Lecture 9:
Accelerating Geometric Queries

Computer Graphics: Rendering, Geometry, and Image Manipulation
Stanford CS248A, Winter 2023
Last time: intersecting a ray with individual primitives

- **Ray-sphere**
- **Ray-plane**
- **Ray-triangle**
Applying what you learned

Consider interesting a ray with a cylinder with radius \( R \) and length \( L \)!
(centered at the origin)

I’ll give you: the implicit form of a circle in 2D

\[ x^2 + y^2 = R^2 \]

From last class you know:

Explicit form for a ray:
\[ \mathbf{r}(t) = \mathbf{o} + t\mathbf{d} \]

Implicit form for a plane:
\[ \mathbf{N}^T \mathbf{x} = c \]

Q. What if the cylinder is centered at \((x_o,y_o,z_o)\) instead of the origin?
Motivation (ray tracing)
Recall the problem of computing triangle visibility

Question 1: what samples does the triangle overlap? ("coverage")

Question 2: what triangle is closest to the camera in each sample? ("occlusion")
The visibility problem: as rasterization

- What scene geometry is visible at each screen sample?
  - What scene geometry projects onto screen sample points? (coverage)
  - Which geometry is visible from the camera at each sample? (occlusion)
The visibility problem: as ray casting

- In terms of casting rays from the camera:
  - Is a scene primitive hit by a ray originating from a point on the virtual sensor and traveling through the aperture of the pinhole camera? (coverage)
  - What primitive is the first hit along that ray? (occlusion)
In other words...

- Rasterization is an efficient implementation of ray casting where:
  - Ray-scene intersection is computed for a batch of rays
  - All rays in the batch originate from the same origin
  - Rays are distributed uniformly in the plane of projection
    (Note: not uniform distribution in angle... angle between rays is smaller away from the view direction)
Generality of ray-scene queries

What object is visible to the camera?
What light sources are visible from a point on a surface (is a surface in shadow?)
What reflection is visible on a surface?
Direct illumination + reflection + transparency

Image credit: Henrik Wann Jensen
“Global illumination” solution

Image credit: Henrik Wann Jensen
Direct illumination
Sixteen-bounce global illumination
Accelerating ray-scene queries
Ray-scene intersection

Given a scene defined by a set of $N$ primitives and a ray $r$, find the closest point of intersection of $r$ with the scene

“Find the first primitive the ray hits”

```
p_closest = NULL
t_closest = inf
for each primitive p in scene:
  t = p.intersect(r)
  if t >= 0 && t < t_closest:
    t_closest = t
    p_closest = p
```

Complexity? $O(N)$

**Can we do better?**

(Assume p.intersect(r) returns value of $t$ corresponding to the point of intersection with ray $r$)
One simple idea

- “Early out” — Skip ray-primitive test if it’s computationally easy to determine that ray does not intersect primitives

- E.g., A ray cannot intersect a primitive if it doesn’t intersect the bbox containing it!

Note: early out does not change asymptotic complexity of ray-scene intersection. But it reduces cost by a constant if ray is far from most triangles.
Ray-axis-aligned-box intersection

What is ray’s closest/farthest intersection with axis-aligned box?

Find intersection of ray with all planes of box:

$$N^T(o + td) = c$$

Math simplifies greatly since plane is axis aligned (consider \(x=x_0\) plane in 2D):

$$N^T = [1 \ 0]^T$$

$$c = x_0$$

$$t = \frac{x_0 - o_x}{d_x}$$

Performance note: it is possible to precompute box independent terms, so computing \(t\) is cheap

$$a = \frac{1}{d_x} \quad b = -\frac{o_x}{d_x}$$

So…

$$t = ax_0 + b$$
So how do we find the closest hit for a 3D box?

1. How do you know there is a hit at all?
2. What is the \( t \) value for that hit?

Figure shows intersections with \( x=x_0 \) and \( x=x_1 \) planes.
Ray-axis-aligned-box intersection

Compute intersections with all planes, take intersection of $t_{\text{min}}/t_{\text{max}}$ intervals

How do we know when the ray misses the box?
Ray-scene intersection with early out

Given a scene defined by a set of \( N \) primitives and a ray \( r \), find the closest point of intersection of \( r \) with the scene

\[
\begin{align*}
p_{\text{closest}} &= \text{NULL} \\
t_{\text{closest}} &= \text{inf} \\
\text{for each primitive } p \text{ in scene:} \\
&\quad \text{if } (!p.\text{bbox}.\text{intersect} (r)) \\
&\quad \quad \text{continue;} \\
&\quad t = p.\text{intersect} (r) \\
&\quad \text{if } t \geq 0 \&\& t < t_{\text{closest}}: \\
&\quad \quad t_{\text{closest}} = t \\
&\quad \quad p_{\text{closest}} = p
\end{align*}
\]

Still \( O(N) \) complexity.

