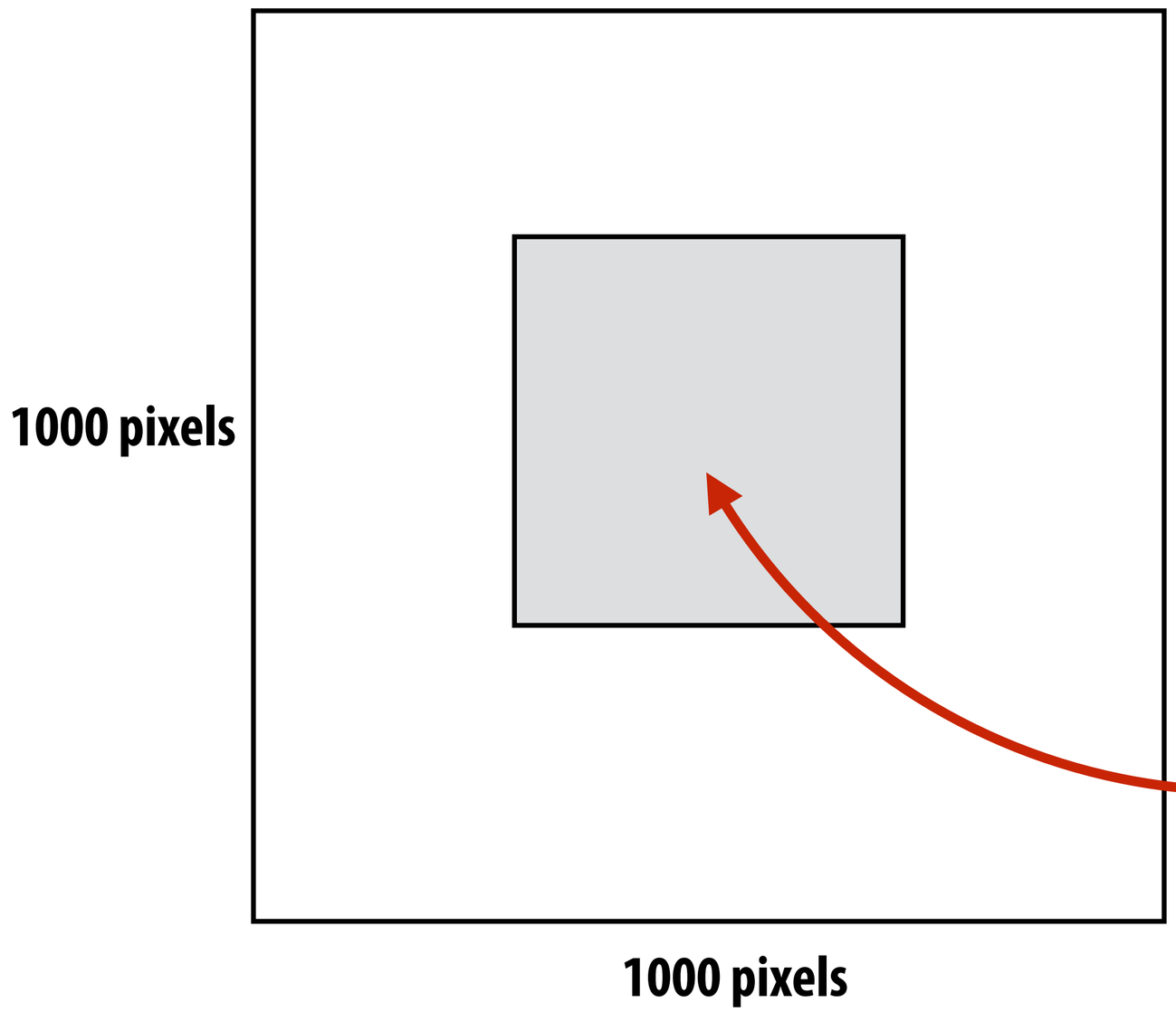






Thought experiment

- Imagine rendering a texture-mapped quadrilateral onto a 1000x1000 pixel image



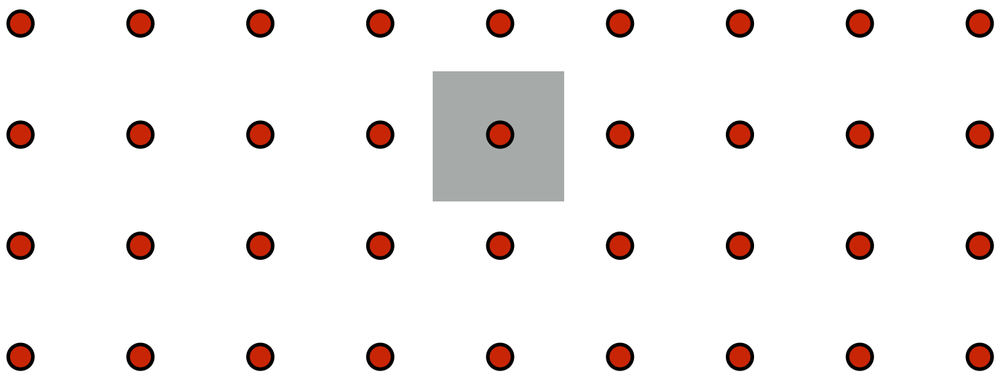
Let's also say the texture image is 1000x1000 as well.

Sampling rate on screen vs in texture: object zoomed out

Rendered image (object zoomed out)

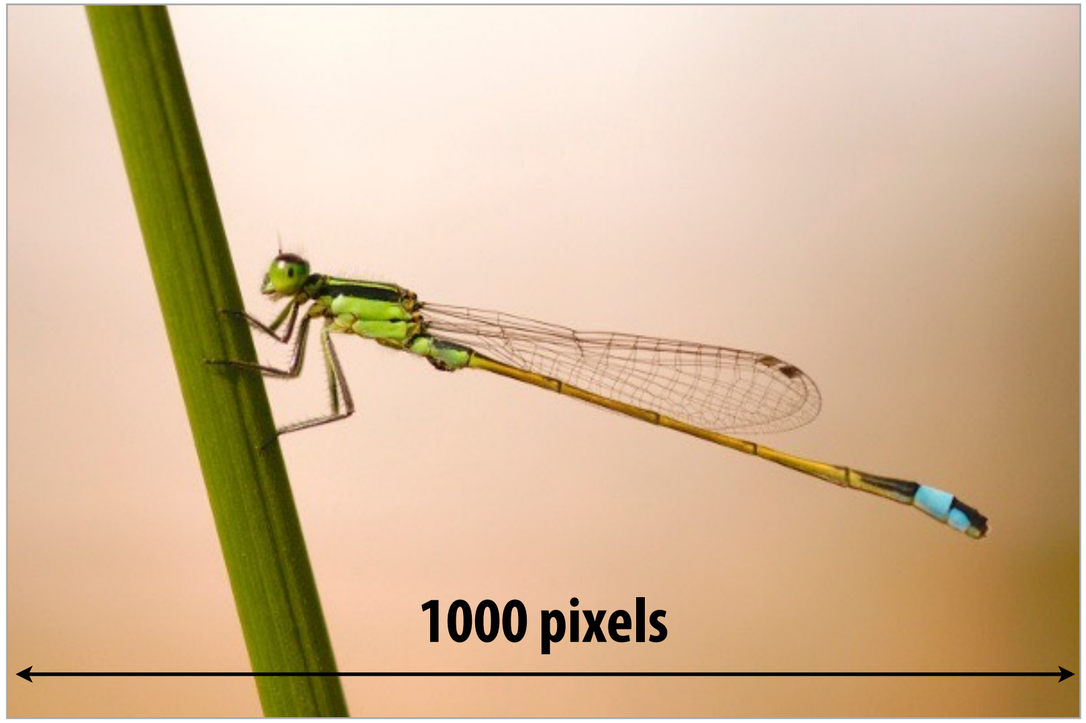
The entire 1000x1000 texture is rendered into a small region of the screen.

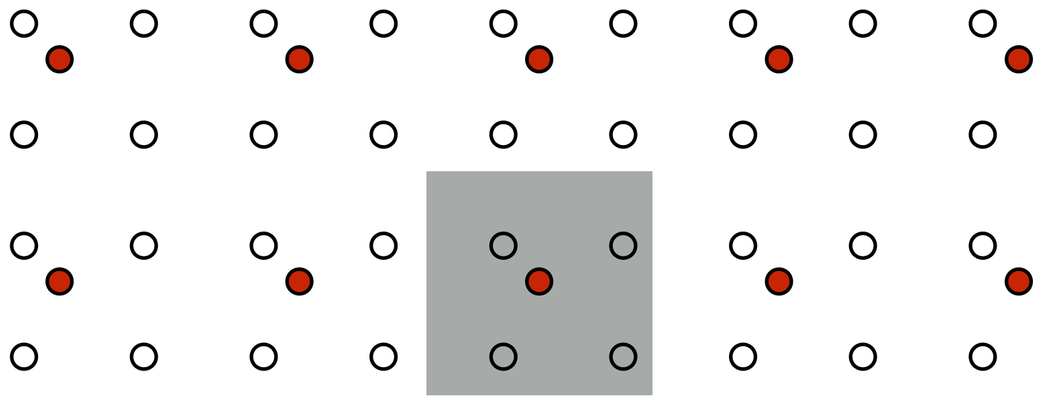
1000 pixels



Screen space (x,y)

Texture Image





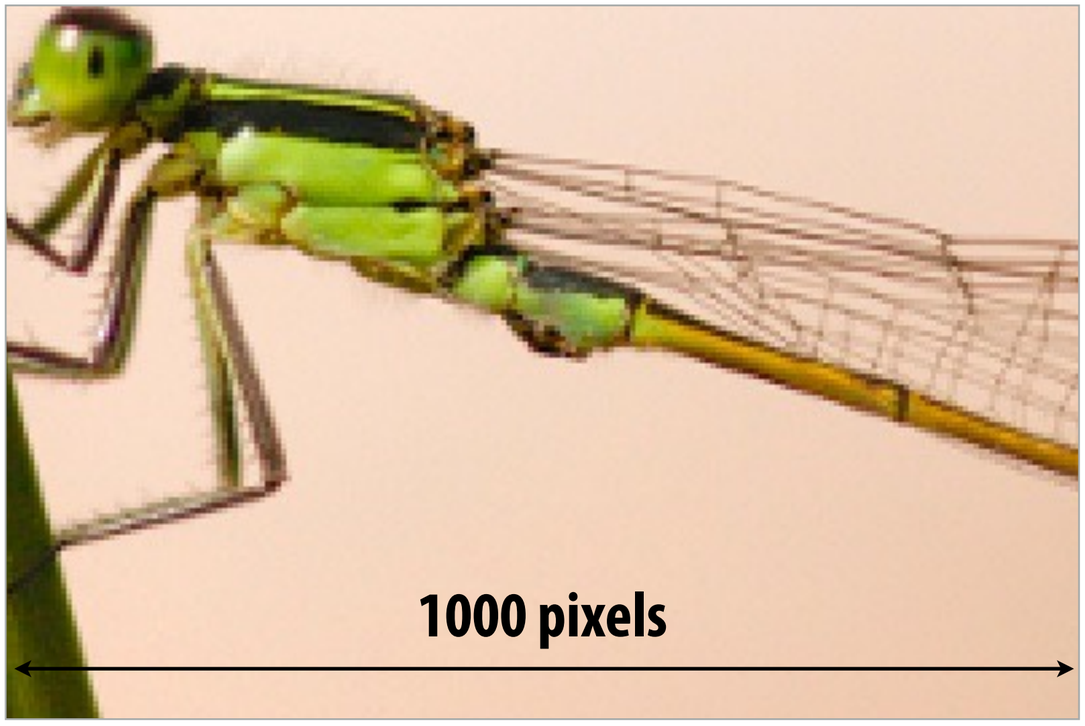
Texture space (u,v)

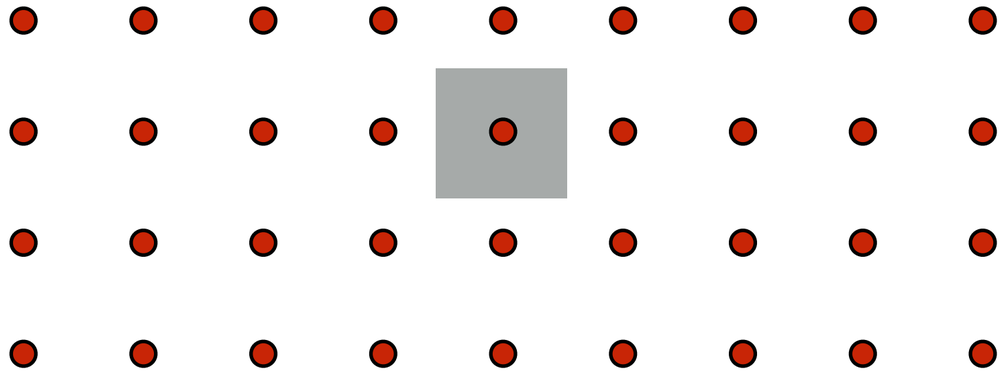
Red dots = samples needed to render
White dots = samples existing in texture map
Gray square = area of a screen pixel

Texture is "minified" on screen

Sampling rate on screen vs in texture: object zoomed in

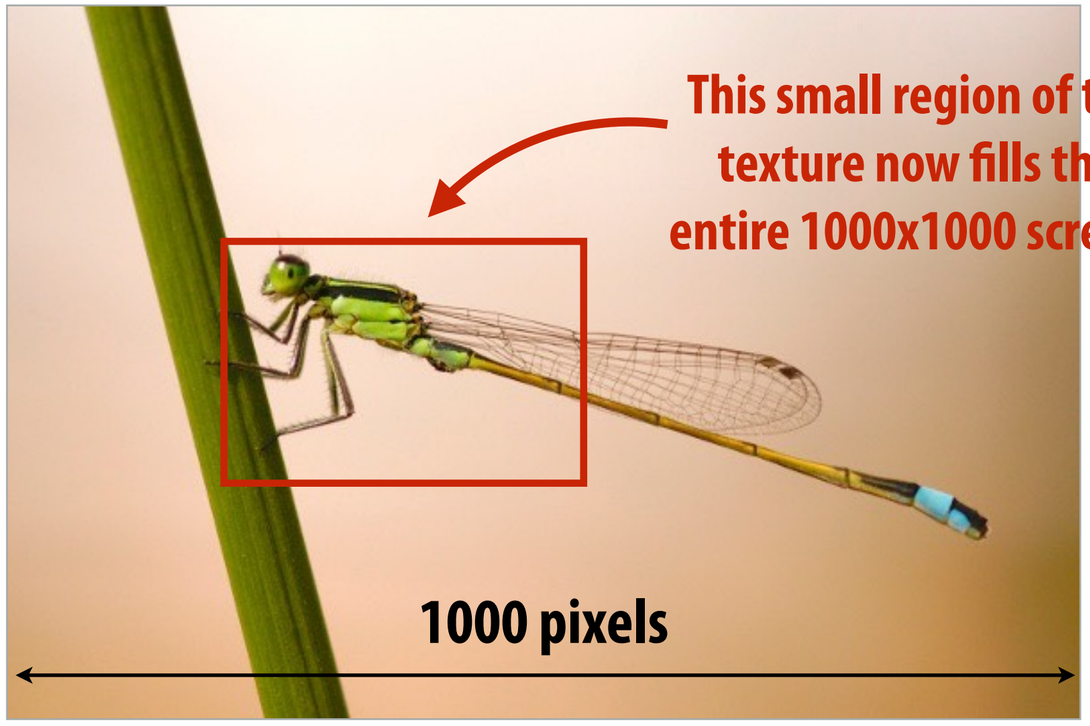
Rendered image (zoomed in)

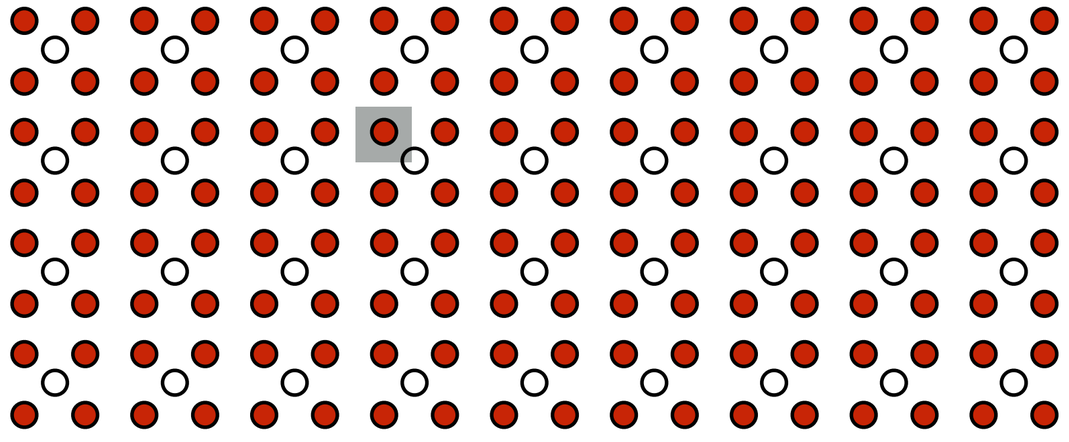




Screen space (x,y)

Texture Image





Texture space (u,v)

Red dots = samples needed to render
White dots = samples existing in texture map
Gray square = area of a screen pixel

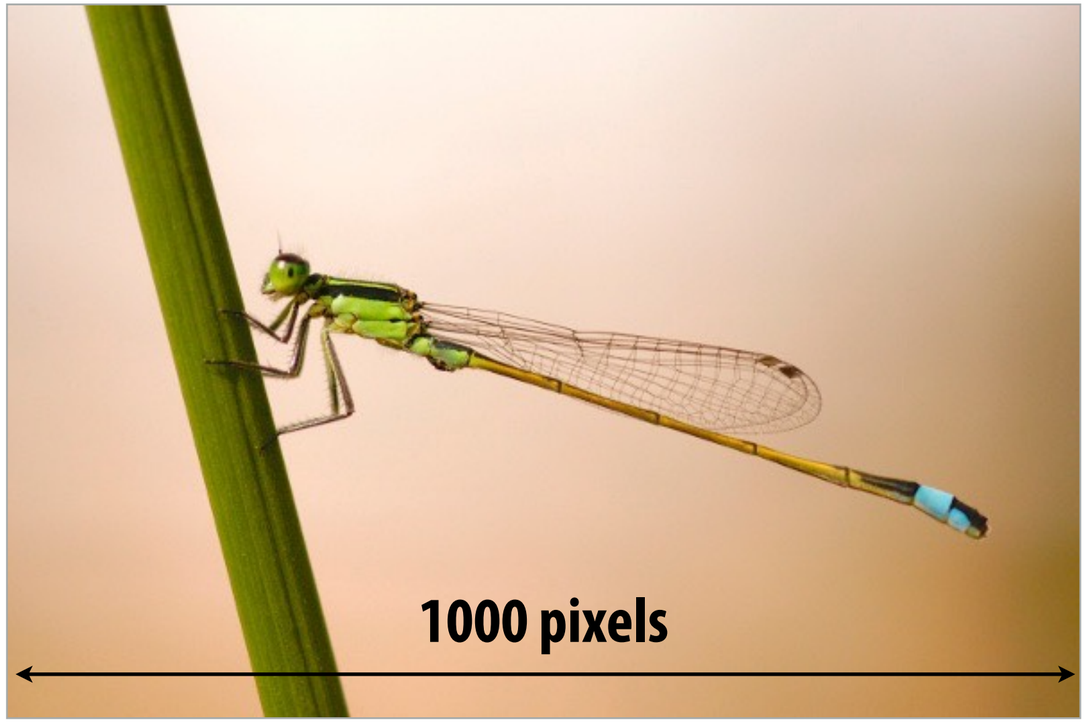
Texture is "magnified" on screen

Sampling rate on screen vs in texture: object rotation

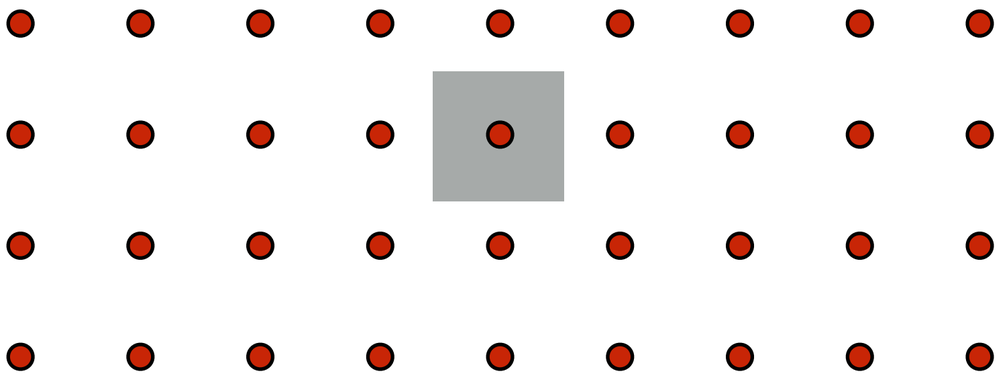
Rendered image (object zoomed out and rotated)

1000 pixels

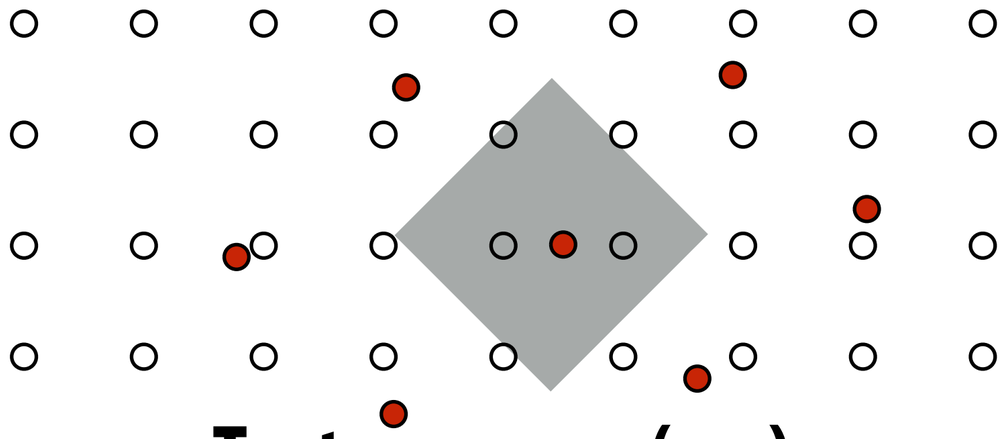
Texture Image



1000 pixels



Screen space (x,y)

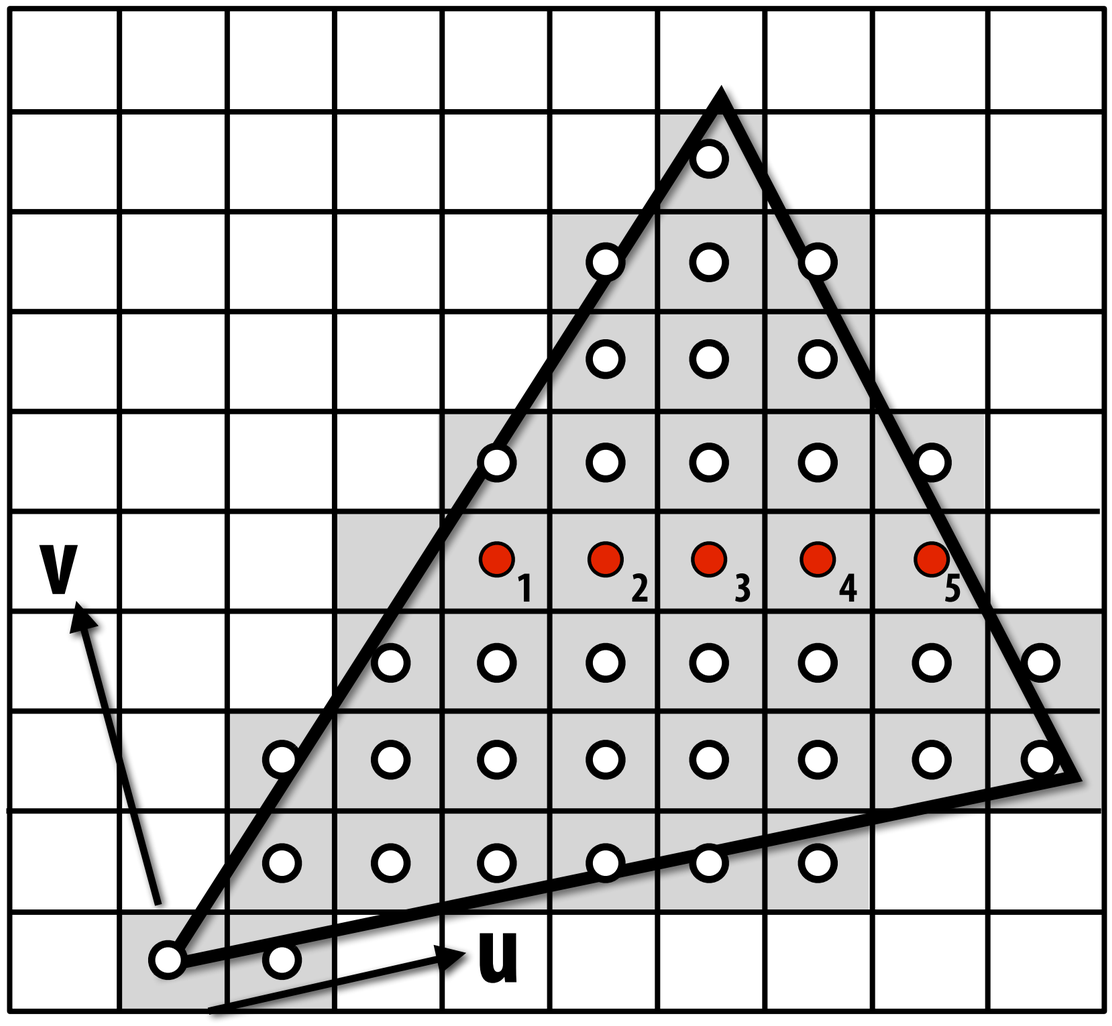


Texture space (u,v)

