Lecture 2:

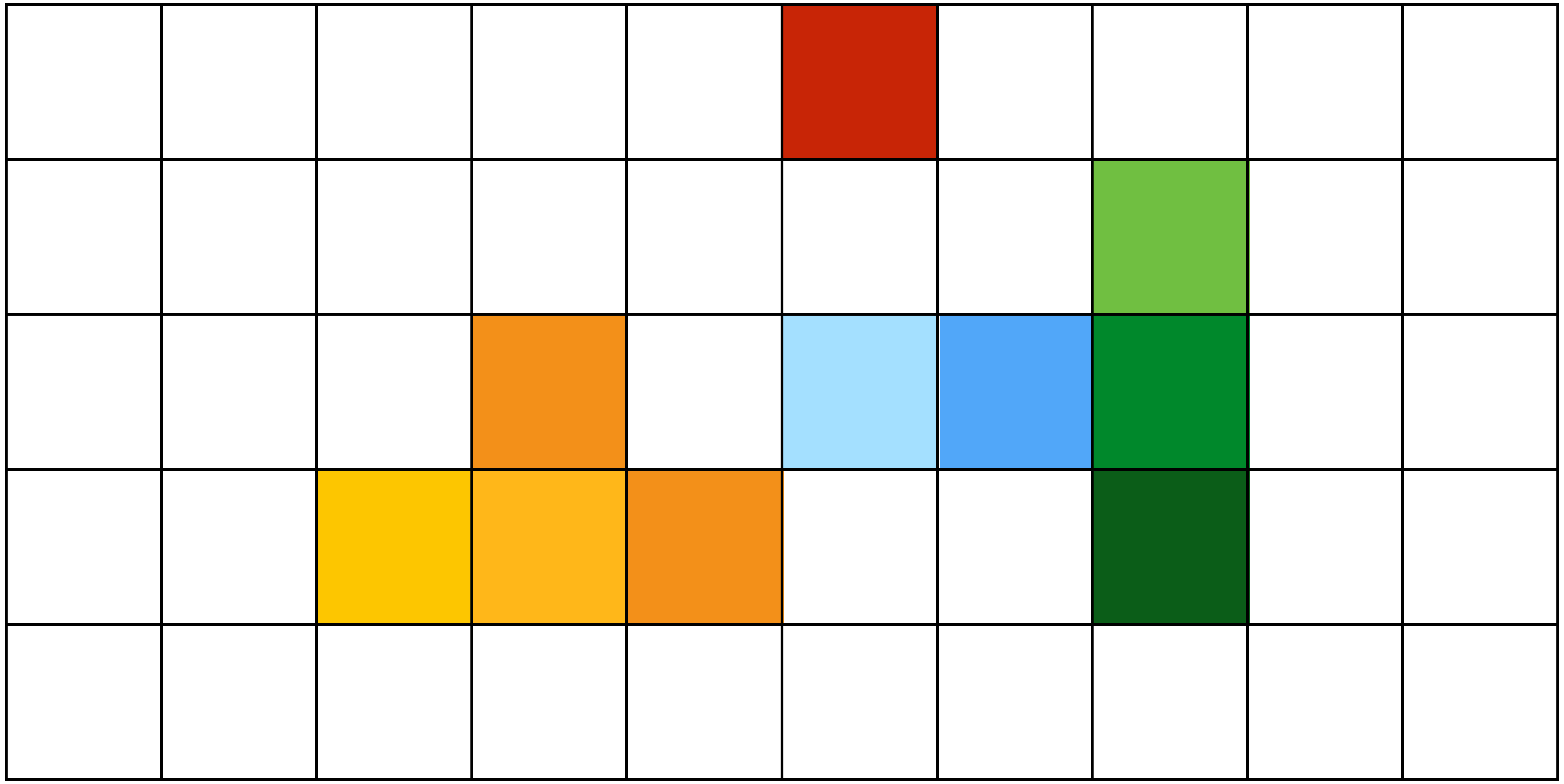
Drawing a Triangle

(+ the basics of sampling and anti-aliasing)

Computer Graphics: Rendering, Geometry, and Image Manipulation
Stanford CS248A, Winter 2025

Last time

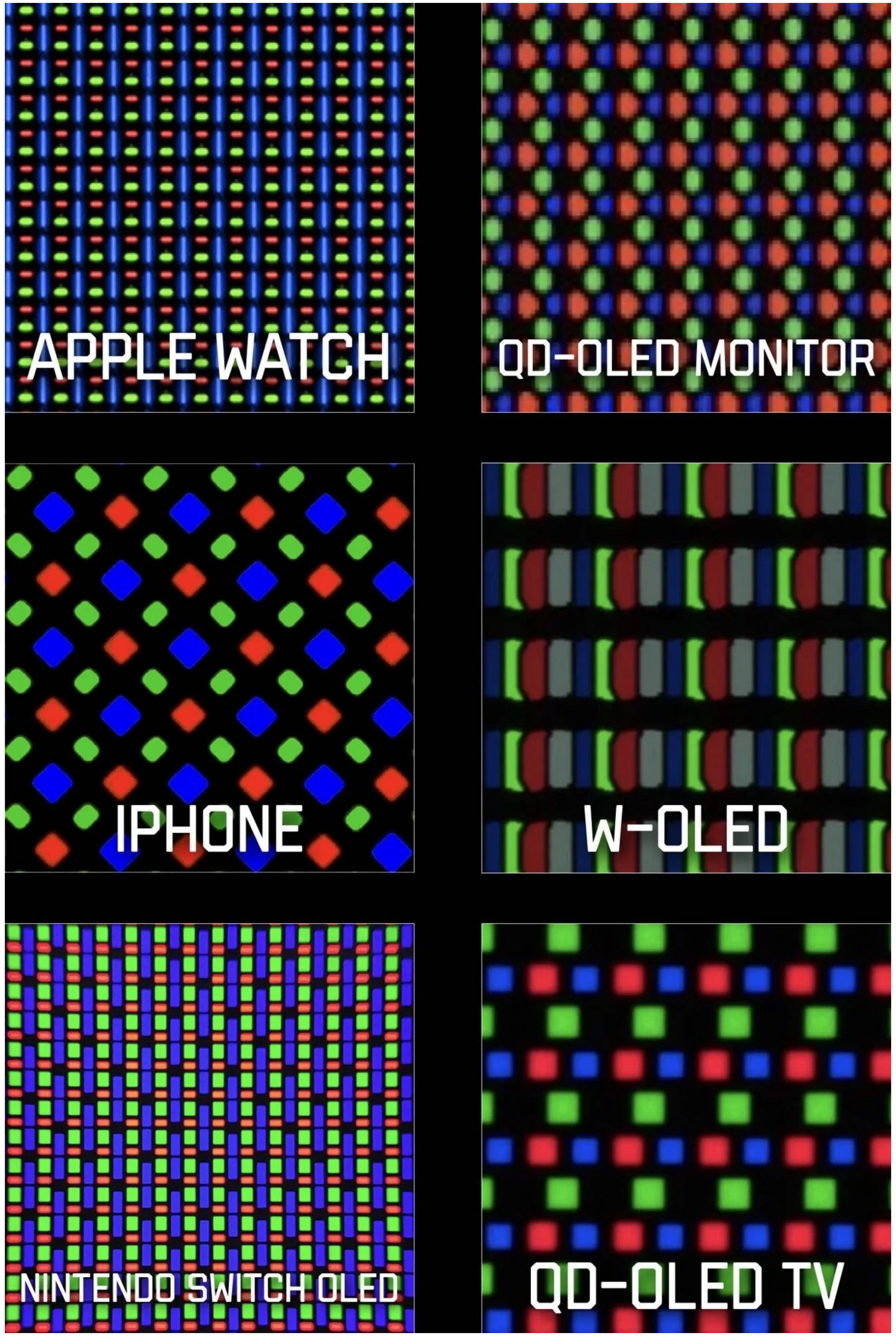
- A very simple notion of digital image representation (that we are about to challenge!)
- An image = a 2D array of color values



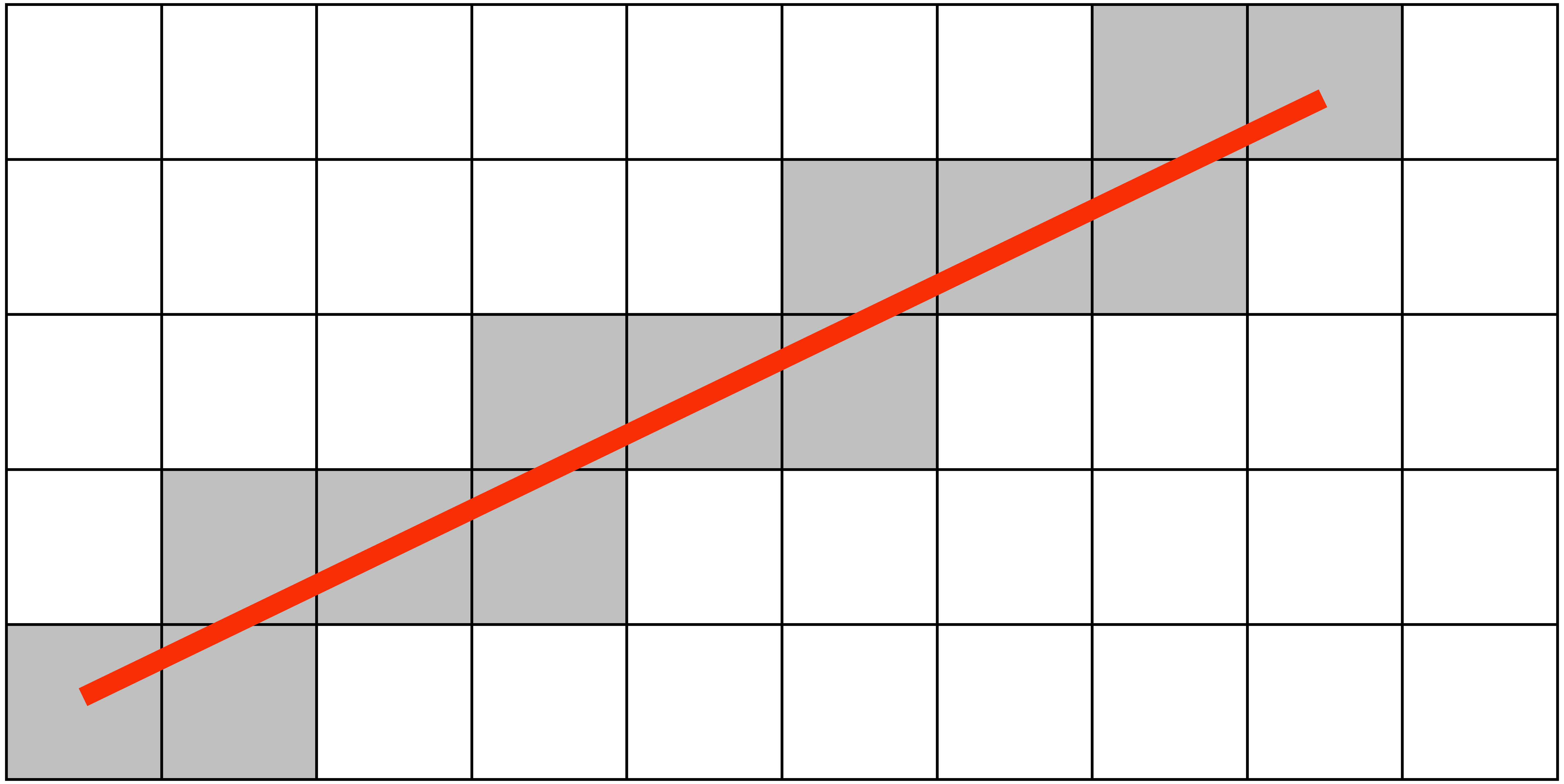
Last time: displaying an image

Aside: other sub pixel layouts

- So what is a pixel, anyway?
- (More on this soon)



Last time: what pixels should we color in to draw a line?



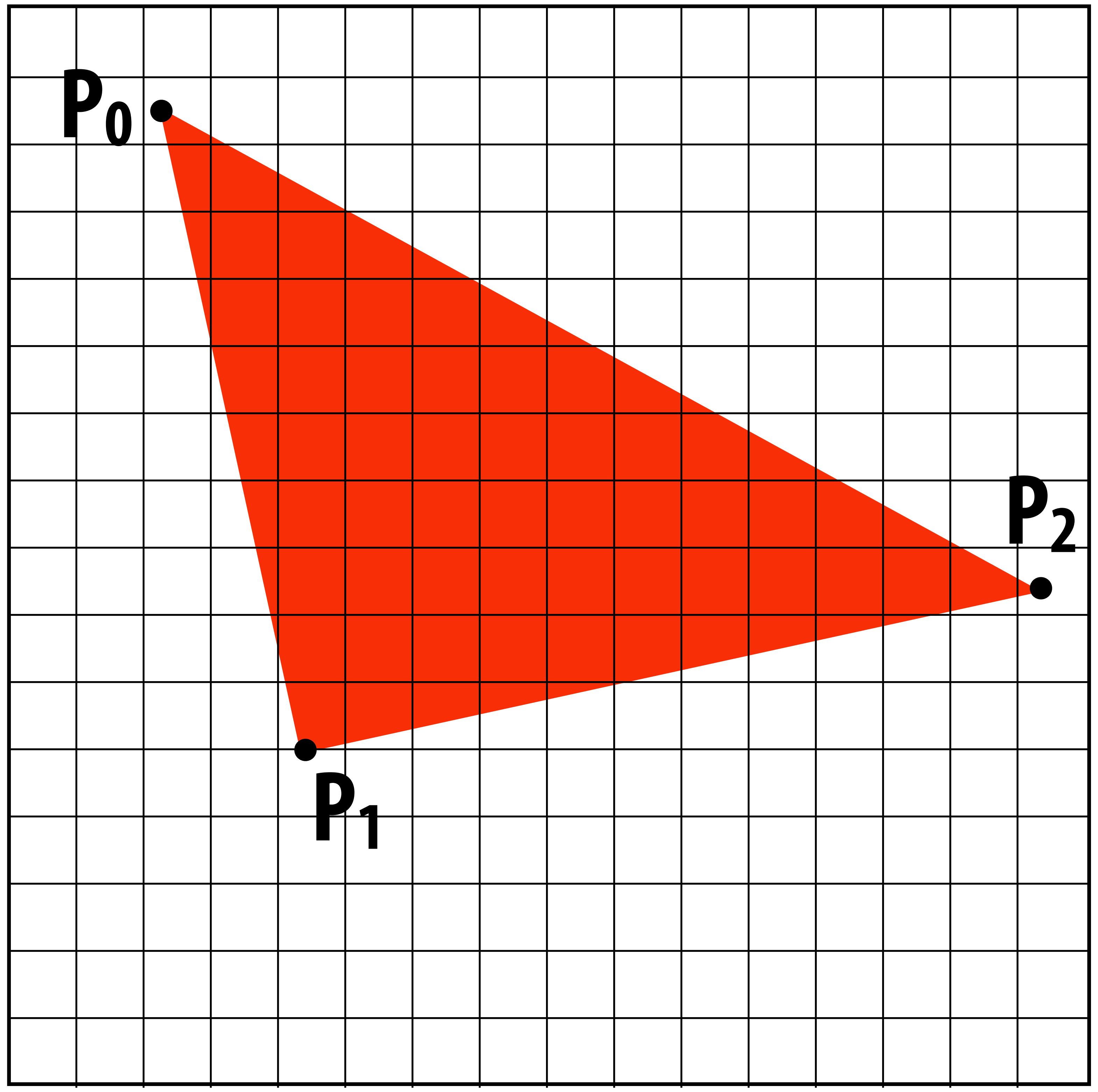
One possible heuristic: light up all pixels intersected by the line?

Today: drawing a triangle

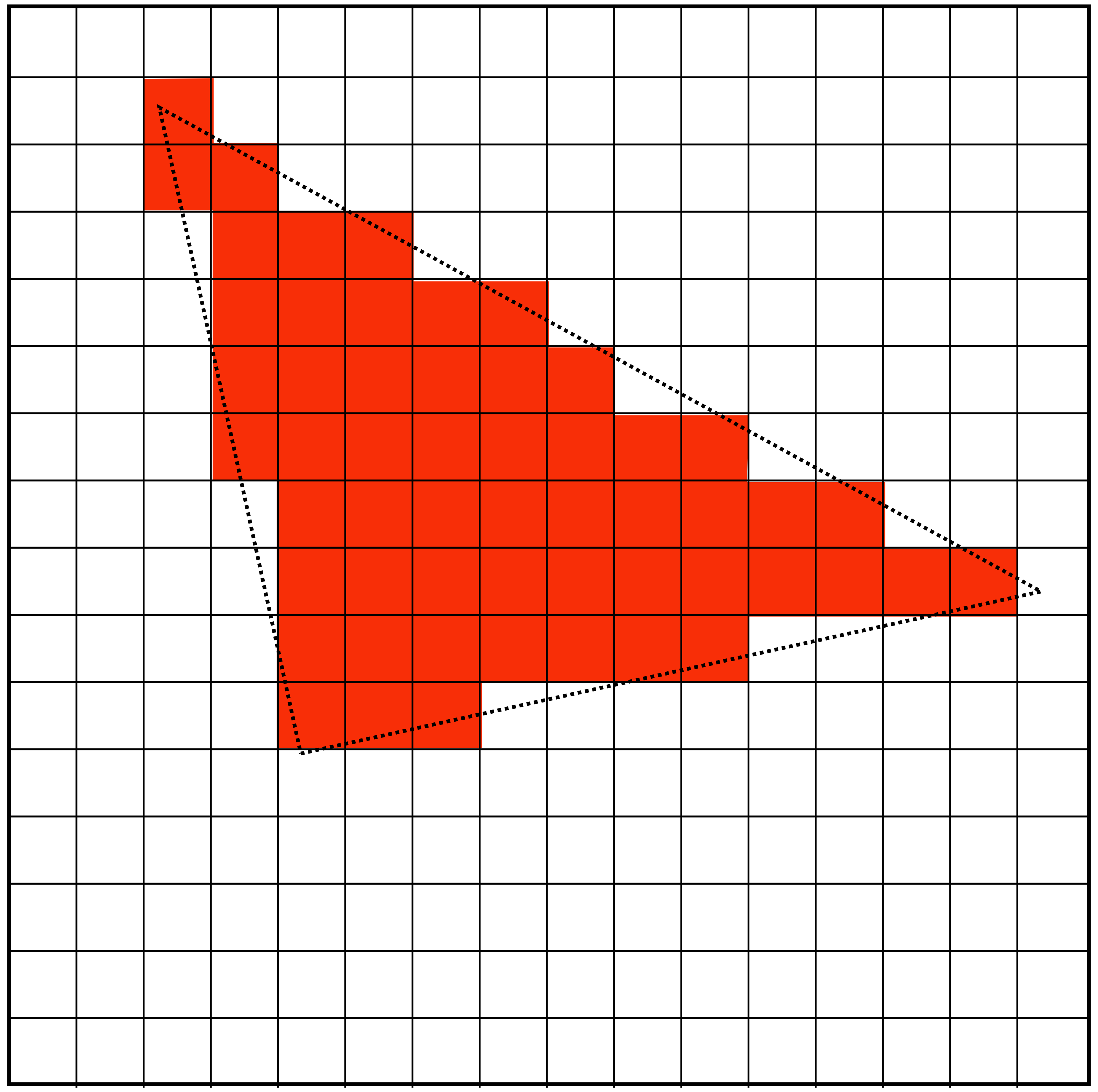
(Converting a representation of a triangle into an image)

"Triangle rasterization"

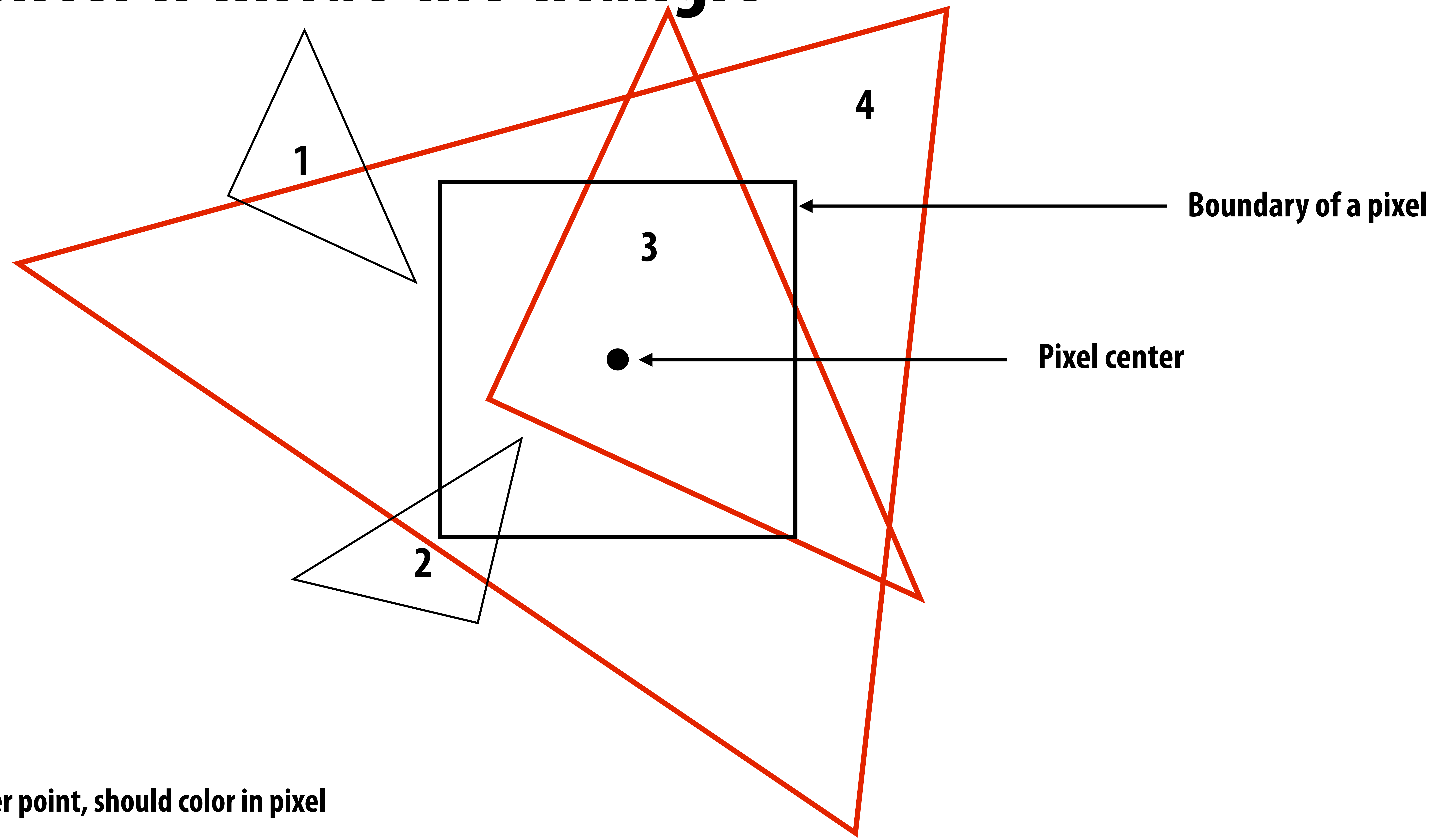
Input:
2D position of triangle vertices: P_0, P_1, P_2

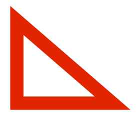


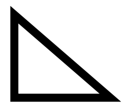
Output:
set of pixels "covered" by the triangle



Idea from last time: let's call a pixel "inside" the triangle if the pixel center is inside the triangle



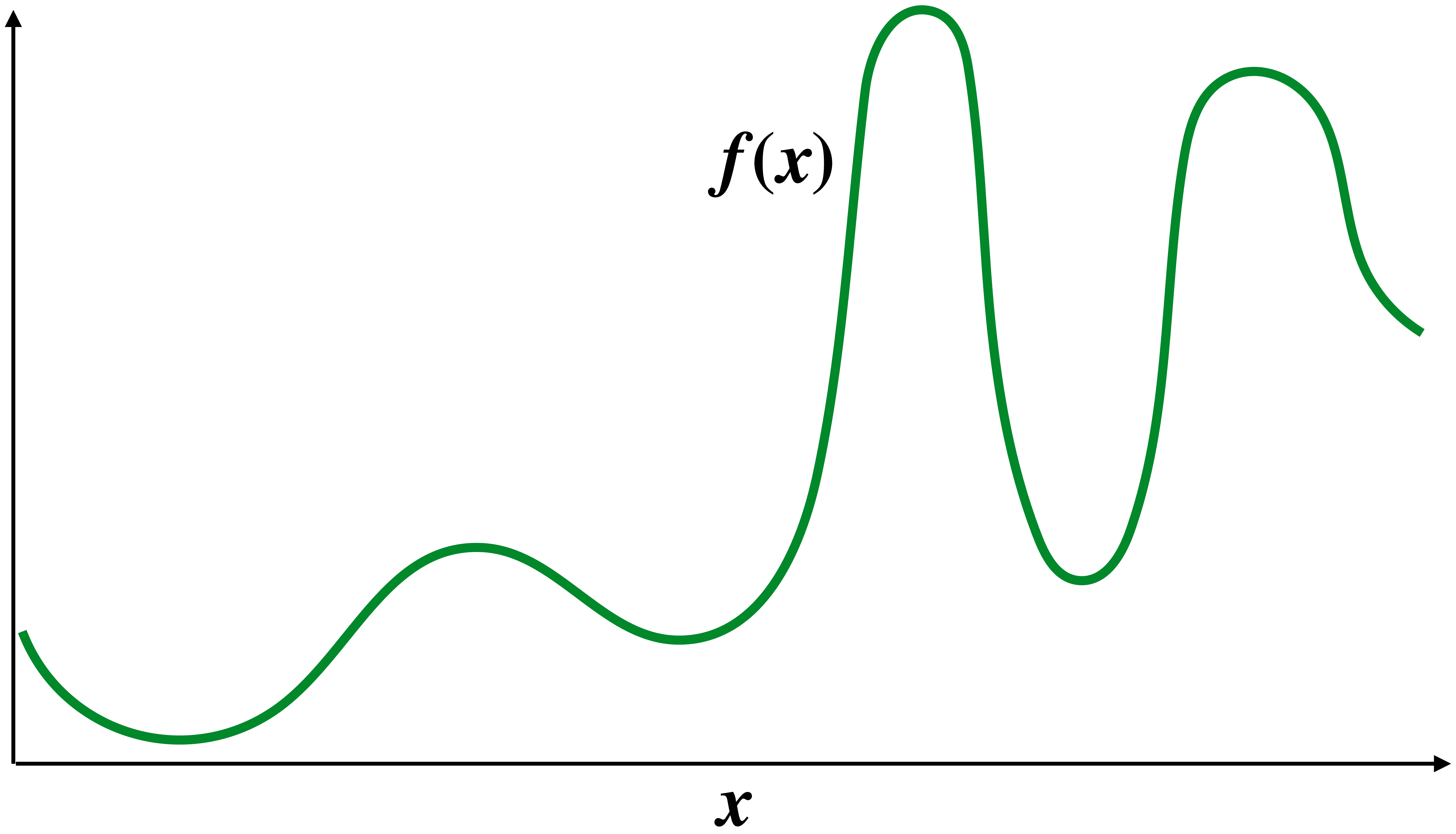
 = triangle covers center point, should color in pixel

 = triangle does not cover center point, do not color in pixel

Today we will draw triangles using a simple method:
point sampling
(testing whether a specific points are inside the triangle)

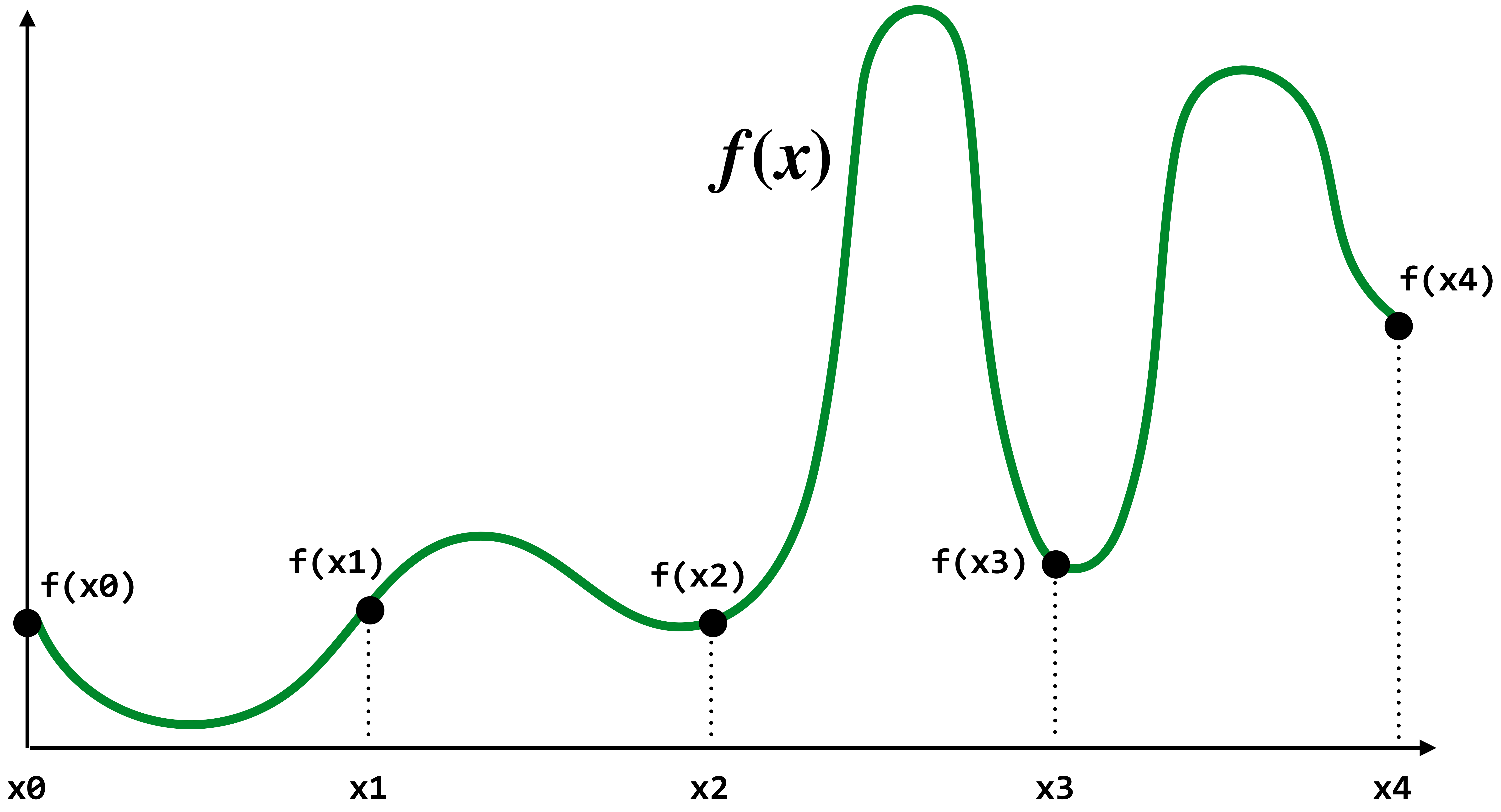
Before talking about sampling in 2D,
let's consider sampling in 1D first…

Consider a 1D signal: f(x)



Sampling: taking measurements of a signal

Below: five measurements ("samples") of $f(x)$



A discrete representation of f(x) is given by the samples $f(x_0)$, $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$

Audio file: stores samples of a 1D signal

Audio is often sampled at 44.1 KHz

Sampling a function

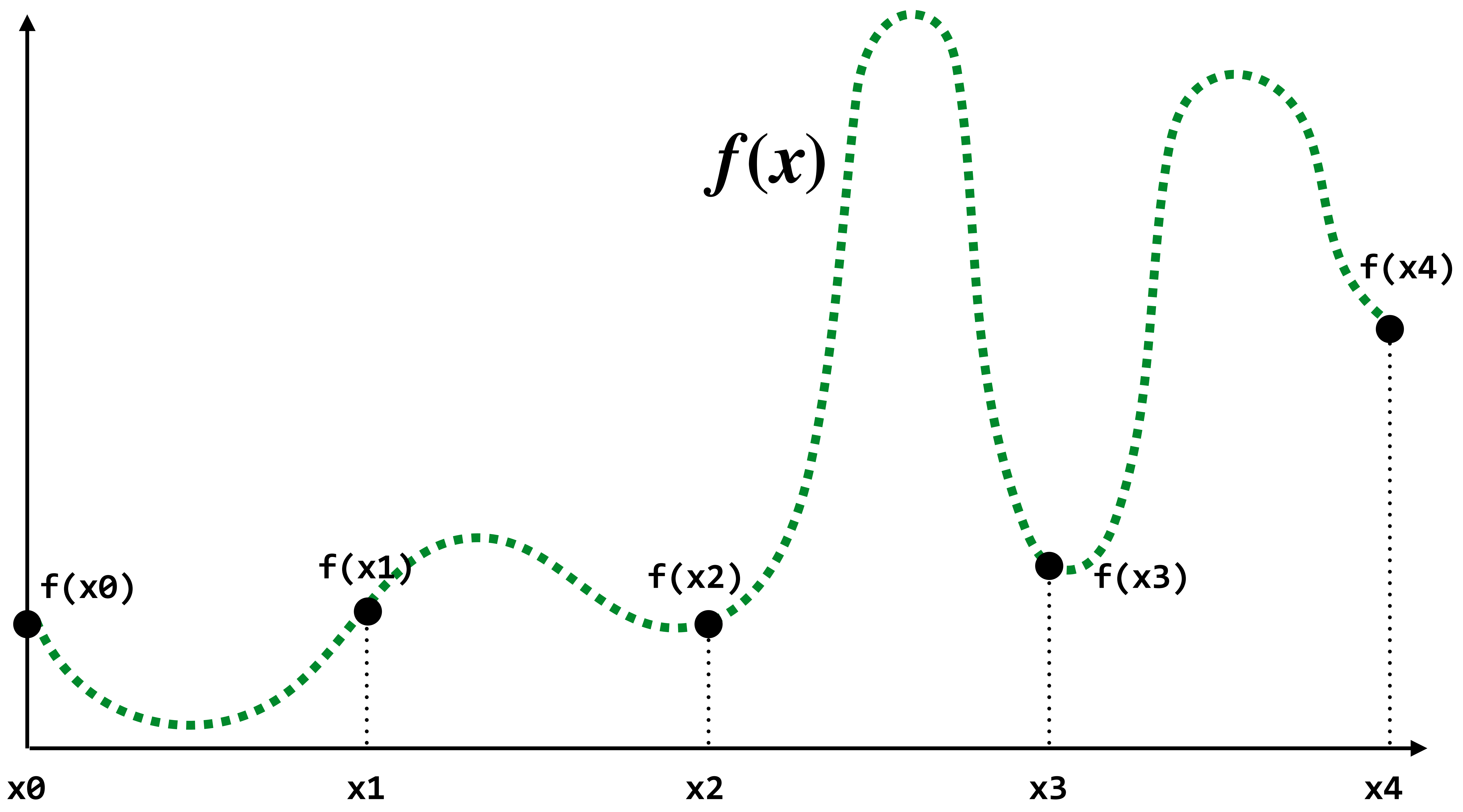
- Evaluating a function at a point is sampling the function's value

- We can discretize a function by periodic sampling

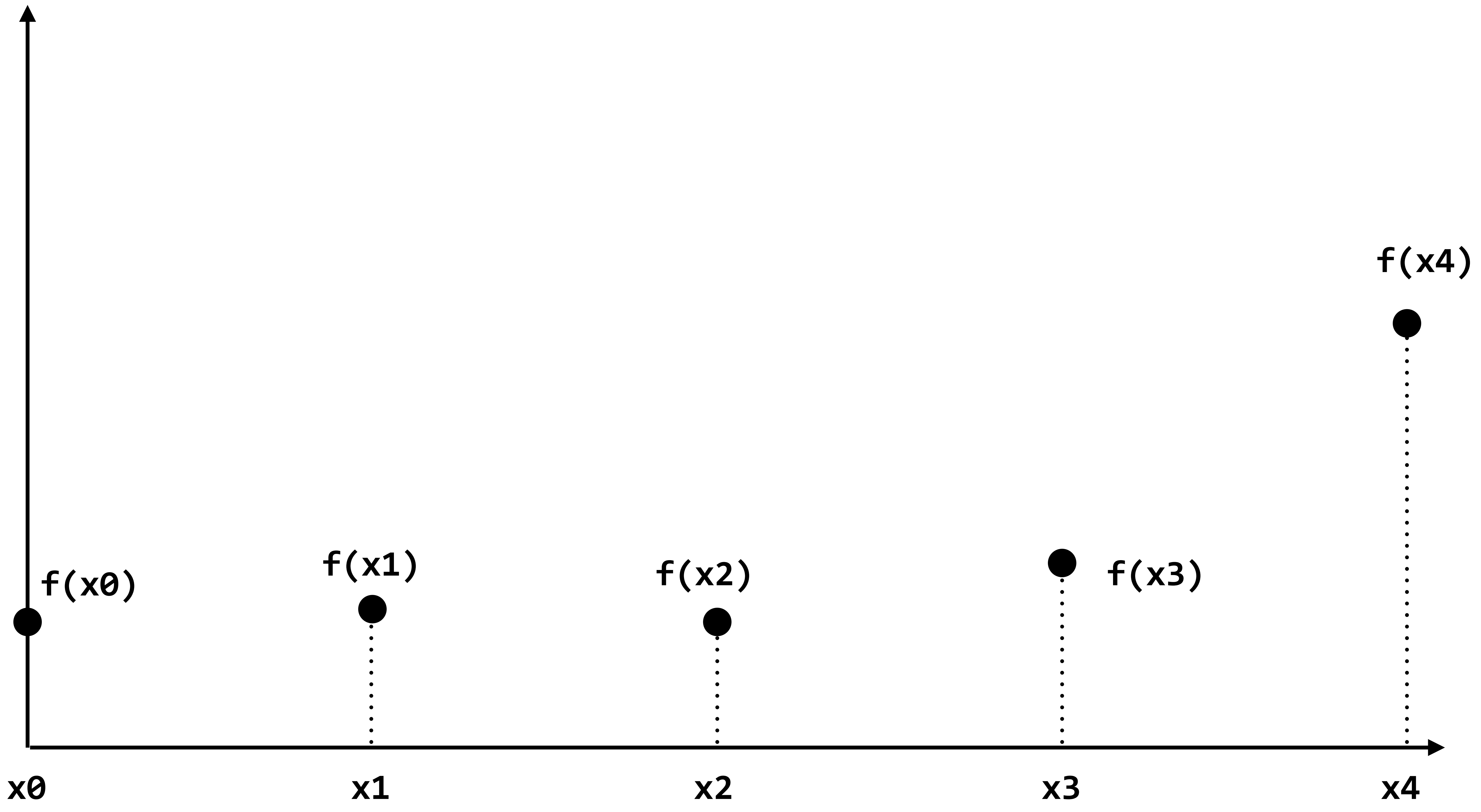
```
for(int x = 0; x < xmax; x++)
    output[x] = f(x);
```

- Sampling is a core idea in graphics. In this class we'll sample signals parameterized by: time (1D), area (2D), angle (2D), volume (3D), paths through a scene (infinite-D) etc …

Reconstruction: given a set of samples, how might we attempt to reconstruct the original (continuous) signal $f(x)$?



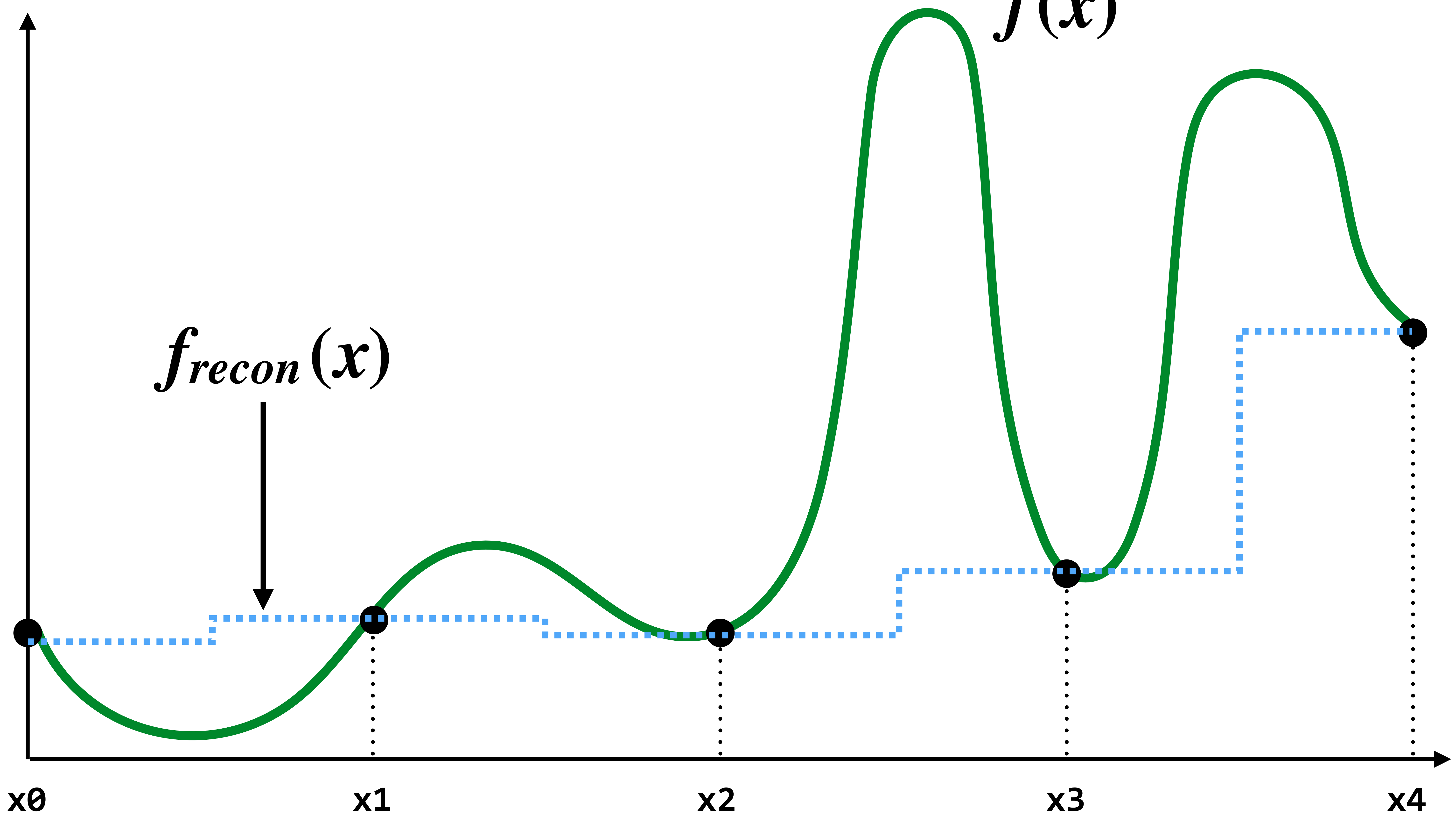
Reconstruction: given a set of samples, how might we attempt to reconstruct the original (continuous) signal $f(x)$?