(Assume \( p.\text{intersect}(r) \) returns value of \( t \) corresponding to the point of intersection with ray \( r \))
Disney Moana scene

Released for rendering research purposes in 2018.
15 billion primitives in scene (more than 90M unique geometric primitives)
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Released for rendering research purposes in 2018.
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Recall optimization in a simple rasterizer

All of your assignment 1 rasterizers skipped sample-in-triangle tests for samples not contained in the bounding box of the triangle. (Analogous to skipping ray-hits-3d triangle test if ray does not hit 3D bbox of a triangle)

```
initialize z_closest[] to INFINITY  // store closest-surface-so-far for all samples
initialize color[]                 // store scene color for all samples
for each triangle t in scene:     // loop 1: over triangles
    t_proj = project_triangle(t)
    for each 2D sample s in frame buffer:  // loop 2: over visibility samples
        if (t_proj covers s)
            compute color of triangle at sample
            if (depth of t at s is closer than z_closest[s])
                update z_closest[s] and color[s]
```

Cull samples not within bbox
(if sample not in bbox don’t attempt more expensive point in triangle test)
Data structures for reducing $O(N)$ complexity of ray-scene intersection

*Given ray, find closest intersection with set of scene triangles.*

*We are also interested in: Given ray, find if there is any intersection with scene triangles*
A simpler problem

Imagine I have a set of integers $S$

Given an integer, say $k=18$, find the element of $S$ closest to $k$:

$\begin{array}{cccccccccccc}
10 & 123 & 2 & 100 & 6 & 25 & 64 & 11 & 200 & 30 & 950 & 111 & 20 & 8 & 1 & 80
\end{array}$

What’s the cost of finding $k$ in terms of the size $N$ of the set?

Can we do better?

Suppose we first sort the integers:

$\begin{array}{cccccccccccc}
1 & 2 & 6 & 8 & 10 & 11 & 20 & 25 & 30 & 64 & 80 & 100 & 111 & 123 & 200 & 950
\end{array}$

How much does it now cost to find $k$ (including sorting)?

Cost for just ONE query: $O(n \log n)$  
Amortized cost over many queries: $O(\log n)$  

worse than before! :-(

much better!
Can we also reorganize scene primitives to enable fast ray-scene intersection queries?
Simple case (rays miss bounding box of scene)

Ray misses bounding box of all primitives in scene

Cost (misses box):
preprocessing: $O(n)$
ray-box test: $O(1)$
amortized cost*: $O(1)$

*amortized over many ray-scene intersection tests
Another (should be) simple case

Cost (hits box):  
preprocessing: $O(n)$  
ray-box test: $O(1)$  
triangle tests: $O(n)$  
amortized cost*: $O(n)$

Still no better than naïve algorithm  
(test all triangles)!

*amortized over many ray-scene intersection tests
Q: How can we do better?

A: Apply this strategy hierarchically
Bounding volume hierarchy (BVH)
Bounding volume hierarchy (BVH)

- BVH partitions each node’s primitives into disjoint sets
  - Note: the sets can overlap in space (see example below)
Bounding volume hierarchy (BVH)
Bounding volume hierarchy (BVH)

- **Leaf nodes:**
  - Contain *small* list of primitives

- **Interior nodes:**
  - Proxy for a *large* subset of primitives
  - Stores bounding box for all primitives in subtree
Bounding volume hierarchy (BVH)

