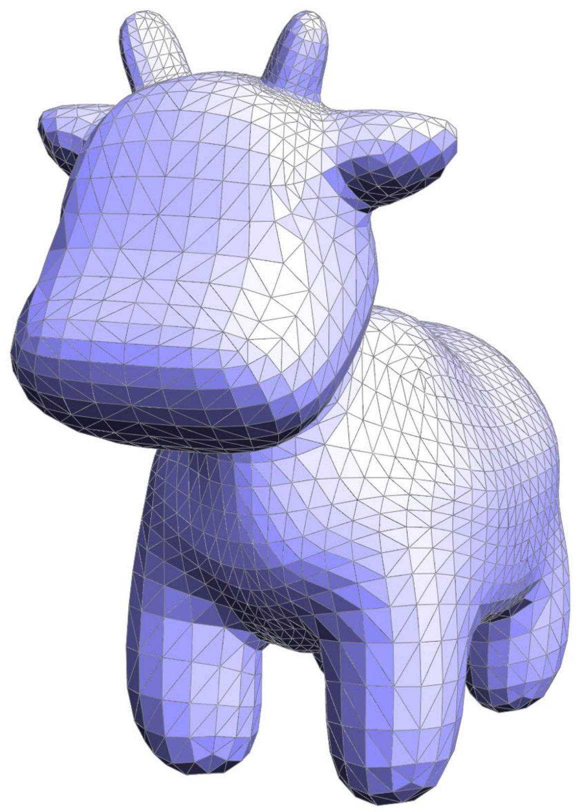
Lecture 10:

Radiometry, BRDFs, and the Reflection Equation

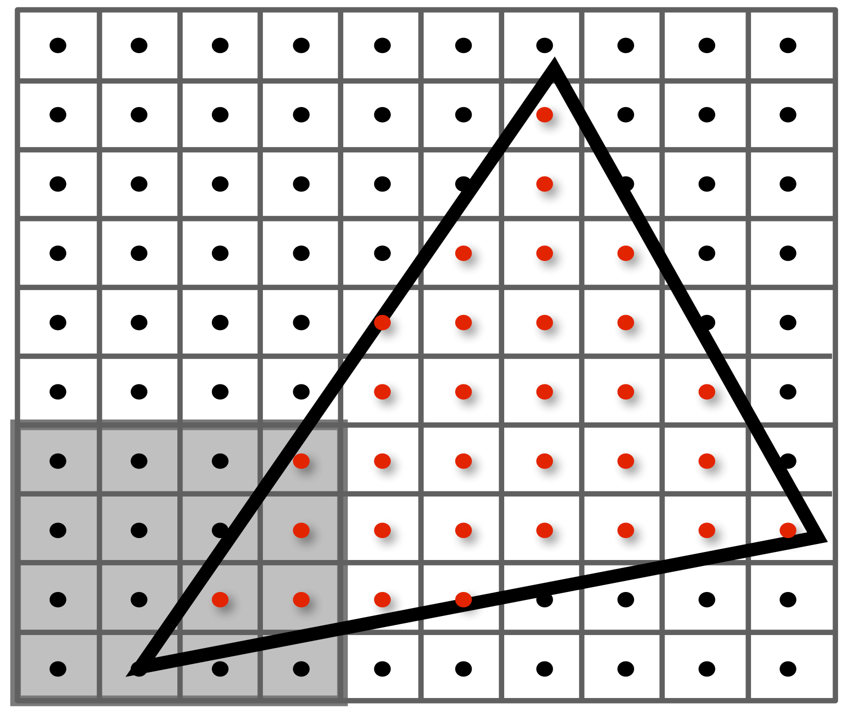
Interactive Computer Graphics
Stanford CS248A, Winter 2023

Things you know so far!

- Representing geometry
  - As triangle meshes, subdivision surfaces, implicit surfaces, etc.



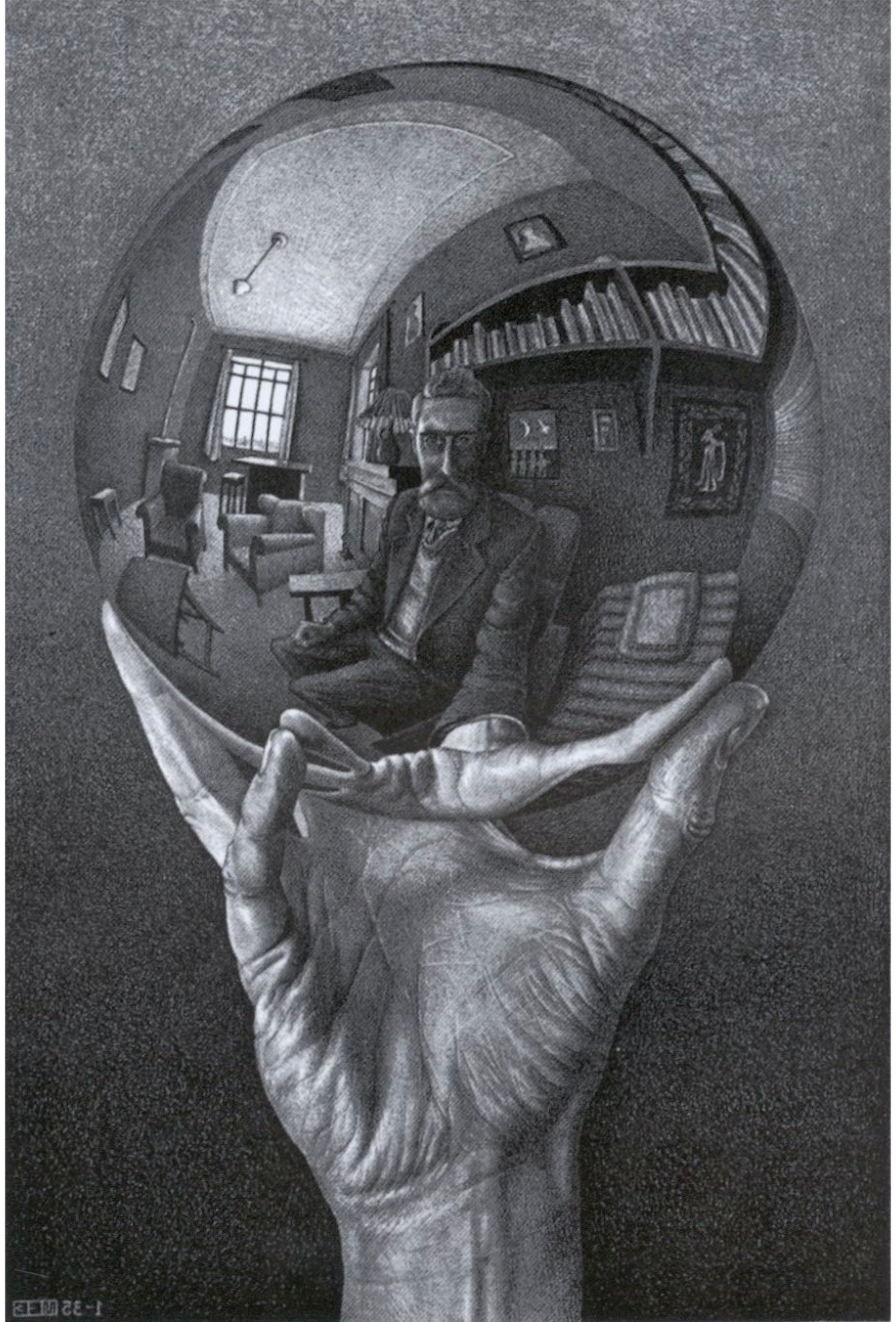
- Visibility and occlusion
  - Rasterization: determining what point on what triangle covers a sample
  - Ray tracing: determining what triangle a ray hits



- Today: basics of lights and materials
  - Computing the "appearance" of the surface at a point

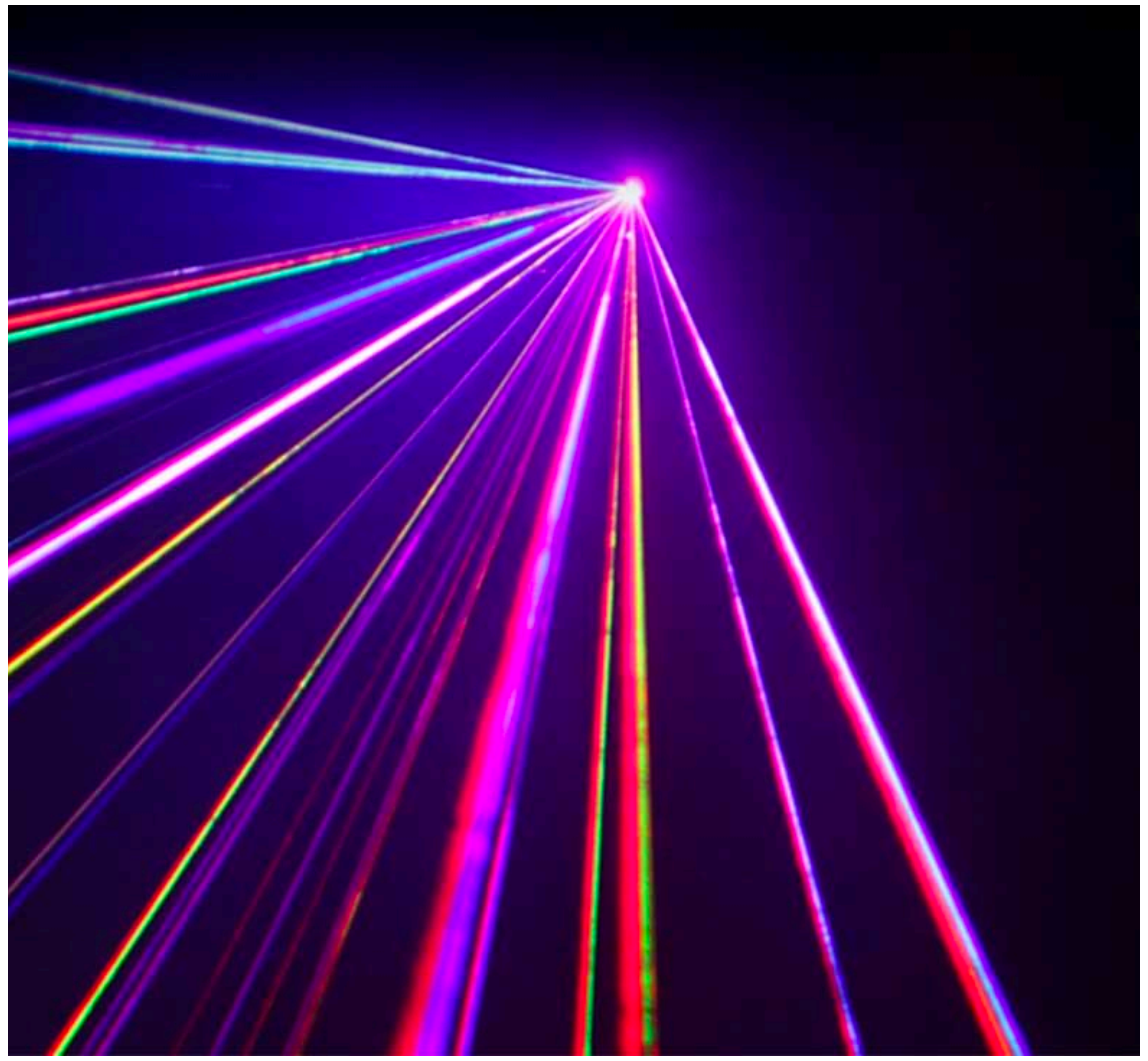
“Shading” in drawing

- Depicting the appearance of the surface
- Due to factors like surface material, lighting conditions



MC Escher pencil sketch

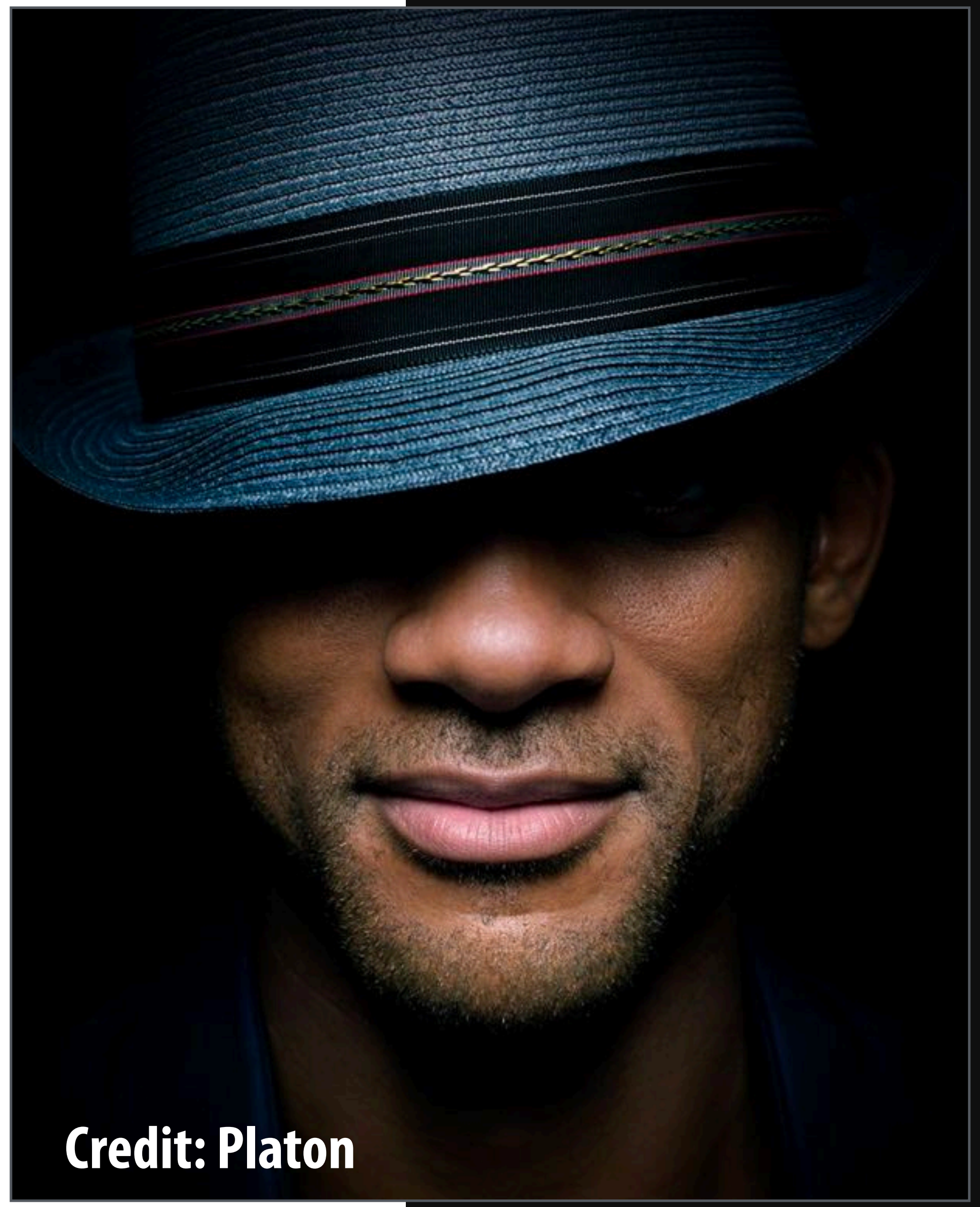
Lighting

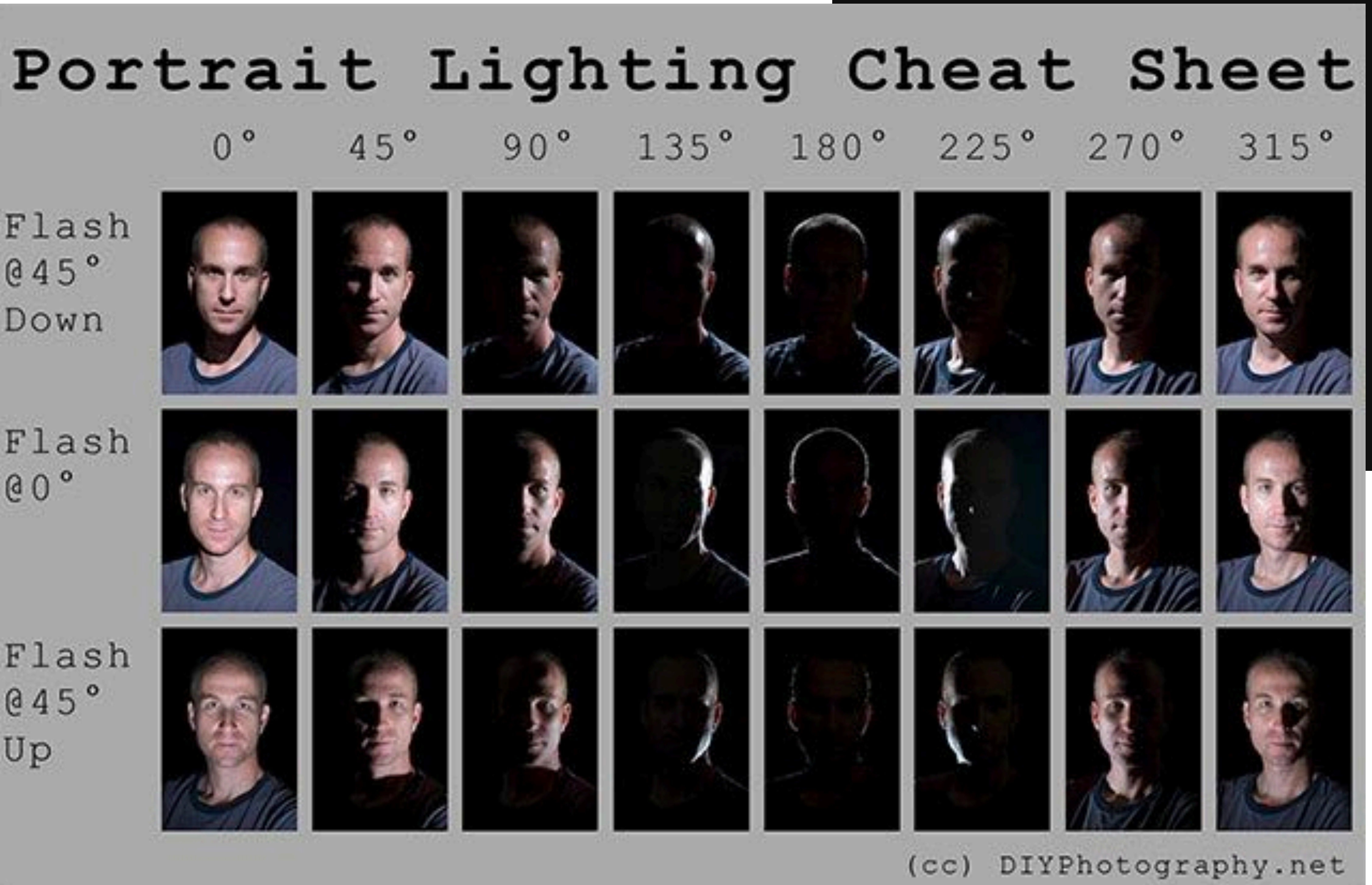


Lighting

Credit: Wikipedia
(Nasir ol Molk Mosque)

Lighting


Credit: Platon


Portrait Lighting Cheat Sheet
0° 45° 90° 135° 180° 225° 270° 315°
Flash @45° Down
Flash @0°
Flash @45° Up
(cc) DIYPhotography.net

Materials: diffuse

Materials: plastic

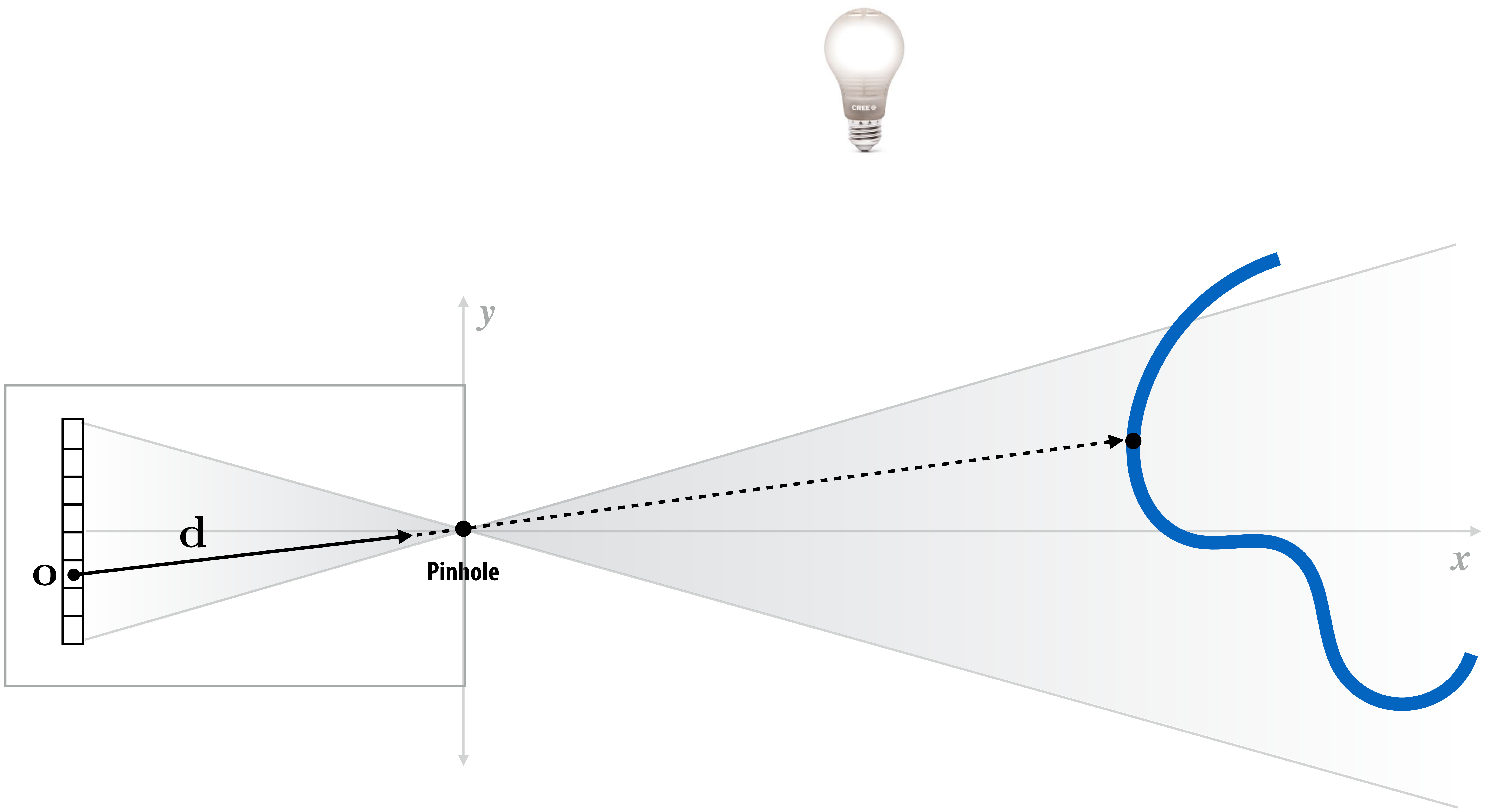
Materials: red semi-gloss paint

Materials: Ford mystic lacquer paint

Materials: mirror

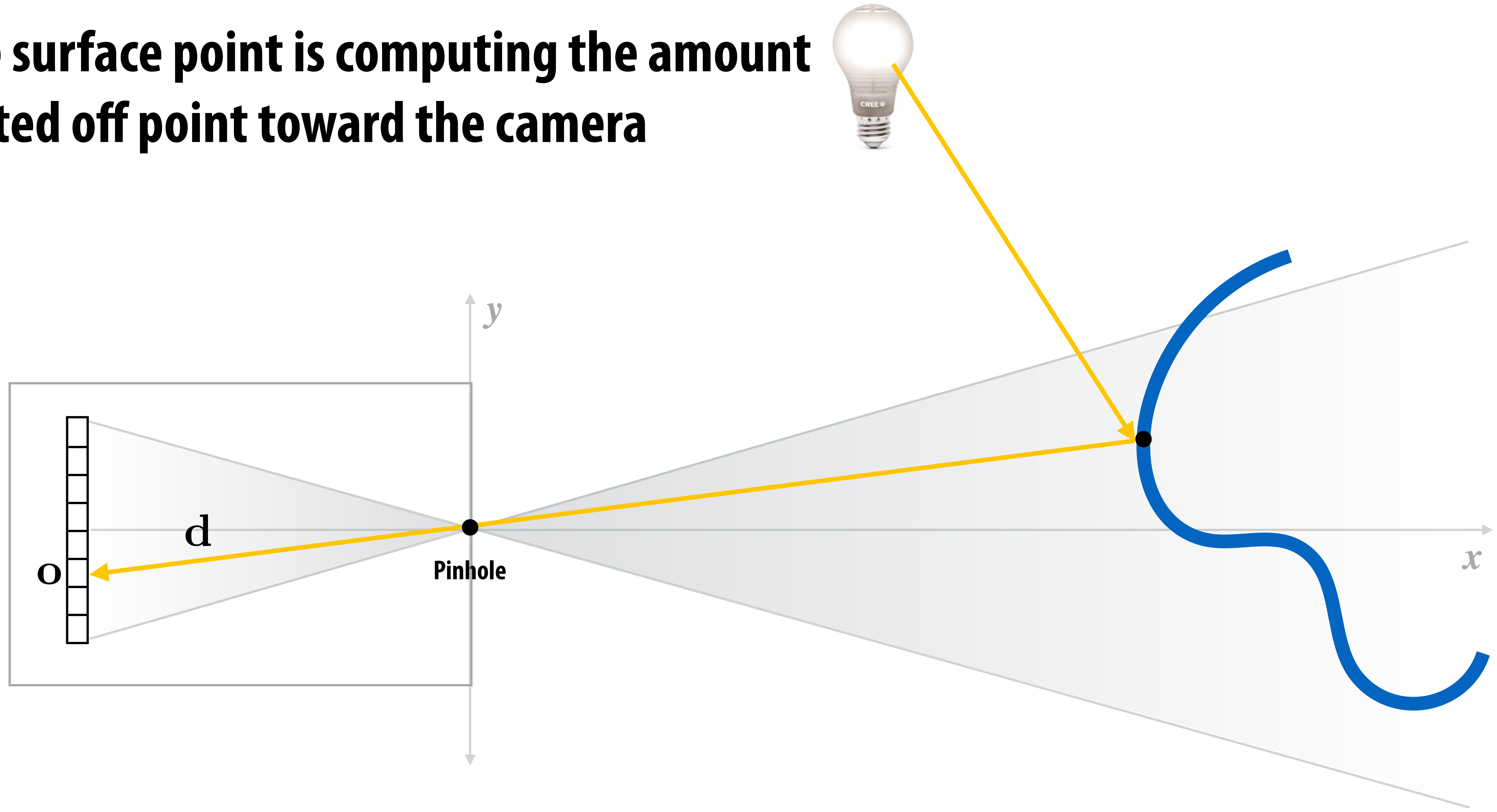
Materials: gold

A renderer measures light energy along a ray

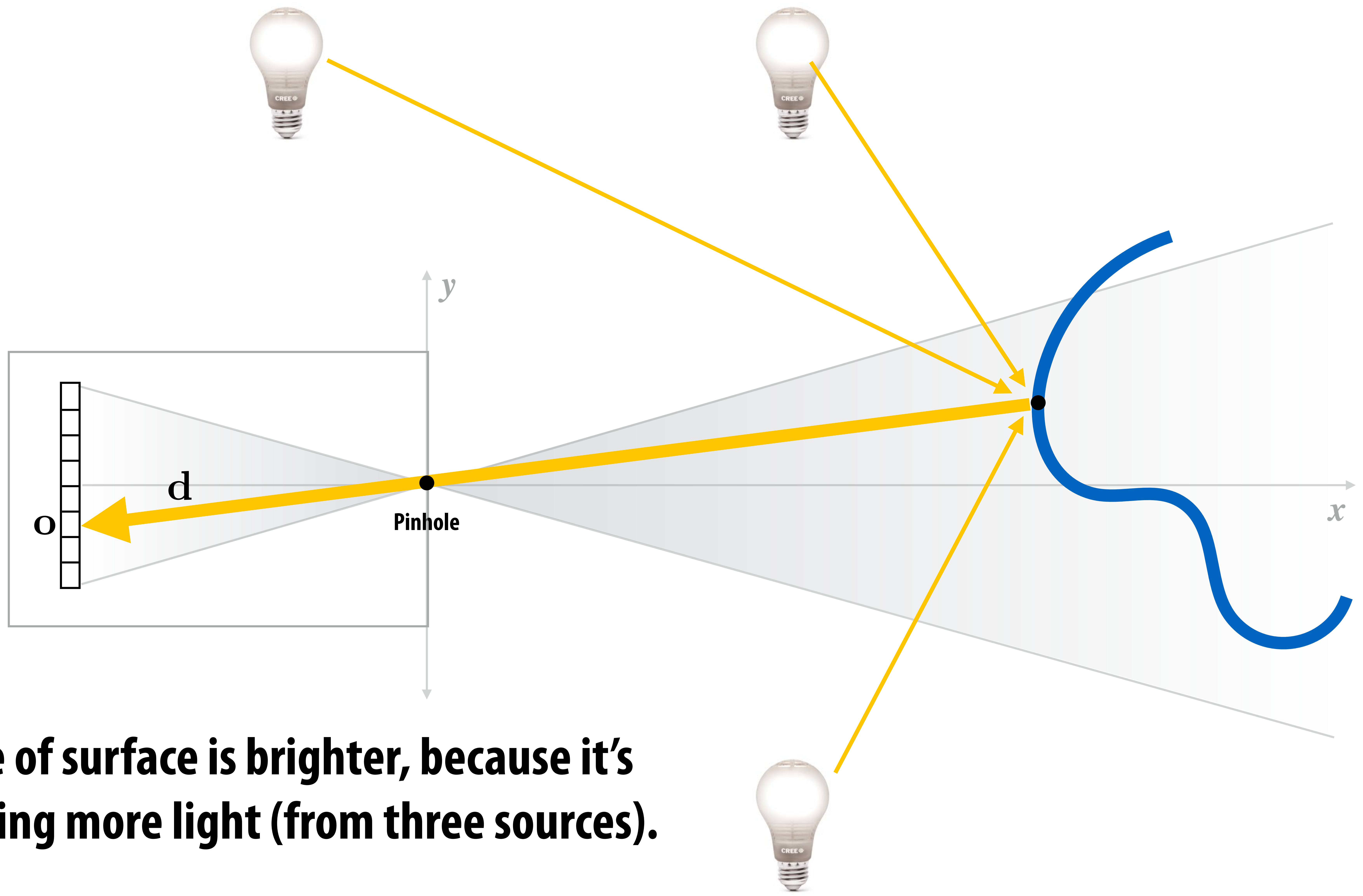


Renderer measures light energy along a ray

"Shading" the surface point is computing the amount of light reflected off point toward the camera



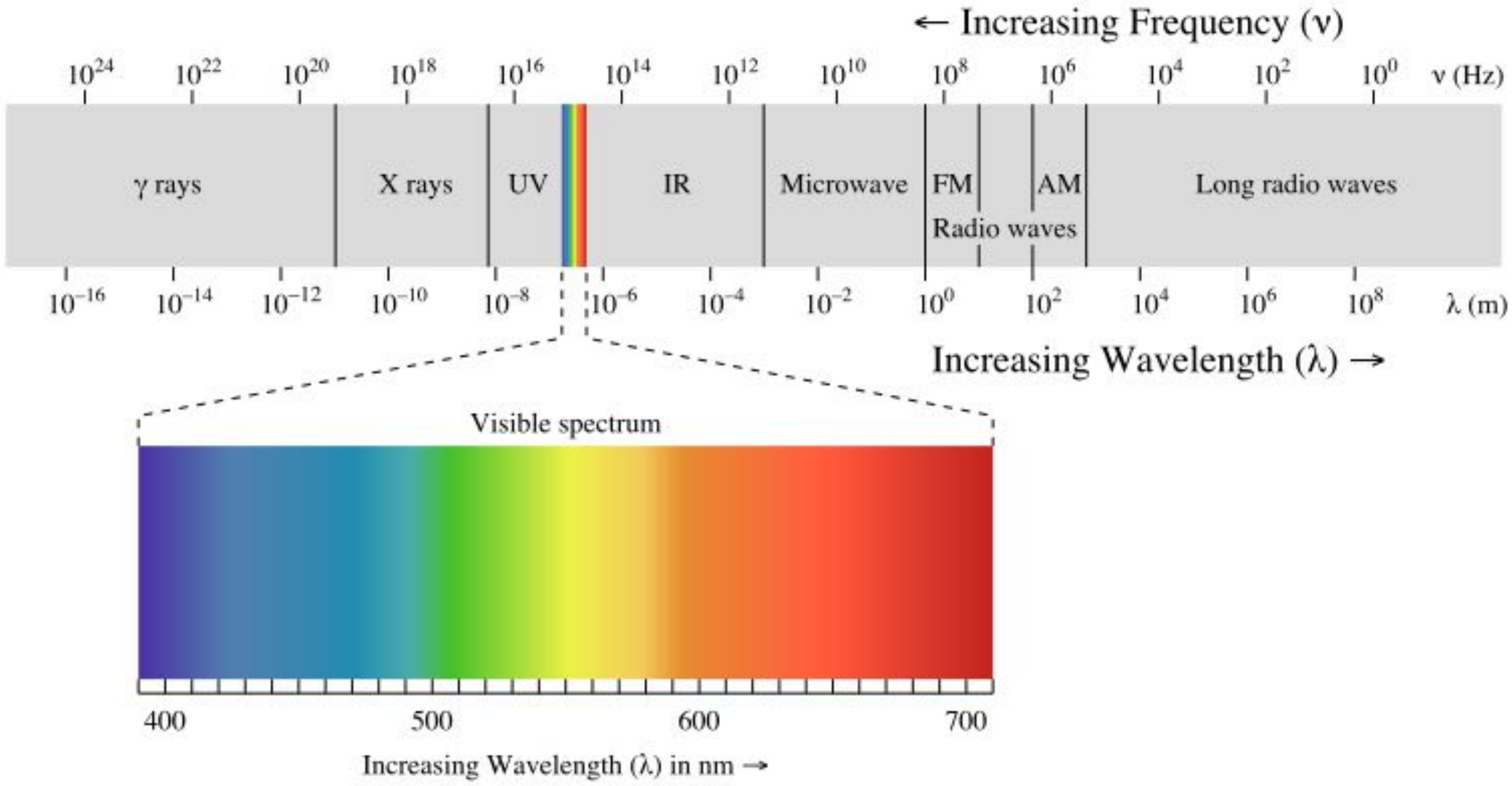
Multiple light sources



Appearance of surface is brighter, because it's now reflecting more light (from three sources).

What is light?

Light is electromagnetic radiation that is visible to the eye



What do lights do?