Piecewise constant approximation

$f_{recon}(x)$ = value of sample closest to x

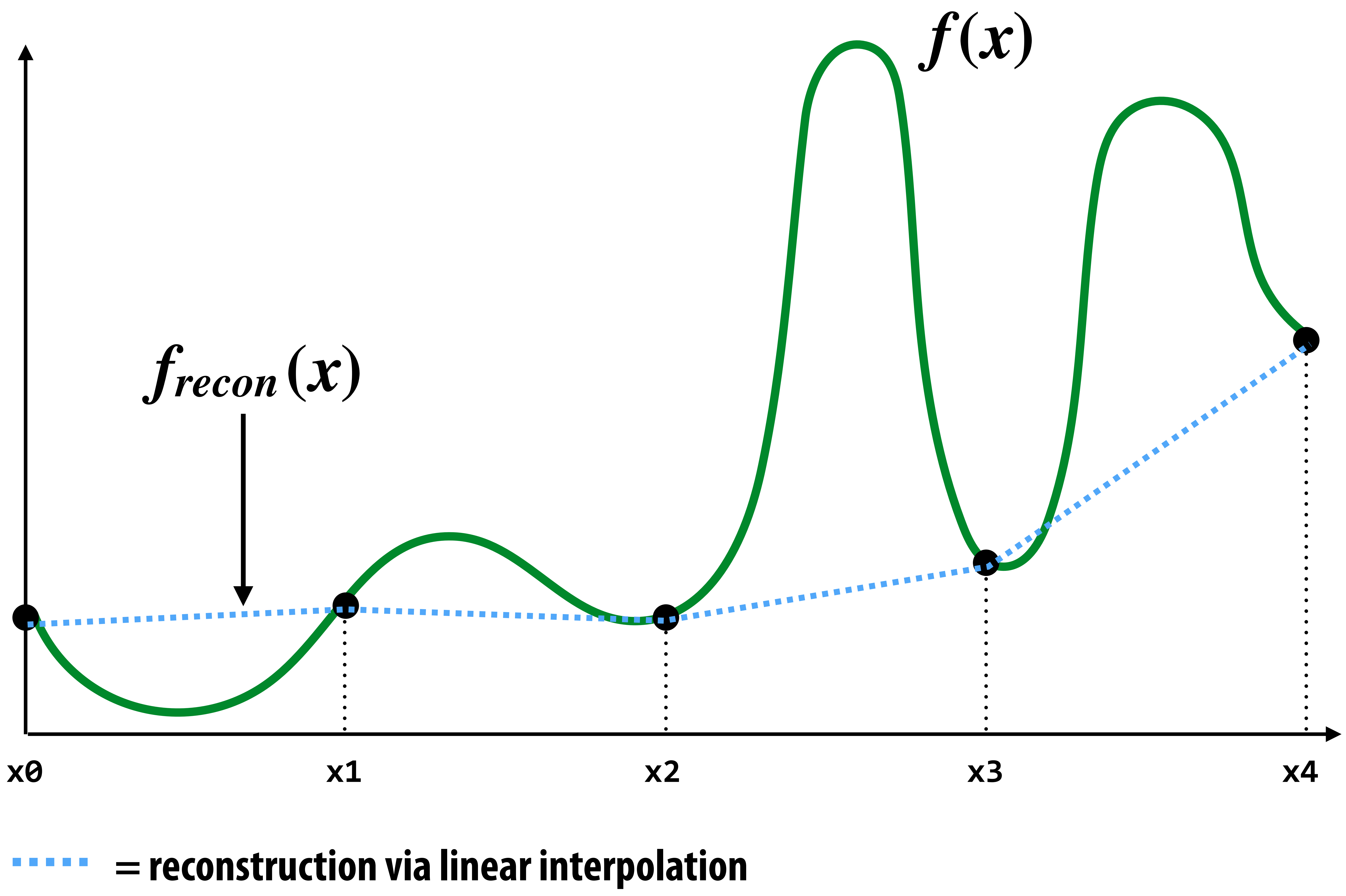
$f_{recon}(x)$ approximates $f(x)$



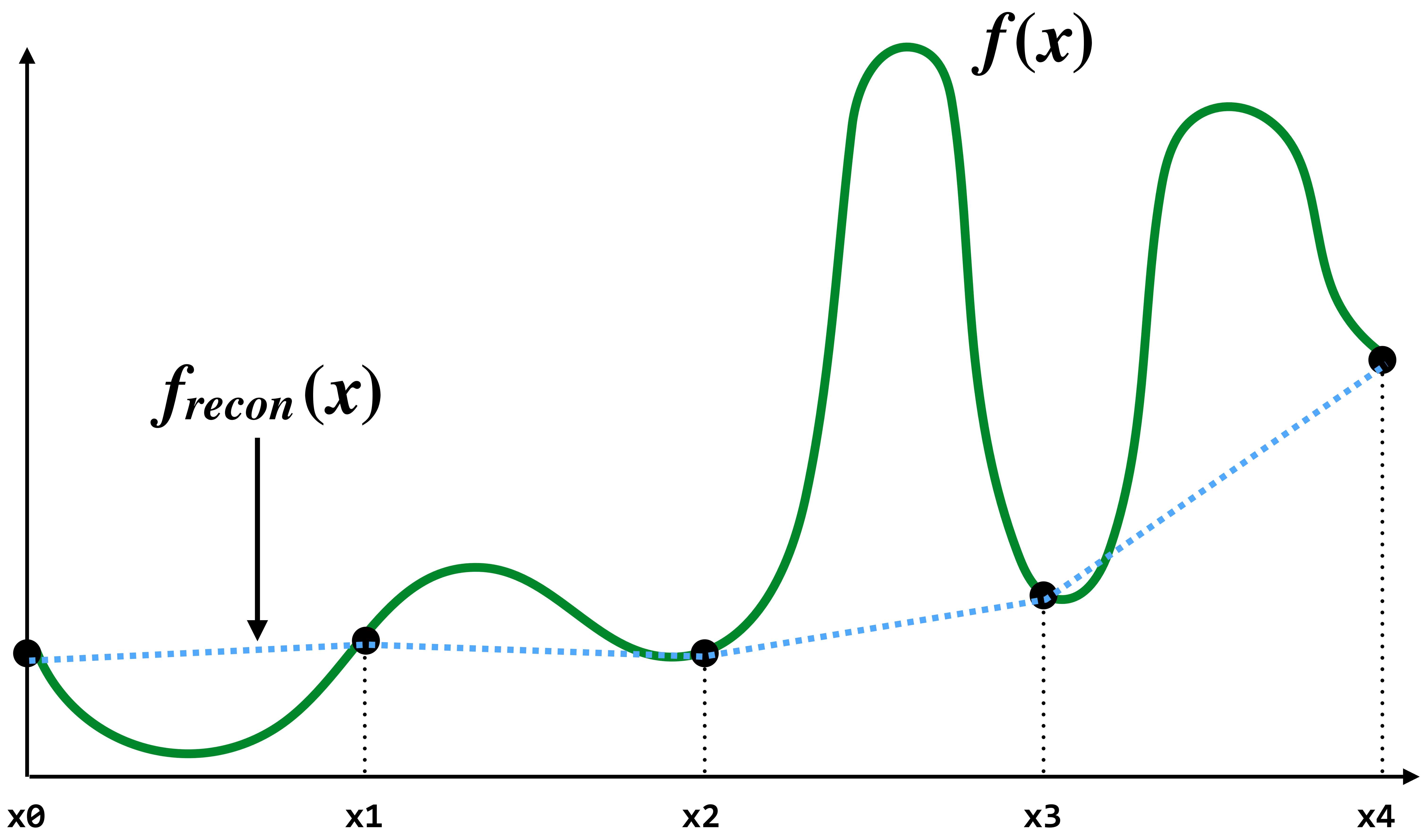
= reconstruction via piece-wise constant interpolation (nearest neighbor)

Piecewise linear approximation

$f_{recon}(x)$ = linear interpolation between values of two closest samples to x



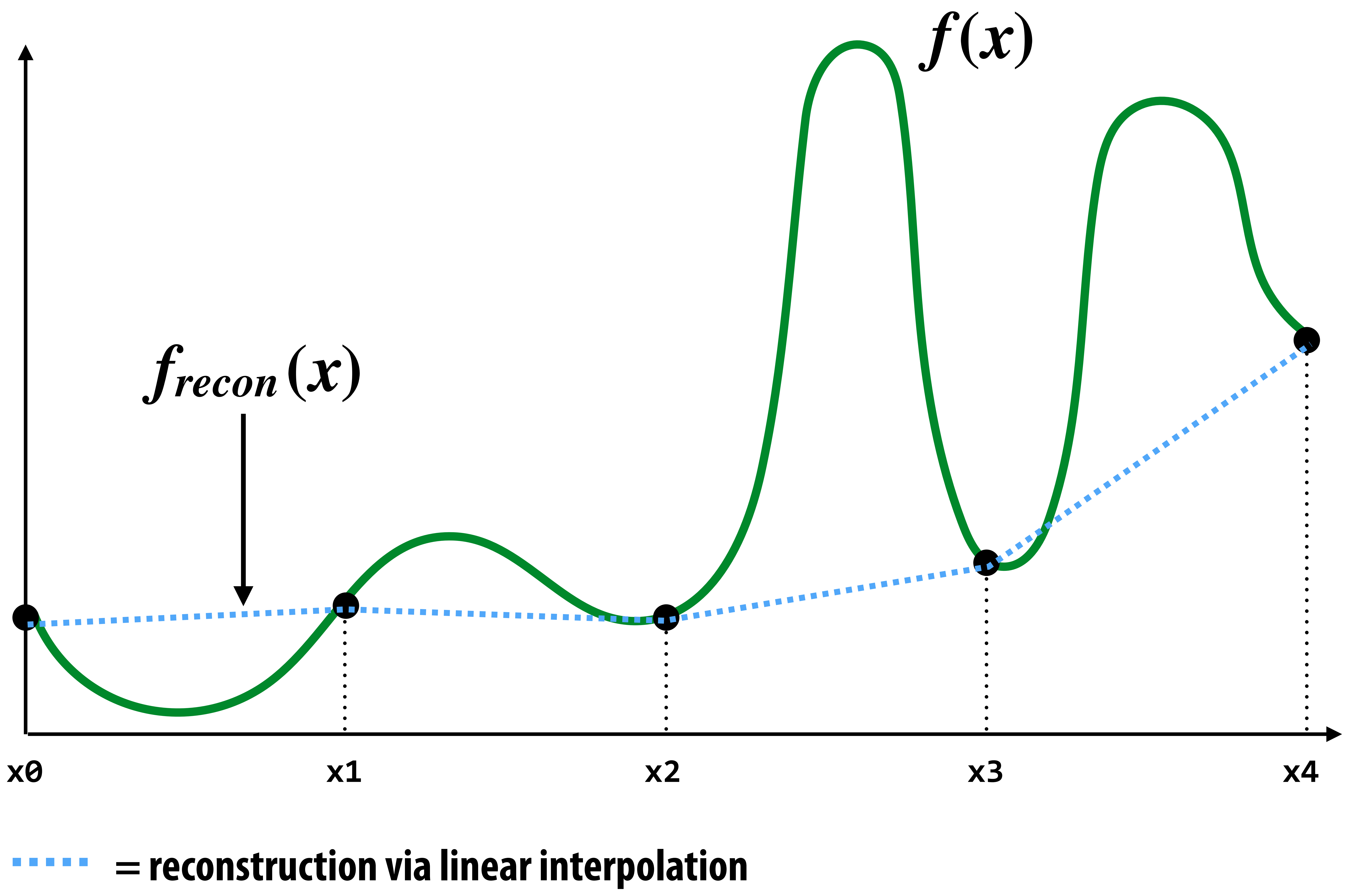
How can we represent the signal more accurately?



Answer: sample signal more densely (increase sampling rate)

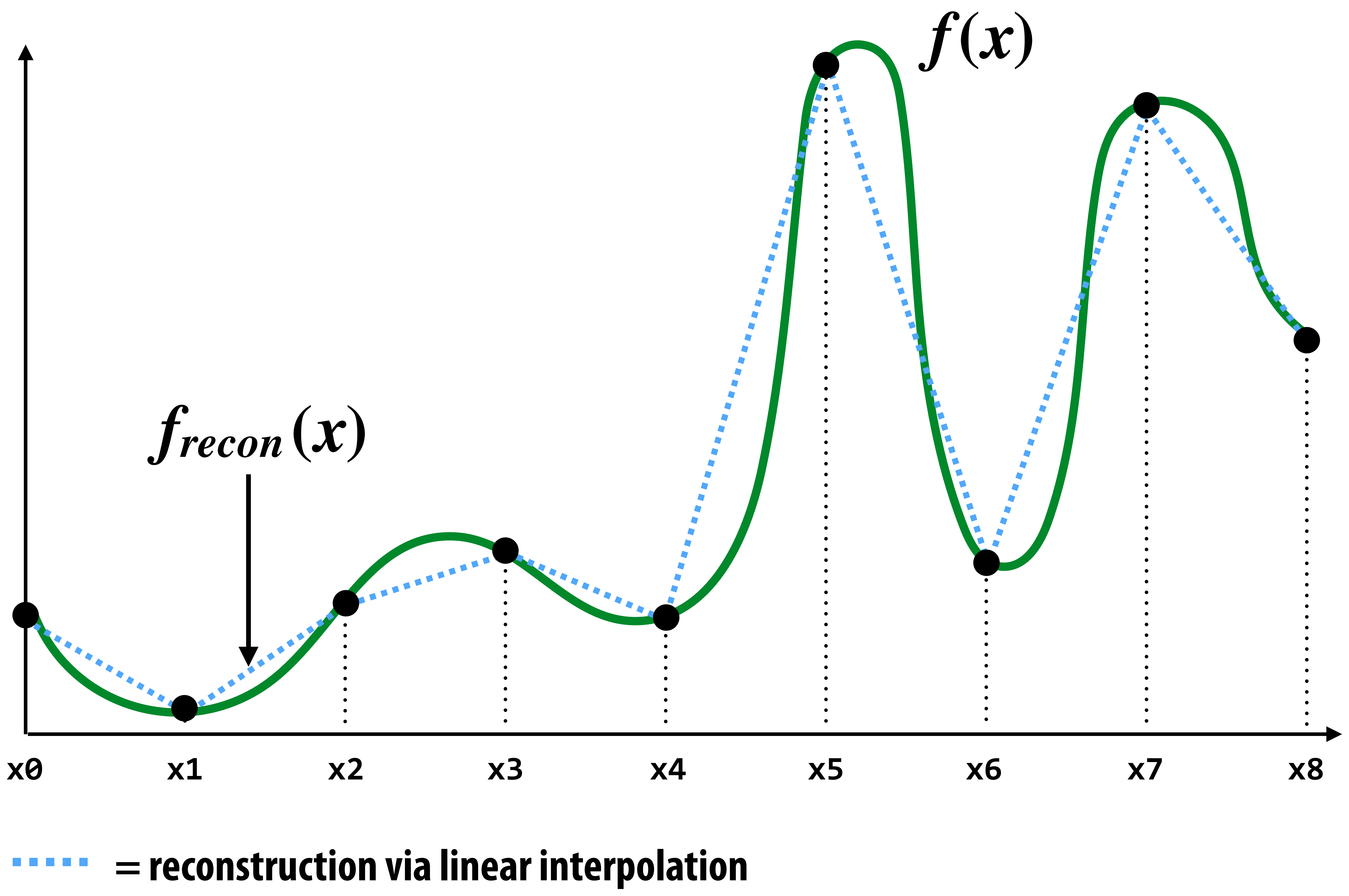
Reconstruction from sparse sampling

(5 samples)



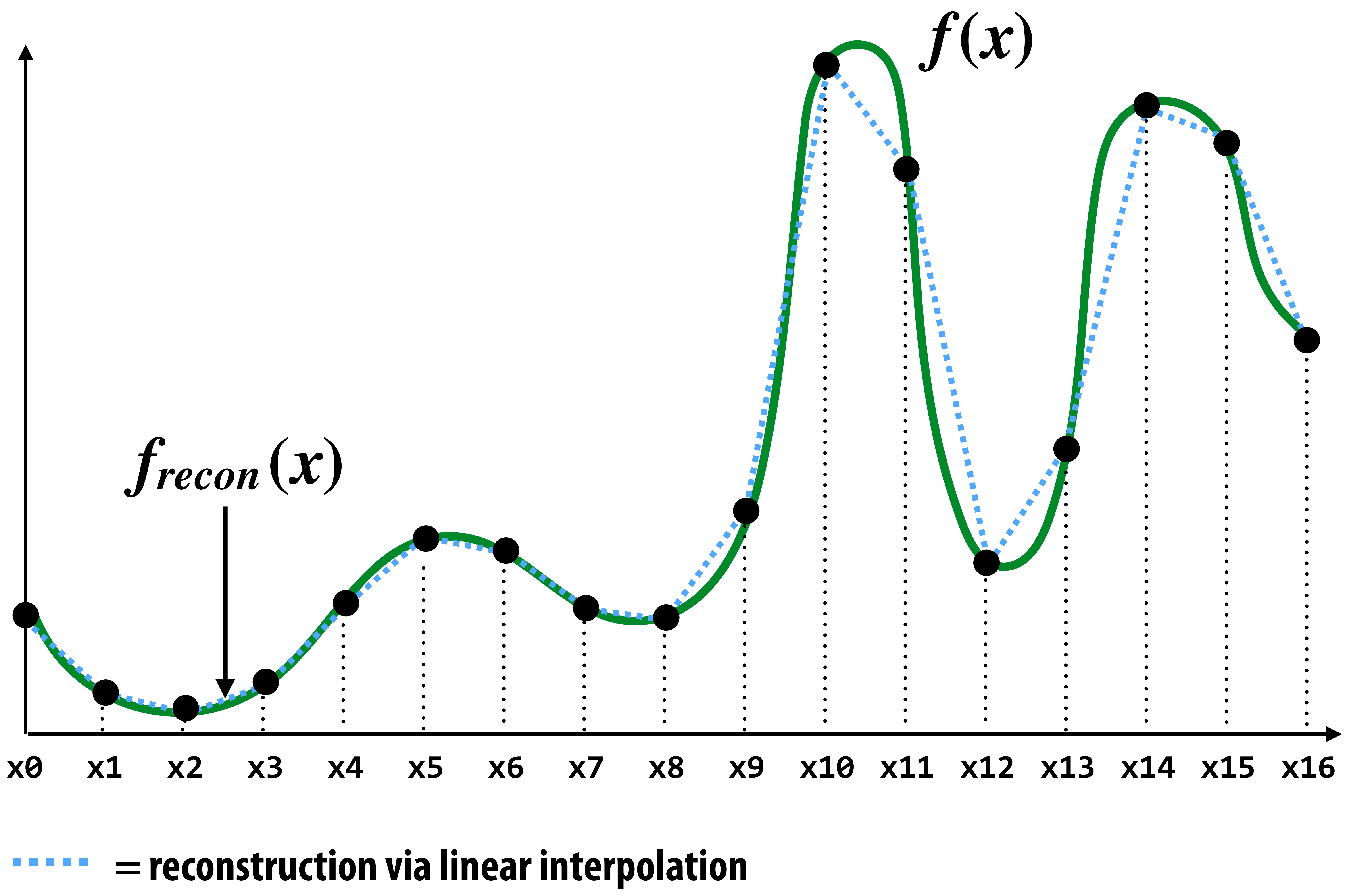
More accurate reconstructions result from denser sampling

(9 samples)



More accurate reconstructions result from denser sampling

(17 samples)



Drawing a triangle by 2D sampling

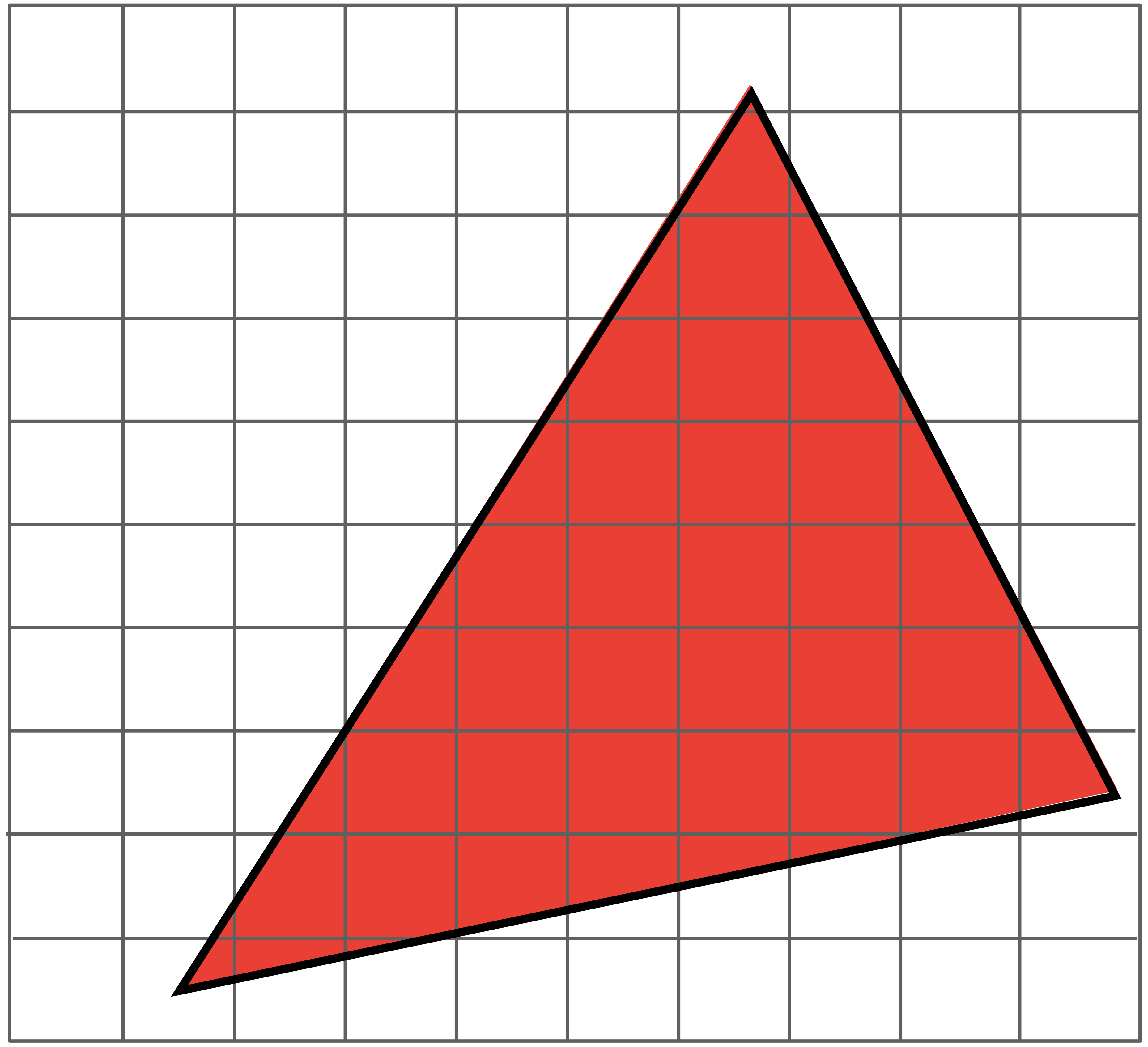
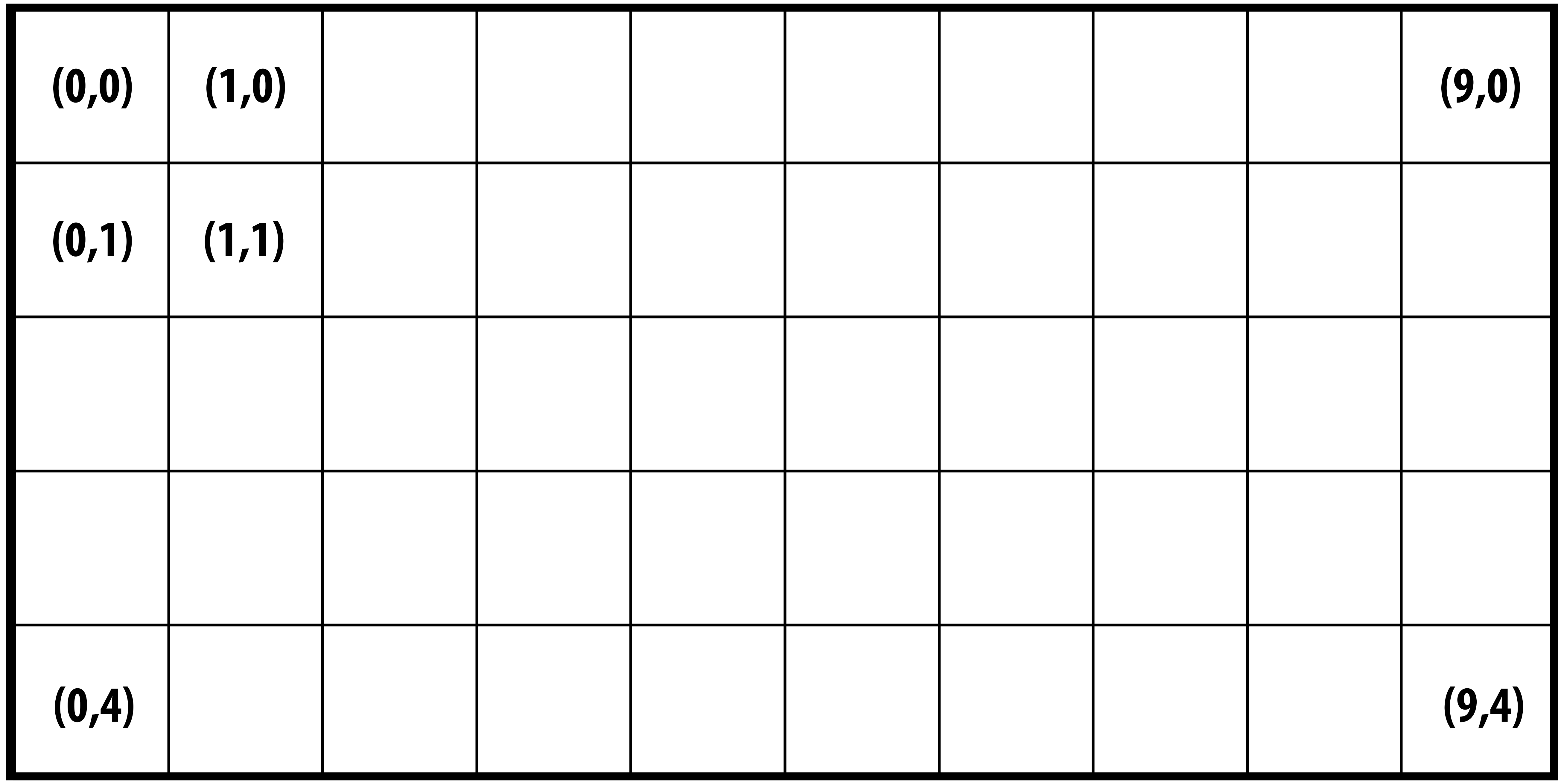


Image as a 2D matrix of pixels

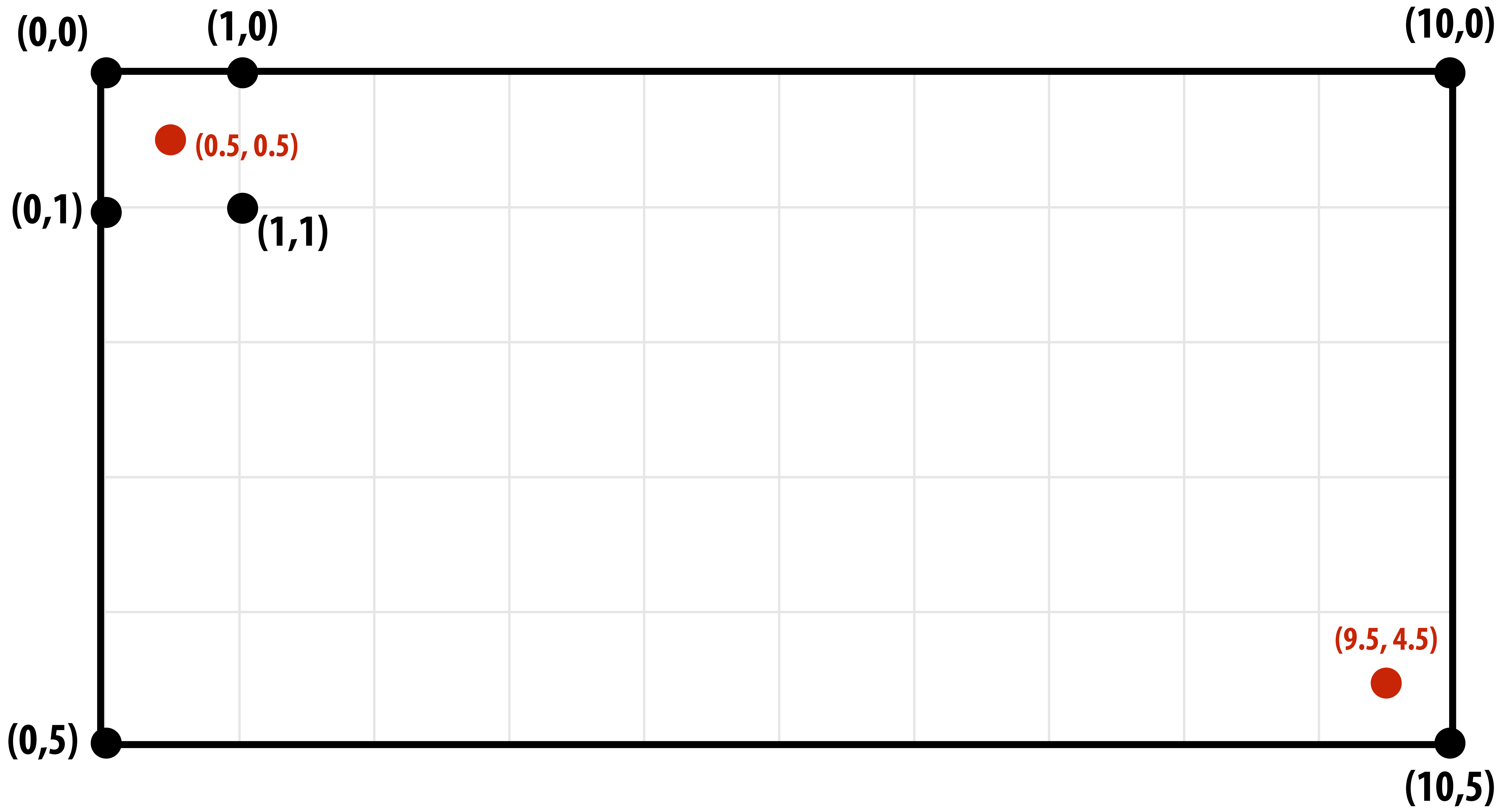
Here I'm showing a 10 x 5 pixel image

Identify pixel by its integer (x,y) coordinates



Continuous coordinate space over image

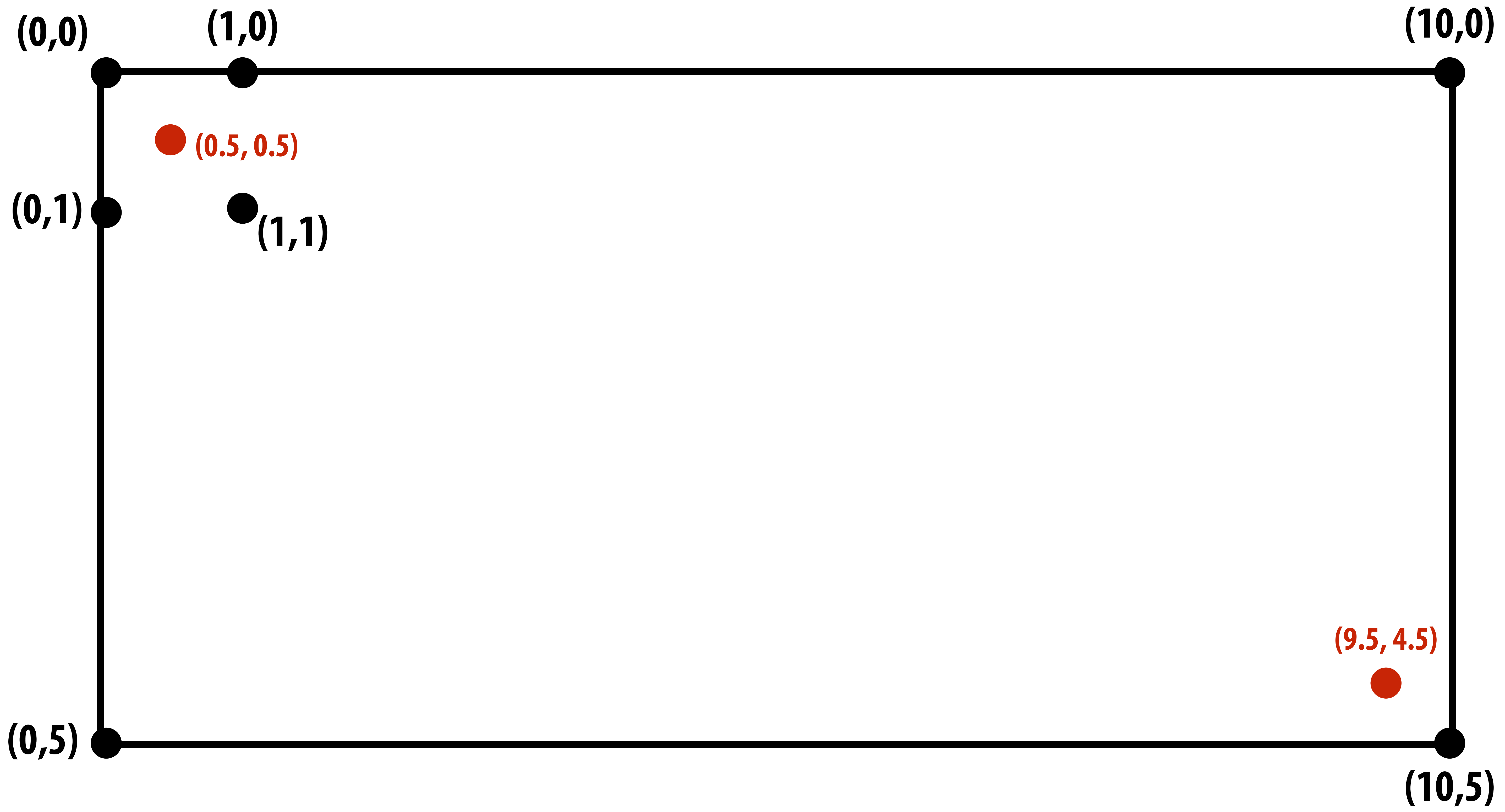
Ok, now forget about pixels!



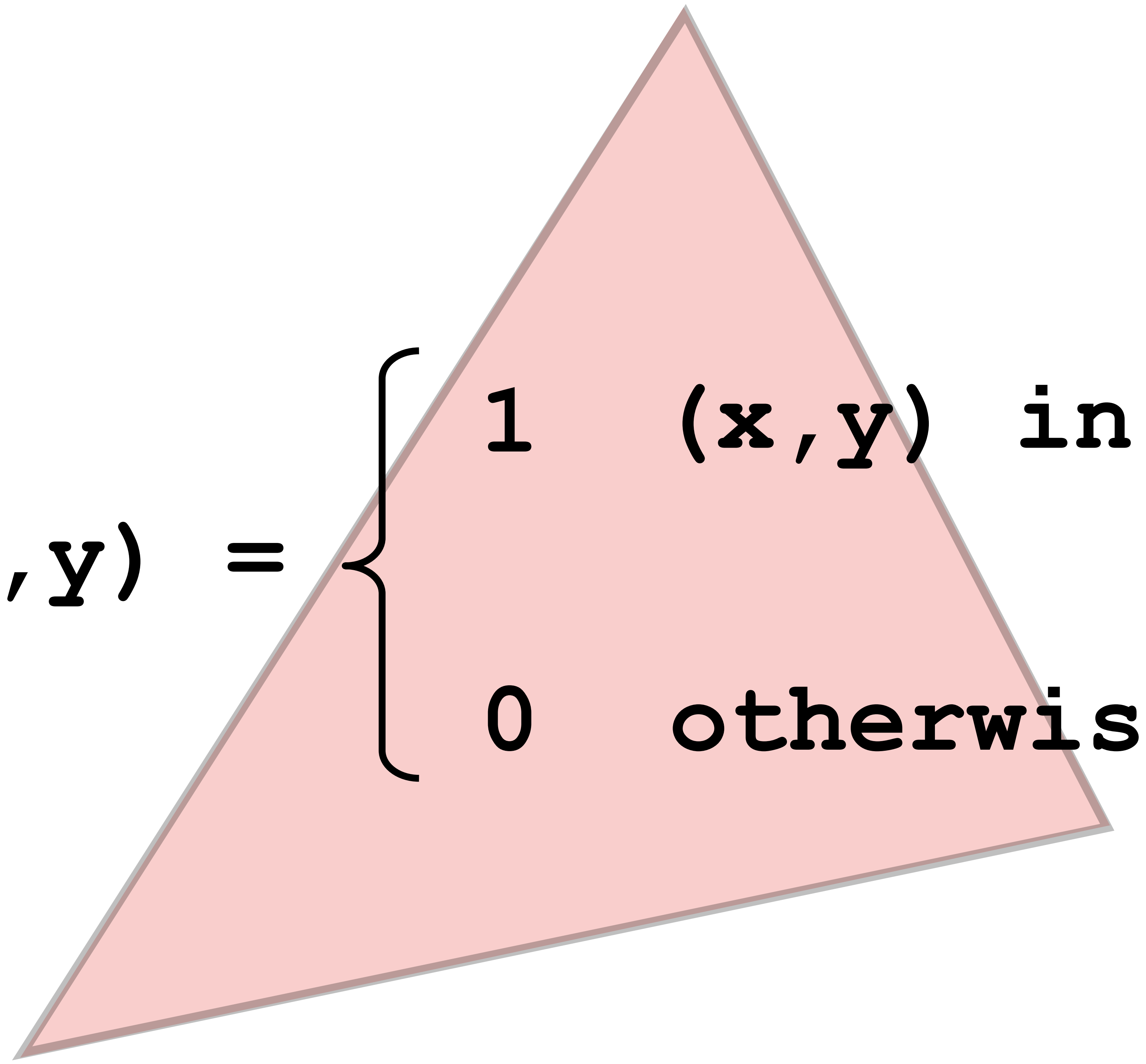
Continuous coordinate space over image

Ok, now forget about pixels!

(I removed pixel boundaries from the figure to encourage you to forget about pixels!)

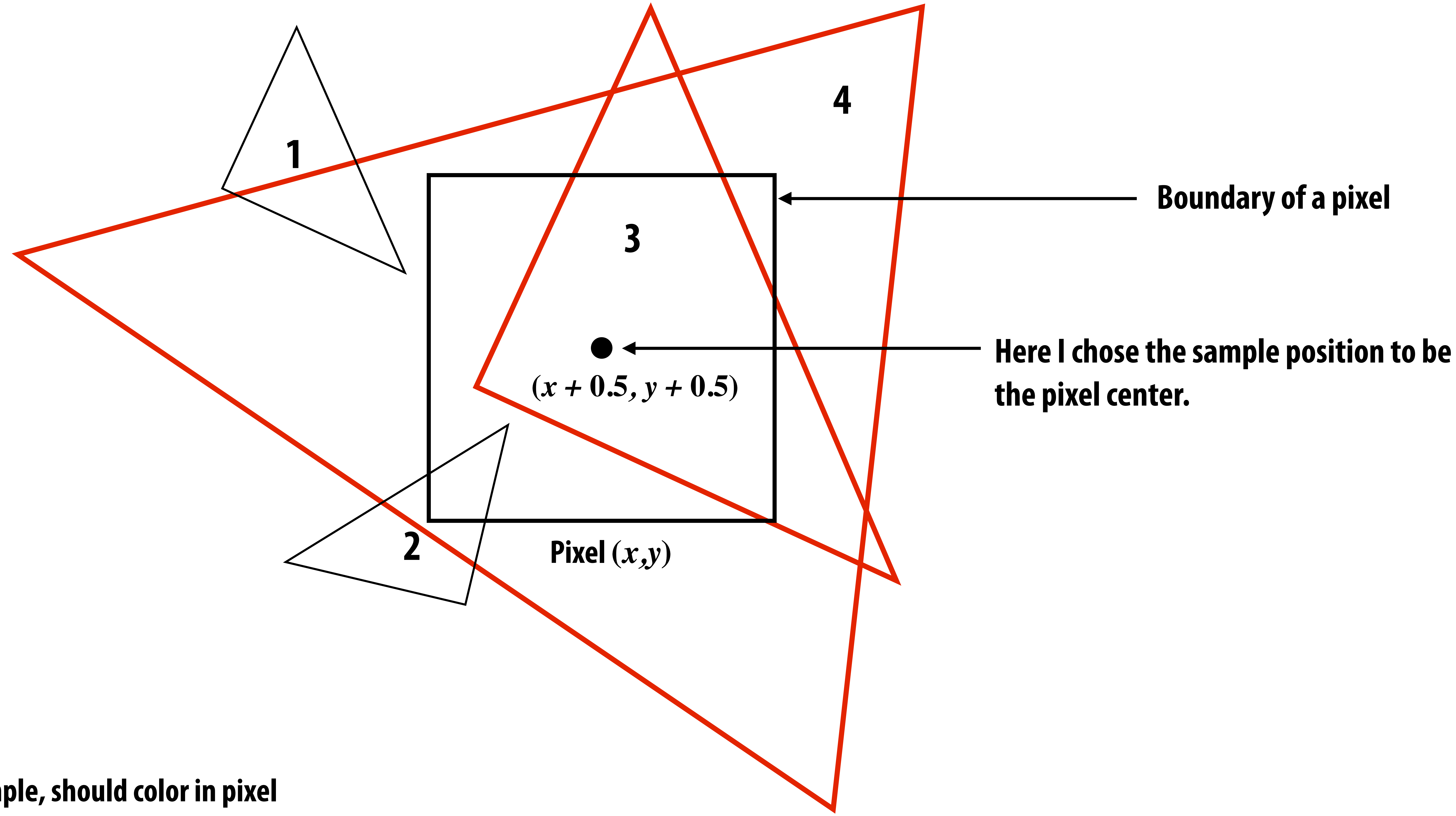


Define binary function: `inside(tri,x,y)`



$$\texttt{inside(t,x,y)} = \begin{cases} \texttt{1} & \texttt{(x,y) in triangle t} \\ \texttt{0} & \texttt{otherwise} \end{cases}$$

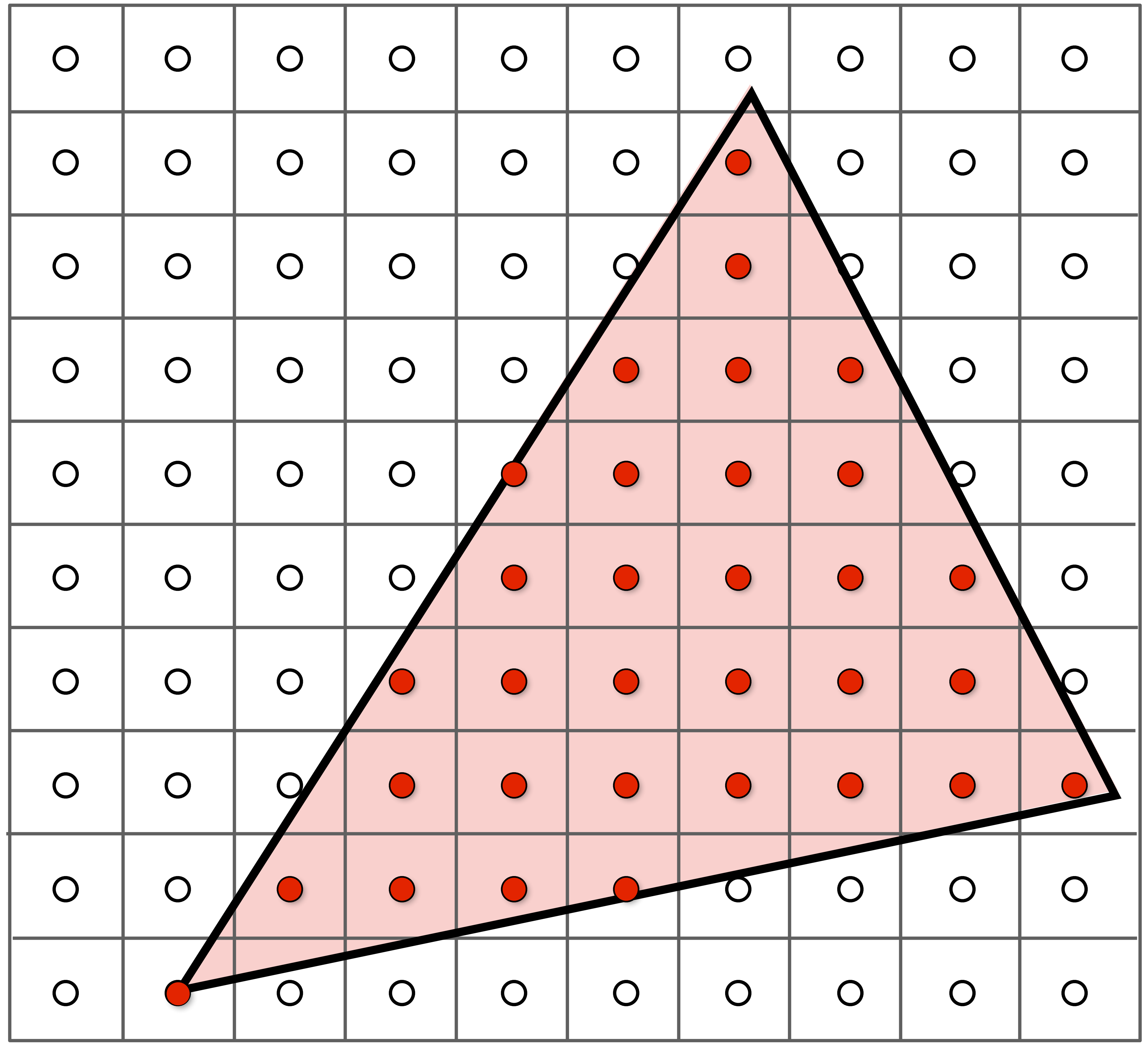
Sampling the binary function: `inside(tri,x,y)`



= triangle covers sample, should color in pixel

= triangle does not cover sample, do not color in pixel

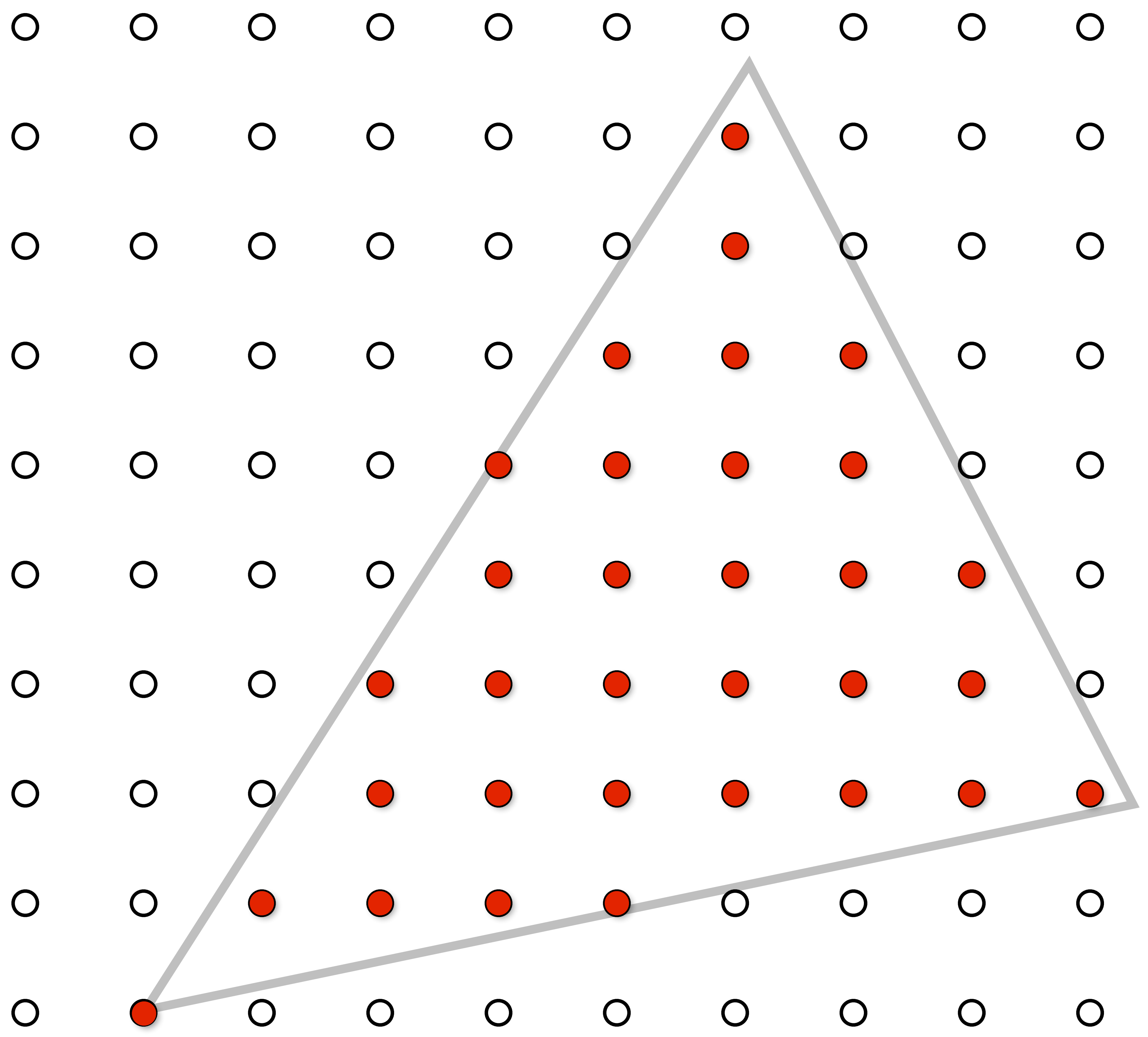
Sample coverage at pixel centers



Sample coverage at pixel centers

I only want you to think about evaluating triangle-point coverage!

NOT TRIANGLE-PIXEL OVERLAP!



