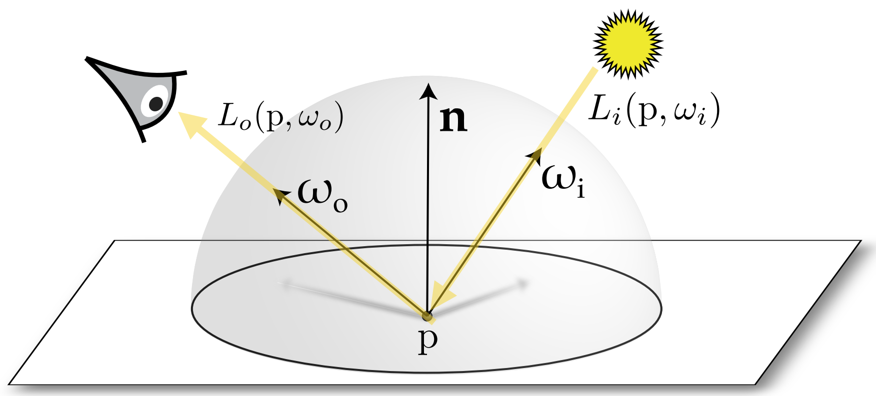
Stanford CS248A: Computer Graphics: Rendering, Geometry, and Image Manipulation
Exercise 5

Describing the Reflection Equation

Problem 1:

In your own words, please describe the terms of the reflection question, provided below. What do the parts A, B, C, D, and E represent?

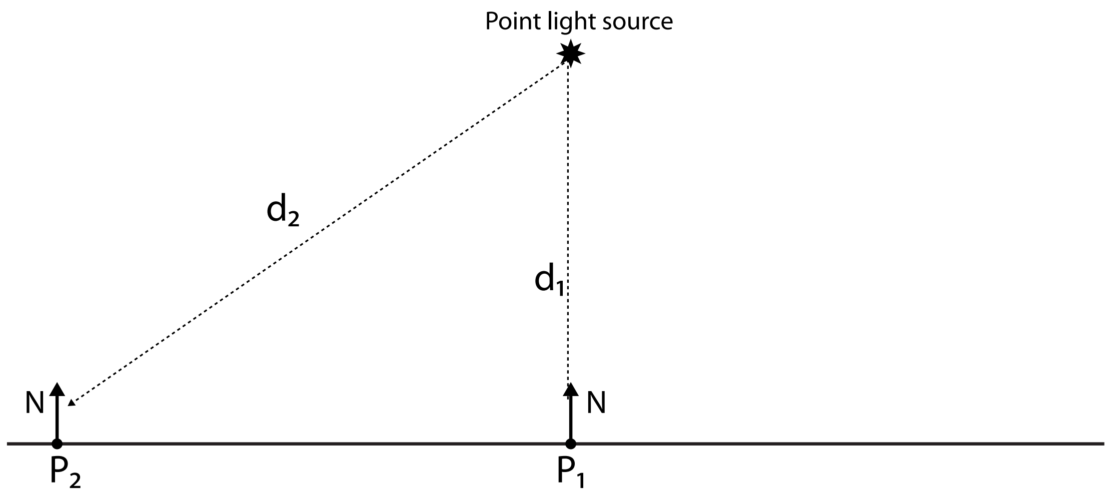


$$\underbrace{L_\mathrm{o}(\mathrm{p}, \omega_\mathrm{o})}_{\textbf{A}} = \underbrace{\int_{\Omega^2}}_{\textbf{B}} \underbrace{f_\mathrm{r}(\mathrm{p}, \omega_\mathrm{i} \rightarrow \omega_\mathrm{o})}_{\textbf{C}} \underbrace{L_\mathrm{i}(\mathrm{p}, \omega_\mathrm{i})}_{\textbf{D}} \underbrace{\cos\theta_\mathrm{i}}_{\textbf{E}} \underbrace{\mathrm{d}\omega_\mathrm{i}}_{\textbf{B}}$$

Problem 2:

Consider a point light source that emits uniformly in all directions. Specifically, it emits **equal power per unit solid angle** ($\frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$=C). The light is shining on the floor as shown in the figure below. Give **TWO REASONS** why the **irradience (E) incident on the floor** at point P_1, which is a distance d_1 from the light, is greater than the irradiance on the floor at point P_2, which is a distance d_2 from the light. Recall that irradience on a surface is power per unit surface area ($\frac{\mathrm{d}\Phi}{\mathrm{d}A}$.)

Hint: consider the definition of a differential solid angle $\mathrm{d}\omega$ in terms of a subtended patch of surface area on a sphere with radius r. What is the power of the light source per unit surface area on the sphere? Then consider the orientation of that surface patch on the sphere compared to the orentiation of the floor.