Cree 11 W LED light bulb
("60 Watt" incandescent replacement)

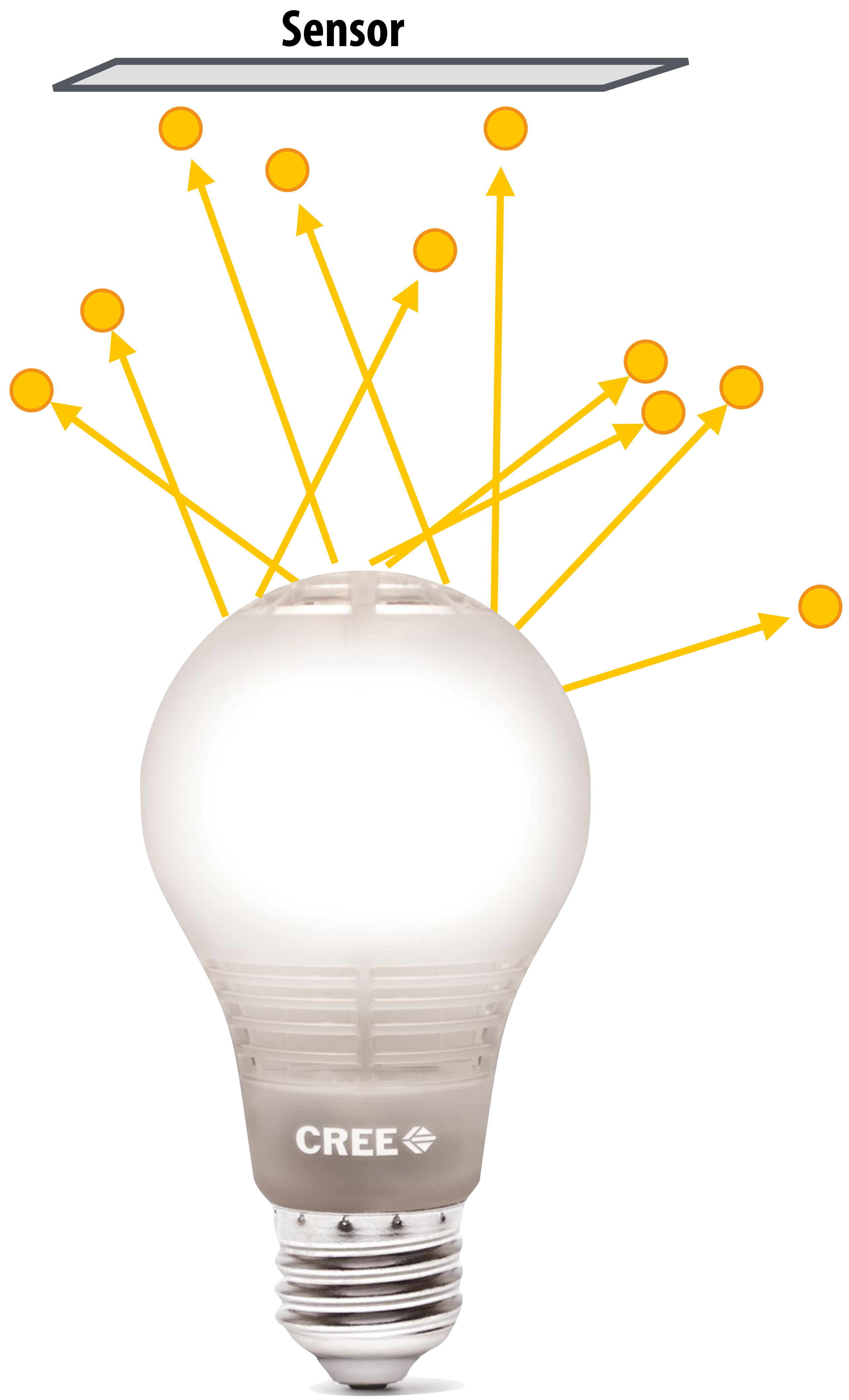
- Physical process converts input energy into photons
  - Each photon carries a small amount of energy
- Over some amount of time, light fixture consumes some amount of energy, Joules
  - Some input energy is turned into heat, some into photons
- Energy of photons hitting an object ~ exposure
  - Film, sensors, sunburn, solar panels, …
- In graphics we generally assume "*steady state*" process
  - Rate of energy consumption = power, Watts (Joules/second)

Measuring illumination: radiant flux (power)

- Given a sensor, we can count how many photons reach it
  - Over a period of time, gives the power received by the sensor
- Given a light, consider counting the number of photons emitted by it
  - Over a period of time, gives the power emitted by the light
- Energy carried by a photon:

$$Q = \frac{hc}{\lambda}$$

$$h \approx 6.626 \times 10^{-34}$$



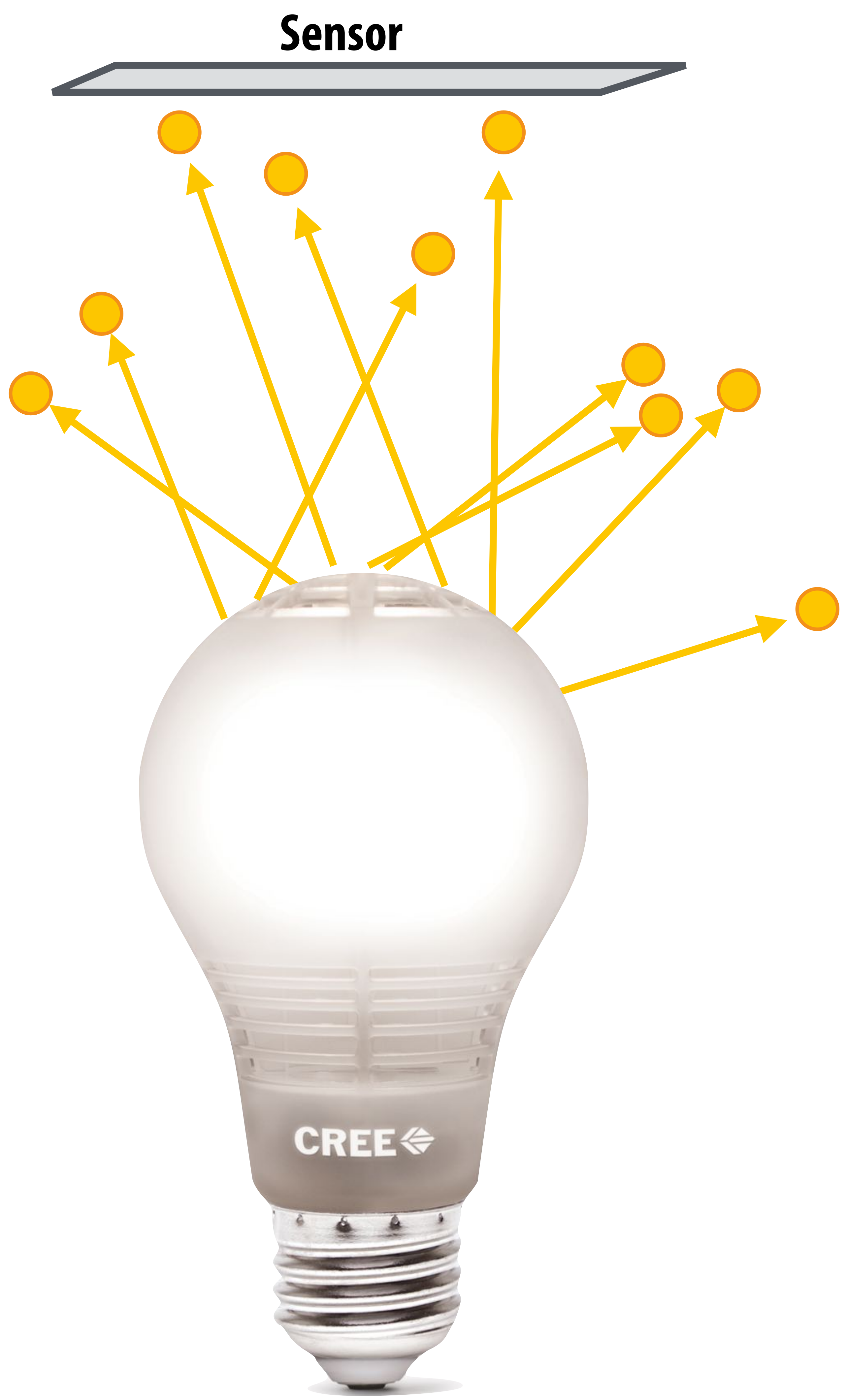
Measuring illumination: radiant flux (power)

- **Flux: energy per unit time (Watts) received by the sensor (or emitted by the light)**

$$\Phi = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t} \quad \left[\frac{\mathrm{J}}{\mathrm{s}}\right]$$

- **Time integral of flux is total radiant energy**

$$Q = \int_{t_0}^{t_1} \Phi(t)\, \mathrm{d}t$$



Spectral power distribution

Describes distribution of energy by wavelength

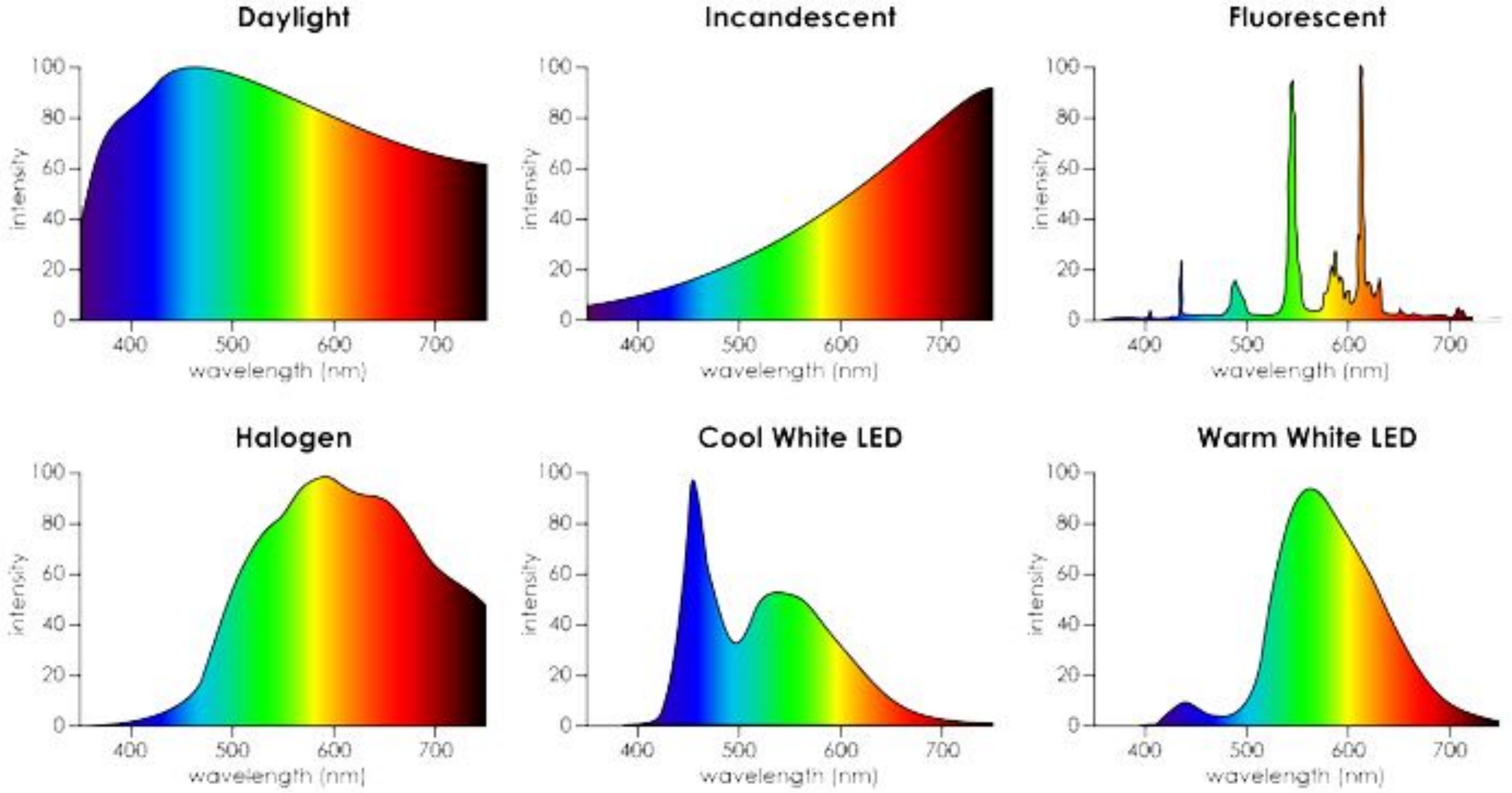


Figure credit:

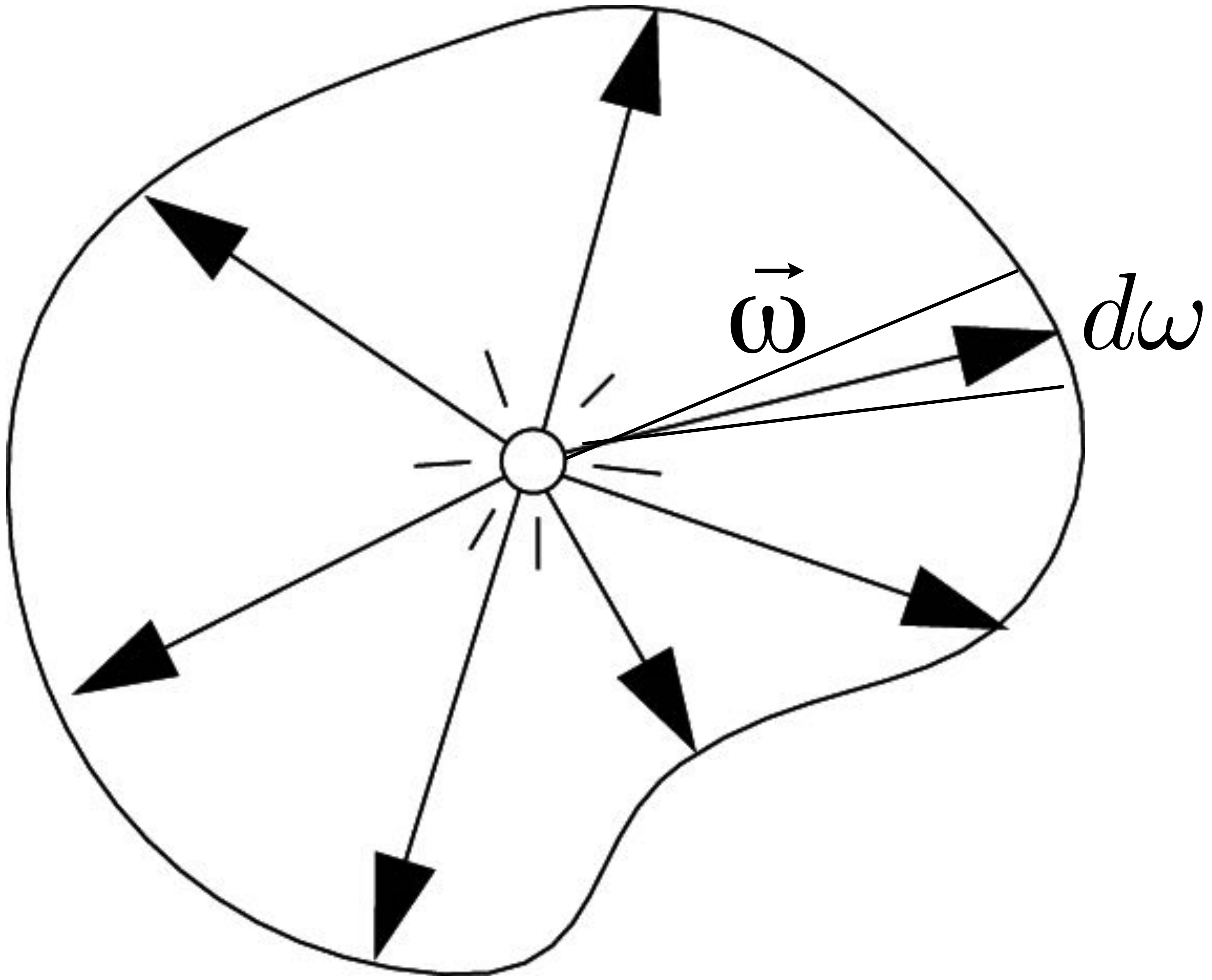
“Warm” vs. “cool” white light LED

Credit: https://www.ledholidaylighting.com/LED-faq.aspx

Radiant intensity

The *radiant intensity* is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$



$$\left[\frac{W}{sr}\right]$$

Units = Watts per steradian

Angles and solid angles

Angle

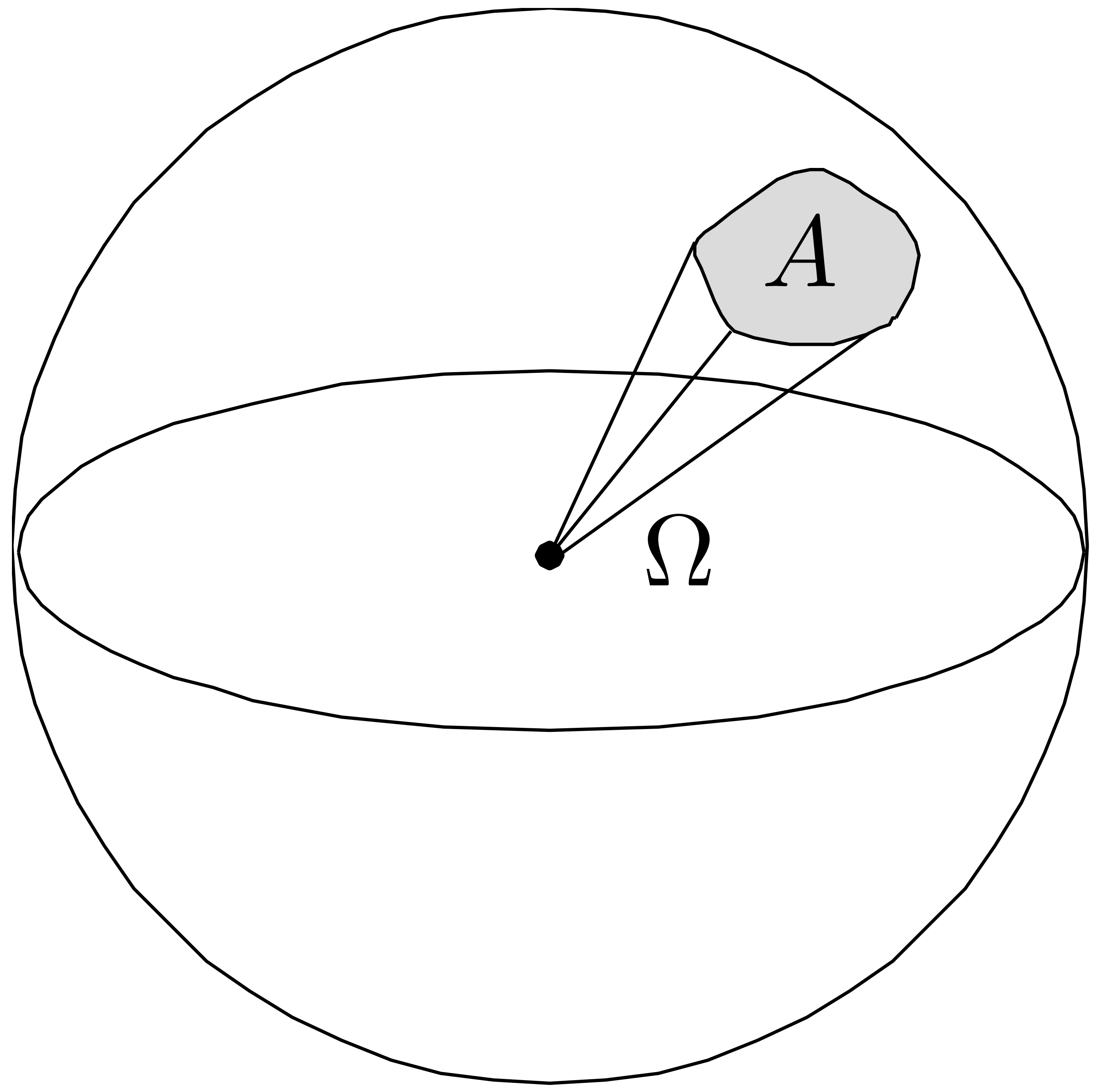
$$\theta = \frac{l}{r}$$

⇒ circle has 2 π radians

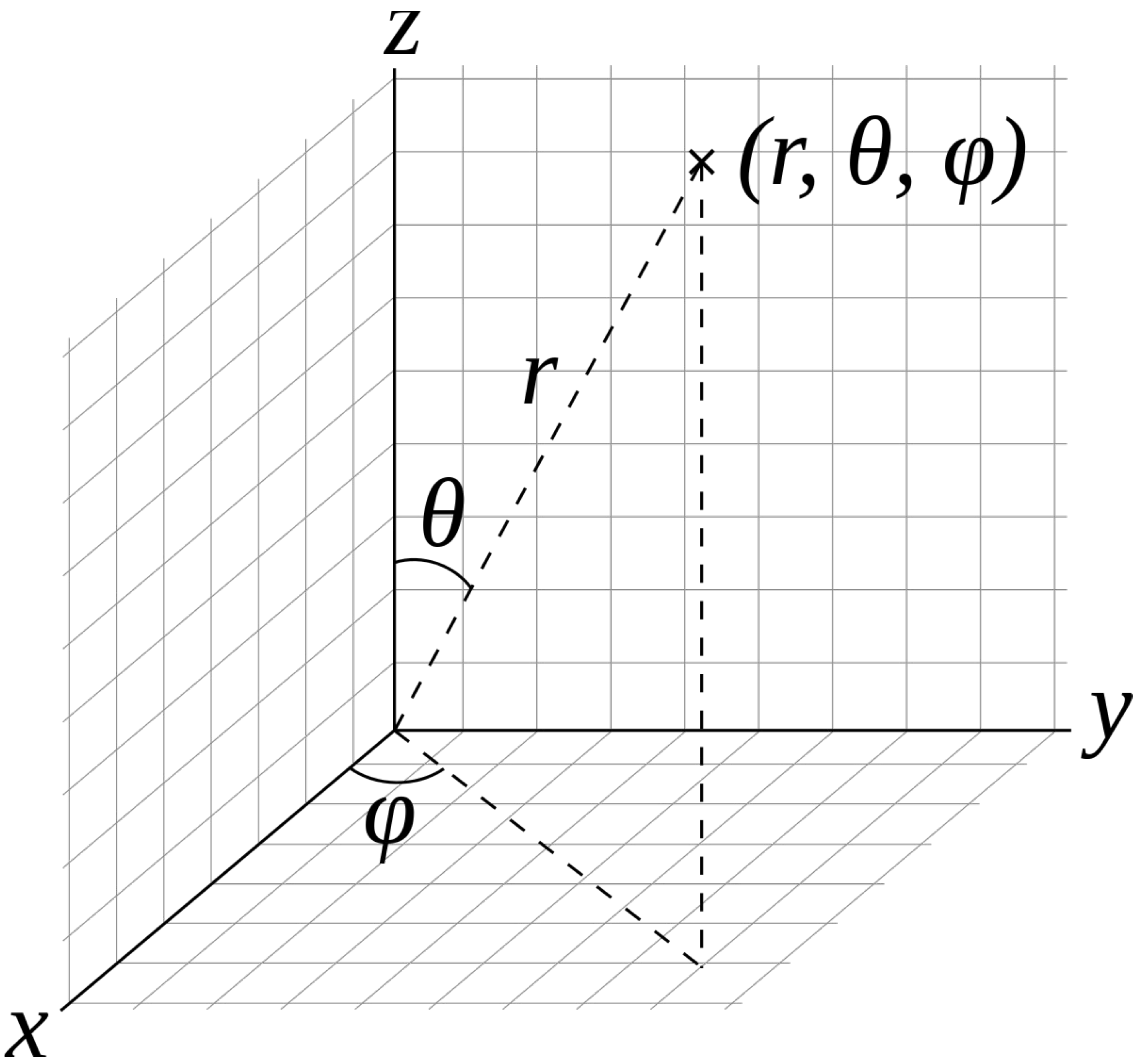
Solid angle

$$\Omega = \frac{A}{R^2}$$

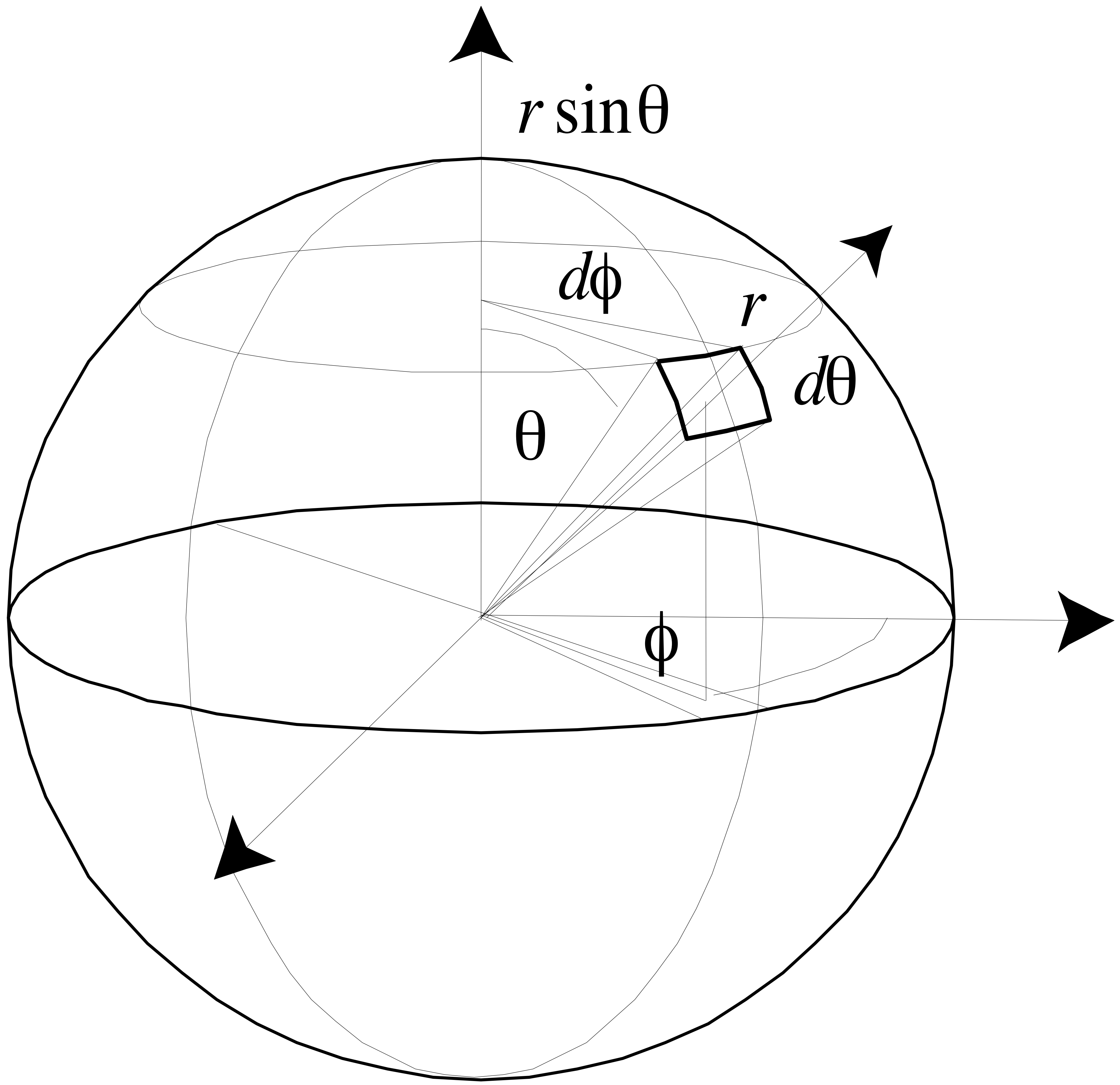
⇒ sphere has 4 π steradians



Review of spherical coordinates

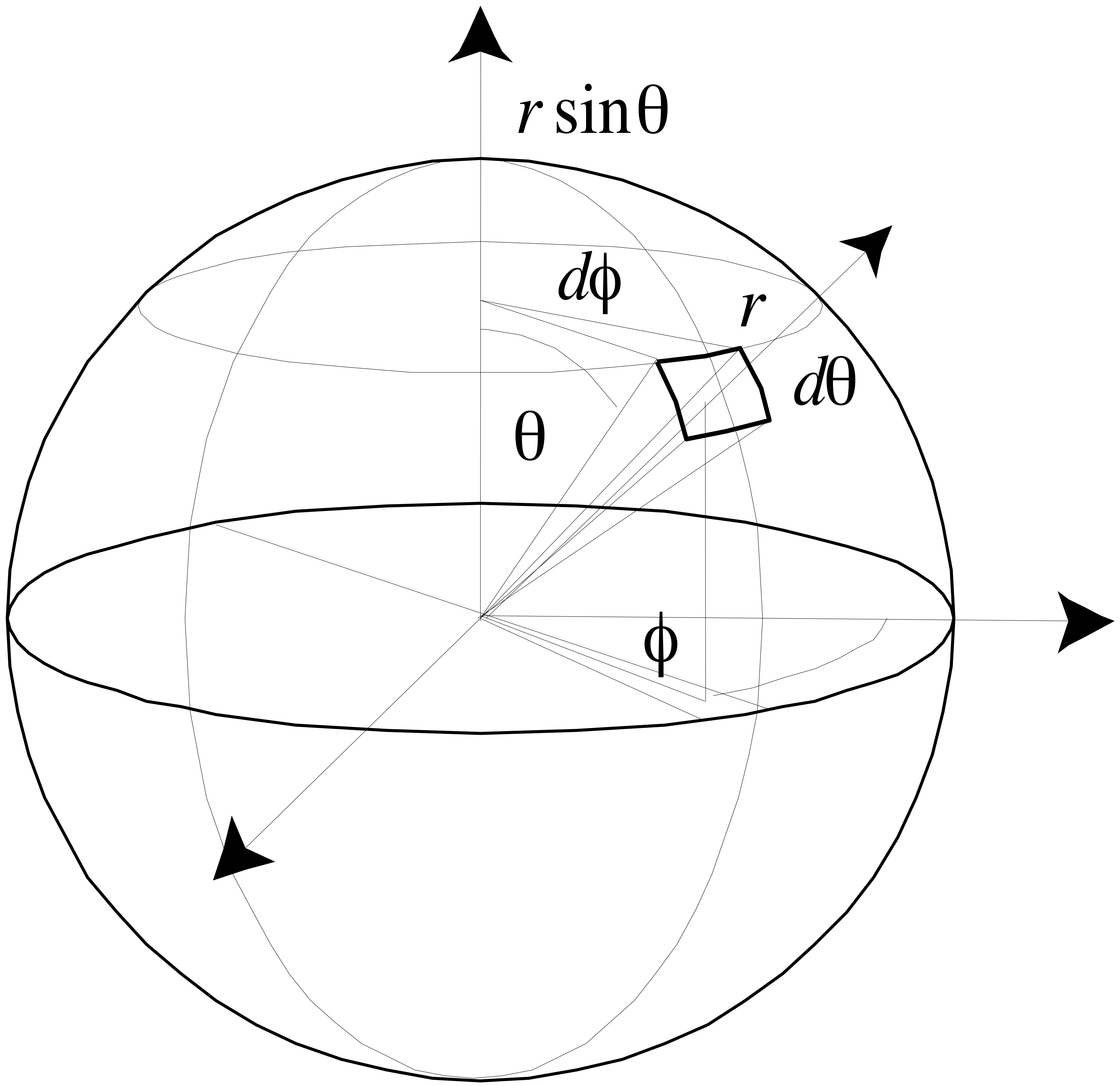


Differential solid angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

Differential solid angles

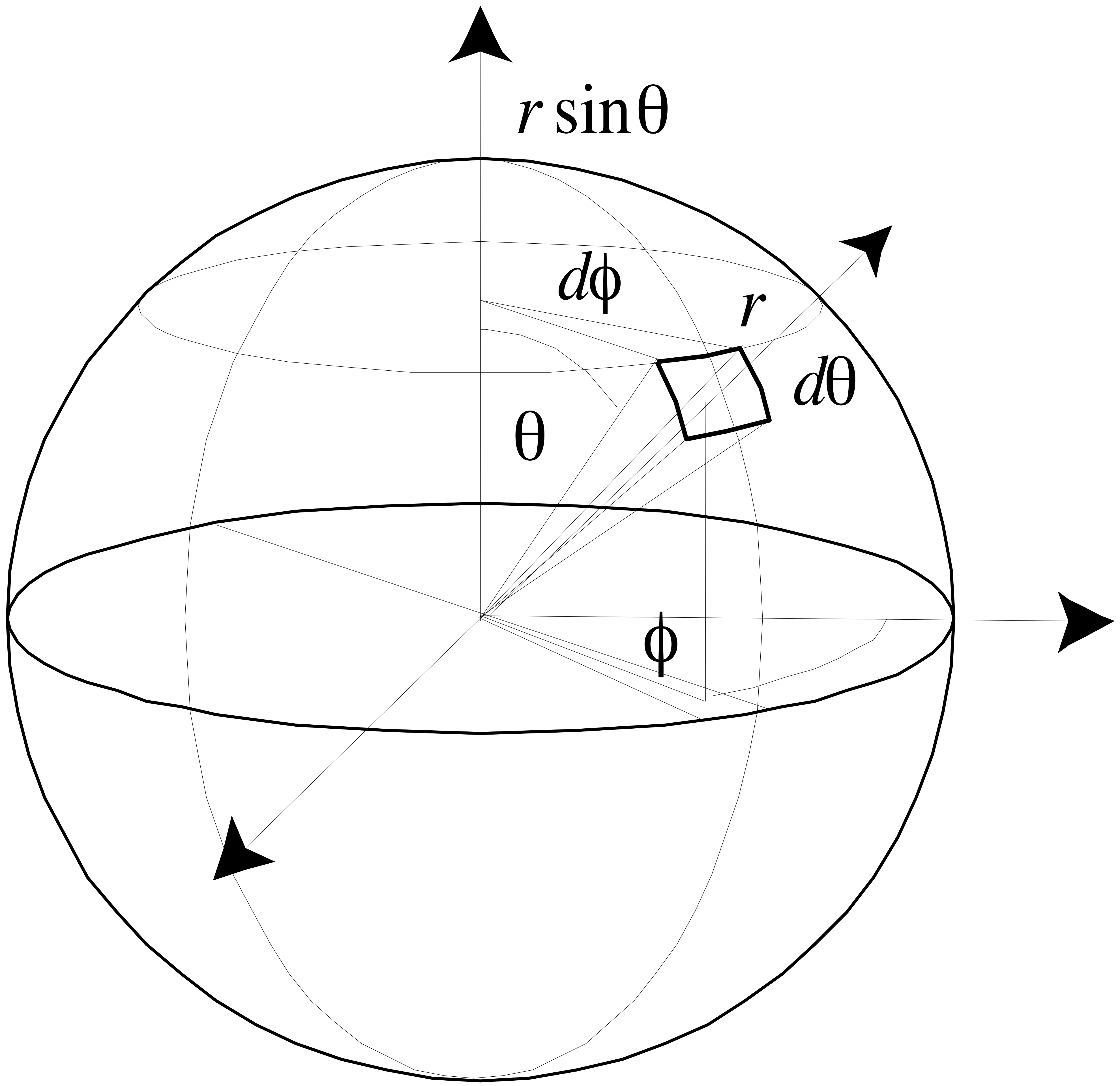


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential solid angles

Sphere S^2



$$d\omega = \sin\theta \, d\theta \, d\phi$$

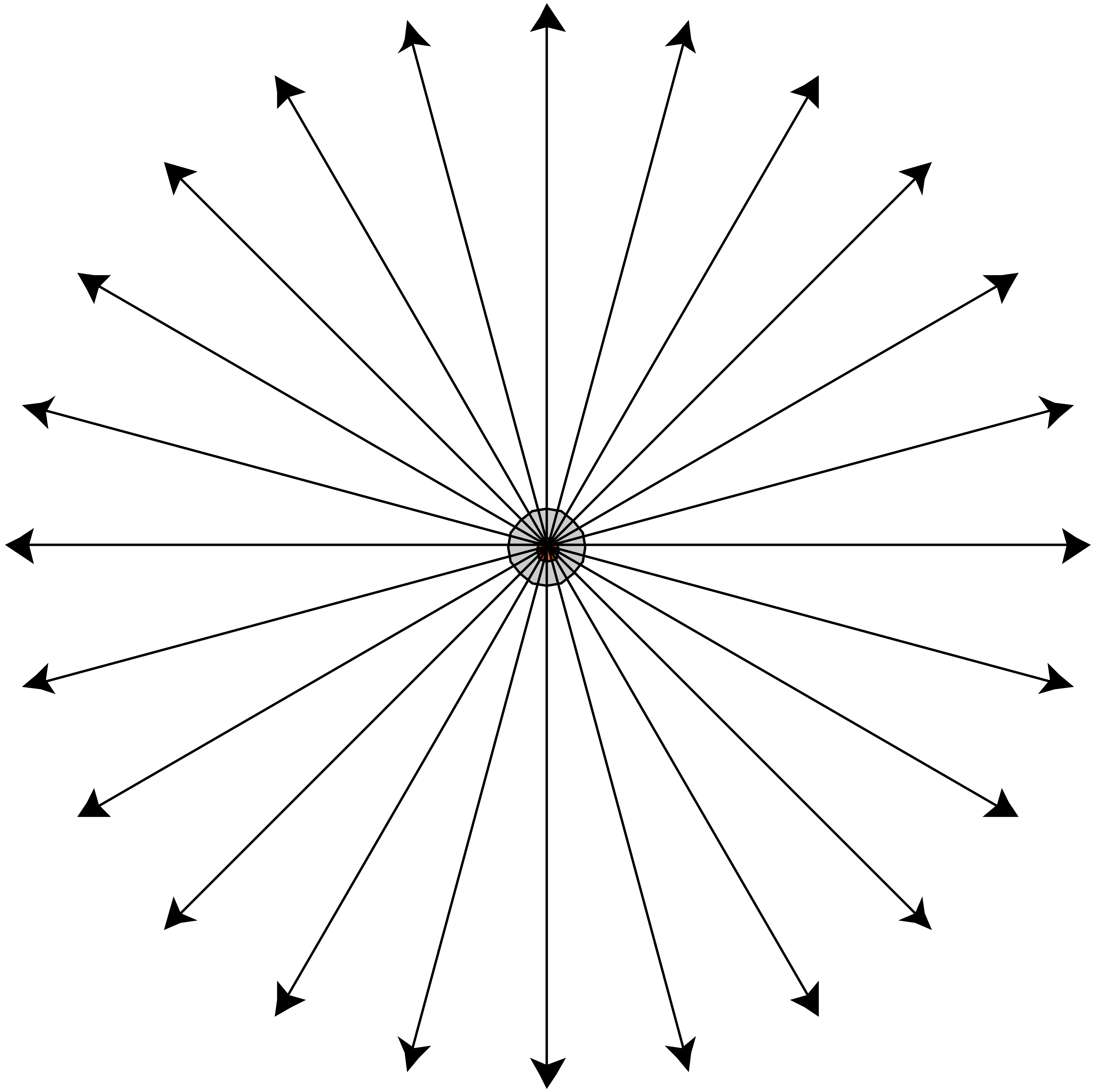
$$\Omega = \int_{S^2} d\omega$$

$$= \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi$$

$$= 4\pi$$

Isotropic point source

Radiating total power Φ. Radiating with same intensity *I* in all directions.