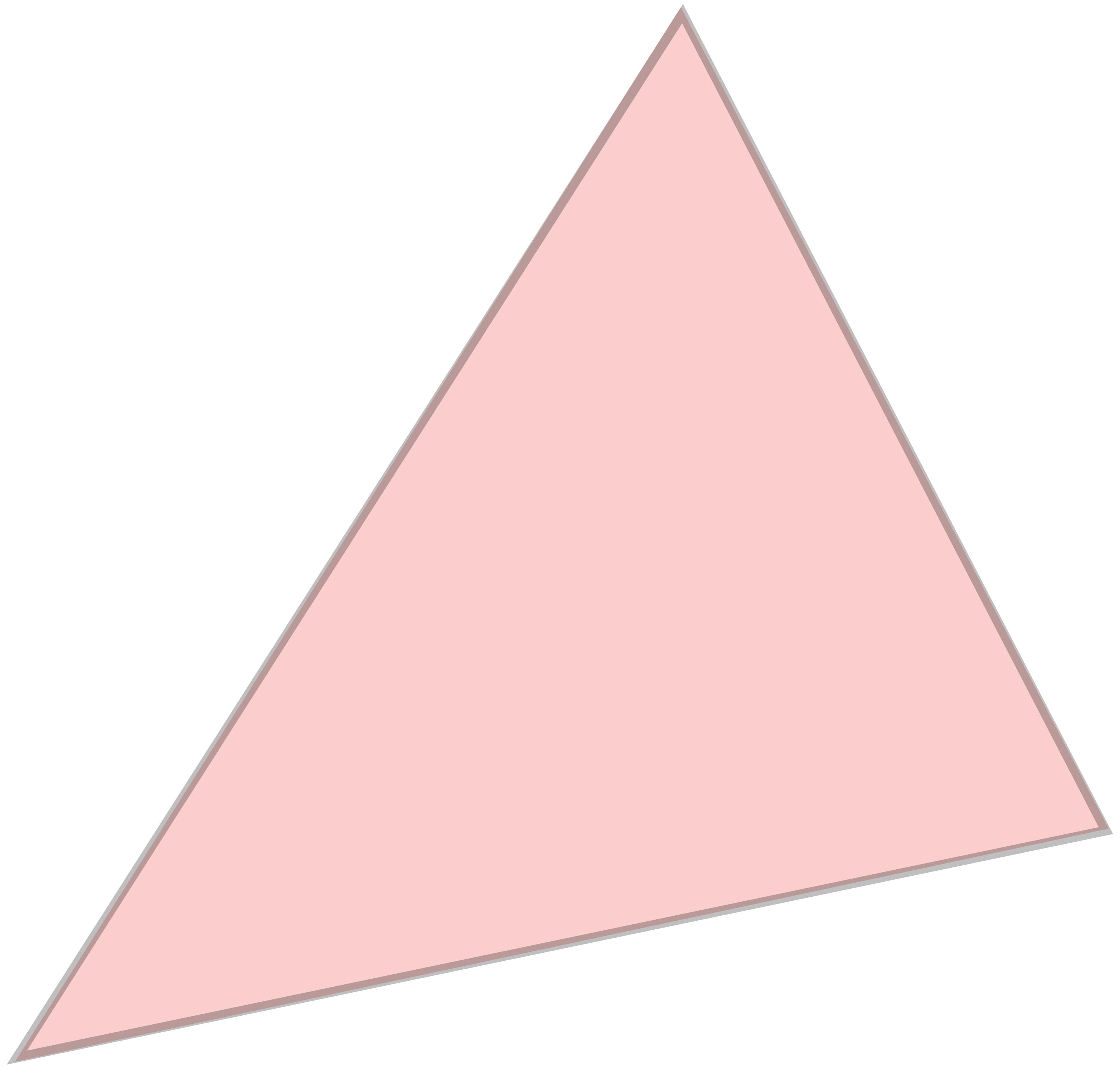
Rasterization = sampling a 2D binary function

- **Rasterize triangle `tri` by sampling the function**

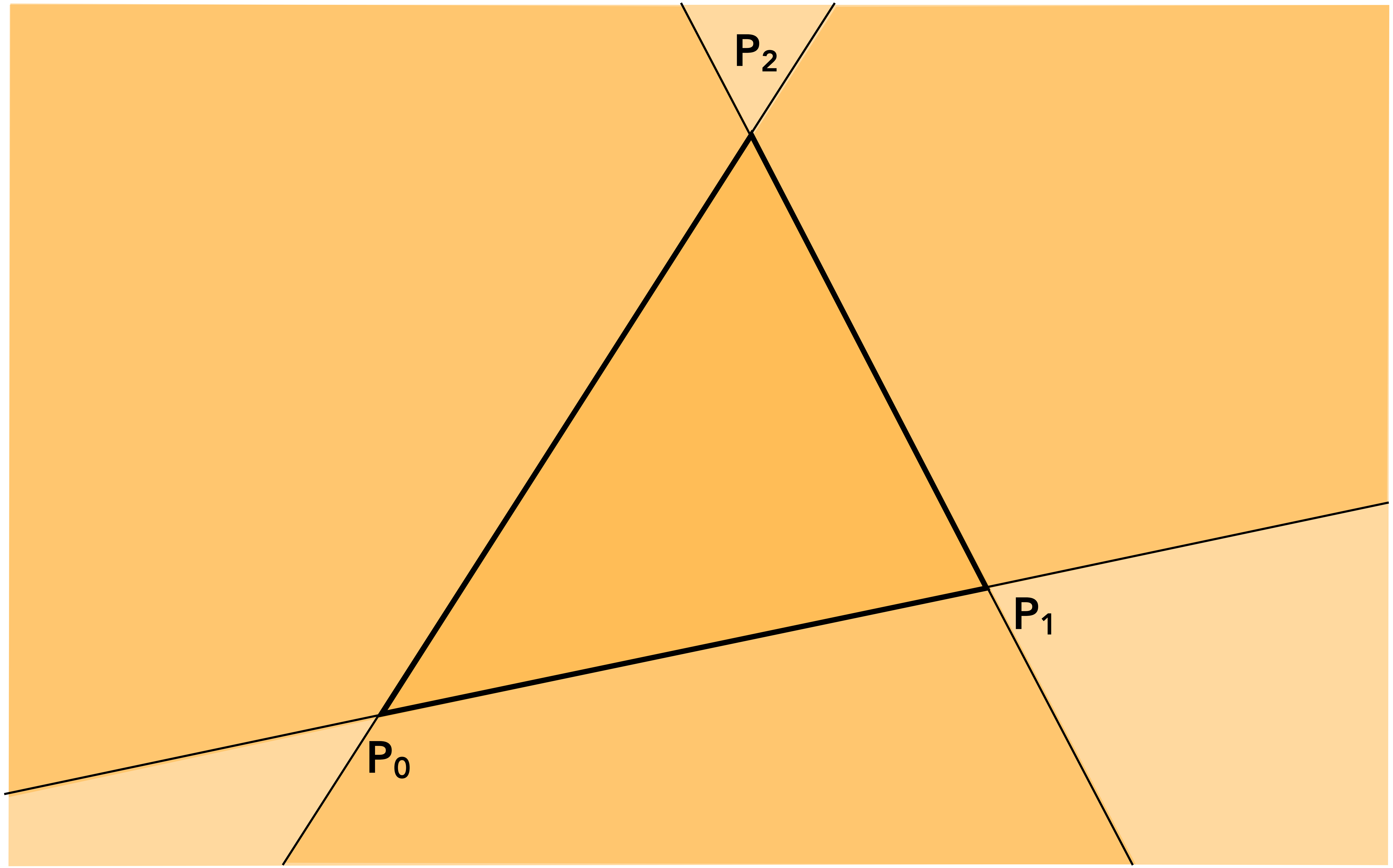
  `f(x,y) = inside(tri,x,y)`

```
for (int x = 0; x < xmax; x++)
  for (int y = 0; y < ymax; y++)
    image[x][y] = f(x + 0.5, y + 0.5);
```

Evaluating `inside(tri,x,y)`



Triangle = intersection of three half planes

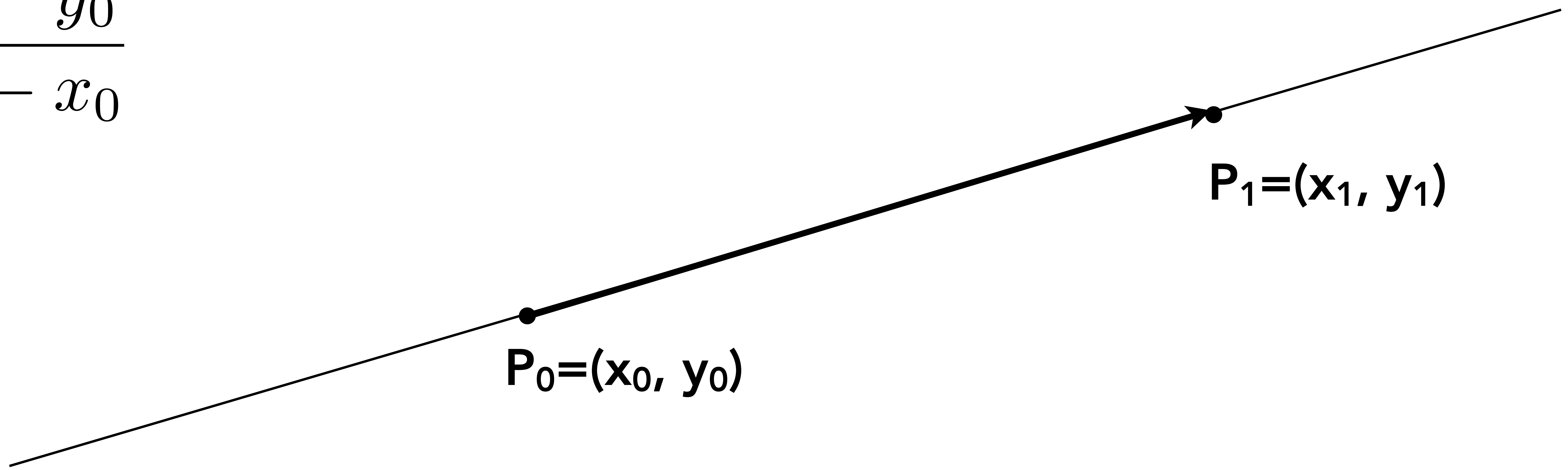


Point-slope form of a line

(You might have seen this in high school)

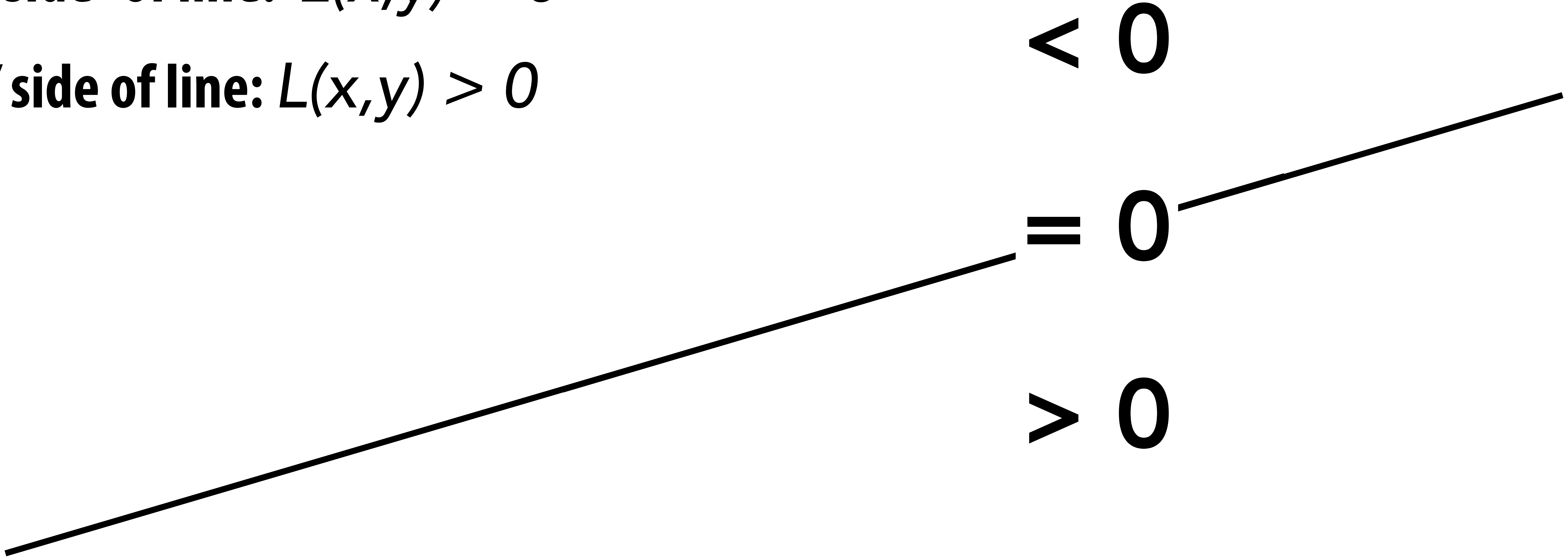
$$y - y_0 = m(x - x_0)$$

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

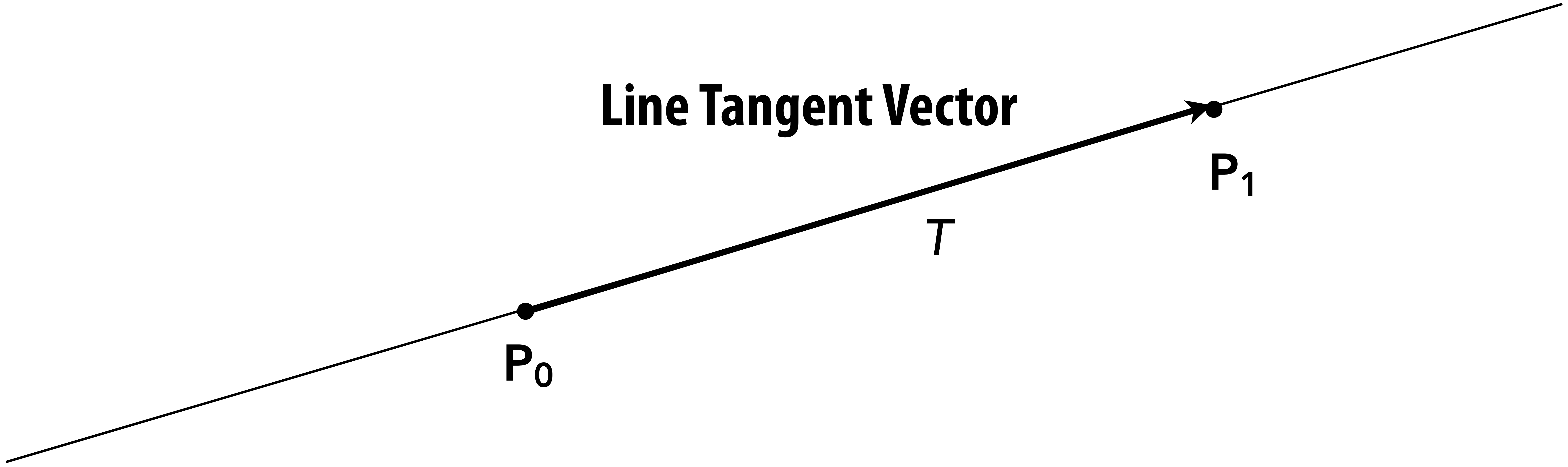


Each line defines two half-planes

- **Implicit line equation**
  - $L(x,y) = Ax + By + C$
  - **On the line:** $L(x,y) = 0$
  - **"Negative side" of line:** $L(x,y) < 0$
  - **"Positive" side of line:** $L(x,y) > 0$

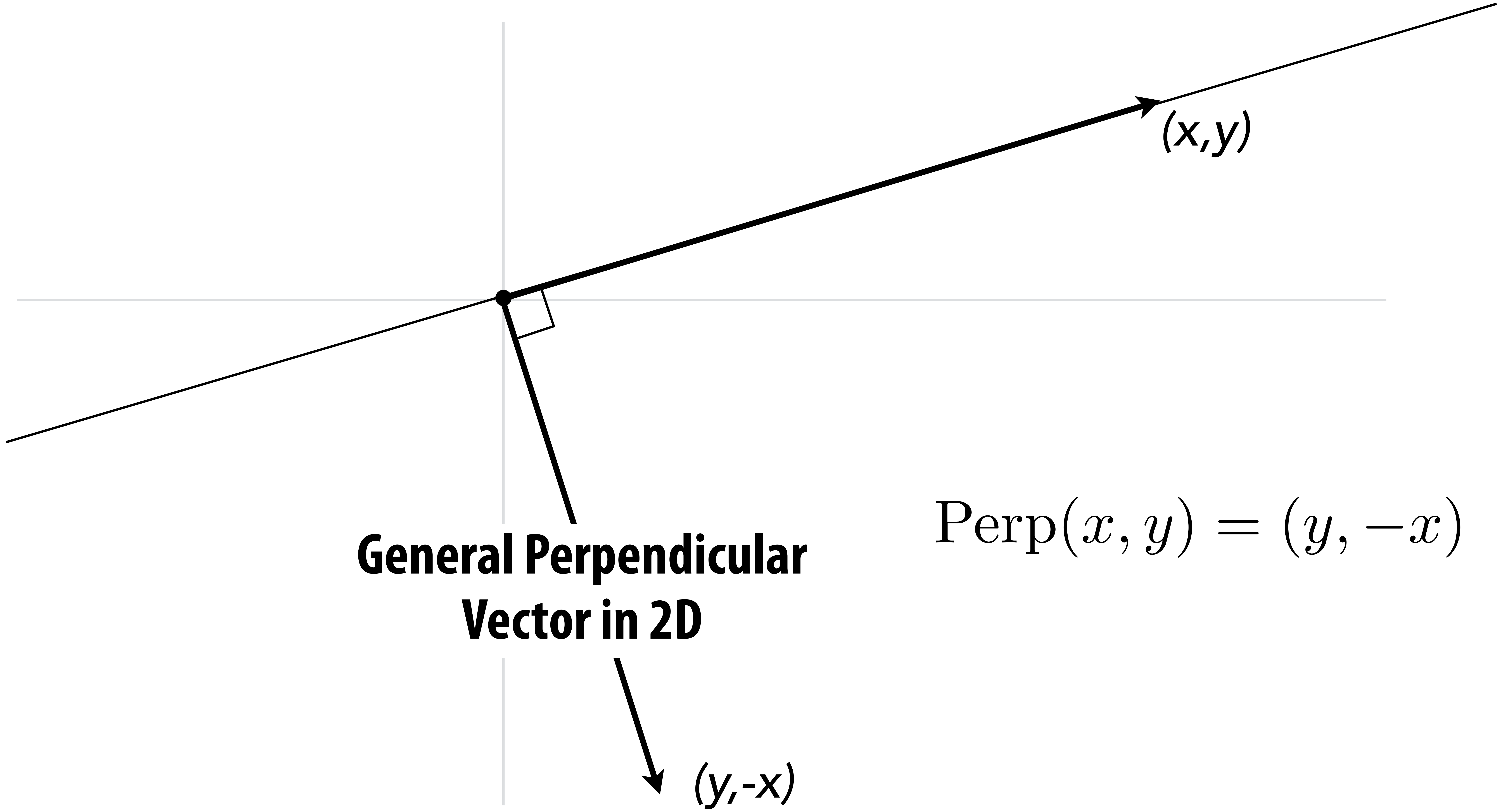


Line equation derivation



$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

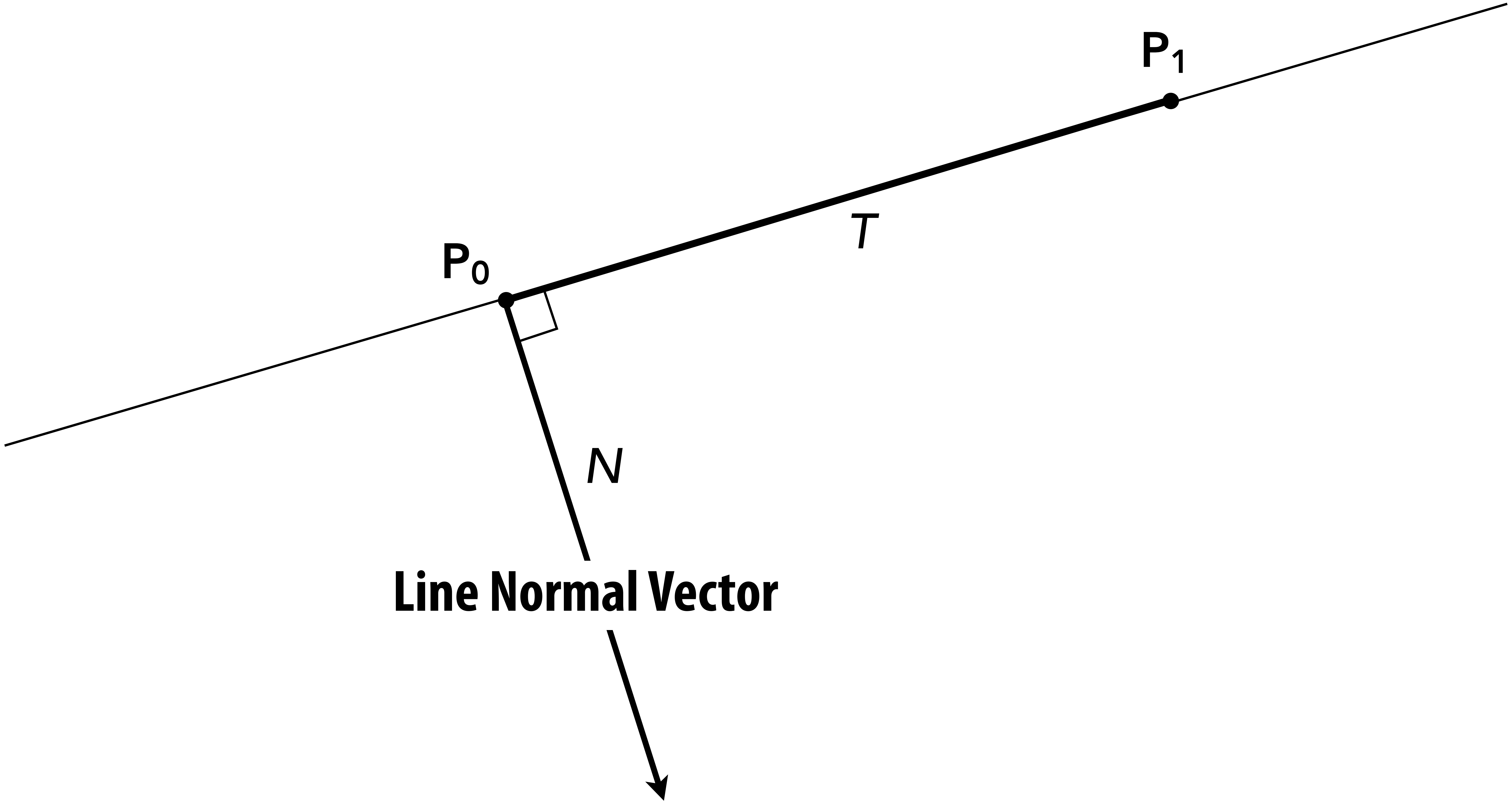
Line equation derivation



$$\mathrm{Perp}(x, y) = (y, -x)$$

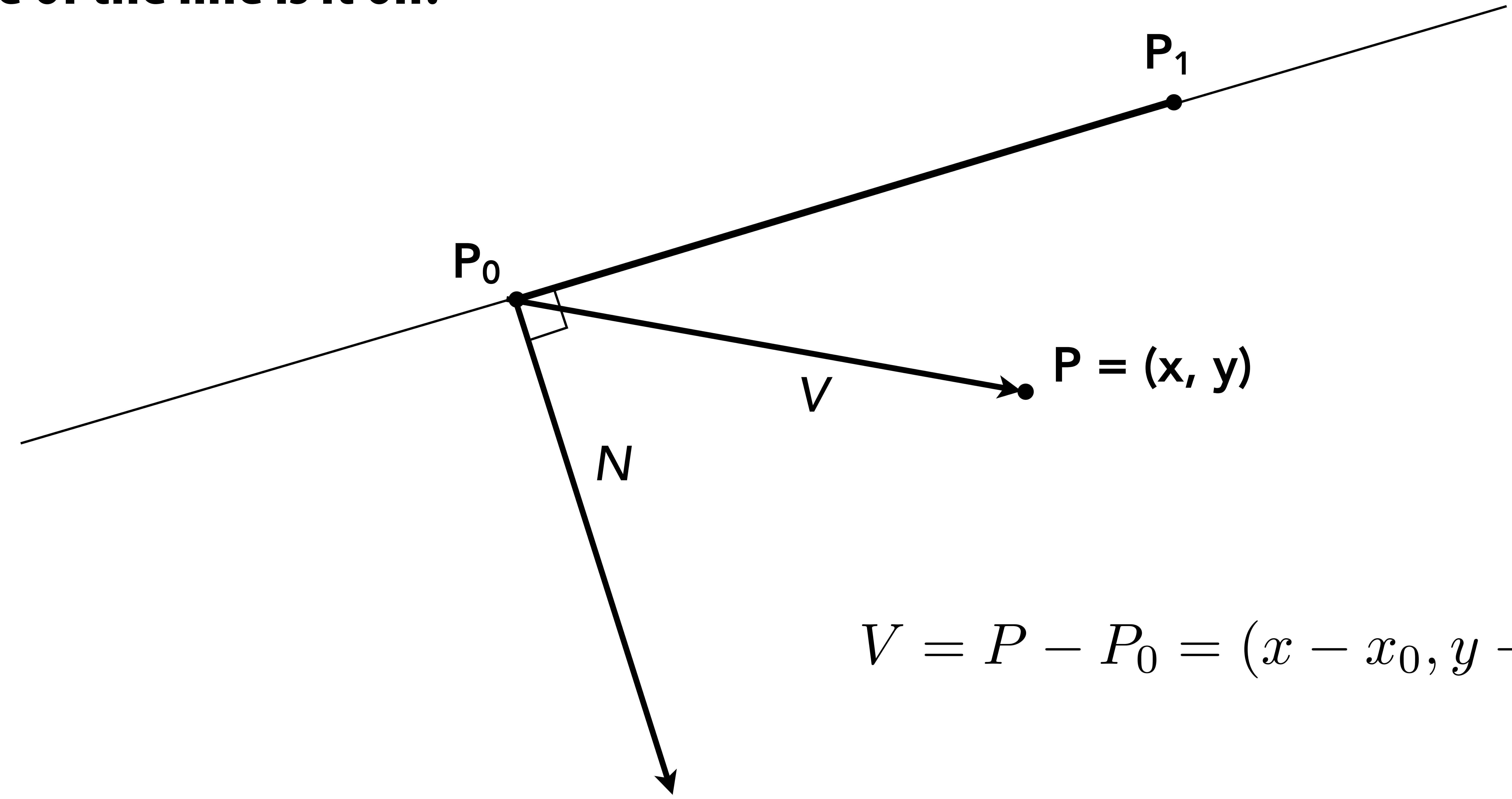
Line equation derivation

$$N = \mathrm{Perp}(T) = (y_1 - y_0, -(x_1 - x_0))$$



Line equation derivation

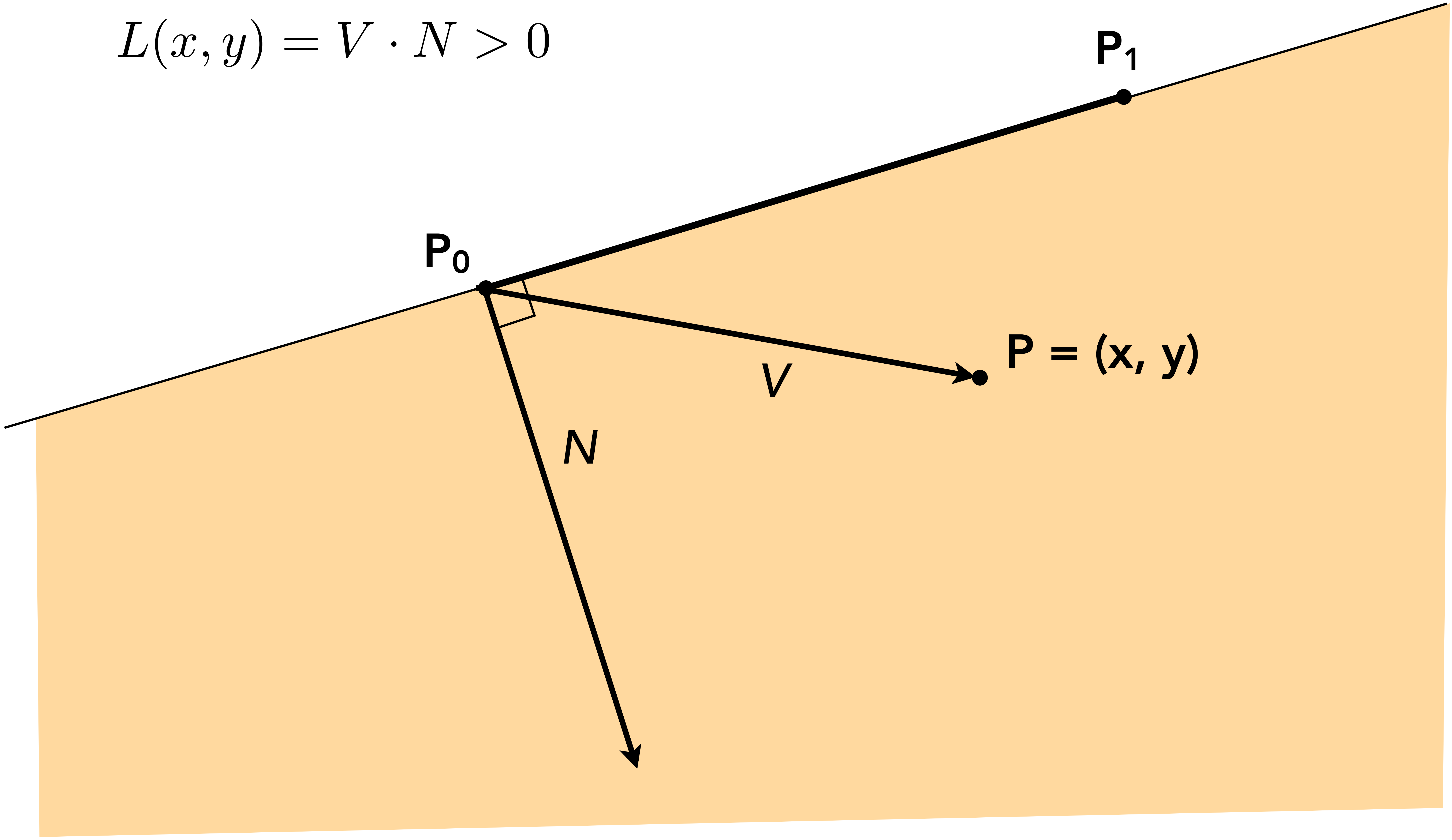
Now consider a point *P*=(x,y).
Which side of the line is it on?



$$V = P - P_0 = (x - x_0, y - y_0)$$

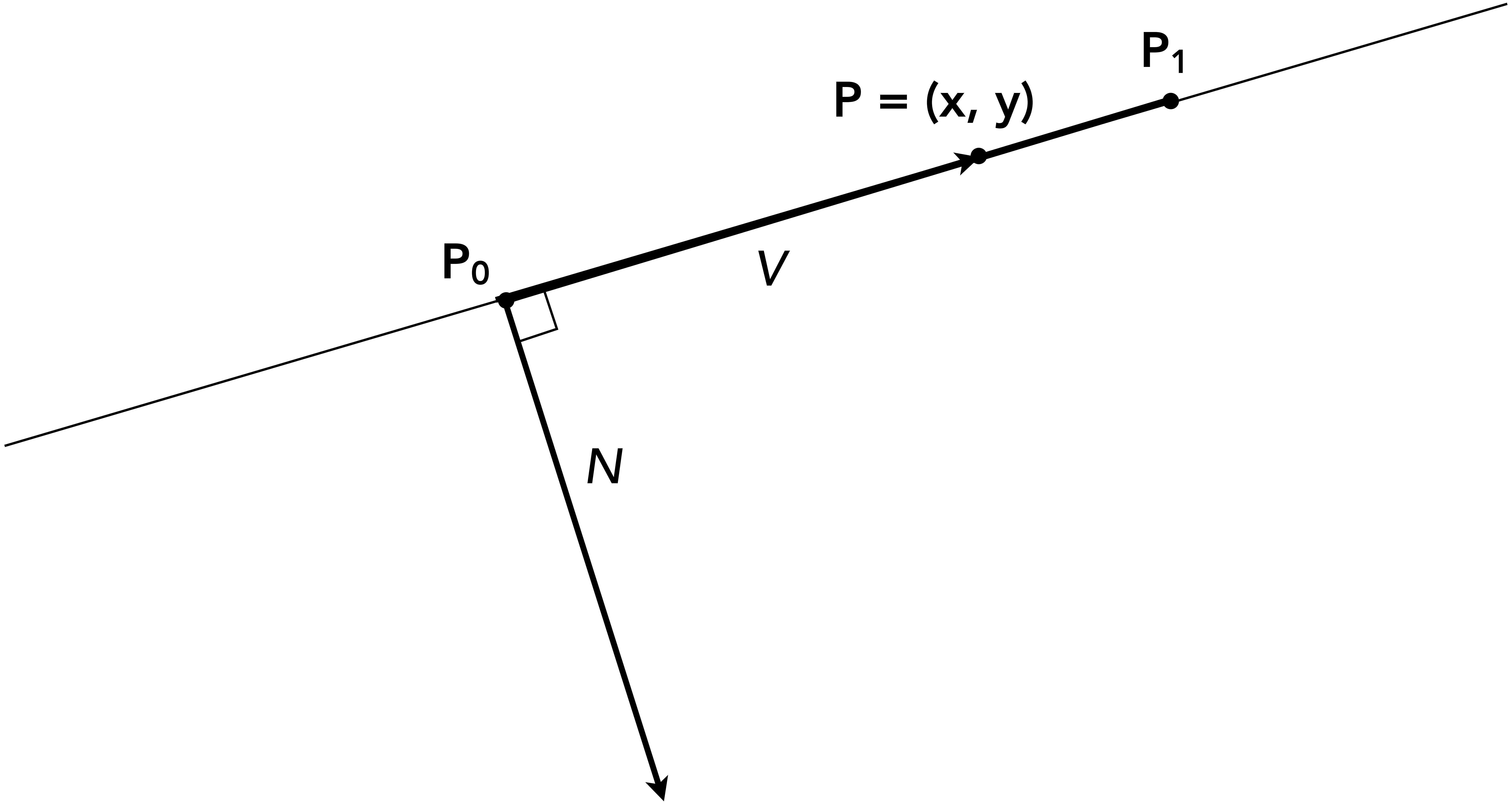
Line equation tests

$$L(x, y) = V \cdot N > 0$$

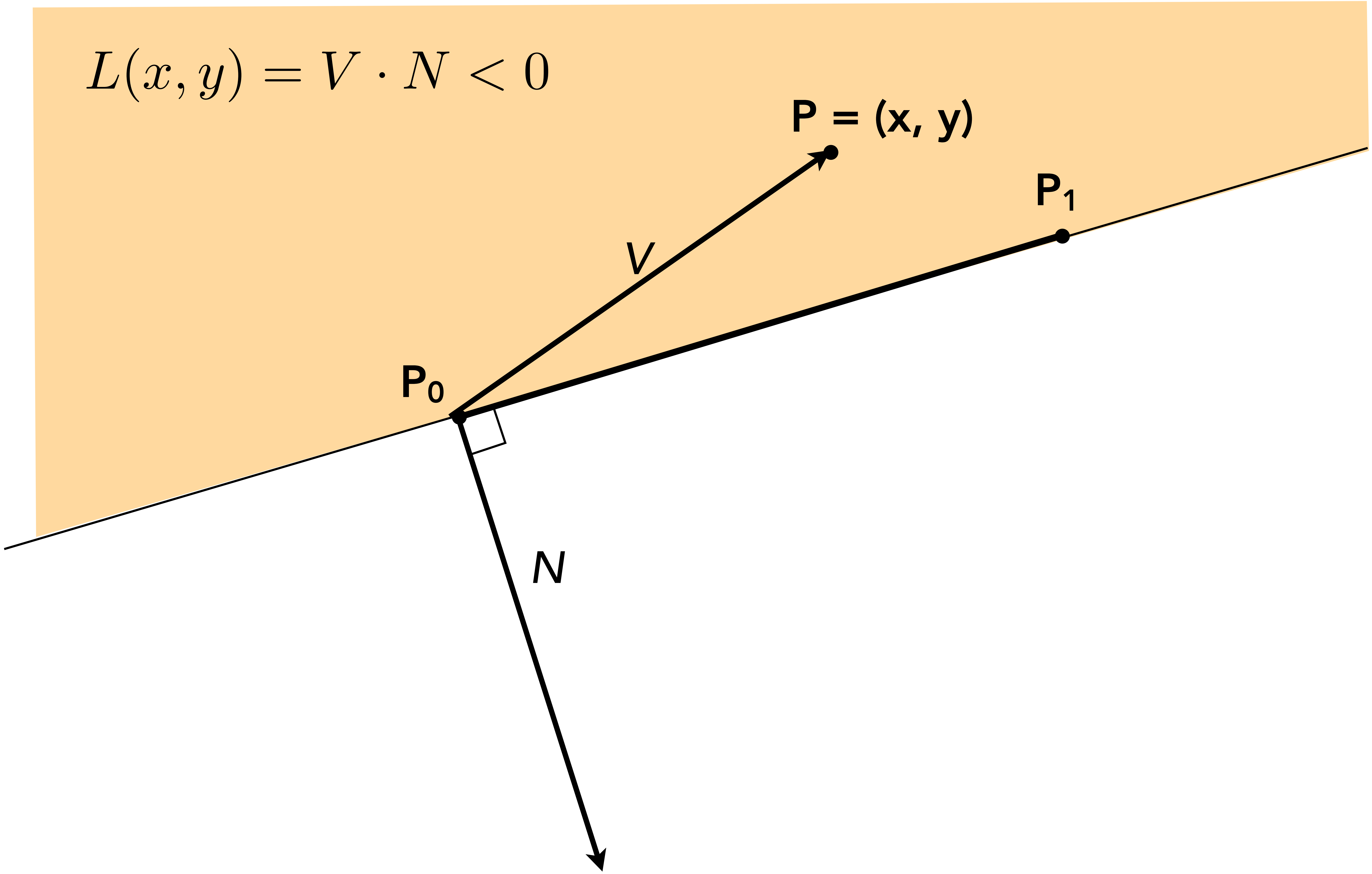


Line equation tests

$$L(x, y) = V \cdot N = 0$$

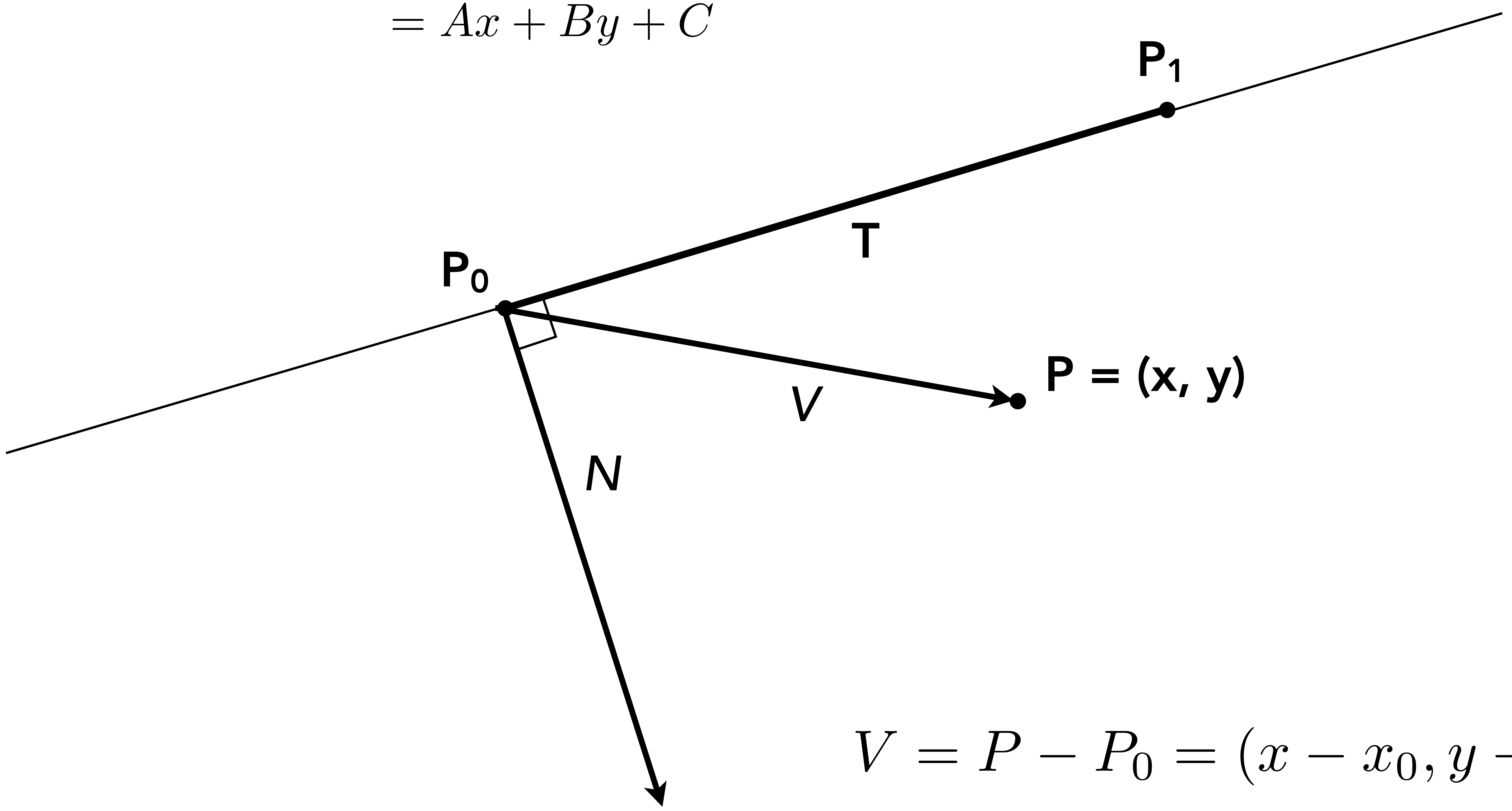


Line equation tests



Line equation derivation

$$
\begin{aligned}
L(x,y) = V \cdot N &= -(y-y_0)(x_1-x_0) + (x-x_0)(y_1-y_0) \\
&= (y_1-y_0)x - (x_1-x_0)y + y_0(x_1-x_0) - x_0(y_1-y_0) \\
&= Ax + By + C
\end{aligned}
$$



$$V = P - P_0 = (x - x_0, y - y_0)$$

$$N = \text{Perp}(T) = (y_1 - y_0, -(x_1 - x_0))$$

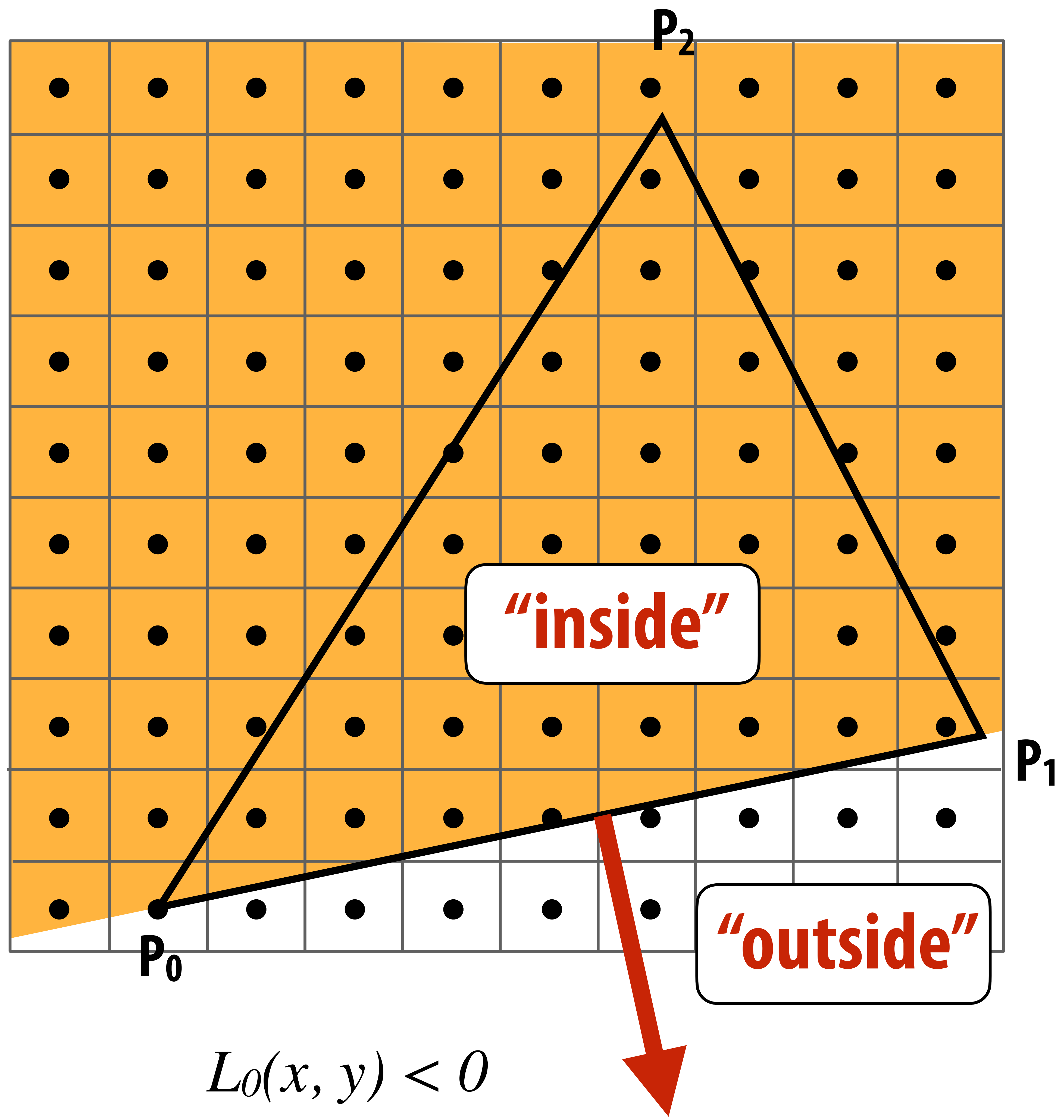
Point-in-triangle test

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = -dX_i = X_i - X_{i+1}$
$C_i = Y_i(X_{i+1} - X_i) - X_i(Y_{i+1} - Y_i)$

$L_i(x, y) = A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
> 0 : outside edge
< 0 : inside edge