Red dots = samples needed to render
White dots = samples existing in texture map
Gray square = area of a screen pixel

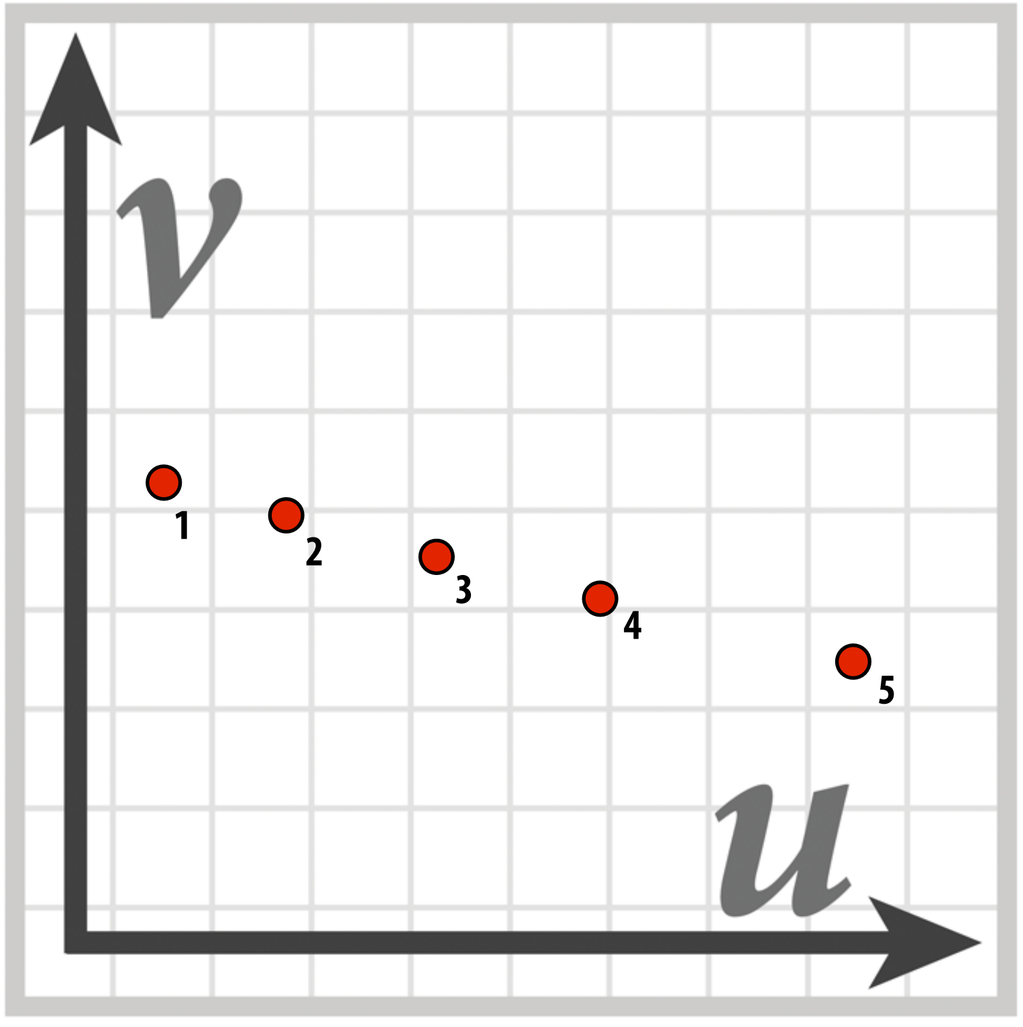
Equally spaced samples on screen != equally spaced samples in texture space

Sample positions in XY screen space



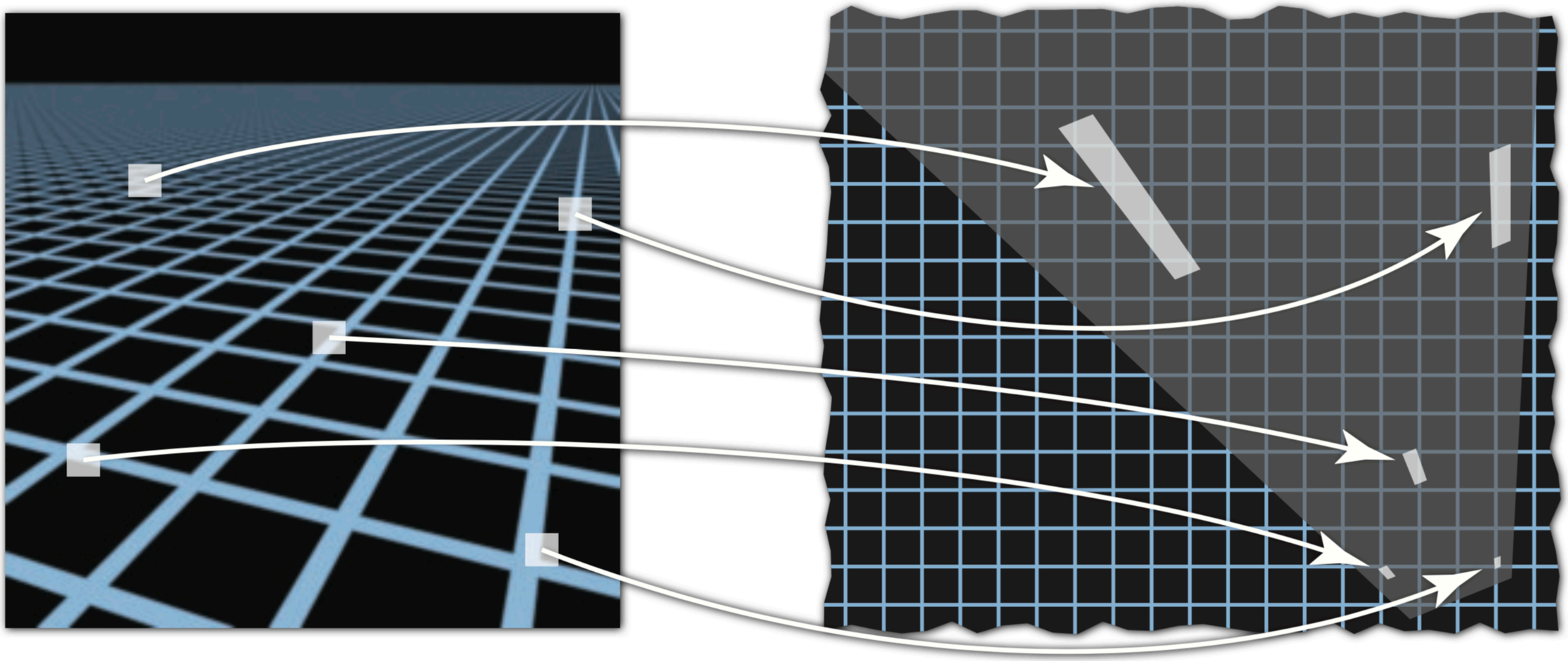
Sample positions are uniformly distributed in screen space (rasterizer samples triangle's appearance at these locations)

Sample positions in texture space



Texture sample positions in texture space (texture function is sampled at these locations)

Screen pixel footprint in texture space

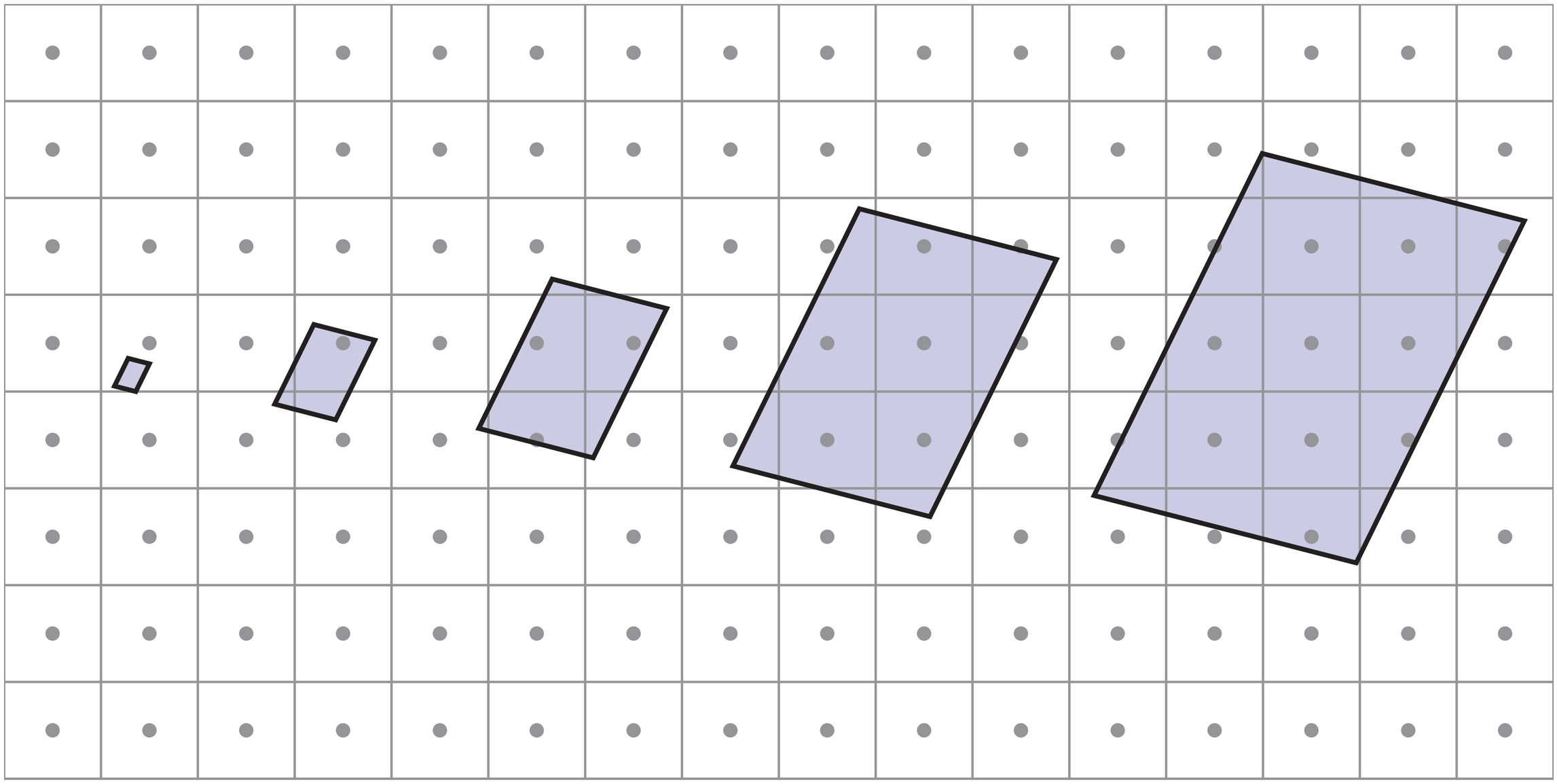


Screen space

Texture space

Texture sampling pattern not rectilinear or isotropic

Screen pixel footprint in texture space



Upsampling (Magnification)

Camera zoomed in close to object

Downsampling (Minification)

Camera far away from object

Screen pixel area vs texel area

- At optimal viewing size:
  - 1:1 mapping between pixel sampling rate and texel sampling rate
  - Dependent on screen and texture resolution!
- When pixel area is larger than texel area (texture minification)
  - Think: zoom far out from object
  - One pixel sample per multiple texel samples
- When pixel area is smaller than texel area (texture magnification)
  - Think: zoom in on an object
  - Multiple pixel samples per texel sample

Texture magnification

Texture magnification (nearest)

Texture magnification (nearest)

Texture magnification (nearest)

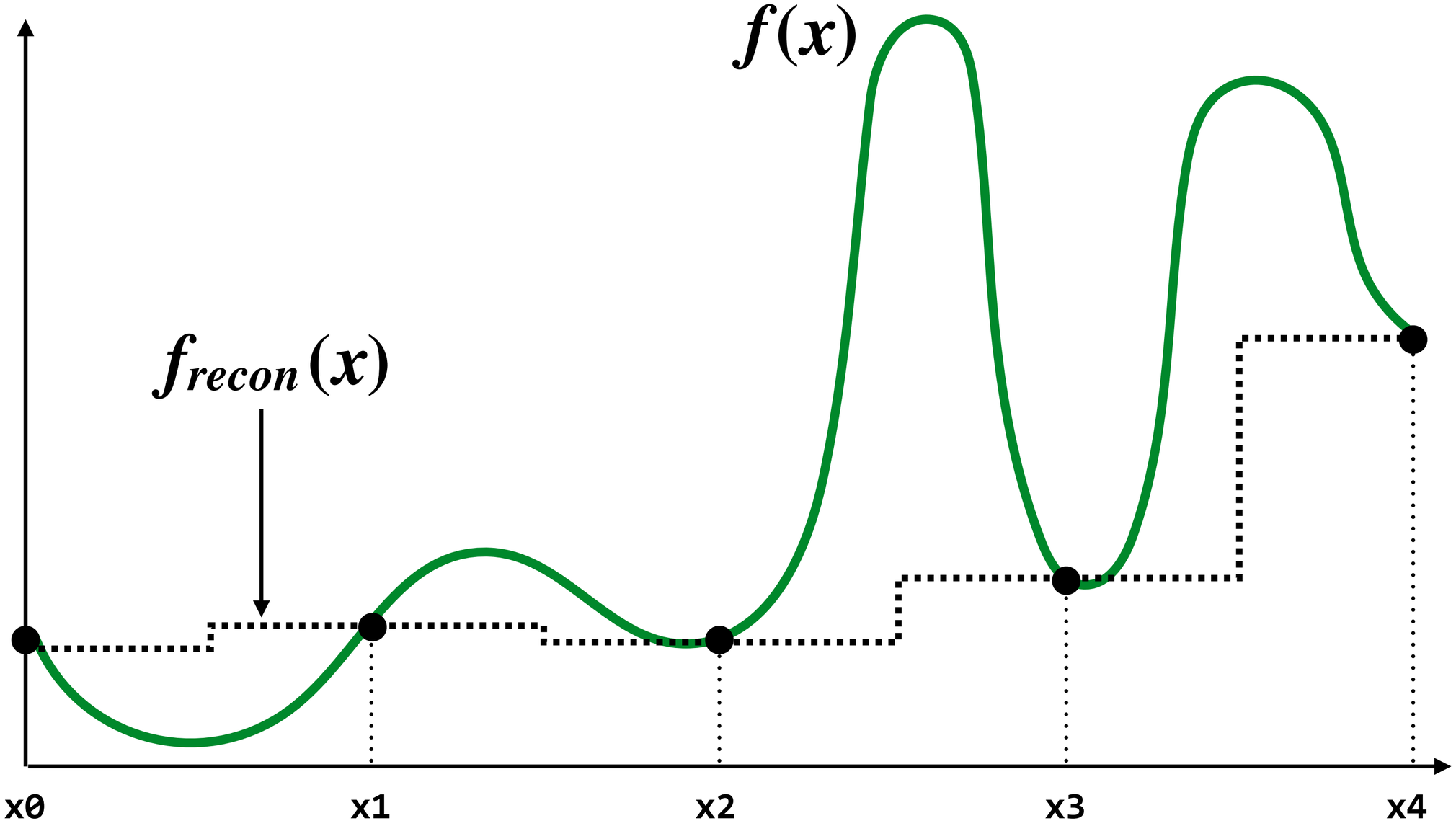
Texture magnification (nearest)

Texture magnification (nearest)

Review: piecewise constant approximation

$f_{recon}(x)$ = value of sample closest to x

$f_{recon}(x)$ approximates $f(x)$



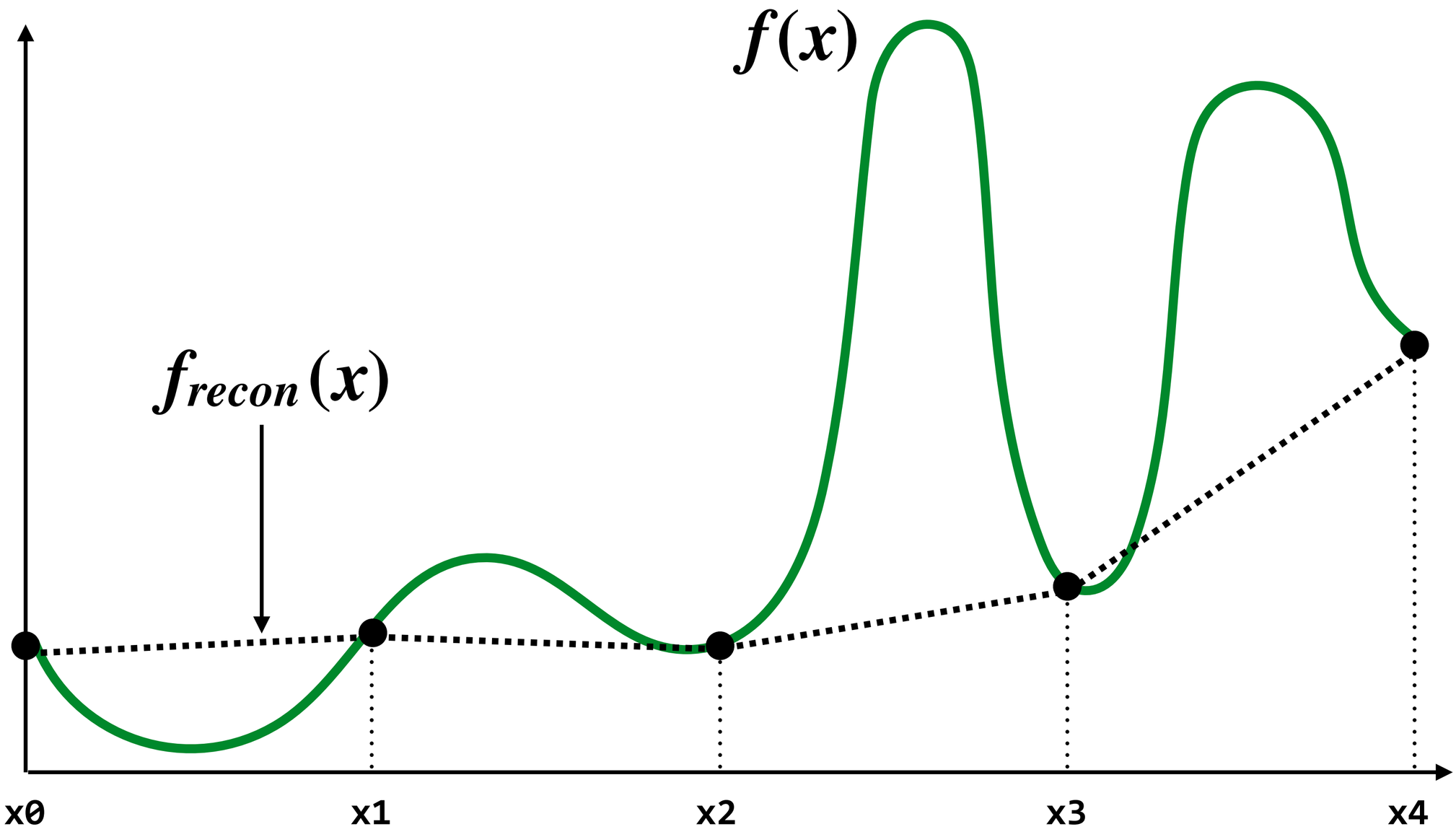
Texture magnification

- Generally don't want this situation — it means we have insufficient texture resolution
- Magnification involves interpolation of values in texture map (below: three different interpolation kernel functions)

Nearest | Bilinear | Bicubic

Review: piecewise linear approximation

$f_{recon}(x)$ = linear interpolation between values of two closest samples to x



Texture magnification (bilinear)

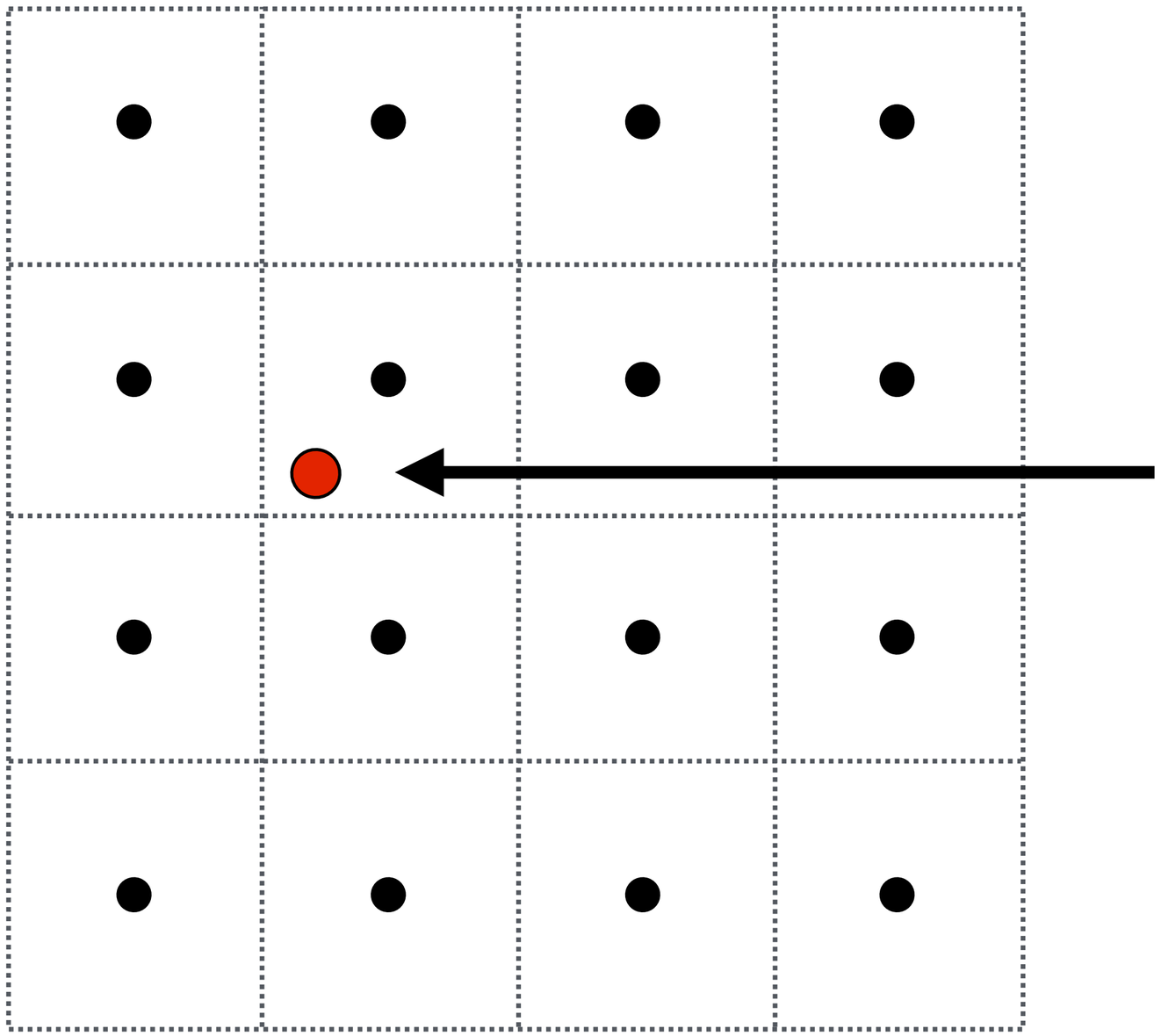
Texture magnification (bilinear)