Two different BVH organizations of the same scene containing 22 primitives. Is one BVH better than the other?
Ray-scene intersection using a BVH

```c
struct BVHNode {
    bool leaf;  // true if node is a leaf
    BBox bbox;  // min/max coords of enclosed primitives
    BVHNode* child1;  // “left” child (could be NULL)
    BVHNode* child2;  // “right” child (could be NULL)
    Primitive* primList;  // for leaves, stores primitives
};

struct HitInfo {
    Primitive* prim;  // which primitive did the ray hit?
    float t;  // at what t value along ray?
};

void find_closest_hit(Ray* ray, BVHNode* node, HitInfo* closest) {
    HitInfo hit = intersect(ray, node->bbox);  // test ray against node’s bounding box
    if (hit.t > closest.t)  // don’t update the hit record
        return;
    if (node->leaf) {
        for (each primitive p in node->primList) {
            hit = intersect(ray, p);
            if (hit.prim != NULL && hit.t < closest.t) {
                closest.prim = p;
                closest.t = t;
            }
        }
    } else {
        find_closest_hit(ray, node->child1, closest);
        find_closest_hit(ray, node->child2, closest);
    }
}
```

Can this occur if ray hits the box?
(assume hit.t is INF if ray misses box)
Improvement: “front-to-back” traversal

New invariant compared to last slide:
assume `find_closest_hit()` is only called for nodes where ray intersects bbox.

```c
void find_closest_hit(Ray* ray, BVHNode* node, HitInfo* closest) {
    if (node->leaf) {
        for (each primitive p in node->primList) {
            HitInfo hit = intersect(ray, p);
            if (hit.prim != NULL && t < closest.t) {
                closest.prim = p;
                closest.t = t;
            }
        }
    } else {
        HitInfo hit1 = intersect(ray, node->child1->bbox);
        HitInfo hit2 = intersect(ray, node->child2->bbox);
        NVHNode* first = (hit1.t <= hit2.t) ? child1 : child2;
        NVHNode* second = (hit1.t <= hit2.t) ? child2 : child1;
        find_closest_hit(ray, first, closest);
        if (second child’s t is closer than closest.t)
            find_closest_hit(ray, second, closest);
    }
}
```

“Front to back” traversal.

Traverse to closest child node first. Why?

Why might we still need to traverse to second child if there was a hit with geometry in the first child?
Aside: another type of query: any hit

Sometimes it is useful to know if the ray hits ANY primitive in the scene at all (don’t care about distance to first hit)

```cpp
bool find_any_hit(Ray* ray, BVHNode* node) {

    if (!intersect(ray, node->bbox))
        return false;

    if (node->leaf) {
        for (each primitive p in node->primList) {
            hit = intersect(ray, p);
            if (hit.prim)
                return true;
        } else {
            return (find_closest_hit(ray, node->child1, closest) ||
                    find_closest_hit(ray, node->child2, closest));
        }
    }
}
```

Interesting question of which child to enter first. How might you make a good decision?
Why “any hit” queries?

Shadow computations!
For a given set of primitives, there are many possible BVHs
(\(\sim 2^N\) ways to partition \(N\) primitives into two groups)

Q: How do we build a high-quality BVH?
How would you partition these triangles into two groups?
What about these?
Intuition about a “good” partition?

Partition into child nodes with equal numbers of primitives

Better partition
Intuition: want small bounding boxes that minimize overlap between children, avoid bboxes with significant empty space
What are we really trying to do?

A good partitioning minimizes the expected cost of finding the closest intersection of a ray with the scene primitives in the node.

If a node is a leaf node (no partitioning):

$$C = \sum_{i=1}^{N} C_{\text{isect}}(i)$$  

Where $C_{\text{isect}}(i)$ is the cost of ray-primitive intersection for primitive $i$ in the node.

$$= NC_{\text{isect}}$$

(Common to assume all primitives have the same cost)
Cost of making a partition

The expected cost of ray-node intersection, given that the node’s primitives are partitioned into child sets A and B is:

\[ C = C_{\text{trav}} + p_A C_A + p_B C_B \]

\( C_{\text{trav}} \) is the cost of traversing an interior node (e.g., load data + bbox intersection check)

\( C_A \) and \( C_B \) are the costs of intersection with the resultant child subtrees

\( p_A \) and \( p_B \) are the probability a ray intersects the bbox of the child nodes A and B

Primitive count is common approximation for child node costs:

\[ C = C_{\text{trav}} + p_A N_A C_{\text{isect}} + p_B N_B C_{\text{isect}} \]

Remaining question: how do we get the probabilities \( p_A, p_B \)?
Estimating probabilities