Point-in-triangle test

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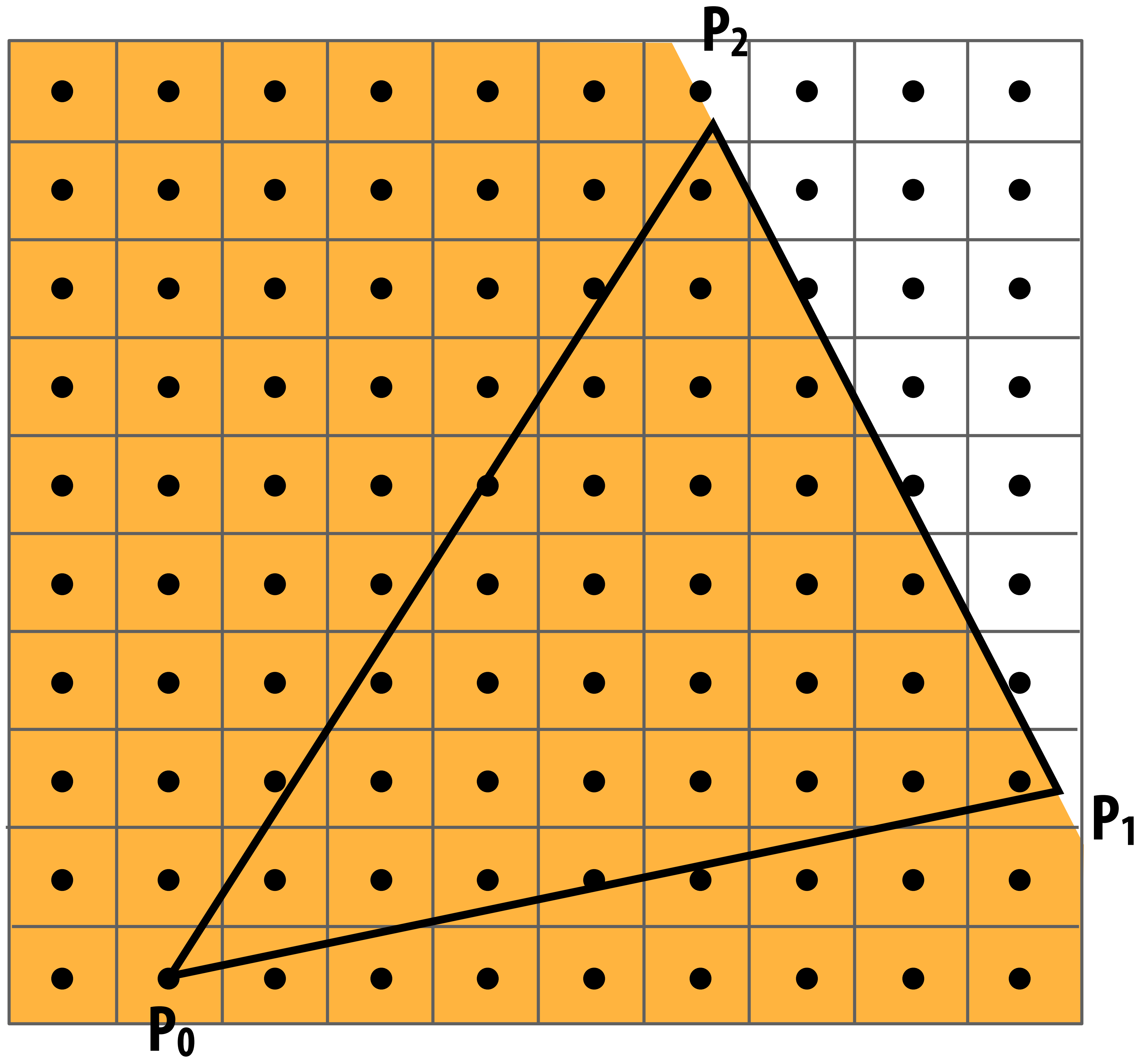

$L_1(x, y) < 0$

Point-in-triangle test

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = -dX_i = X_i - X_{i+1}$
$C_i = Y_i(X_{i+1} - X_i) - X_i(Y_{i+1} - Y_i)$

$L_i(x, y) = A_i x + B_i y + C_i$

$L_i(x, y) = 0$: point on edge
> 0 : outside edge
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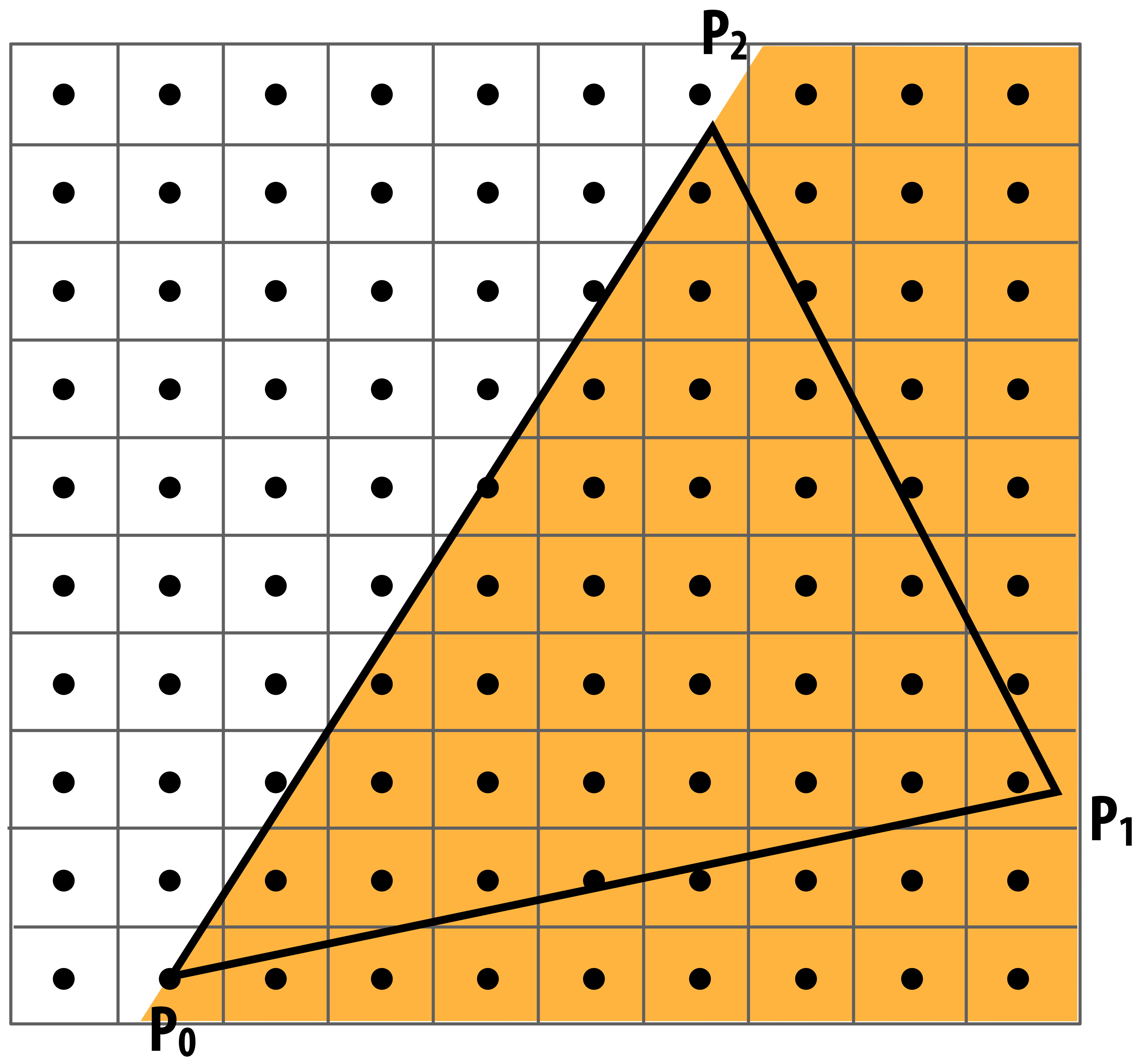


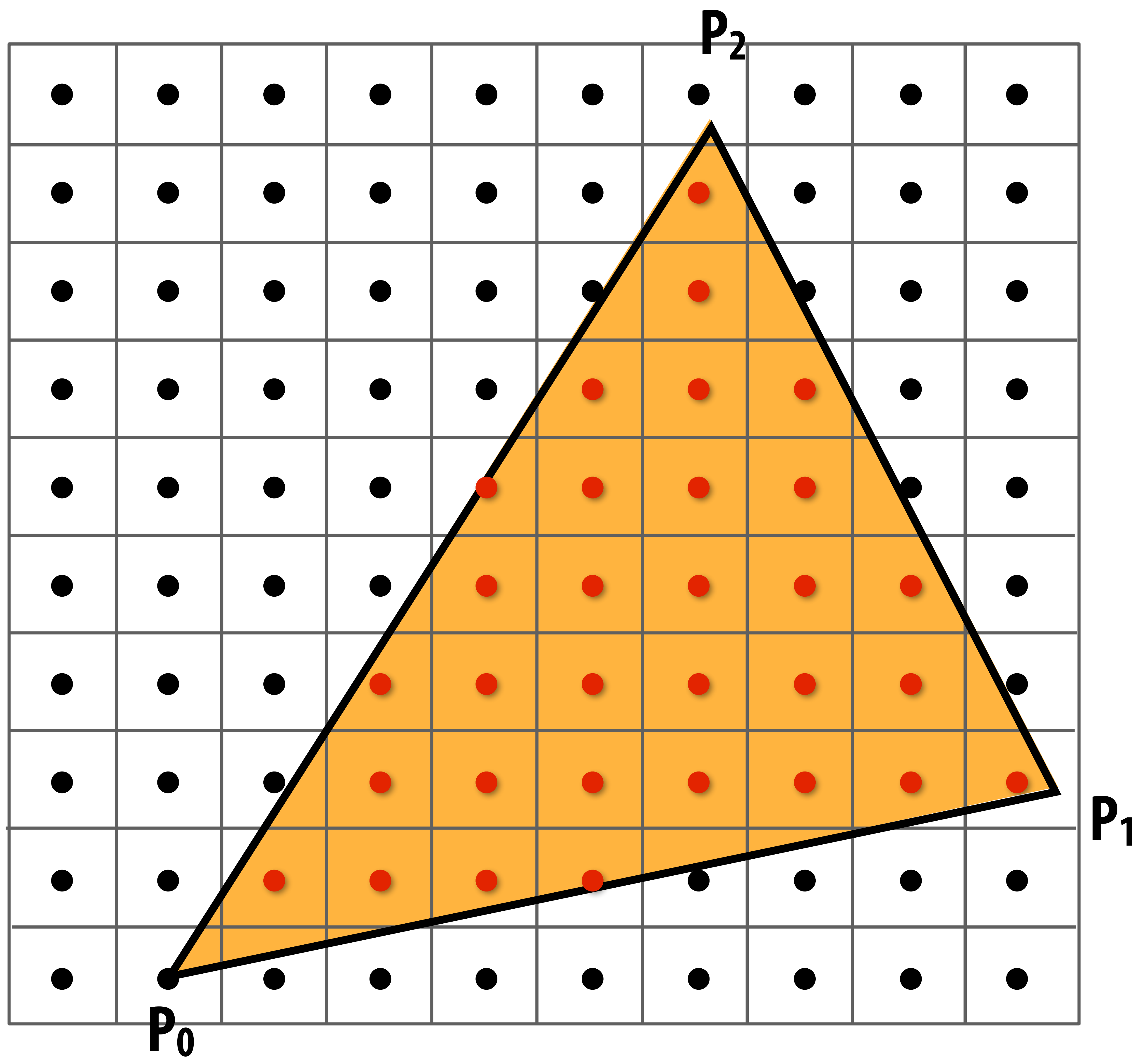
$L_2(x, y) < 0$

Point-in-triangle test

Sample point $s = (sx, sy)$ **is inside the triangle if it is inside all three edges.**

$$inside(sx, sy) =$$
$$L_0(sx, sy) < 0 \ \&\&$$
$$L_1(sx, sy) < 0 \ \&\&$$
$$L_2(sx, sy) < 0$$

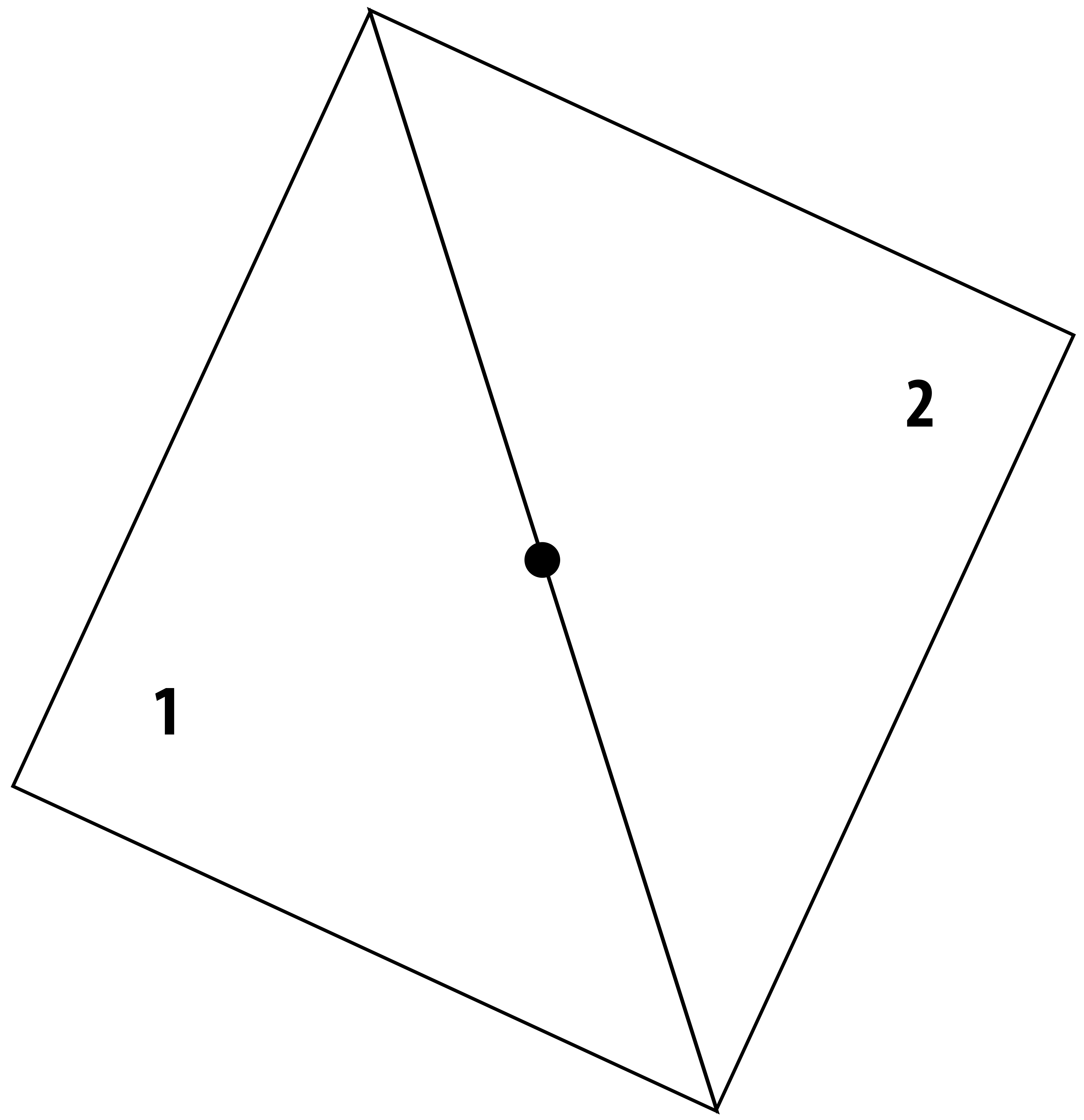
Note: actual implementation of $inside(sx,sy)$ **involves ≤ checks based on the triangle coverage edge rules (see next slide)**



Sample points inside triangle are highlighted red.

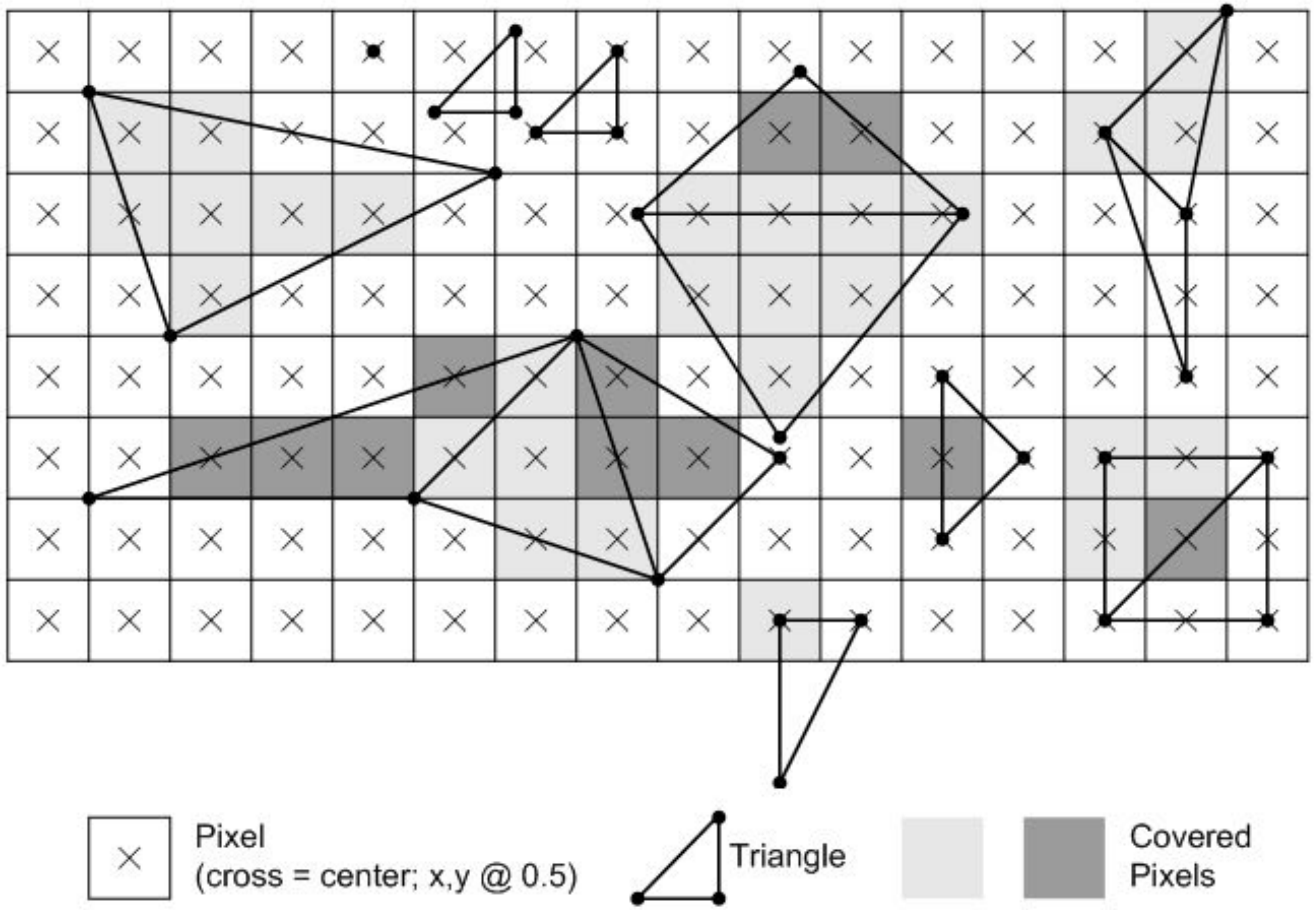
Edge cases (literally)

Is this sample point covered by triangle 1? or triangle 2? or both?



A detail: rasterization “edge rules”

- When edge falls directly on a screen sample point, the sample is classified as within triangle if the edge is a “top edge” or “left edge”
  - Top edge: horizontal edge that is above all other edges
  - Left edge: an edge that is not exactly horizontal and is on the left side of the triangle.(triangle can have one or two left edges)



Finding covered samples: incremental triangle traversal

$P_i = (X_i, Y_i)$

$A_i = dY_i = Y_{i+1} - Y_i$
$B_i = dX_i = X_{i+1} - X_i$
$C_i = Y_i (X_{i+1} - X_i) - X_i (Y_{i+1} - Y_i)$

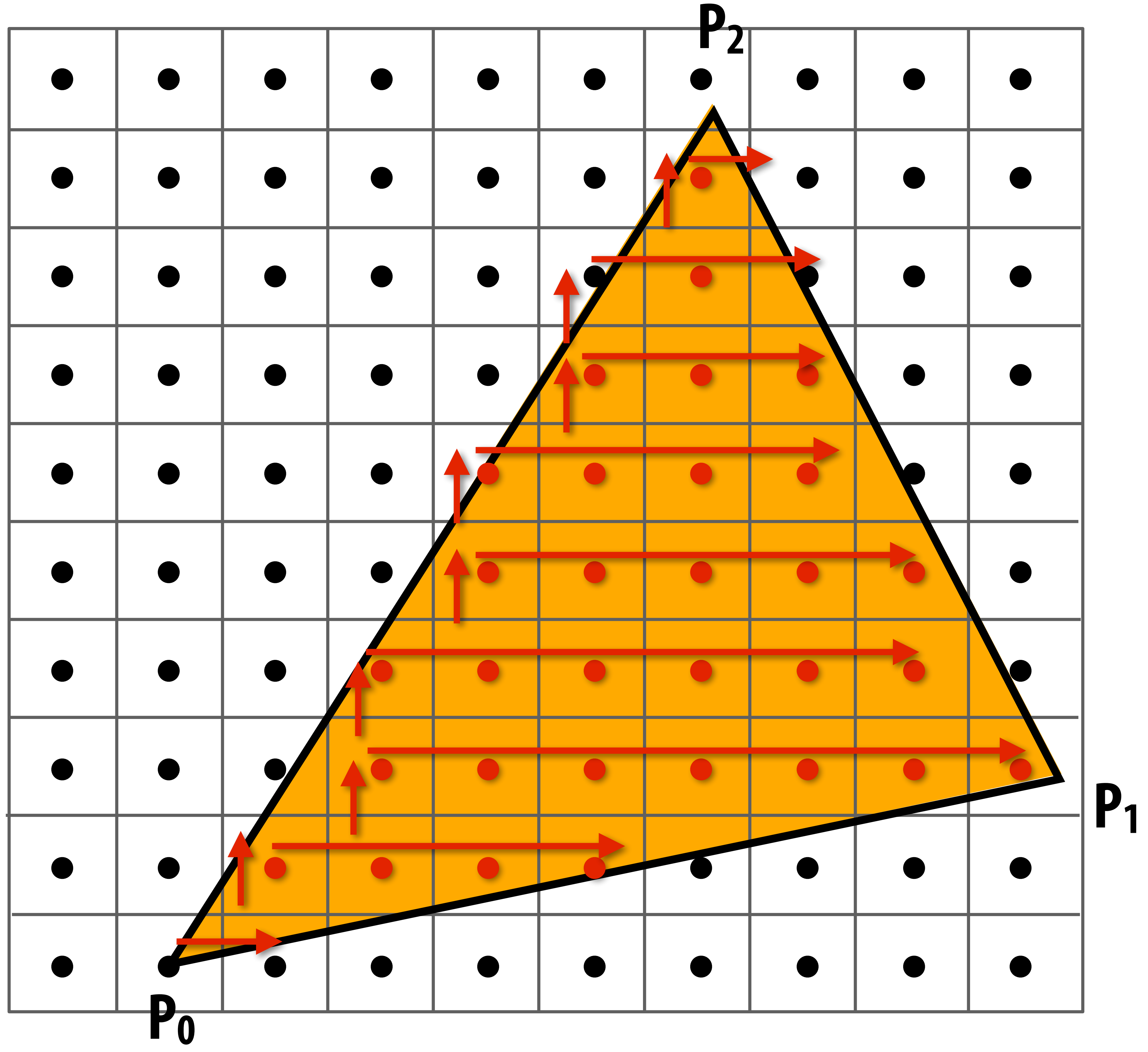
$L_i (x, y) = A_i x + B_i y + C_i$

$L_i (x, y) = 0$: point on edge
> 0 : outside edge
< 0 : inside edge

Efficient incremental update:

$L_i (x+1, y) = L_i (x,y) + dY_i = L_i (x,y) + A_i$
$L_i (x, y+1) = L_i (x,y) - dX_i = L_i (x,y) + B_i$

Incremental update saves computation:
Only one addition per edge, per sample test
Many traversal orders are possible: backtrack, zig-zag, Hilbert/Morton curves



Modern approach: tiled triangle traversal

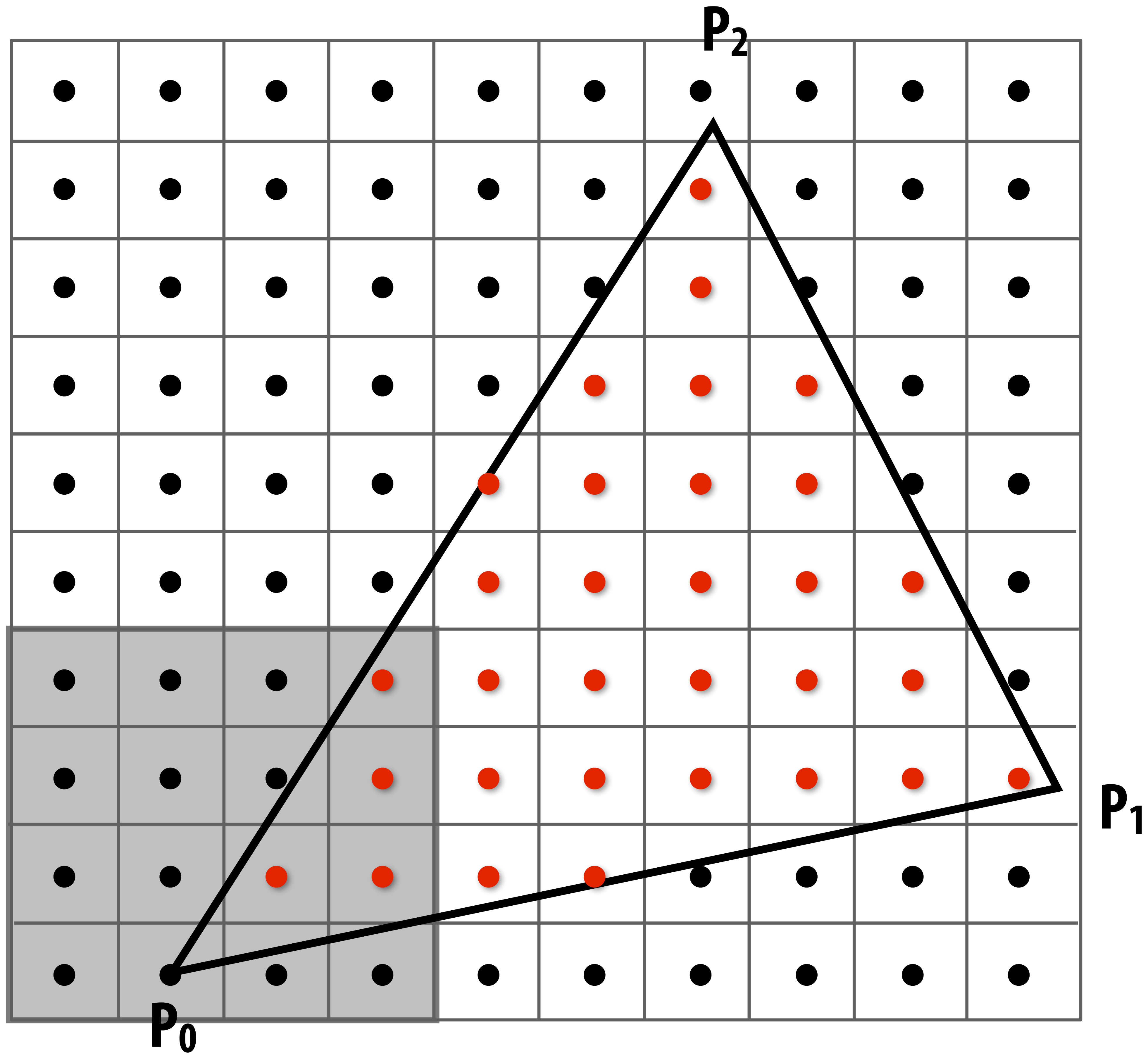
Traverse triangle in blocks

Test all samples in block against triangle in parallel

Advantages:
- Simplicity of parallel execution overcomes cost of extra point-in-triangle tests (most triangles are big enough to cover many samples)

- Can skip sample testing work: entire block not in triangle (“early out”), entire block entirely within triangle (“early in”)

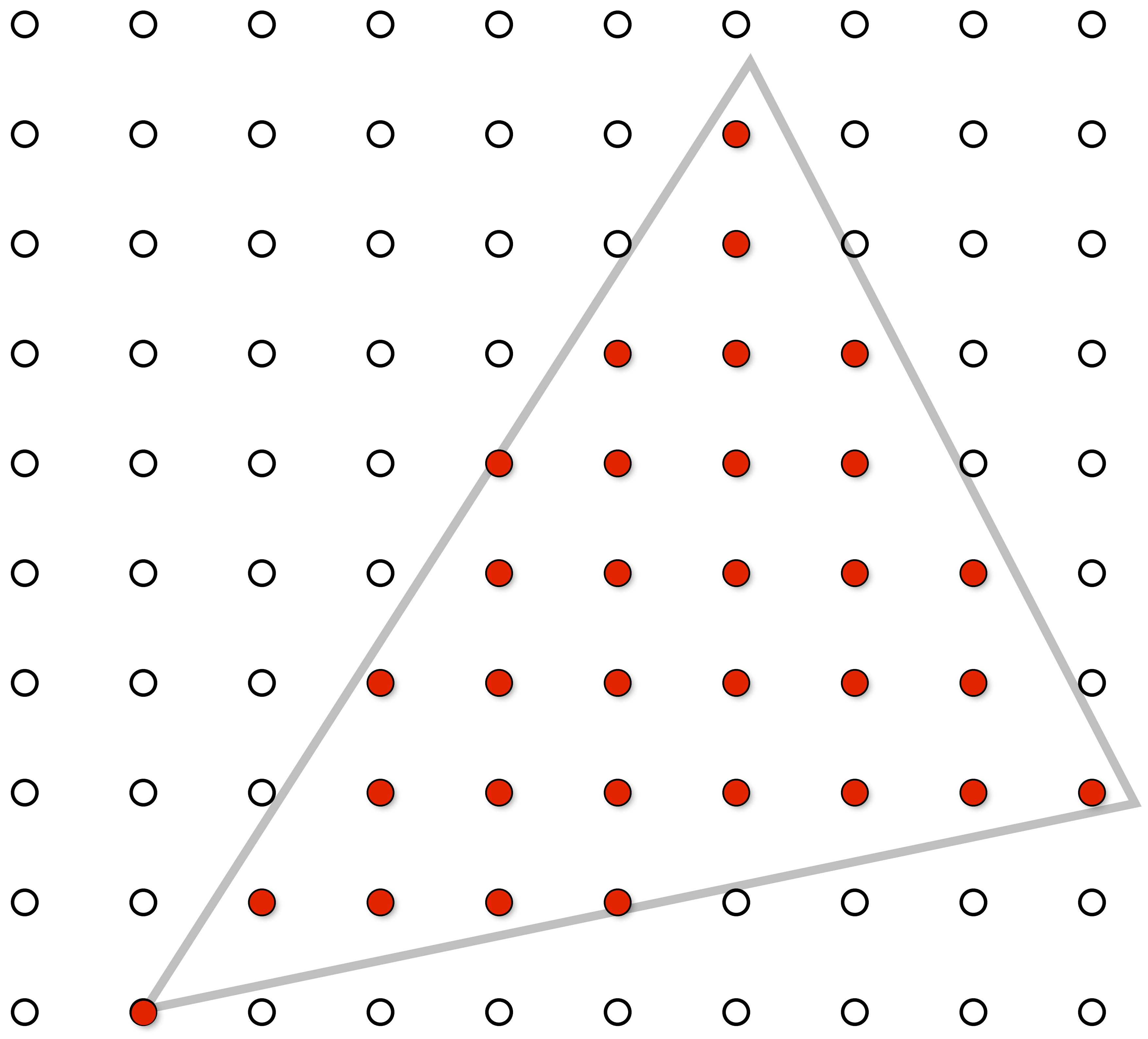
- Additional advantages related to accelerating occlusion computations (not discussed today)



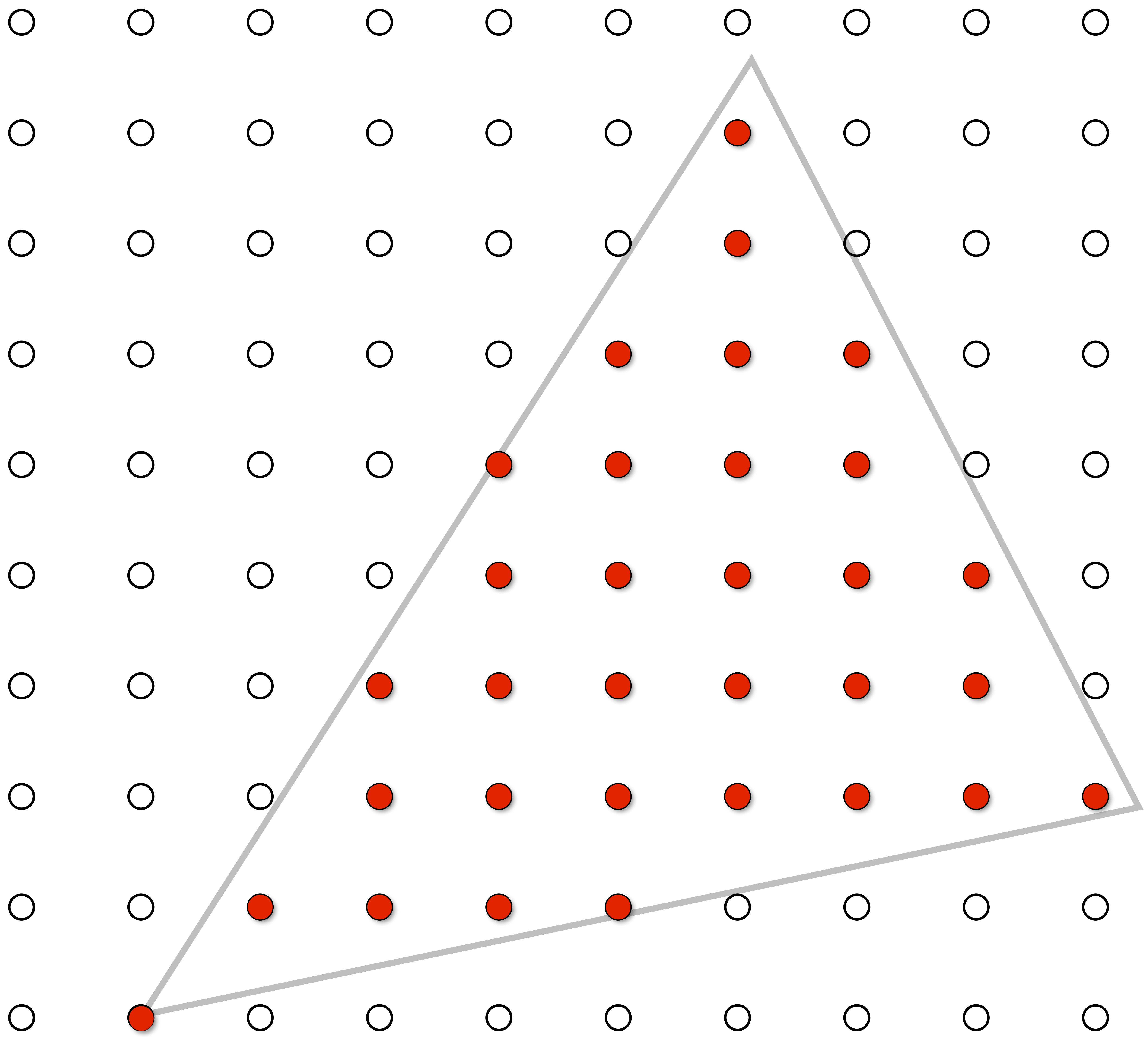
All modern graphics processors (GPUs) have special-purpose hardware for efficiently performing point-in-triangle tests

Where are we now

- We have the ability to determine if any point in the image is inside or outside the triangle



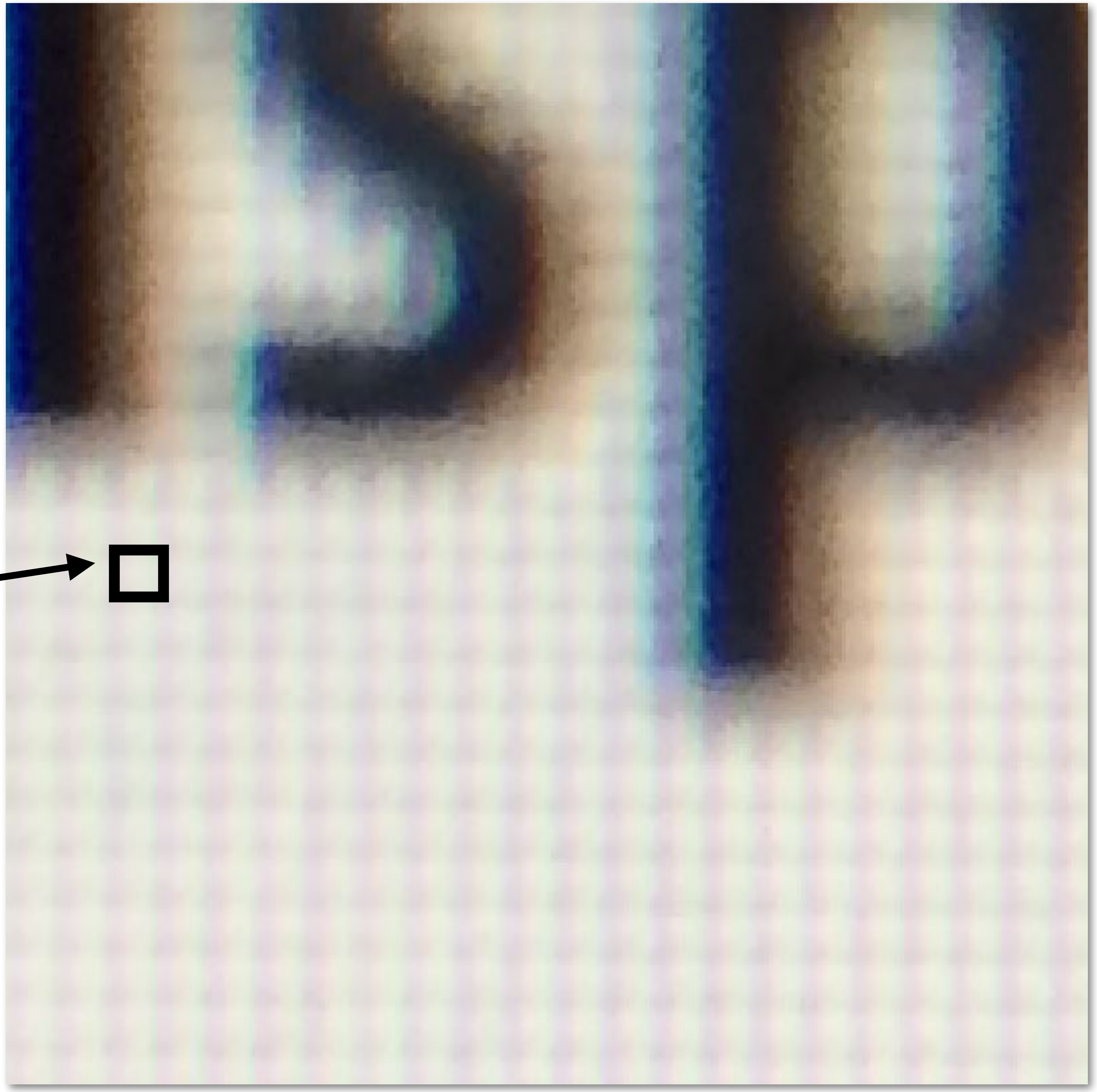
- How to we interpret these results as an image to display?
  (Recall, there's no pixels above, just samples)



Recall: pixels on a screen

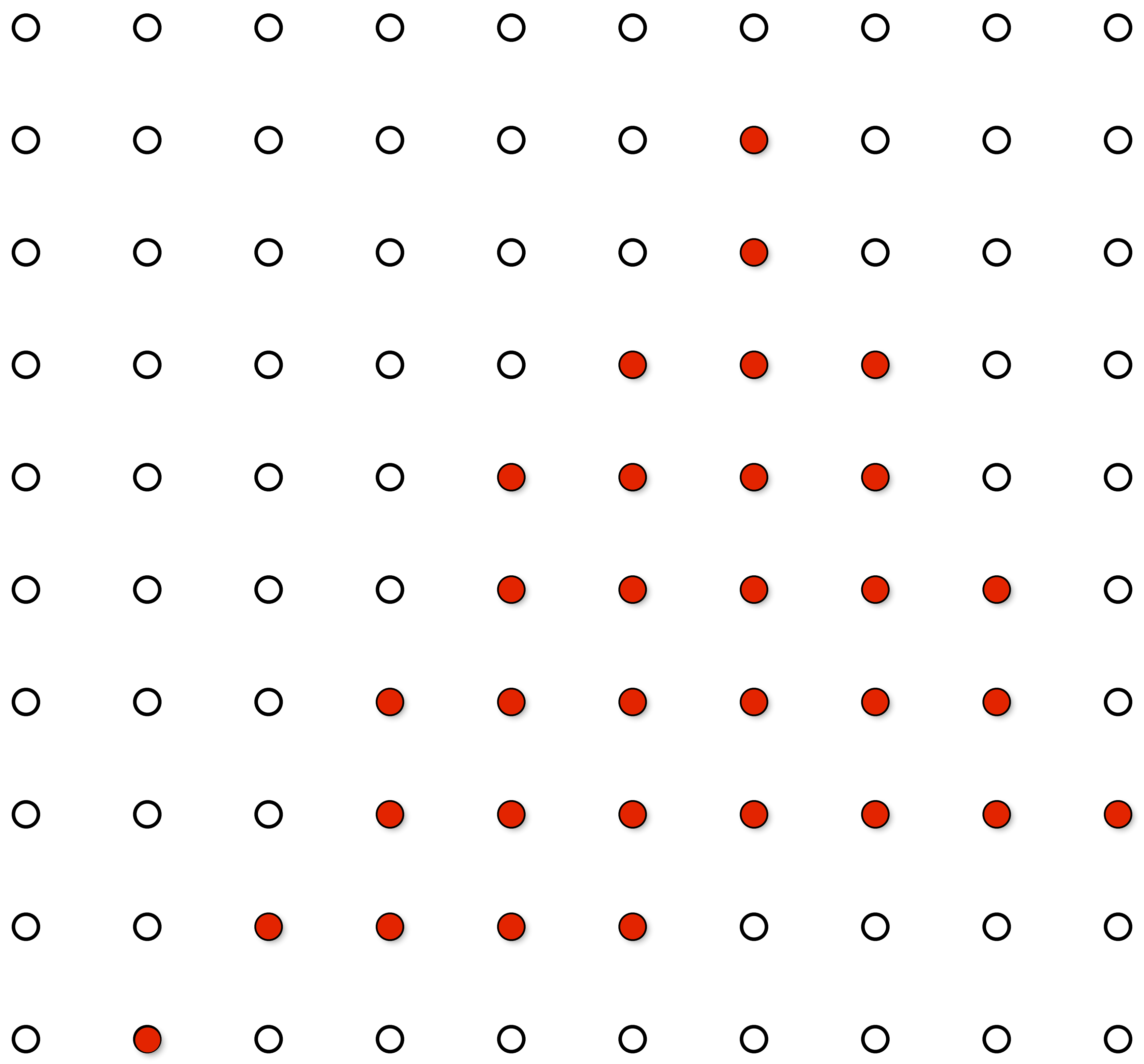
Each image sample sent to the display is converted into a little square of light of the appropriate color:
(a pixel = picture element)

Laptop display pixel



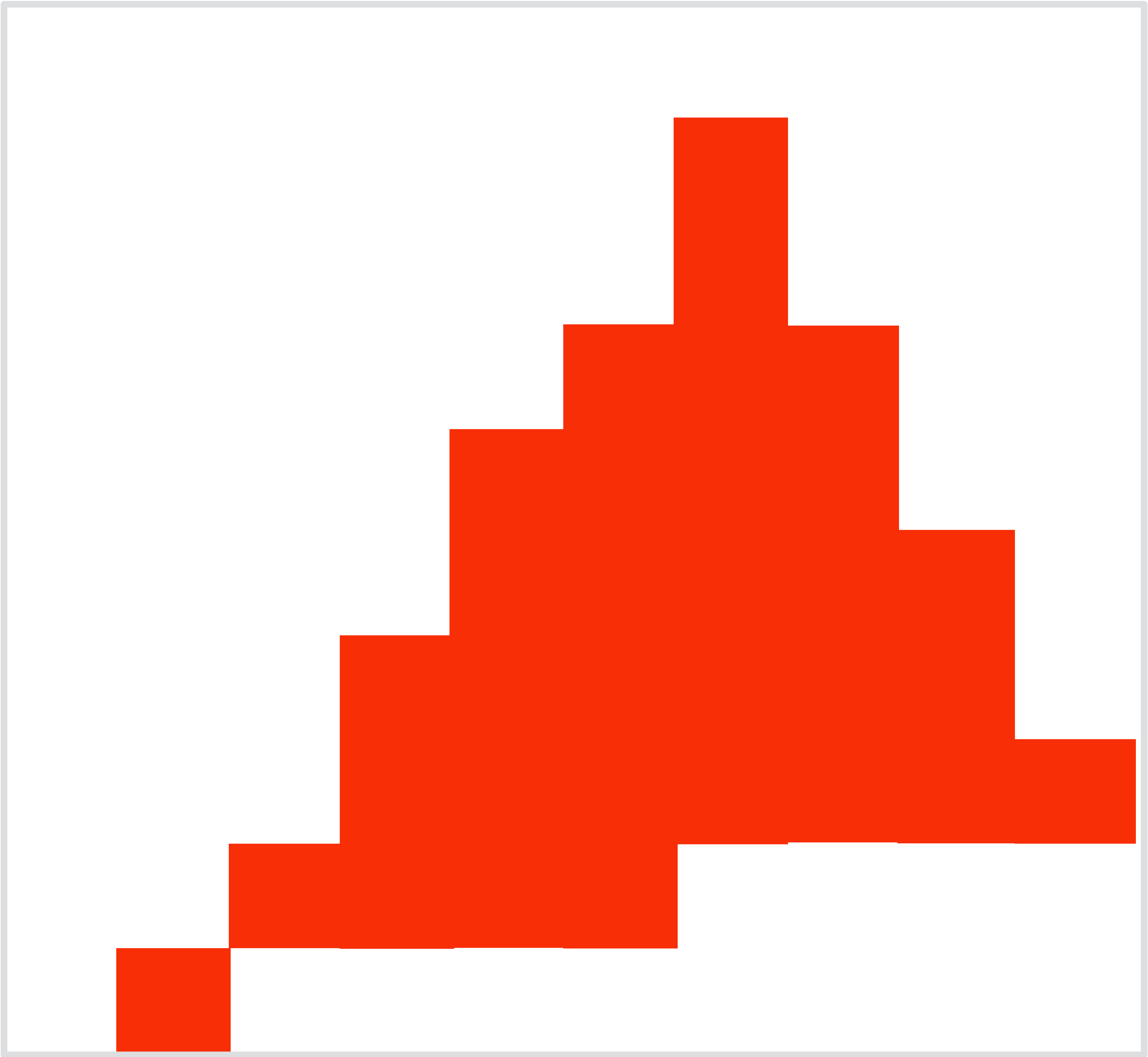
*** Thinking of each screen pixel as emitting a square of uniform intensity light of a single color is a an approximation to how real displays work, but it will do for now.**