Texture magnification (bilinear)

Texture magnification (bilinear)

Texture magnification (bilinear)

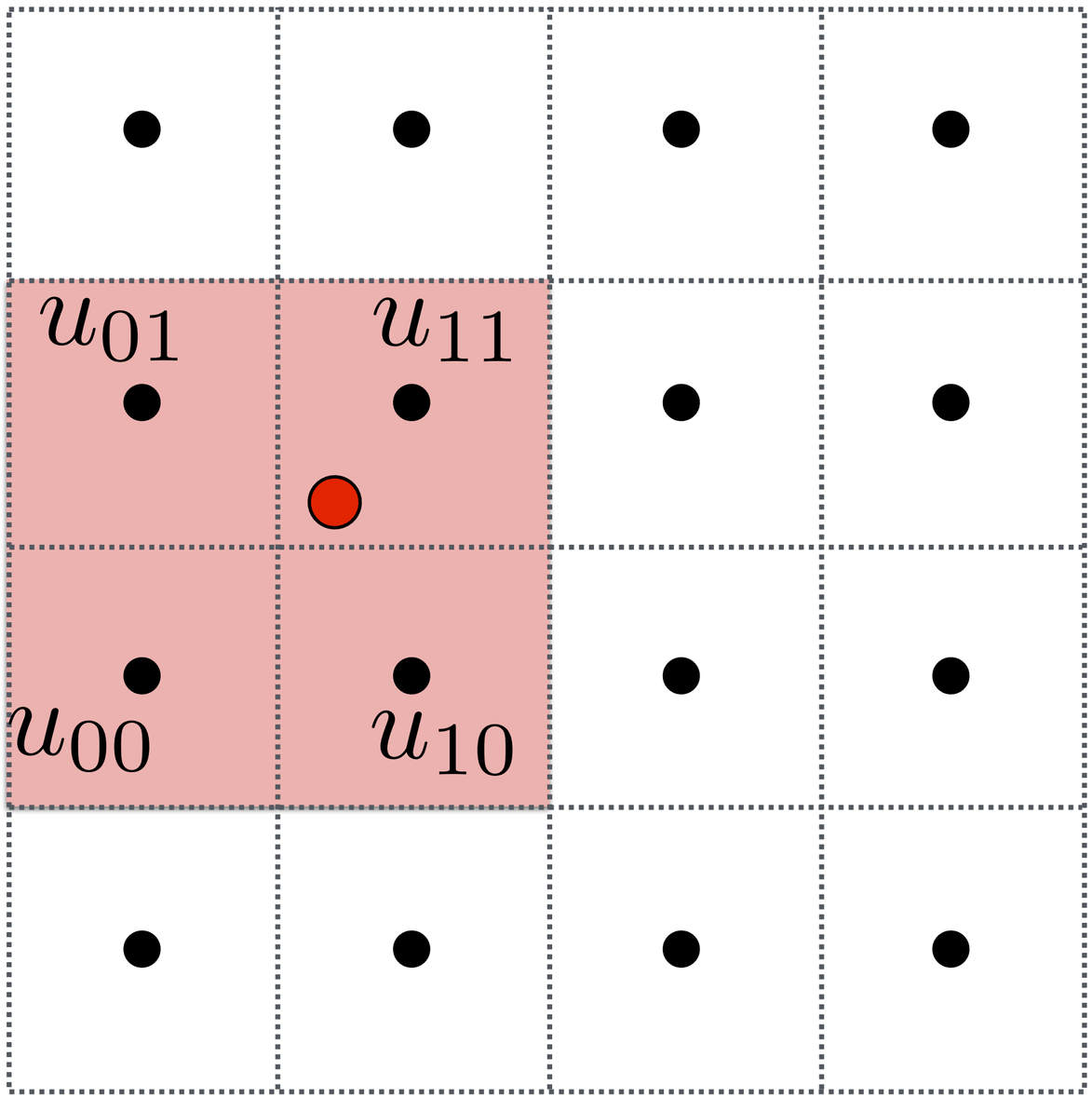
Bilinear interpolation



Want to sample texture value f(x,y) at red point

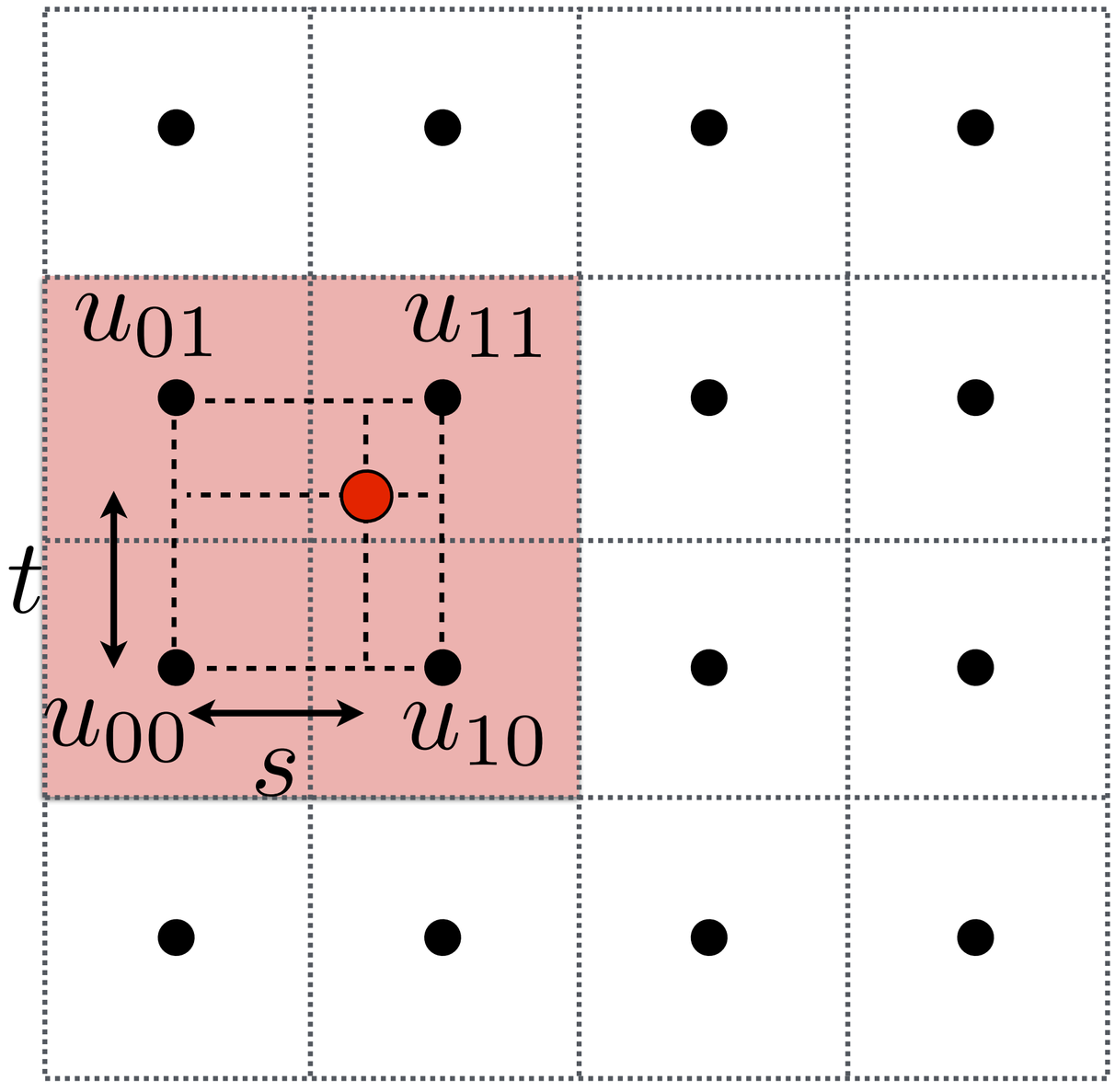
Black points indicate texture sample locations

Bilinear interpolation



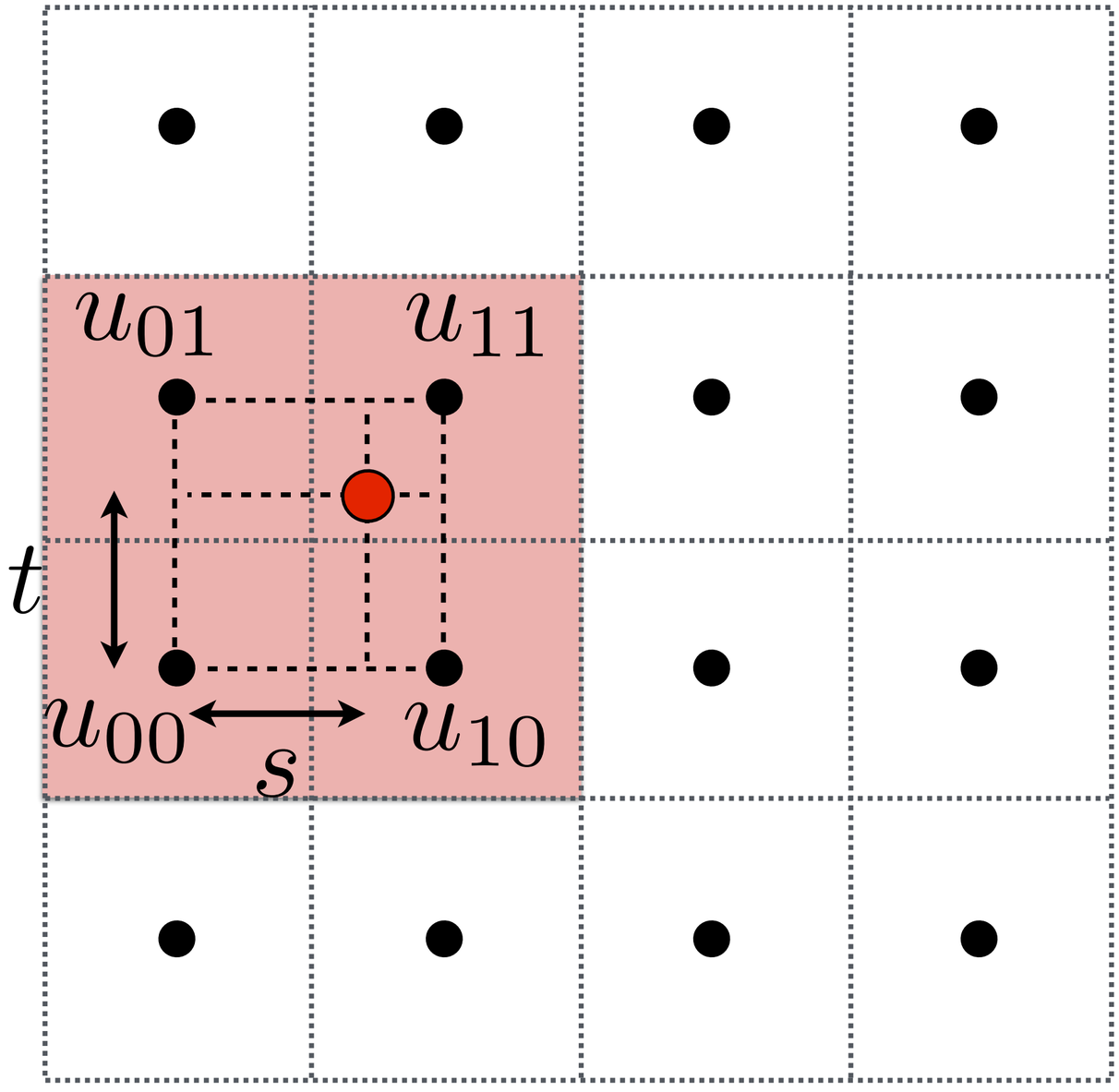
Take 4 nearest sample locations, with texture values as labeled.

Bilinear interpolation



And fractional offsets, (s,t) as shown

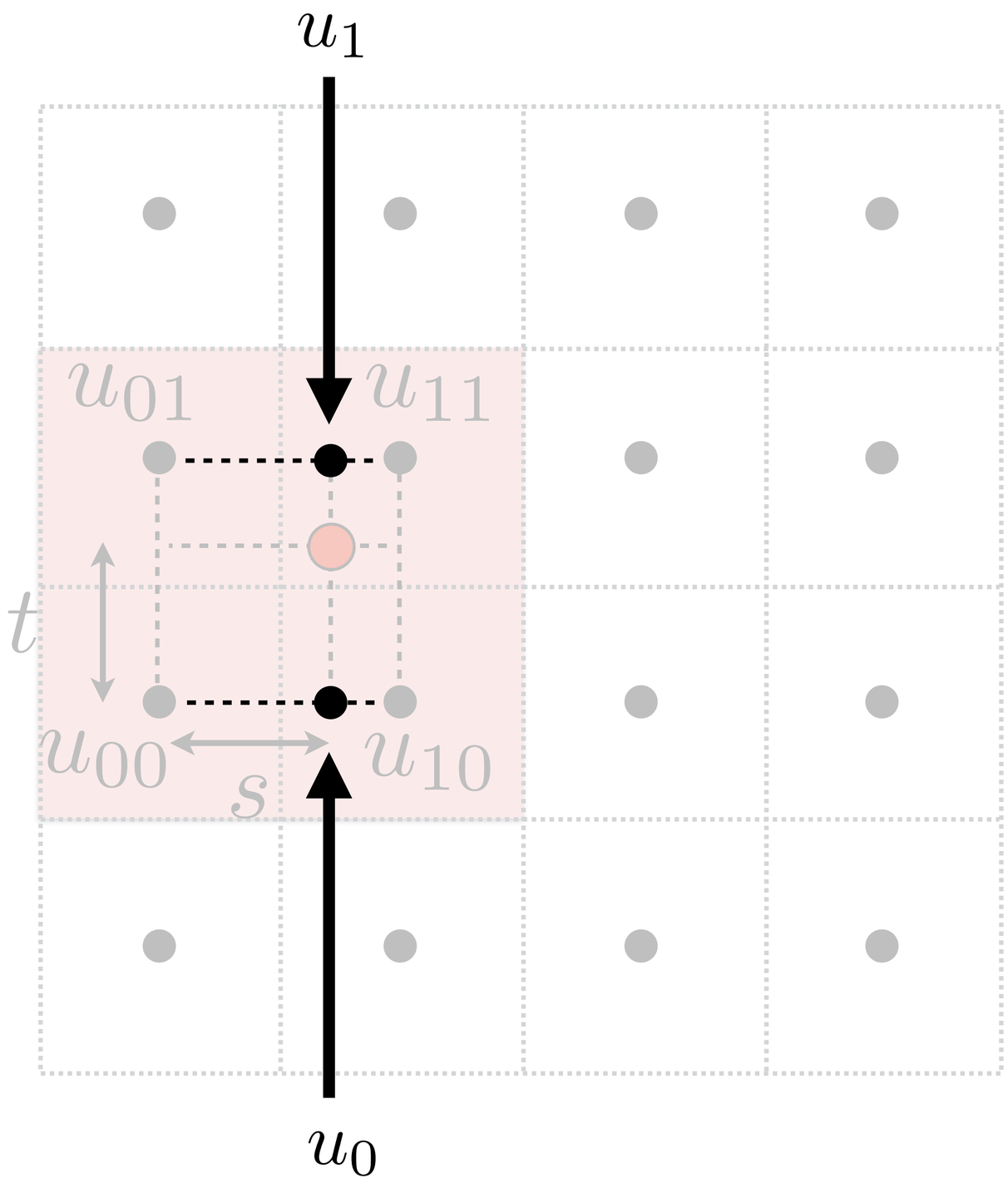
Bilinear interpolation



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Bilinear interpolation



Linear interpolation (1D)

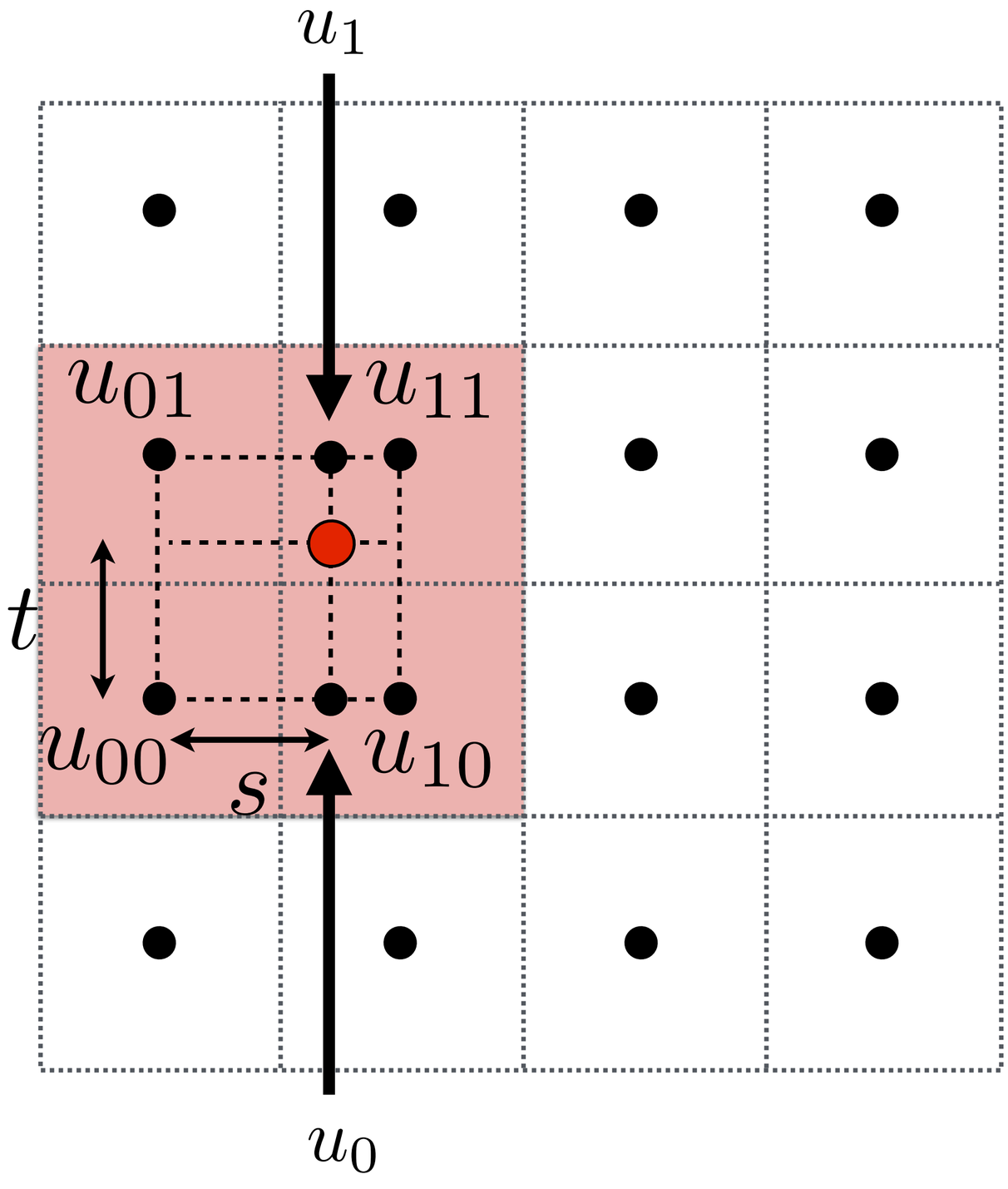
$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps (horizontal)

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

Bilinear interpolation



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

Final vertical lerp, to get result:

$$f(x, y) = \text{lerp}(t, u_0, u_1)$$

Reconstruction filter function

- Magnification involves interpolation of values in texture map
- Interpolation is convolution of sampled values with a filter function
- What is the reconstruction filter k(x,y) for:
  - Nearest neighbor interpolation?
  - Bilinear interpolation?

Nearest

Bilinear

Bicubic

Texture minification

By now I hope you've realized:

Applying textures is a form of sampling!

t(u,v)

Minification of Josephine

Imagine the texture map is 9x9

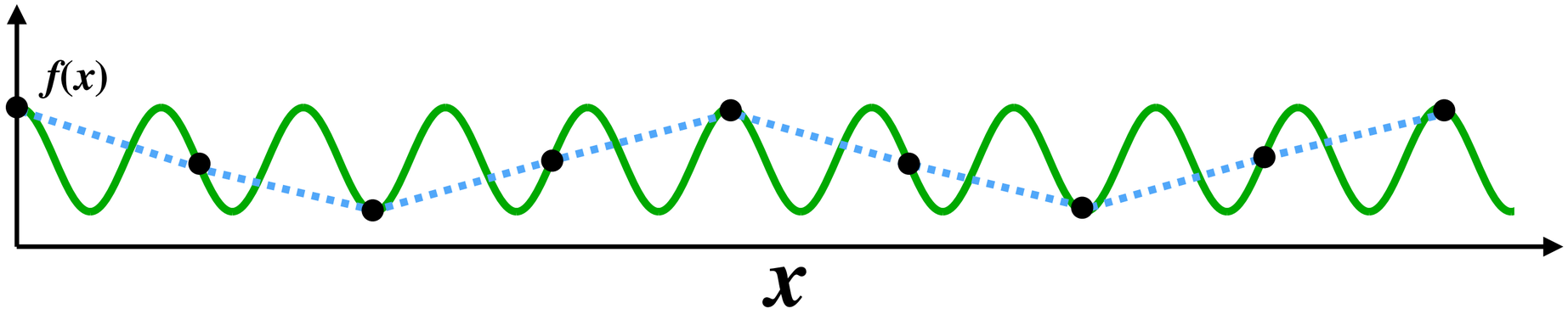
And is applied to a quad that spans a 3x3 pixel region of screen.

When a texture is minimized, the texture map is sampled sparsely!