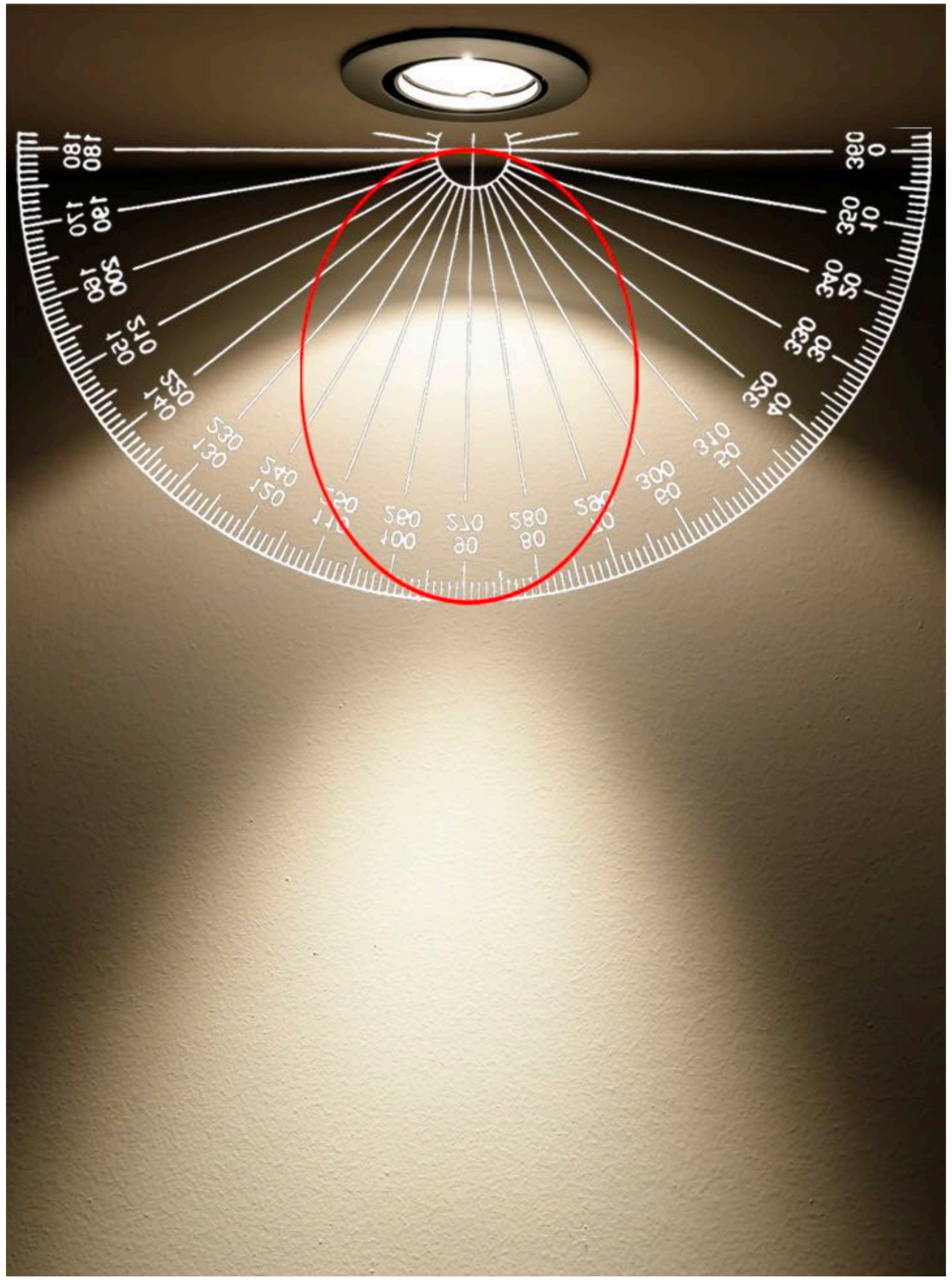


$$\Phi = \int_{S^2} I \, d\omega$$

$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Anisotropic intensity distributions

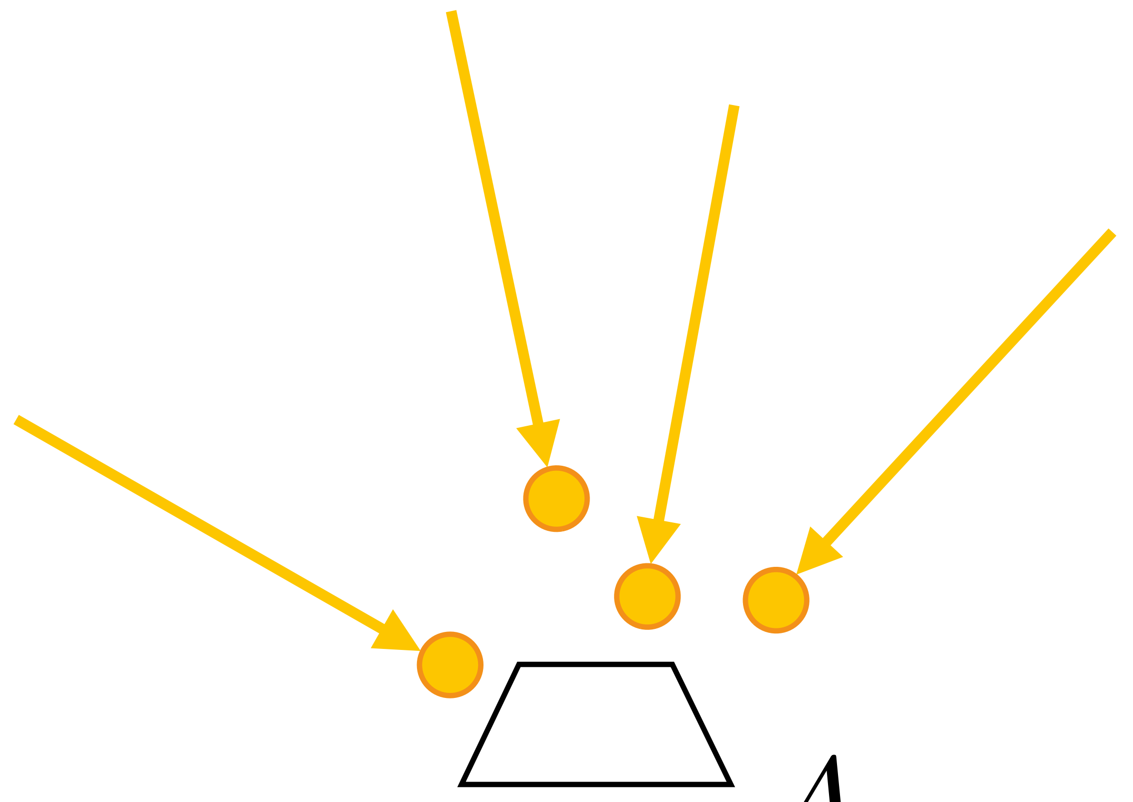


ADJ H20 Dmx Pro Ir Led Water Effect Spotlight

http://www.photometrictesting.co.uk/File/understanding_photometric_data_files.php

Measuring illumination: irradiance

- **Flux: time density of energy**
- **Irradiance: area density of flux**



Given a sensor of with area *A*, we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

Irradiance (E) is given by taking the limit of area at a single point on the sensor:

$$E(\mathrm{p}) = \lim_{\Delta \to 0} \frac{\Delta\Phi(\mathrm{p})}{\Delta A} = \frac{\mathrm{d}\Phi(\mathrm{p})}{\mathrm{d}A} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Units = Watts per area

Beam power in terms of irradiance

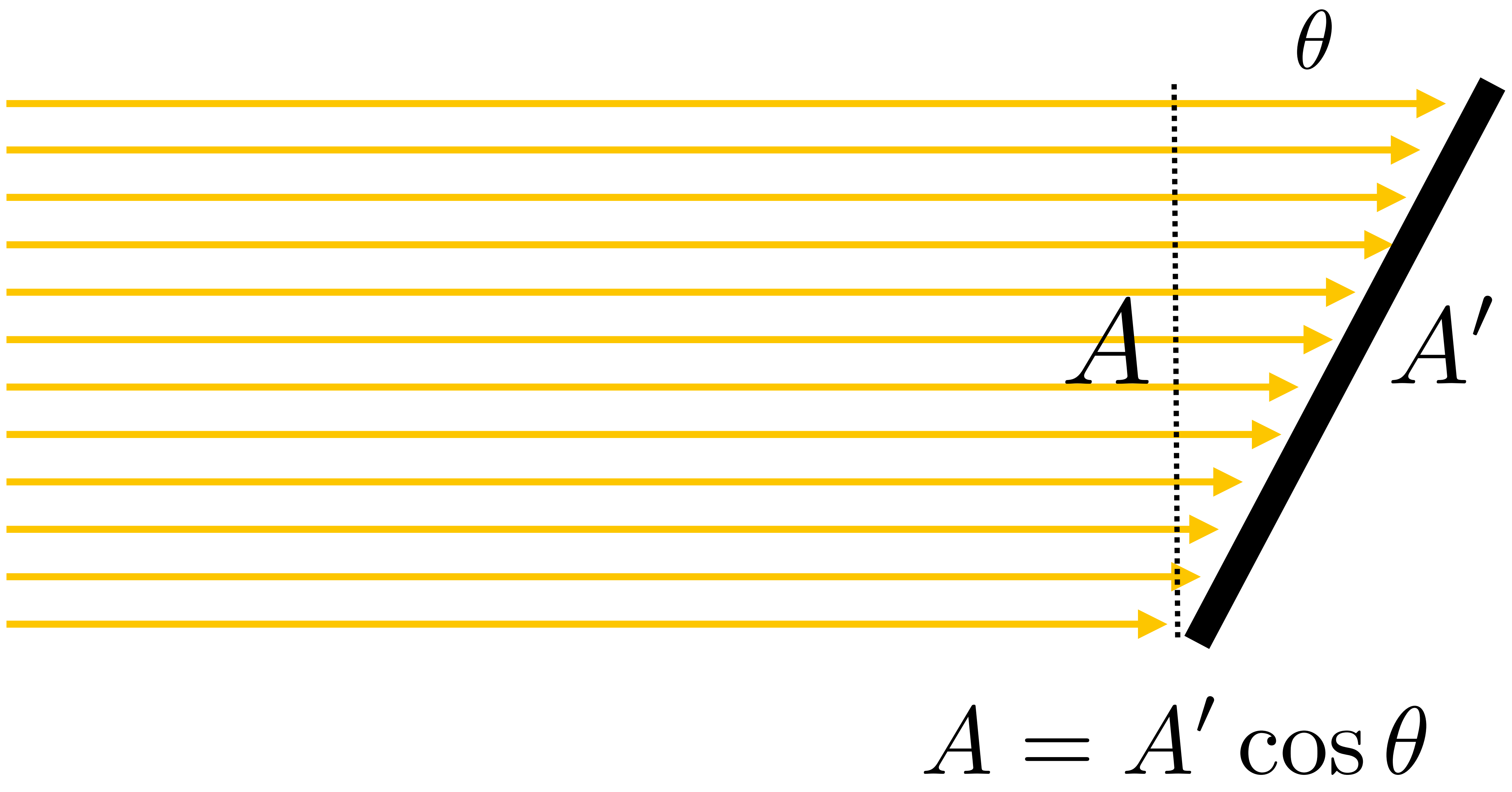
Consider beam with flux Φ incident on surface with area *A*

$$E = \frac{\Phi}{A}$$

$$\Phi = EA$$

Projected area

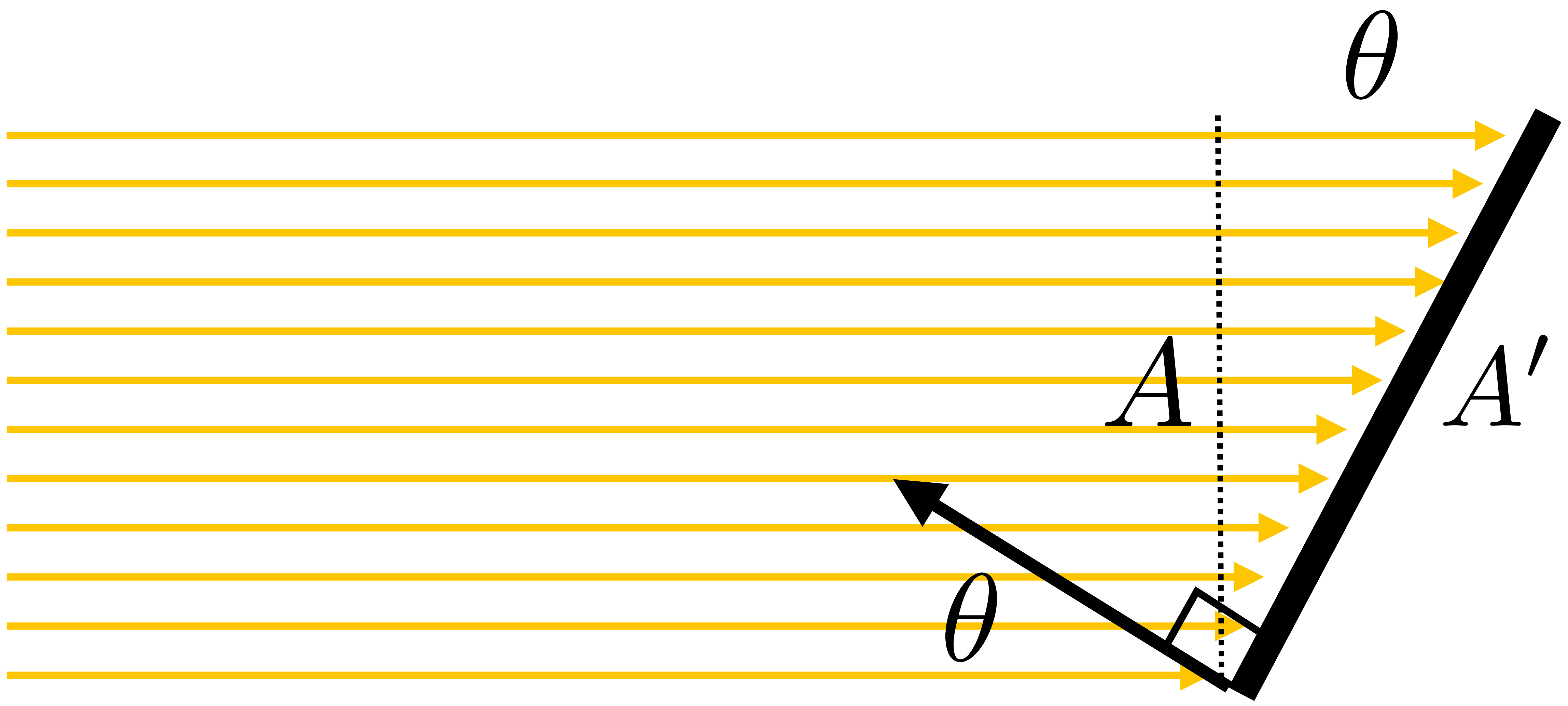
Consider beam with flux Φ incident on angled surface with area *A′*



A = projected area of surface relative to direction of beam

Lambert's Law

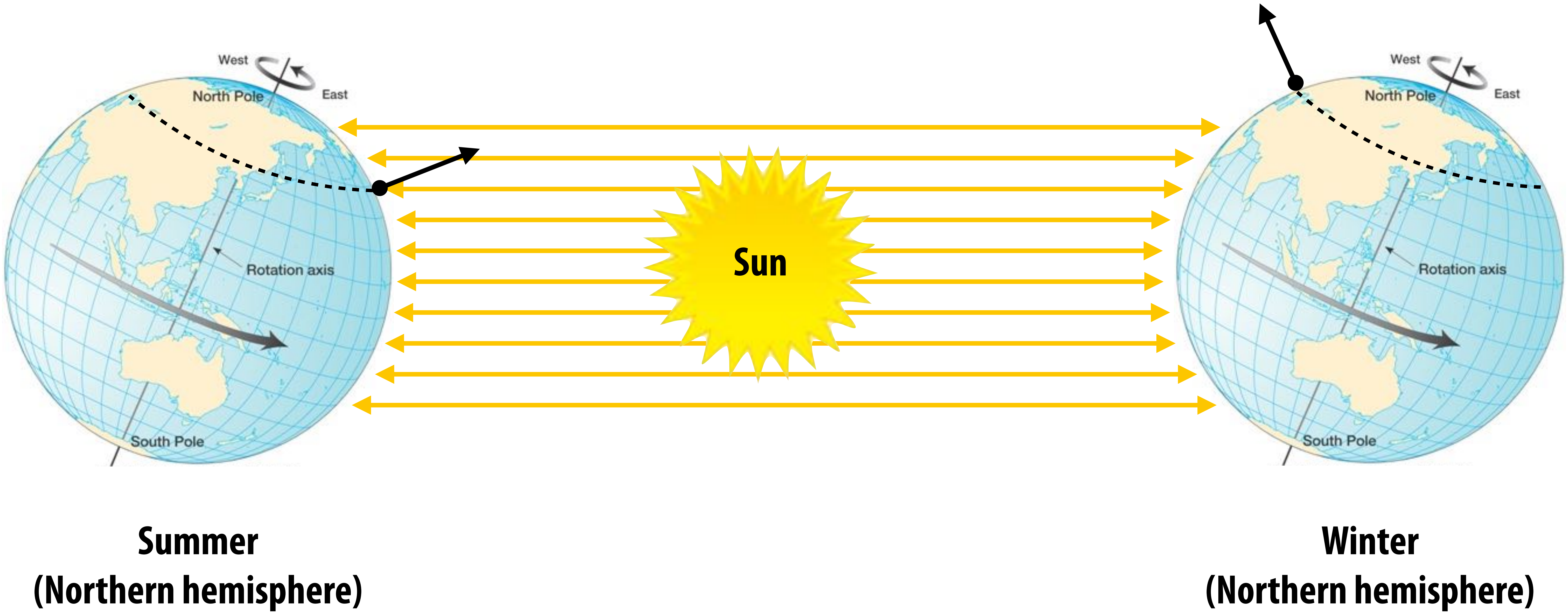
Irradiance at surface is proportional to cosine of angle between light direction and surface normal.



$$A = A' \cos\theta$$

$$E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$$

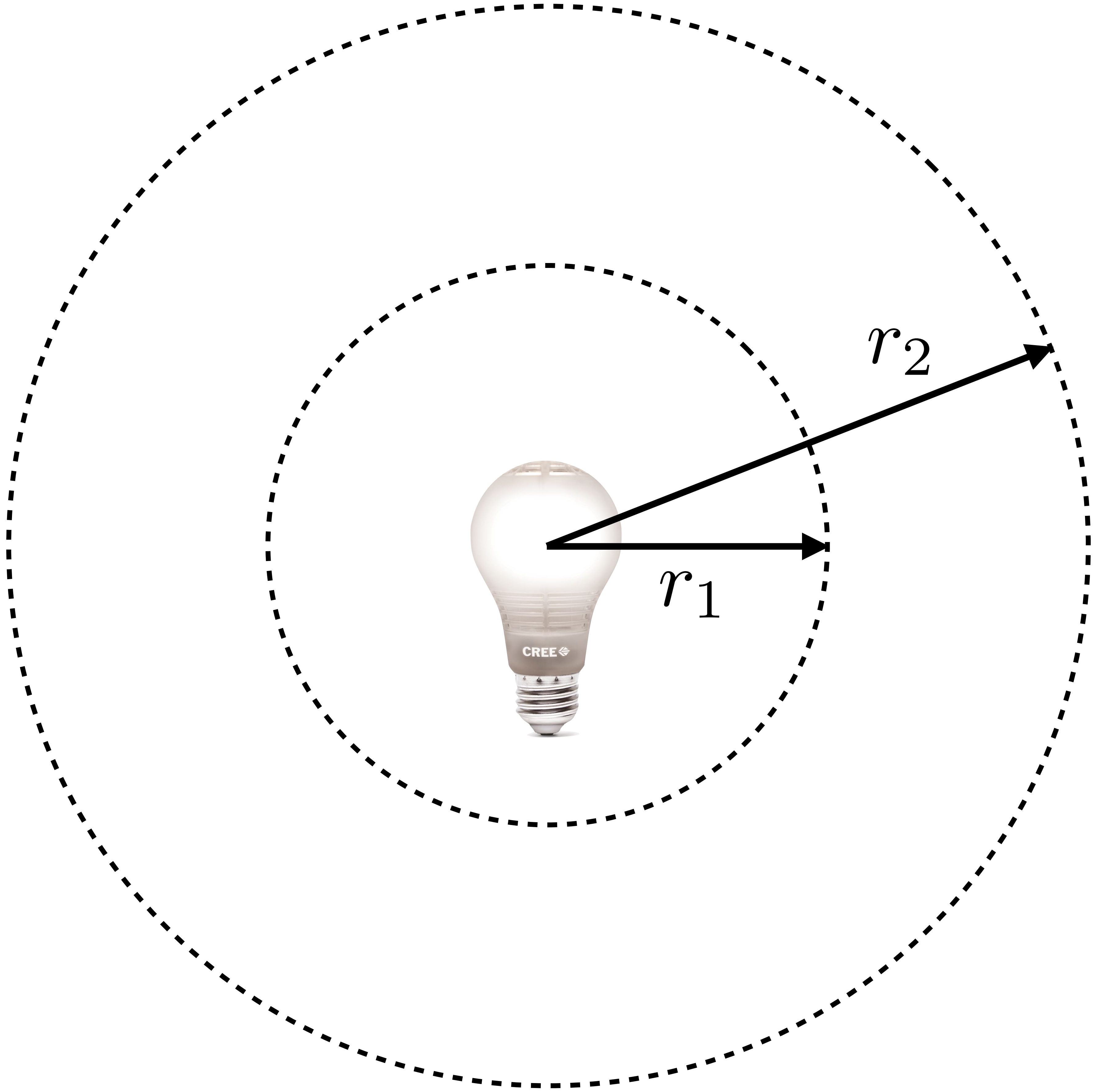
Why do we have seasons?



Earth's axis of rotation: ~23.5° off axis

[Image credit: Pearson Prentice Hall]

Irradiance falloff with distance



Assume light is emitting flux Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:

$$E_1 = \frac{\Phi}{4\pi r_1^2}$$

$$E_2 = \frac{\Phi}{4\pi r_2^2}$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$

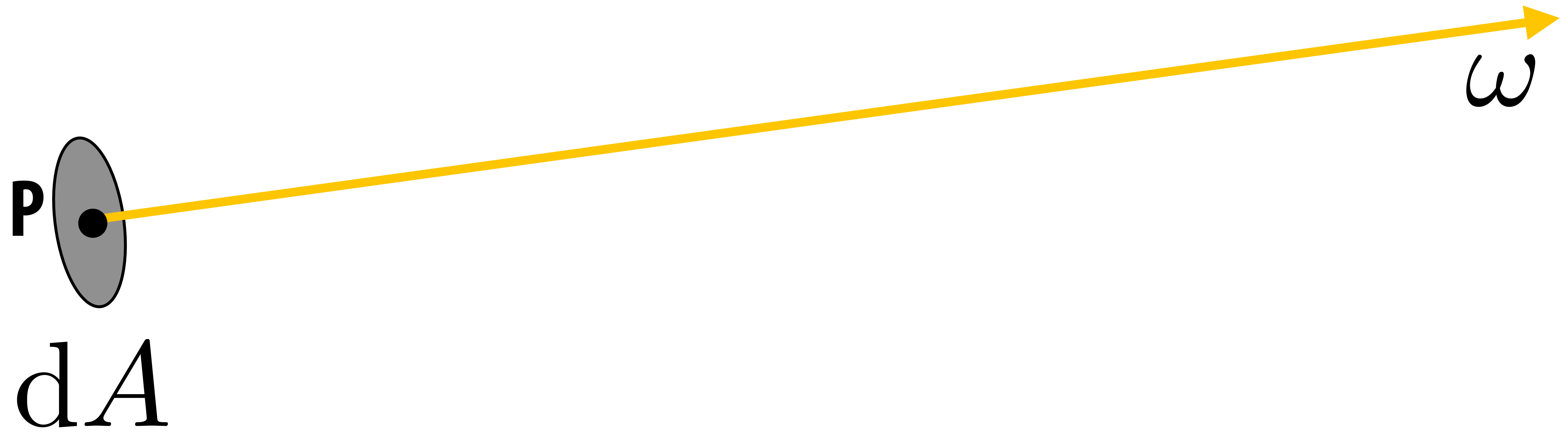
Why does a room get darker farther from a light source?

Image credit: LeRamz on Flickr

Measuring illumination: radiance

- **Radiance (L) is the solid angle density of irradiance (irradiance per unit direction)**

where ω denotes that the differential surface area is oriented to face in the direction



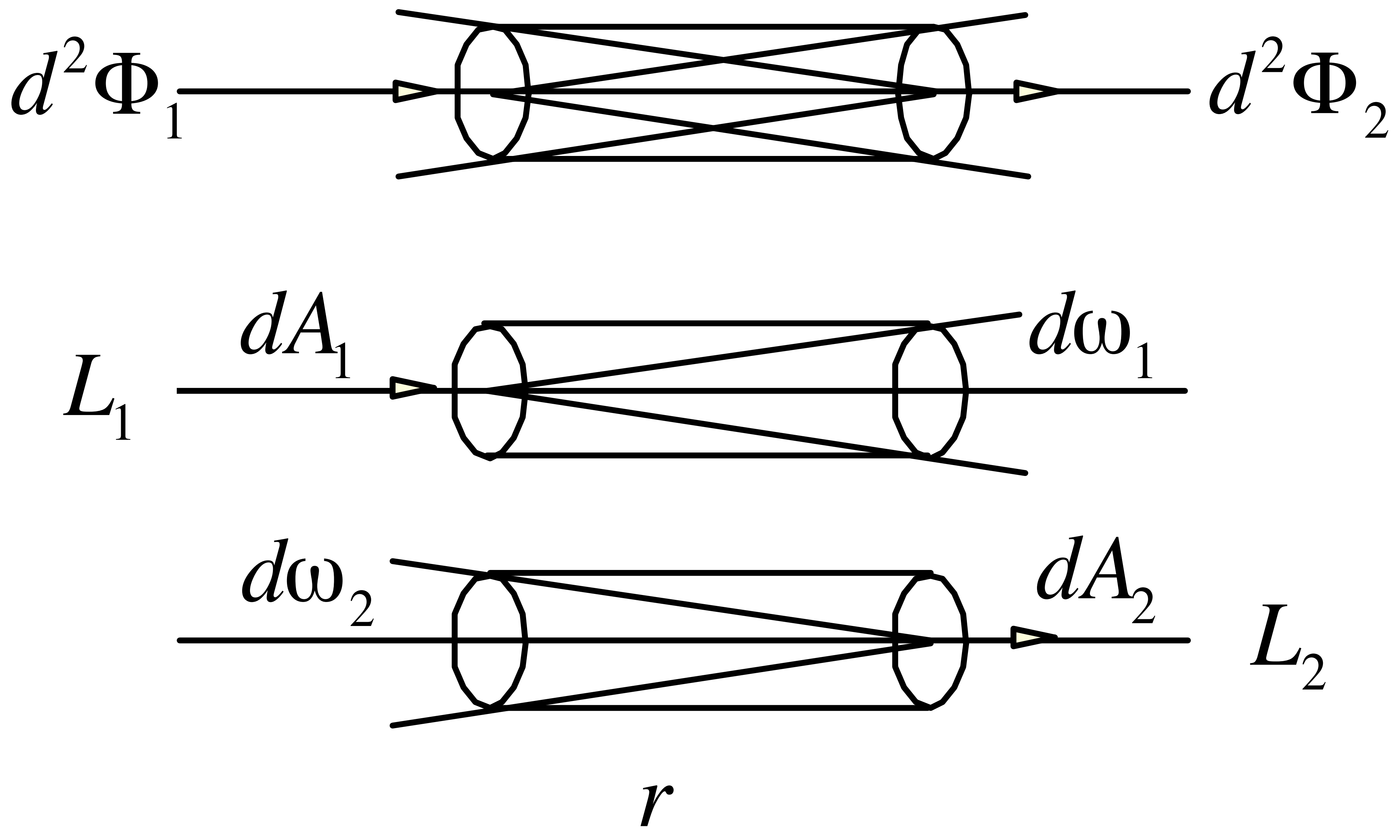
In other words, radiance is energy along a ray defined by origin point *p* and direction ω

Properties of radiance

- Fundamental field quantity that characterizes the distribution of light in an environment
  - Radiance is the quantity associated with a ray
  - Ray tracers compute the radiance
- Radiance is invariant along a ray
  - Reduces parameters from 6D to 4D

1st Law: conservation of radiance

The radiance in the direction of a light ray remains constant as the ray propagates through empty space



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

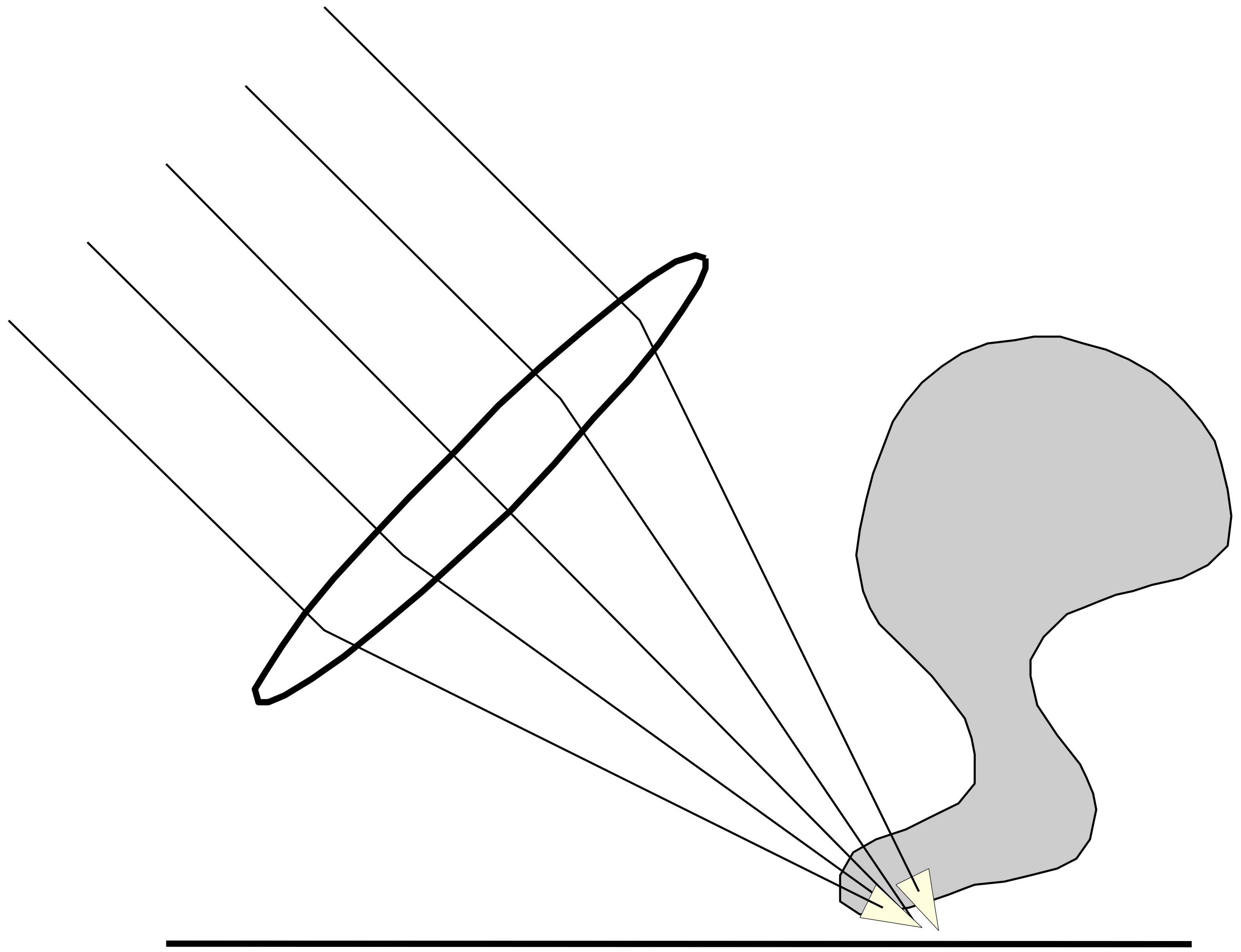
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