For convex object $A$ inside convex object $B$, the probability that a random ray that hits $B$ also hits $A$ is given by the ratio of the surface areas $S_A$ and $S_B$ of these objects.

$$P(\text{hit } A | \text{hit } B) = \frac{S_A}{S_B}$$

Leads to surface area heuristic (SAH):

$$C = C_{\text{trav}} + \frac{S_A}{S_N} N_A C_{\text{isect}} + \frac{S_B}{S_N} N_B C_{\text{isect}}$$

Assumptions of the SAH (which may not hold in practice!):

- Rays are randomly distributed
- Rays are not occluded
Implementing partitions

- Constrain search for good partitions to axis-aligned spatial partitions
  - Choose an axis; choose a split plane on that axis
  - Partition primitives by the side of splitting plane their centroid lies
  - SAH changes only when split plane moves past triangle boundary
  - Have to consider large number of possible split planes... $O(# \text{ objects})$
Efficiently implementing partitioning

Efficient modern approximation: split spatial extent of primitives into B buckets (B is typically small: $B < 32$)

For each axis: $x, y, z$:
- initialize bucket counts to 0, per-bucket bboxes to empty
  - For each primitive $p$ in node:
    - $b = \text{compute\_bucket}(p.\text{centroid})$
    - $b.\text{bbox}.\text{union}(p.\text{bbox})$
    - $b.\text{prim\_count}++$
  - For each of the $B-1$ possible partitioning planes evaluate SAH
- Use lowest cost partition found (or make node a leaf)
Troublesome cases

All primitives with same centroid (all primitives end up in same partition)

All primitives with same bbox (ray often ends up visiting both partitions)

In general, different strategies may work better for different types of geometry / different distributions of primitives...
Question

- Imagine you have a valid BVH
- Now I move one of the triangles in the scene to a new location
- How do I “refit” the BVH so it is a valid BVH?

Imagine I moved a triangle in this red leaf node.
Primitive-partitioning acceleration structures vs. space-partitioning structures

- Primitive partitioning (e.g., bounding volume hierarchy): partitions primitives into disjoint sets (but sets of primitives may overlap in space)

- Space-partitioning (e.g., grid, K-D tree) partitions space into disjoint regions (primitives may be contained in multiple regions of space)
K-D tree

- Recursively partition space via axis-aligned partitioning planes
  - Interior nodes correspond to spatial splits
  - Node traversal can proceed in strict front-to-back order
  - Unlike BVH, can terminate search after first hit is found.
Challenge: objects overlap multiple tree nodes

Want node traversal to proceed in front-to-back order so traversal can terminate search after first hit found.

Triangle 1 overlaps multiple nodes.
Ray hits triangle 1 when in highlighted leaf cell.
But intersection with triangle 2 is closer!
(Haven’t traversed to that node yet)

Solution: require primitive intersection point to be within current leaf node.

(primitives may be intersected multiple times by same ray *)

* Caching hit info (“mailboxing”) can be used to avoid repeated intersections.
Uniform grid (a very simple hierarchy)
Uniform grid

- Partition space into equal sized volumes (volume-elements or “voxels”)
- Each grid cell contains primitives that overlap the voxel.
  - Cheap to construct acceleration structure
- Walk ray through volume in order
  - Efficient implementation possible (think: 3D line rasterization)
  - Only consider intersection with primitives in voxels the ray intersects
Consider tiled triangle rasterization

initialize \( z_{\text{closest}}[] \) to INFINITY // store closest-surface-so-far for all samples
initialize color[] // store scene color for all samples

for each triangle \( t \) in scene: // loop 1: triangles
  \( t_{\text{proj}} = \text{project}\_\text{triangle}(t) \)
  for each 2D tile of screen samples touching bbox of triangle: // loop 2: tiles
    if (triangle does not overlap tile)
      continue;
    for each 2D sample \( s \) in tile: // loop 3: visibility samples
      if (\( t_{\text{proj}} \) covers \( s \))
        compute color of triangle at sample
        if (depth of \( t \) at \( s \) is closer than \( z_{\text{closest}}[s] \))
          update \( z_{\text{closest}}[s] \) and color[s]

For each TILE of image
  If triangle overlaps tile, check all samples in tile

What does this strategy remind you of? :-)

Sample = 2D point

Coverage: 2D triangle/sample tests (does projected triangle cover 2D sample point)

Occlusion: depth buffer
What should the grid resolution be?