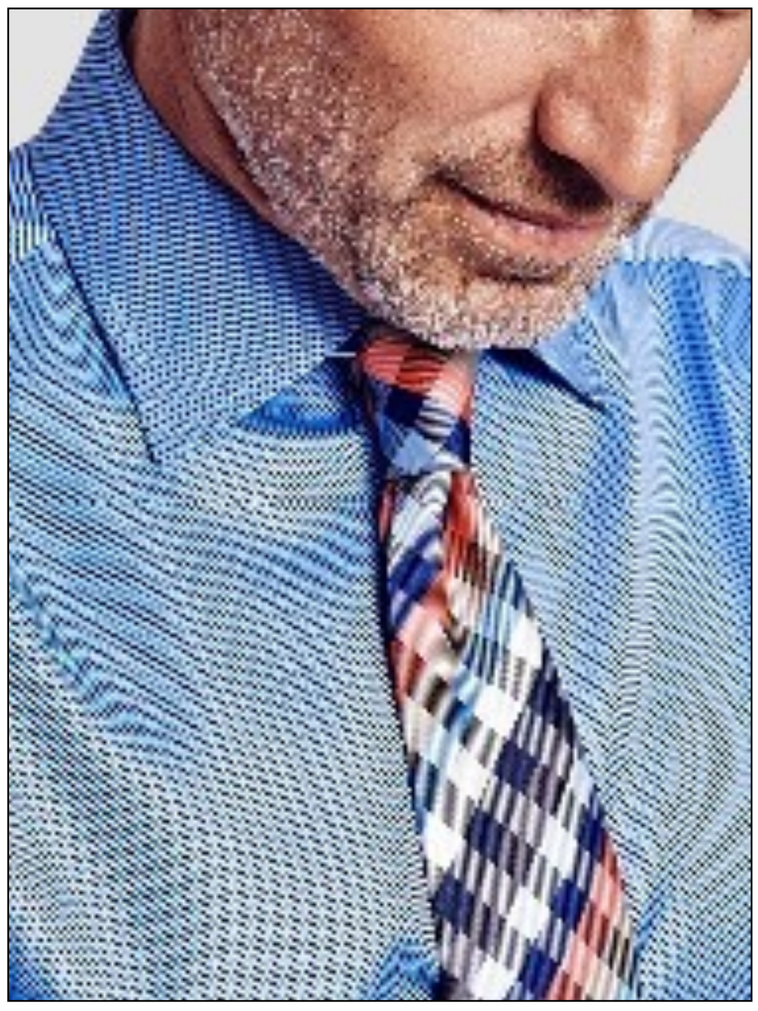
Red dots = samples needed to render
White dots = samples existing in texture map

Recall: aliasing

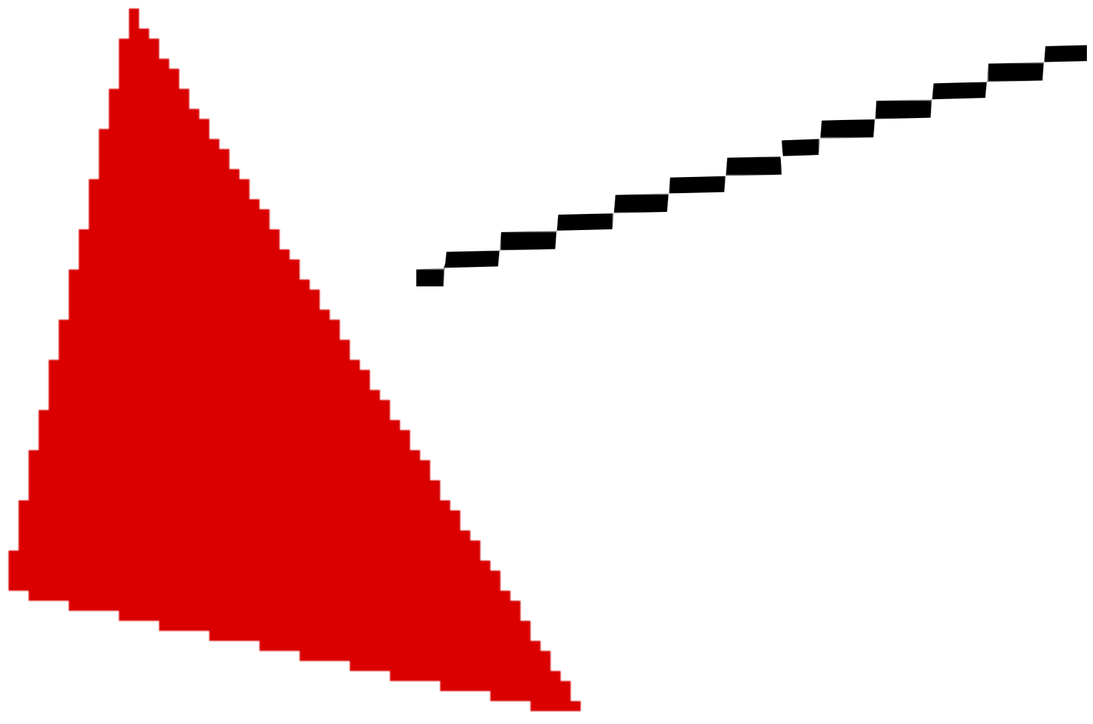
Undersampling a high-frequency signal can result in aliasing



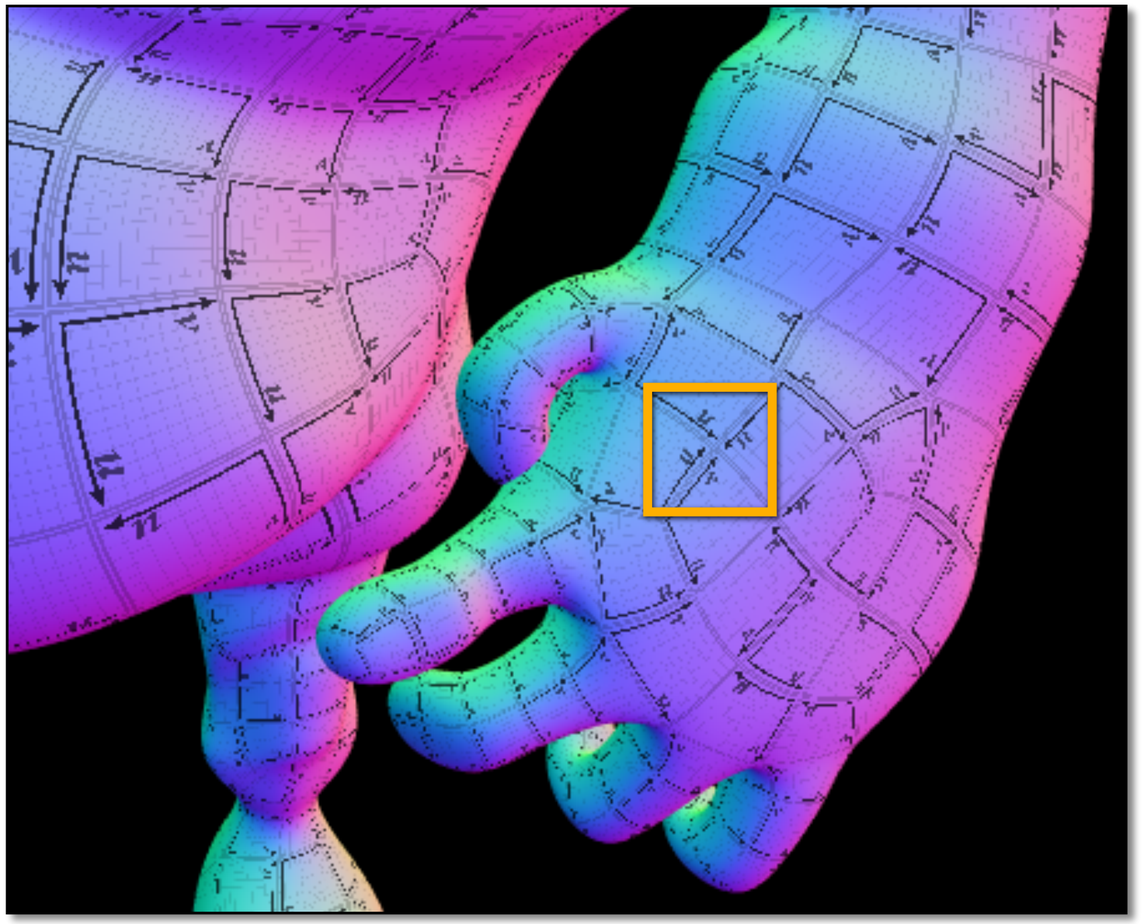
1D example



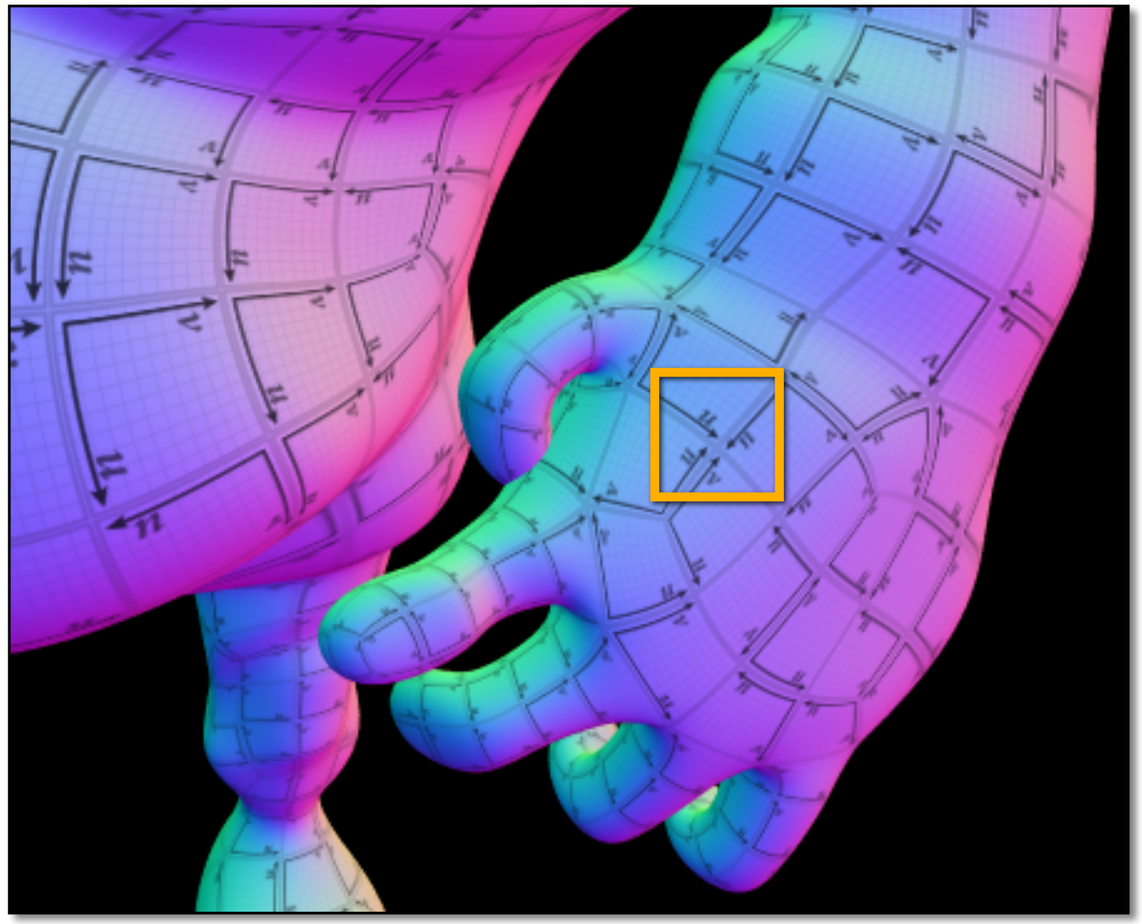
2D examples:
Moiré patterns, jaggies



Aliasing due to undersampling texture

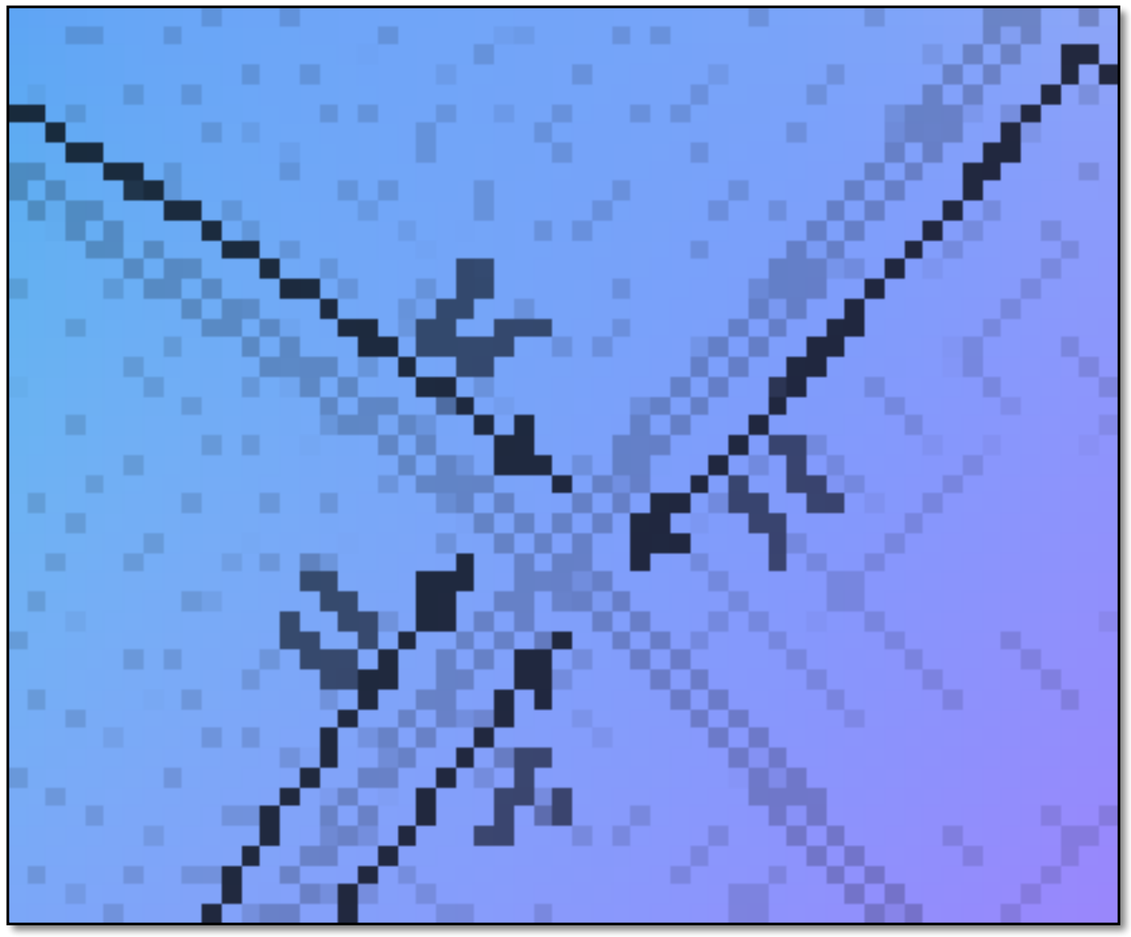


One texture sample per pixel
(aliasing!)

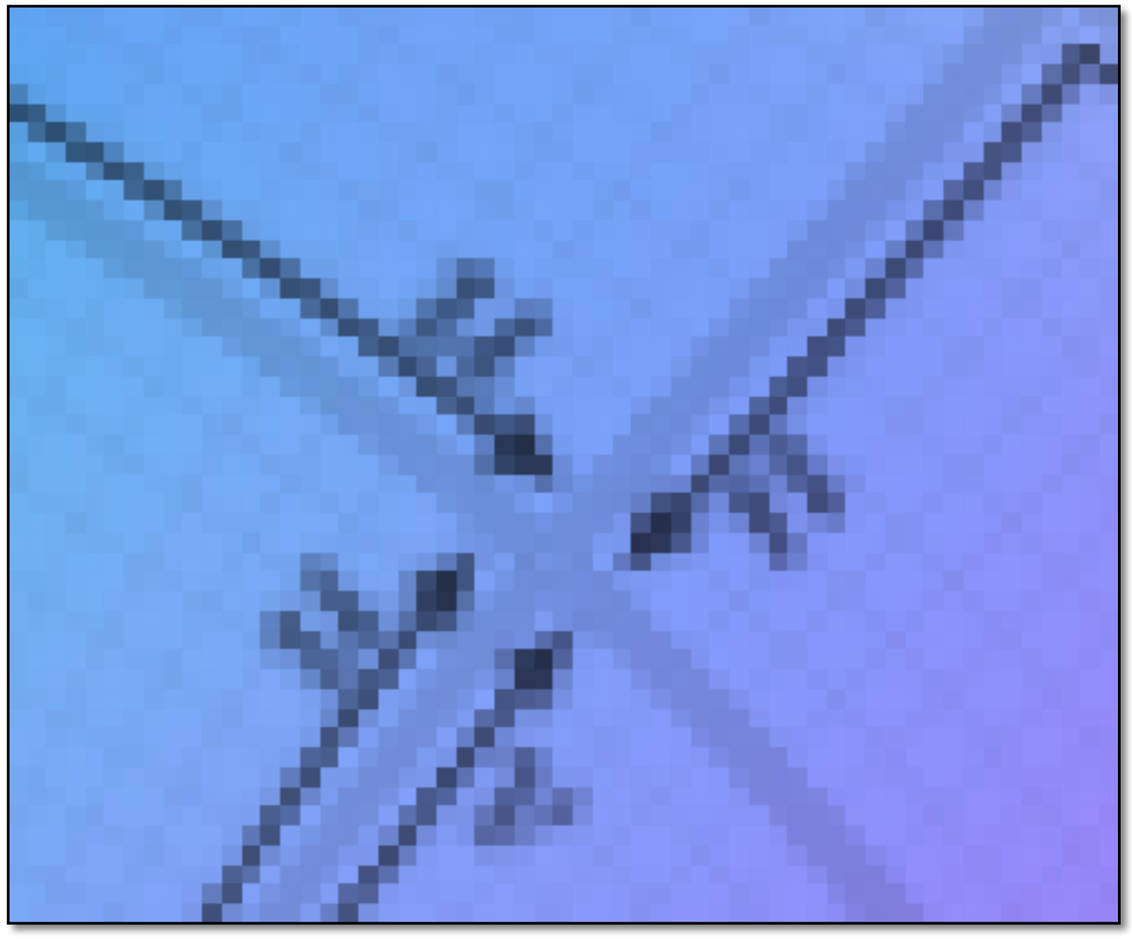


Anti-aliased texture sampling

Aliasing due to undersampling (zoom)

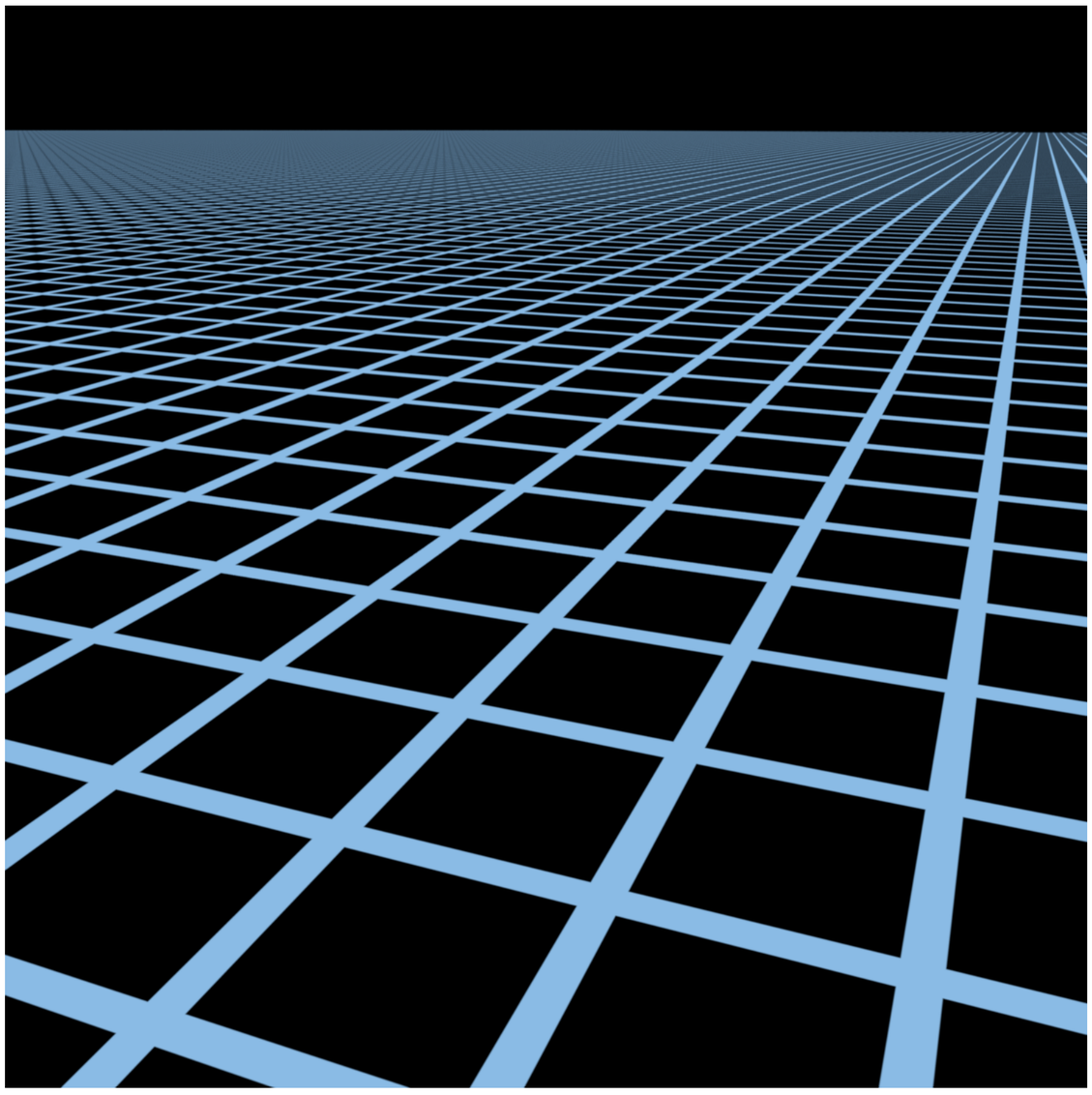


One texture sample per pixel
(aliasing!)

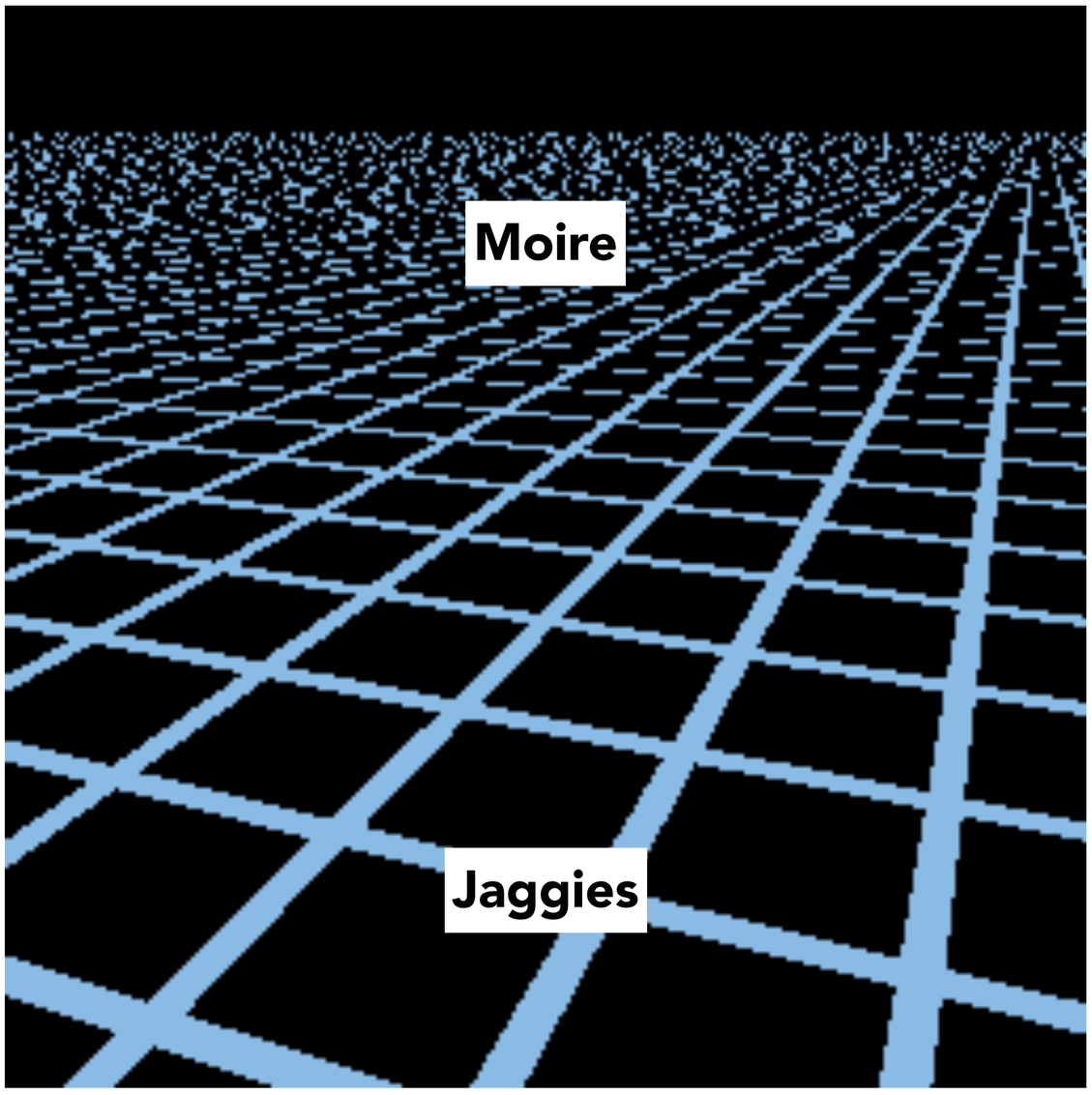


Anti-aliased texture sampling

Another example



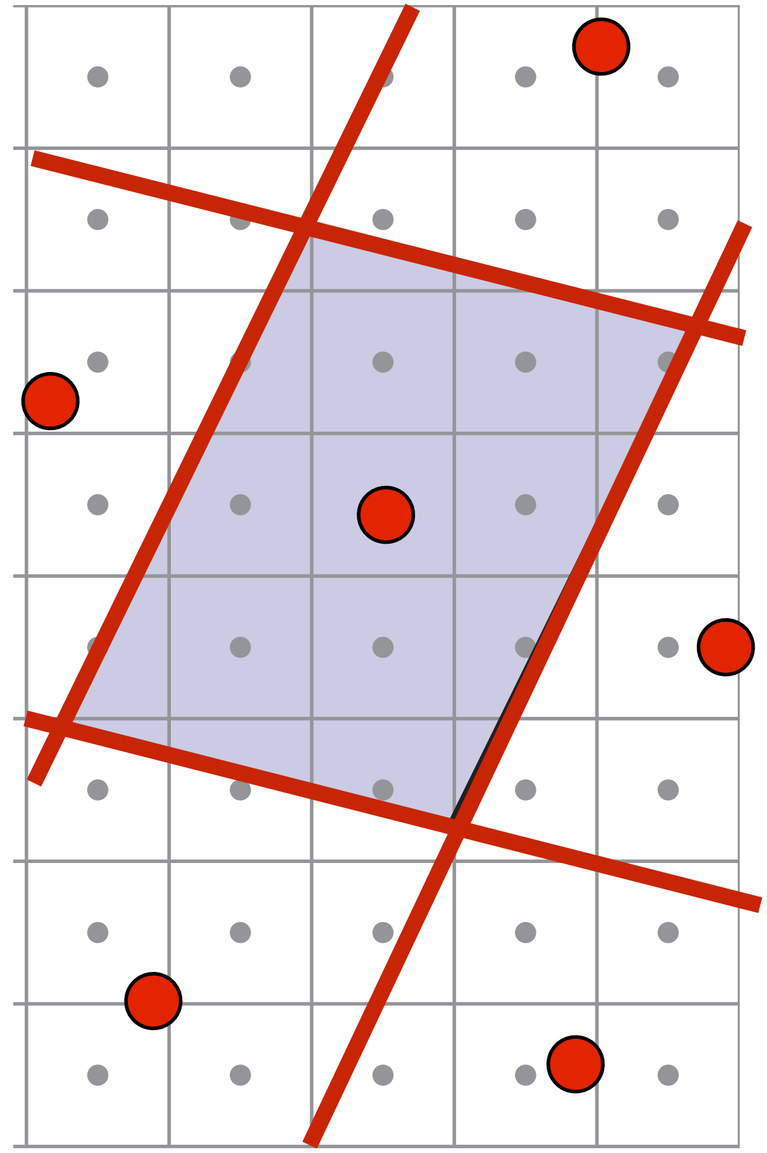
Anti-aliased result



Rendered image: 256x256 pixels

Texture minification - hard case

- Challenge:
  - Many texels contribute to color of an output image pixel (sampling only one of them could yield aliasing)
  - Shape of pixel footprint can be complex