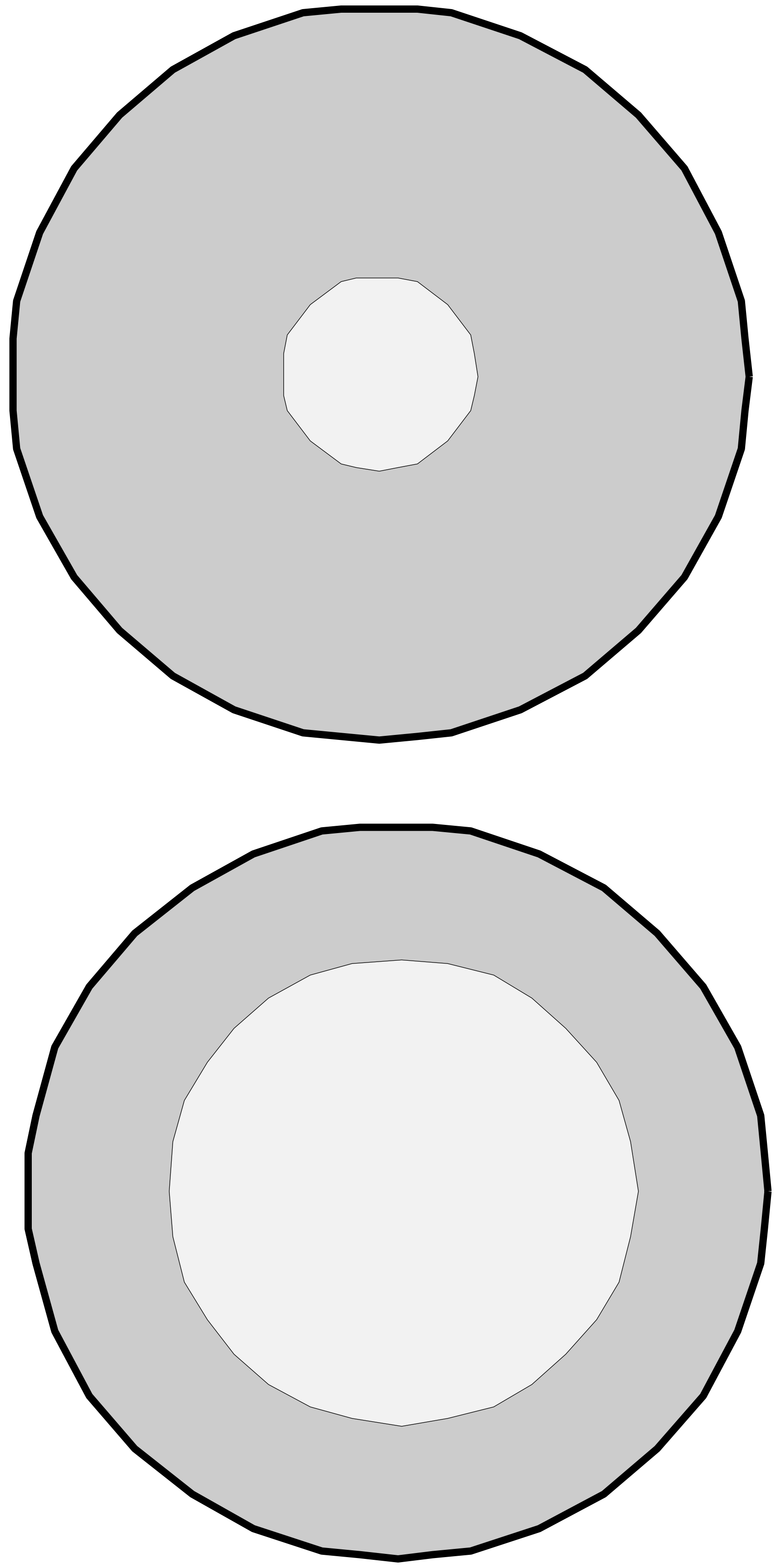
$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



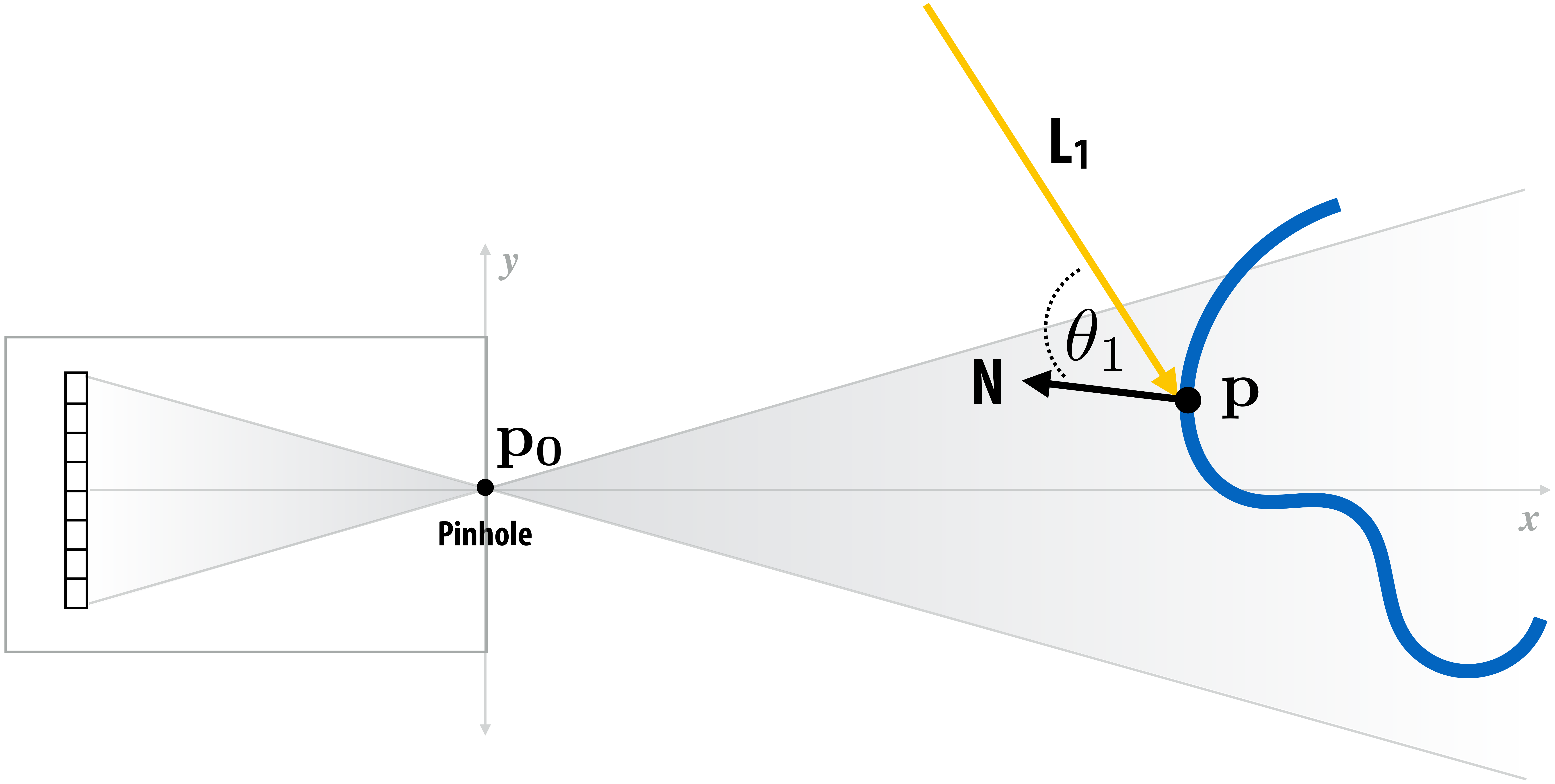
No!!



How much light hits the surface at point p

(irradiance at point P)

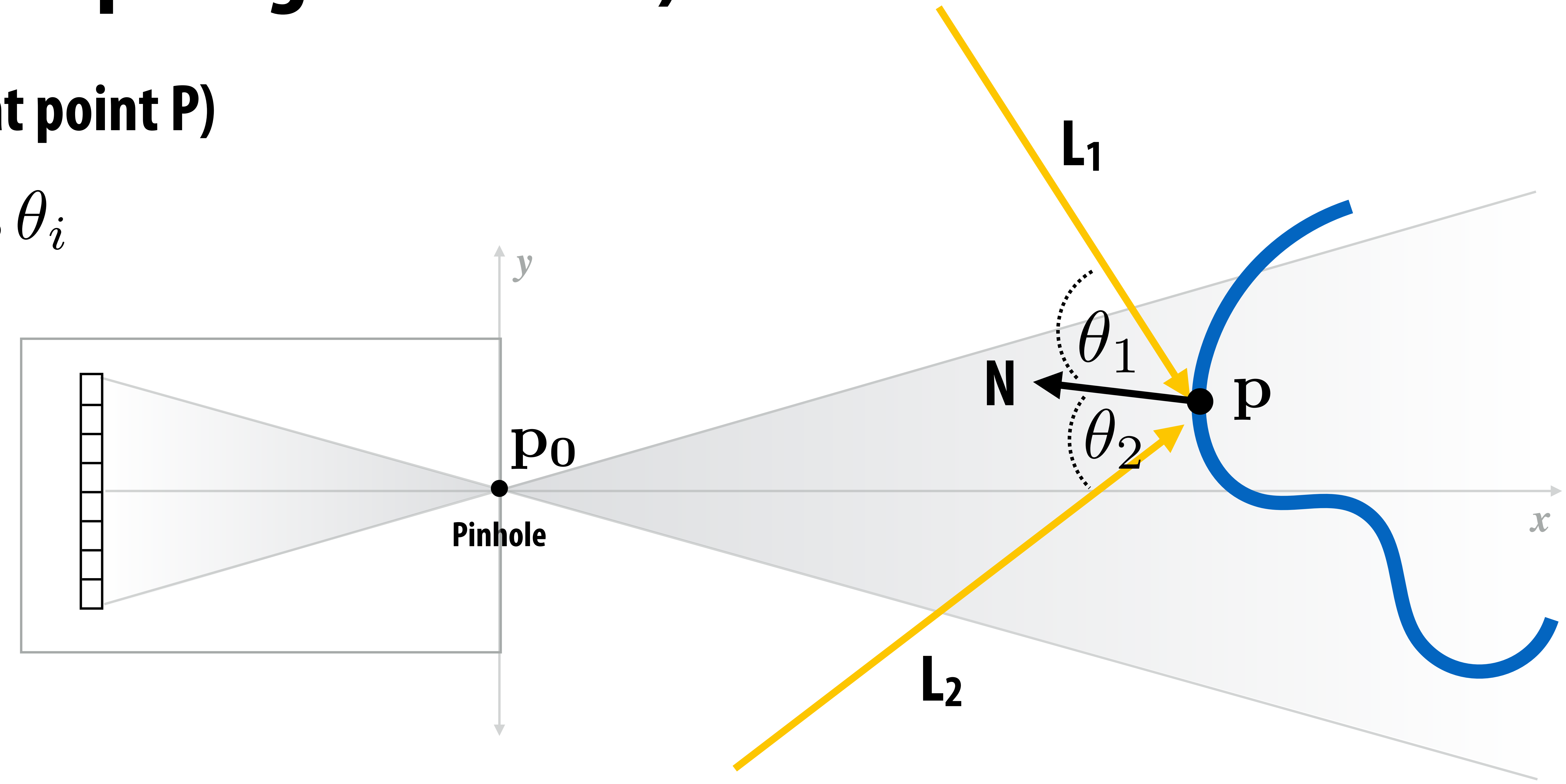
$$L_1 \cos \theta_1$$



How much light hits the surface at point p? (from multiple light sources)

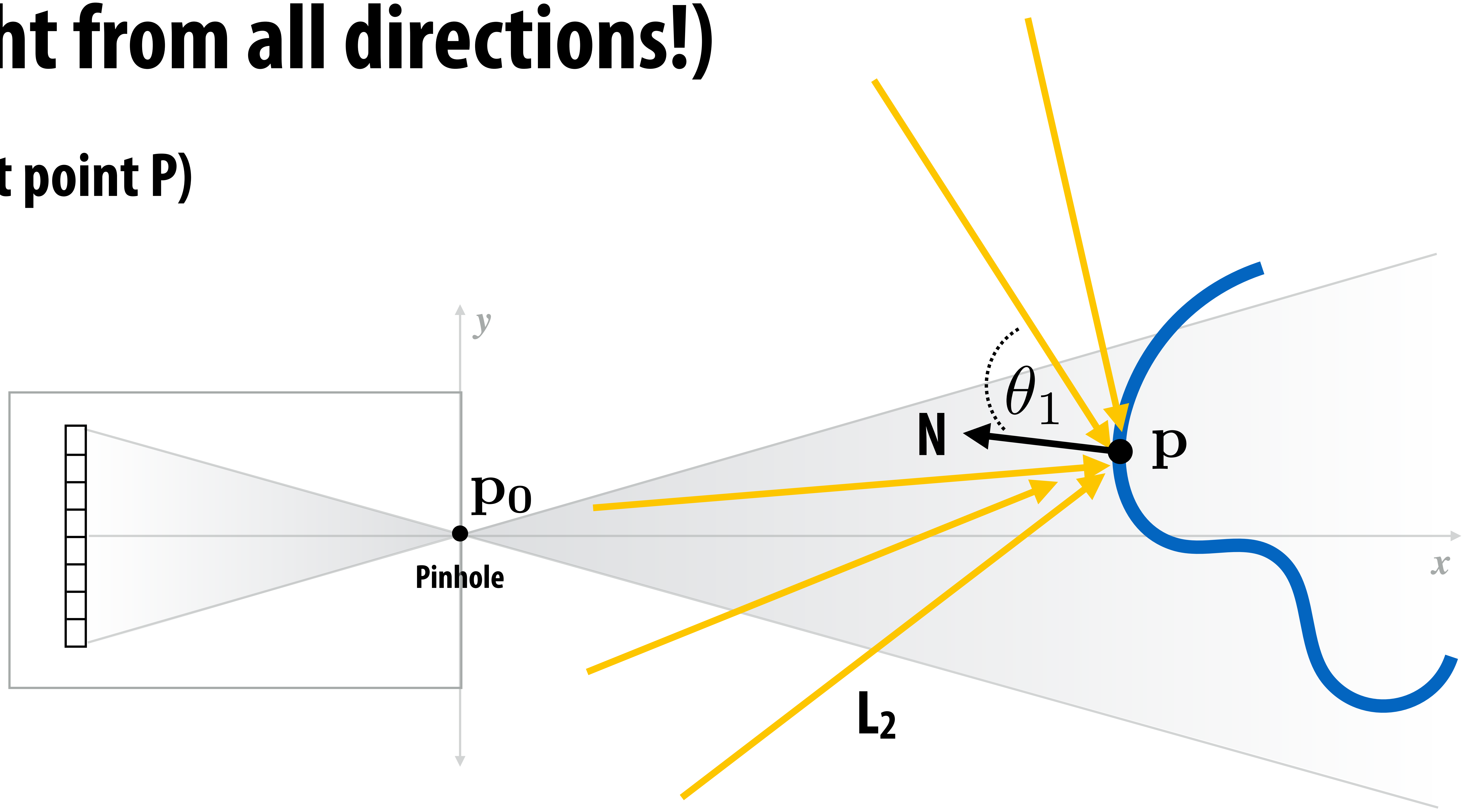
(irradiance at point P)

$$\sum_i L_i \cos\theta_i$$



How much light hits the surface at point p?
(from light from all directions!)

(irradiance at point P)

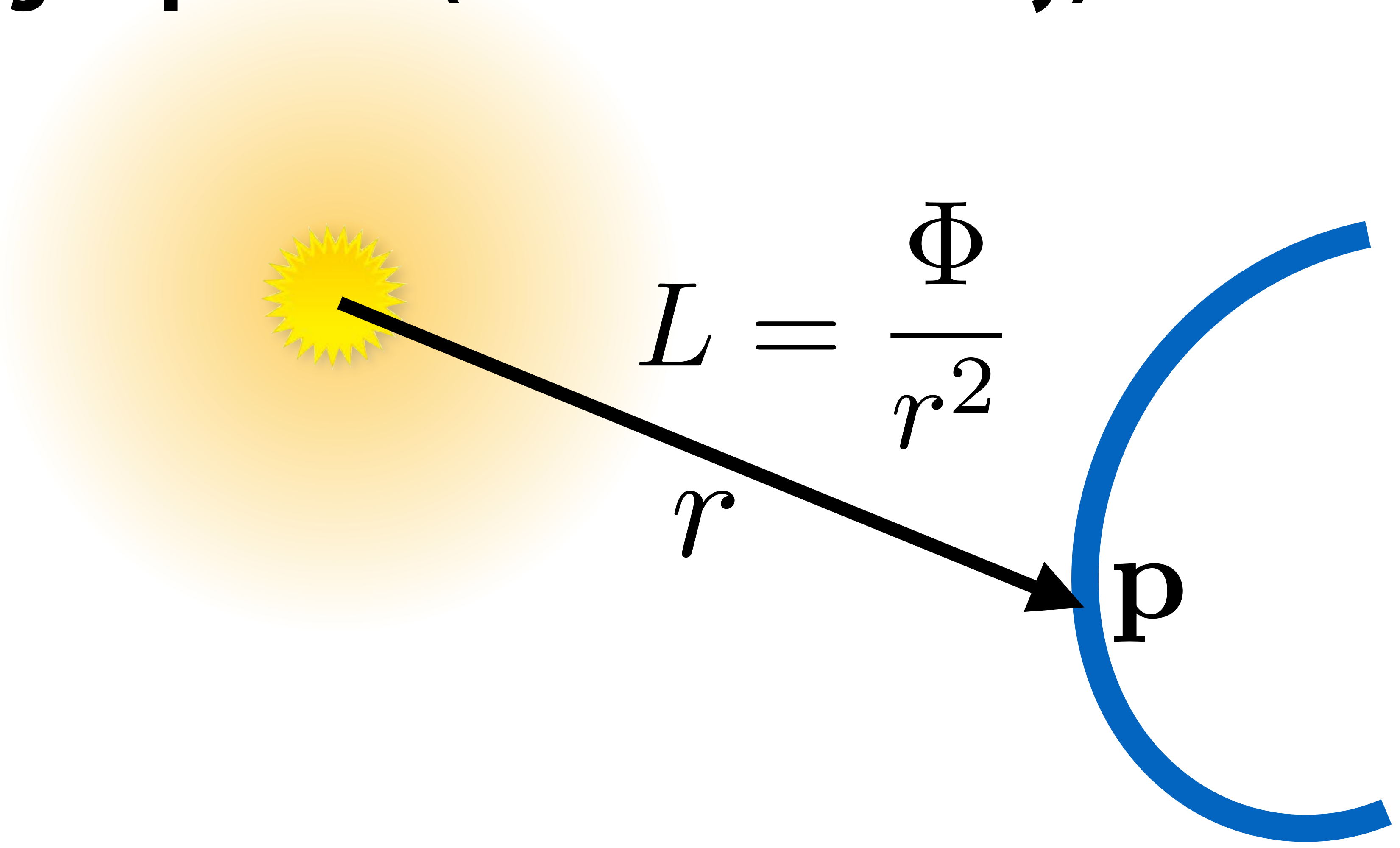


$$\int_{S^2} L_i(\omega_i)\cos\theta_i d\omega = \int_0^{2\pi}\int_0^{\pi} L_i(\omega_i)\cos\theta_i \sin\theta_i d\theta d\phi$$

Types of lights

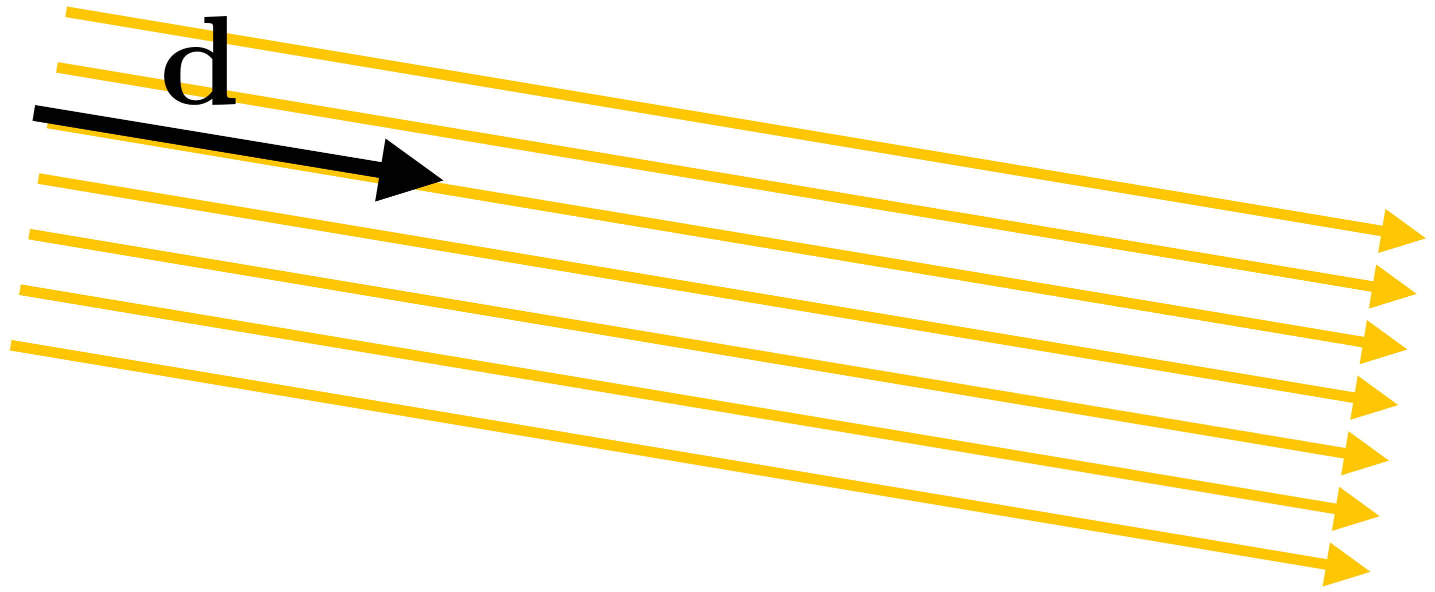
- **Attenuated omnidirectional point light**

  (emits equally in all directions, energy arriving at point P (radiant intensity) falls off with distance: $1/R^2$ falloff)



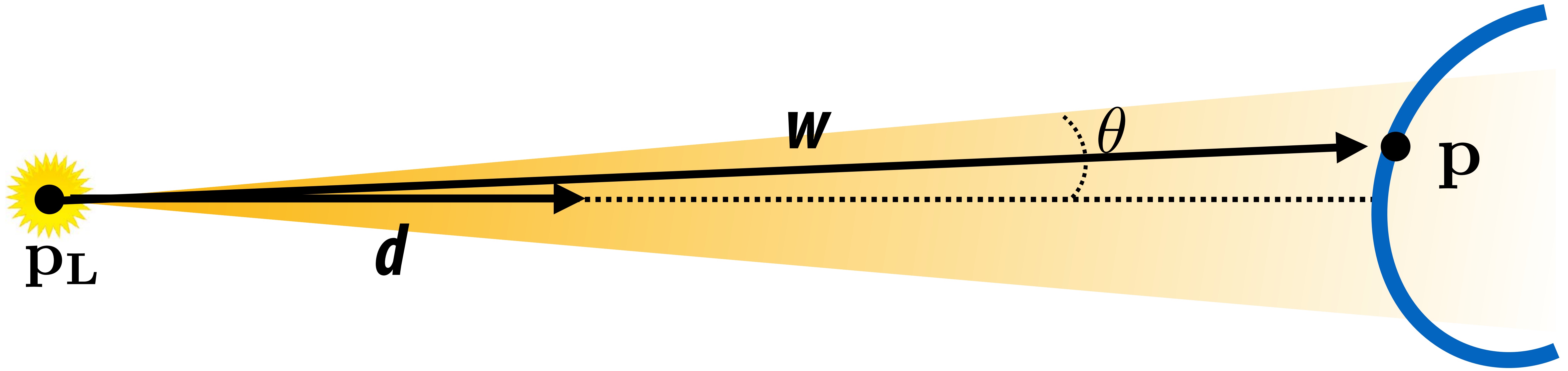
- **Infinite directional light in direction *d***

  (infinitely far away, all points in scene receive light with radiance L from direction *d*



Spot light

Does not emit equally in all directions… intensity falls off in directions away from main spotlight direction



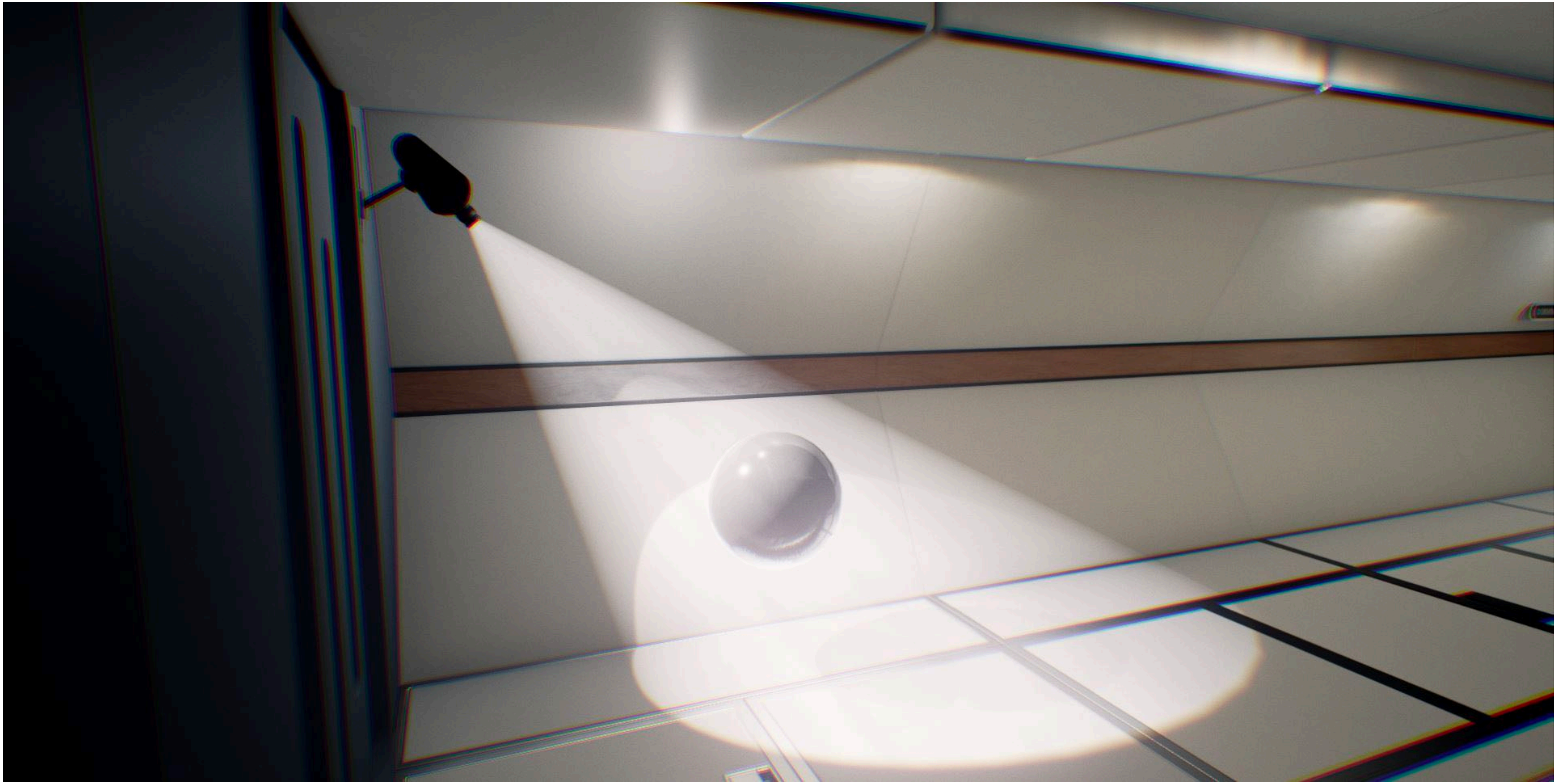
$$\mathbf{w} = \text{normalize}(\mathbf{p} - \mathbf{p_L})$$

$$L(\mathbf{w}) = 0 \quad \textbf{if} \quad \mathbf{w} \cdot \mathbf{d} > \cos\theta$$
$$= L_0 \quad \textbf{otherwise}$$

Or, if spotlight intensity falls off from direction *d*

$$L(\mathbf{w}) \approx \mathbf{w} \cdot \mathbf{d}$$

Spot light



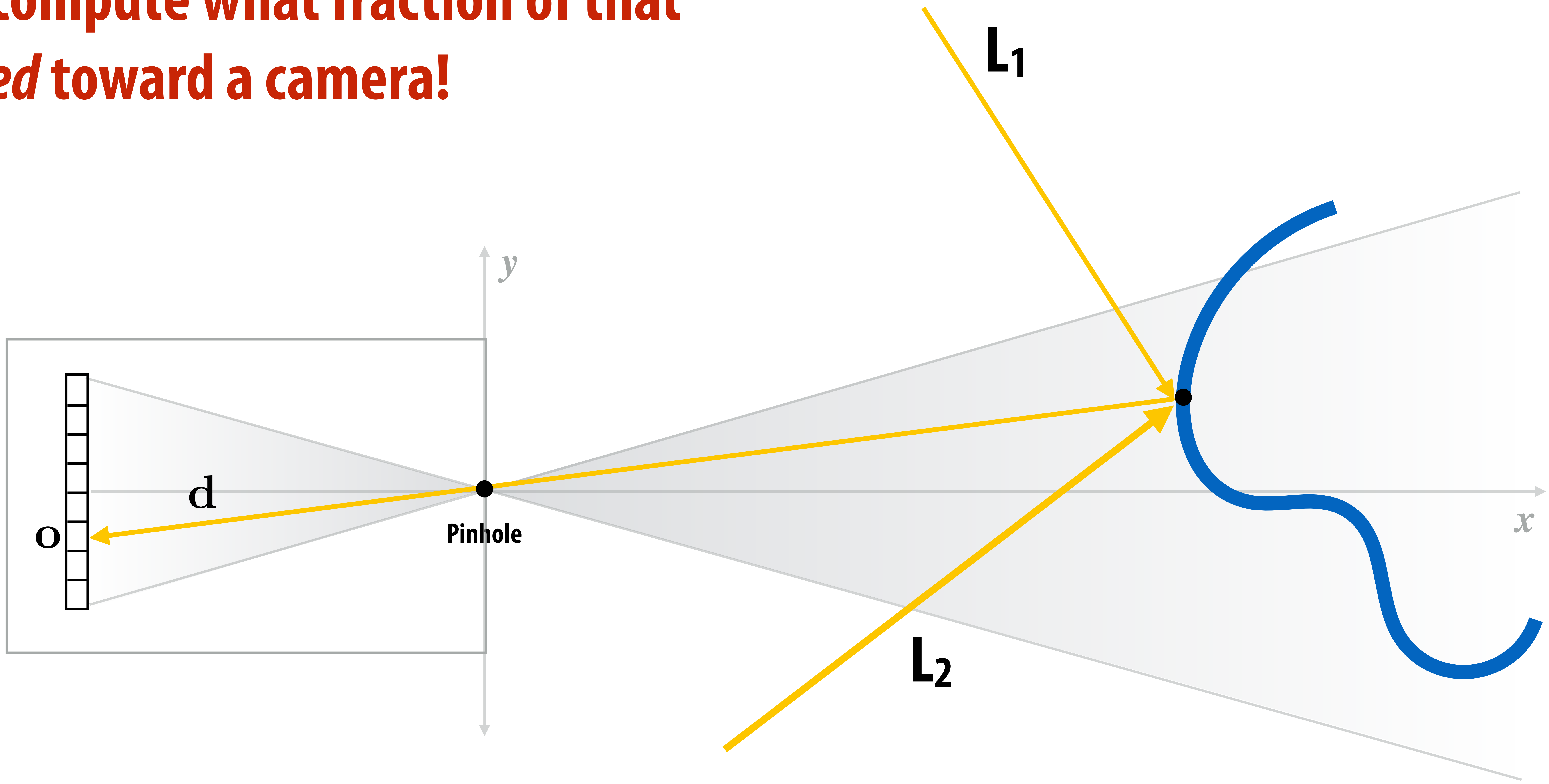
Environment light (represented by texture map)

Pixel (x,y) stores radiance L from direction (ϕ, θ)

Image credit: USC High-Resolution Light Probe Image Gallery

So far… we've discussed how to compute the light arriving at a surface point (radiance along incoming ray)

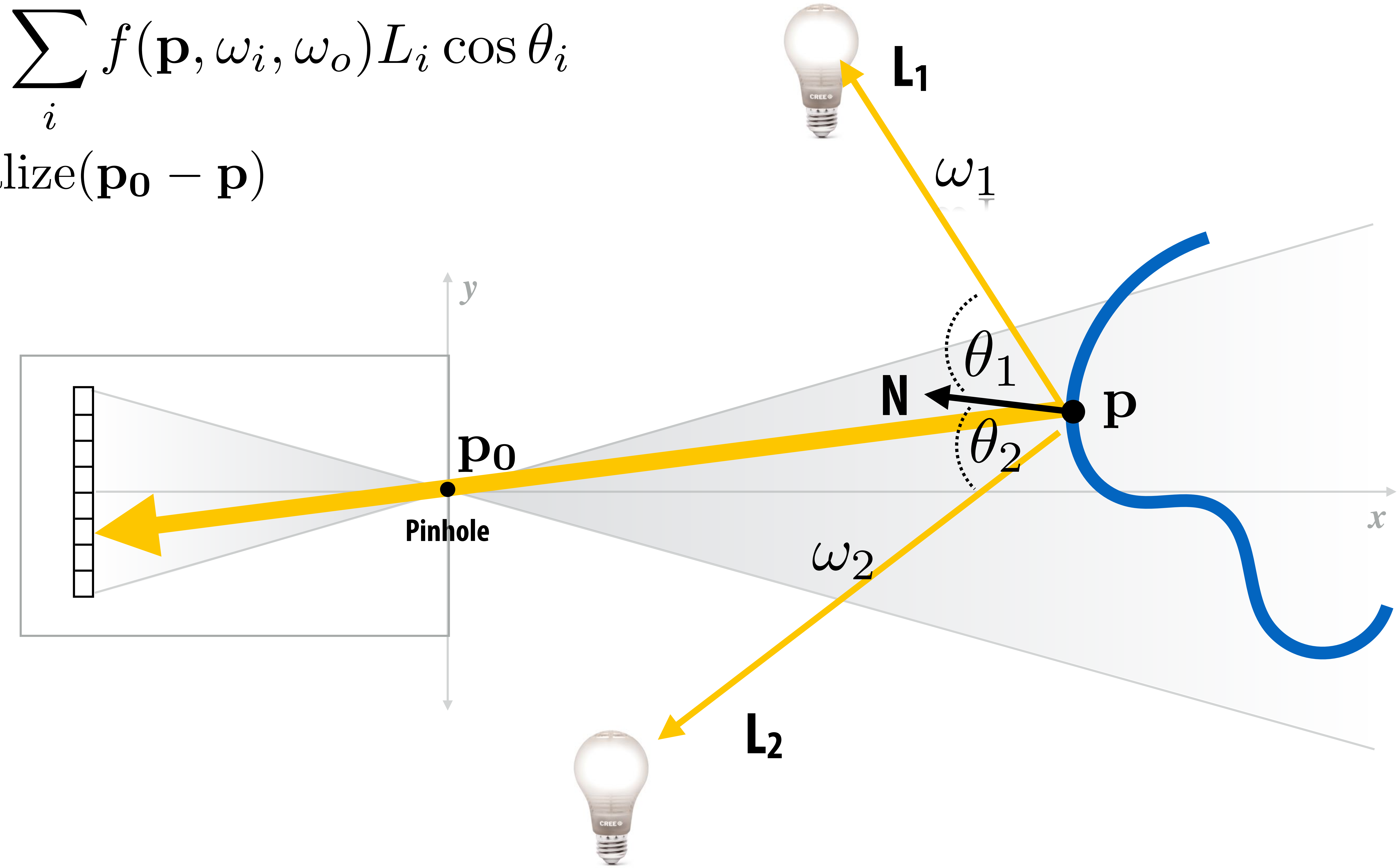
But goal is to compute what fraction of that light is *reflected* toward a camera!