Too few grids cell: degenerates to brute-force approach

Too many grid cells: incur significant cost traversing through cells with empty space
Grid size heuristic

Choose number of cells $\sim$ total number of primitives
(yields constant prims per cell for any scene size — assuming uniform distribution of primitives)

Intersection cost: $O\left(\sqrt[3]{N}\right)$
(assuming 3D grid)

(Q: Which grows faster, cube root of N or log(N)?)
When uniform grids work well: uniform distribution of primitives in scene

Terrain / height fields:
[Image credit: Misuba Renderer]

Field of grass:
[Image credit: www.kevinboulanger.net/grass.html]
Uniform grids cannot adapt to non-uniform distribution of geometry in scene

(Unlike K-D tree, location of spatial partitions is not dependent on scene geometry)

"Teapot in a stadium problem"

Scene has large spatial extent.
Contains a high-resolution object that has small spatial extent (ends up in one grid cell)
When uniform grids do not work well:
non-uniform distribution of geometric detail
When uniform grids do not work well:
non-uniform distribution of geometric detail
Quad-tree / octree

Quad-tree: nodes have 4 children (partitions 2D space)

Octree: nodes have 8 children (partitions 3D space)

Like uniform grid: easy to build (don’t have to choose partition planes)

Has greater ability to adapt to location of scene geometry than uniform grid.

But lower intersection performance than K-D tree (the structure only has limited ability to adapt to distribution of scene geometry)
Understanding BVH Performance
Recall: Moana scene
Moana costs

Number of nodes visited

Num ray-triangle tests
Another example
BVH # Nodes Visited

- 24
- 77
BVH # Ray-Tri Tests
Another example: diagonal geometry

Number of nodes visited

Num ray-triangle tests

??
Axis-alignment and performance

Wall and its bounding box

Rotated wall and its bounding box
Original scene

Rendering time: 27m 38s
Same scene transformed (rotated in world space)

Rendering time: 1h 55m 45s
Axis-alignment and performance

Rotated wall and its bounding box

Work-around: refine bounding boxes

Note: this introduces back the idea of partitioning space!
(Recall octree, KD-tree)
Summary of spatial acceleration structures: 

*Choose the right structure for the job!*

- **Primitive vs. spatial partitioning:**
  - Primitive partitioning: partition sets of objects
    - Bounded number of BVH nodes, *simpler to update if primitives in scene change position*
  - Spatial partitioning: partition space into non-overlapping regions
    - Traverse space in order (first intersection is closest intersection), may intersect primitive multiple times

- **Adaptive structures (BVH, K-D tree)**
  - More costly to construct (must be able to amortize cost over many geometric queries)
  - Better intersection performance under non-uniform distribution of primitives

- **Non-adaptive accelerations structures (uniform grids)**
  - Simple, cheap to construct
  - Good intersection performance if scene primitives are uniformly distributed

- Many, many combinations thereof...
Bonus material:
A few words on fast ray tracing
This image was rendered in real-time on a single high-end GPU

Microsoft’s DirectX Ray Tracing support / NVIDIA’s DXR announced in April 2018
Real time ray tracing

Image credit: Unreal Engine 4
Hardware support for ray tracing

- Accelerate ray tracing by building hardware to perform operations like ray-triangle intersection and ray-BVH intersection

- Long academic history of papers...

- 2018: NVIDIA's RTX GPUs — 10B rays/sec
Ray tracing dynamic scenes

- Challenge #1: scenes have millions of triangles, many objects are in motion
- Challenge #2: relatively few rays traced per frame

- For real time, can allow a few ms / frame for BVH build
  - e.g. @10M tris, 60fps, need 600M tris / second

⇒ Hierarchy construction efficiency really matters!
A BVH is an intersectable primitive

- It has a bounding box
- It supports ray-primitive intersection
- So it can be used as a primitive in another BVH.
Two-level acceleration structures

- 2-level hierarchy

Top-level acceleration structure

Bottom-level acceleration Structures (Primitives in top-level BVH)
Hierarchical BVH build

At scene load...
Build one BVH for each object
Each frame...
Build top-level BVH of BVH’s based on current object positions.

(Scene may contain millions of triangles, but only hundreds of objects.)
Acknowledgements

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