Monte Carlo Estimator

Problem 3:

Consider the reflection equation for a surface with a constant BRDF: $f(\omega_i, \omega_o) = C$

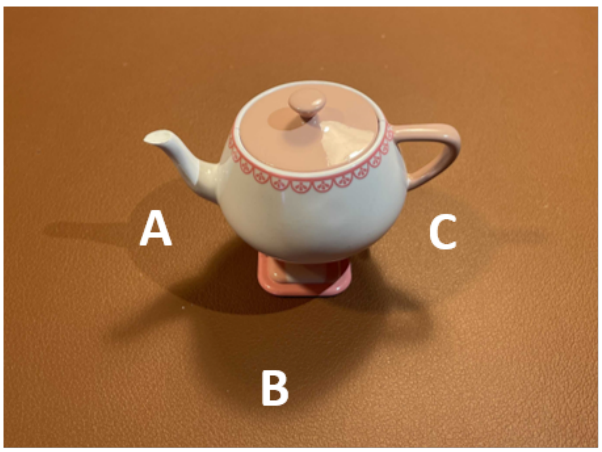
$$L_o(\omega) = \int_{\Omega} C\, L(\omega_i)\, \cos(\theta_i)\, \mathrm{d}\omega_i$$

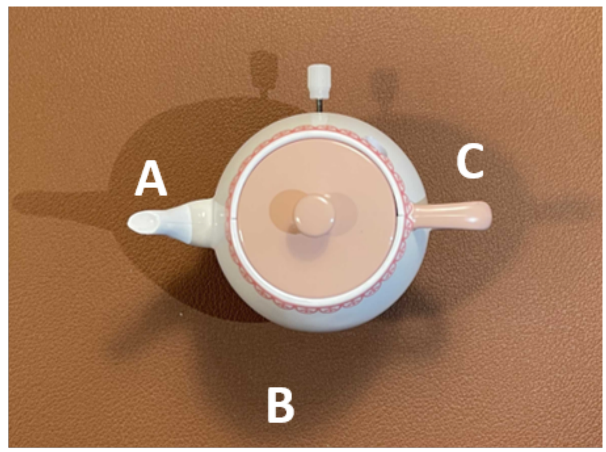
Assuming that we draw samples uniformly from the hemisphere of directions about the hemisphere, please write down an expression for the Monte Carlo estimator F_N for $L_o(w)$. Recall that N is the number of samples used for the estimate, and the expected value of F_N is equal to the integral given above. Also recall that there are 2π steradians in the hemisphere.

A Photo of a Teapot

Problem 4:

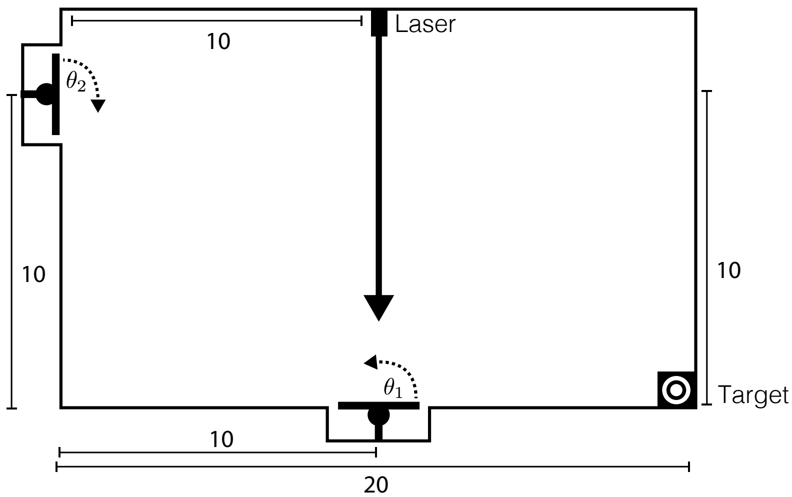
One night Kayvon decides to take a photo of a cute teapot model he has on his desk. To illuminate the teapot he uses three light sources. The first two are identical size desk lamps each with a large light emitting bulb–they are *area light* sources. But one lamp is placed closer to the teapot than the other). The third source is a tiny LED flashlight on his cellphone. Below are two photos of the teapot under these lighting conditions (a front view and top view). Which shadow (A,B,C) comes from which light source? Please explain why. Note that there is also a base level of ambient illumination in the room.

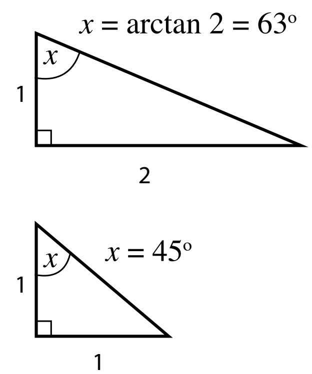




OPTIONAL PRACTICE PROBLEM 1:

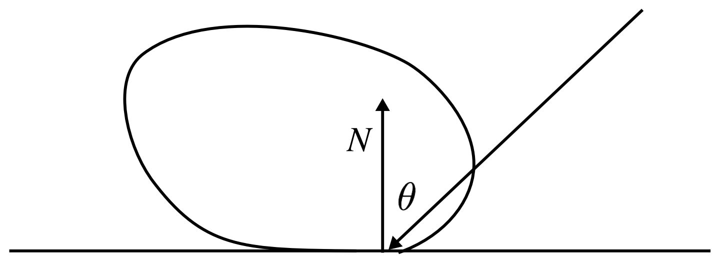
A mad scientist decides to design a fun physics experiment to amuse students in class. The goal of the experiment is to use two **perfectly reflective mirrors** to direct a laser beam, positioned downward from the top of the box, to hit a target in the bottom right corner of the box. The two mirrors can be rotated about their center point by the angles θ_1 and θ_2 as shown in the figure.