How much light is REFLECTED from p toward p_0?

$$L(\mathbf{p}, \omega_o) = \sum_i f(\mathbf{p}, \omega_i, \omega_o) L_i \cos\theta_i$$

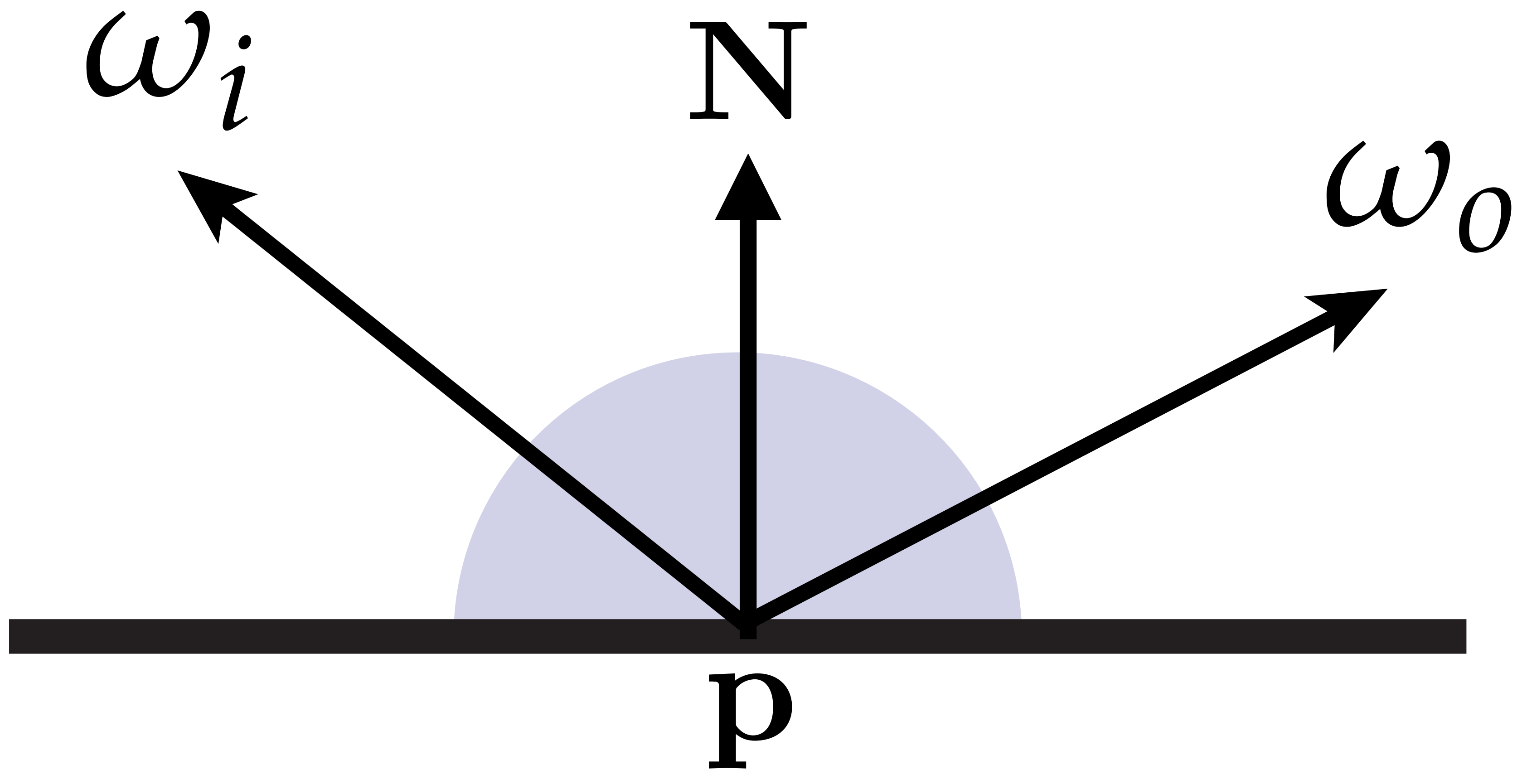
$$\omega_o = \text{normalize}(\mathbf{p_0} - \mathbf{p})$$



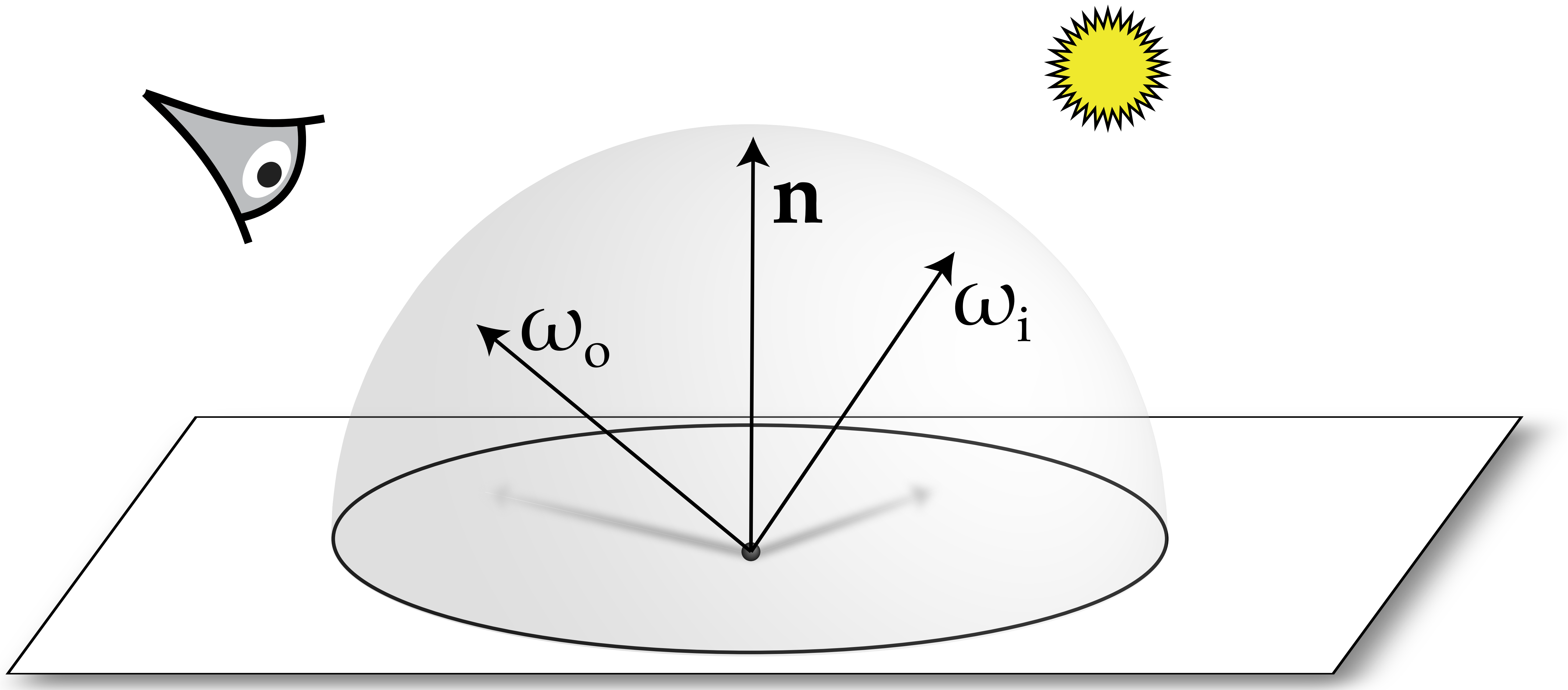
Bidirectional reflectance distribution function (BRDF)

- Gives fraction of light arriving at surface point P from direction w_i is reflected in the direction w_0

$$f(\mathbf{p}, \omega_i, \omega_o)$$



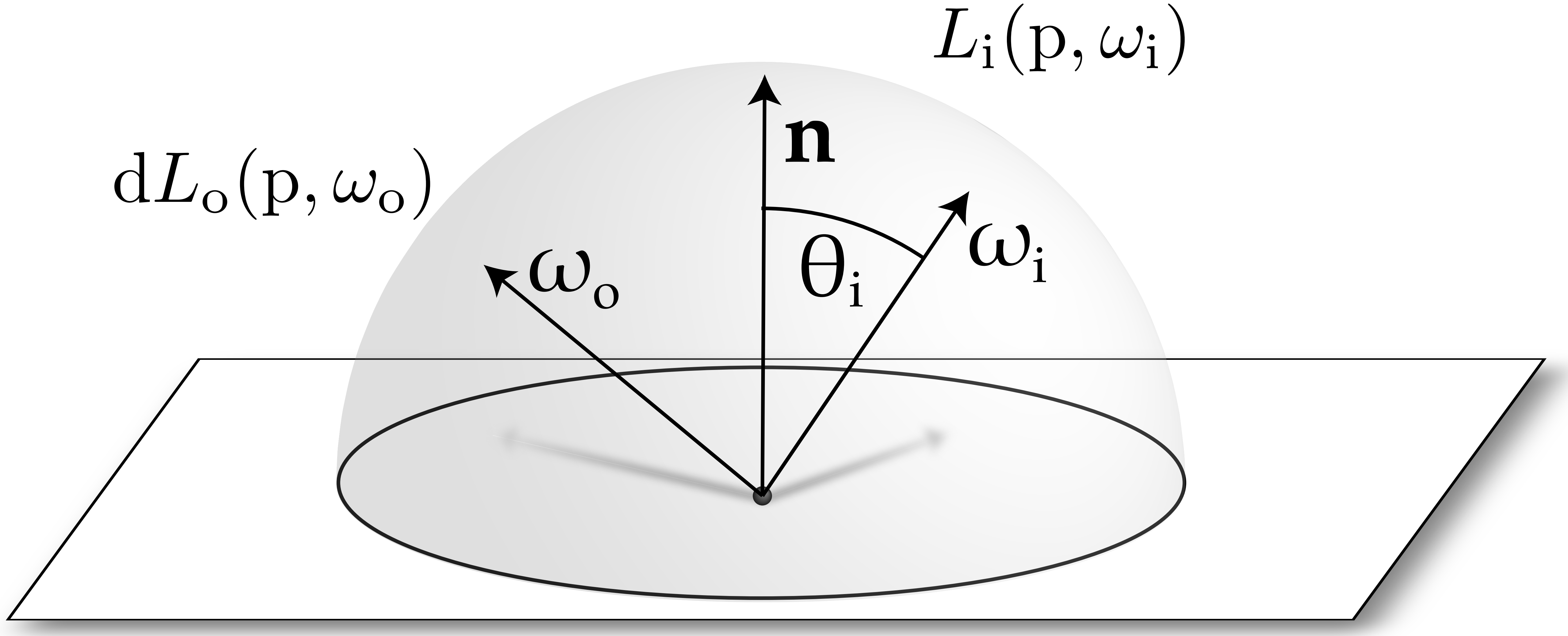
The reflection equation



$$L_o(p, \omega_o) = \int_{\Omega^2} \underbrace{f_r(p, \omega_i \rightarrow \omega_o)}_{\textbf{BRDF}} \underbrace{L_i(p, \omega_i)\ \cos\theta_i}_{\textbf{Illumination}} \, d\omega_i$$

The BRDF

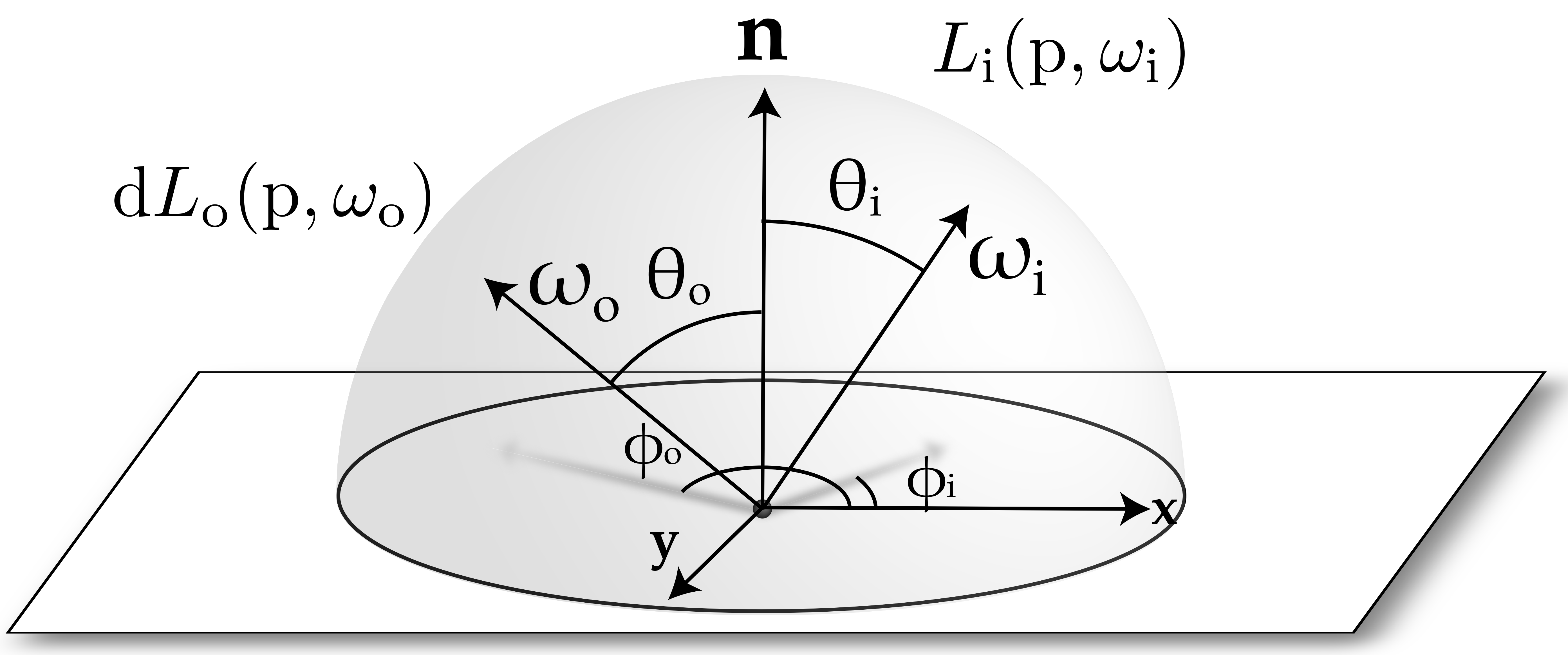
Bidirectional Reflectance-Distribution Function



$$f_\mathrm{r}(\omega_\mathrm{i} \rightarrow \omega_\mathrm{o}) \equiv \frac{\mathrm{d}L_\mathrm{o}(\omega_\mathrm{o})}{\mathrm{d}E_\mathrm{i}(\omega_\mathrm{i})} \quad \left[\frac{1}{sr}\right]$$

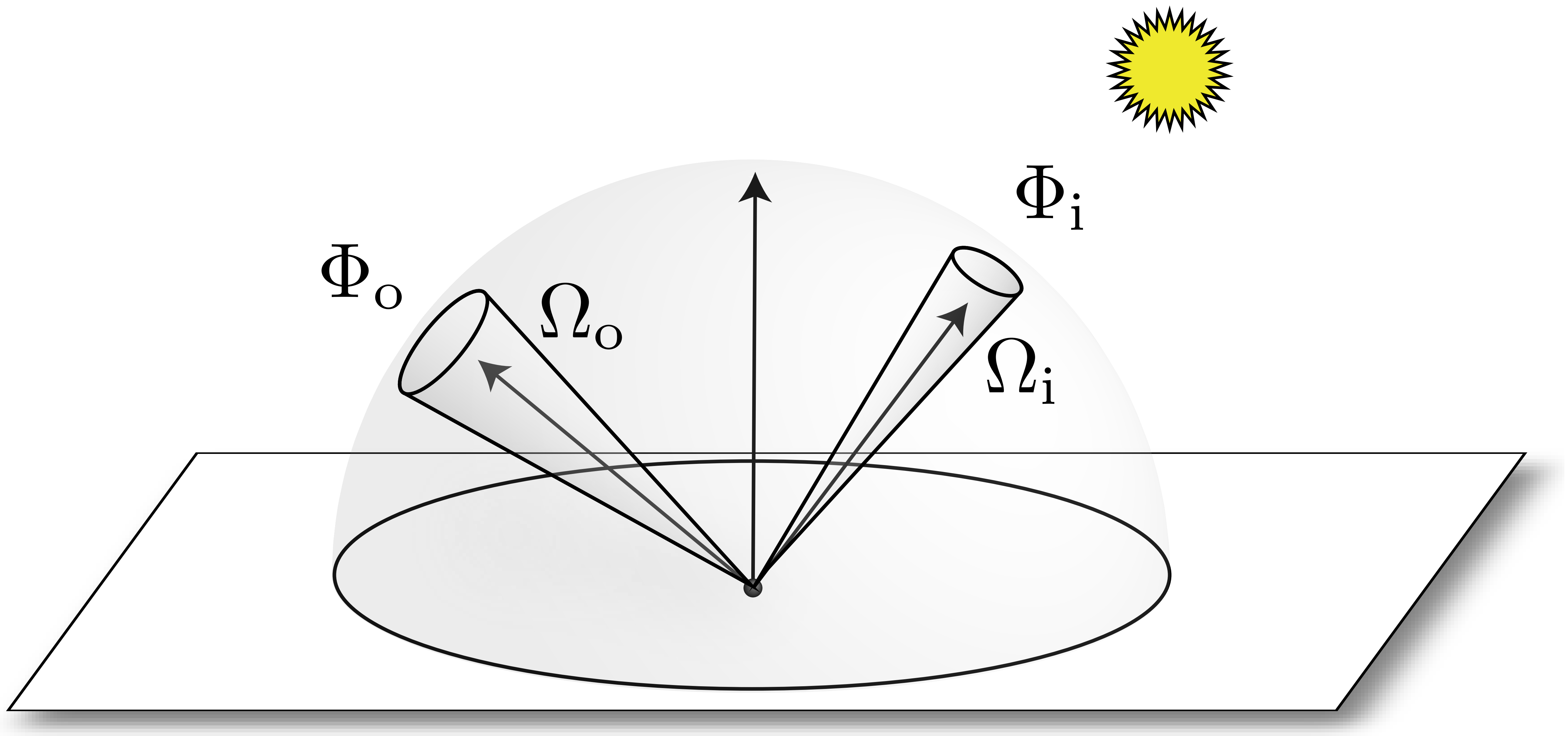
The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \rightarrow \omega_o) \equiv \frac{dL_o(\omega_o)}{dE_i(\omega_i)} \quad \left[\frac{1}{sr}\right]$$

BRDF energy conservation

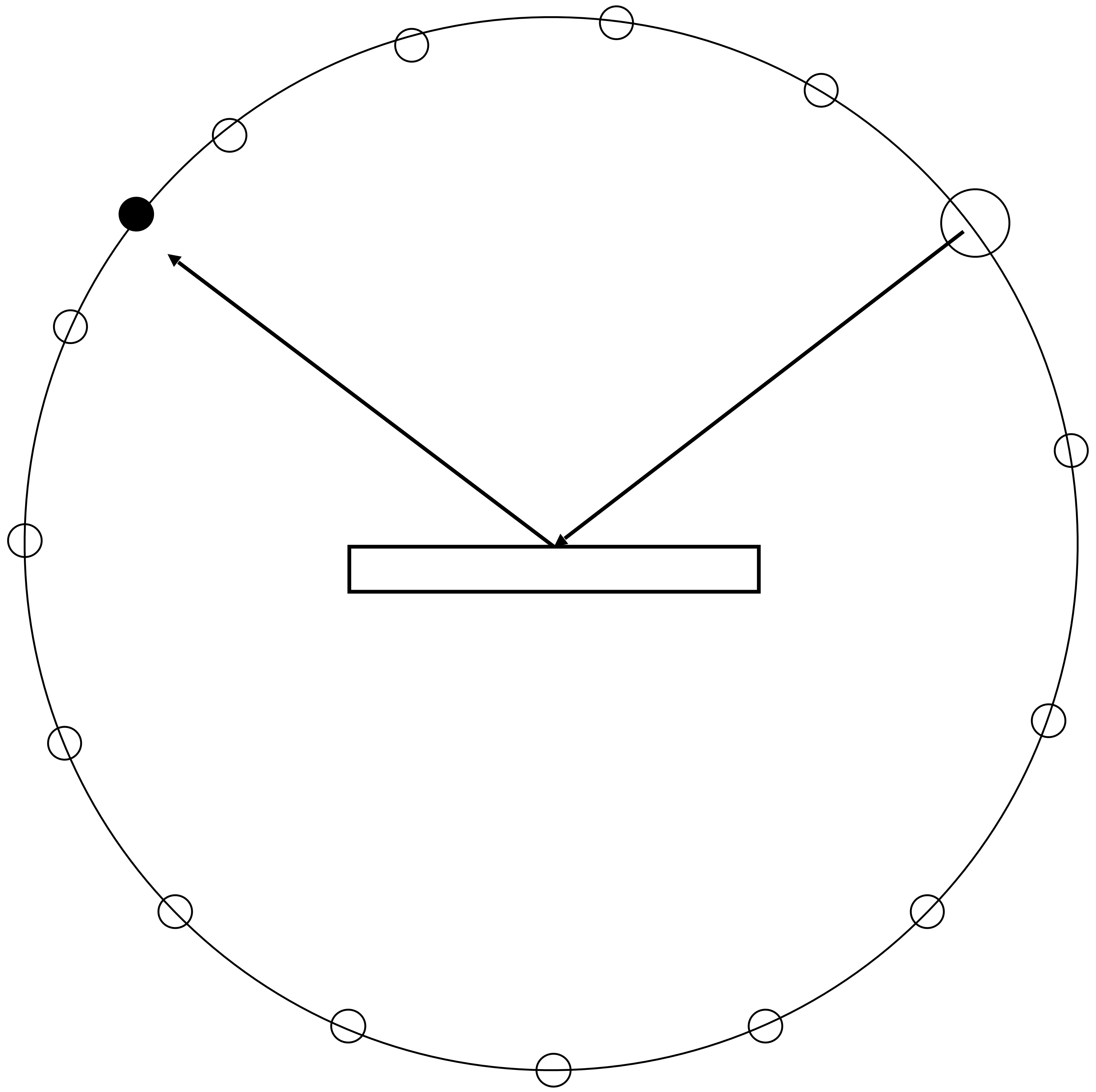


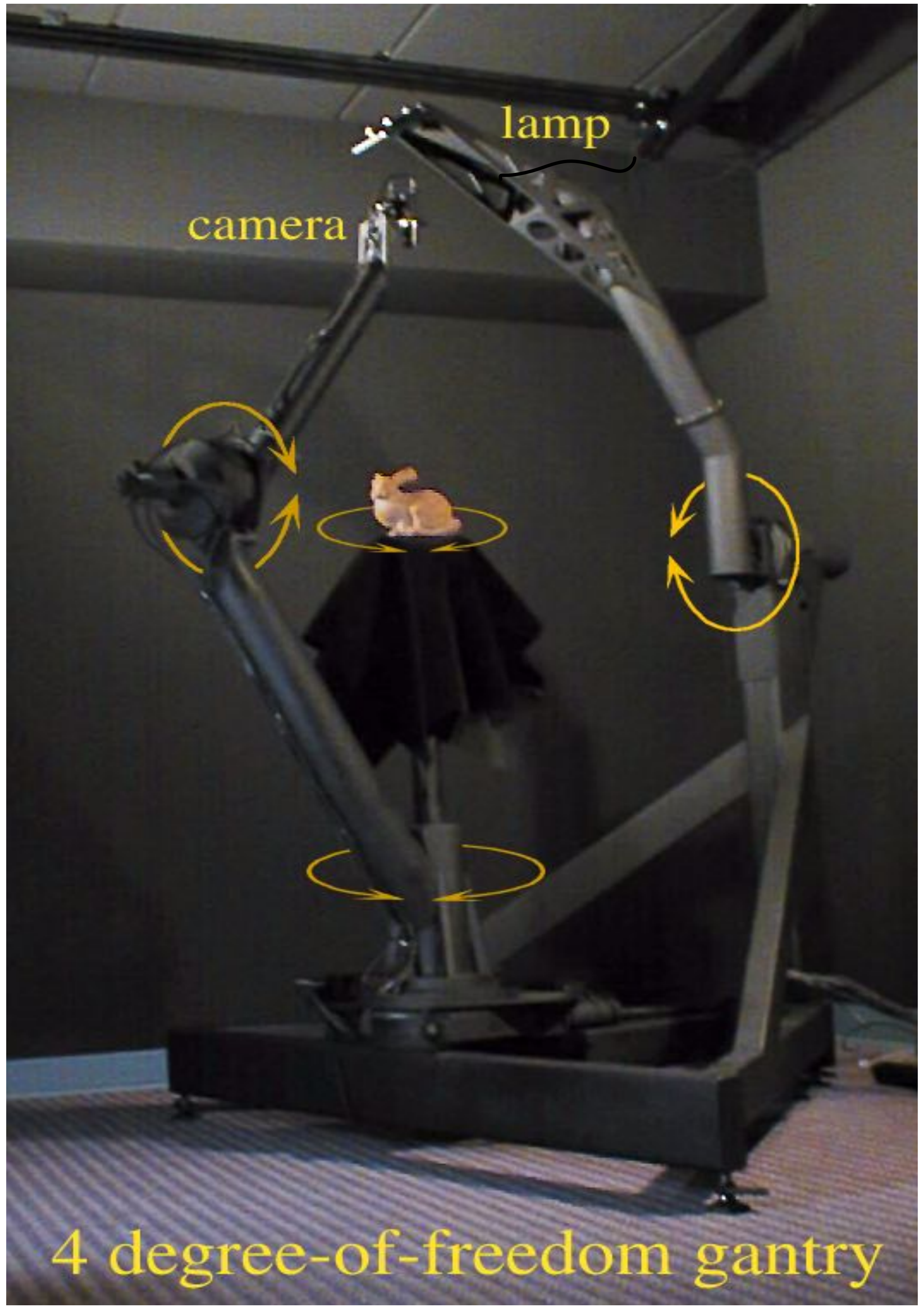
Reflectance

$$\rho = \frac{\Phi_o}{\Phi_i} = \frac{\int_{\Omega_o} L_o(\omega_o) \cos\theta_o \, d\omega_o}{\int_{\Omega_i} L_i(\omega_i) \cos\theta_i \, d\omega_i}$$

$$0 \leq \rho \leq 1$$

Stanford Gonioreflectometer

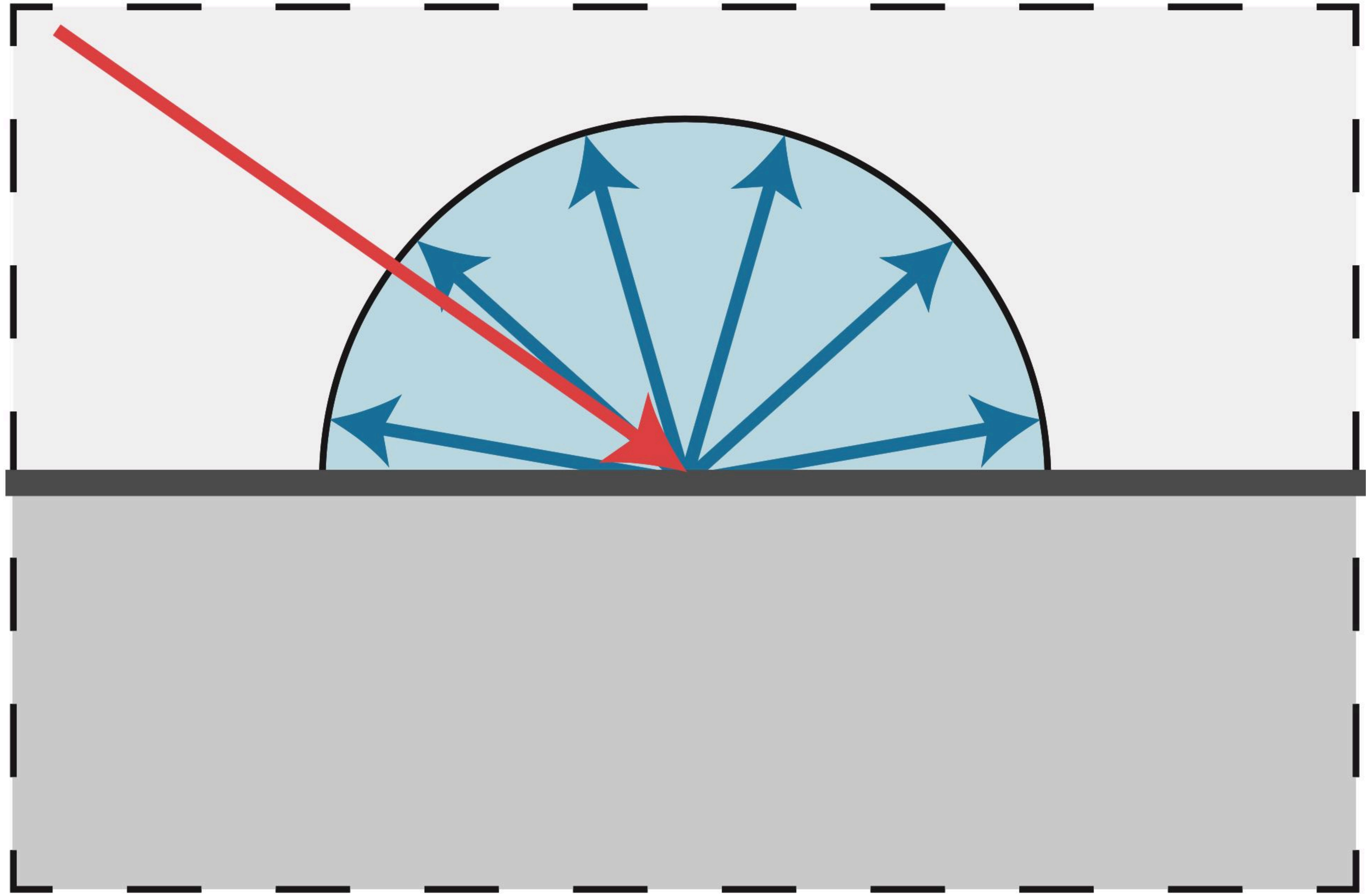




Reflection models

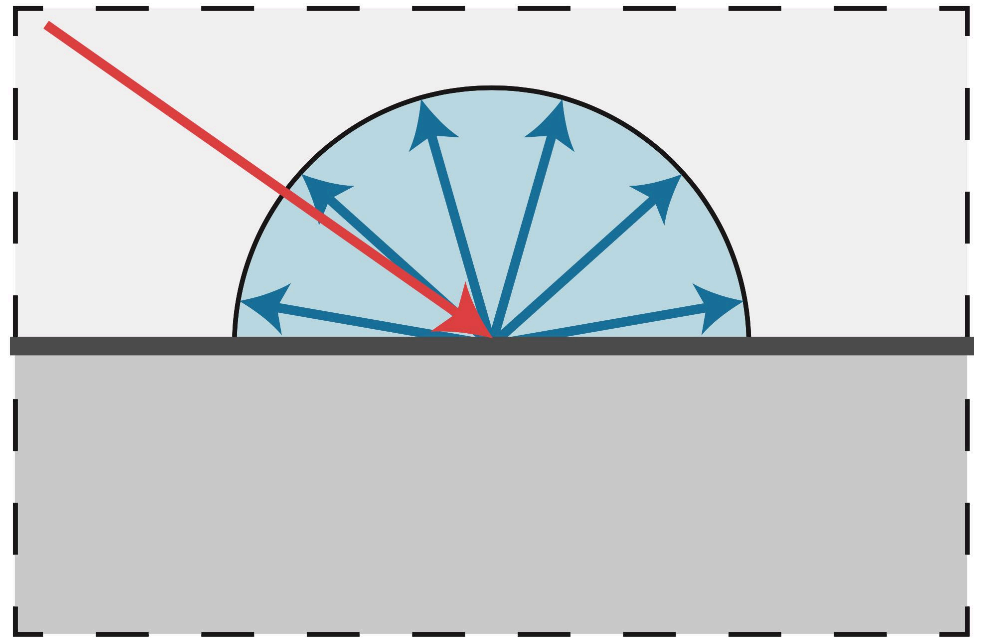
- *Reflection* is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency
- Choice of reflection function determines surface appearance

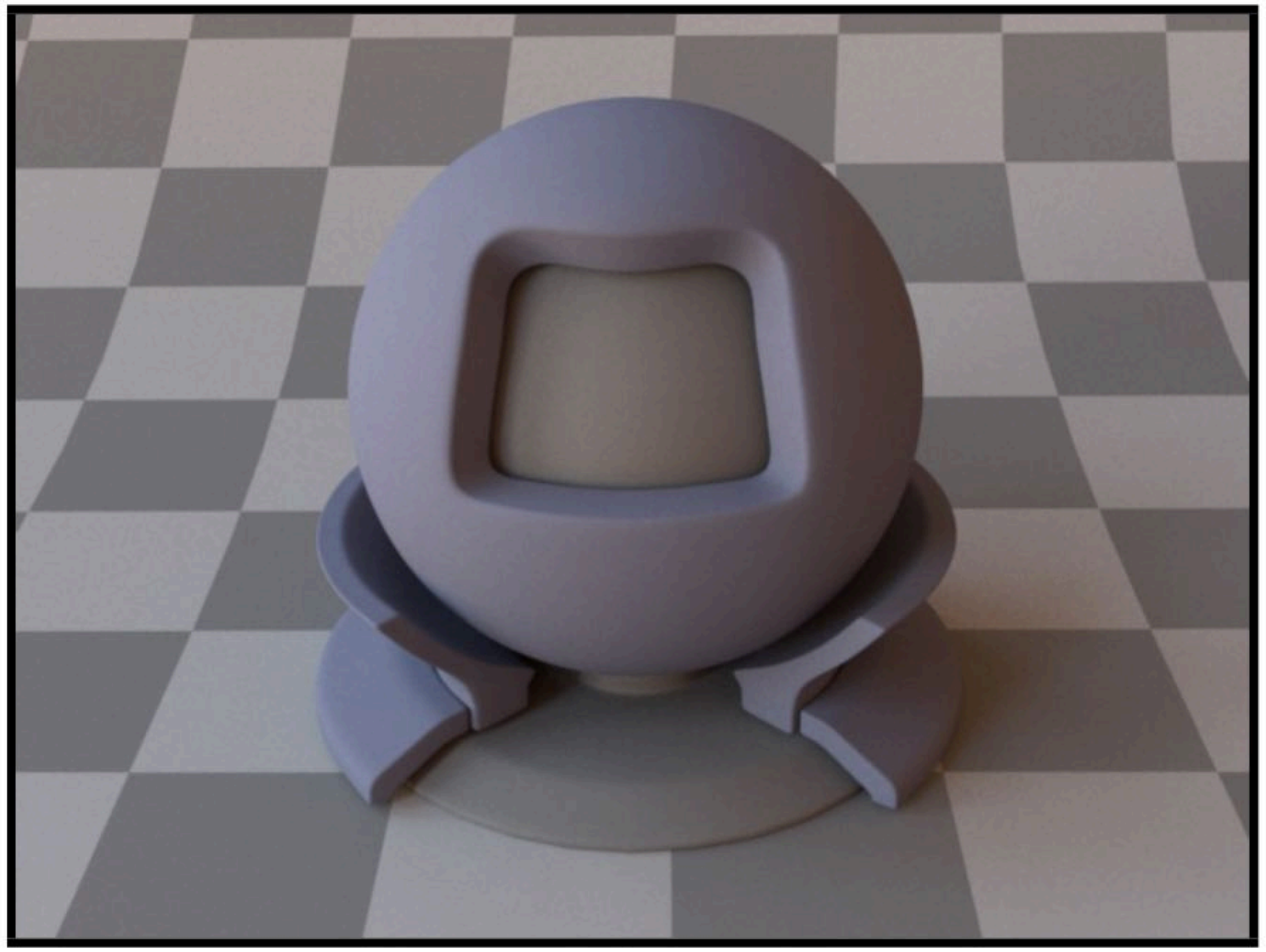
What is this material?



