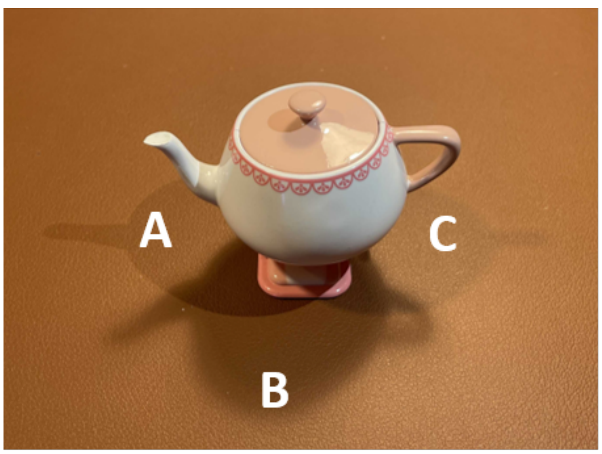
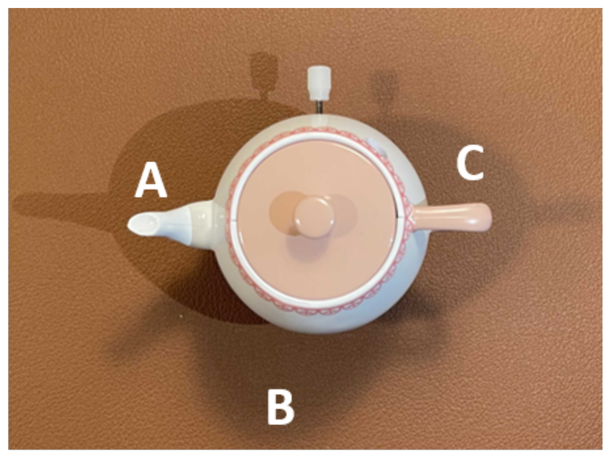
Stanford CS248A: Computer Graphics
Participation Exercise 6

This exercise is graded for CREDIT ONLY. Serious attempts to answer the problems will be given full credit, even if the answers are incorrect.

Problem 1: A Photo of a Teapot

One night Kayvon decides to take a photo of a cute teapot model he has on his desk. To illuminate the teapot he uses three light sources. The first two are identical size desk lamps each with a large light emitting bulb–they are *area light* sources. But one lamp is placed closer to the teapot than the other). The third source is a tiny LED flashlight on his cellphone. Below are two photos of the teapot under these lighting conditions (a front view and top view). Which shadow (A,B,C) comes from which light source? Please explain why. Note that there is also a base level of ambient illumination in the room.





Problem 2: Uniform Sampling of the Sphere

In class we talked about situations where we want to sample the unit sphere uniformly with *respect to solid angle*. But we never talked about how to compute random ray directions that correspond to uniform solid angle sampling. Let's do that here. Recall the definition of solid angle (on the unit sphere, so radius $r = 1$):

$$d\omega = \sin\theta \mathrm{d}\theta \mathrm{d}\phi$$

And since there are 4π steradians in the sphere (you can prove this to yourself by evaluating $\int_{S^2} \mathrm{d}\omega$), then uniform random sampling implies that:

$$p(\omega) = \frac{1}{4\pi}$$

The question is how to generate a random ray direction that corresponds to sampling the sphere equally with respect to solid angle. One way to do this would be to generate two random numbers from the uniform distribution $U(0,1)$, convert these numbers to values of (θ, ϕ), then convert polar coordinates into a ray direction (x, y, z).

The challenge is that sampling uniformly in θ doesn't result in sampling equally in solid angle, since $\mathrm{d}\theta$ and $\mathrm{d}\omega$ are related by a factor of $\sin(\theta)$.

$$\begin{aligned} p(\omega)\mathrm{d}\omega &= p(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi \\ p(\omega)\sin\theta\mathrm{d}\theta\mathrm{d}\phi &= p(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi \\ p(\omega)\sin\theta &= p(\theta,\phi) \end{aligned}$$

As a result:

$$p(\theta,\phi) = \frac{\sin\theta}{4\pi}$$

We'd like to use the inversion method to compute a random ray direction $\mathrm{d} = (x, y, z)$ from (ξ_1, ξ_2) where the resuting rays sample the sphere uniformly with respect to solid angle. Let's start by identifying that the joint pdf $p(\theta, \phi)$ is seperable into the product of two 1D pdfs $p(\theta)$ and $p(\phi)$.

$$p(\theta,\phi) = \frac{\sin\theta}{4\pi} = p(\theta)p(\phi)$$

where:

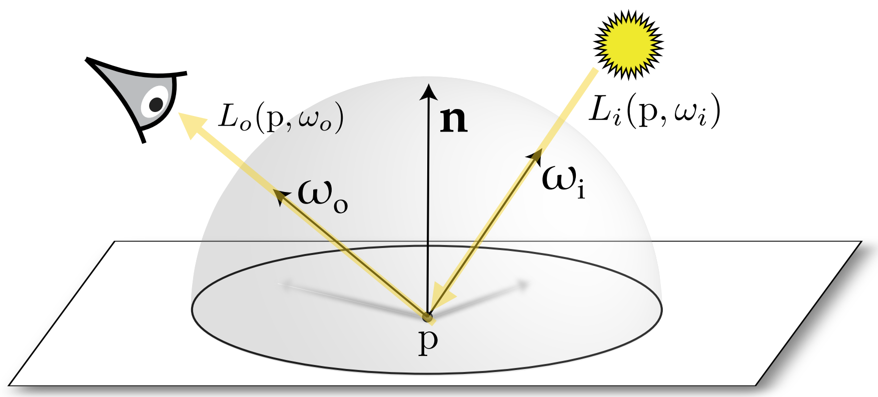
$$\begin{aligned} p(\phi) &= \frac{1}{2\pi} \\ p(\theta) &= \frac{\sin\theta}{2} \end{aligned}$$

Now, given the pdfs $p(\theta)$ and $p(\phi)$ from the previous page please use inversion to produce a ray direction (x, y, z) from (ξ_i, ξ_2). Recall that to compute a sample x drawn from distribution $p(x)$ you need to:

- Compute $P(x)$, the CDF of the pdf $p(x)$.
- Invert $P(x)$ so you can compute $x = P^{-1}(\xi)$.

Problem 3: Describing the Reflection Equation

In your own words, please describe the terms of the reflection question, provided below. What do the parts A, B, C, D, and E represent?



$$\underbrace{L_\mathrm{o}(\mathrm{p}, \omega_\mathrm{o})}_{\textbf{A}} = \underbrace{\int_{\Omega^2}}_{\textbf{B}} \underbrace{f_\mathrm{r}(\mathrm{p}, \omega_\mathrm{i} \to \omega_\mathrm{o})}_{\textbf{C}} \, \underbrace{L_\mathrm{i}(\mathrm{p}, \omega_\mathrm{i})}_{\textbf{D}} \, \underbrace{\cos\theta_\mathrm{i}}_{\textbf{E}} \, \underbrace{\mathrm{d}\omega_\mathrm{i}}_{\textbf{B}}$$