A. Please compute positive values of θ_1 and θ_2 to hit the box. Some helpful triangles are given for you, which may or may not be useful. Hint: first determine how to orient the first mirror to direct the beam to hit the second mirror. Then orient the second mirror to hit the target. Please review the slides from lecture about *perfect specular reflection.*

B. One challenge with perfect mirrors is that if you don't get them tilted just right, the laser will miss the target. One of the students, frustrated they couldn't hit the target, takes out a piece of sandpaper and scruffs up the two mirrors. The result is that the mirrors now have a BRDF that is almost fully diffuse, as given by the plot below. Note the surface reflects non-zero incoming light in all directions, but the fraction of light reflected in each of these directions is angle dependent. (More light is still reflected in the direction of perfect specular reflection.)

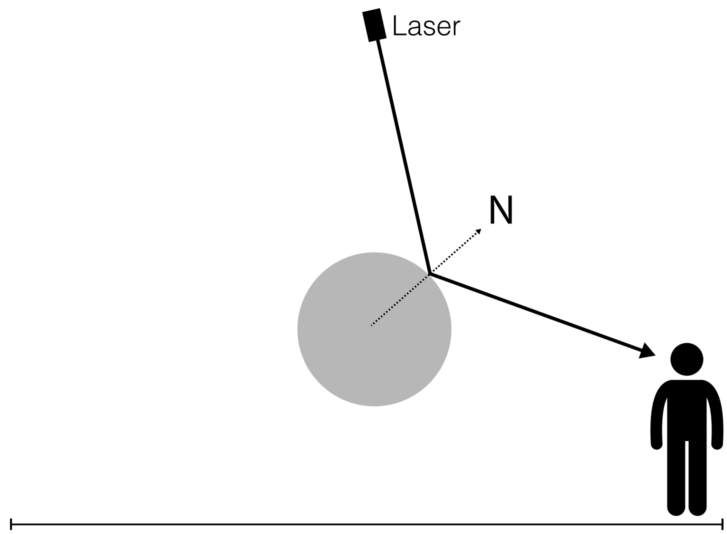


Assuming that (1) the mirrors are set so that $\theta_1 = \theta_2 = 0$ and that all the walls of the room reflect no light (they are perfectly black), does any laser light hit the target? If your answer is no, explain why not. If your answer is yes, please explain why, and also state whether target is brighter or darker compared to what it would look like in the case of well-aligned perfect mirrors from part A.

Reasoning about BRDFs

OPTIONAL PRACTICE PROBLEM 2:

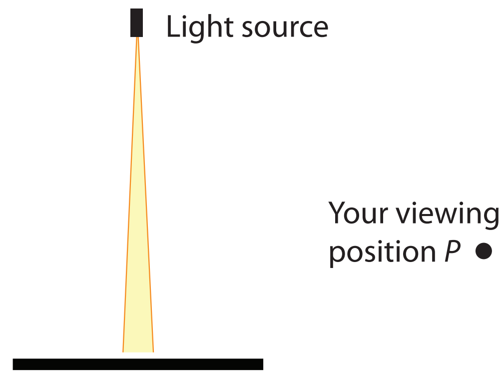
A. Consider the following setup where a red laser (a point light source emitting light in a single direction) is shining on sphere that is a **perfectly reflective mirror surface**. The light reflects off the sphere directly into the eyes of a person standing in the room. Assuming that there no other light sources in the room, and all other surfaces of the room are black, what does the person see if they lower the position of their head by sitting on the floor? Why? (Hint: a good answer will refer to the amount of light arriving at the viewer's eyes.)



B. You are standing in a room with a floor that is completely flat, but covered with two different types of materials (denoted as 1 and 2 in the figure below). The room only has one light source, a far away point light that is directly above the floor, and shines light directly down on the floor. (You can think of it as a very, very narrow spotlight, shining straight down.) Material 1 has a **completely diffuse BRDF** (white material) and Material 2 is a **perfect mirror** that reflects all light that hits it. If you are standing on the side of the floor at point P (as shown in the figure) what do you see? In particular be precise about what the mirror tiles look like. (Can you see anything in them, yes or no?) *Remember, there is no other light source in the room other than the spotlight from above..*

2	1	2	1
1	2	1	2
2	1	2	1
1	2	1	2

Top down view of floor



Side view of floor