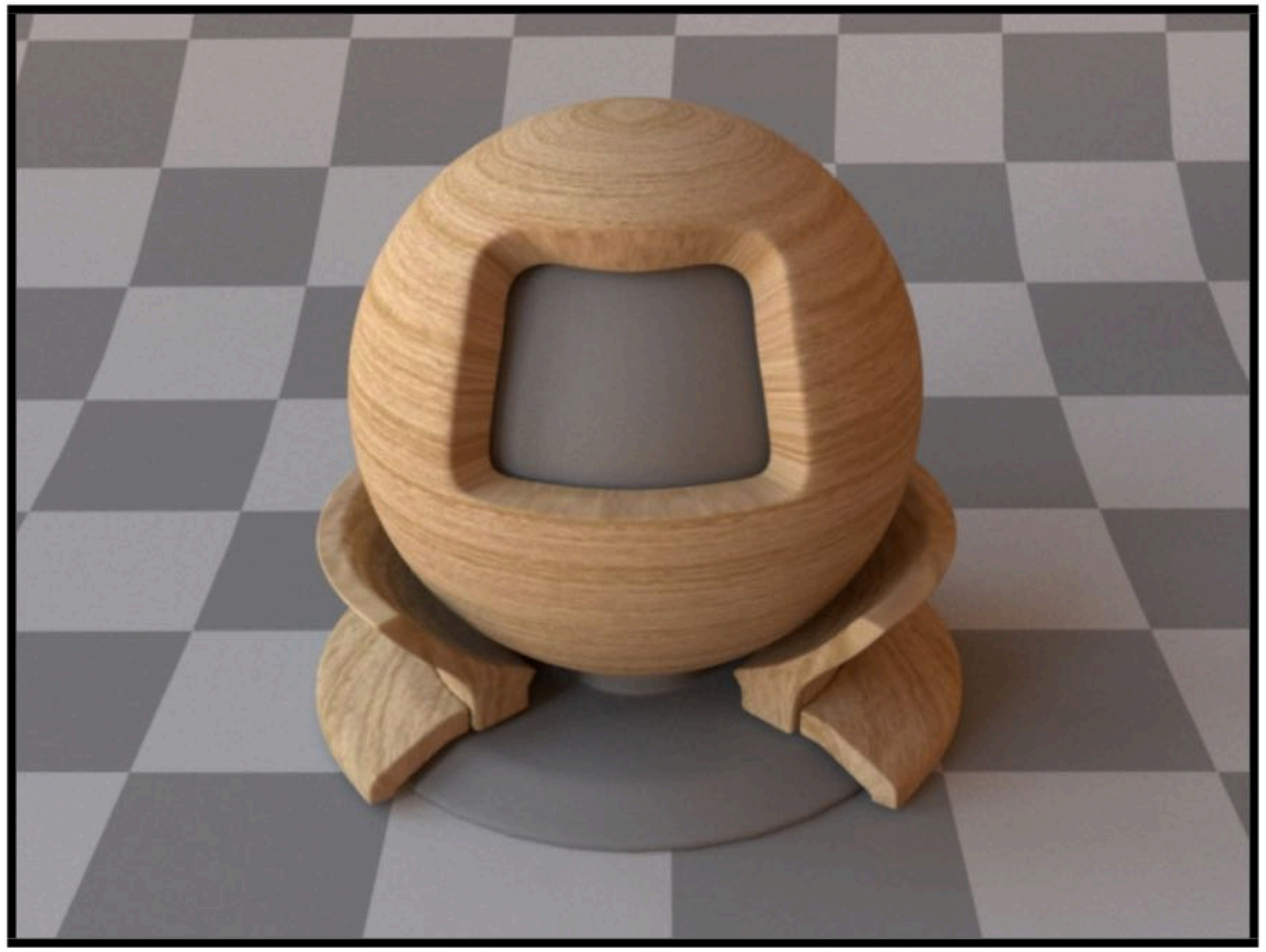
Light is scattered equally in all directions

Diffuse / Lambertian material





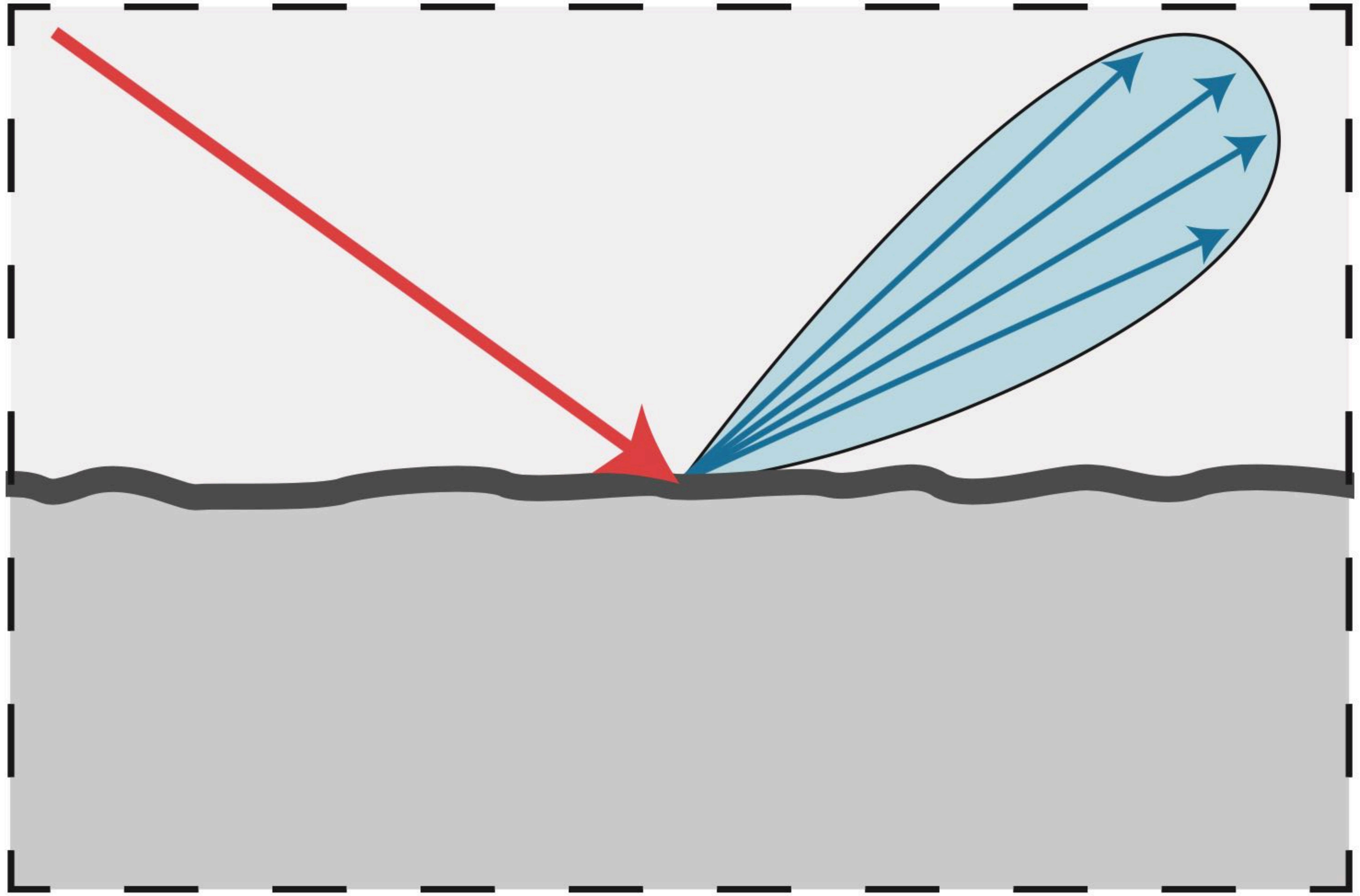
Uniform colored diffuse BRDF
Albedo (fraction of light reflected) is same
for all surface points p



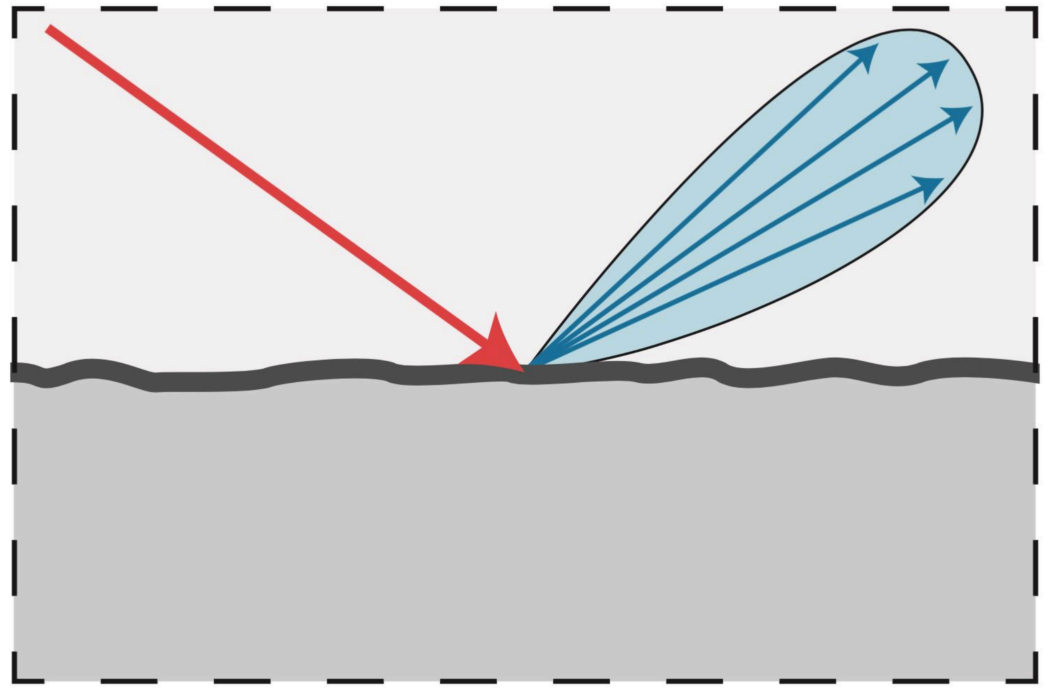
Textured diffuse BRDF
Albedo is spatially varying,
and is encoded in texture map.

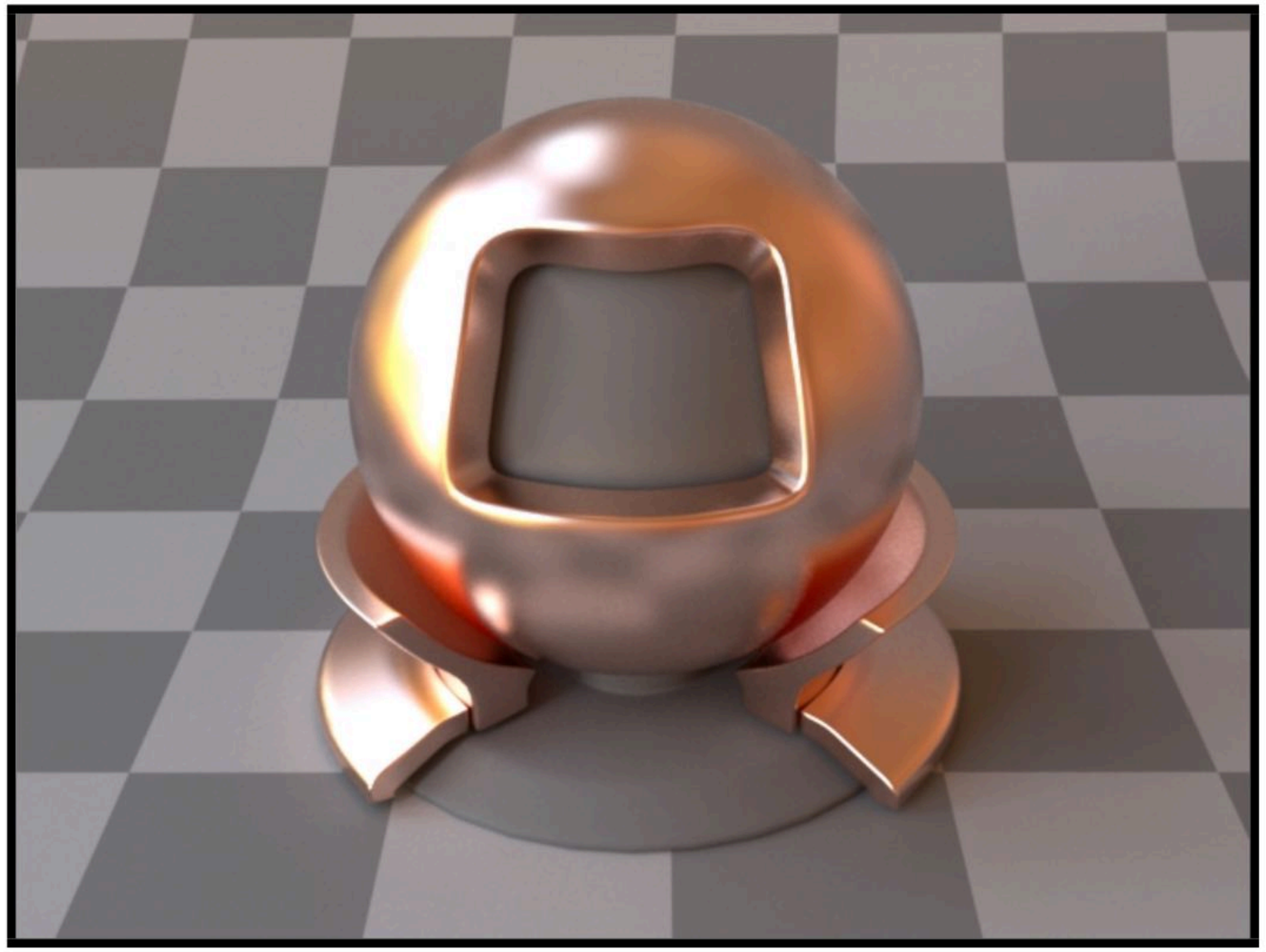
[Mitsuba renderer, Wenzel Jakob, 2010]

What is this material?

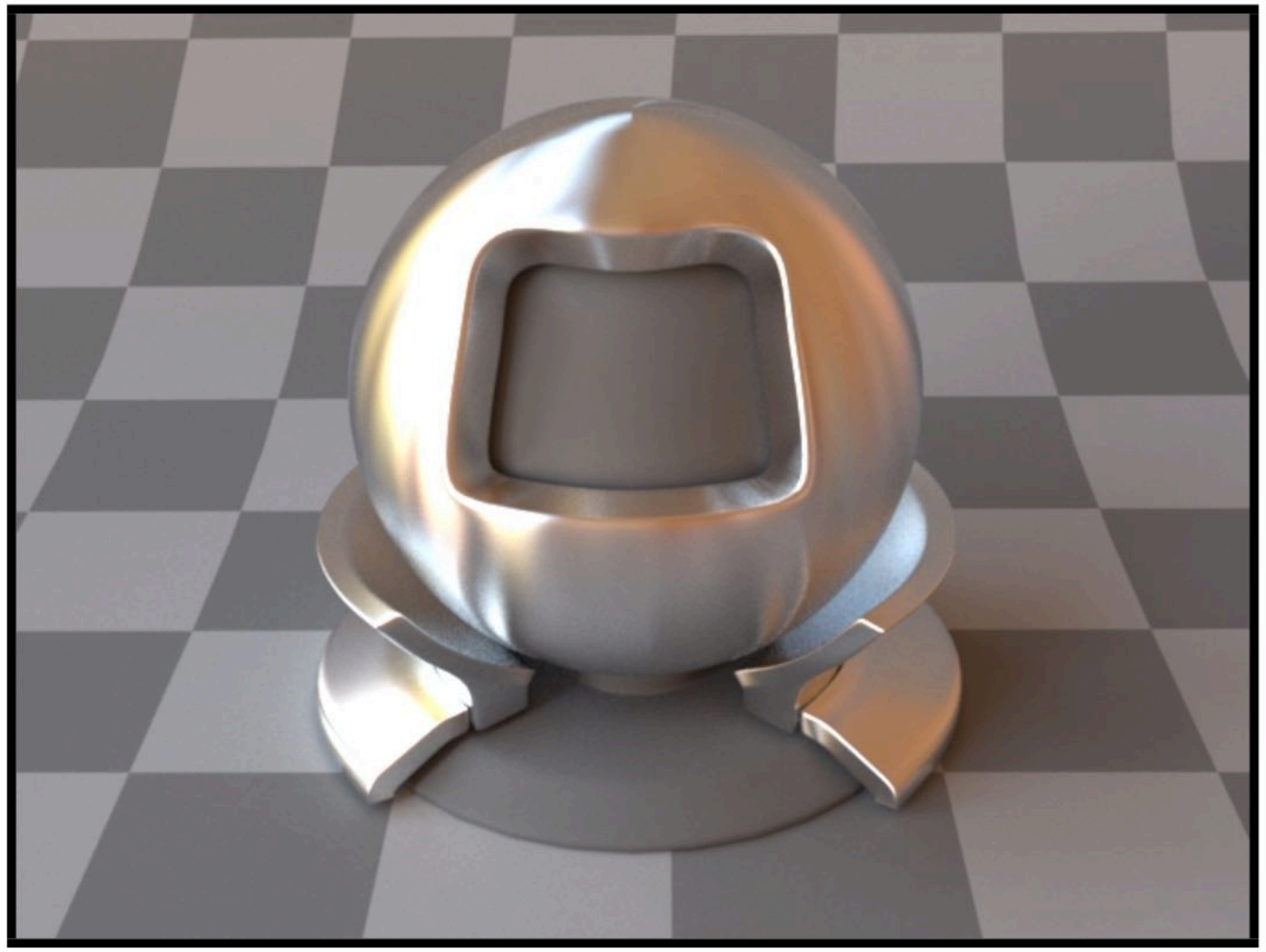


Glossy material (BRDF)





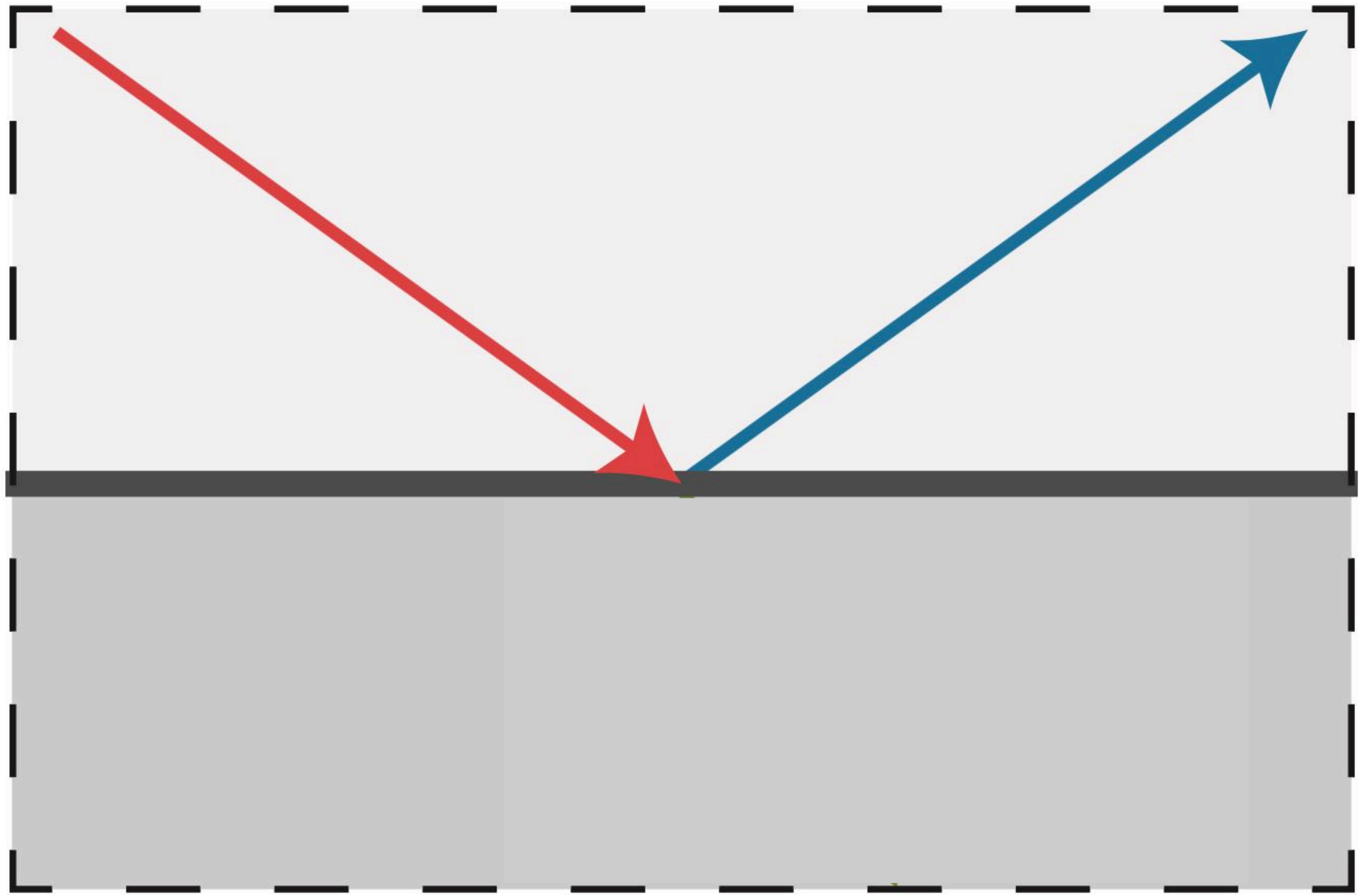
Copper



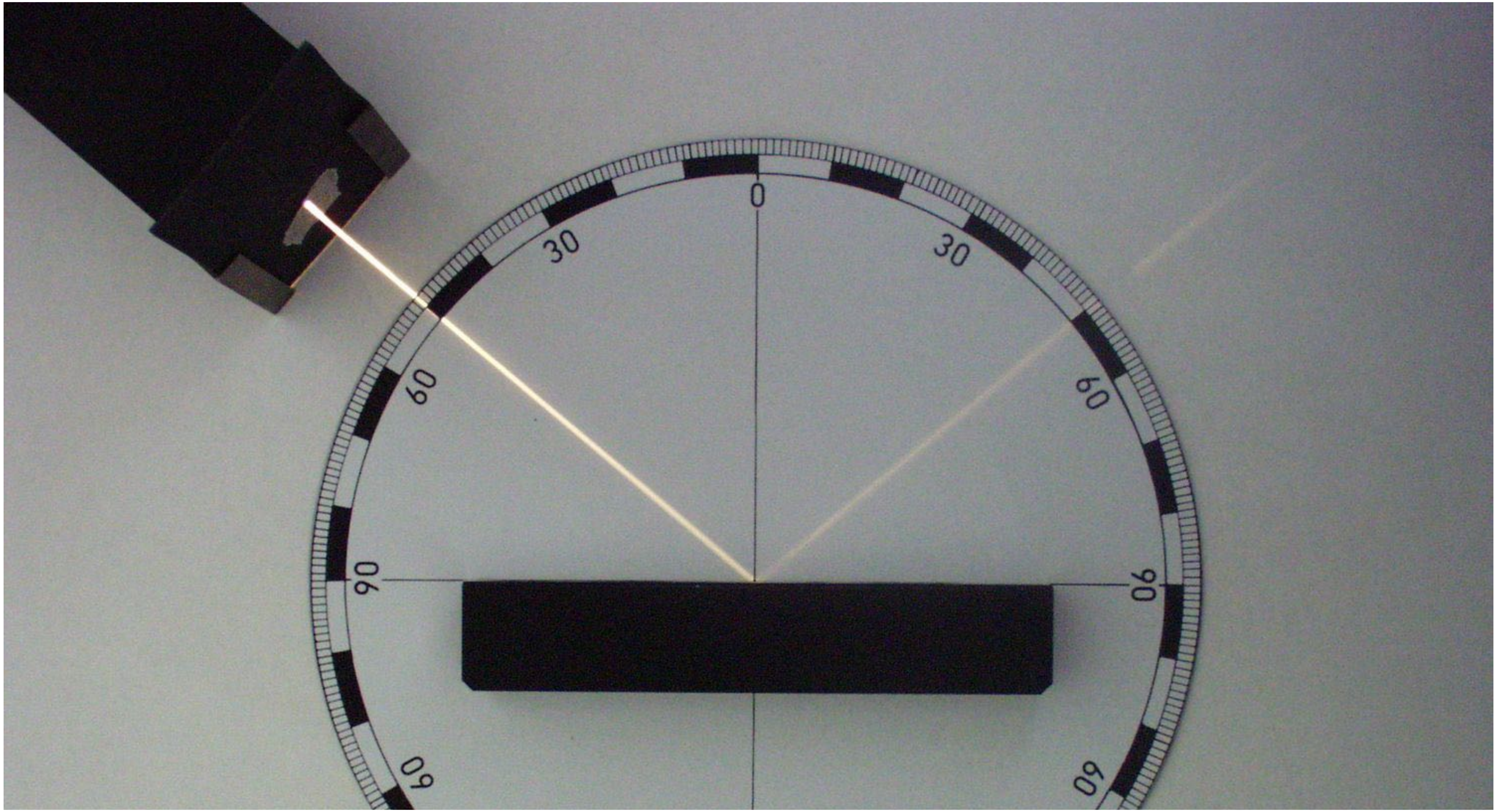
Aluminum

[Mitsuba renderer, Wenzel Jakob, 2010]

What is this material?



Perfect specular reflection



[Zátonyi Sándor]

Perfect specular reflection

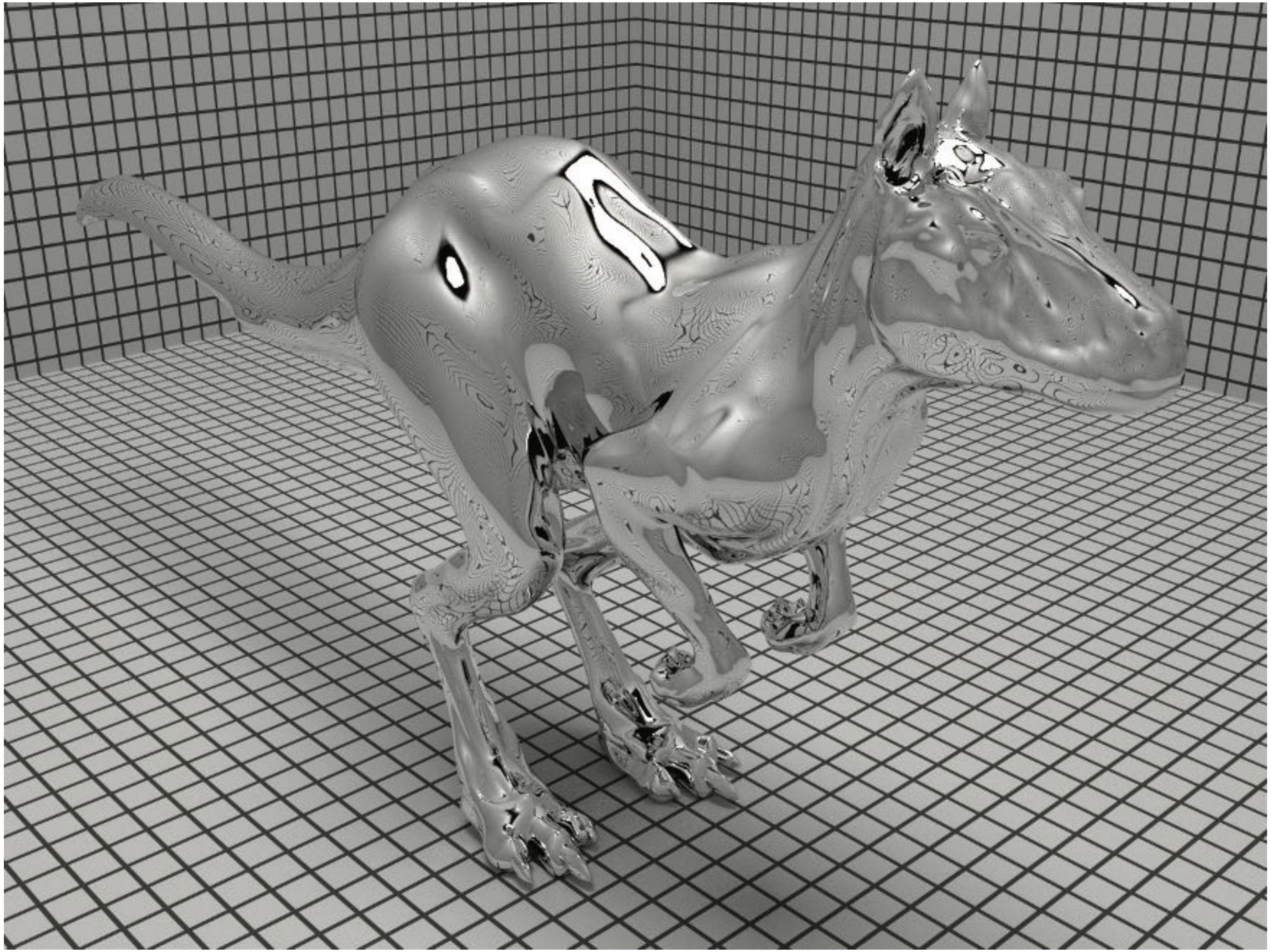
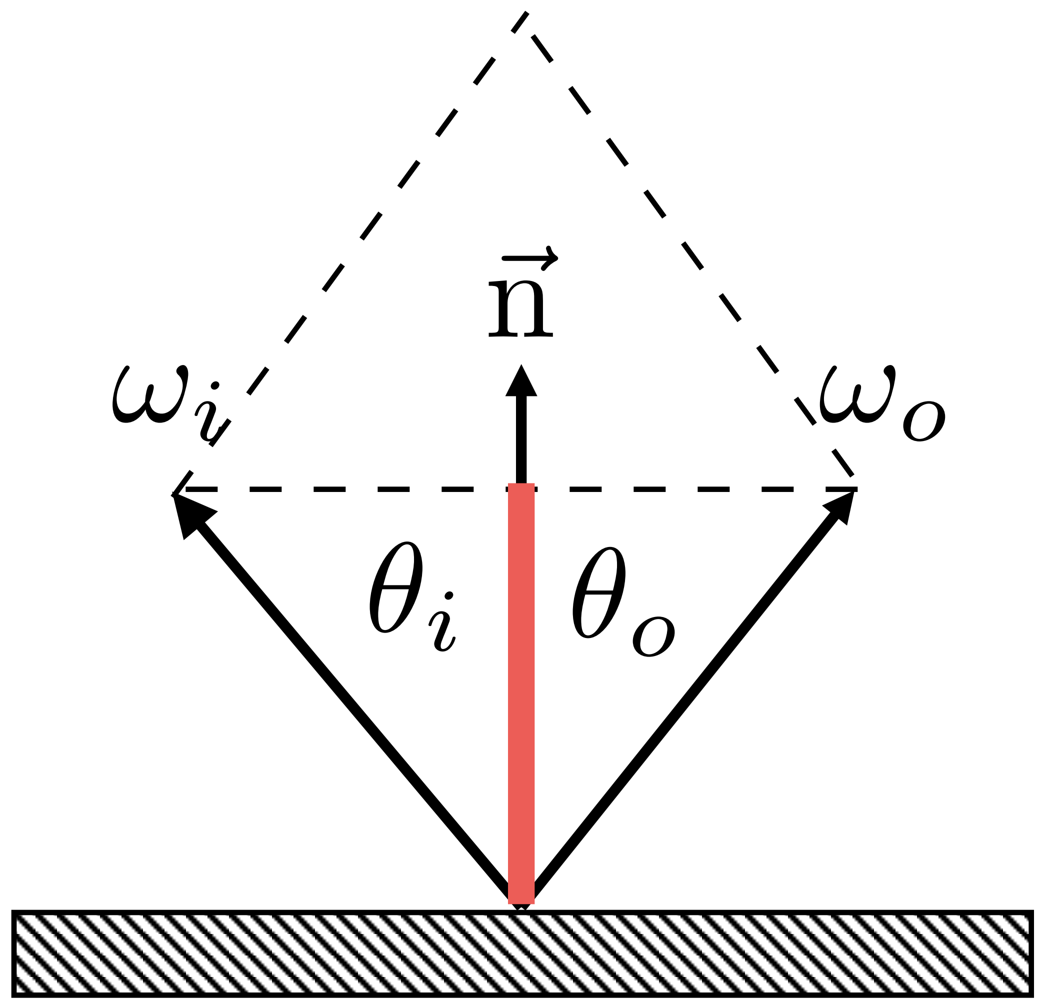
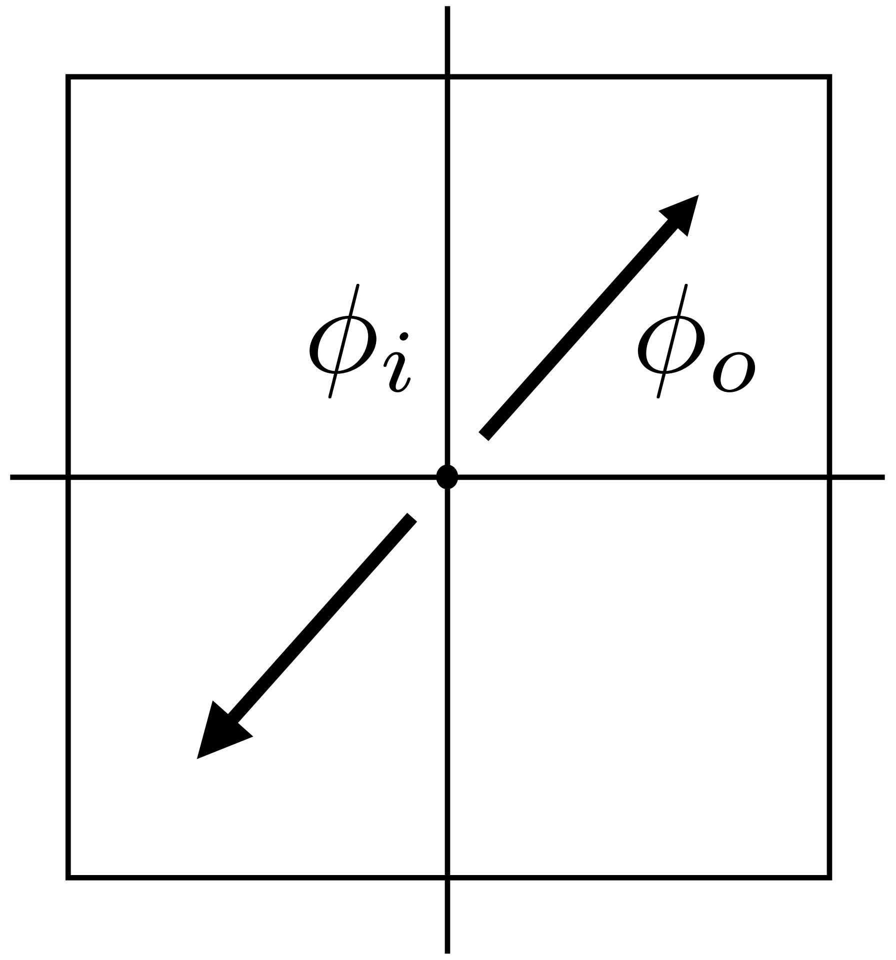


Image credit: PBRT

Calculating direction of specular reflection



$$\theta = \theta_o = \theta_i$$



$$\phi_o = (\phi_i + \pi) \bmod 2\pi$$

$$\omega_o + \omega_i = 2\cos\theta\,\vec{n} = 2(\omega_i \cdot \vec{n})\vec{n}$$

$$\omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n}$$

How might you render a specular surface

- Compute direction from surface point *p* to camera = w_o
- Given normal at *p*, compute reflection direction w_i
- Light reflected in direction w_o is light arriving from direction w_i
- How do you measure light arriving from w_i?