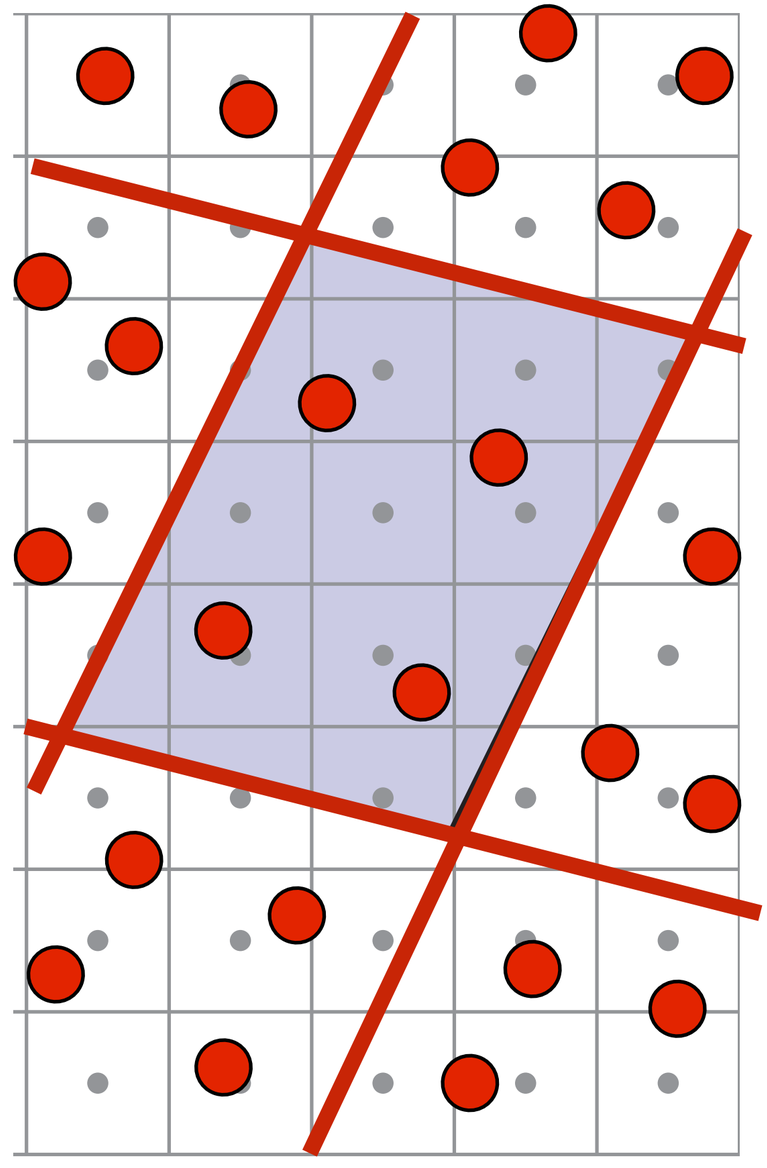


Shaded region = pixel area
Red lines = screen pixel boundaries
Red dots = texture space sample points for adjacent pixels

Texture minification - hard case

- Challenge:
  - Many texels contribute to color of an output image pixel (sampling only one of them could yield aliasing)
  - Shape of pixel footprint can be complex

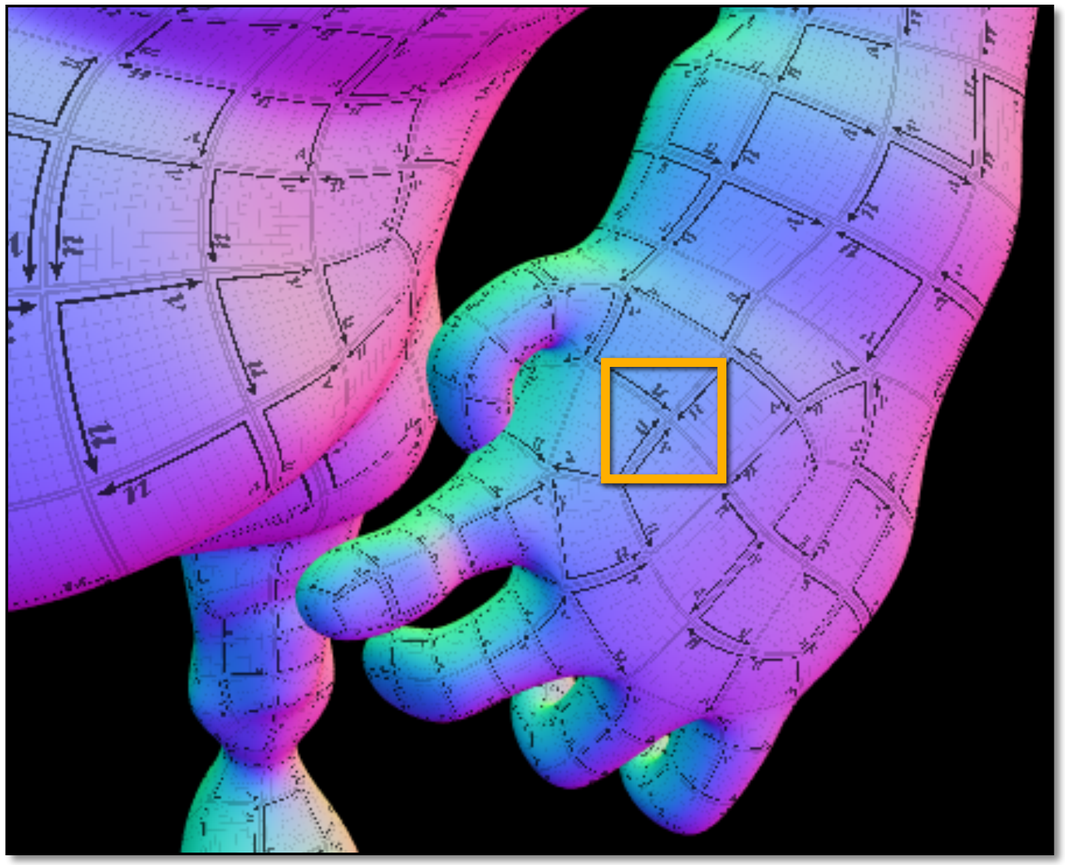
- One solution that you already know: supersampling
  - Averaging many texture samples per pixel can approximate result of convolving texture map with pixel-area sized filter
  - Problem?



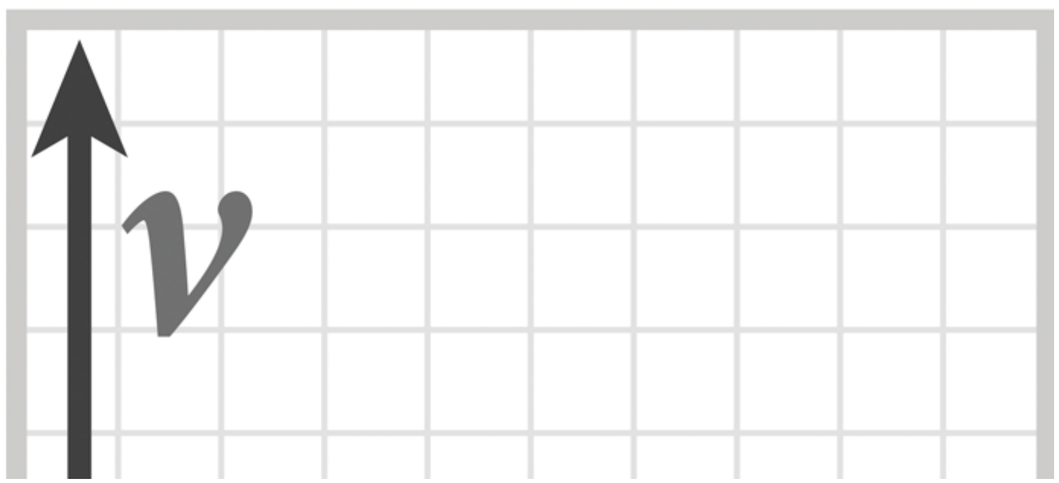
Shaded region = pixel area
Red lines = screen pixel boundaries
Red dots = texture space sample points for adjacent pixels

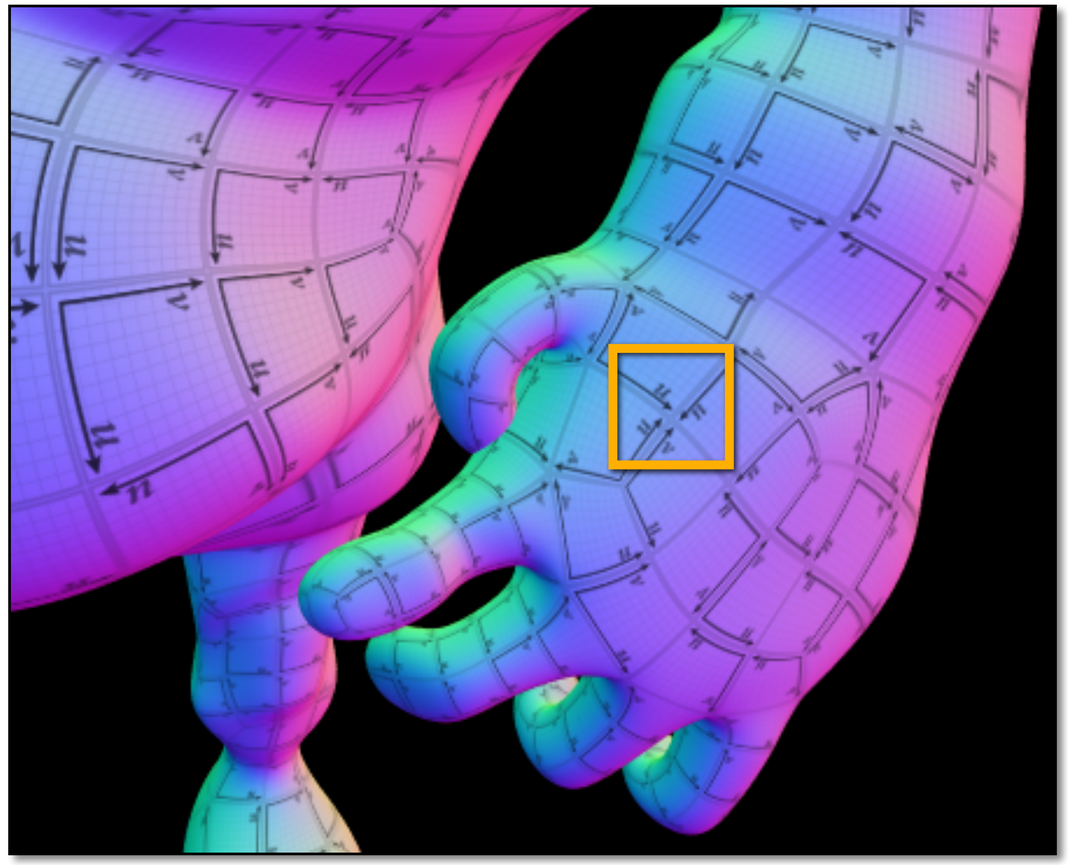
Alternative solution: remove high frequency from texture to reduce aliasing!

Pre-filtering texture map reduces aliasing



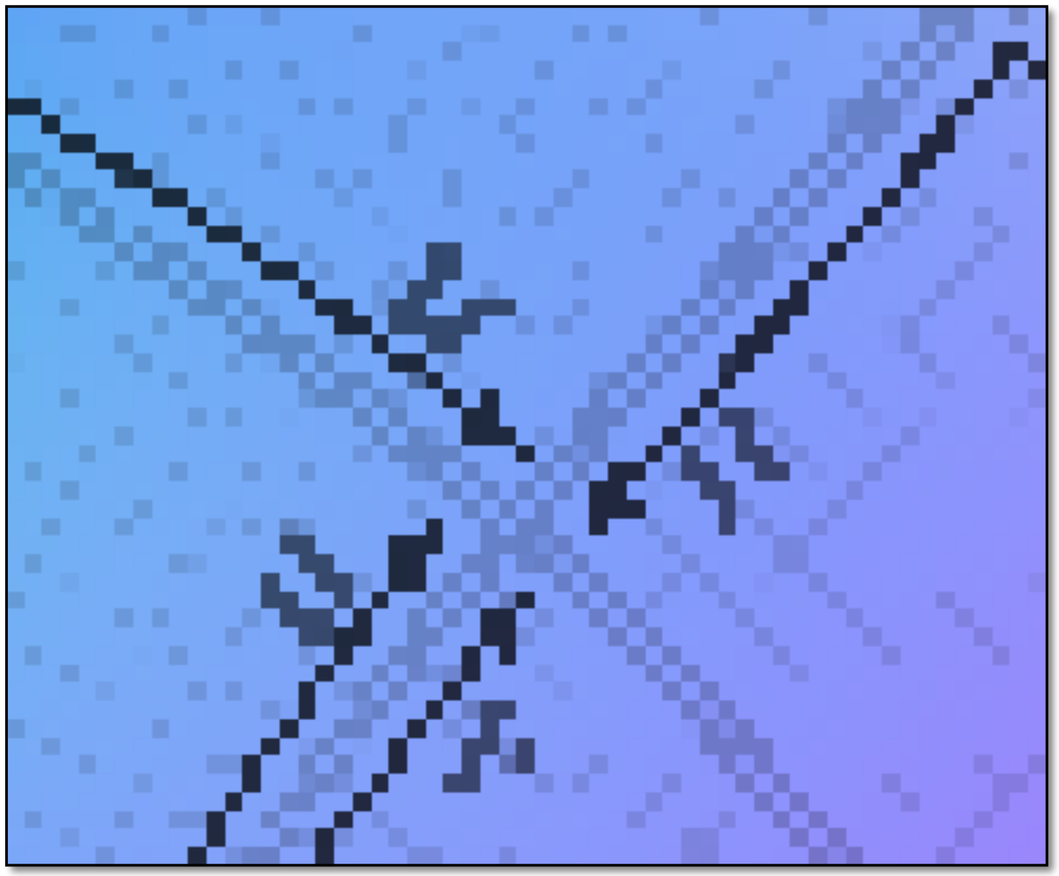
One texture sample per pixel
(aliasing!)



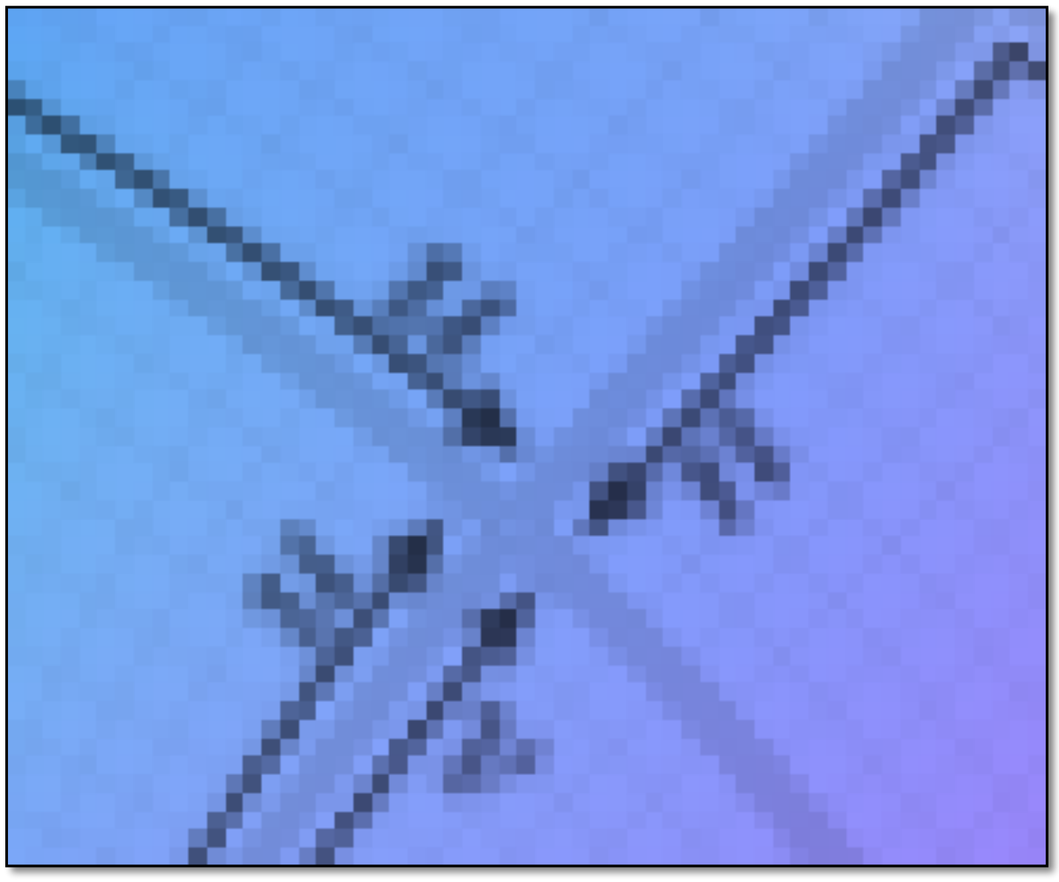


Pre-filtered texture map
(high frequencies removed)

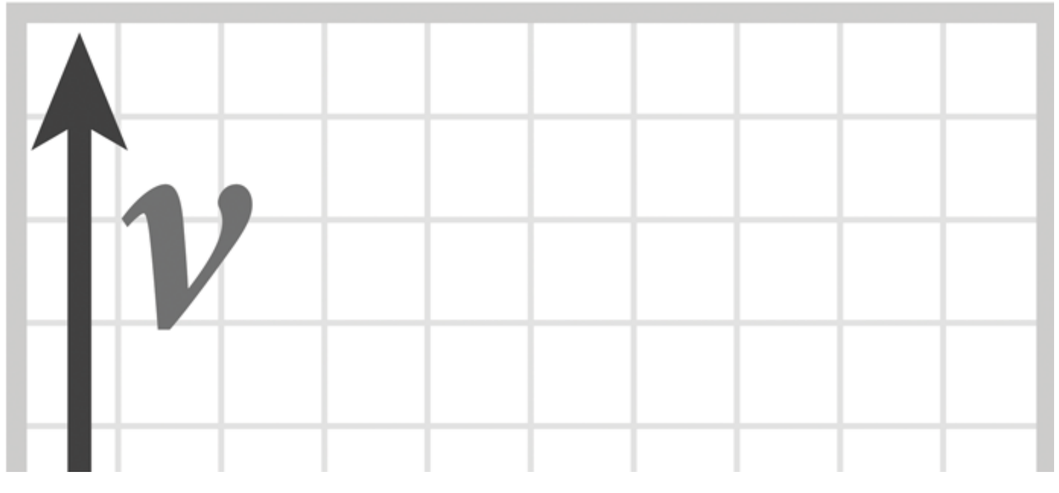
Pre-filtering texture map reduces aliasing



No pre-filtering of texture data
(resulting image exhibits aliasing)



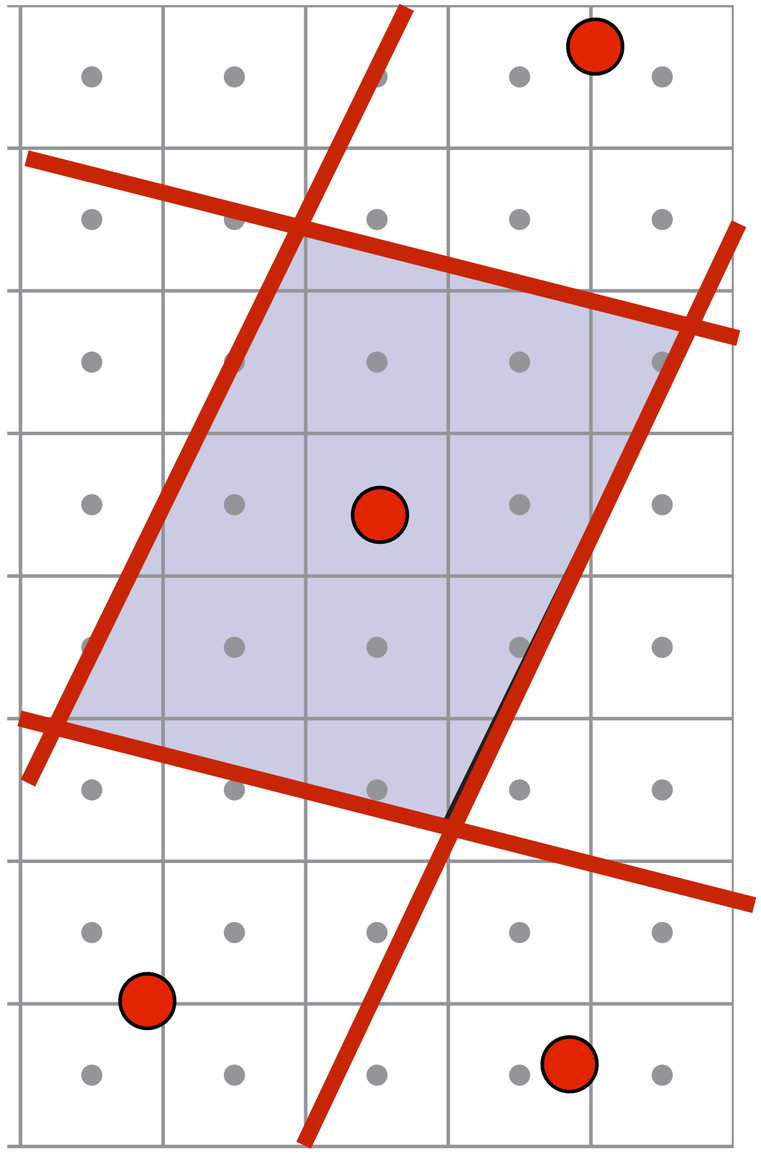
Pre-filtered texture map
(high frequencies removed)



But how much should we pre-filter?