So, if we send the display this sampled signal…



…and each value determines the light emitted from a pixel…

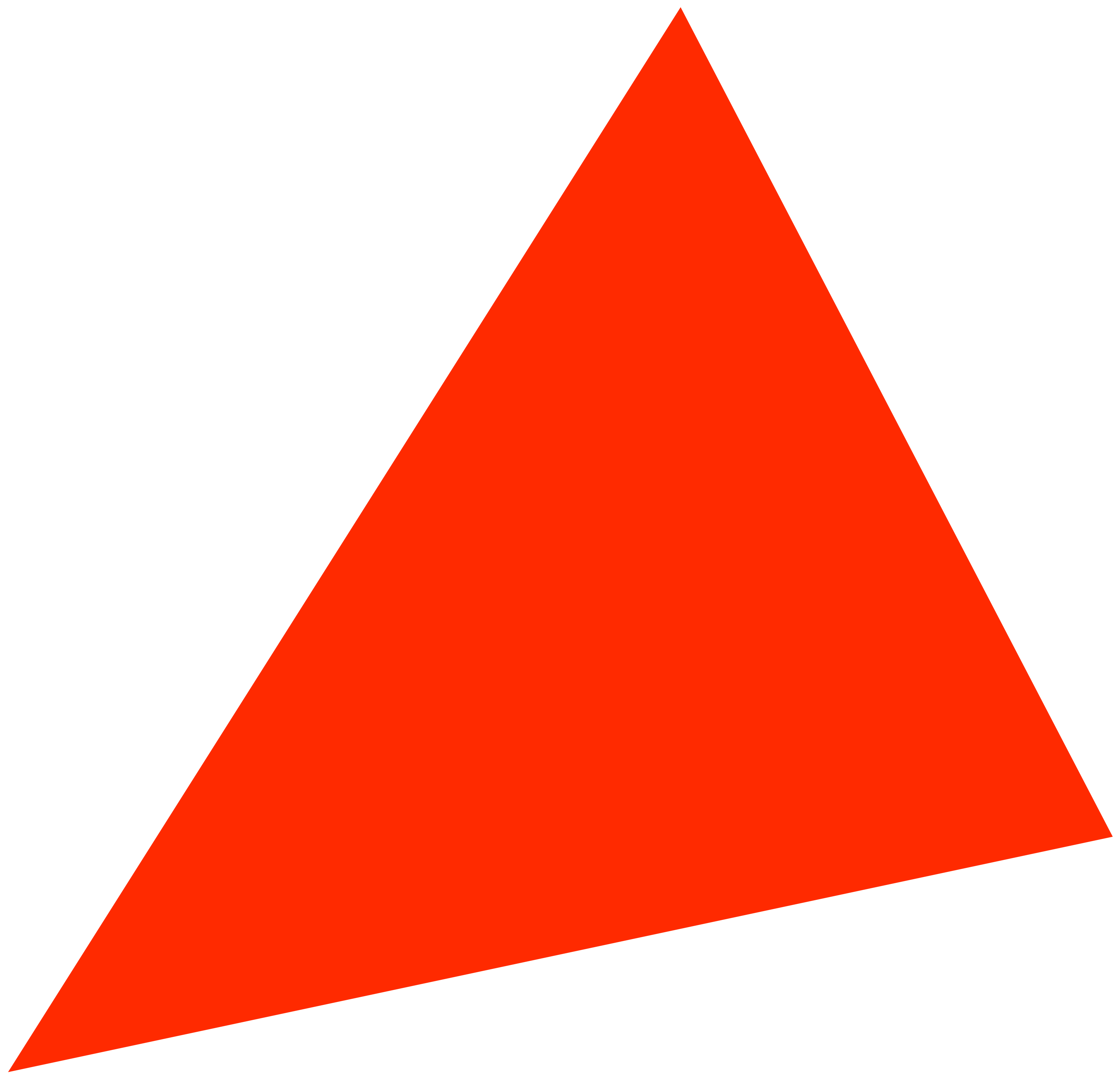
The display physically emits this signal



Given our simplified "square pixel" display assumption, the emitted light is a piecewise constant reconstruction of the samples

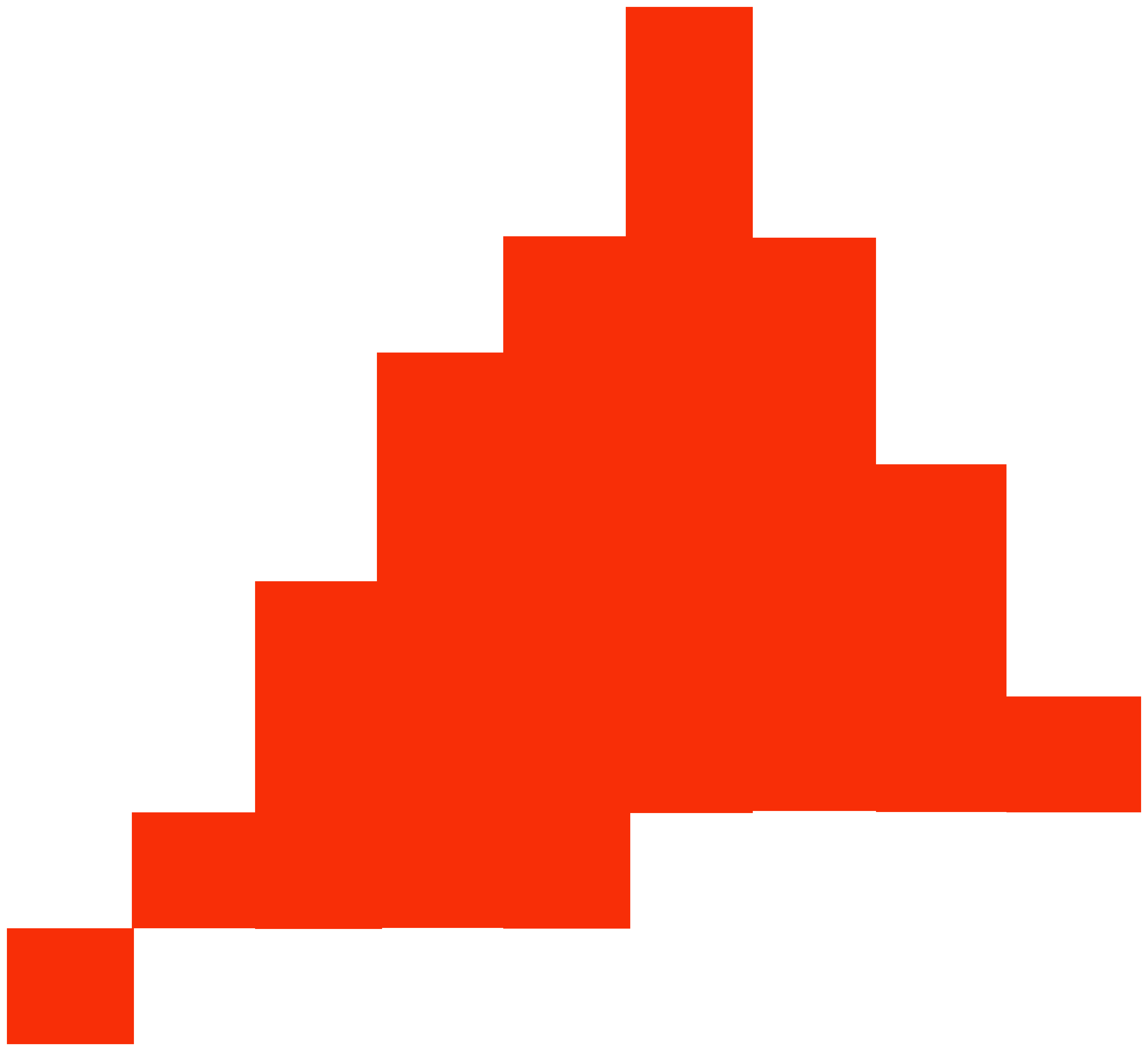
Compare: the continuous triangle function

(This is the function we sampled)



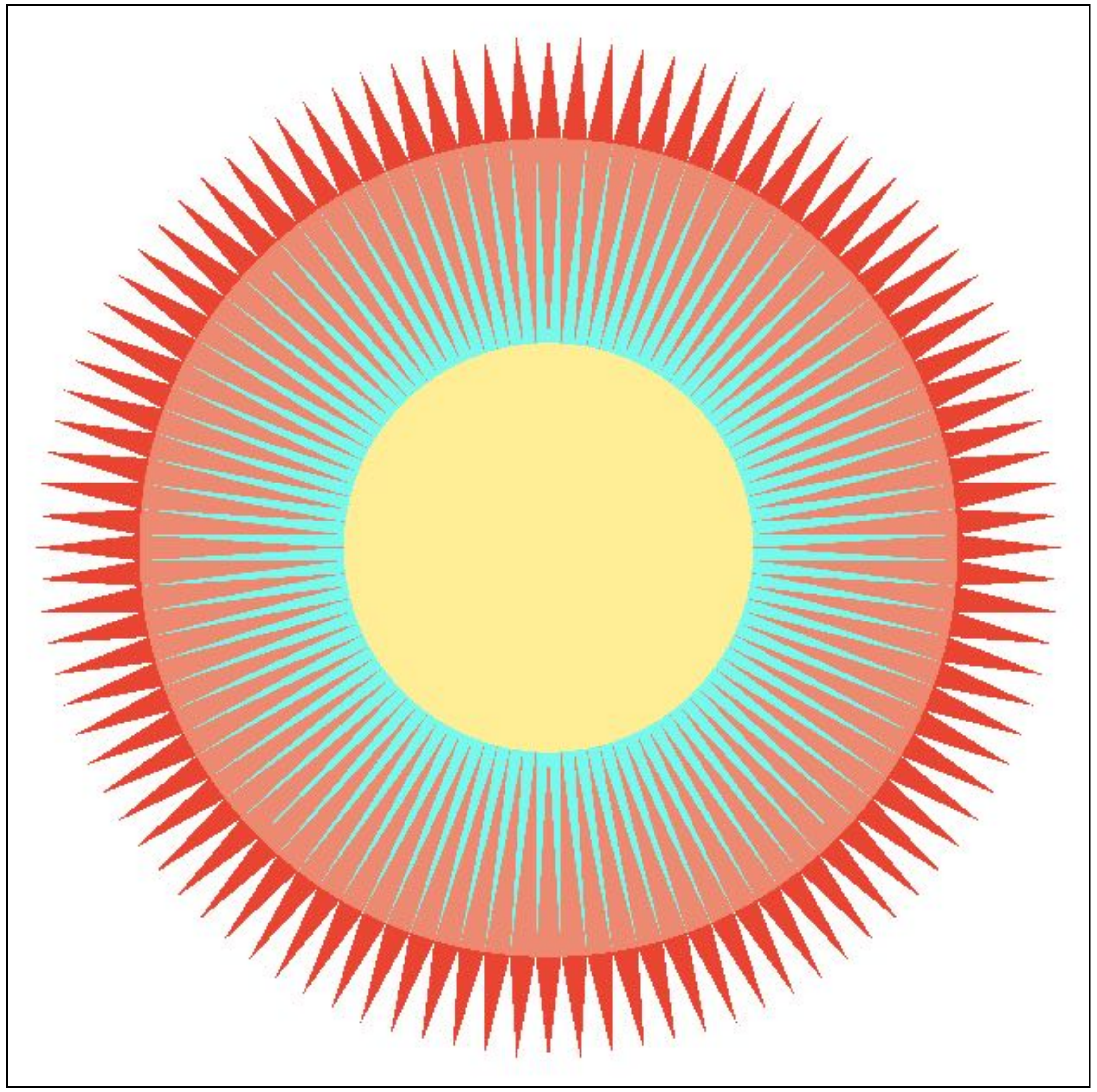
What's wrong with this picture?

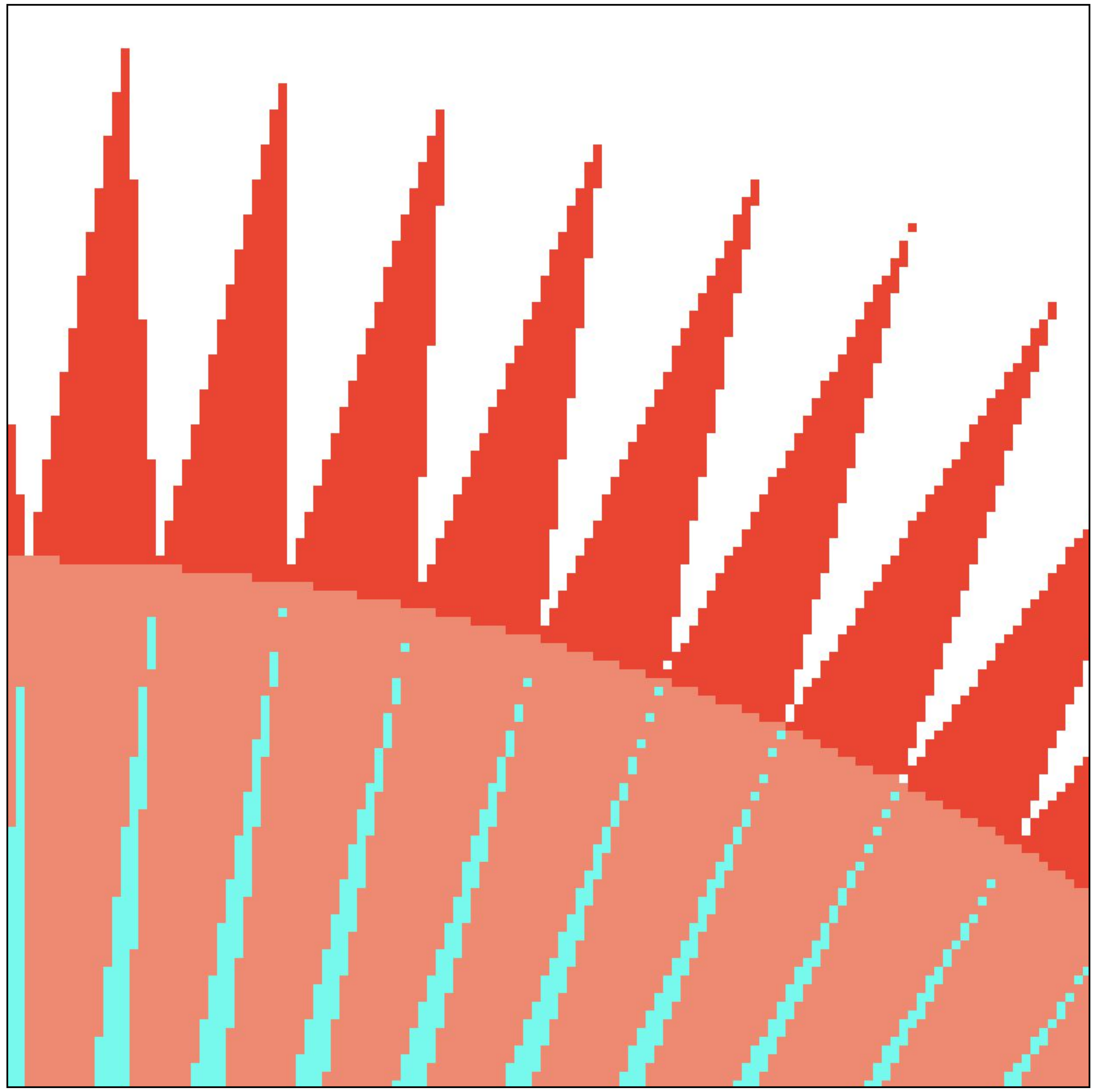
(This is the reconstruction emitted by the display)



Jaggies!

Jaggies (staircase pattern)

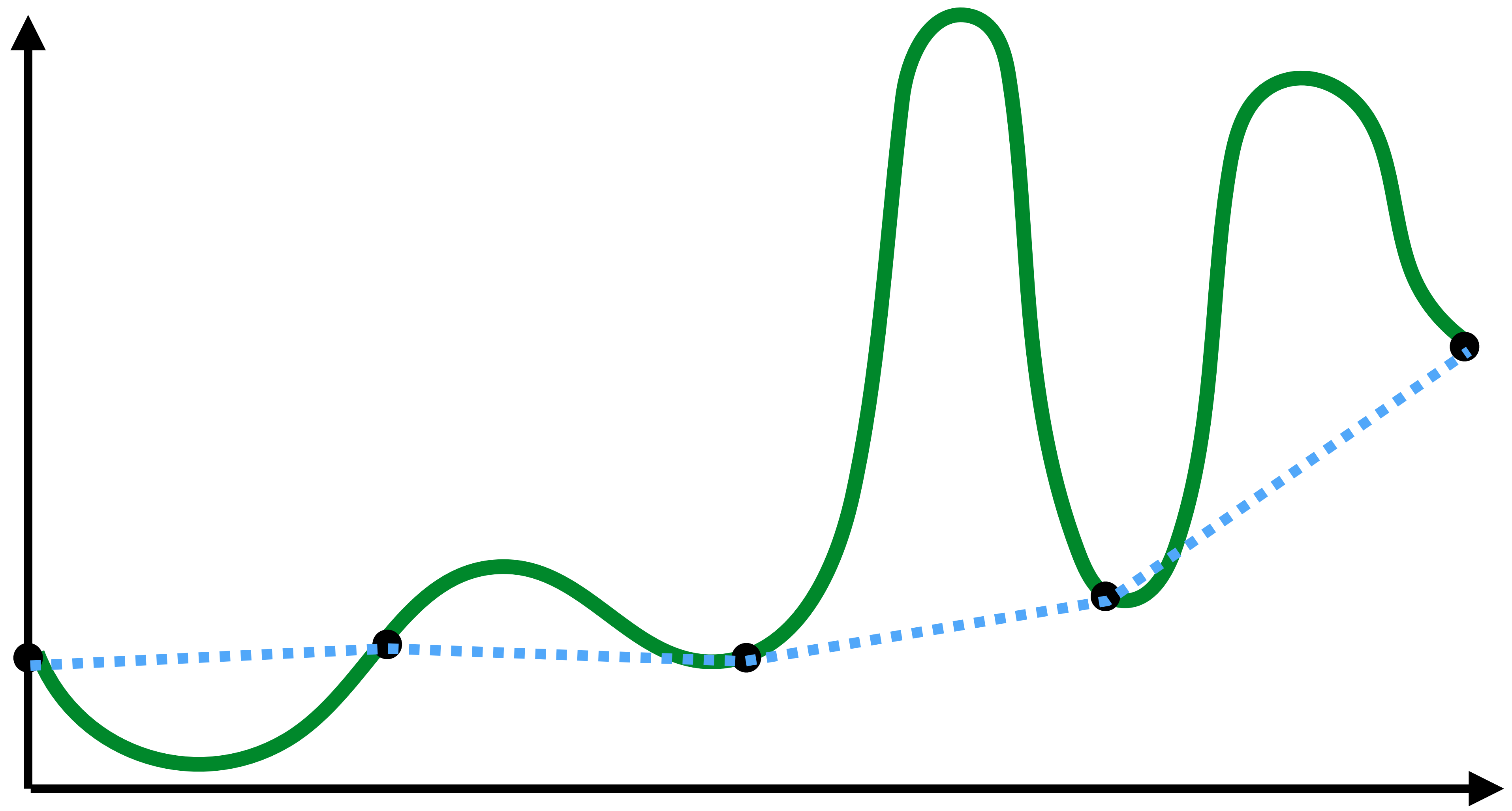




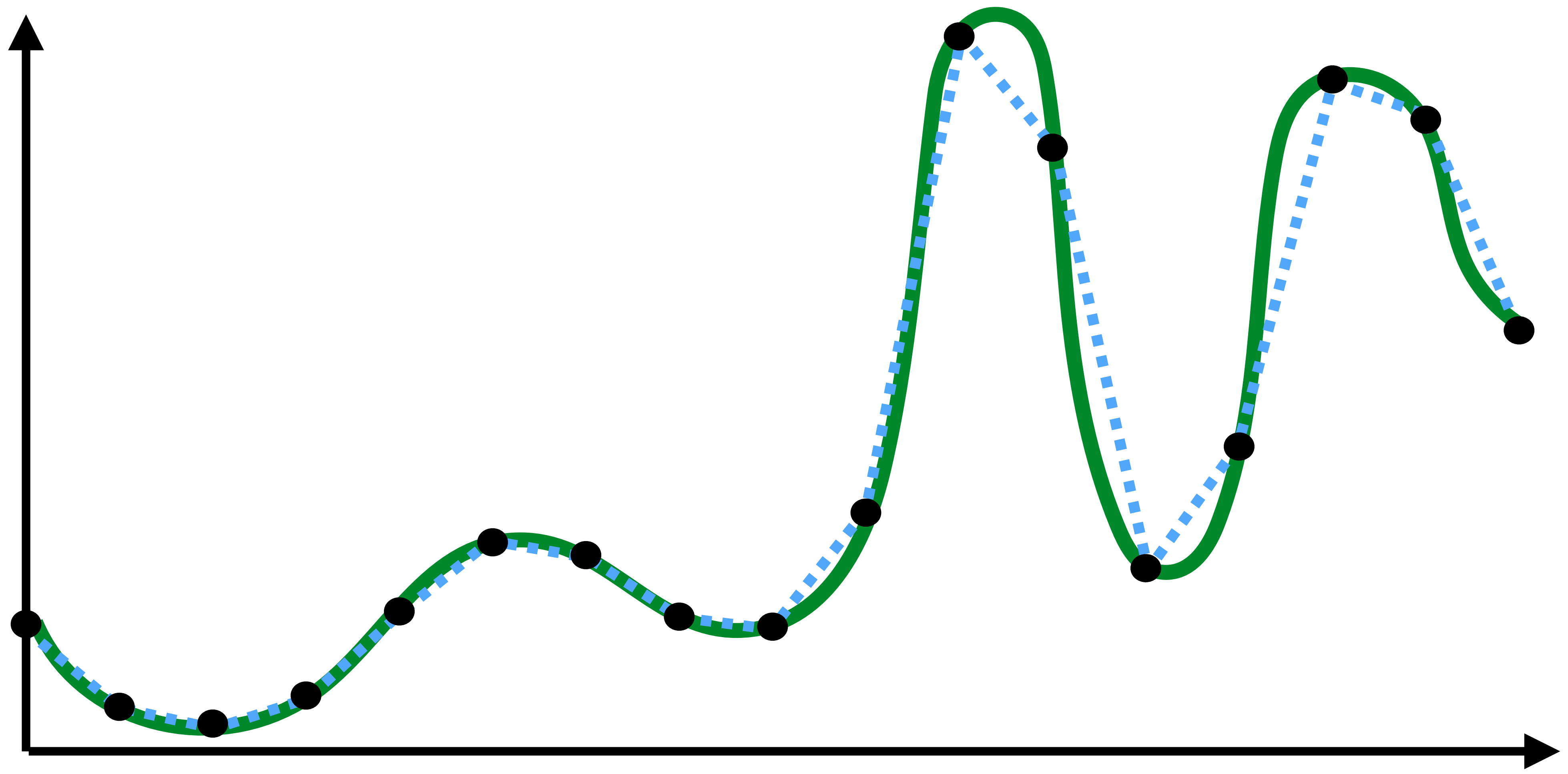
Is this the best we can do?

Reminder: how can we represent a signal more accurately?

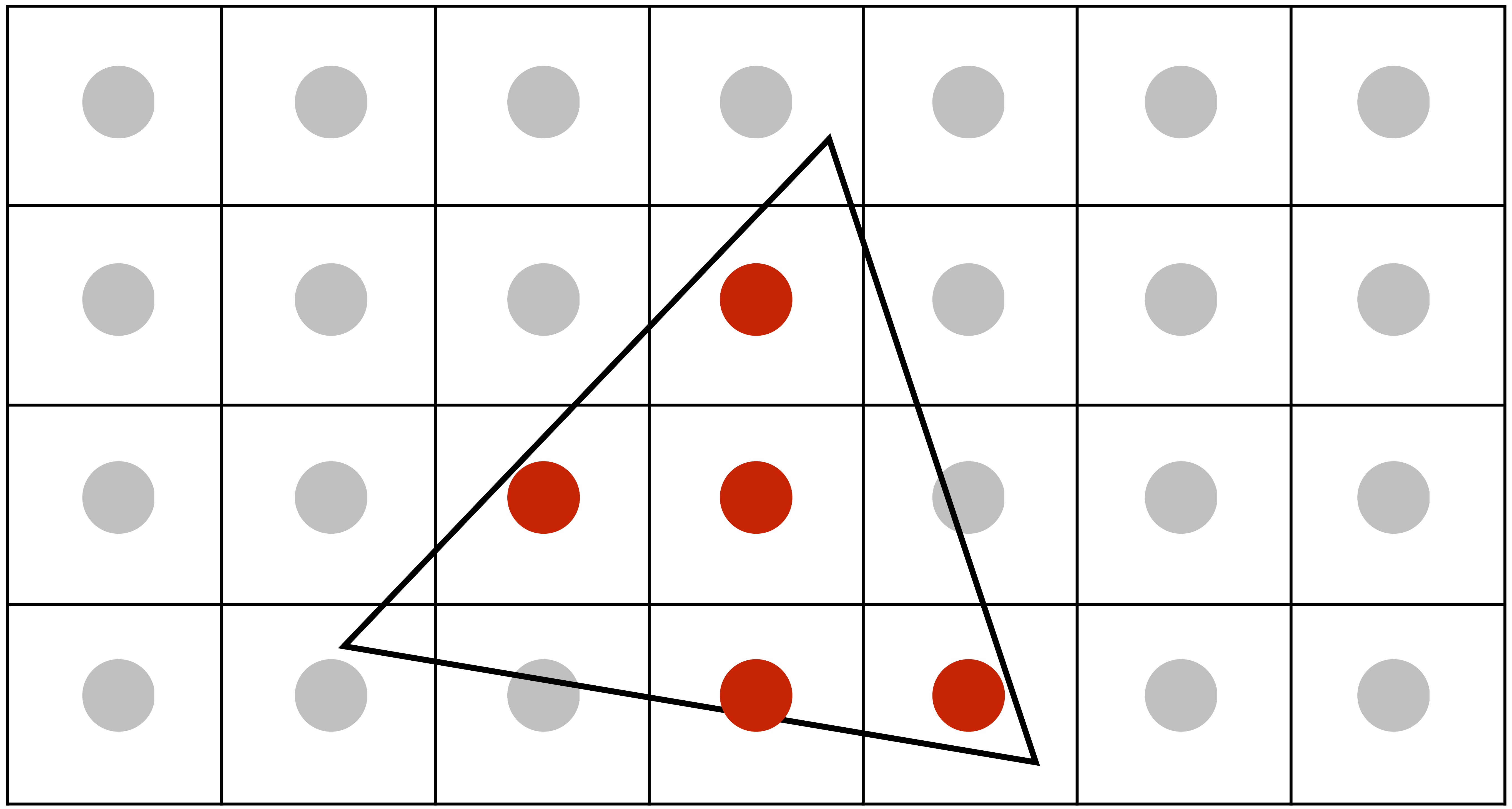
Sample signal more densely! (increase sampling rate)



vs.



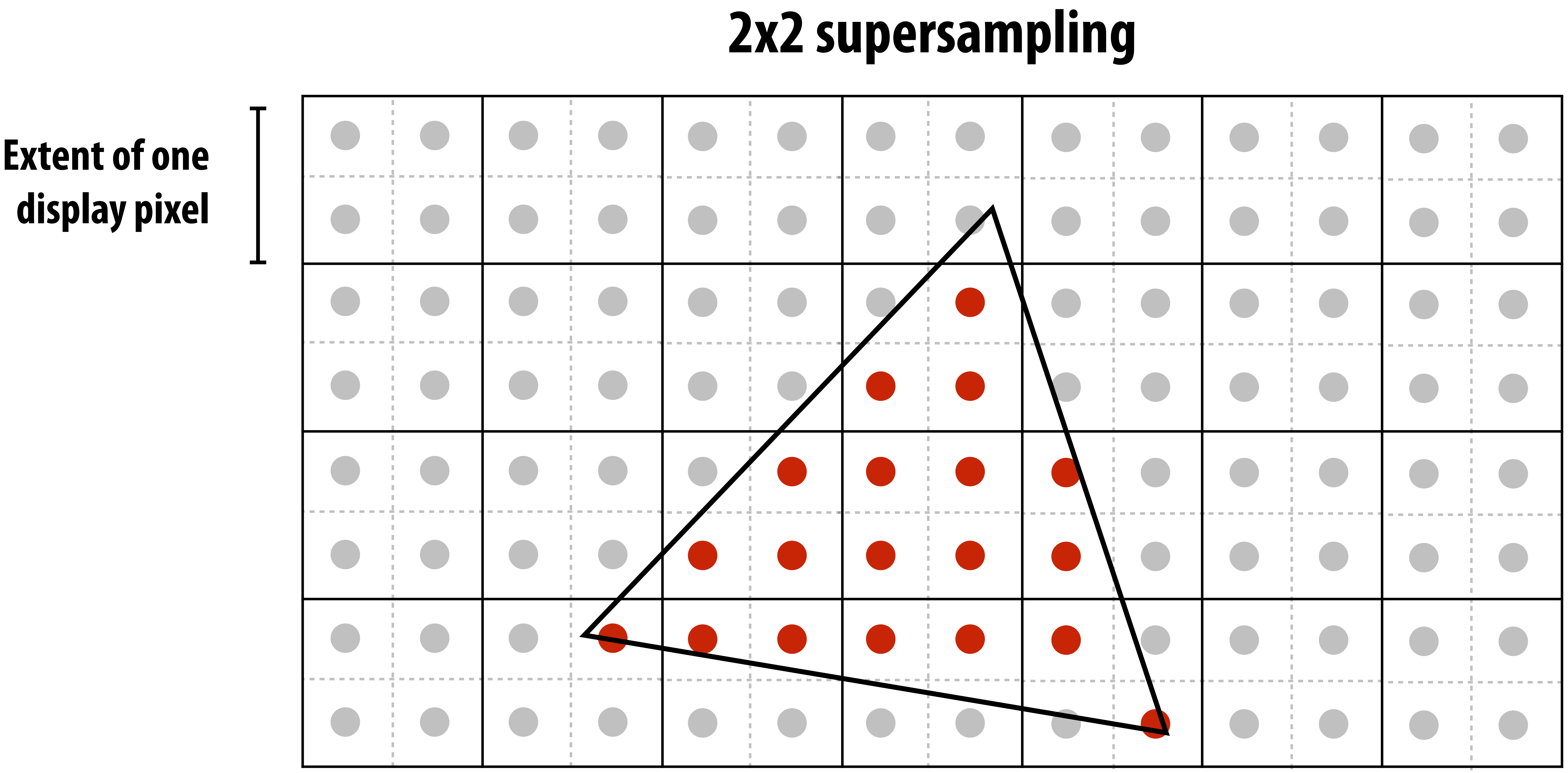
Sampling using one sample per pixel



Supersampling: step 1

Sample the input signal more densely in the image plane

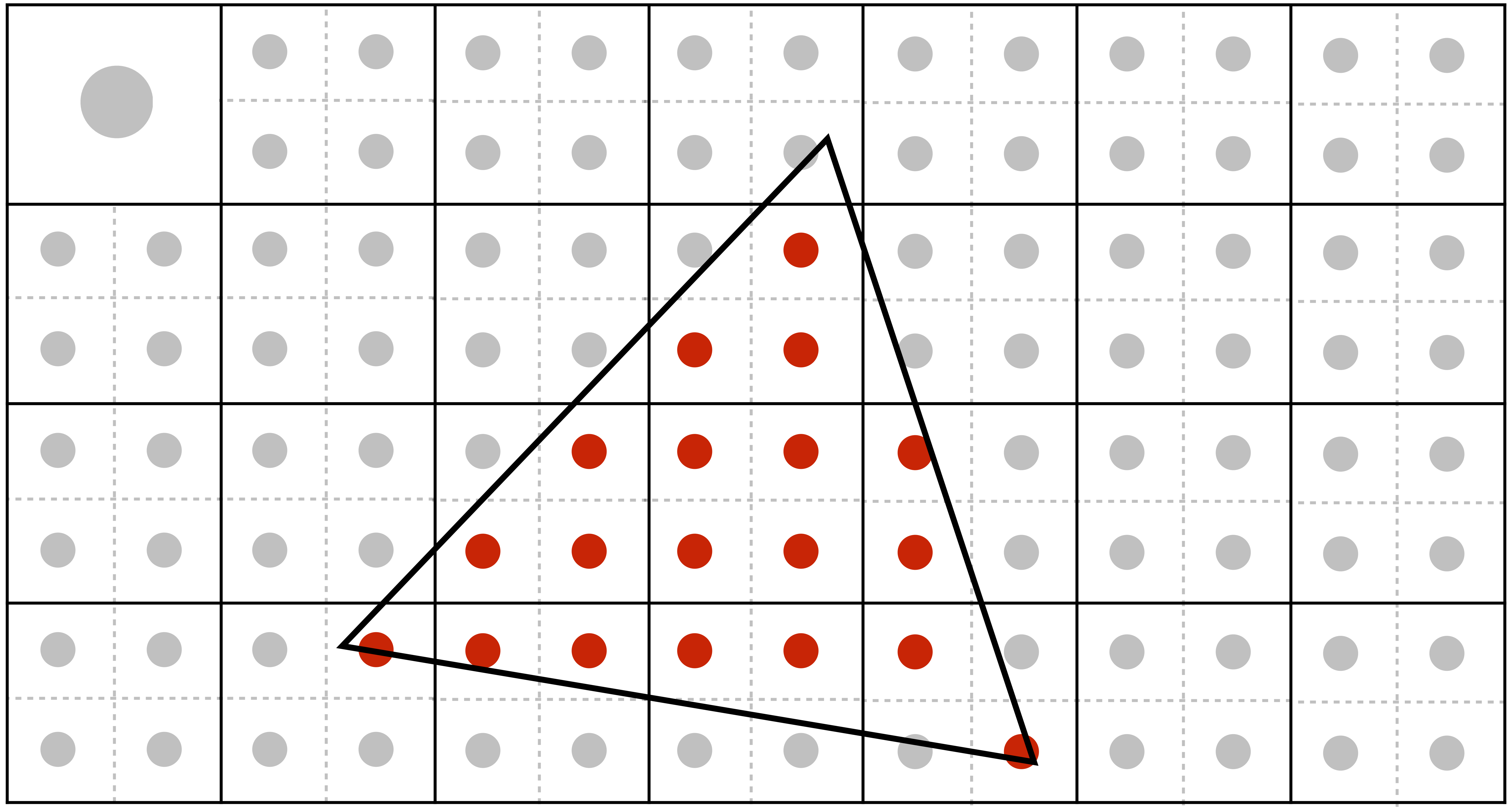
In this example: take 2 x 2 samples in the area spanned by a pixel



But how do we use these samples to drive a display, since there are four times more samples than display pixels! 🤔

Supersampling: step 2

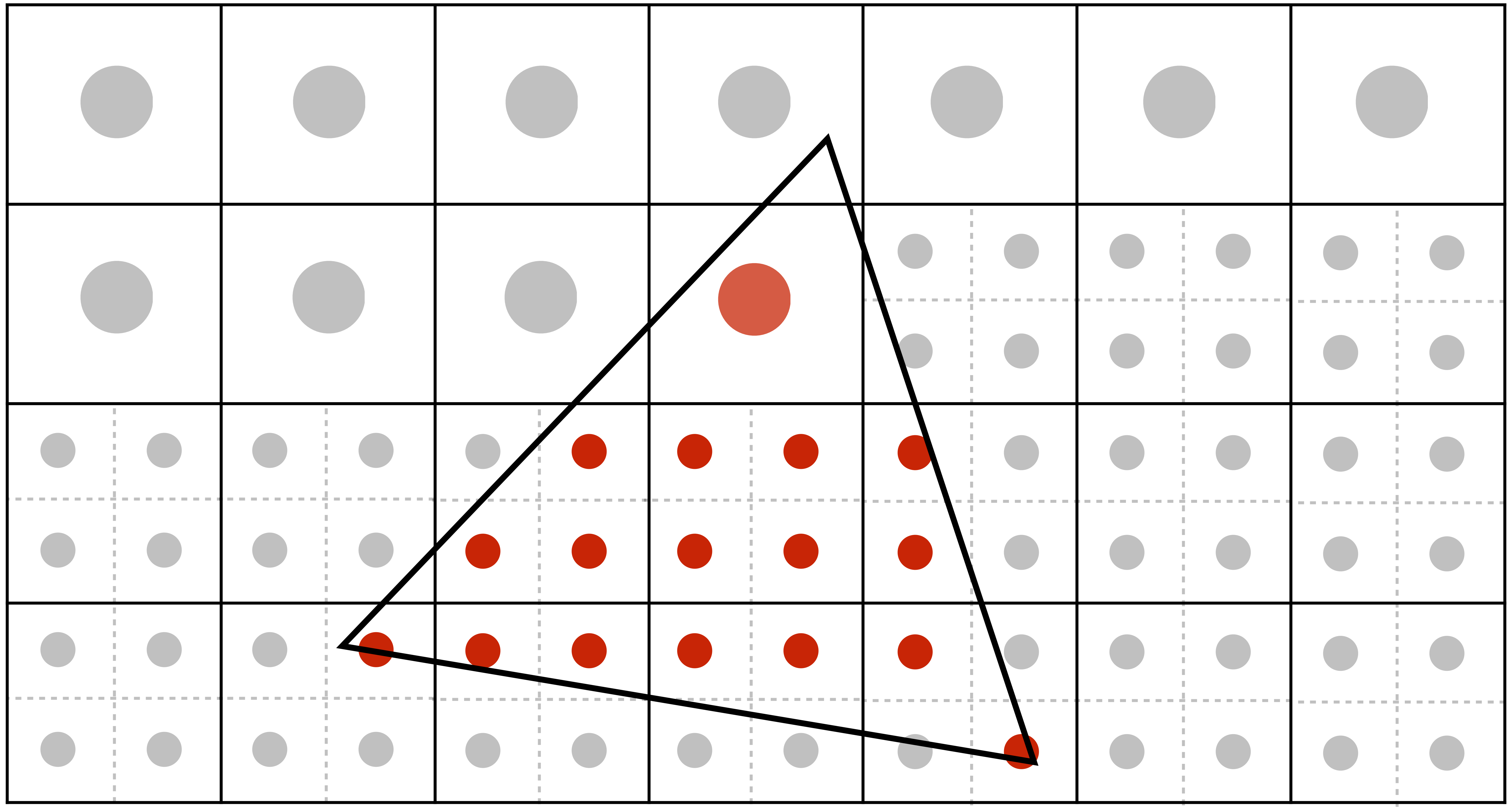
Average the N x N samples “inside” each pixel



Averaging down

Supersampling: step 2

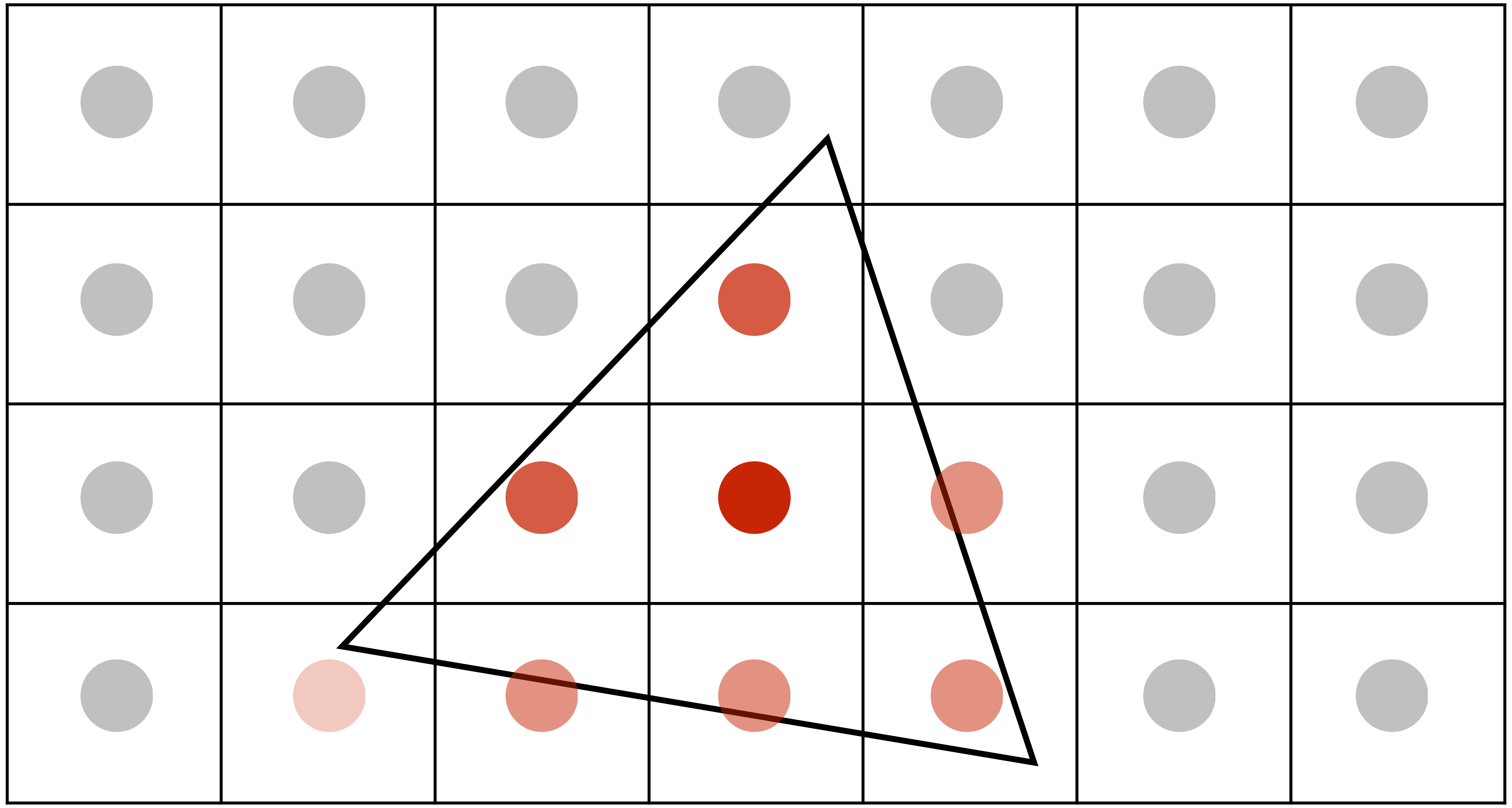
Average the N x N samples “inside” each pixel