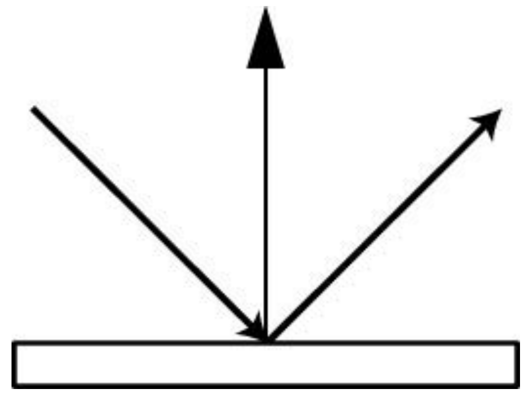
One idea…
look up amount in environment map!
(more on this later)

Pixel (x,y) stores radiance L from direction (ϕ, θ)

Some basic reflection functions

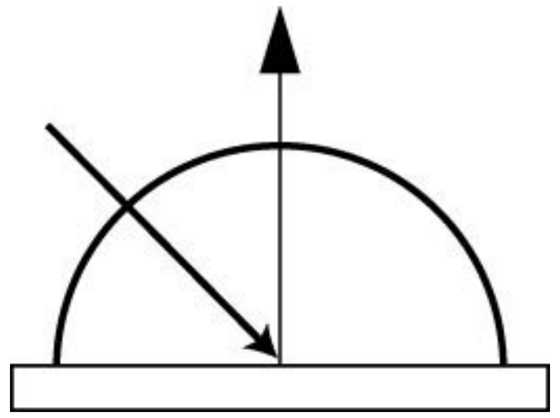
- **Ideal specular**

  Perfect mirror



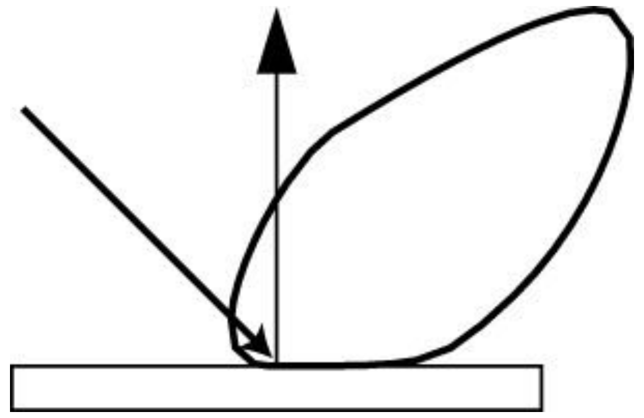
- **Ideal diffuse**

  Uniform reflection in all directions



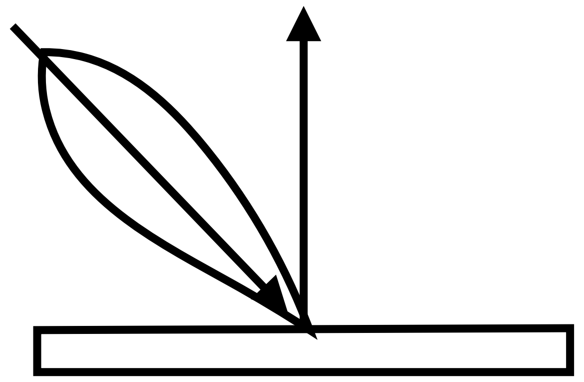

- **Glossy specular**

  Majority of light distributed in reflection direction



- **Retro-reflective**

  Reflects light back toward source



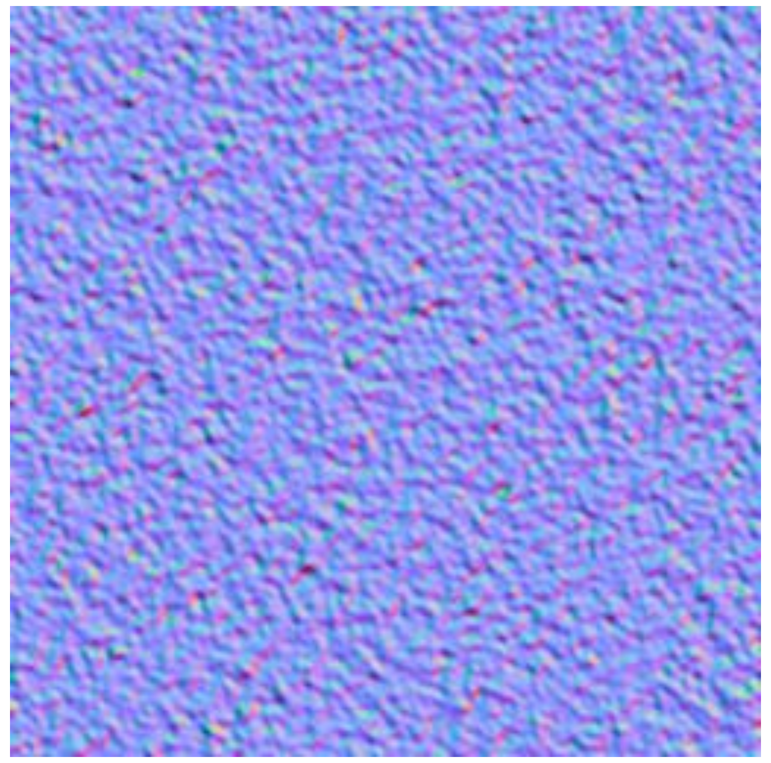

Diagrams illustrate how incoming light energy from given direction is reflected in various directions.

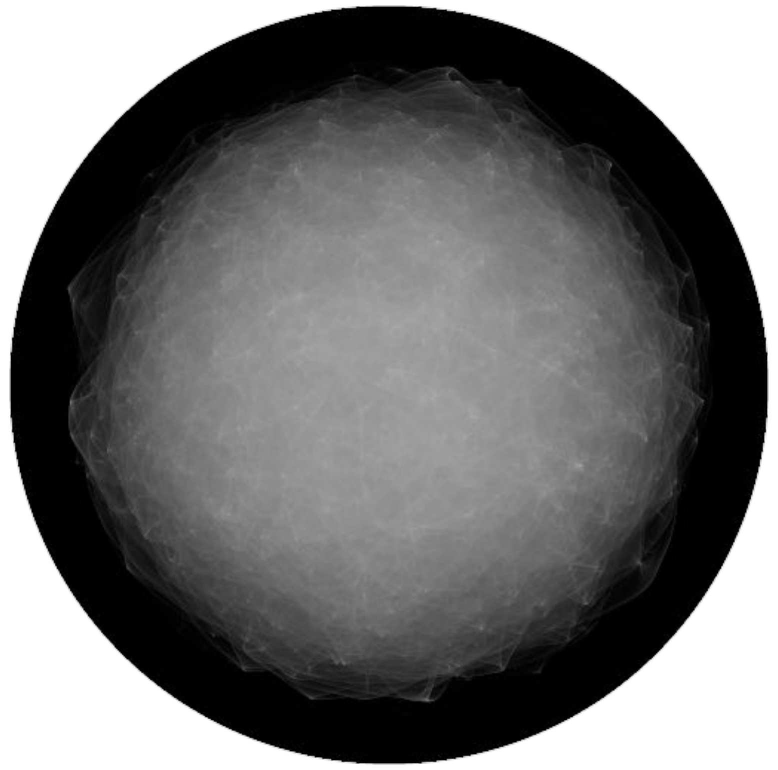
More complex materials

Isotropic / anisotropic materials (BRDFs)

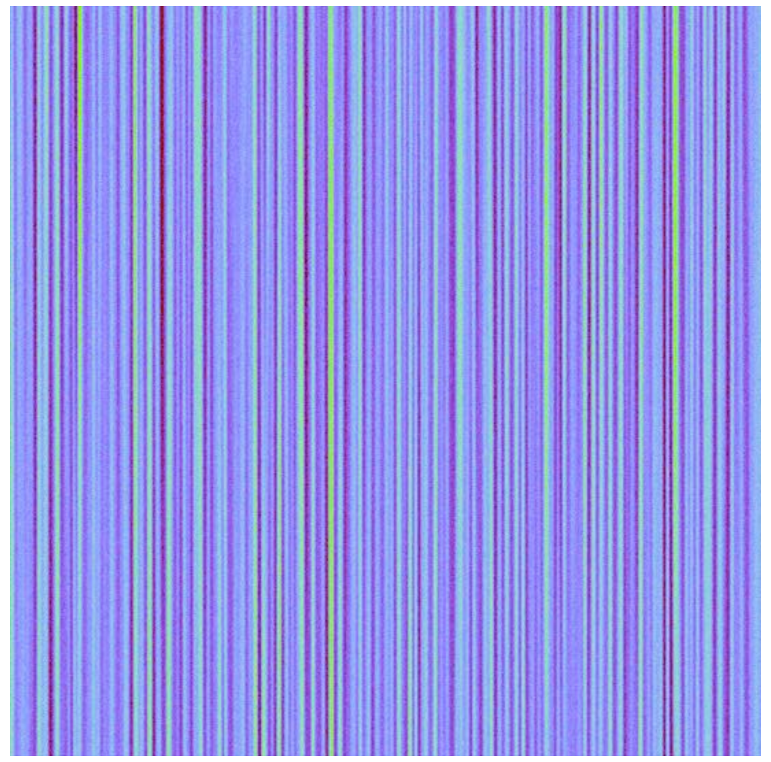
Key: directionality of underlying surface

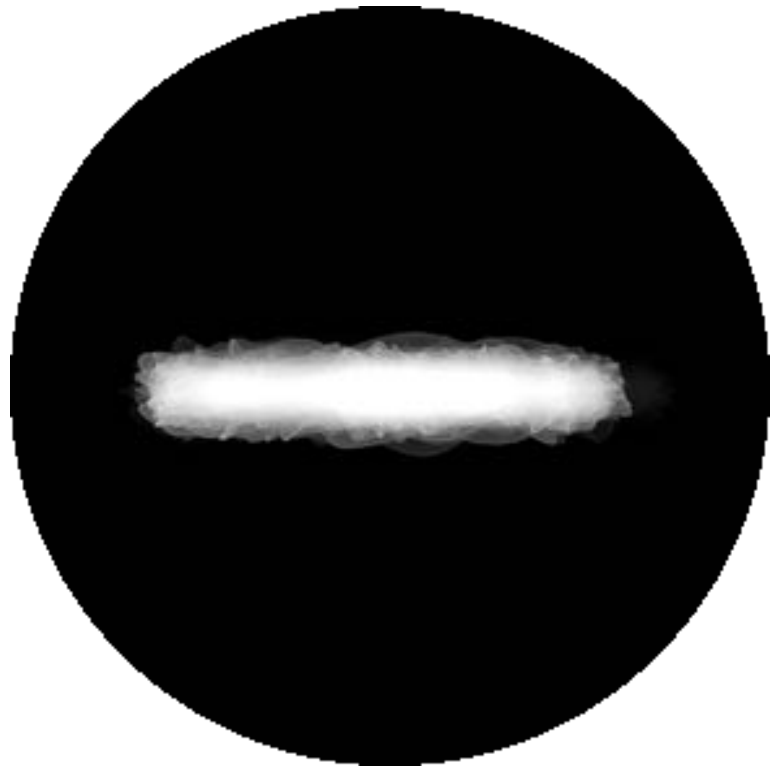
Isotropic





Anisotropic





Surface (normals)

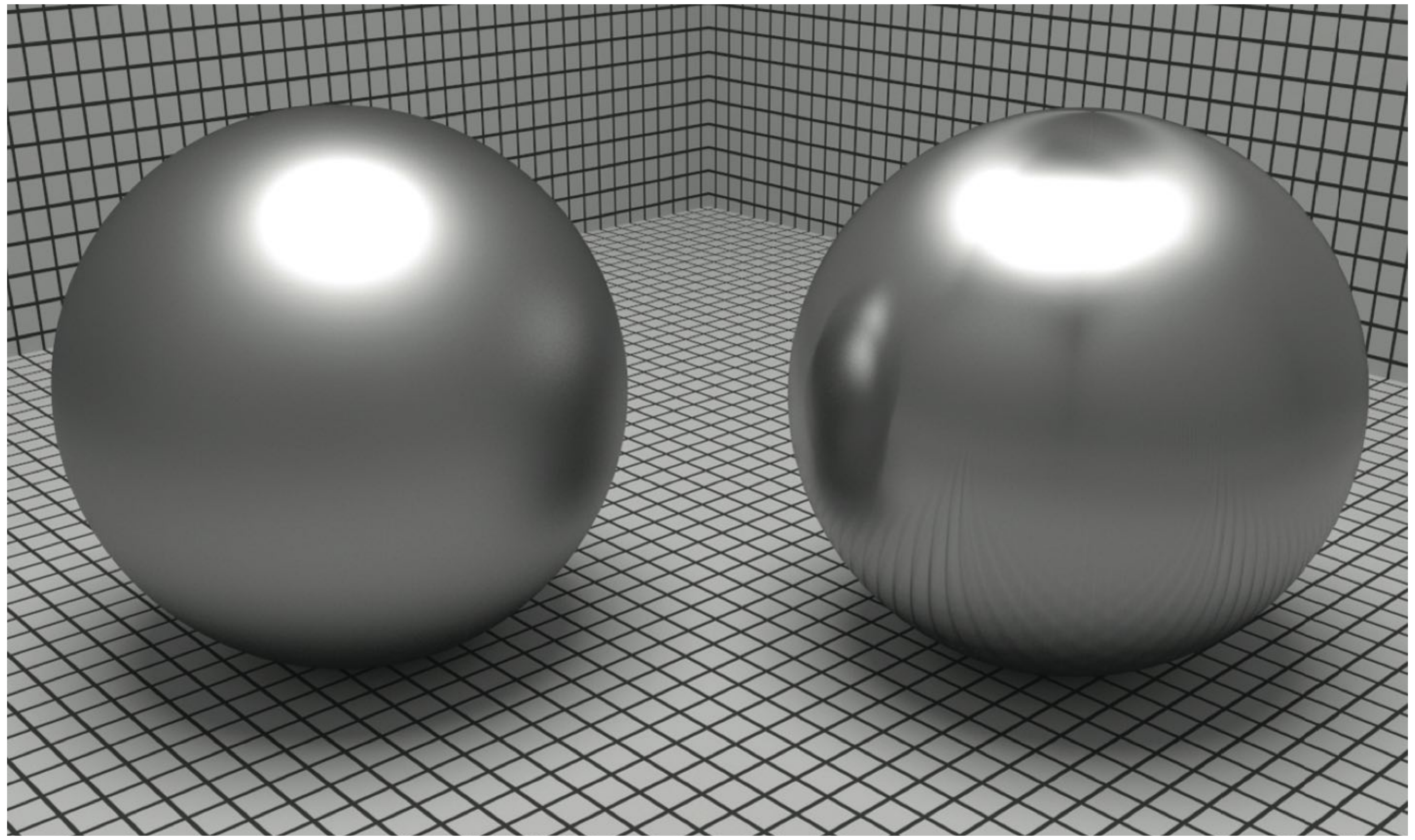
BRDF (fix wi, vary wo)

Anisotropic BRDFs

Reflection depends on azimuthal angle ϕ

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$

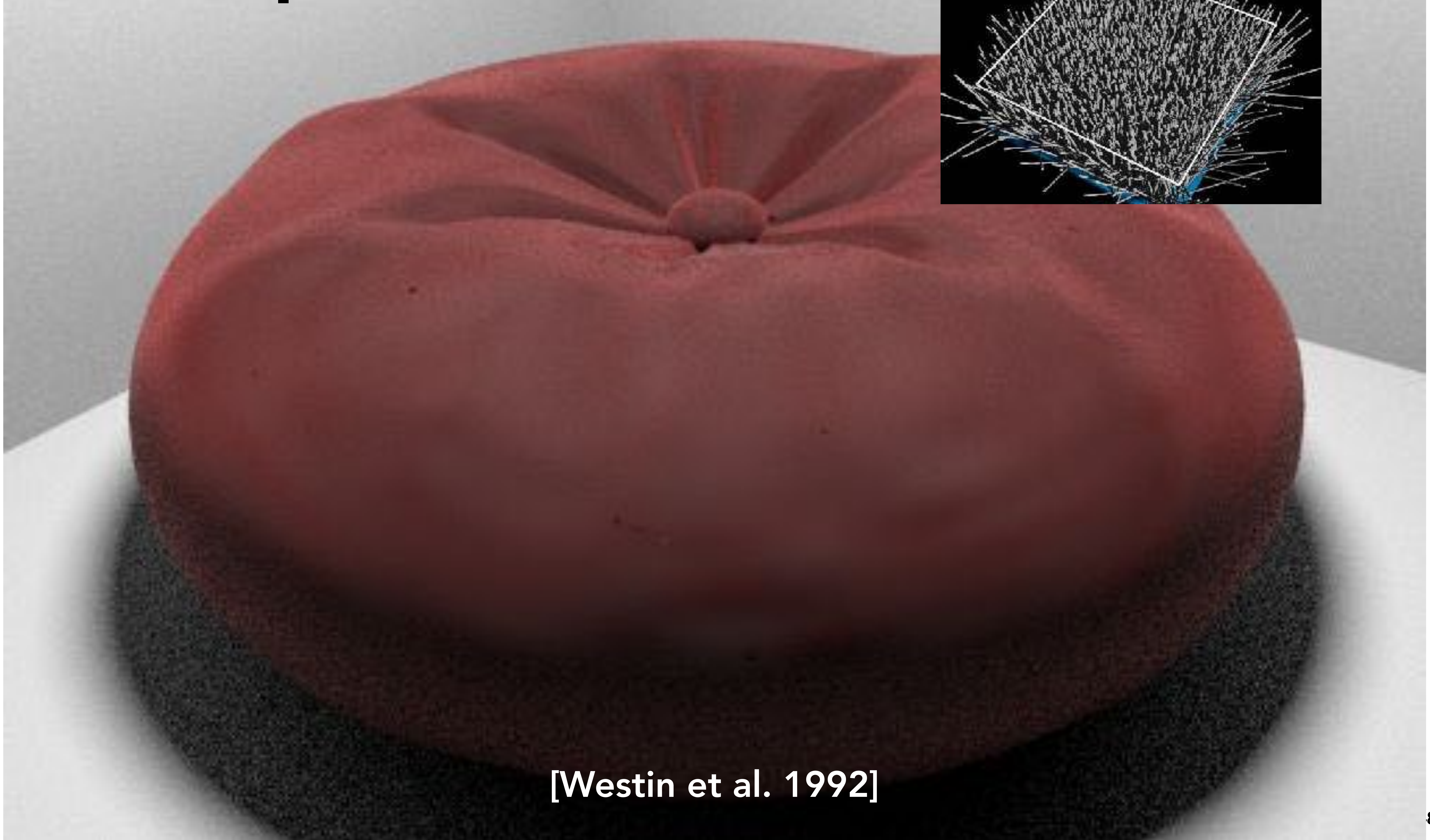
Results from oriented microstructure of surface, e.g., brushed metal



Anisotropic BRDF: Nylon

[Westin et al. 1992]

Anisotropic BRDF: Velvet

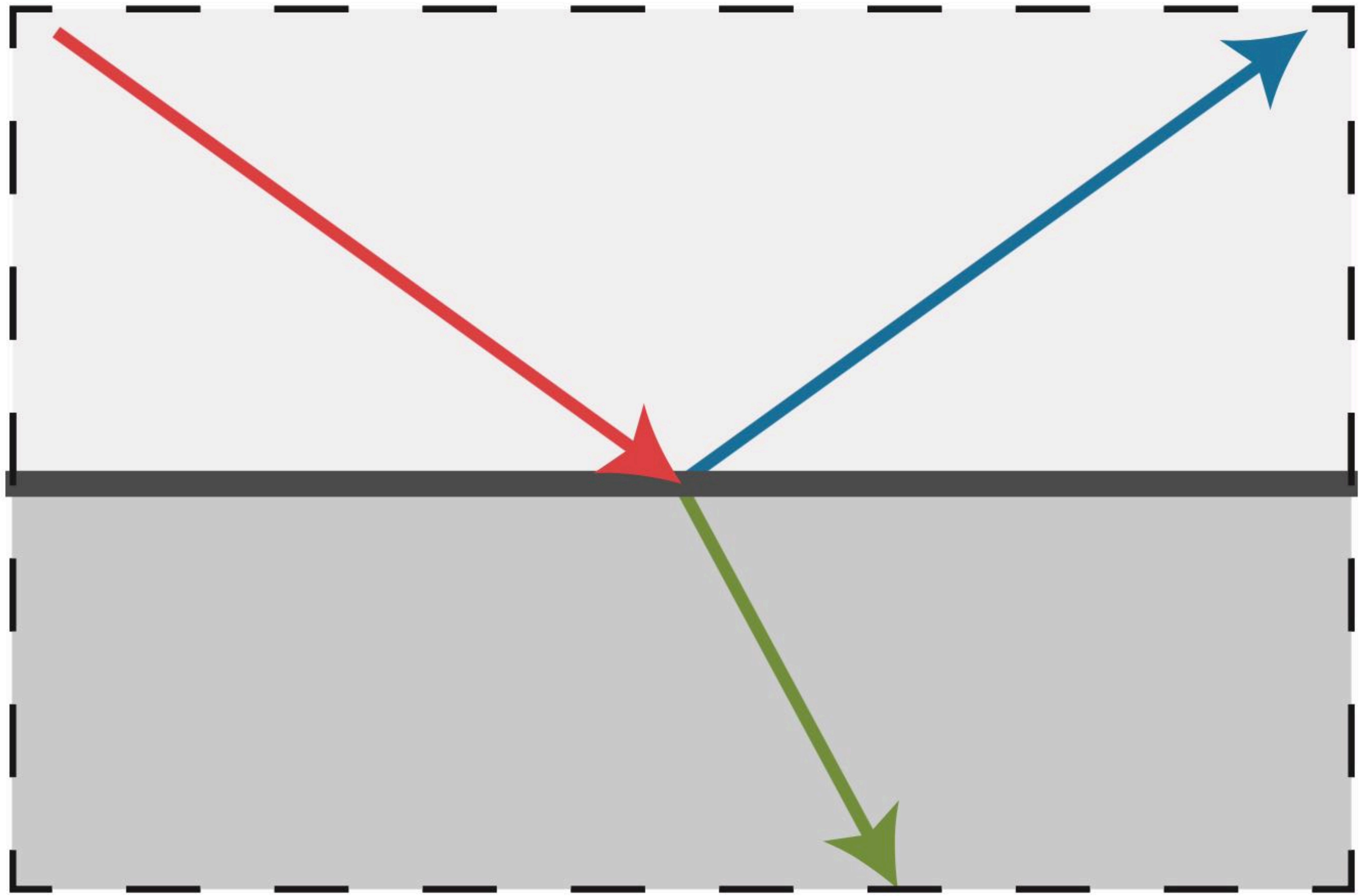


[Westin et al. 1992]

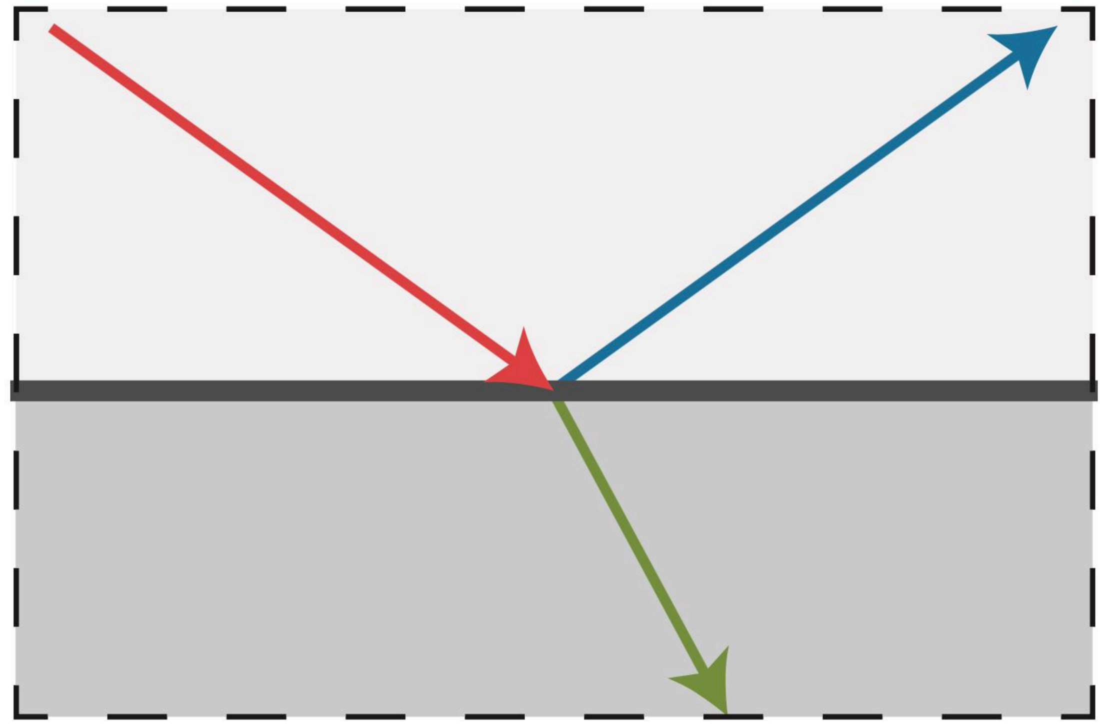
Anisotropic BRDF: Velvet

[https://www.youtube.com/watch?v=2hjoW8TYTd4]

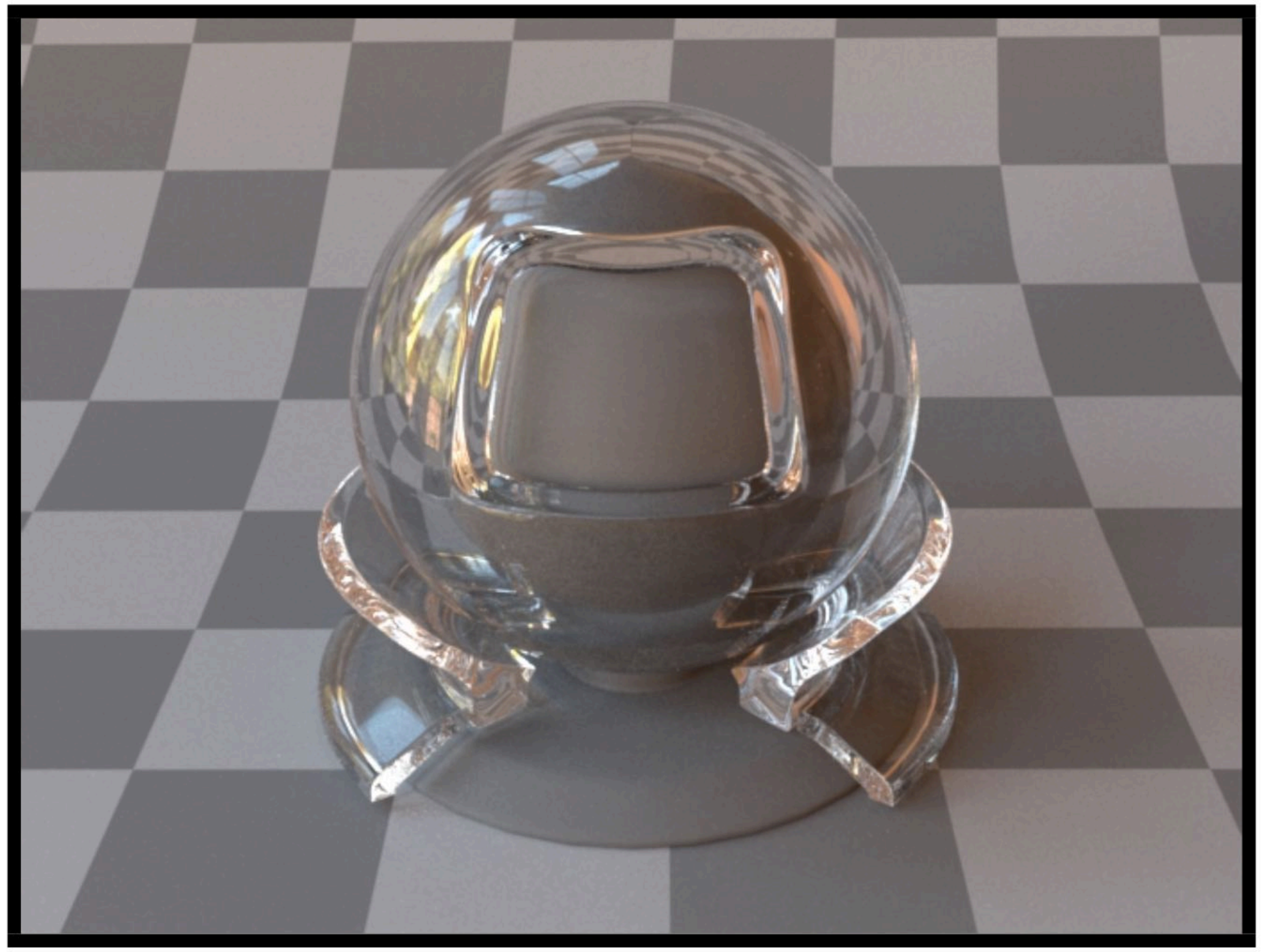
What is this material?



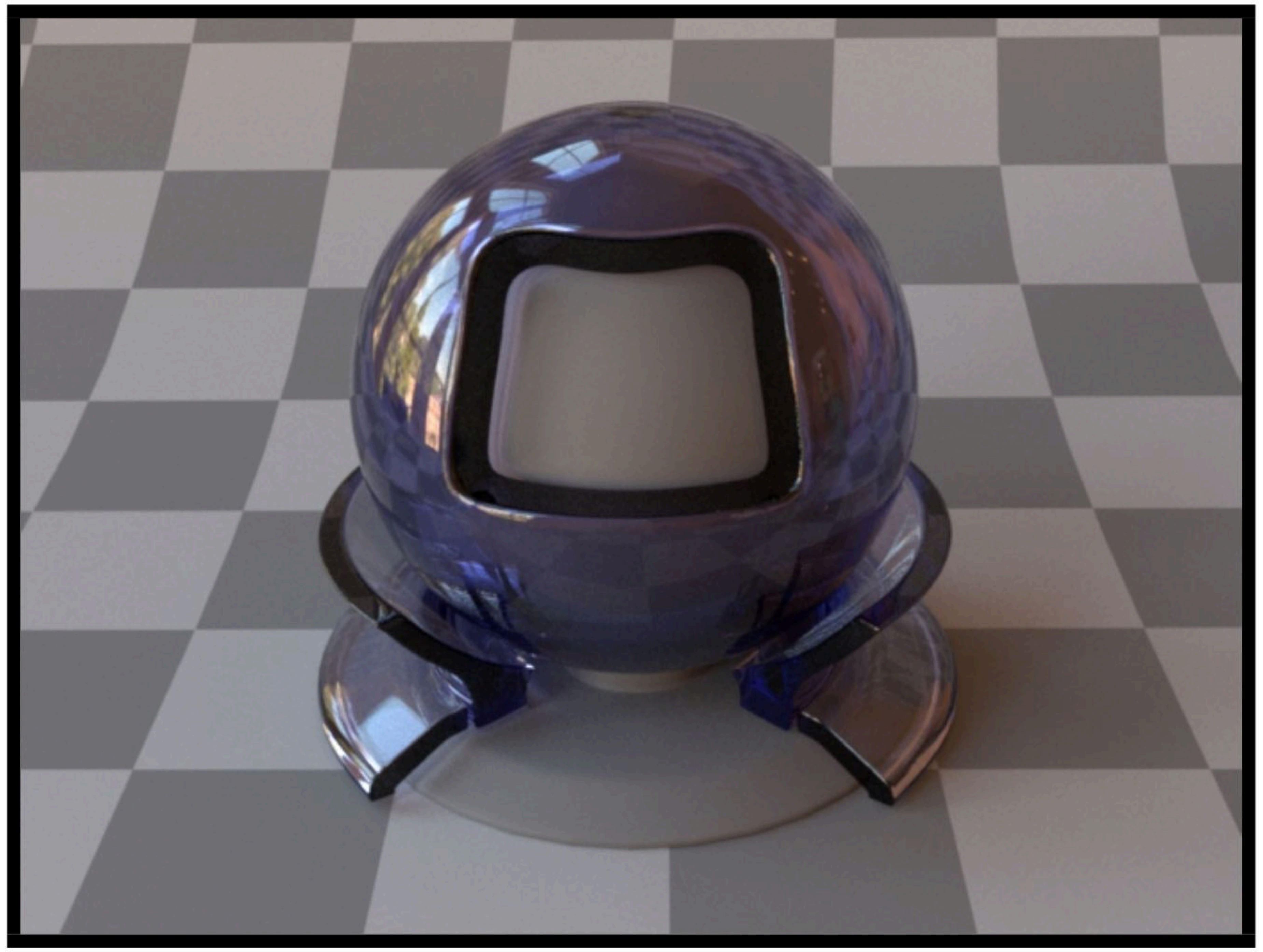
Ideal reflective / refractive material (BxDF *)



[Mitsuba renderer, Wenzel Jakob, 2010]



Air <-> water interface



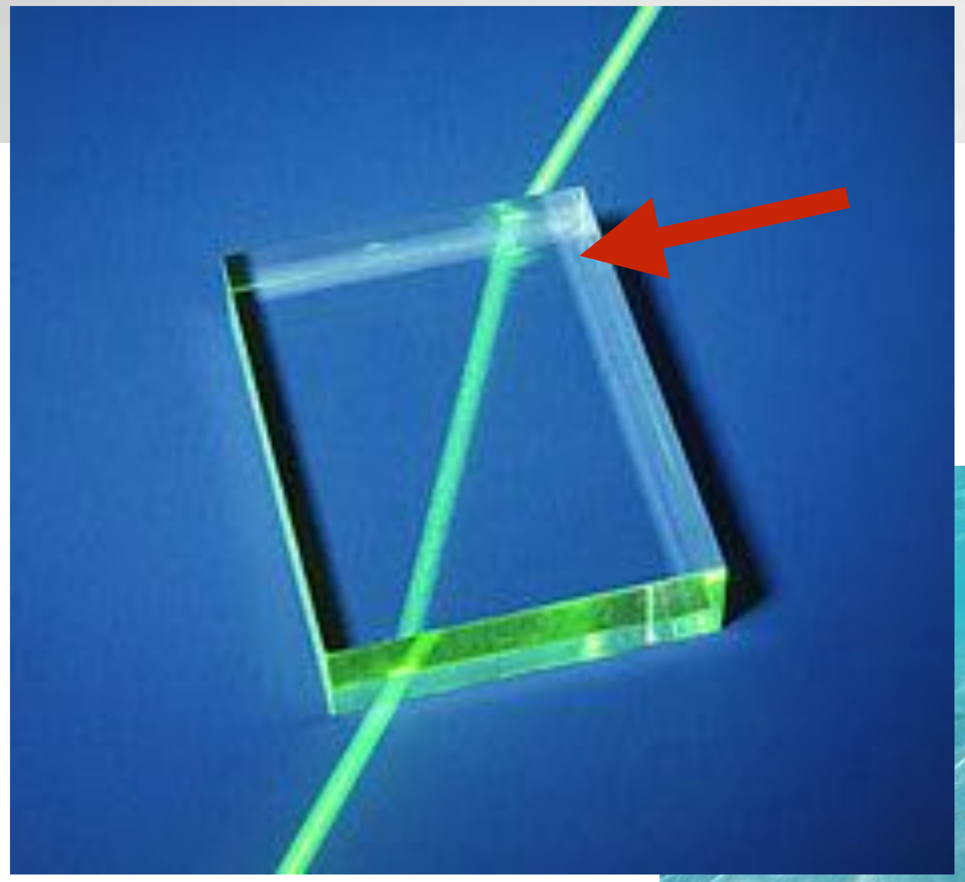
Air <-> glass interface
(with absorption)

* X stands in for reflectance “r”, scattering, transmission, etc.