- Amount of pre-filtering depends on how far away the object is:
  - minor minification: image pixel extreme magnification: image pixel spans large region of texture

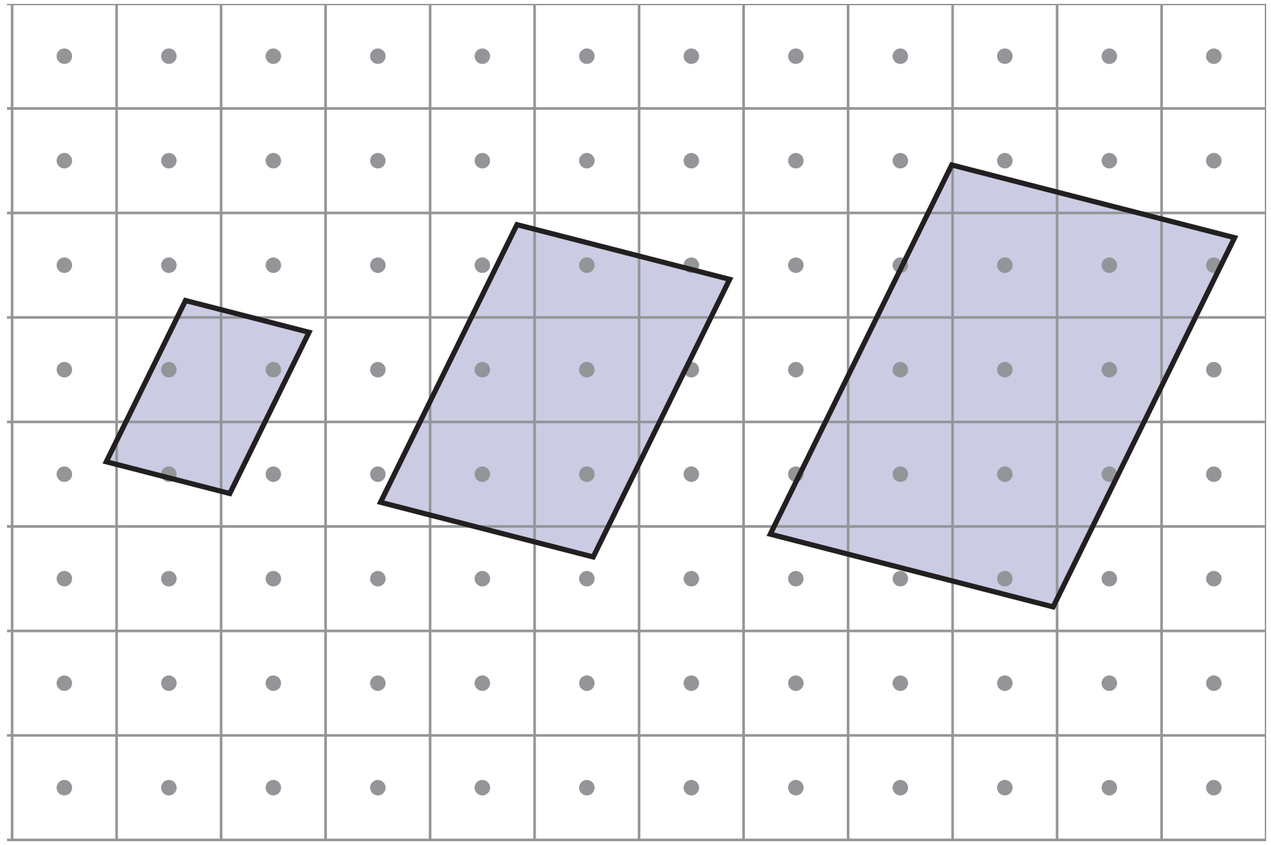
- Idea:
  - Low-pass filter and downsample texture file, and store successively lower resolutions
  - For each sample, use the texture file whose resolution approximates the screen



Shader region = pixel area
Red lines = screen pixel boundaries
Red dots = texture space sample points for adjacent pixels

But how much should we pre-filter?

- Amount of pre-filtering necessary depends on how far away the object is

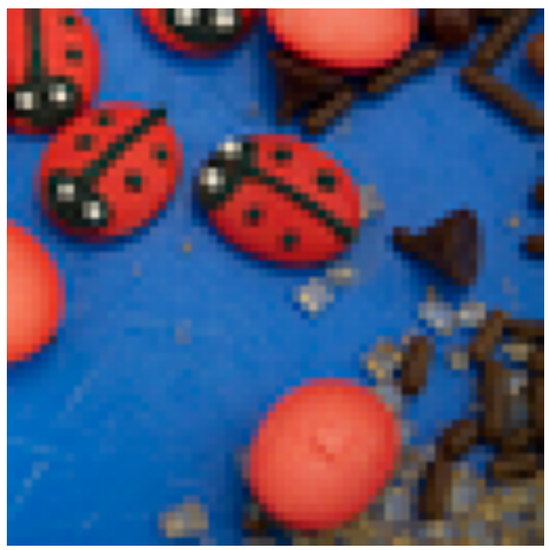


- Idea: pre-compute and store different versions of the texture with different amounts of prefiltering
  - Low-pass filter and downsample texture file, and store successively lower resolutions
  - When sampling texture, use the texture file whose prefiltering amount matches the desired sampling rate

Mipmap (L. Williams 83)

Each mipmap level is downsampled (low-pass filtered) version of the previous

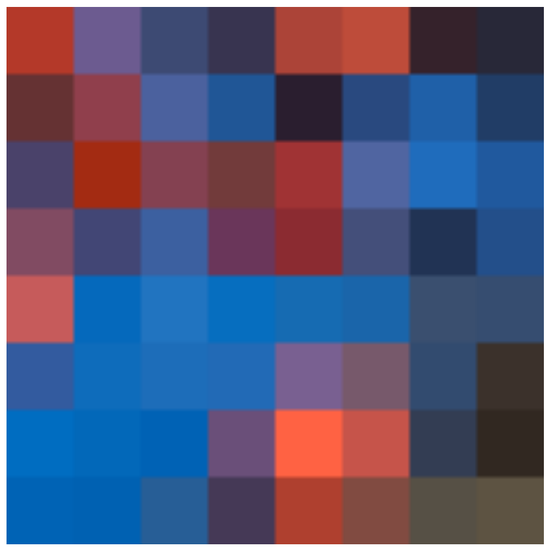
Level 0 = 128x128



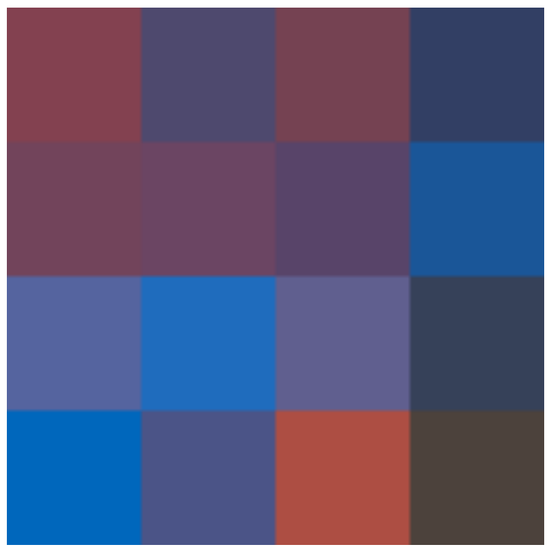
Level 1 = 64x64

Level 2 = 32x32

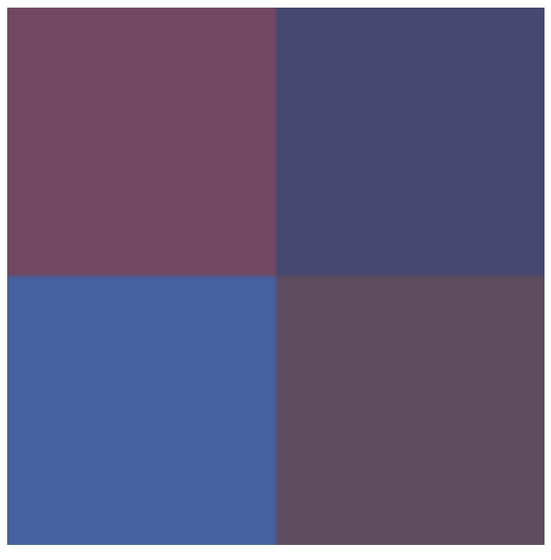
Level 3 = 16x16



Level 4 = 8x8



Level 5 = 4x4

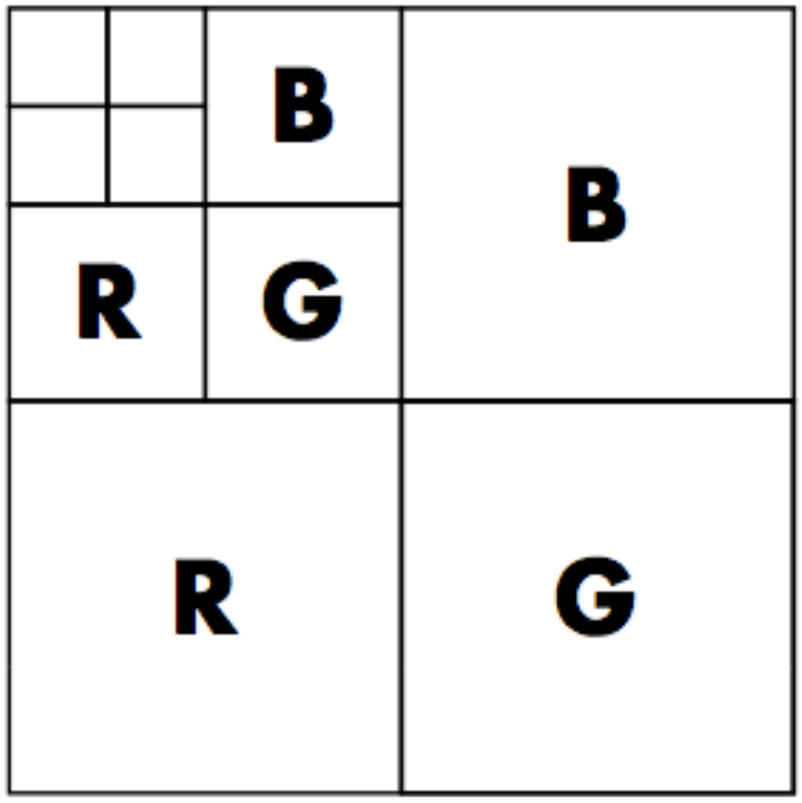


Level 6 = 2x2

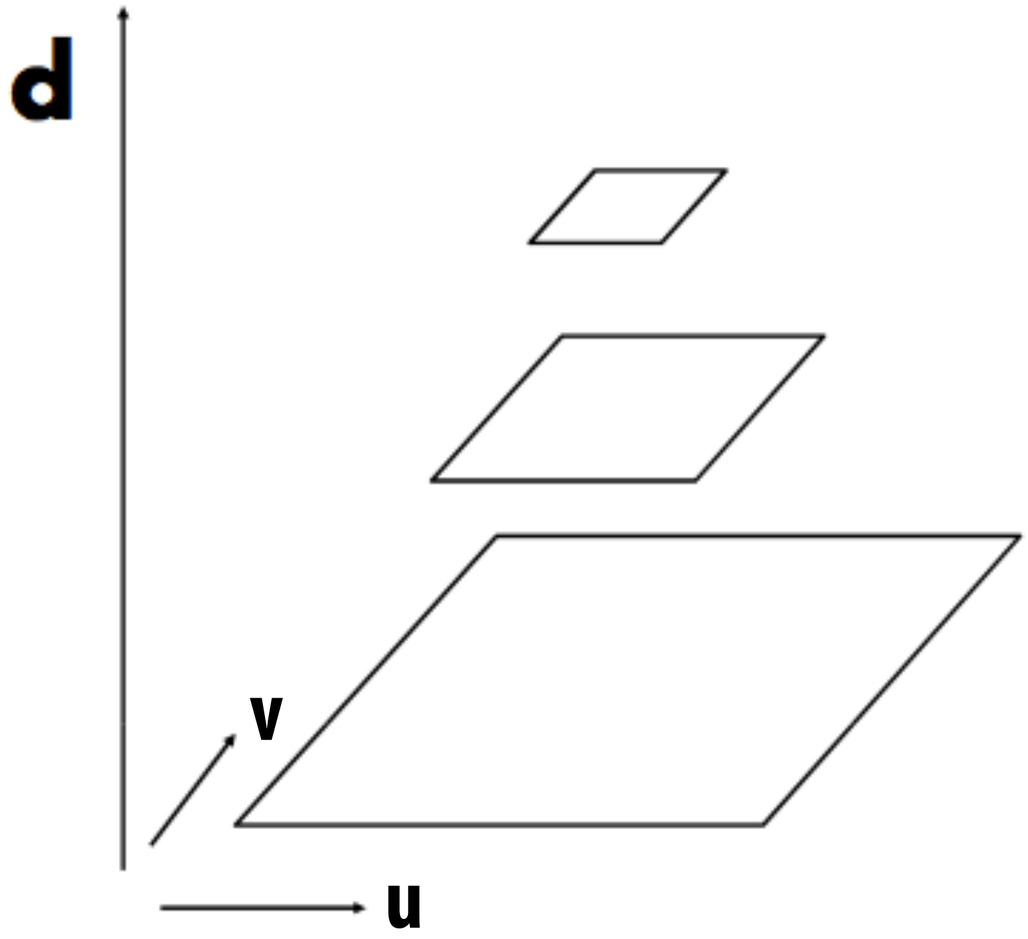
Level 7 = 1x1

“Mip” comes from the Latin “multum in parvo", meaning a multitude in a small space

Mipmap (L. Williams 83)



Williams' original proposed mip-map layout



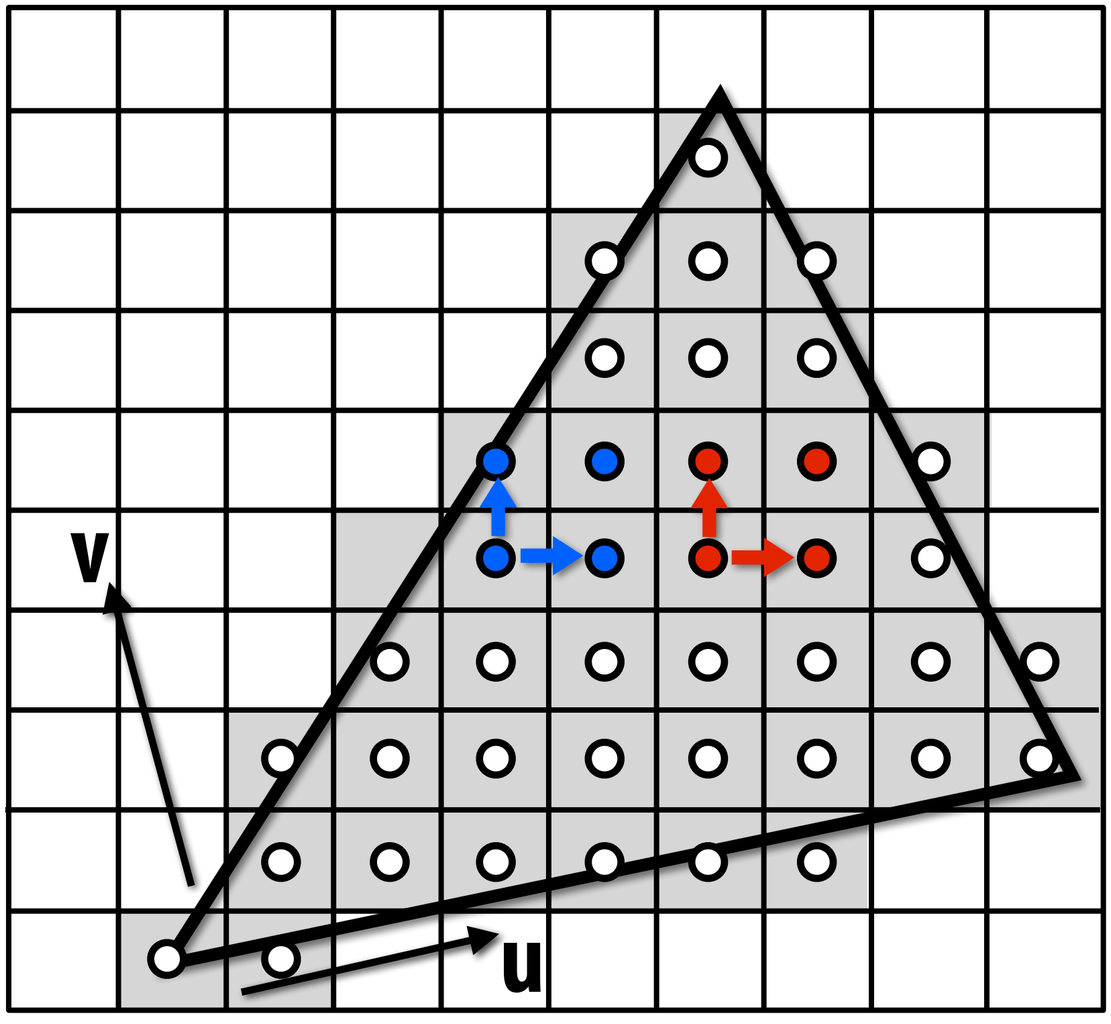
"Mip hierarchy" level = *d*

What is the storage overhead of a mipmap?

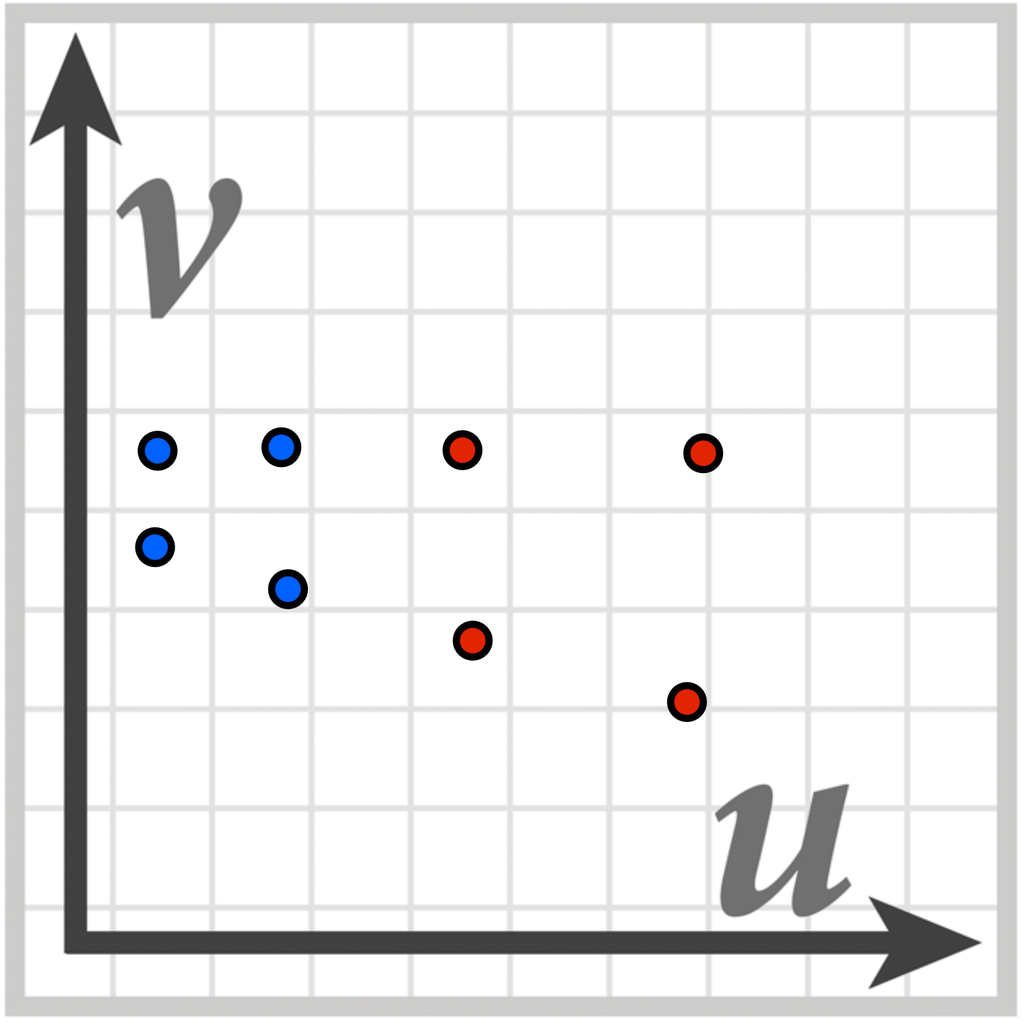
Slide credit: Akeley and Hanrahan

Computing mipmap level

Compute differences between texture coordinate values of neighboring screen samples



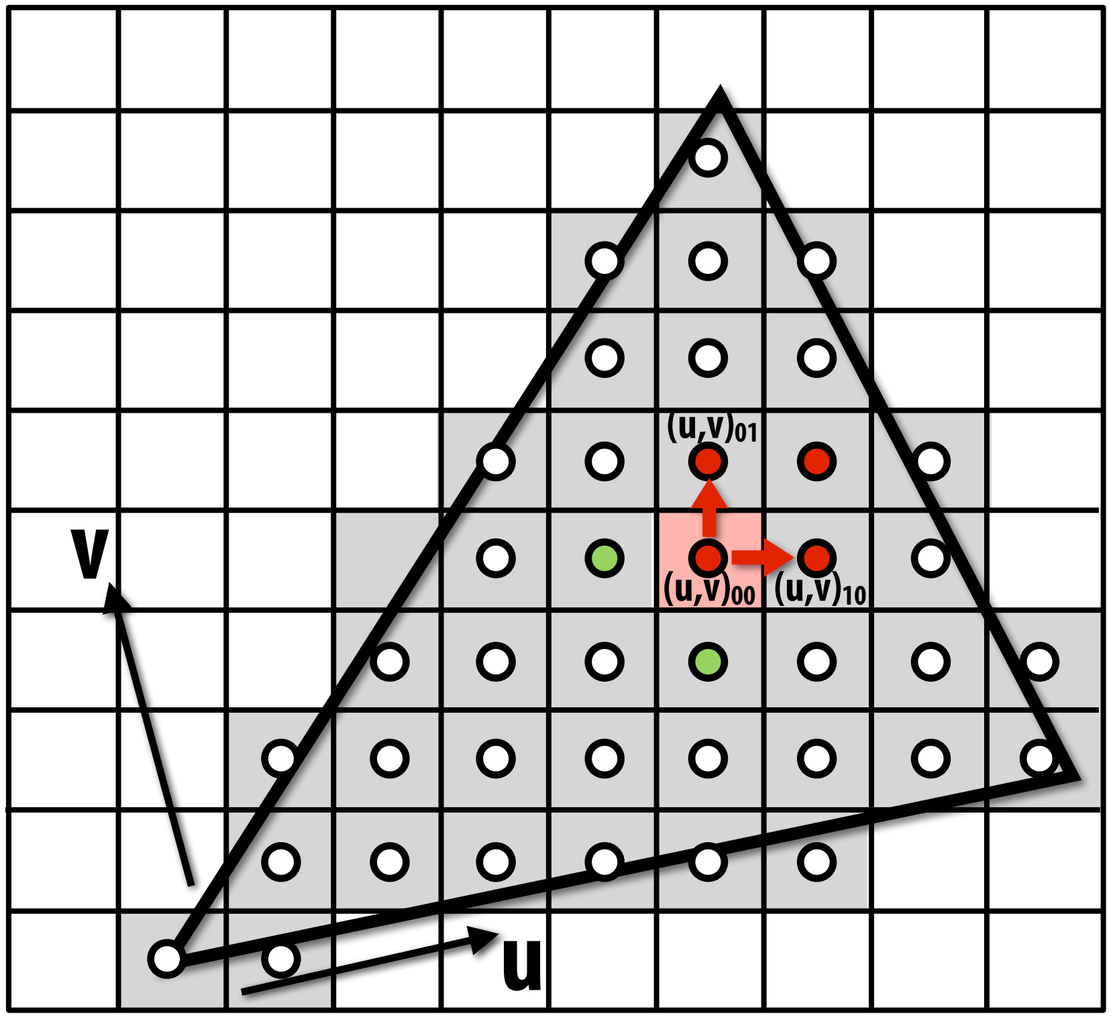
Screen space

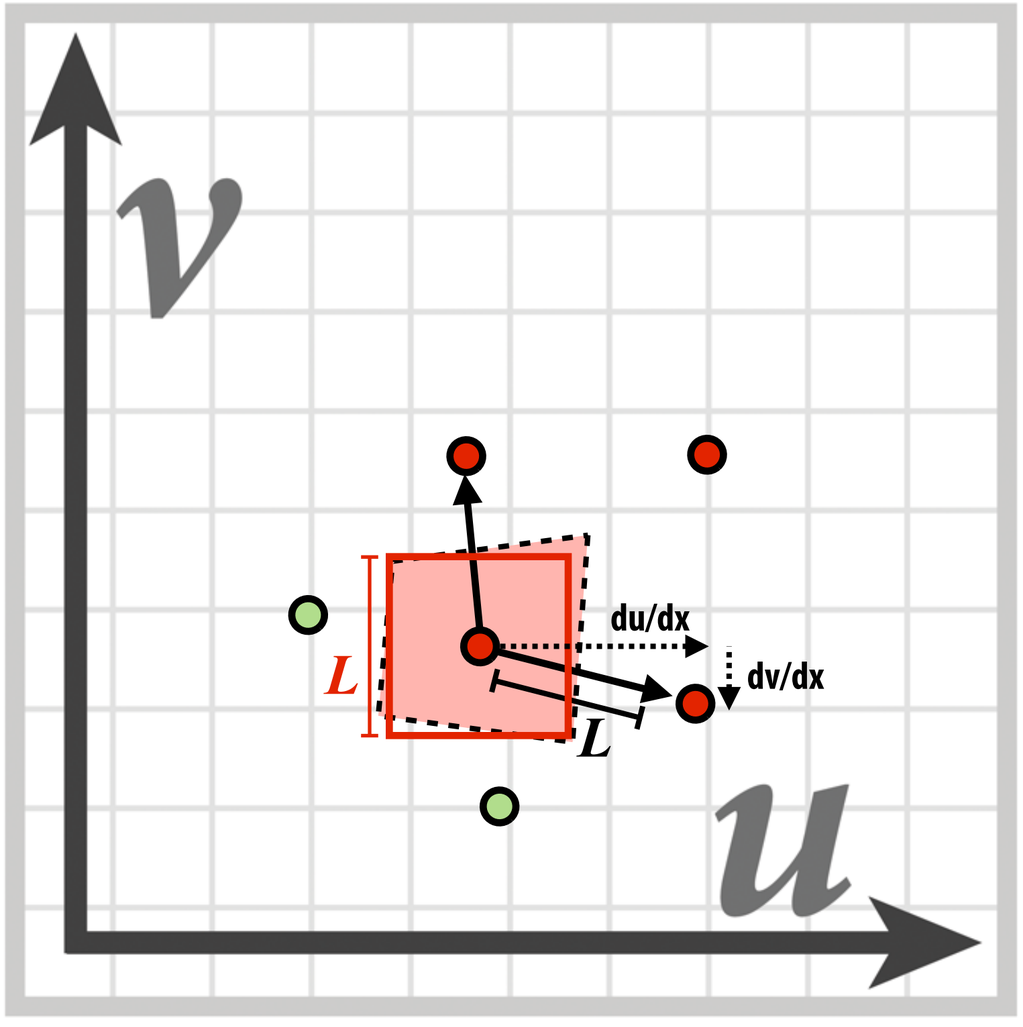


Texture space

Computing mipmap level

Compute differences between texture coordinate values of neighboring screen samples