Averaging down

Supersampling: step 2

Average the N x N samples “inside” each pixel

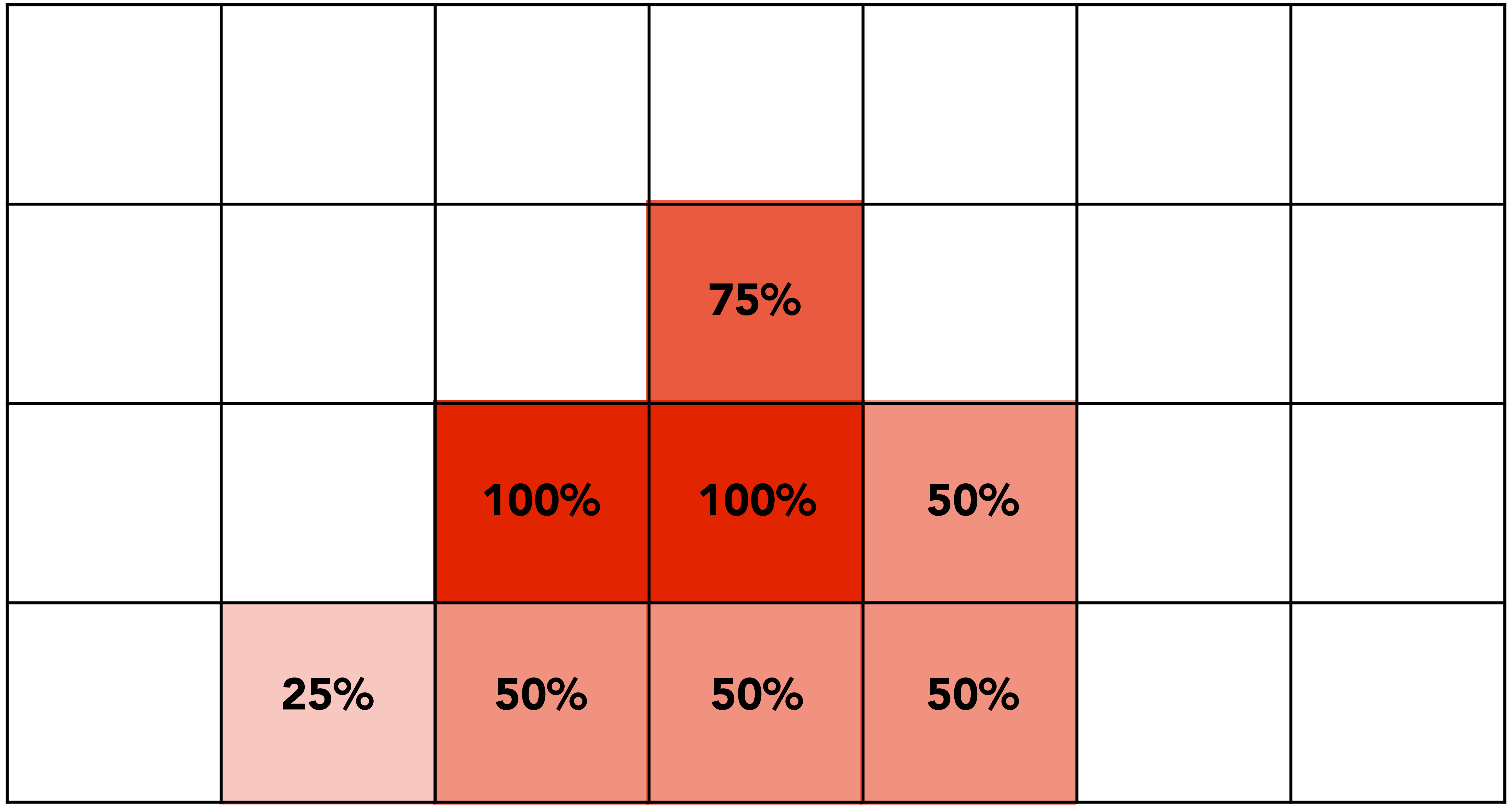


Averaging down

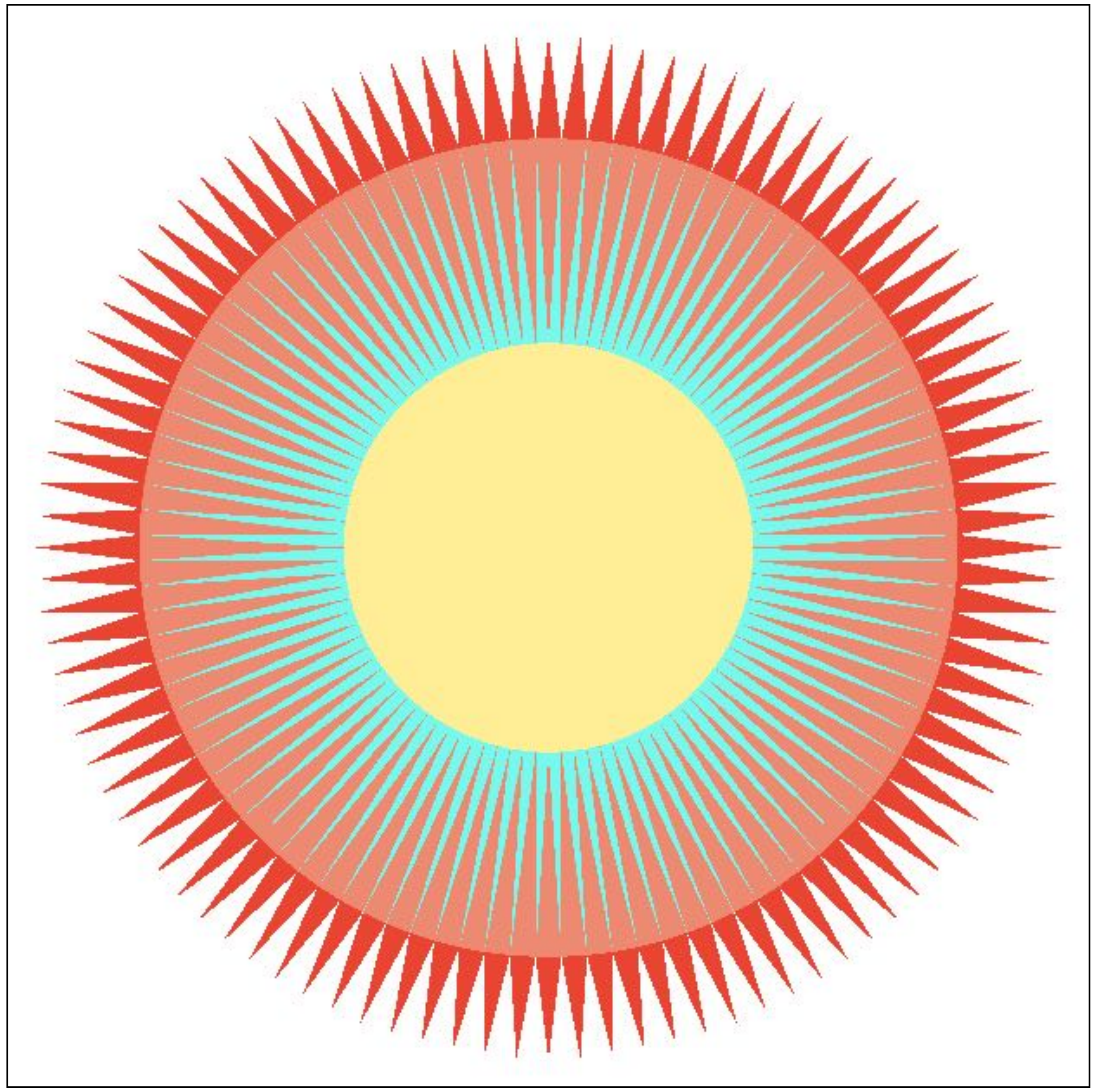
Displayed result

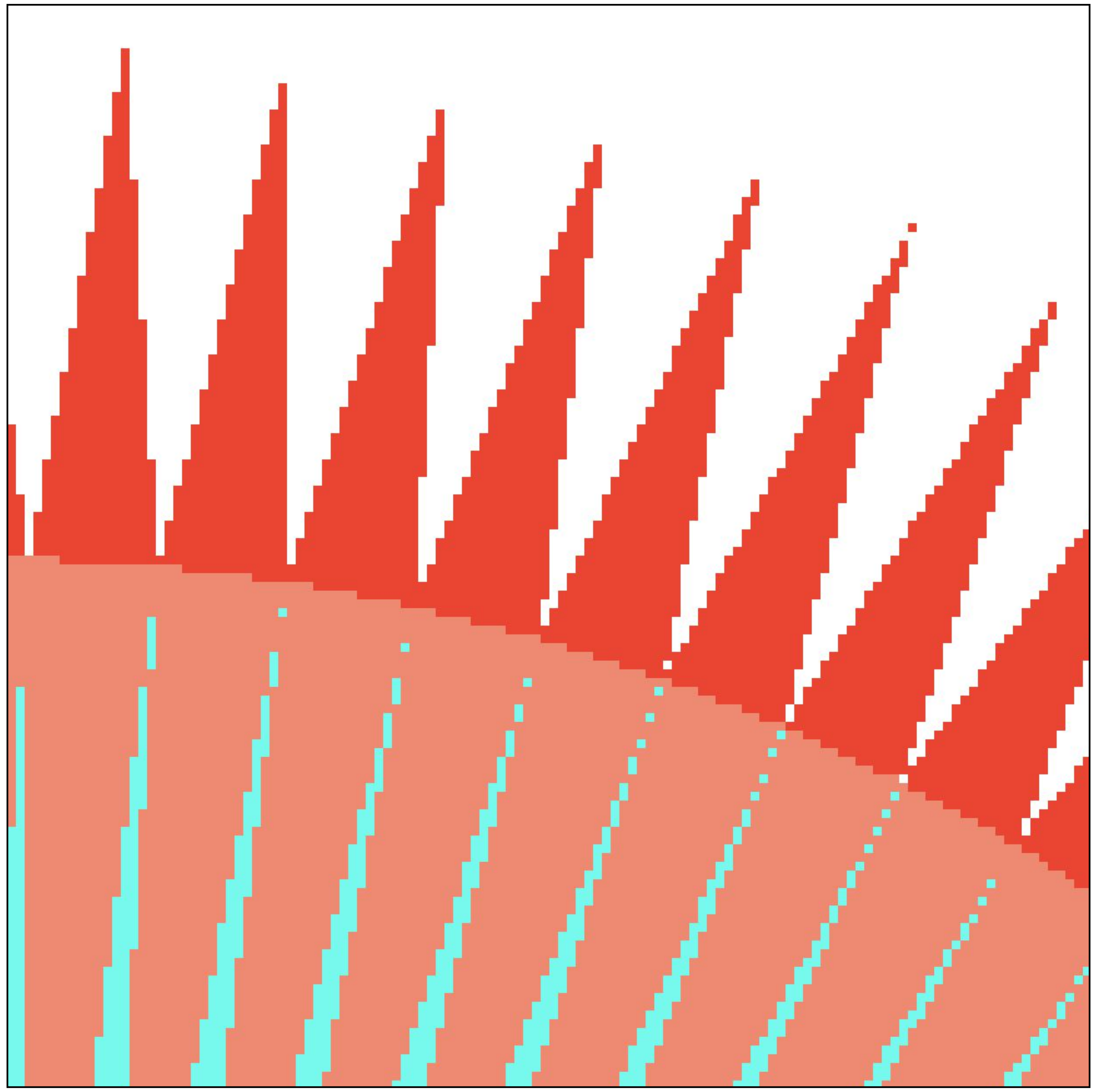
This is the corresponding signal emitted by the display

(value provided to each display pixel is the average of the values sampled in that region)



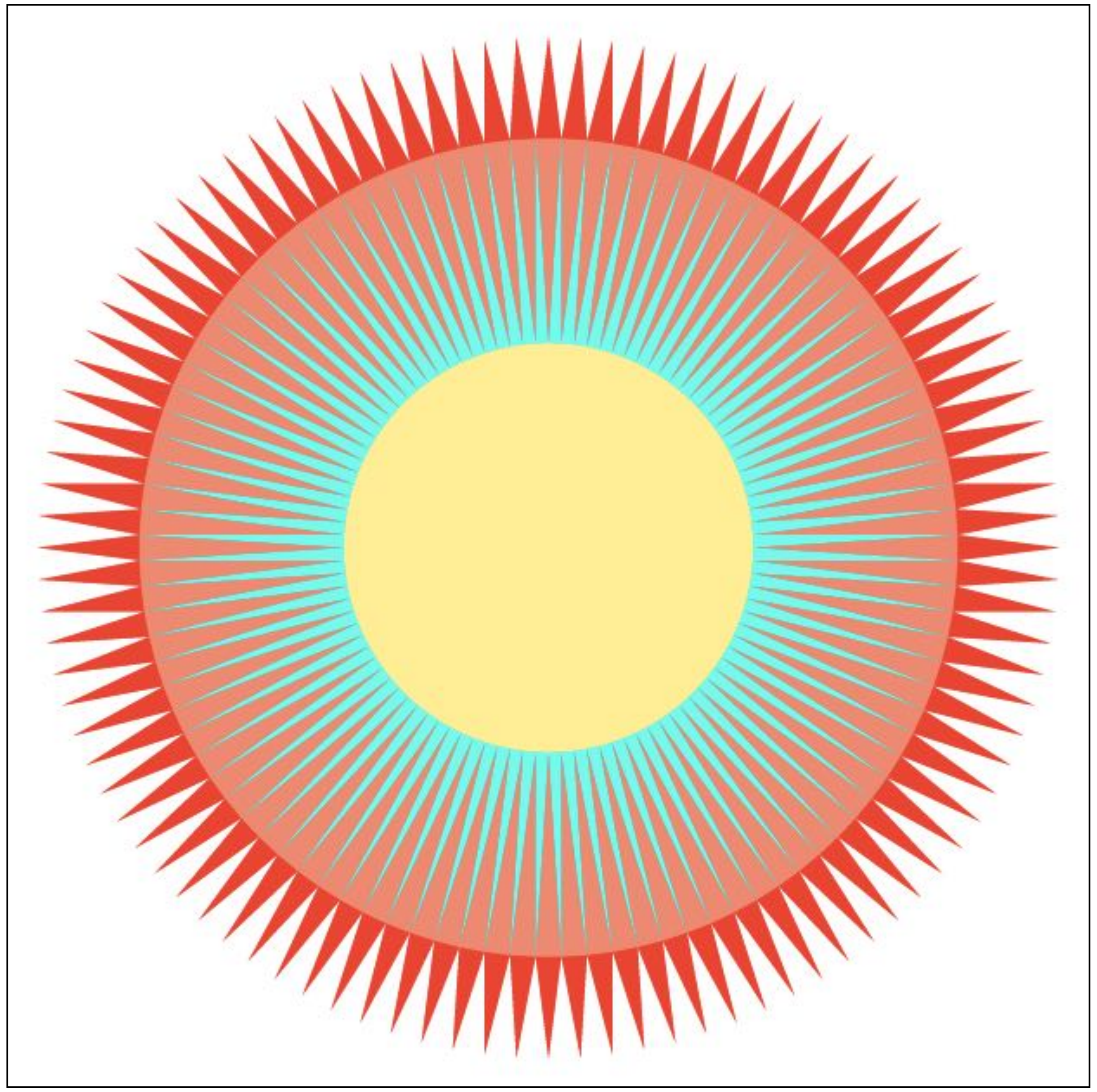
Images rendered using one sample per pixel

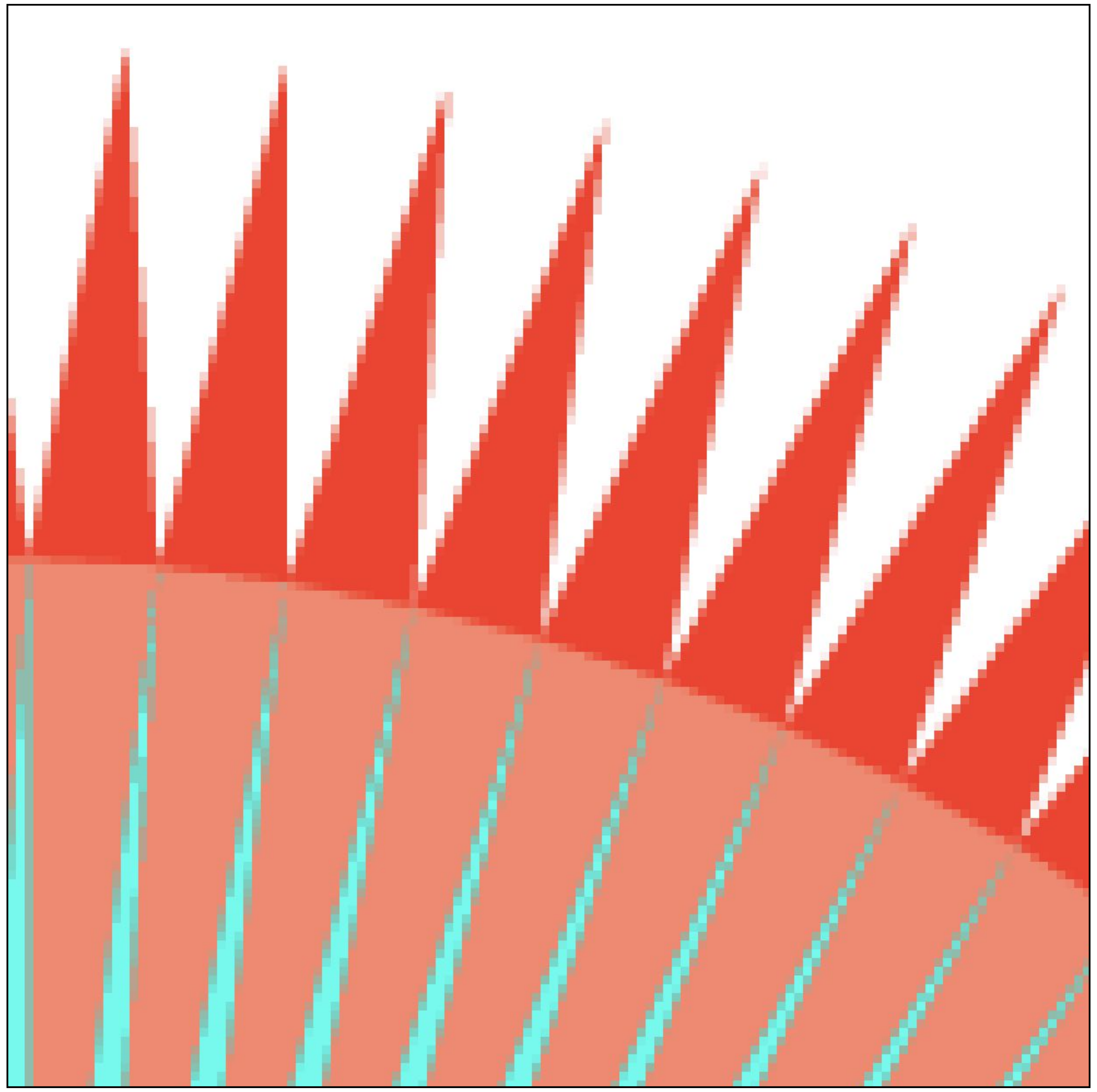




4x4 supersampling + downsampling

(16 samples per pixel)



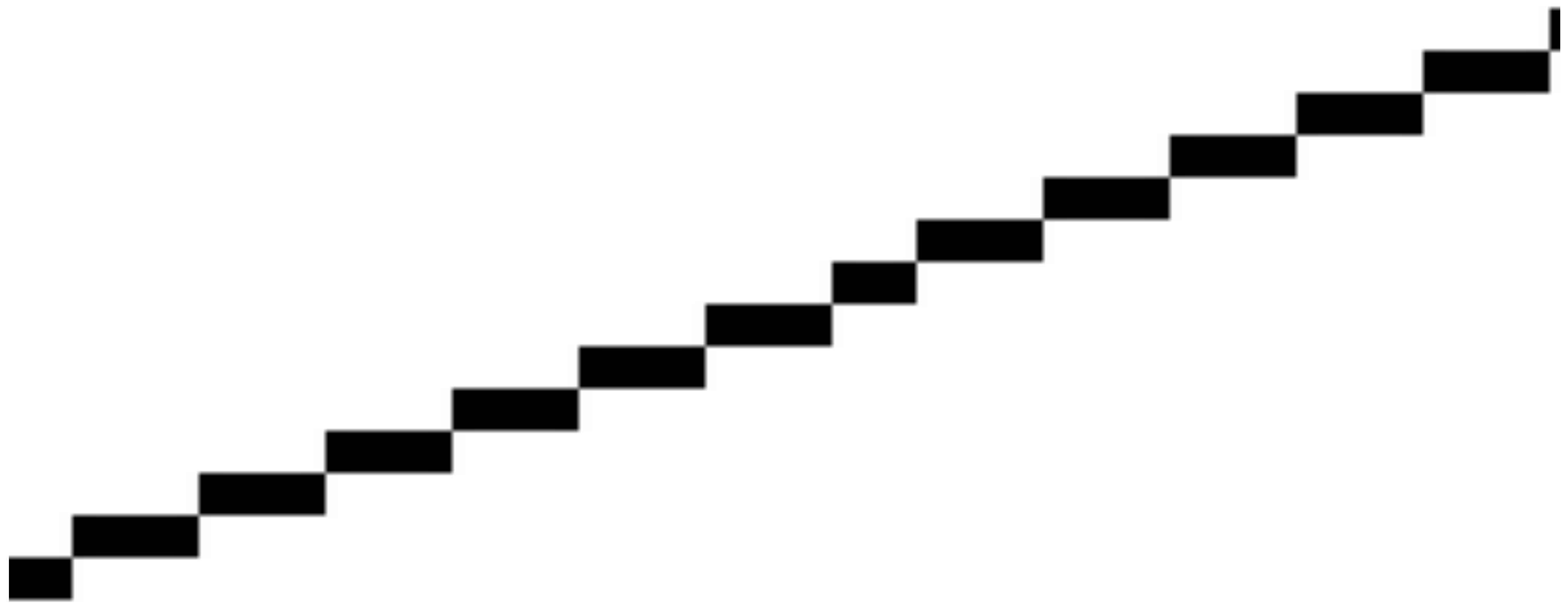


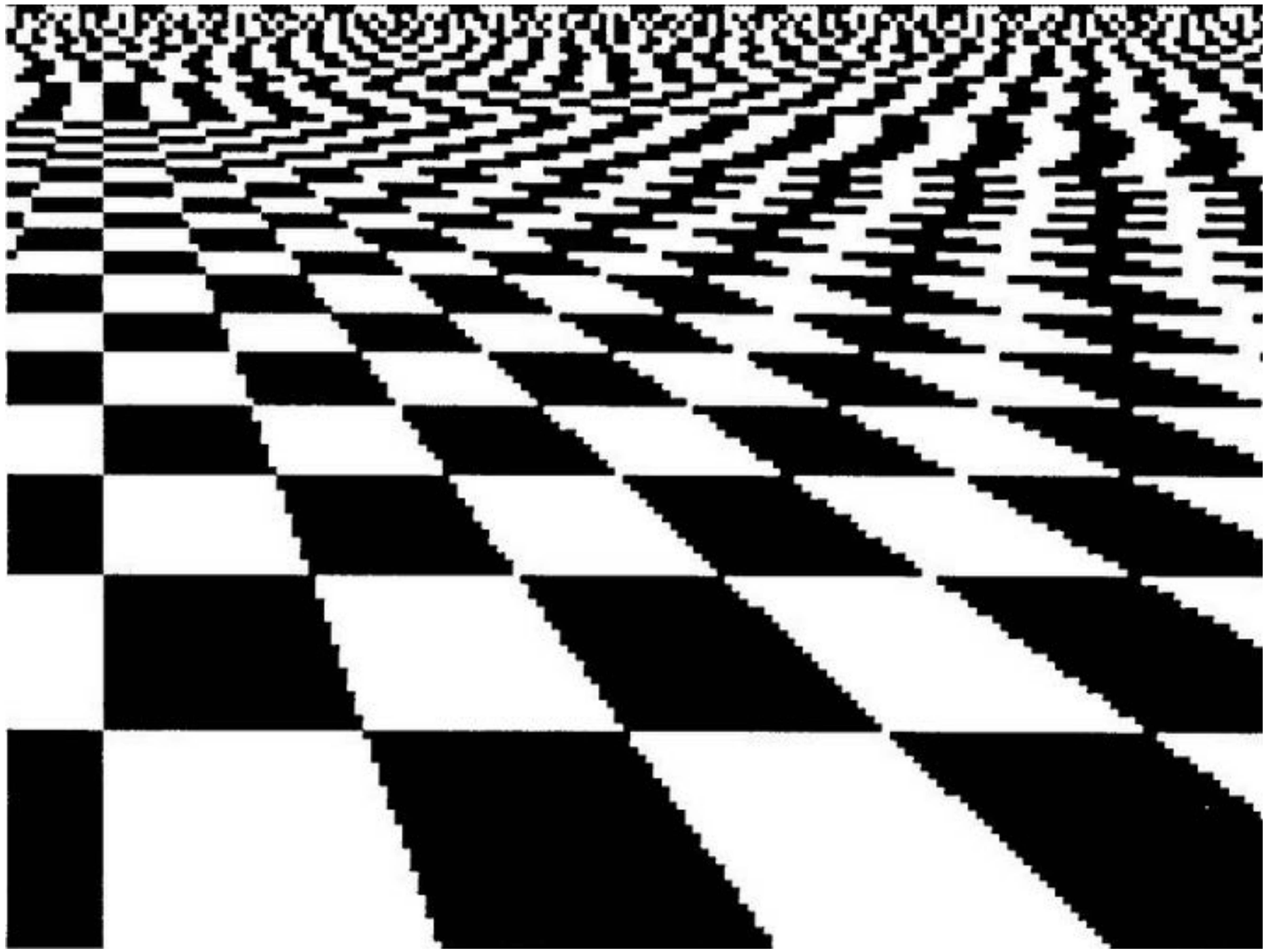
Each pixel's value is the average of the values of the 4x4 samples per pixel

Let’s understand what just happened in a more principled way

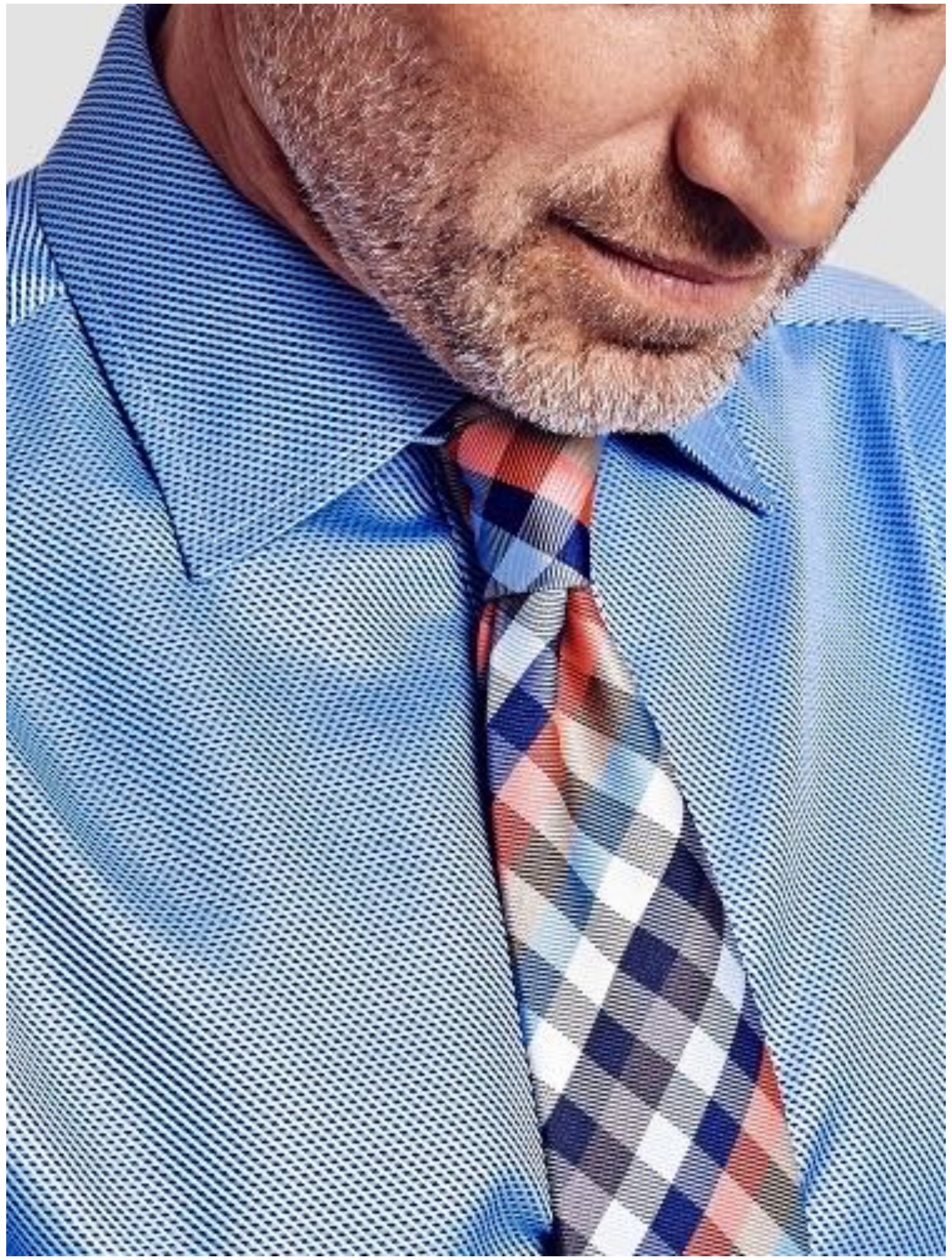
More examples of sampling artifacts in computer graphics

Jaggies (staircase pattern)

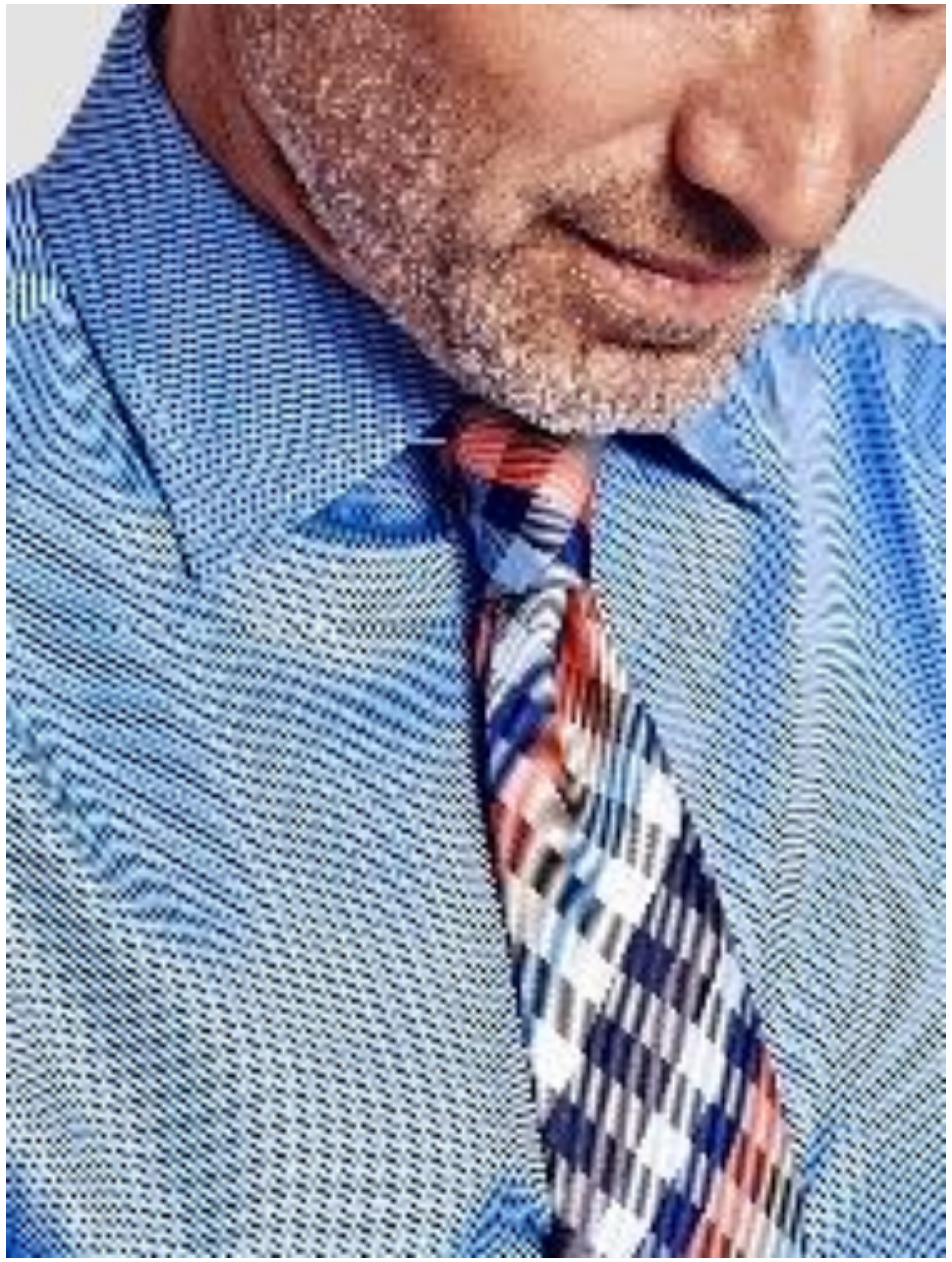




Moiré patterns in imaging



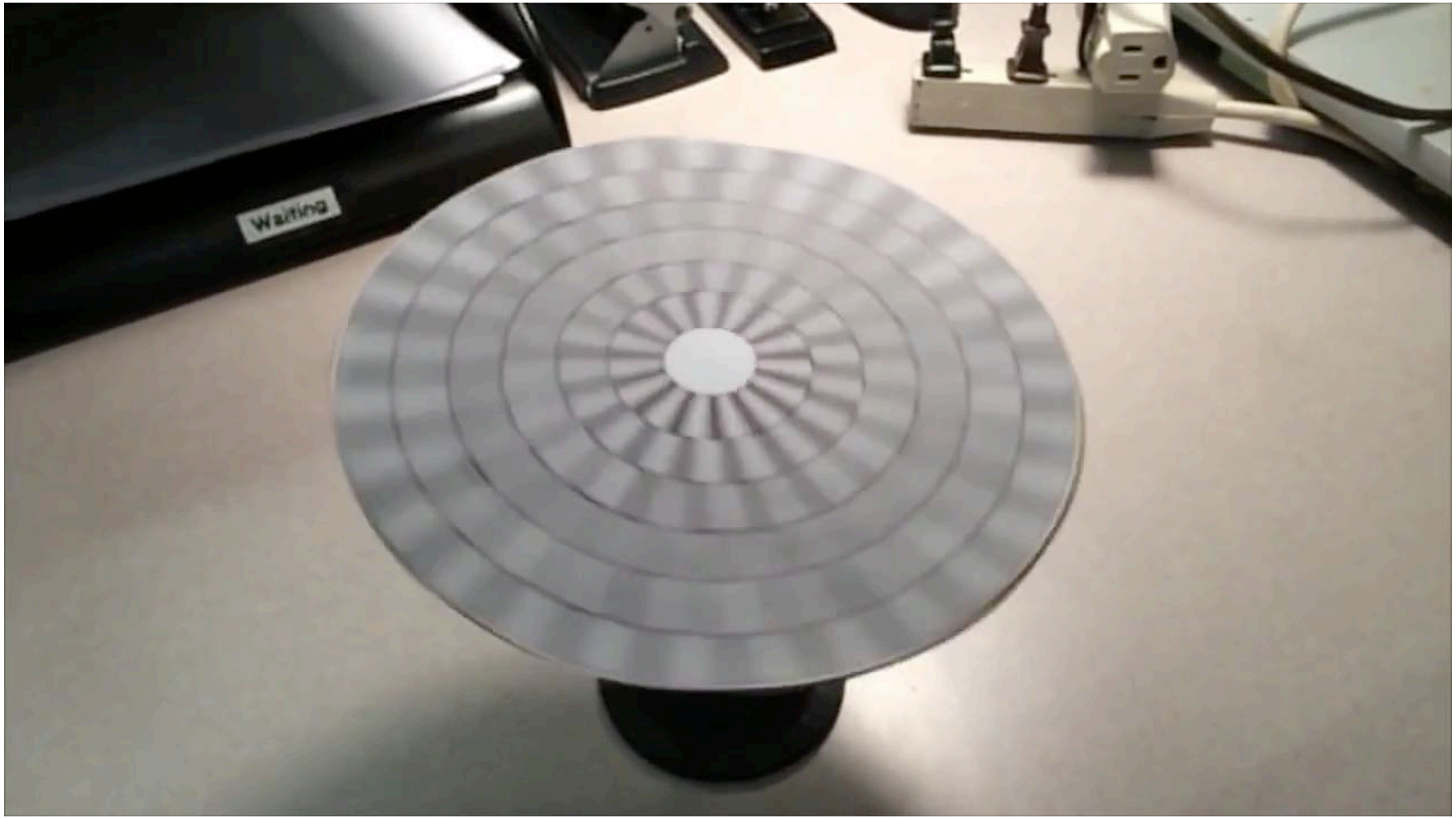
Full resolution image



1/2 resolution image:
skip pixel odd rows and columns

lystit.com

Wagon wheel illusion (false motion)



Camera's frame rate (temporal sampling rate) is too low for rapidly spinning wheel.

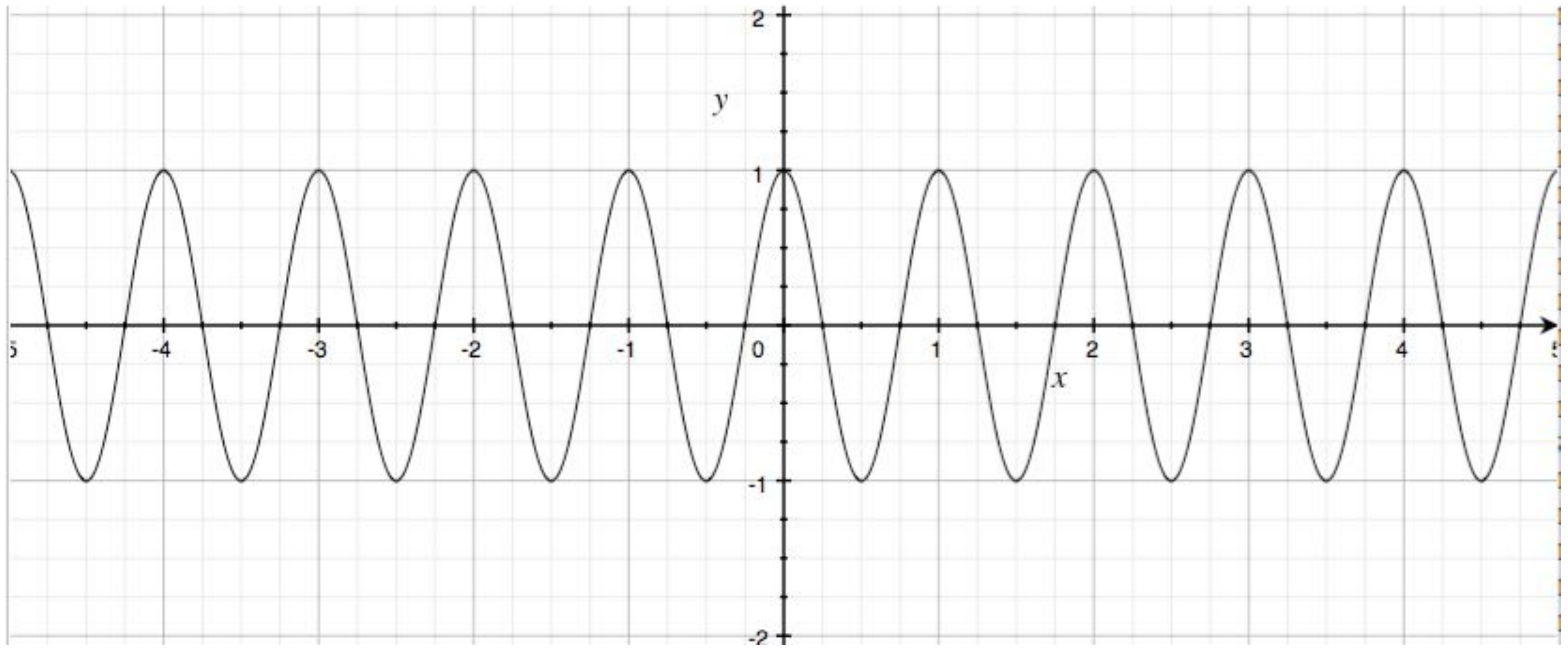
Created by Jesse Mason, https://www.youtube.com/watch?v=QOwzkND_ooU

Sampling artifacts in computer graphics

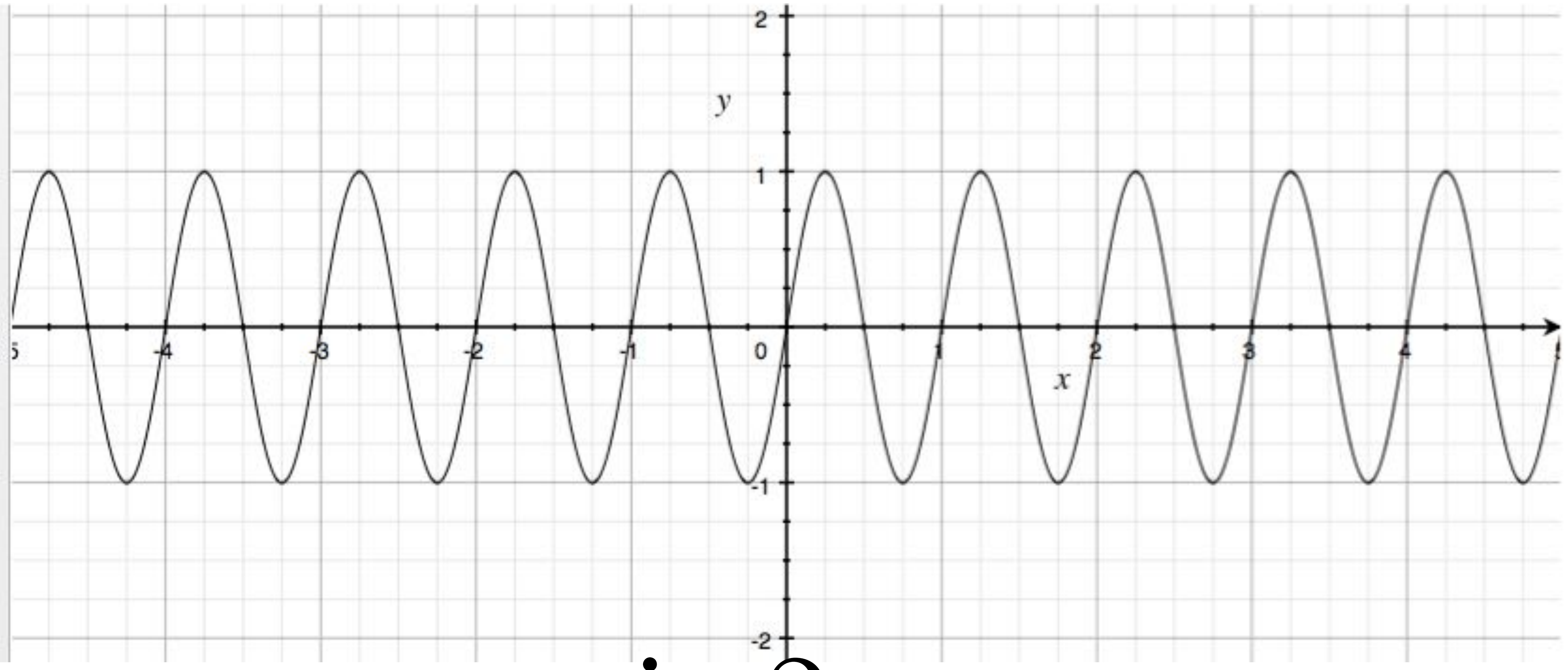
- **Artifacts due to sampling - "Aliasing"**
    - Jaggies – sampling to sparsely in space
    - Wagon wheel effect – sampling to sparsely in time
    - Moire – undersampling images (and texture maps)
    - [Many more] …

- **We notice this in fast-changing signals, when we sample the signal too sparsely**