Transmission

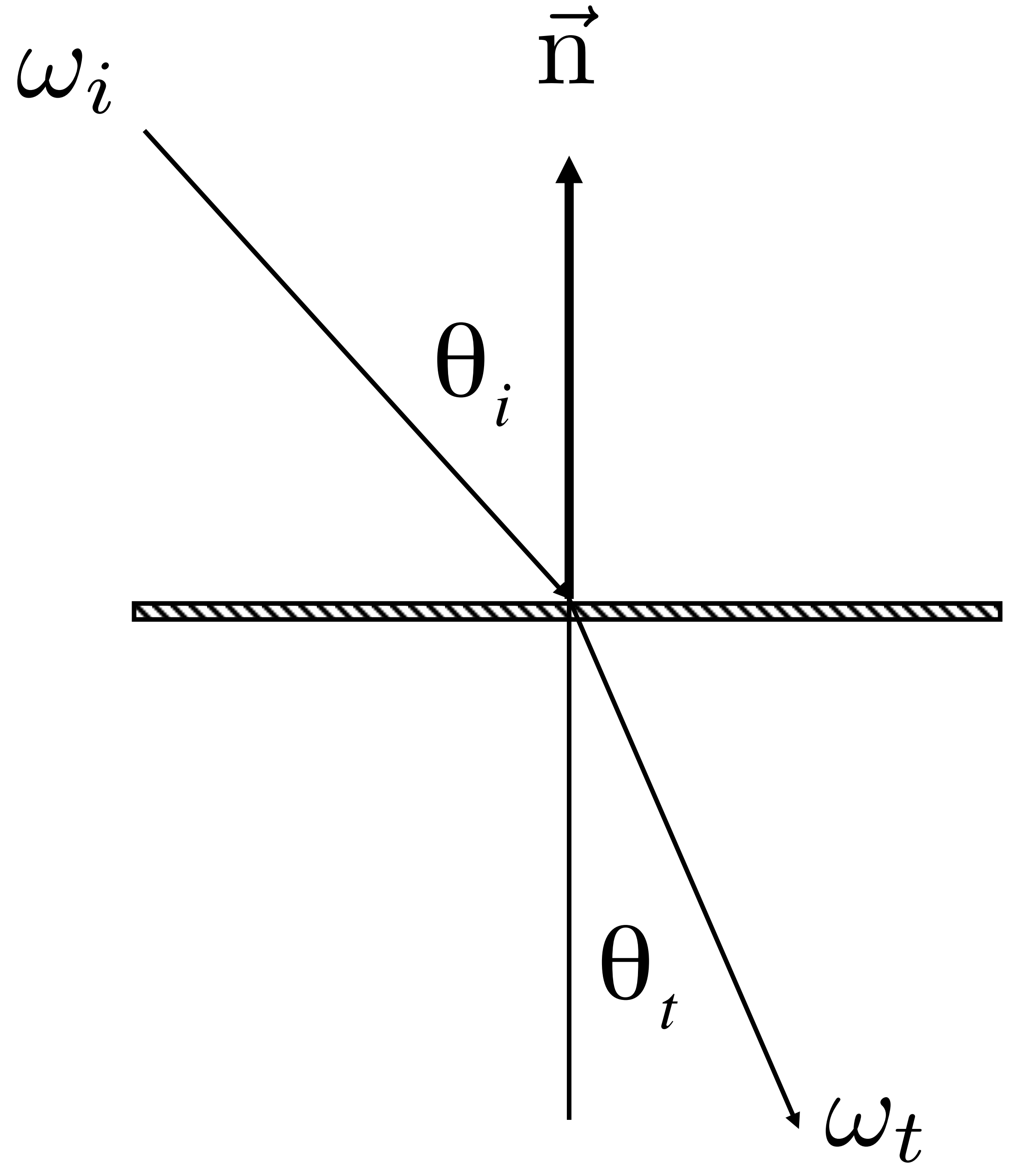
In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.

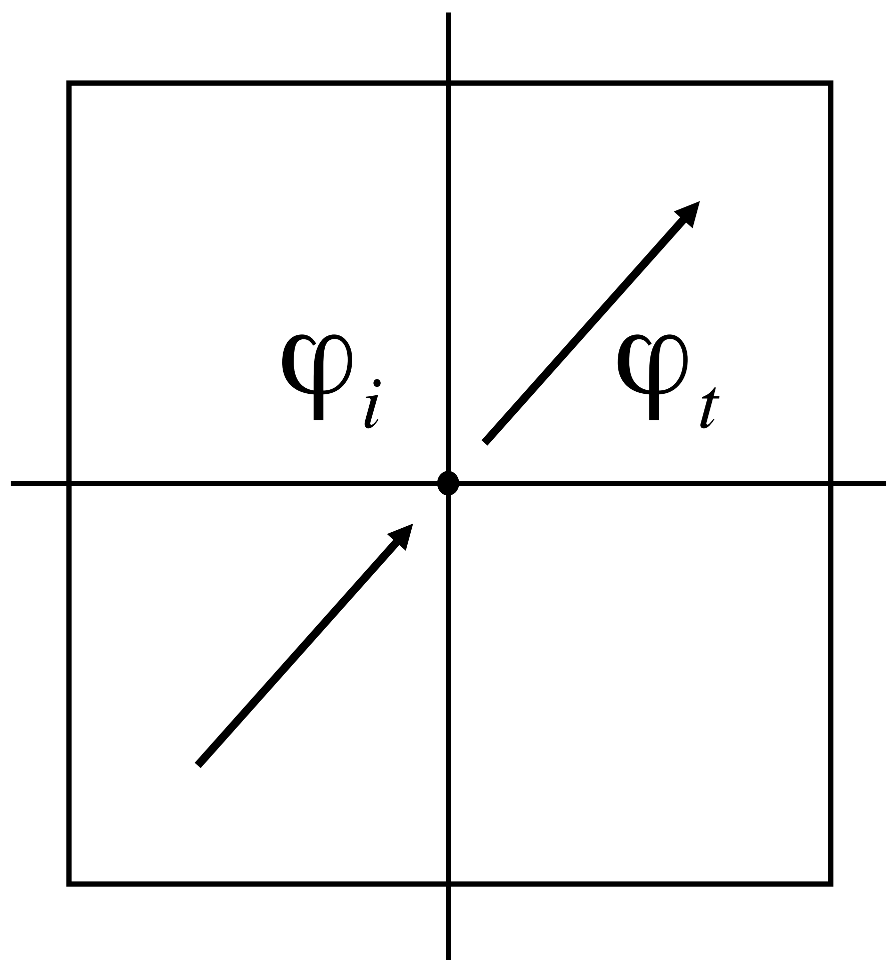


Snell's Law

Transmitted angle depends on index of refraction of medium incident ray is in and index of refraction of medium light is entering.



$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

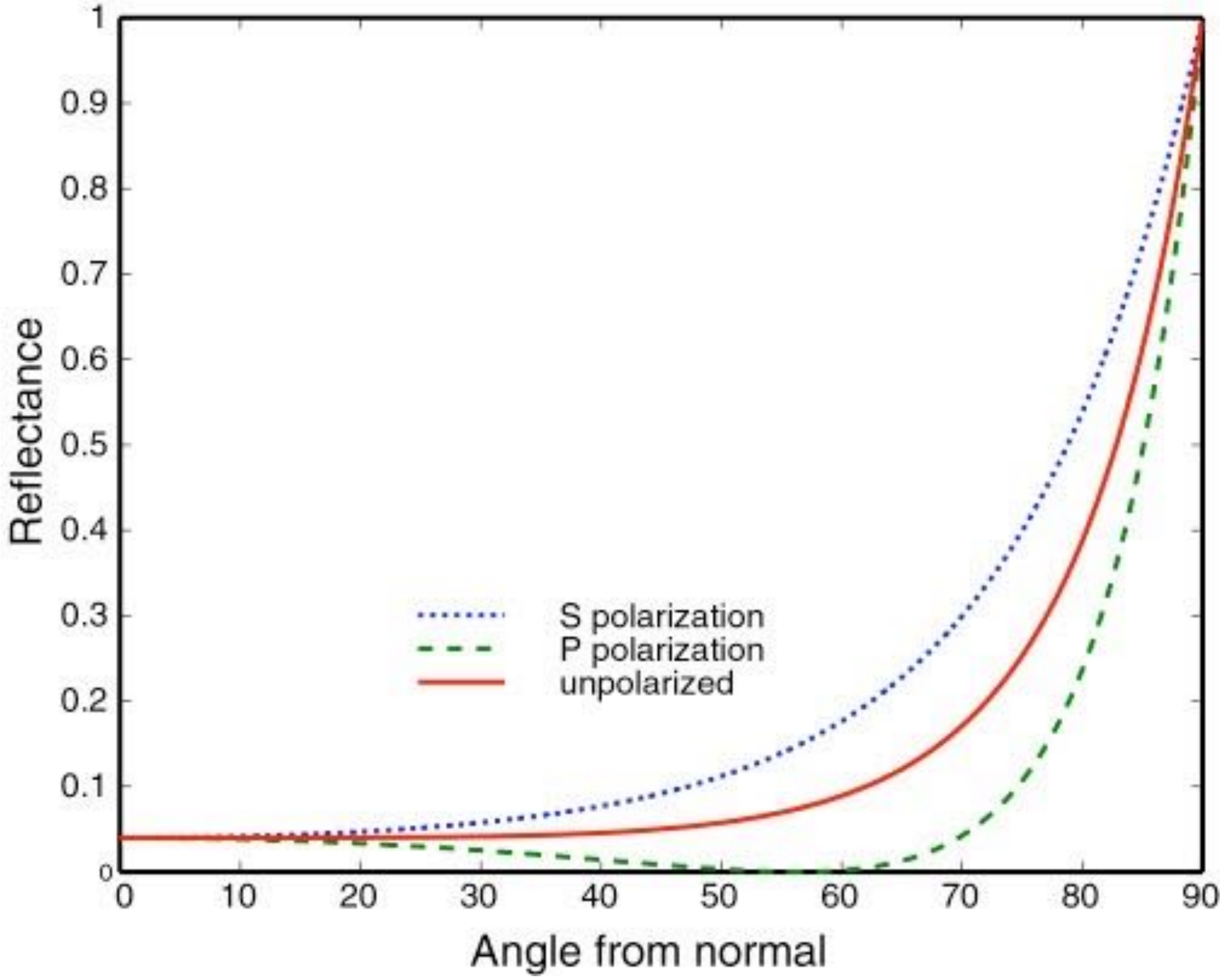


Medium	η*
Vacuum	1.0
Air (sea level)	1.00029
Water (20°C)	1.333
Glass	1.5-1.6
Diamond	2.42

*** index of refraction is wavelength dependent (these are averages)**

Fresnel reflection

Many real materials: reflectance increases w/ viewing angle

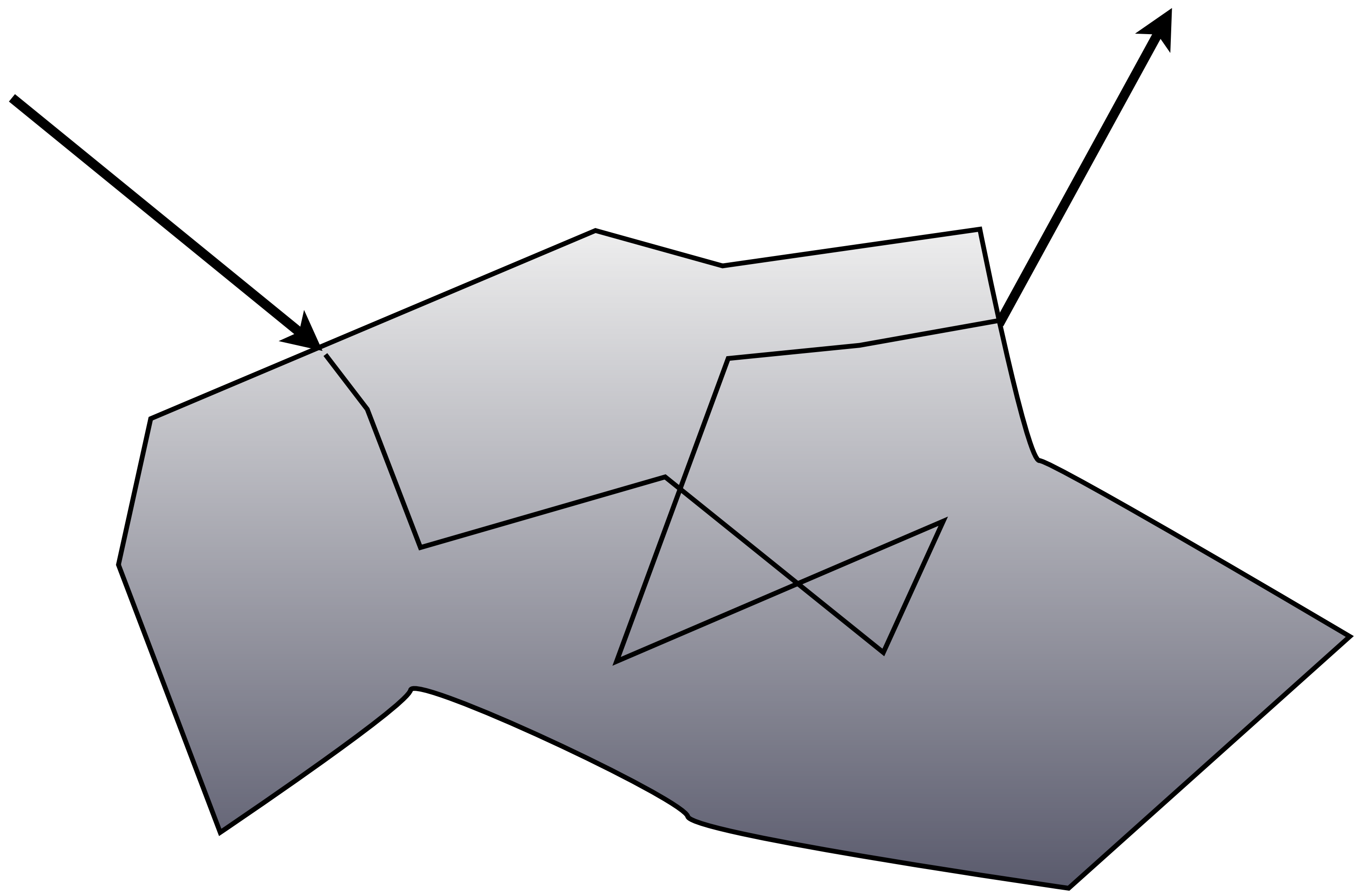


[Lafortune et al. 1997]

Snell + Fresnel: example

Subsurface scattering

- Visual characteristics of many surfaces caused by light entering at different points than it exits
  - Violates a fundamental assumption of the BRDF
  - Need to generalize scattering model (BSSRDF)



[Jensen et al 2001]

[Donner et al 2008]

* BSSRDF = bidirectional subsurface scatting reflectance distribution function

Translucent materials: Jade

Translucent materials: skin

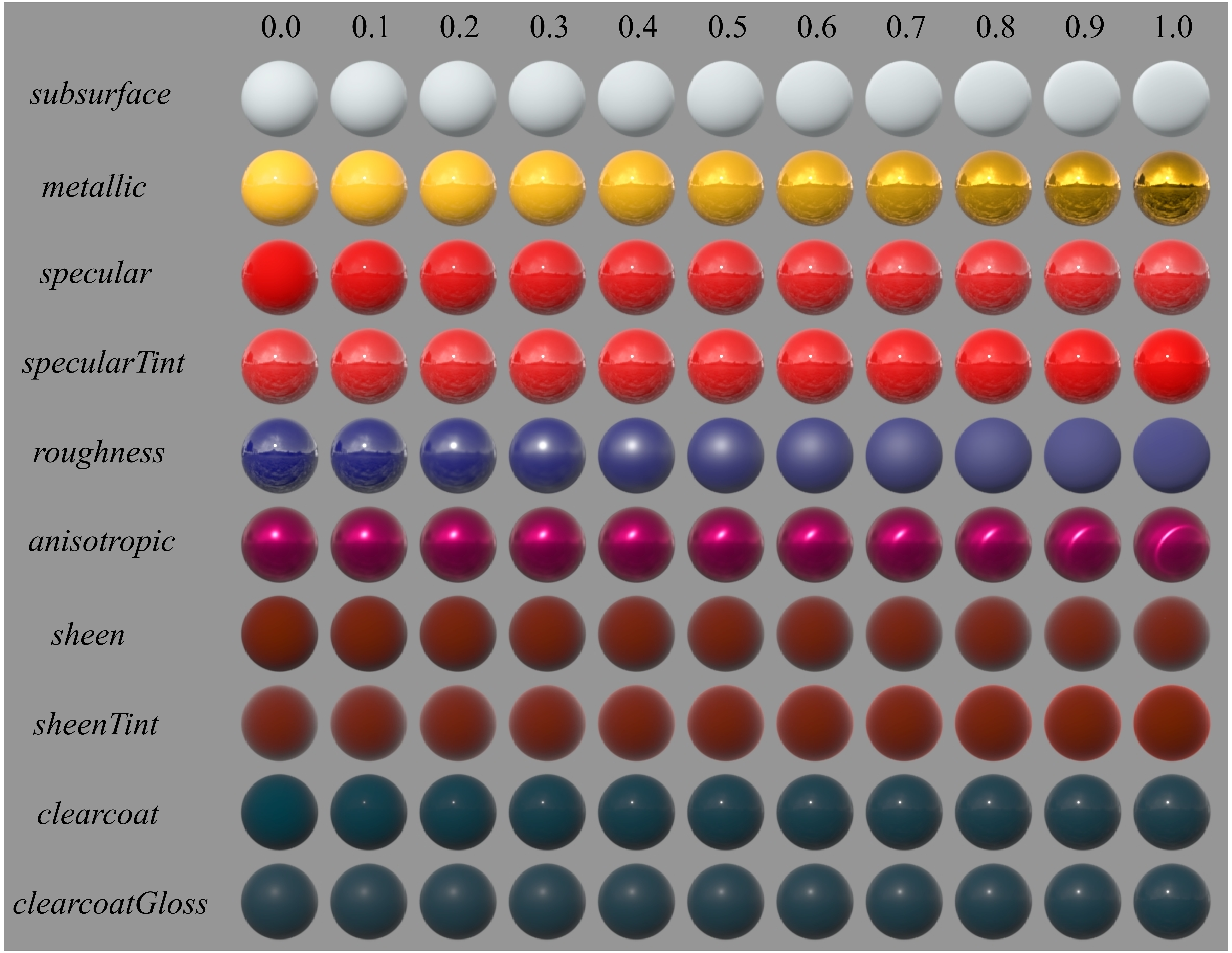
Translucent materials: leaves

BRDF

BSSRDF

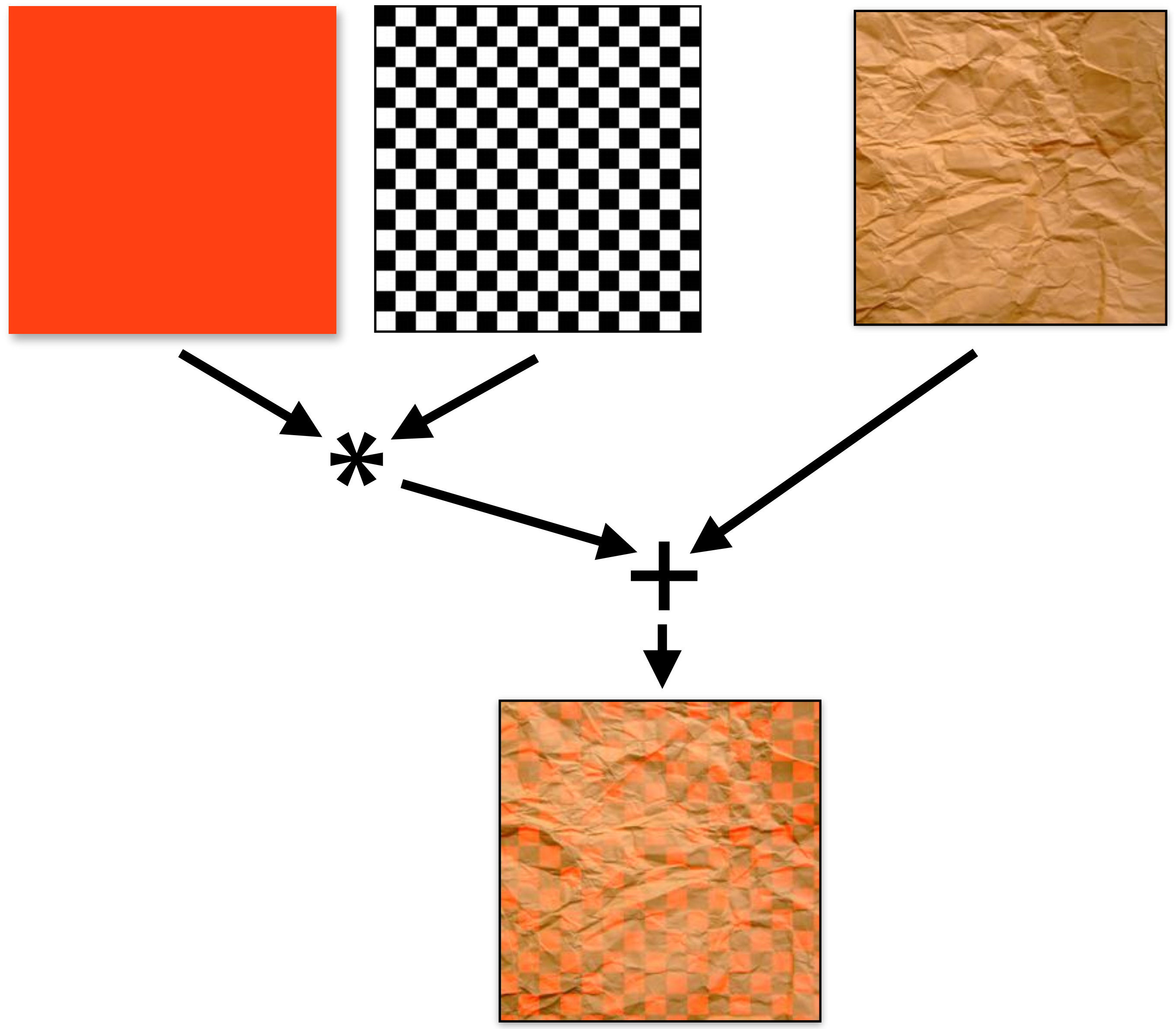
(models subsurface scattering of light)

Parameters to Disney BRDF



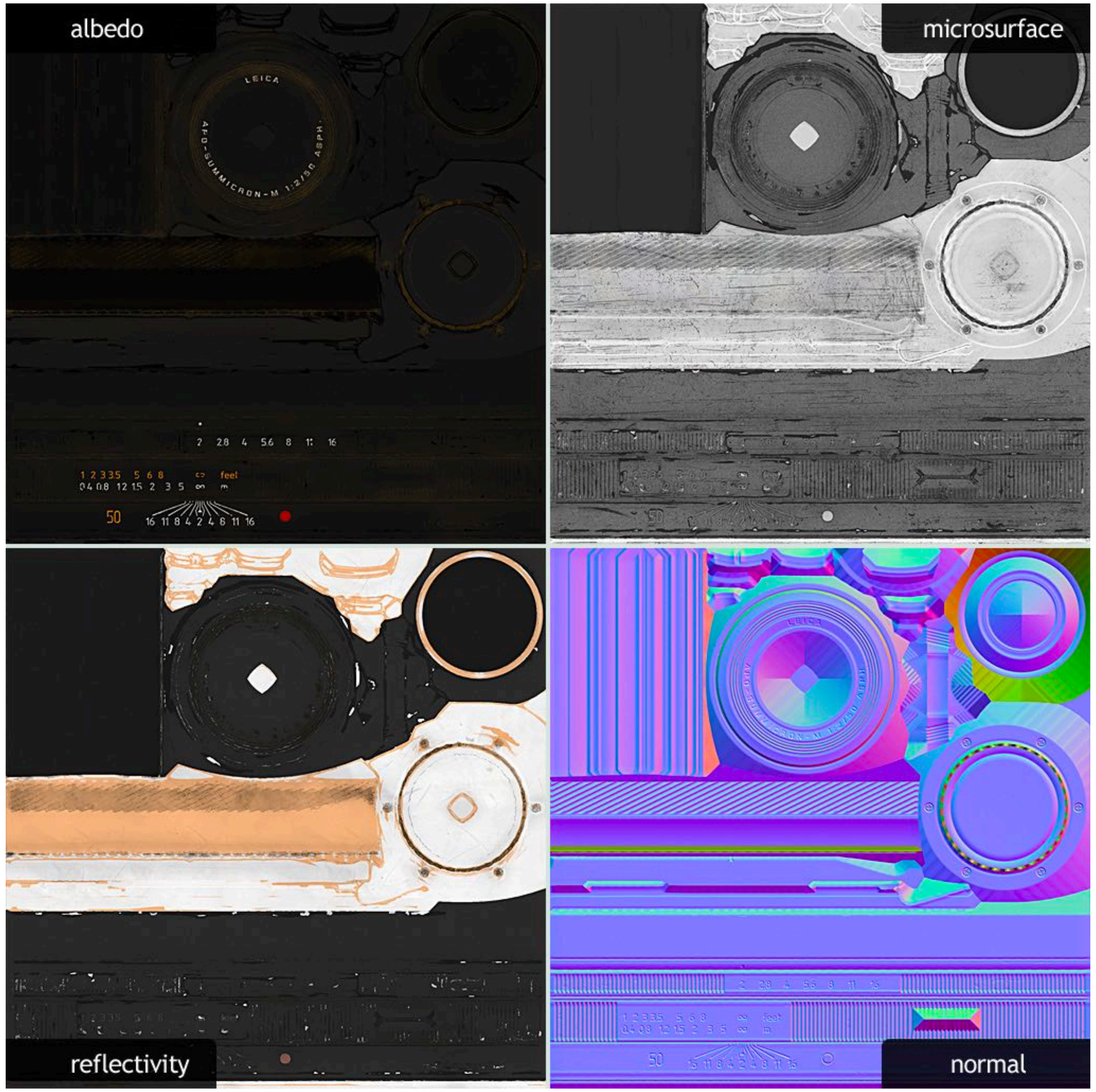
Pattern generation vs. BRDF

In practice, it is convenient to separate computation of spatially varying BRDF parameters (like albedo, shininess, etc.) from the reflectance function itself

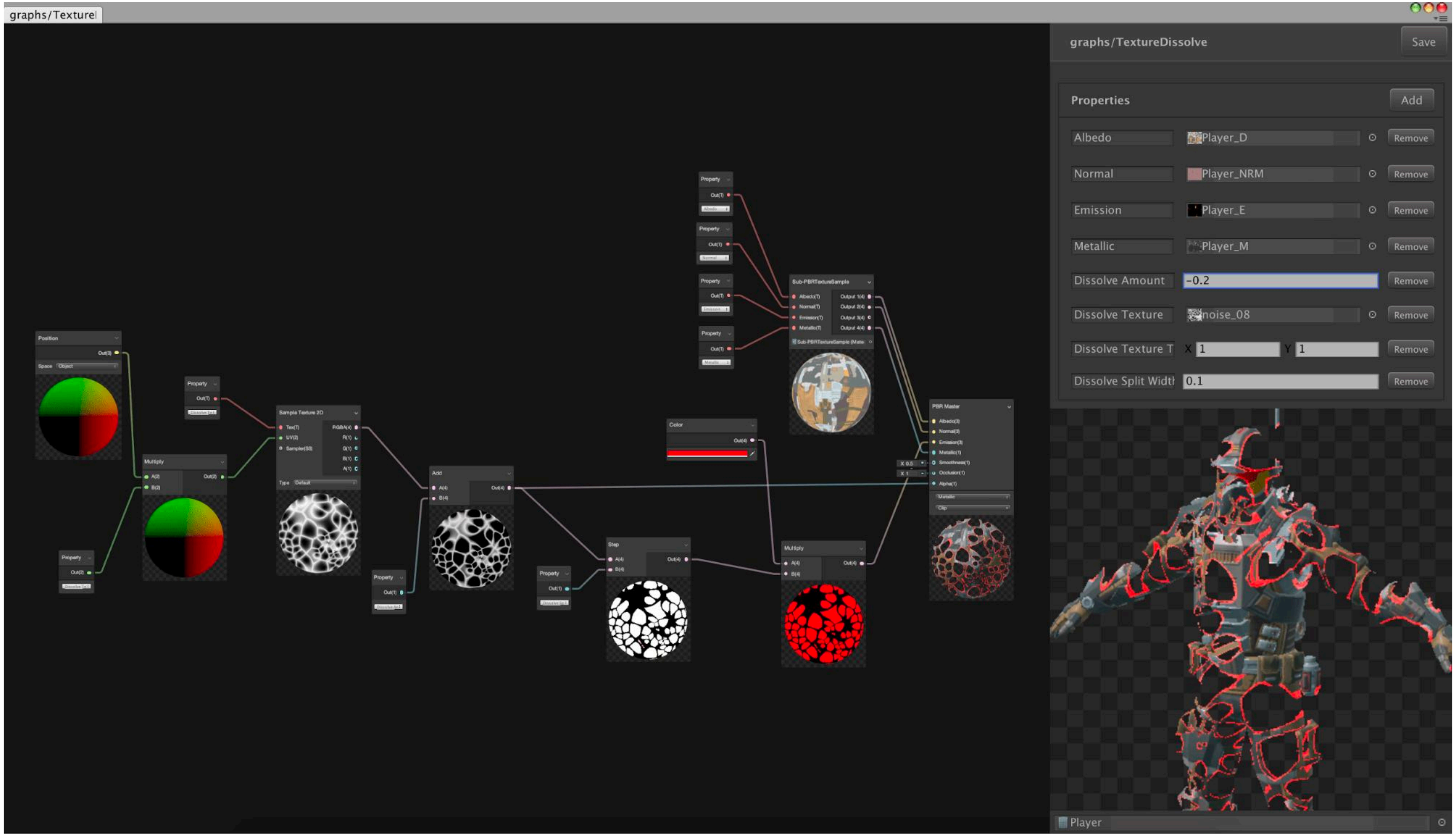


Example 1: albedo value at surface point is given by expression combining multiple textures

Example 2:
Different textures defining different spatially varying BRDF input parameters

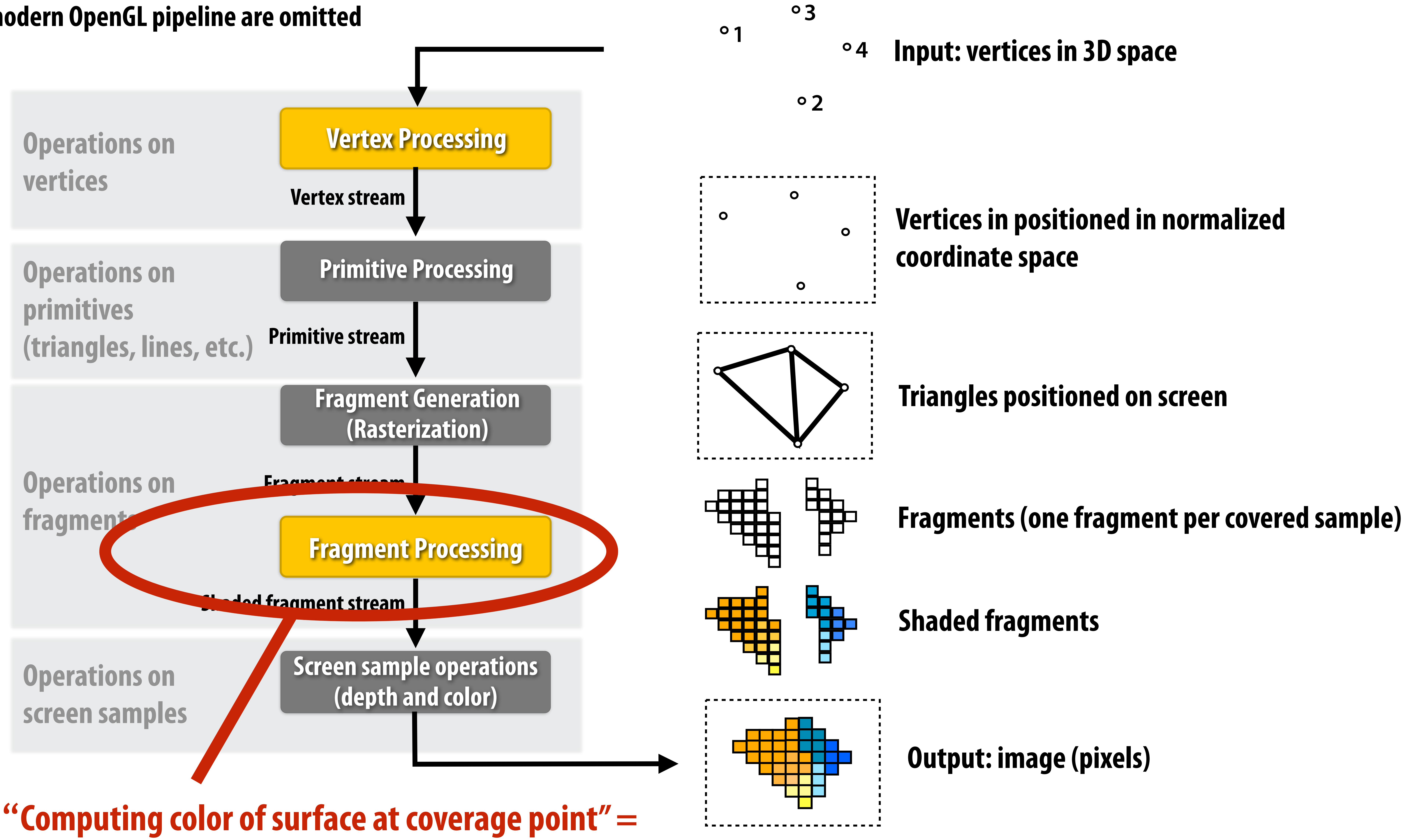


Unity's shader graph



Fragment processing stage of graphics pipeline evaluates surface appearance

* Several stages of the modern OpenGL pipeline are omitted



"Computing color of surface at coverage point" = simulation of lighting and materials

GLSL shader programs

Define behavior of vertex processing and fragment processing stages of pipeline

Describe operation on a single vertex (or single fragment)

Example GLSL fragment shader program

```
uniform sampler2D myTexture;
uniform vec3 lightDir;     // light direction
uniform vec3 Li;           // light intensity

in vec2 uv;
in vec3 norm;
out vec4 fragColor;

void diffuseShader() {
  vec3 kd = texture(myTexture, uv);
  vec3 in_light = Li * clamp(dot(-lightDir, norm), 0.0, 1.0);
  fragColor = vec4(kd * in_light, 1.0);
}
```

Program parameters

Per-fragment attributes (interpolated by rasterizer)

Sample surface albedo (reflectance color) from texture

Output color

Diffuse brdf: f(wo, wi) = kd
incoming light reflected equally in all directions
(fraction reflected = kd)

Shader function executes once per "fragment".

Outputs color of surface at sample point corresponding to fragment.

(this shader performs a texture lookup to obtain the surface's material color at this point, then performs a simple lighting computation)

Summary

- **Appearance of a surface is determined by:**
  - **The amount of light reaching the surface from different directions**
    - **Surface irradiance: the amount of light arriving at a surface point**
    - **Radiance: the amount of light arriving at a surface point from a given direction**
  - **The reflectance properties of the surface:**
    - **BRDF(w_i,w_o): the fraction of energy from direction w_i reflected in direction w_o**

Acknowledgements

- Thanks to Keenan Crane, Ren Ng, Pat Hanrahan and Matt Pharr for presentation resources