$du/dx = u_{10}-u_{00}$ $\quad dv/dx = v_{10}-v_{00}$

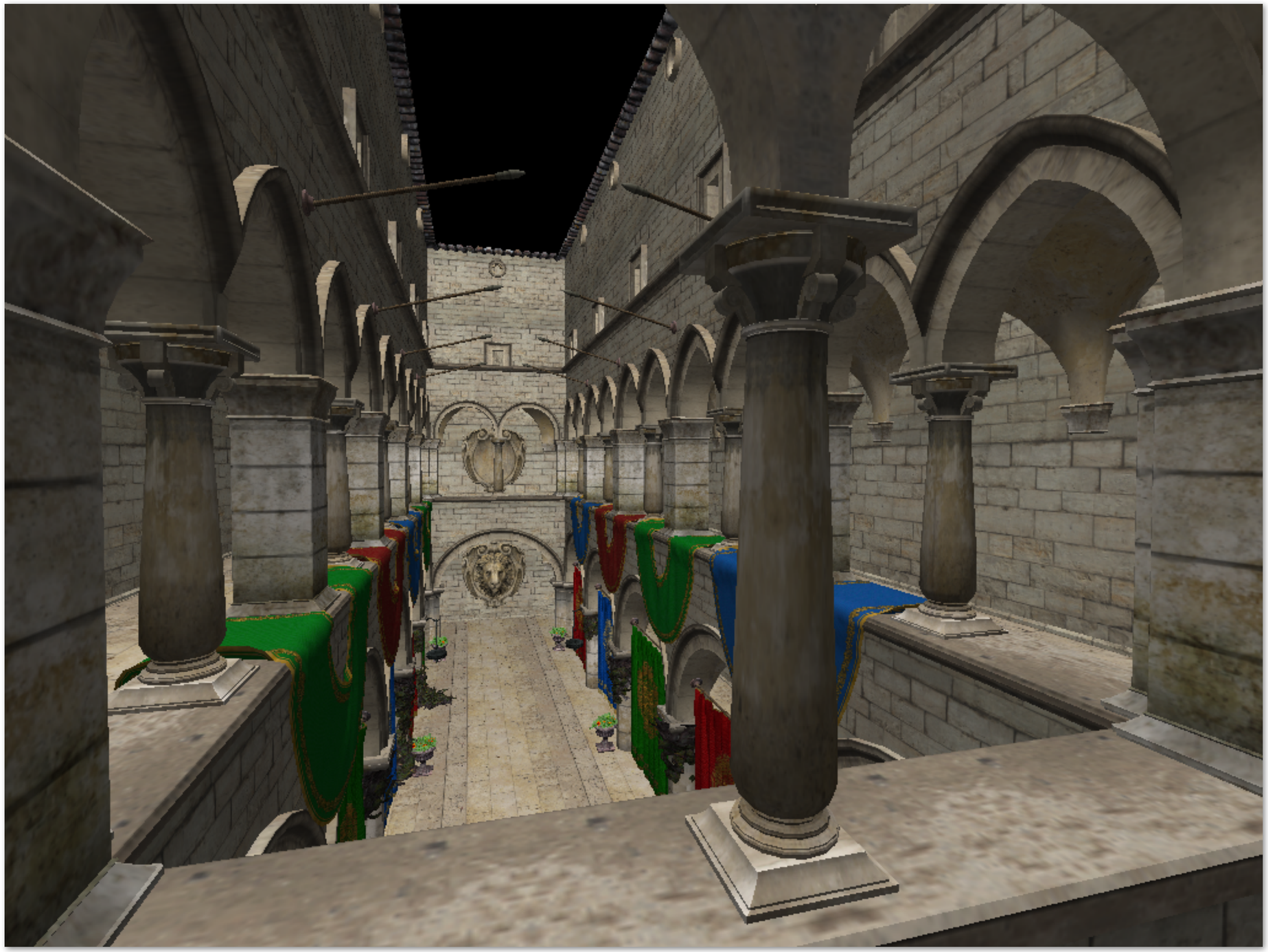
$du/dy = u_{01}-u_{00}$ $\quad dv/dy = v_{01}-v_{00}$

$$L = \max\left(\sqrt{\left(\frac{du}{dx}\right)^2+\left(\frac{dv}{dx}\right)^2}, \sqrt{\left(\frac{du}{dy}\right)^2+\left(\frac{dv}{dy}\right)^2}\right)$$

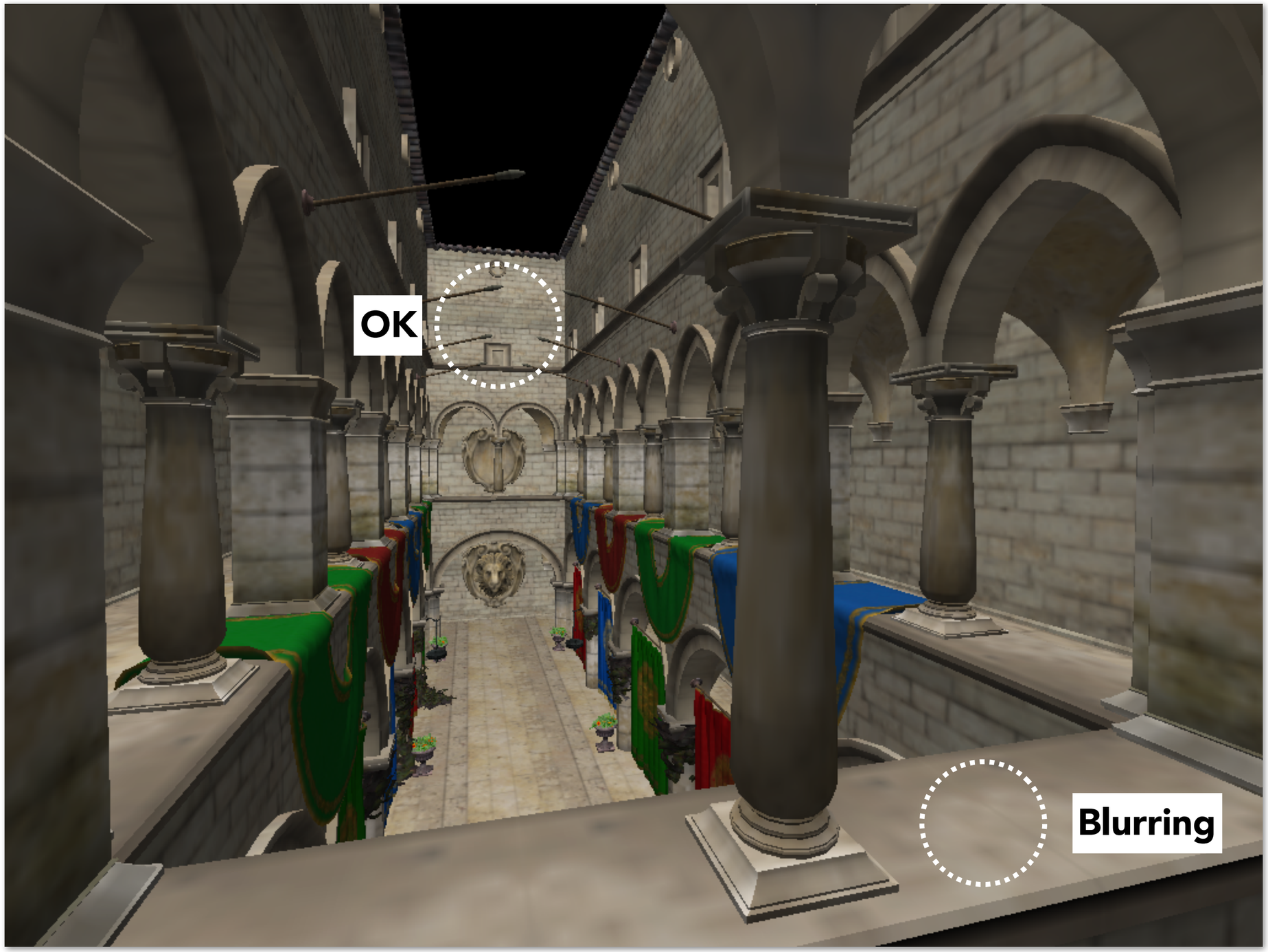
$$\textit{mip-map } d = \log_2 L$$

Bilinear resampling at level 0

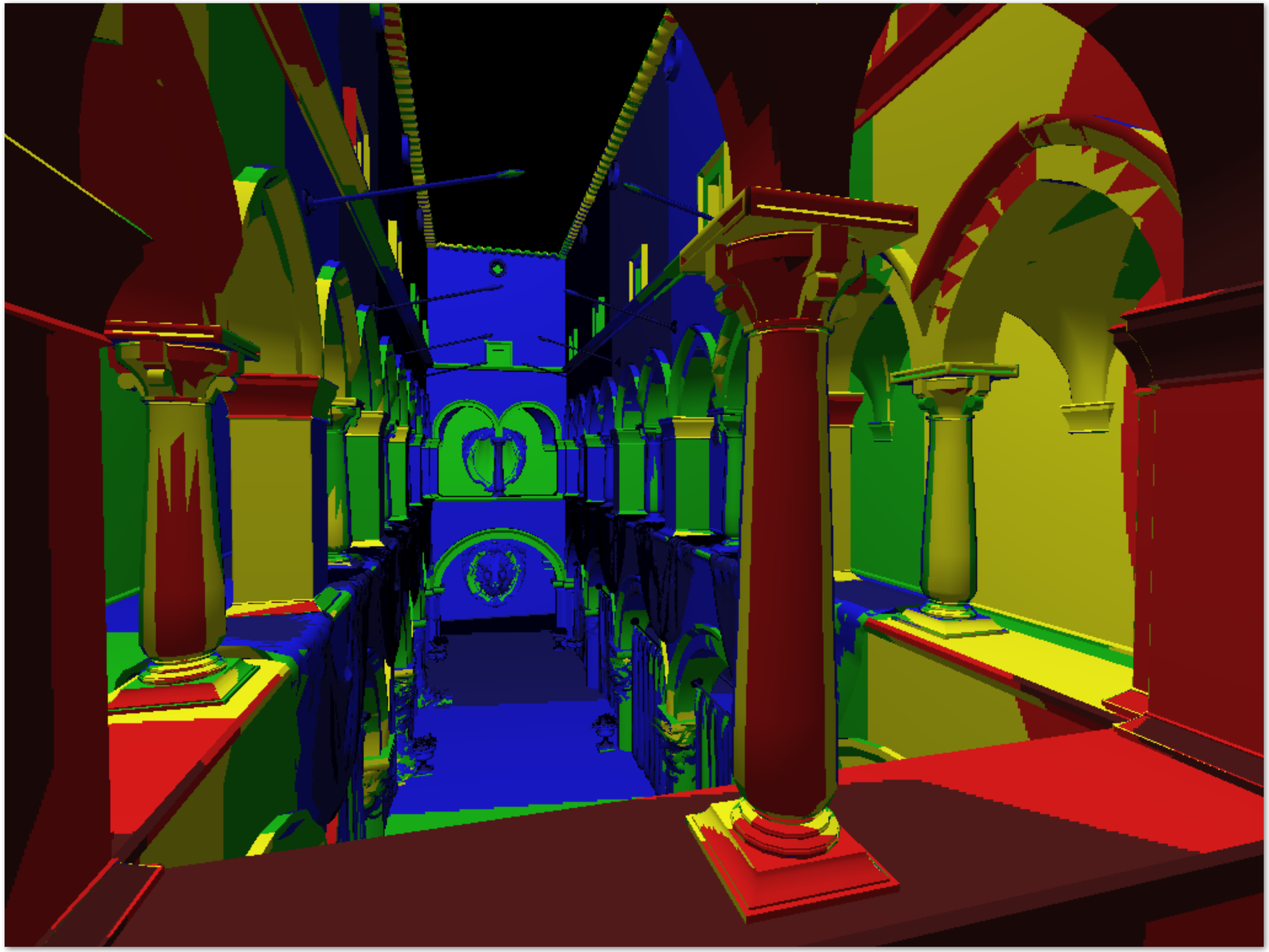
Bilinear resampling at level 2



Bilinear resampling at level 4



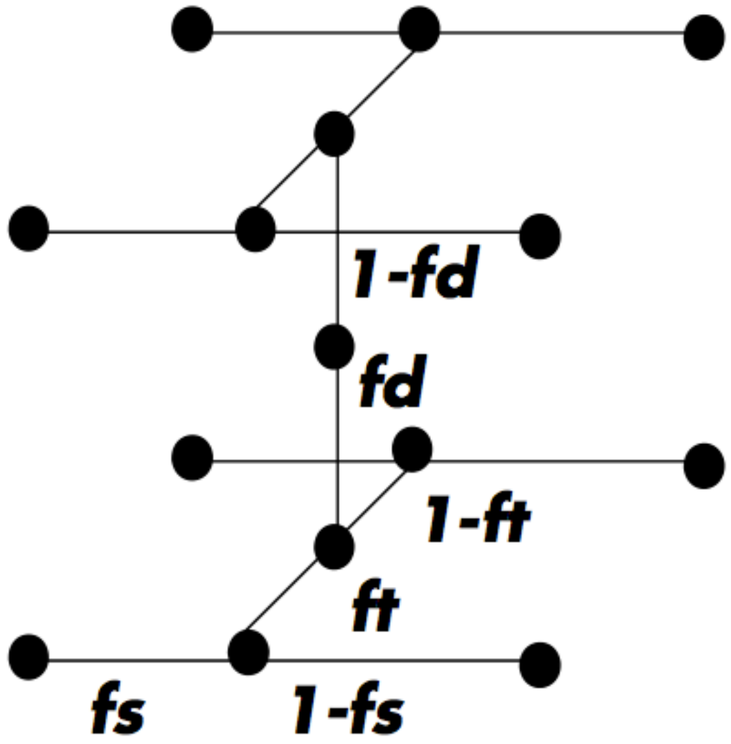
Visualization of mipmap level
(bilinear filtering only: *d* clamped to nearest level)



"Tri-linear" filtering

Linearly interpolate the bilinear interpolation results from two adjacent levels of the mip map.

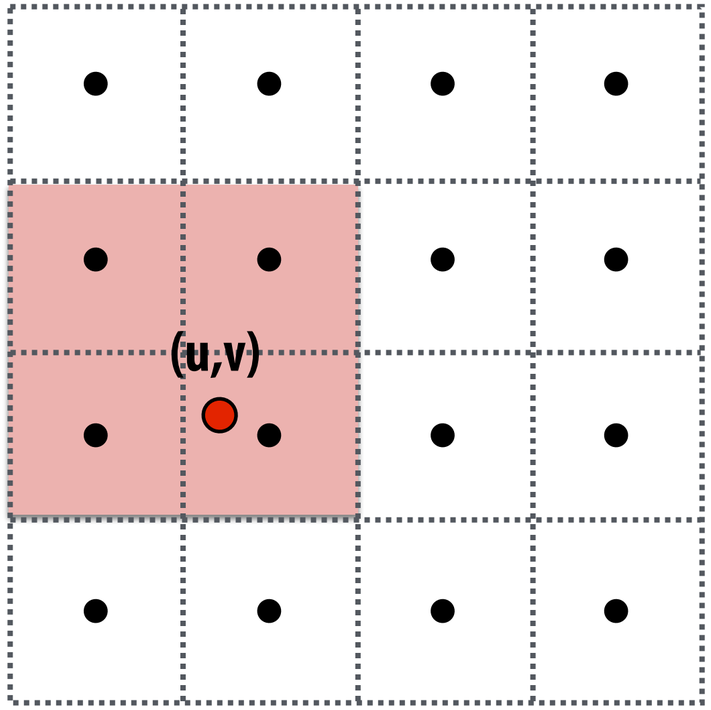
(smoothly transition between different levels of prefiltering)



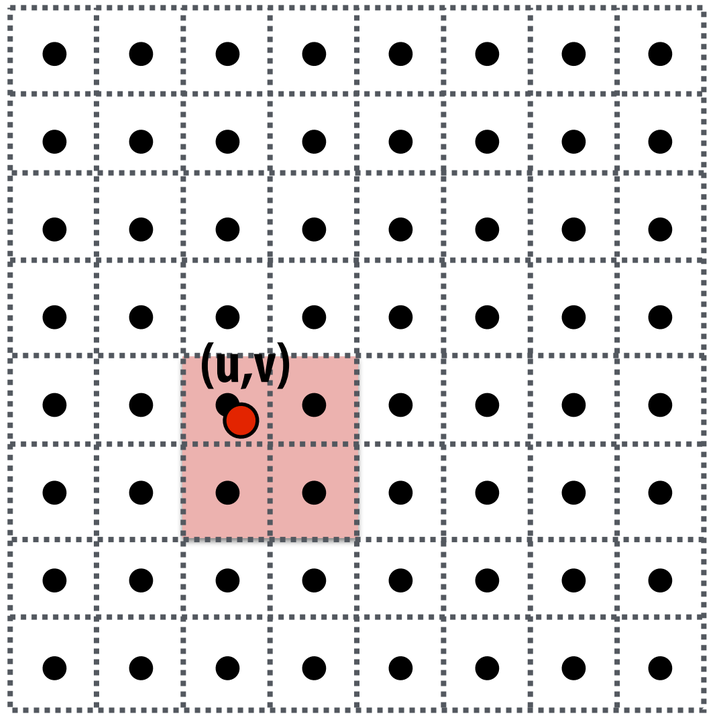
$$lerp(t, v_1, v_2) = v_1 + t(v_2 - v_1)$$

Bilinear resampling:
four texel reads
3 lerps (3 mul + 6 add)

Trilinear resampling:
eight texel reads
7 lerps (7 mul + 14 add)



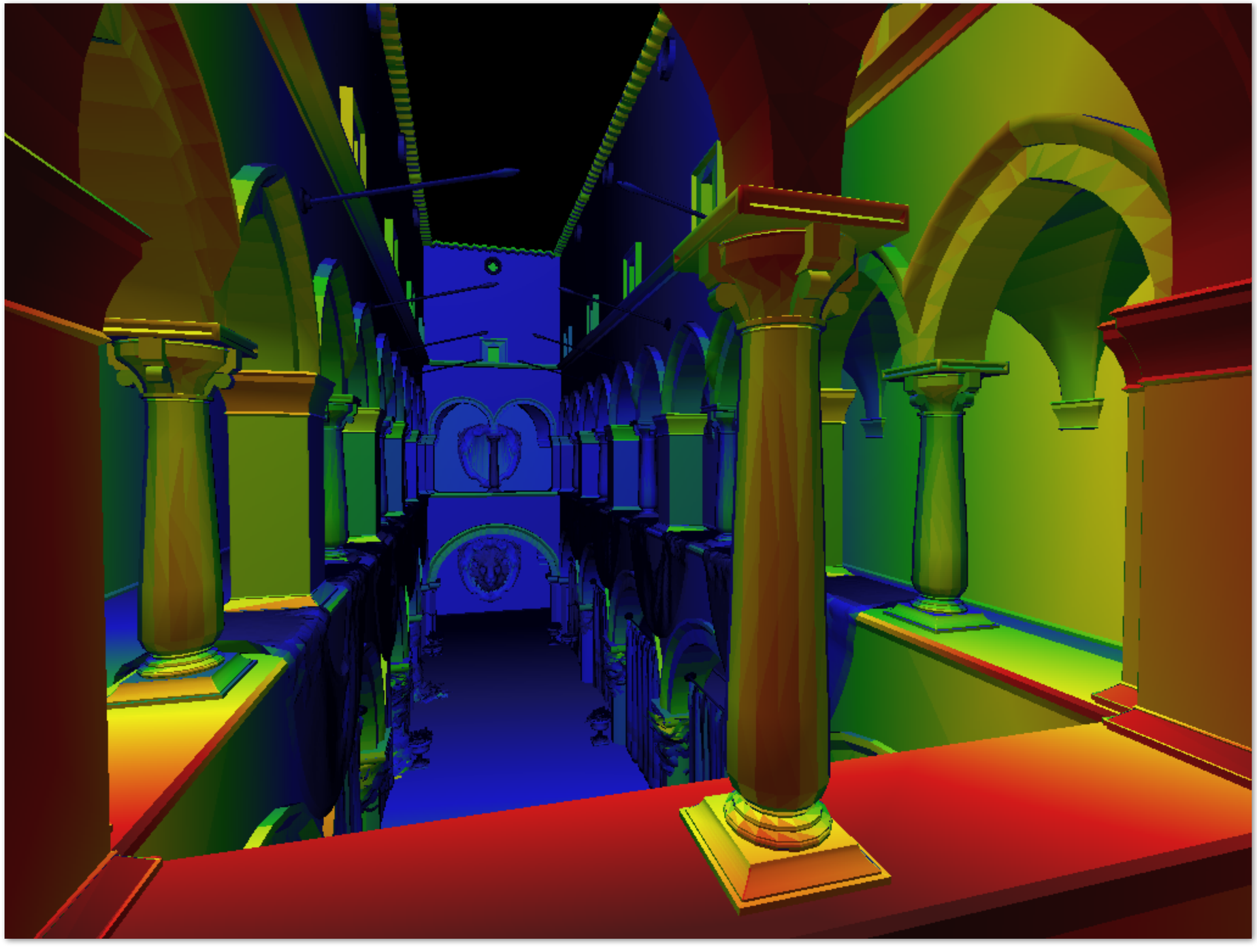
mip-map texels: level d+1



mip-map texels: level d

Figure credit: Akeley and Hanrahan

Visualization of mipmap level
(trilinear filtering: visualization of continuous *d*)