Sines and cosines



$\cos 2\pi x$

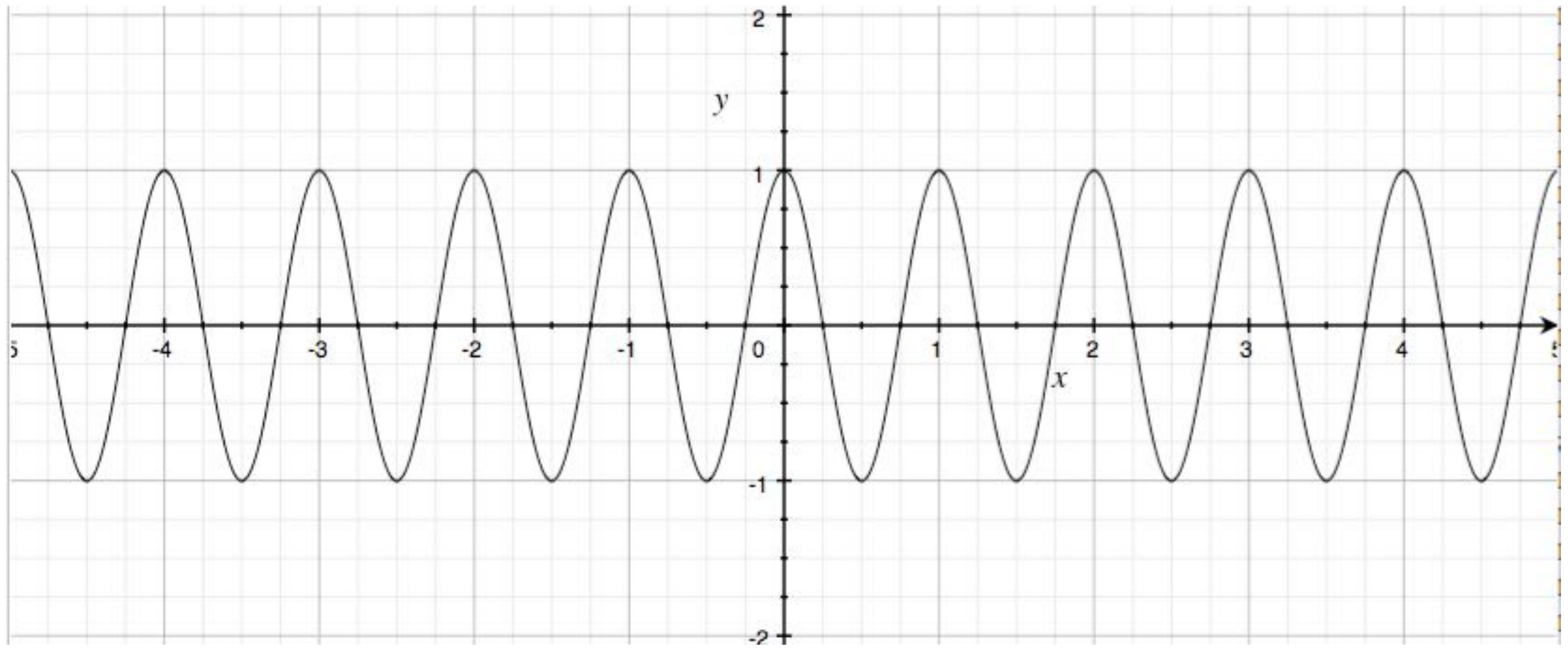


$\sin 2\pi x$

Frequencies

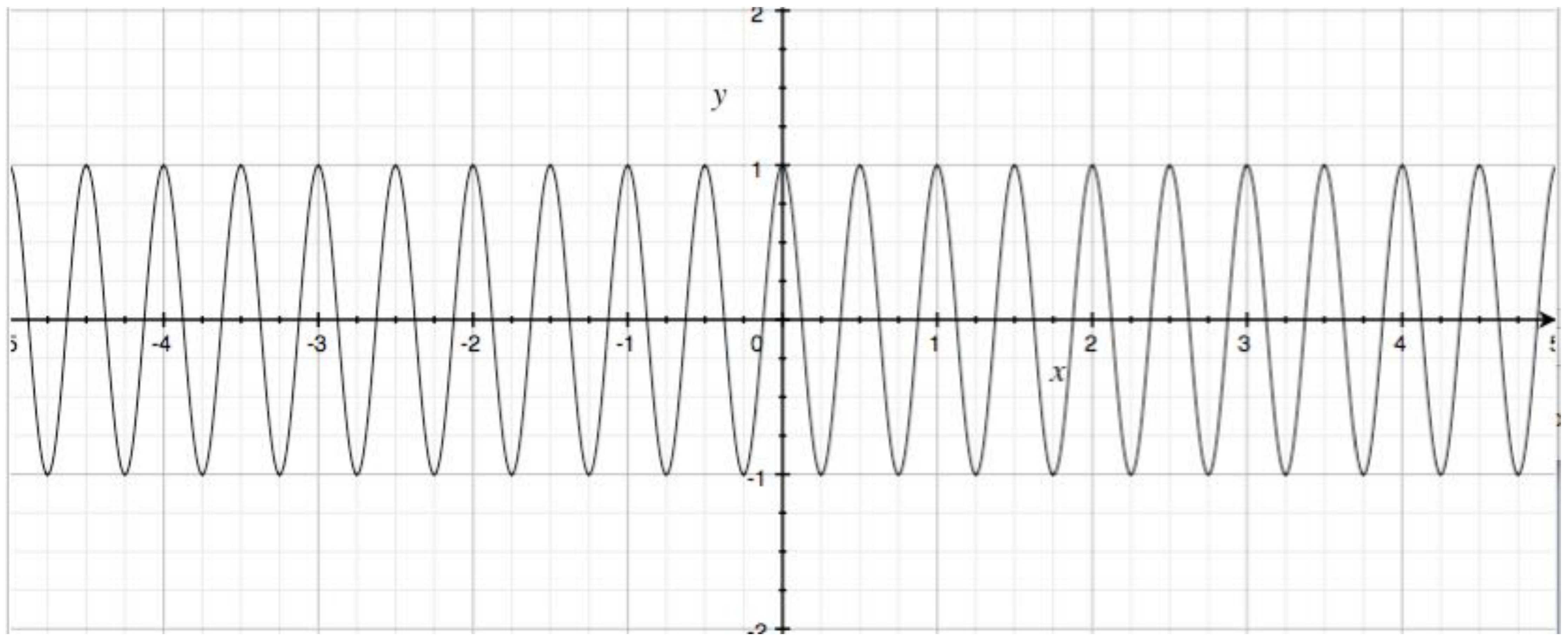
$$\cos 2\pi f x$$

$$f = \frac{1}{T}$$



$$f = 1$$

$$\cos 2\pi x$$

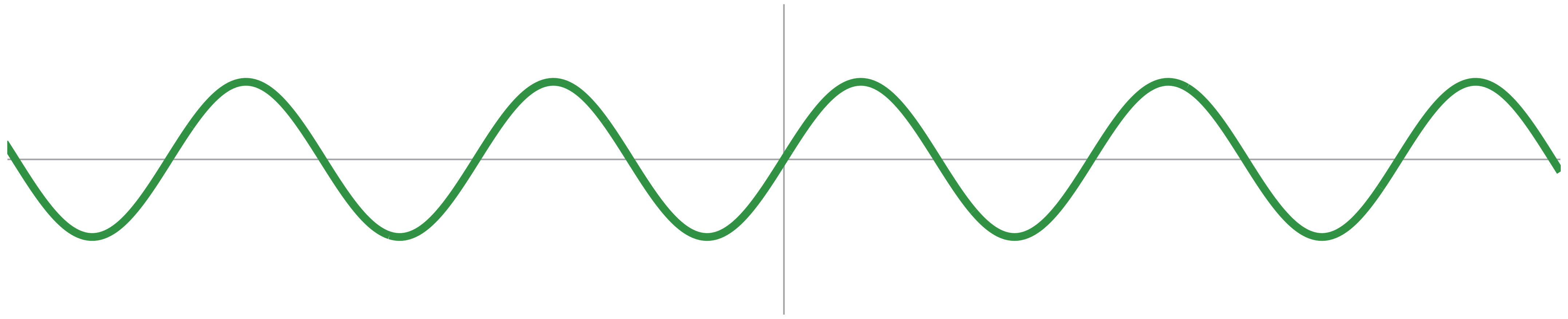


$$f = 2$$

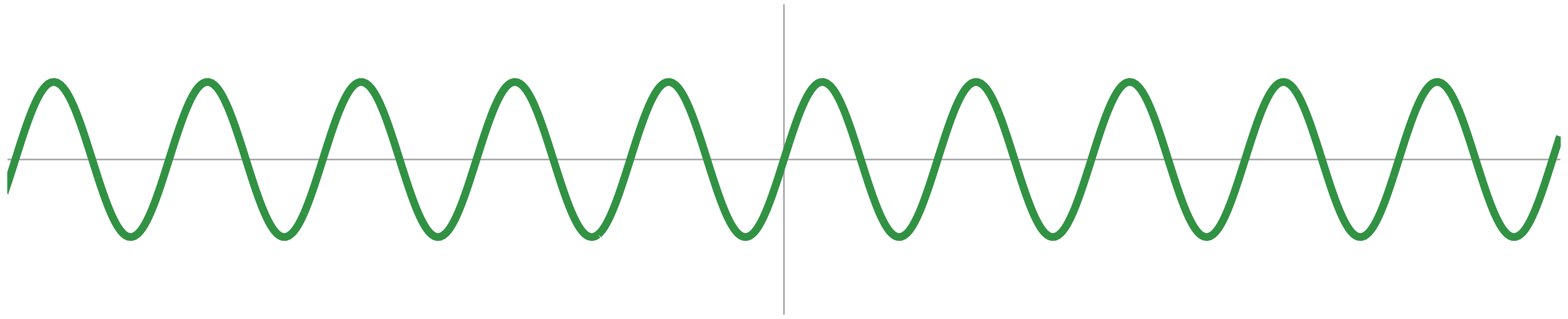
$$\cos 4\pi x$$

Representing sound wave as a superposition (linear combination) of frequencies

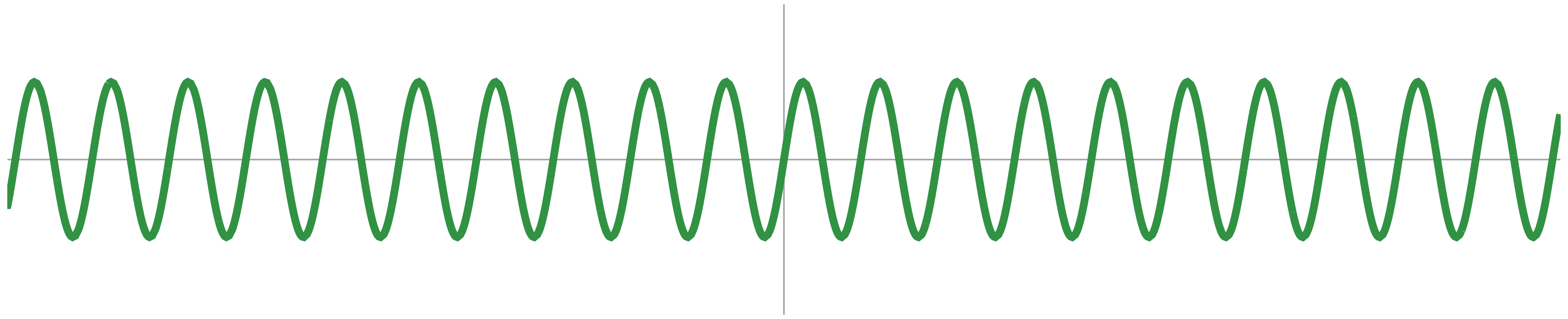
$f_1(x) = sin(\pi x)$



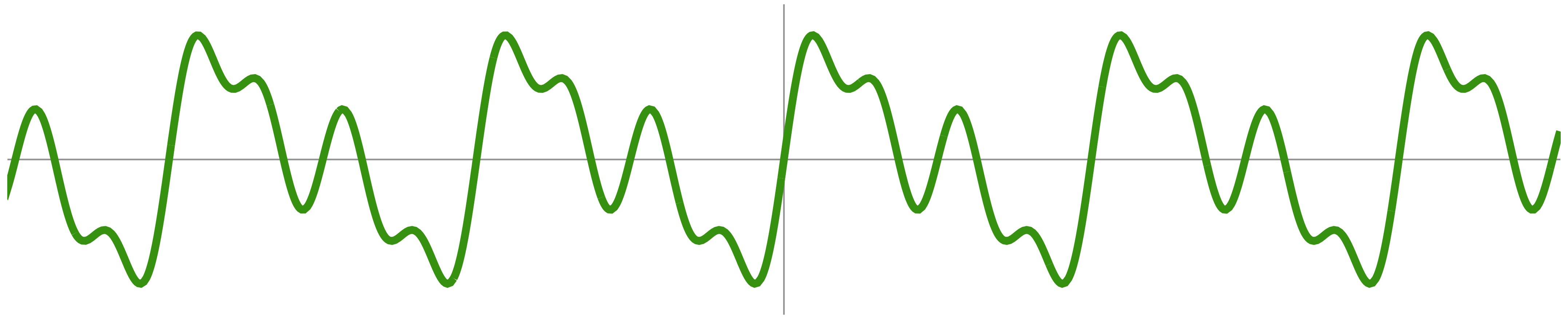
$f_2(x) = sin(2\pi x)$



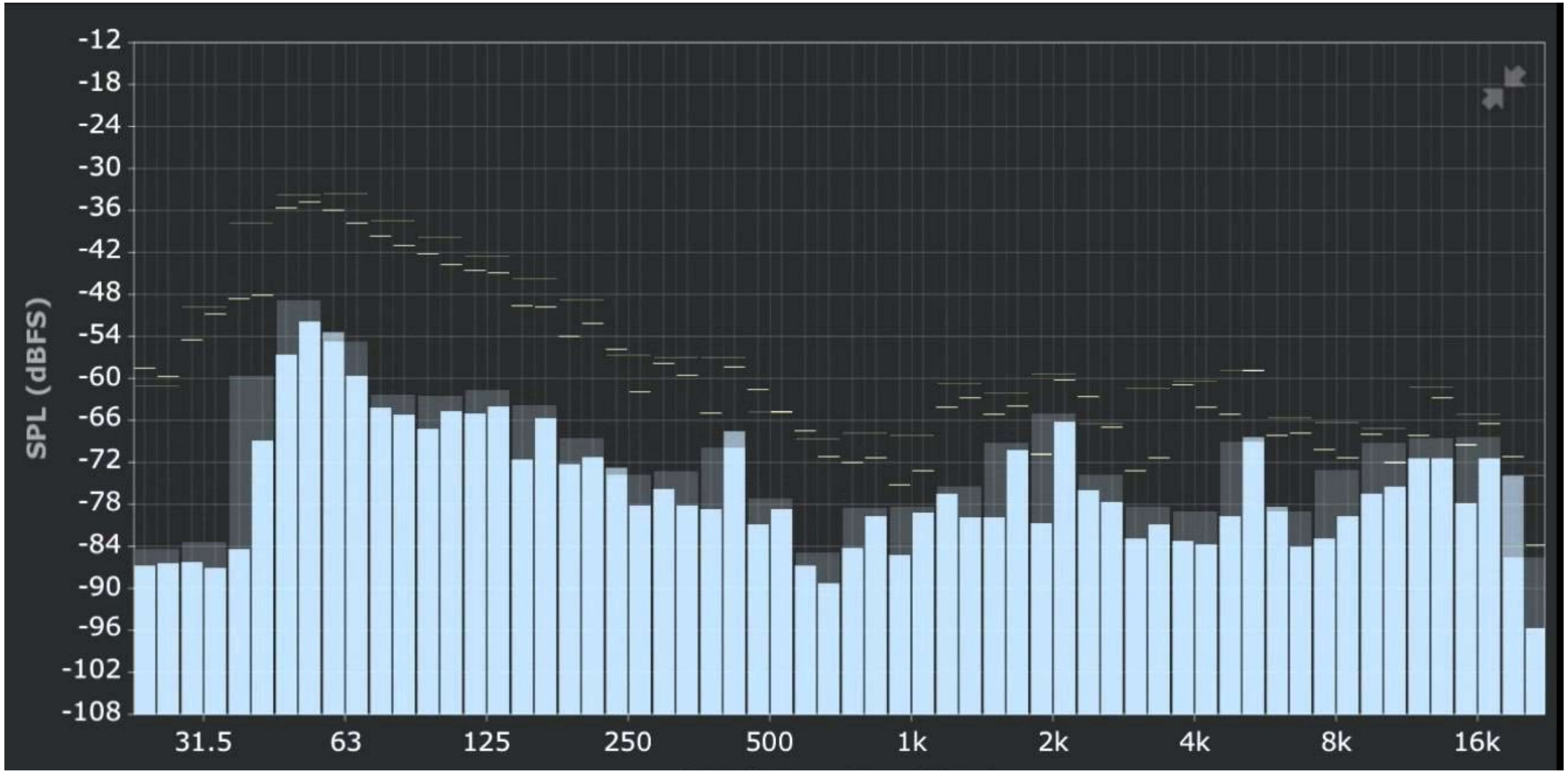
$f_4(x) = sin(4\pi x)$



$f(x) = 1.0\, f_1(x) + 0.75\, f_2(x) + 0.5\, f_4(x)$



Audio spectrum analyzer: representing sound as a sum of its constituent frequencies



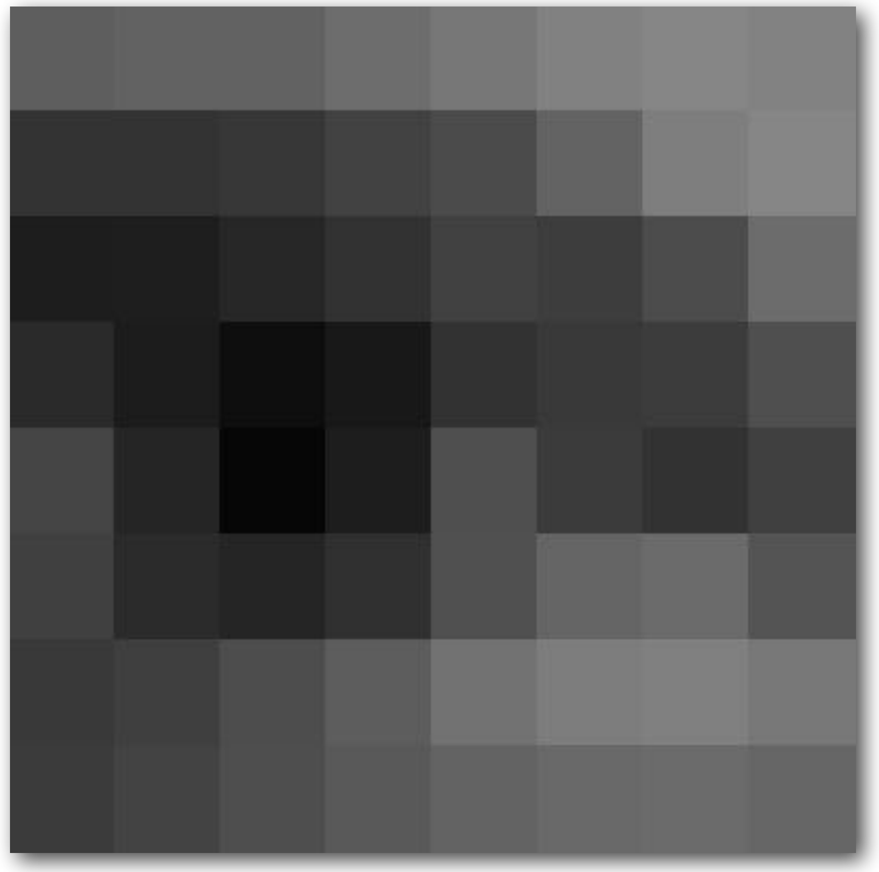
Intensity of low-frequencies (bass)

Intensity of high frequencies

Image credit: ONYX Apps

Images as a superposition of cosines

8x8 images



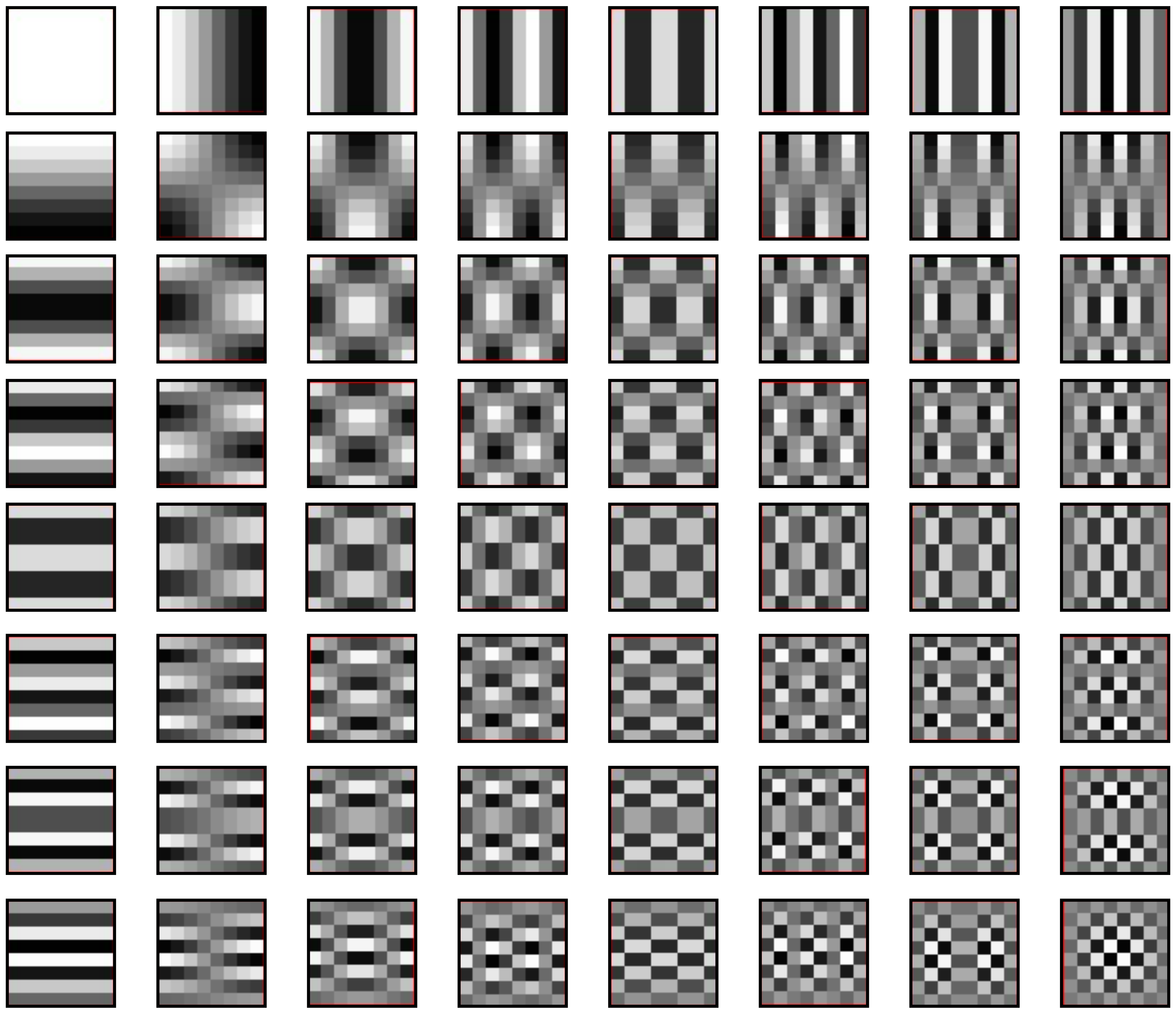
$$= \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

x

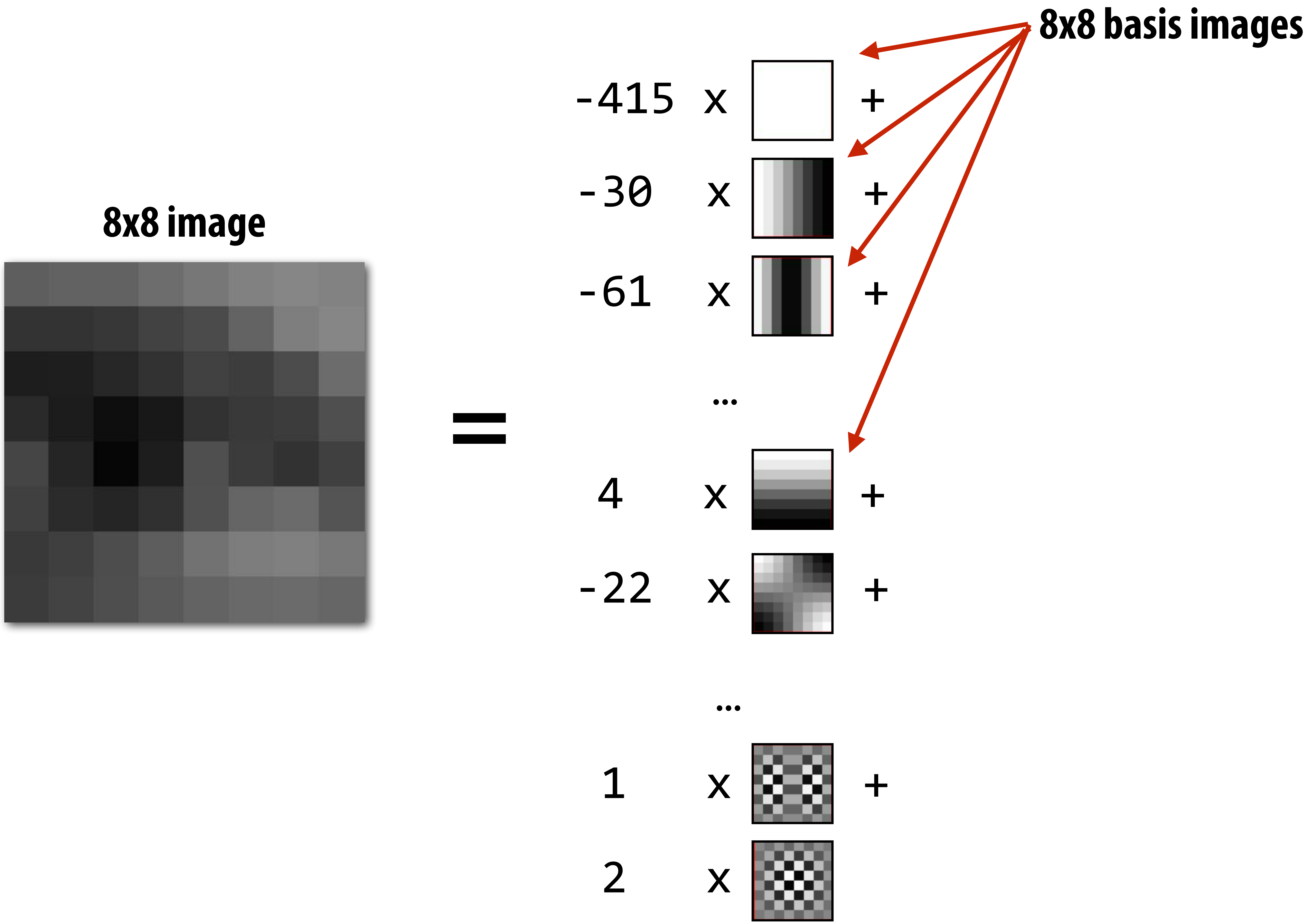
$$\cos\left[\pi \frac{i}{N}\left(x + \frac{1}{2}\right)\right] \times \underset{j}{\cos}\left[\pi \frac{j}{N}\left(y + \frac{1}{2}\right)\right]$$

i=0

j=0



Images as a superposition of cosines

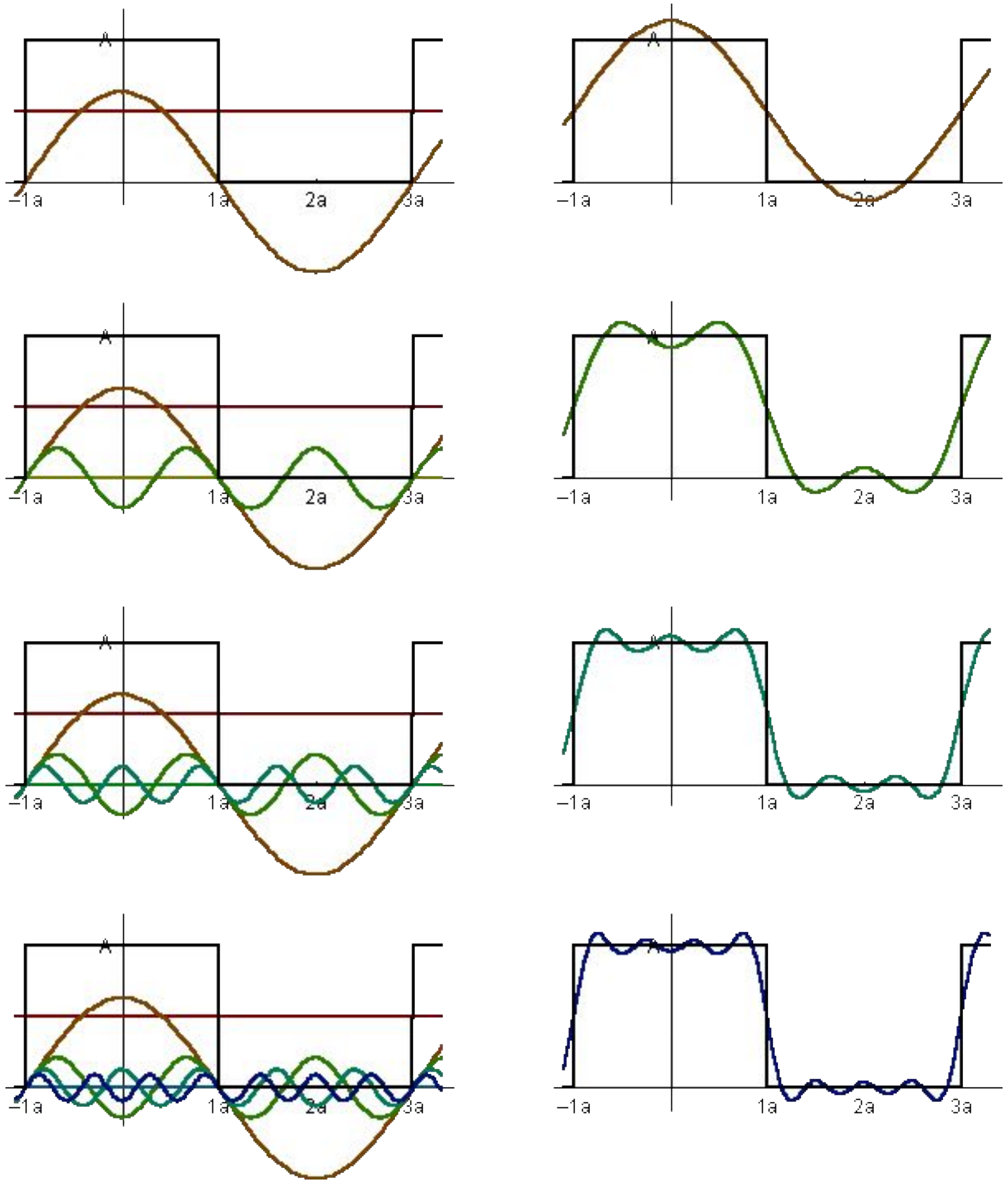


How to compute frequency-domain representation of a signal?

Fourier transform

Represent any function as a weighted sum of sines and cosines

Joseph Fourier 1768 - 1830



Fourier transform

Convert representation of signal from primal domain (spatial/temporal) to frequency domain by projecting signal into its component frequencies

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \omega} dx$$

$$= \int_{-\infty}^{\infty} f(x)(\cos(2\pi\omega x) - i\sin(2\pi\omega x)) dx$$

Recall:

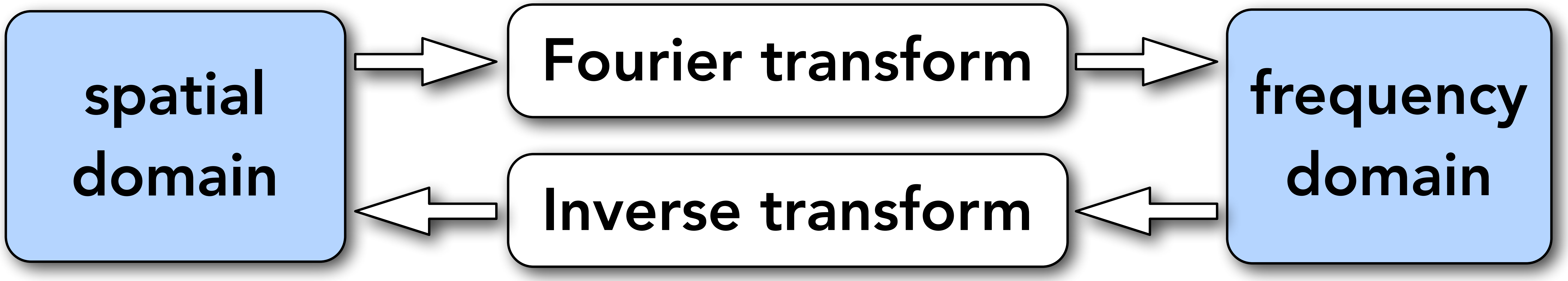
$$e^{ix} = \cos x + i \sin x$$

2D form:

$$F(u,v) = \int\int f(x,y) e^{-2\pi i (ux+vy)} dx dy$$

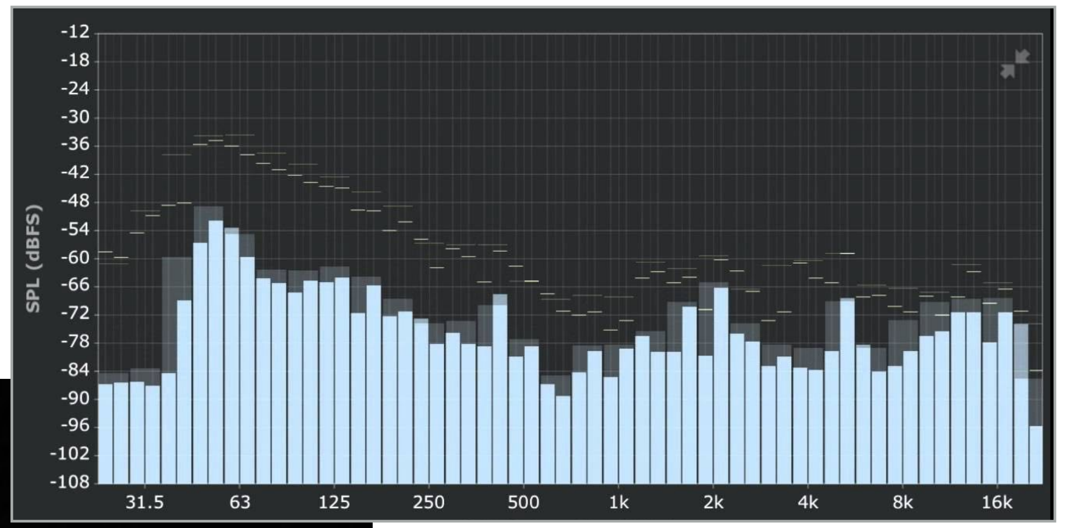
The Fourier transform decomposes a signal into its constituent frequencies

$$f(x) \qquad F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx \qquad F(\omega)$$



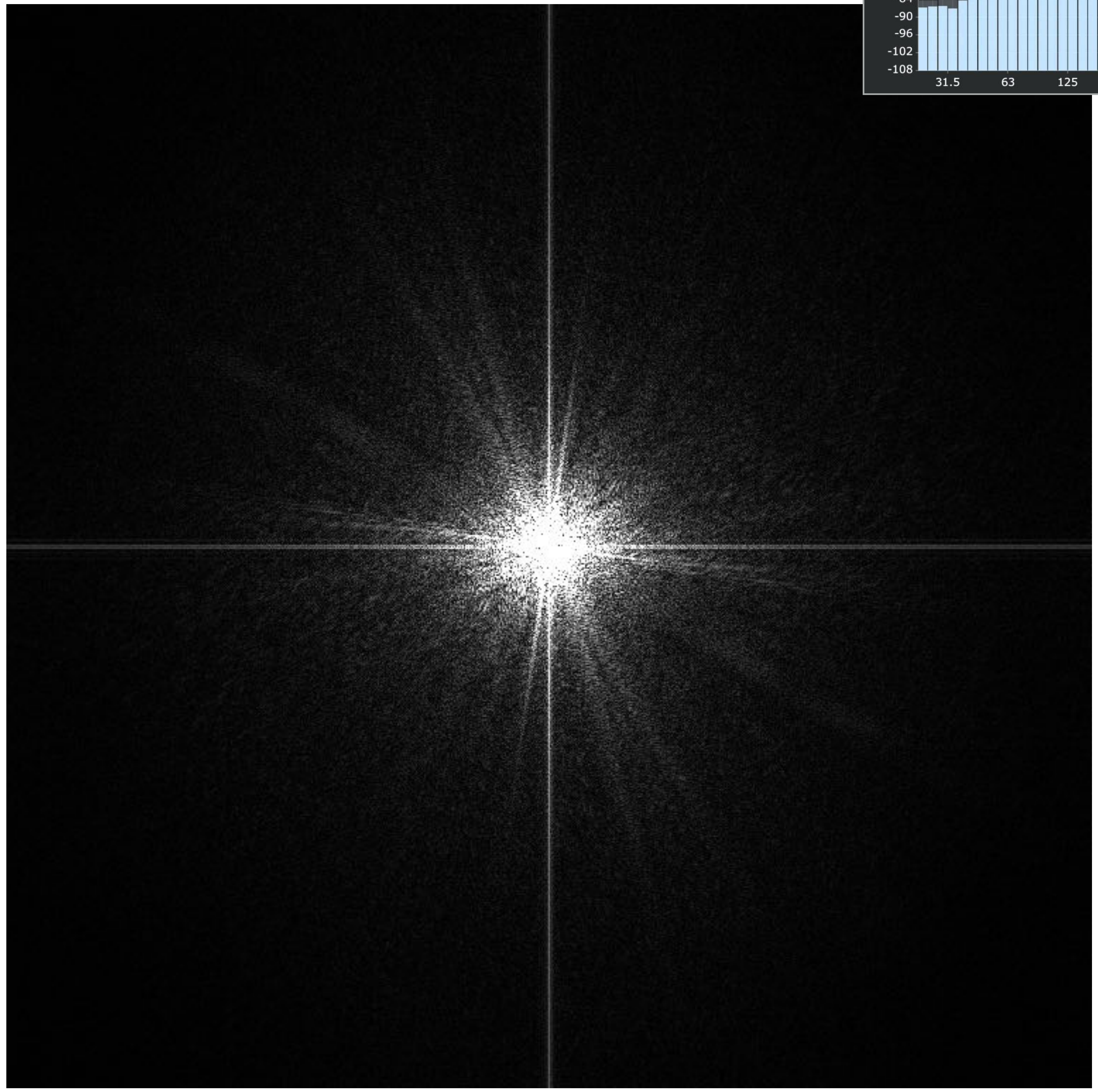
$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

Visualizing the frequency content of images



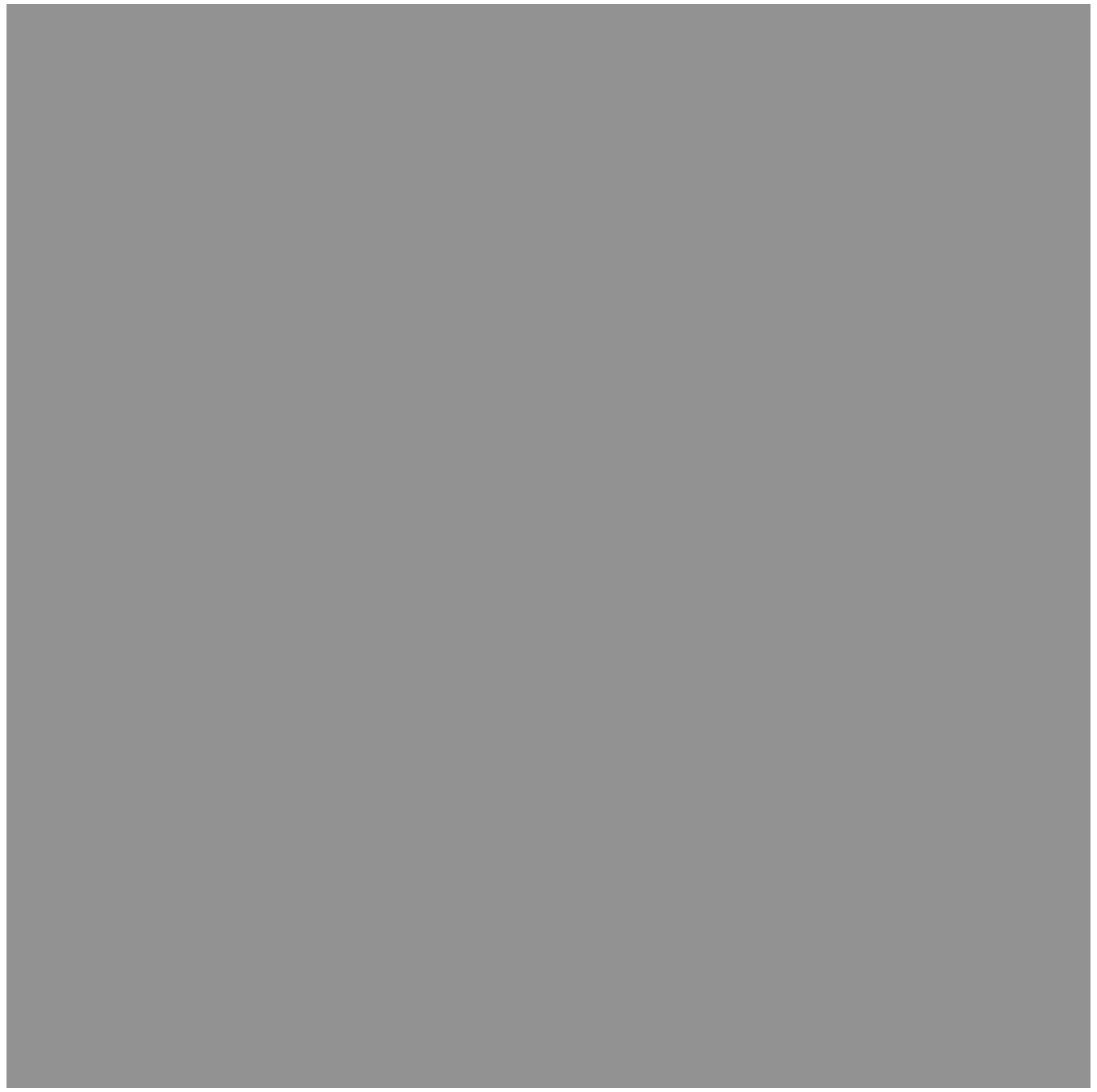
The visualization below is the 2D frequency domain equivalent of the 1D audio spectrum I showed you earlier *

Spatial domain result

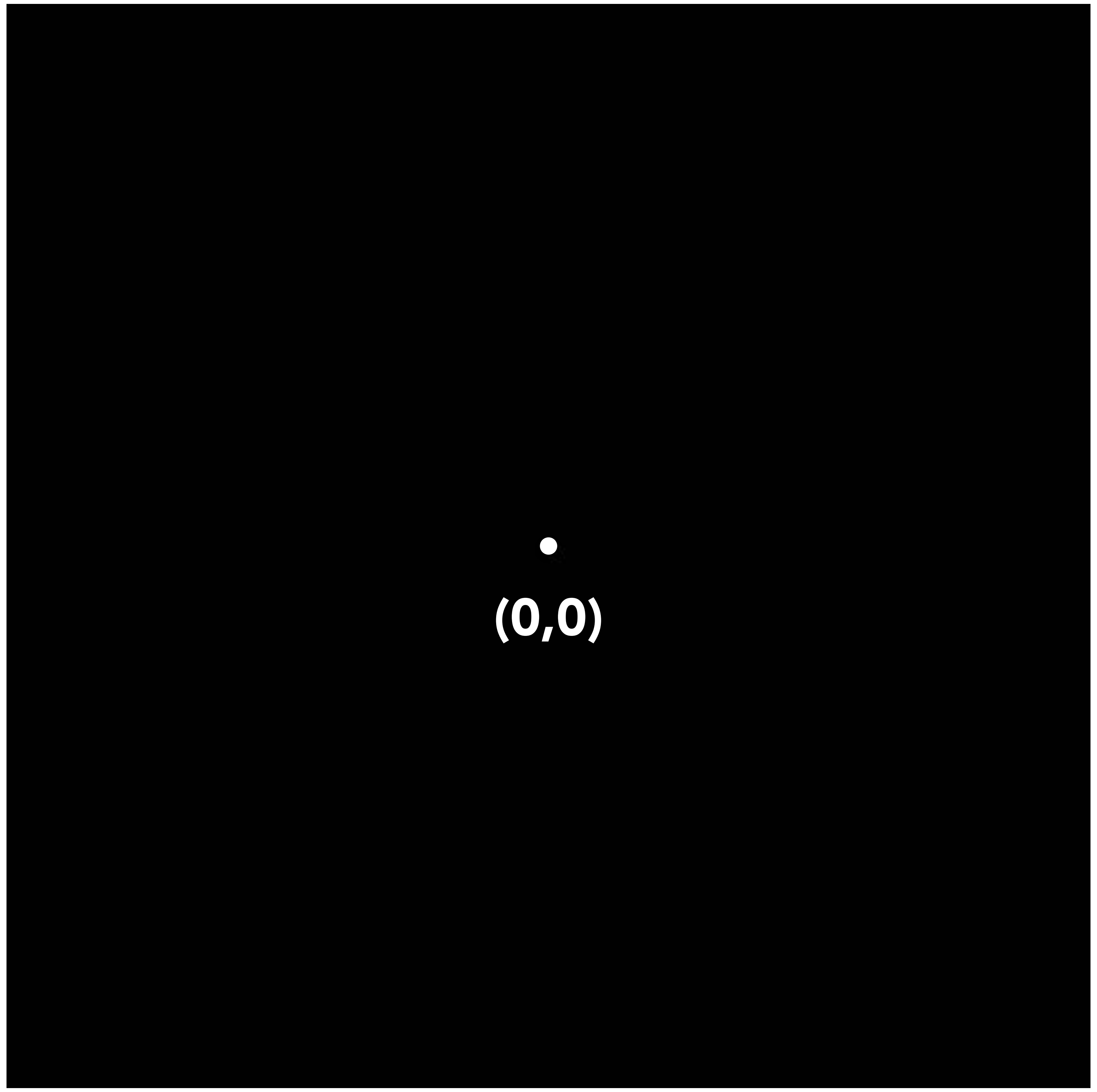


Spectrum

Constant signal (in primal domain)