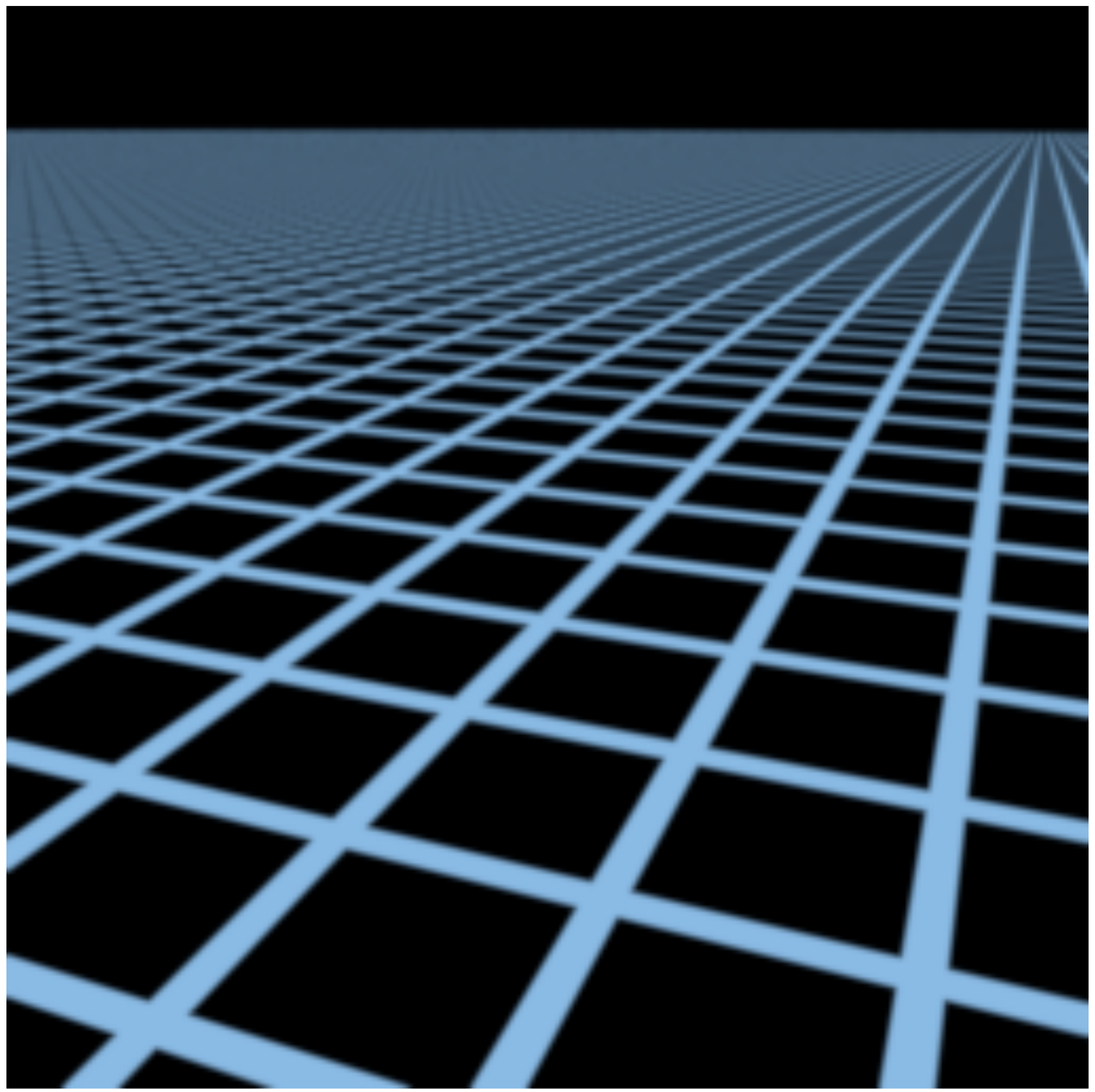


Bilinear vs trilinear filtering cost

- **Bilinear resampling:**
  - **4 texel reads**
  - **3 lerps (3 mul + 6 add)**

- **Trilinear resampling:**
  - **8 texel reads**
  - **7 lerps (7 mul + 14 add)**

Example: mipmap limitations

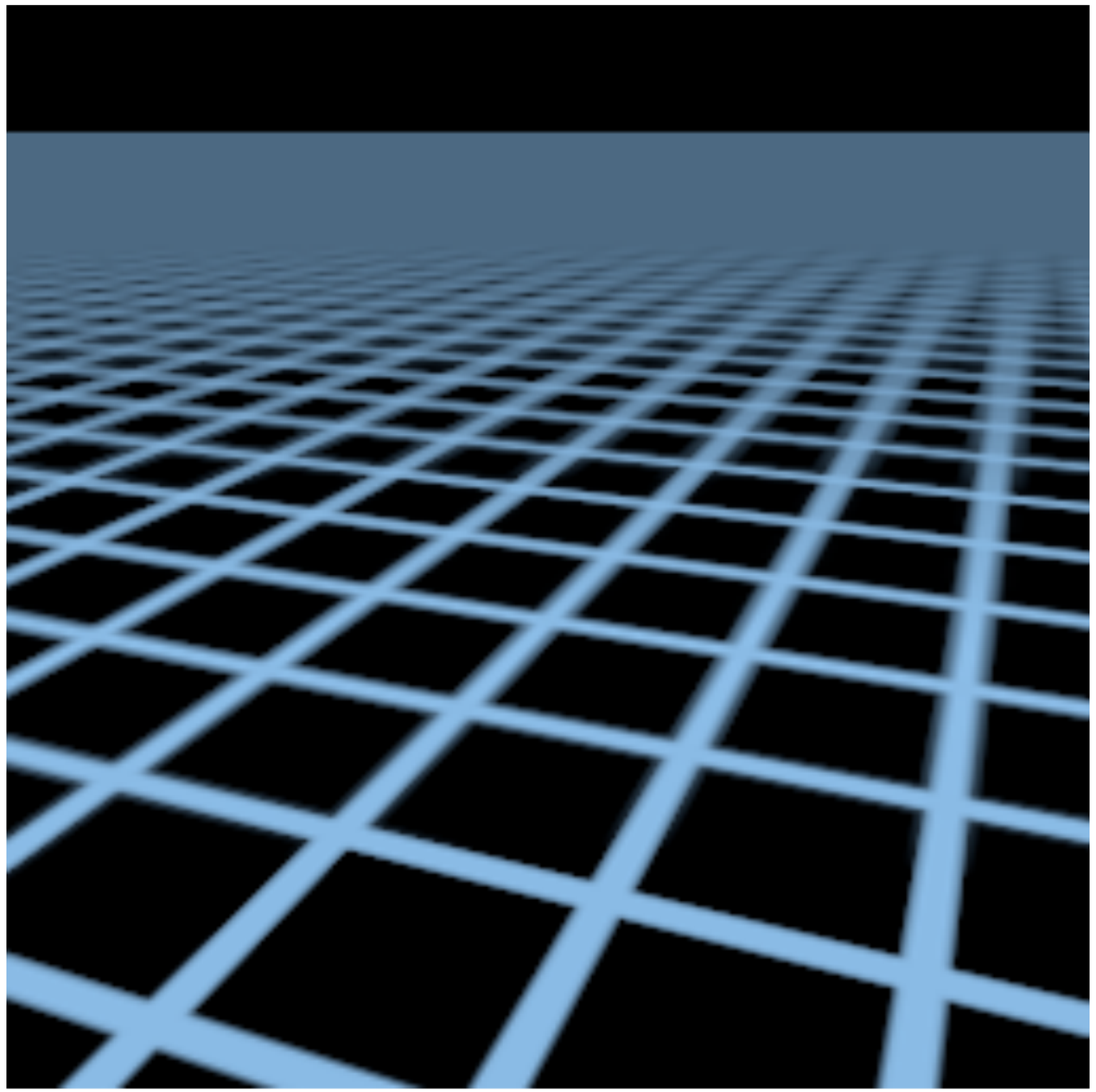


Supersampling: 512 texture samples per pixel (desired answer)

Example: mipmap limitations

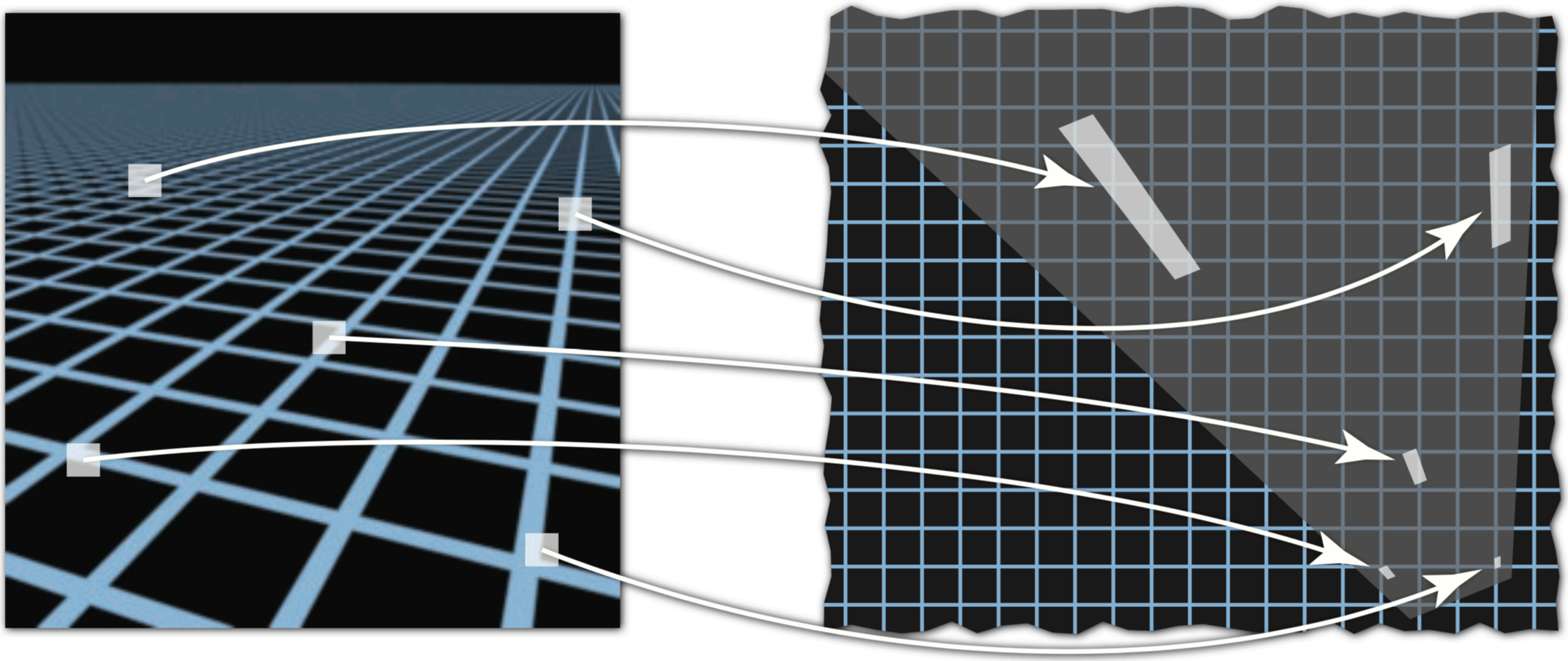
Overblurs

Why?



Mipmap trilinear sampling

Screen pixel footprint in texture space



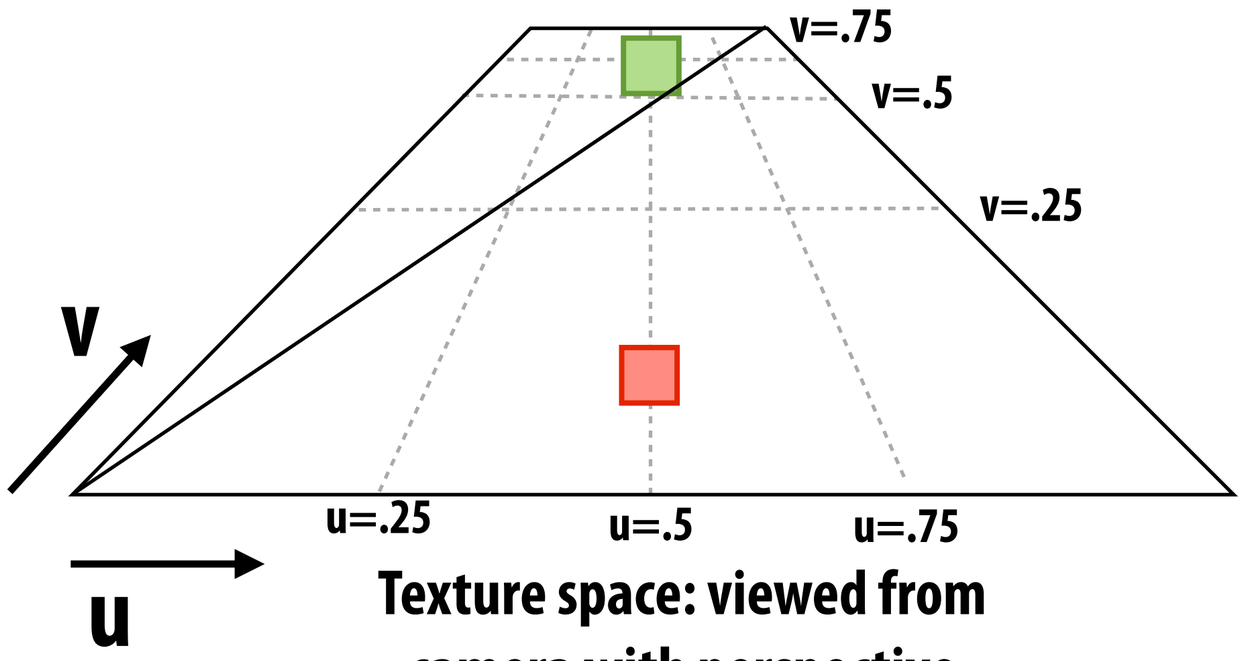
Screen space

Texture space

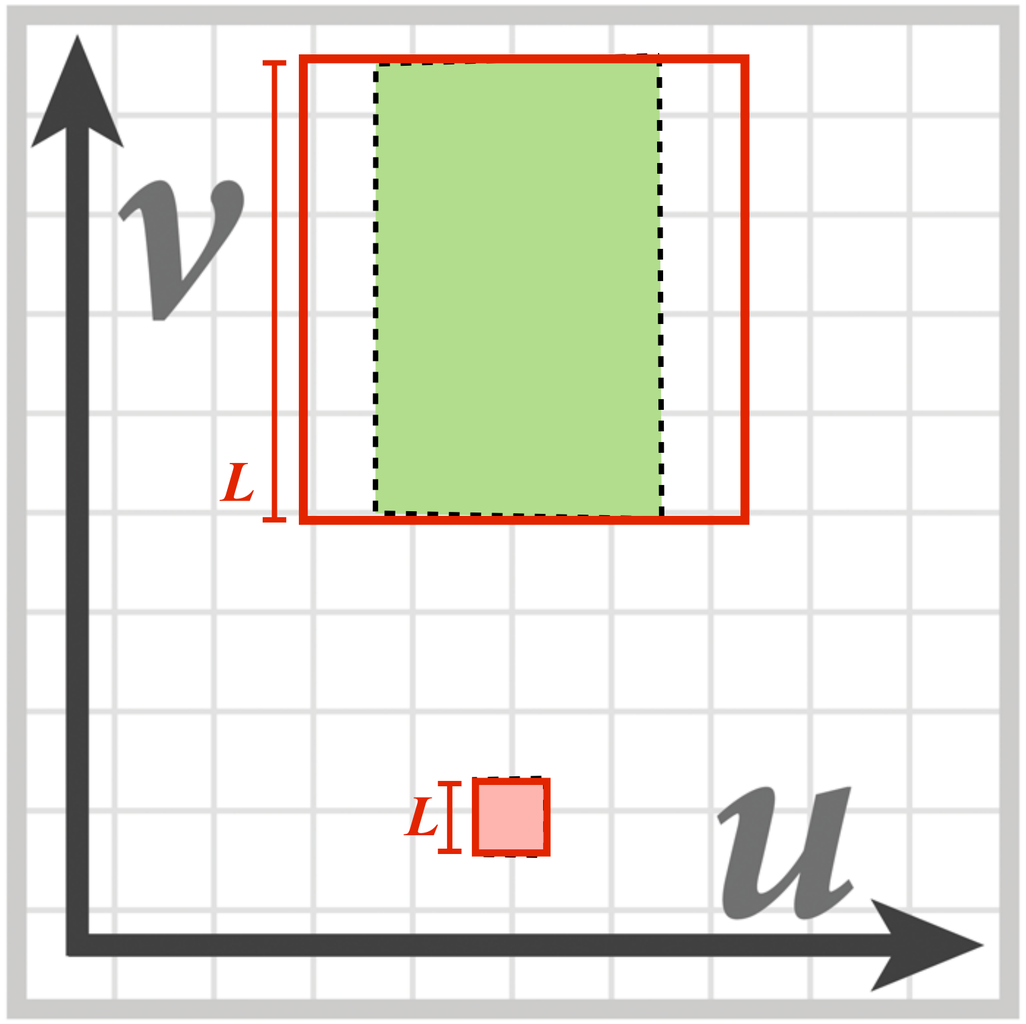
Texture sampling pattern not rectilinear or isotropic

Pixel area may not map to isotropic region in texture space

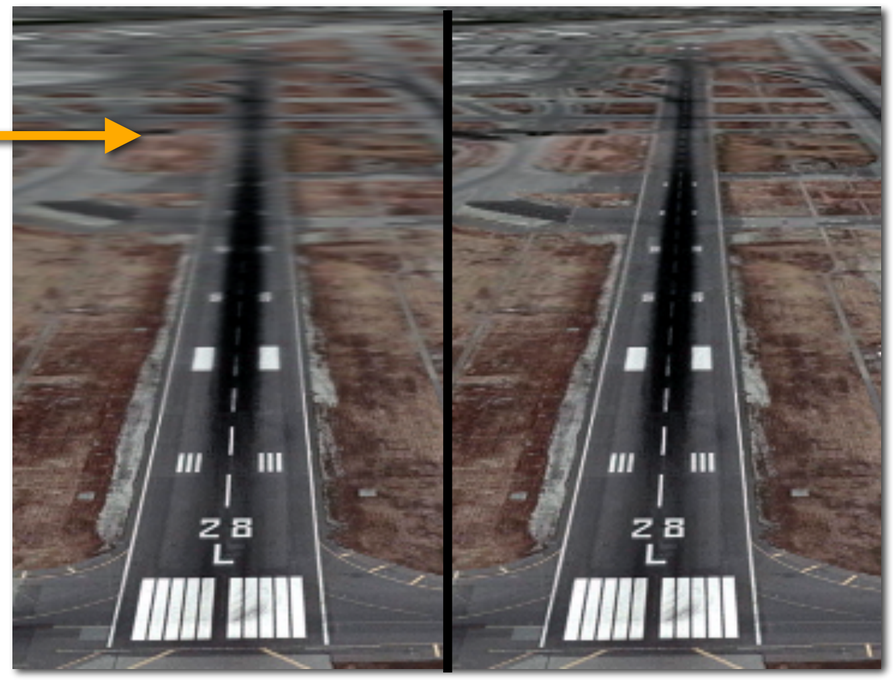
Proper filtering requires anisotropic filter footprint



Texture space: viewed from camera with perspective



Overblurring in *u* direction

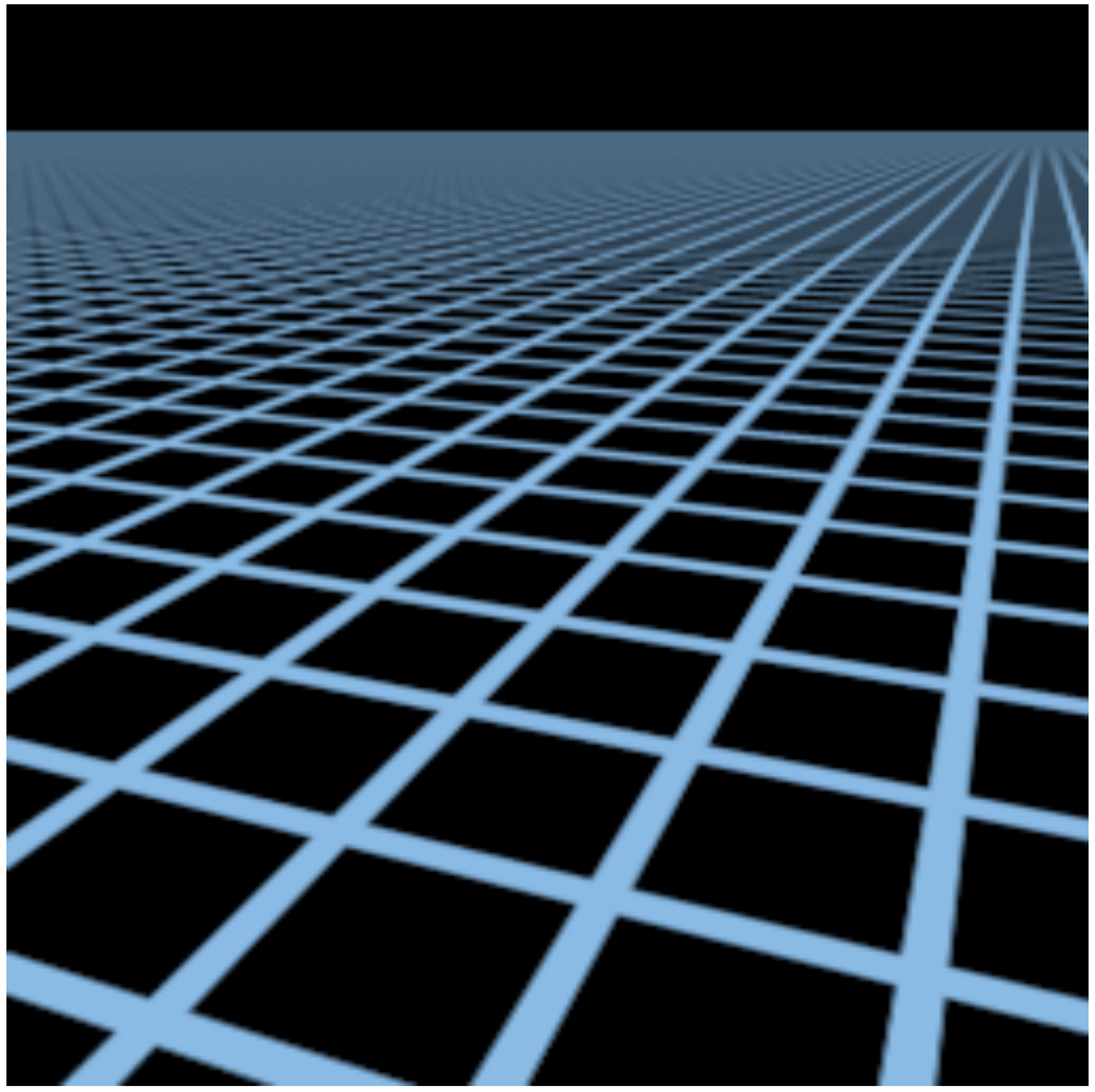


Trilinear (Isotropic) Filtering

Anisotropic Filtering

(Modern anisotropic texture filtering solutions combine multiple mip map samples to approximate integral of texture value over arbitrary texture space regions)

Anisotropic filtering



Elliptical weighted average (EWA) filtering
(uses multiple lookups into mip-map to approximate filter region)

Summary: texture filtering using the mip map

- **Small storage overhead (33%)**
  - Mipmap is 4/3 the size of original texture image

- **For each isotropically-filtered sampling operation**
  - Constant filtering cost (independent of mip map level)
  - Constant number of texels accessed (independent of mip map level)

- **Combat aliasing with *prefiltering*, rather than supersampling**
  - Recall: we used supersampling to address aliasing problem when sampling coverage

- **Bilinear/trilinear filtering is isotropic and thus will "overblur" to avoid aliasing**
  - Anisotropic texture filtering provides higher image quality at higher compute and memory bandwidth cost (in practice: multiple mip map samples)

A full texture sampling operation

1. Compute u and v from screen sample x,y (via evaluation of attribute equations)
2. Compute du/dx, du/dy, dv/dx, dv/dy differentials from screen-adjacent samples.
3. Compute mip map level *d*
4. Convert normalized [0,1] texture coordinate (u,v) to texture coordinates U,V in [W,H]
5. Compute required texels in window of filter
6. Load required texels from memory (need eight texels for trilinear)
7. Perform tri-linear interpolation according to (U, V, d)

Takeaway: a texture sampling operation is not just an image pixel lookup! It involves a significant amount of math.

For this reason, modern GPUs have dedicated fixed-function hardware support for performing texture sampling operations.

Summary: texture mapping

- Texturing: used to add visual detail to surfaces that is not captured in geometry
- Texture coordinates: define mapping between points on triangle's surface (object coordinate space) to points in texture coordinate space
- Texture mapping is a sampling operation and is prone to aliasing
  - Solution: precompute and store multiple multiple resampled versions of the texture image (each with different amounts of low-pass filtering to remove increasing amounts of high frequency detail)
  - During rendering: dynamically select how much low-pass filtering is required based on distance between neighboring screen samples in texture space
    - Goal is to retain as much high-frequency content (detail) in the texture as possible, while avoiding aliasing

Acknowledgements

- Thanks to Ren Ng, Pat Hanrahan, and Keenan Crane for slide materials