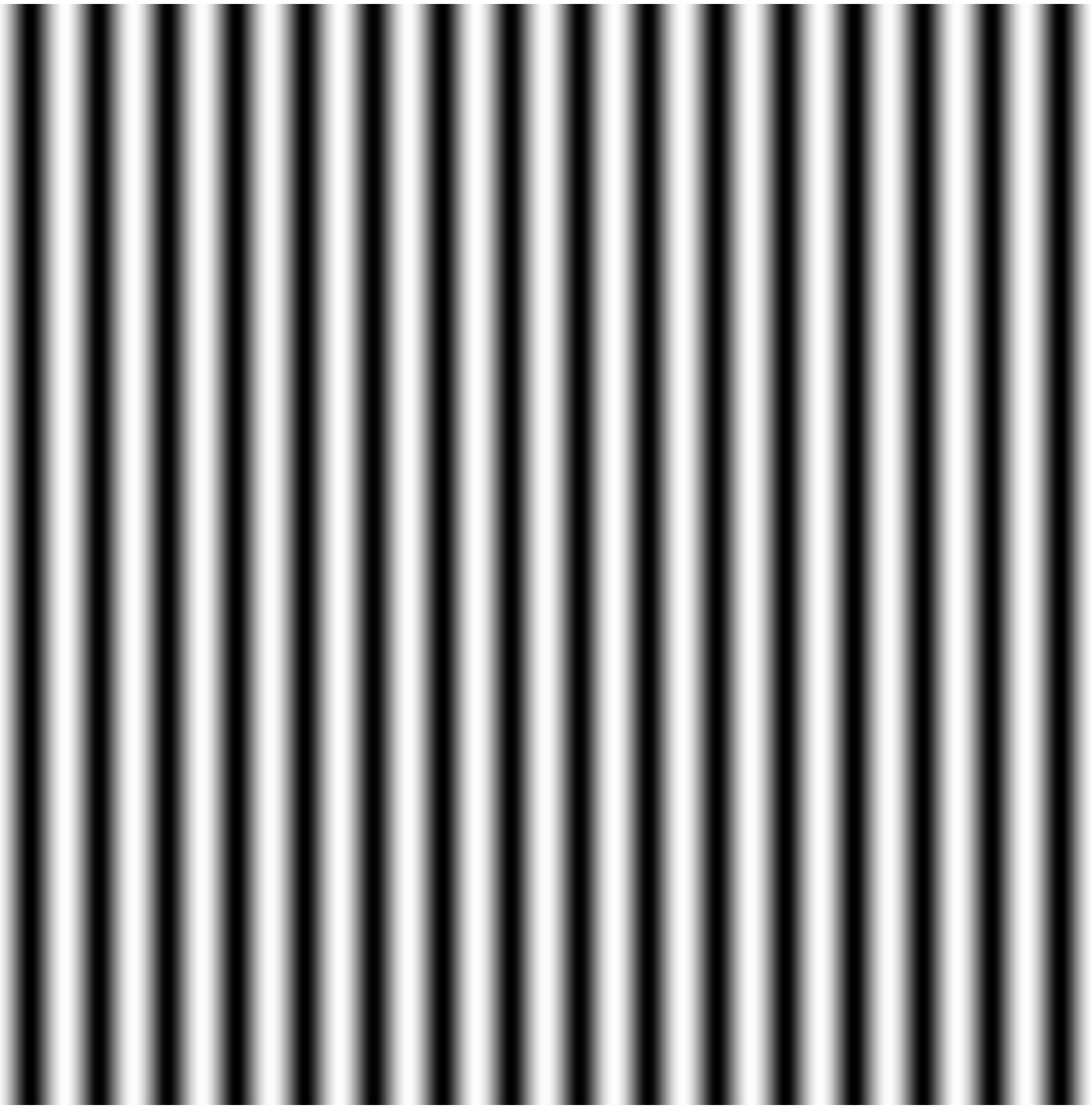


Spatial domain

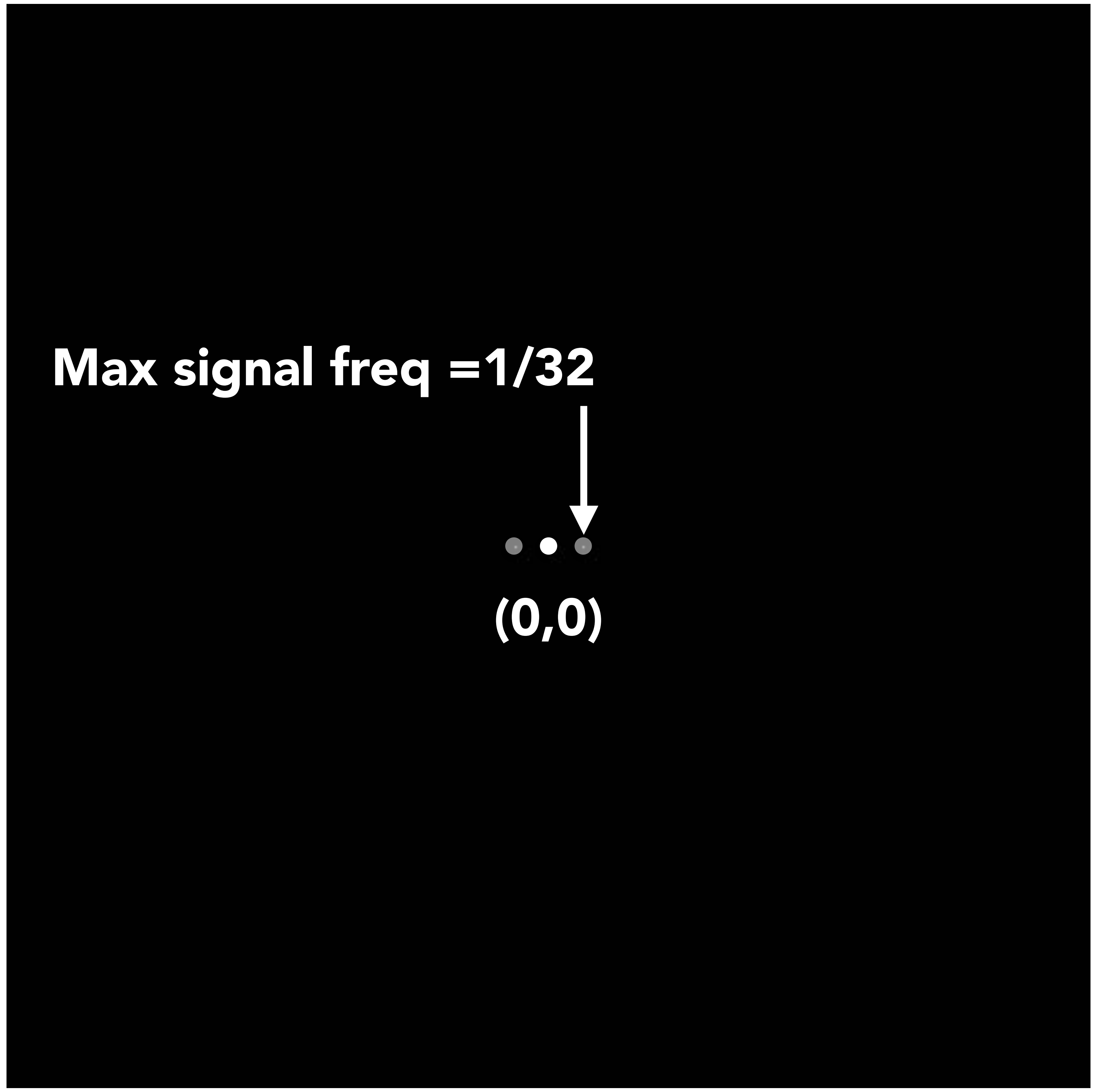


Frequency domain

$\sin(2\pi/32)x$ — frequency 1/32; 32 pixels per cycle

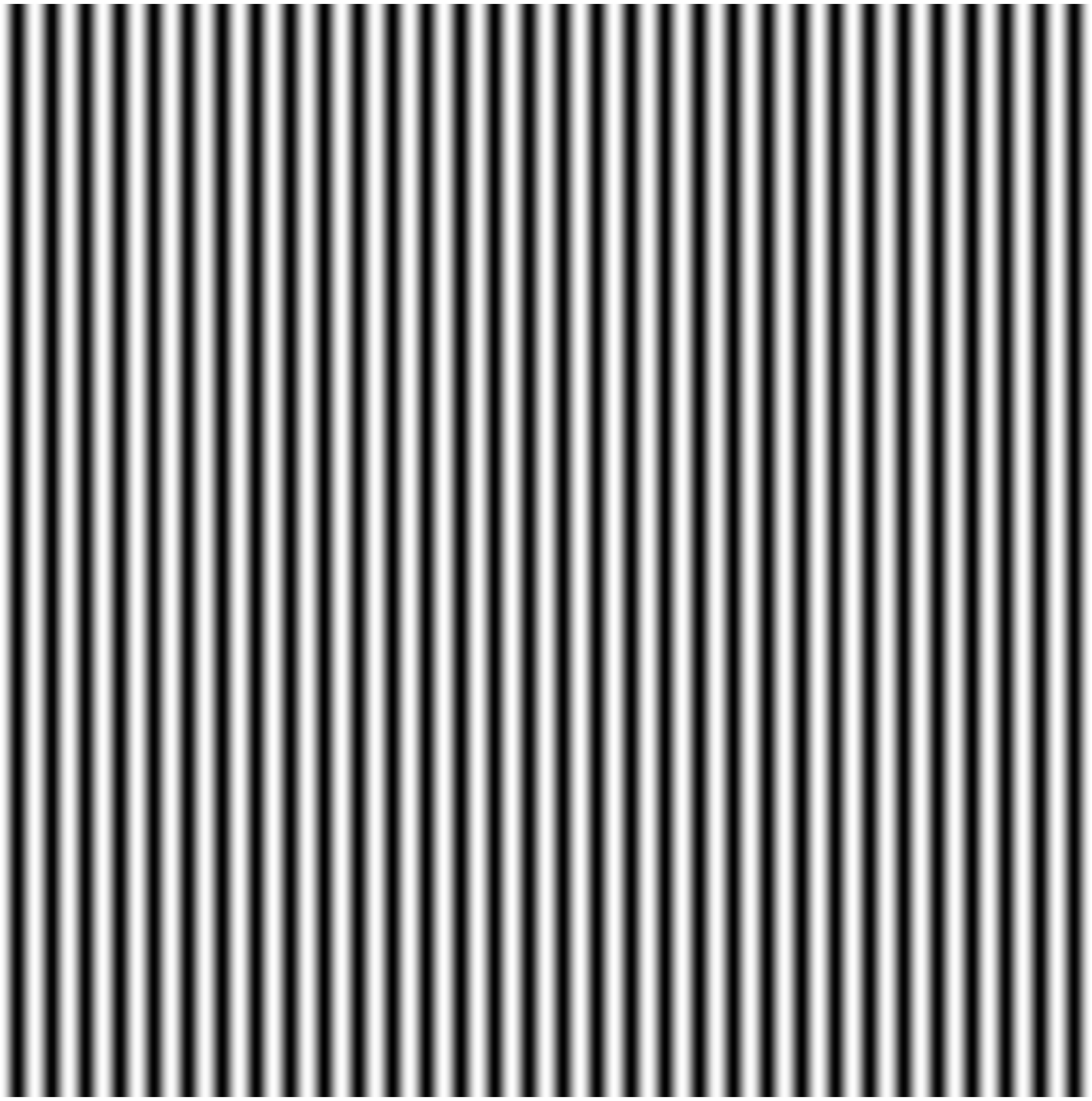


Spatial domain

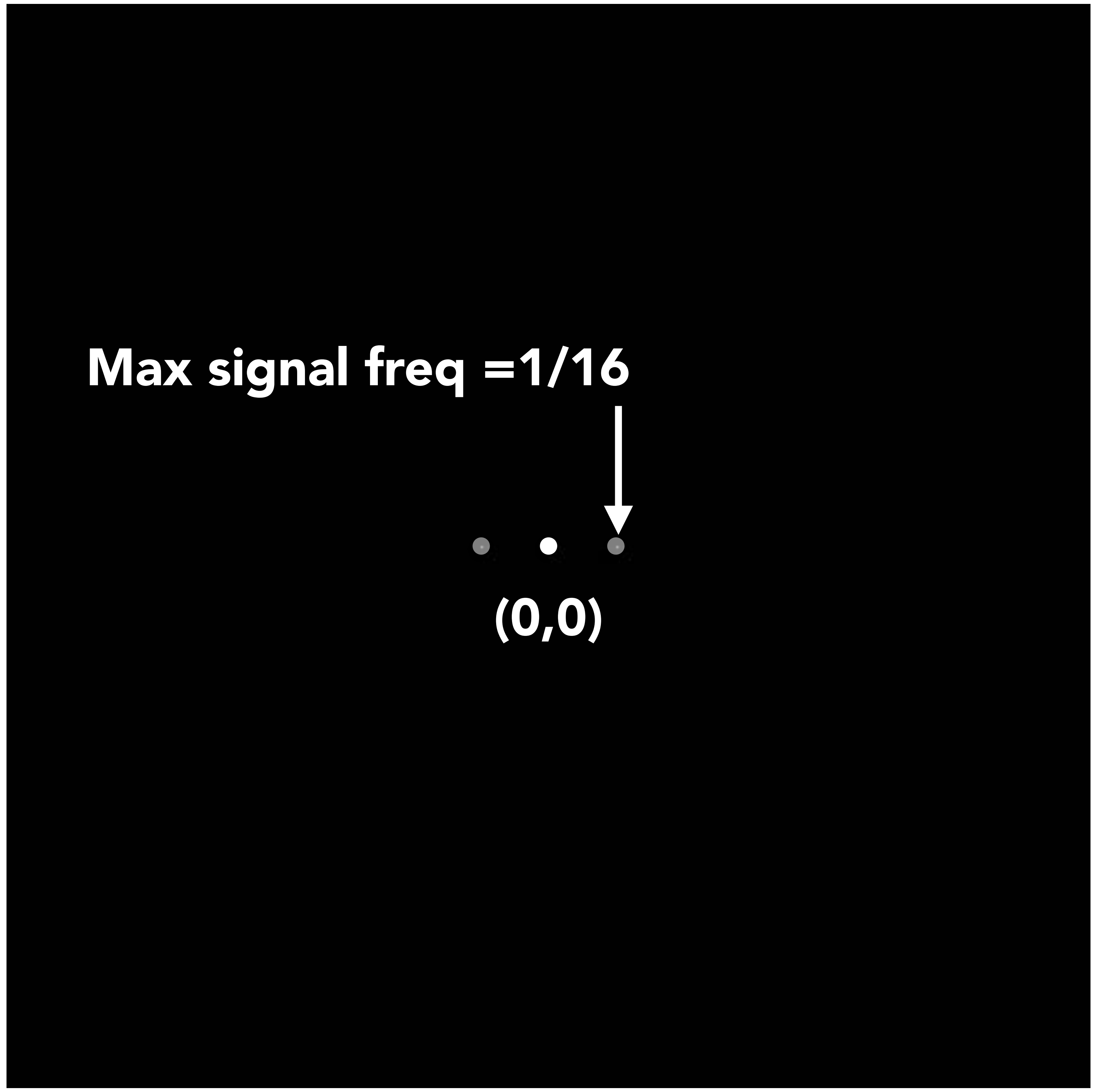


Frequency domain

$\sin(2\pi/16)x$ — frequency 1/16; 16 pixels per cycle

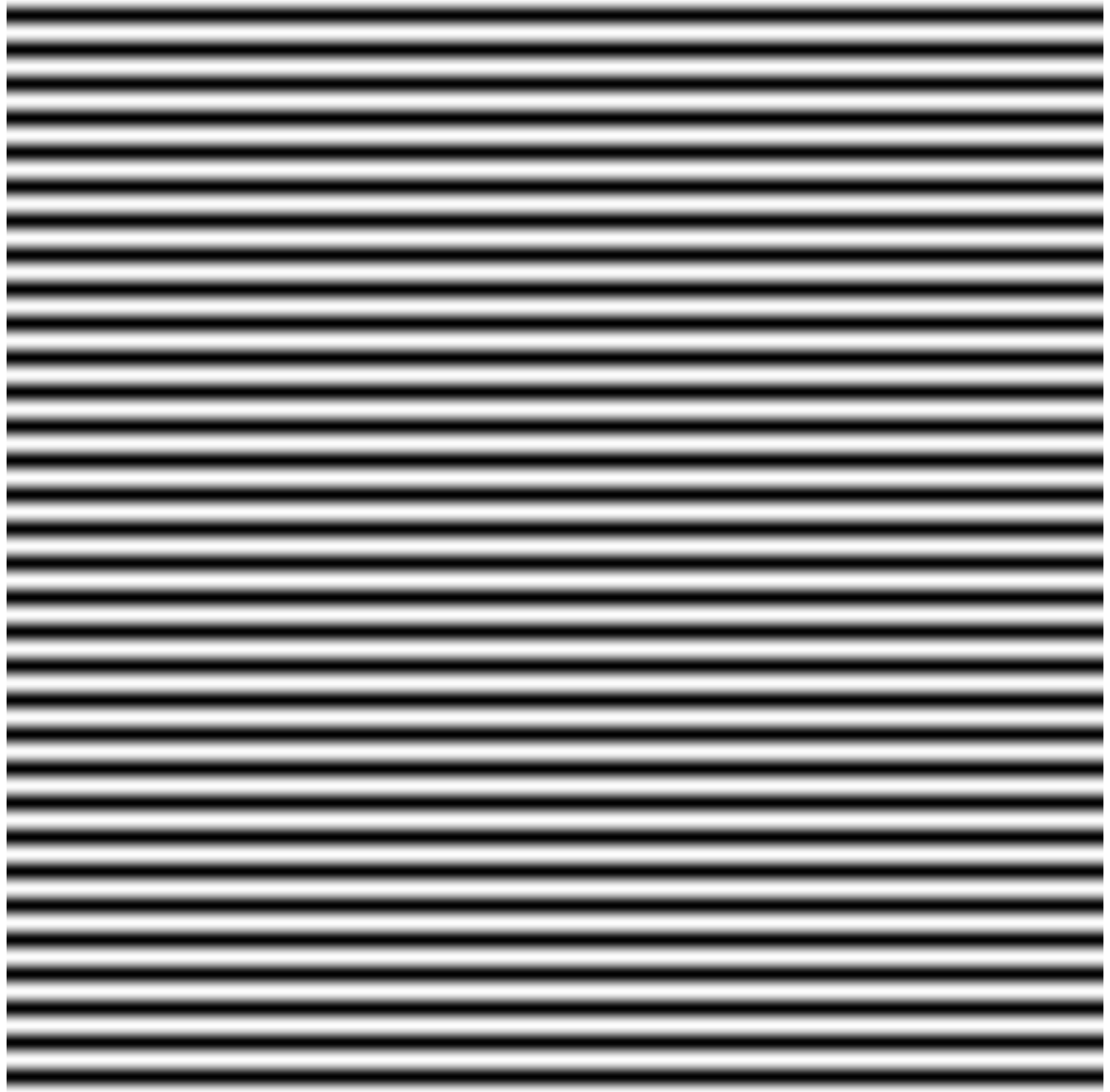


Spatial domain

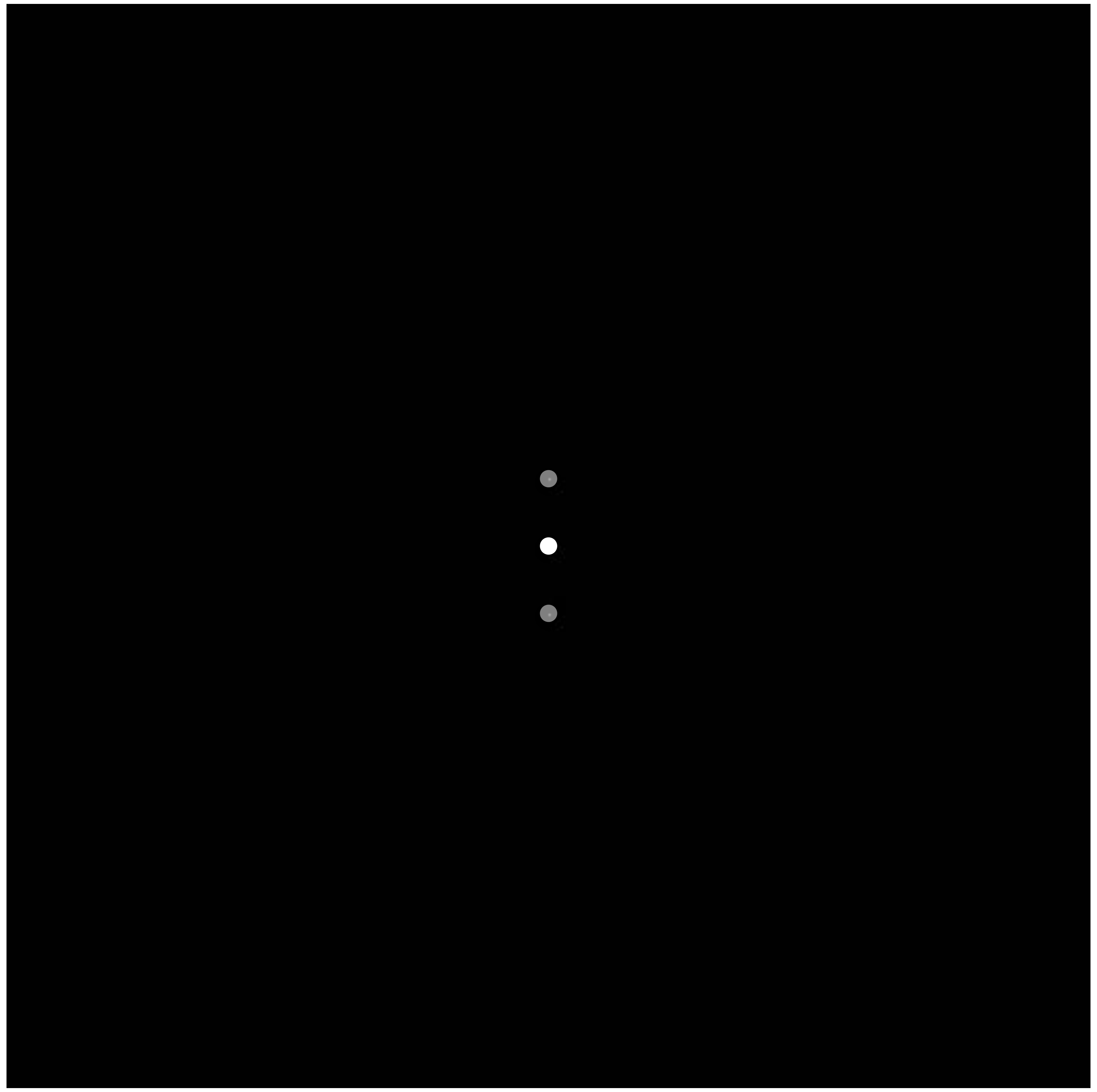


Frequency domain

$$\sin(2\pi/16)y$$

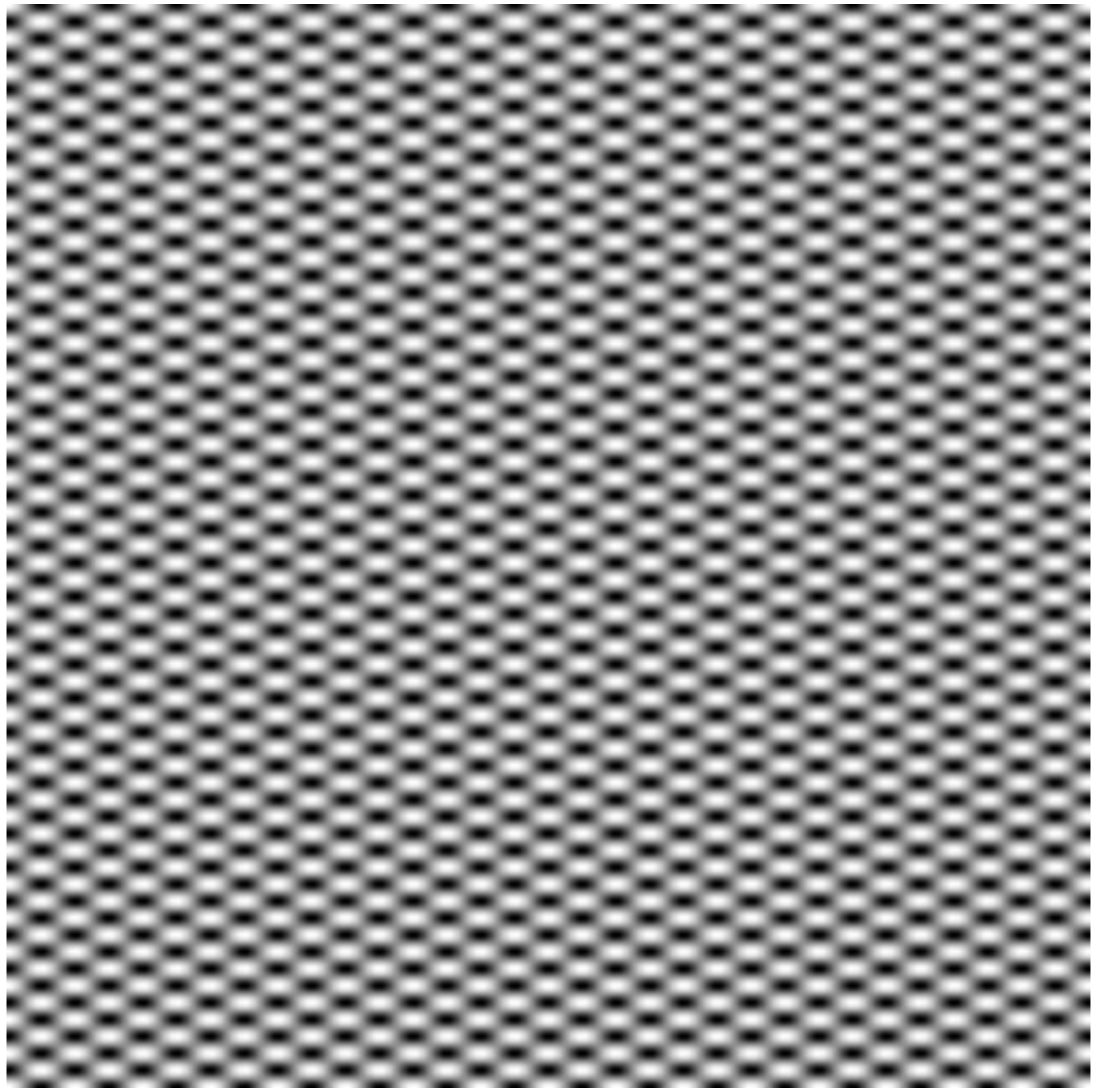


Spatial domain

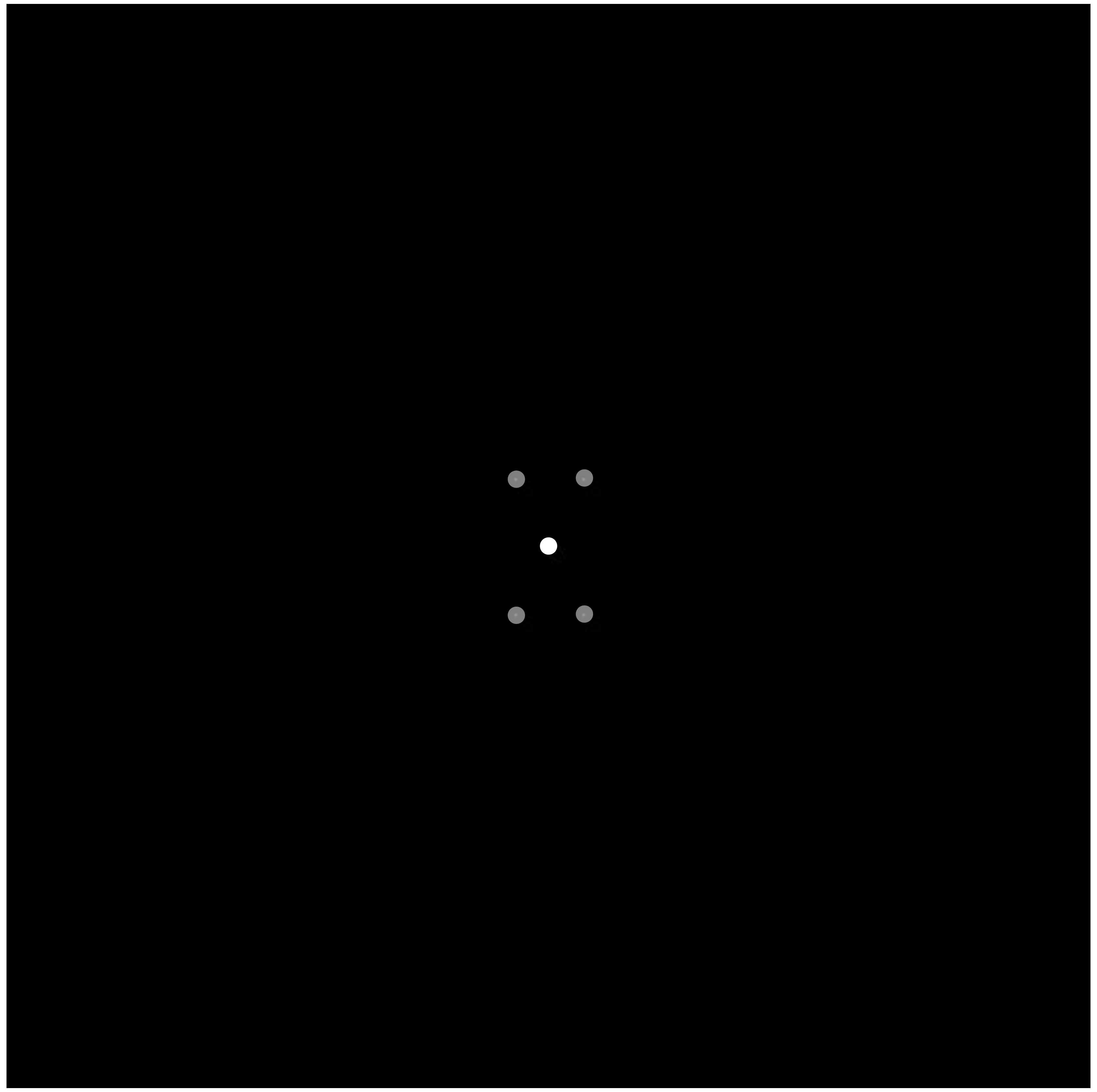


Frequency domain

$$\sin(2\pi/32)x \times \sin(2\pi/16)y$$

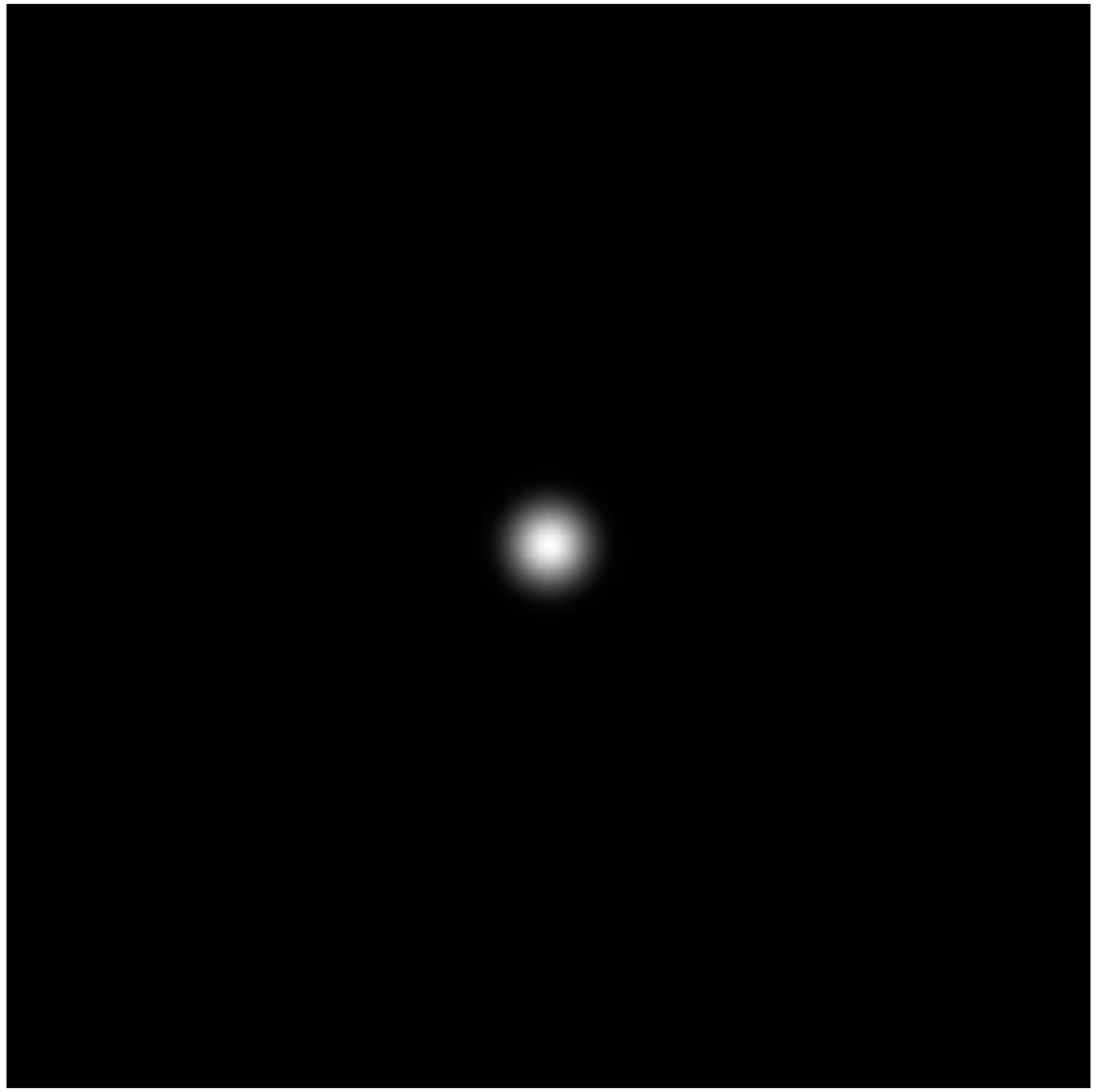


Spatial domain

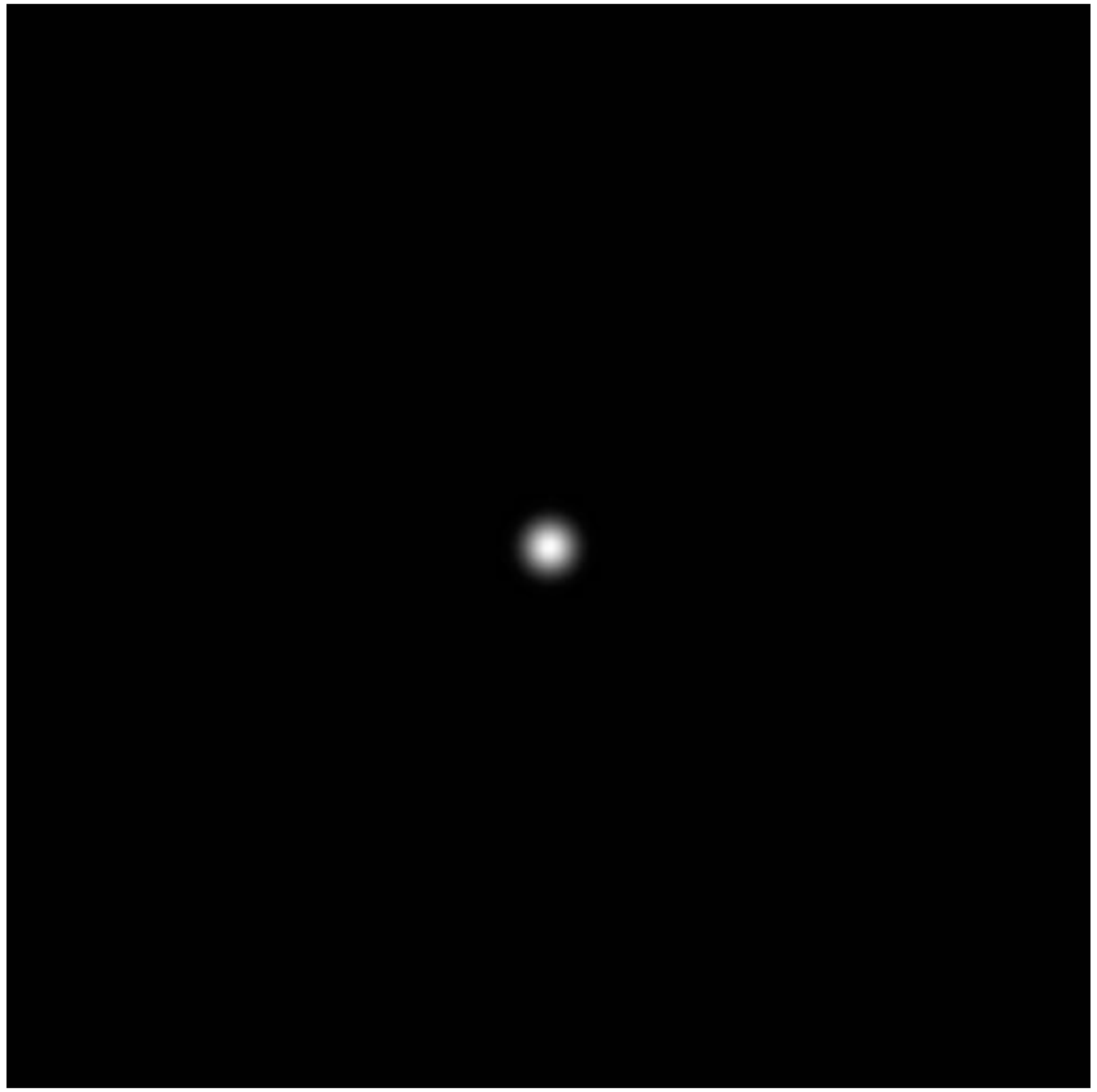


Frequency domain

$$\exp(-r^2/16^2)$$

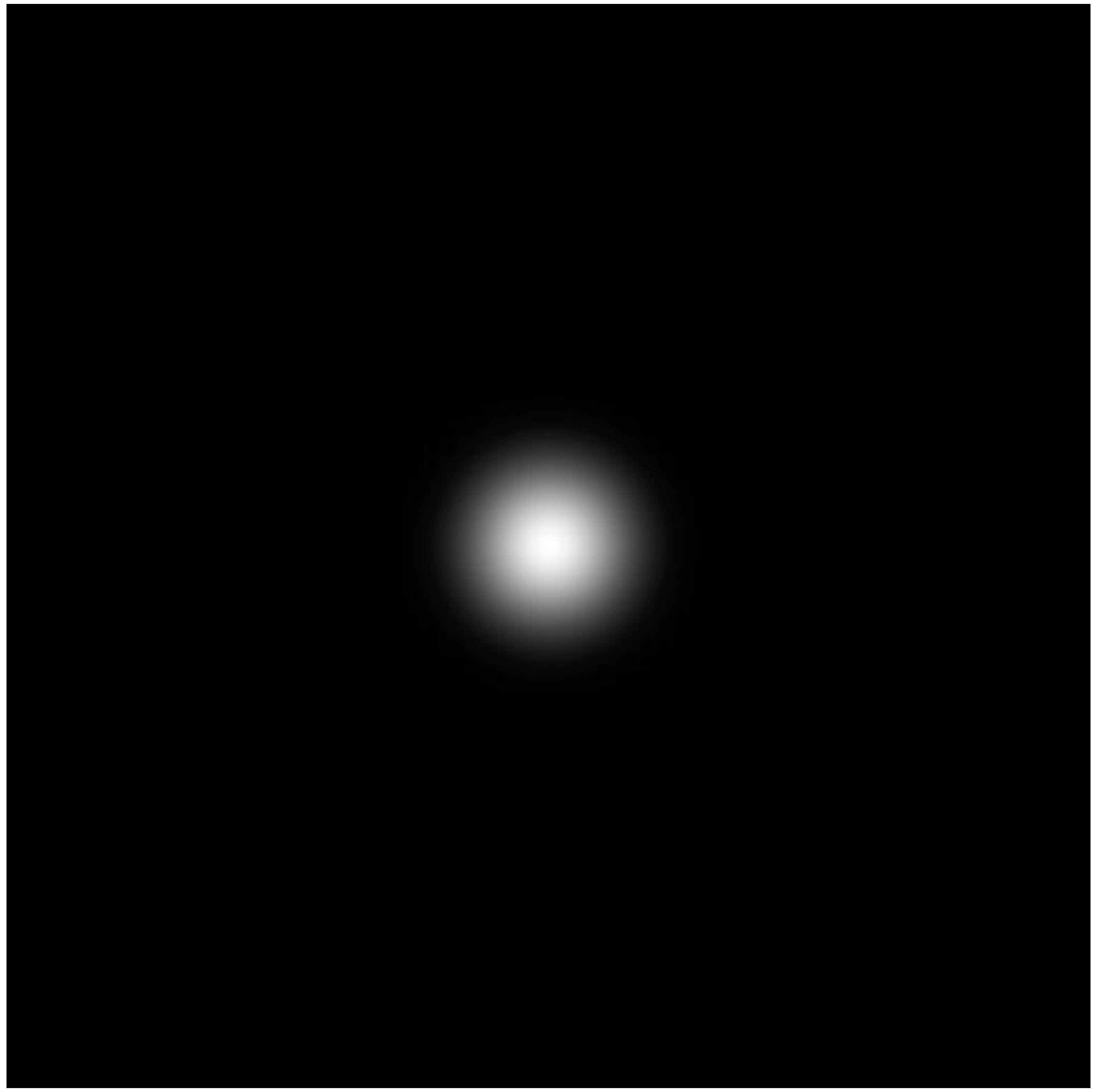


Spatial domain

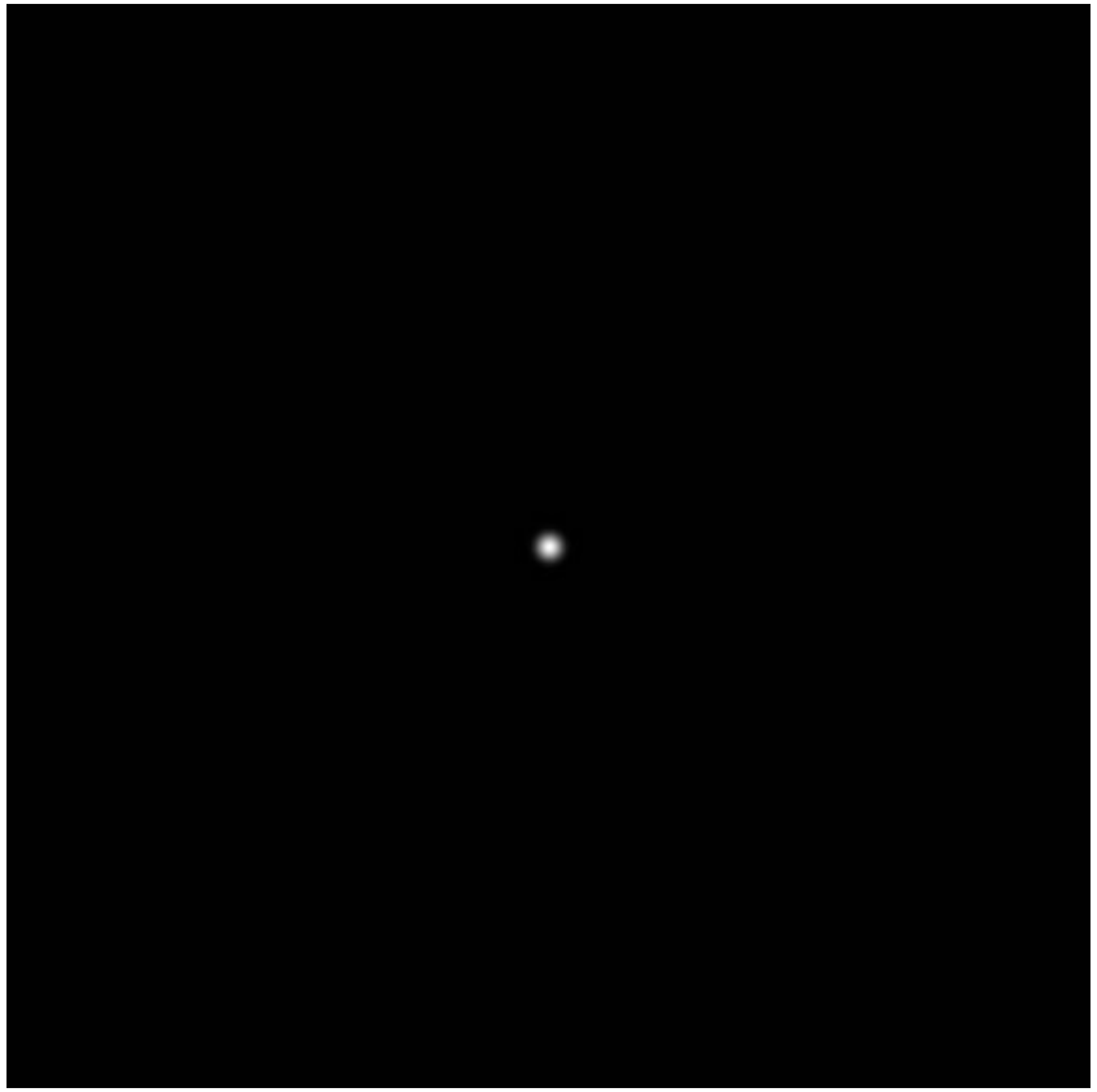


Frequency domain

$$\exp(-r^2/32^2)$$



Spatial domain



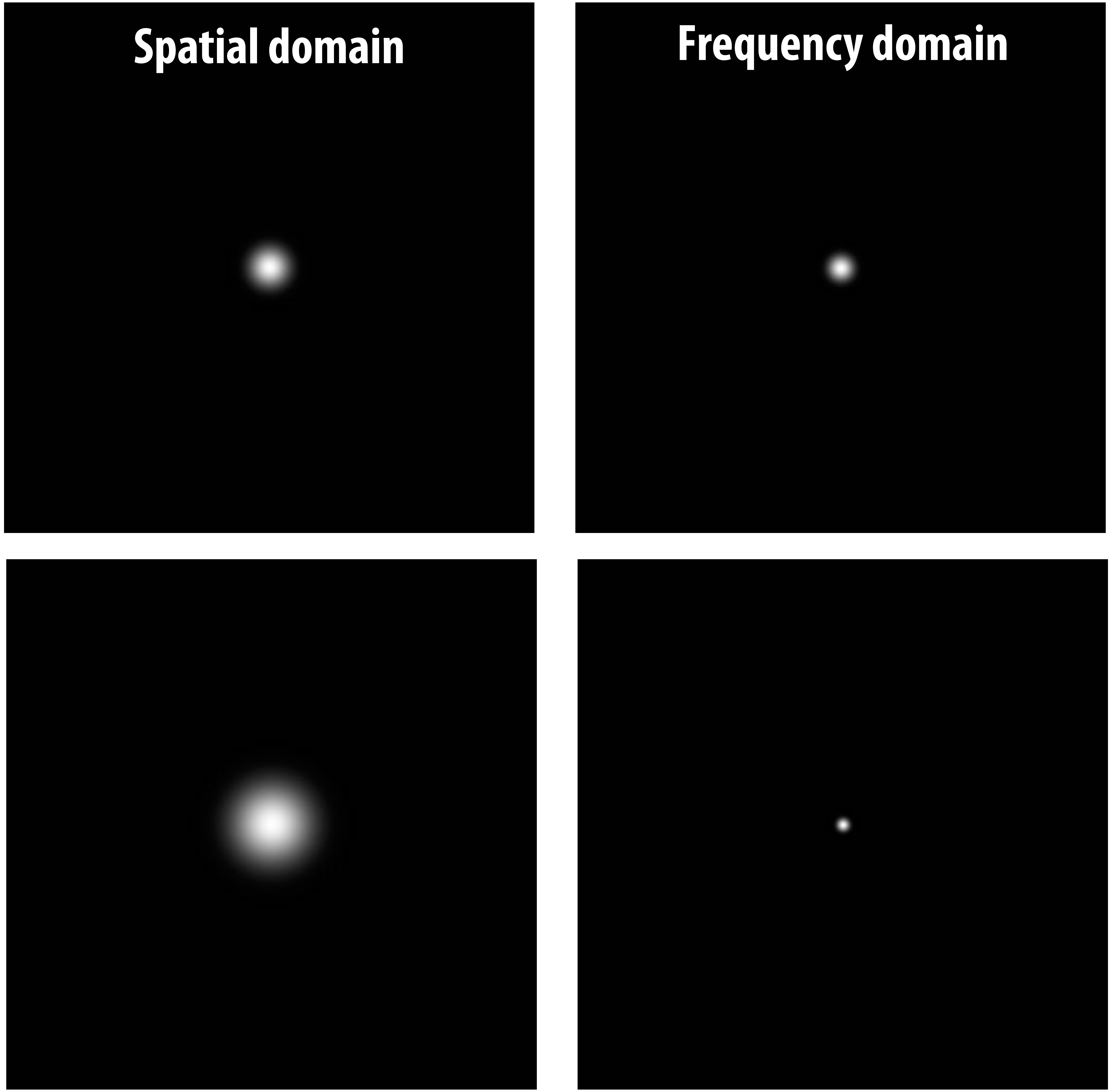
Frequency domain

Question:

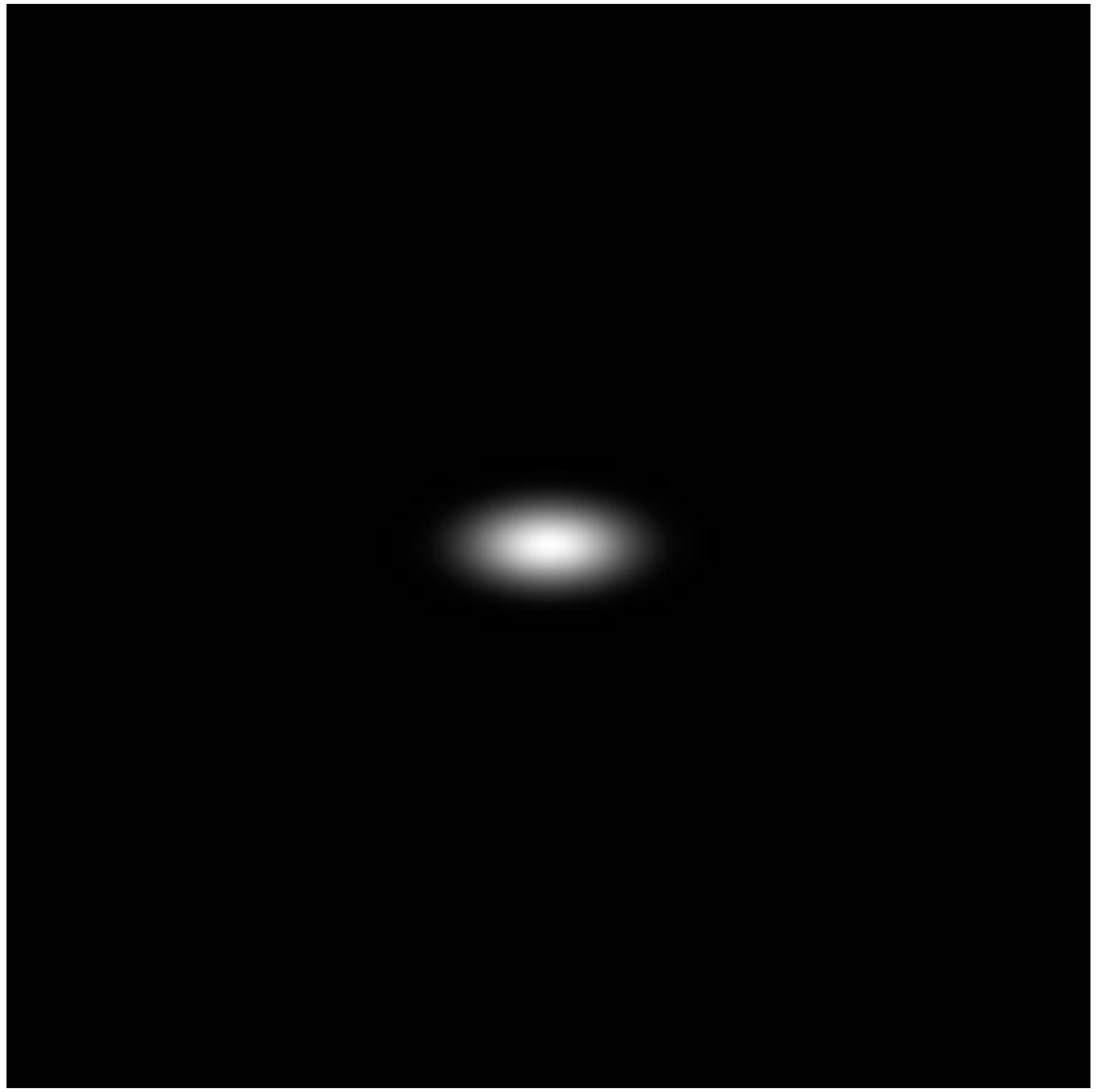
$$\exp(-r^2/16^2)$$

Why does a "smoother" exponential function in the spatial domain look "more compact" in the frequency domain?

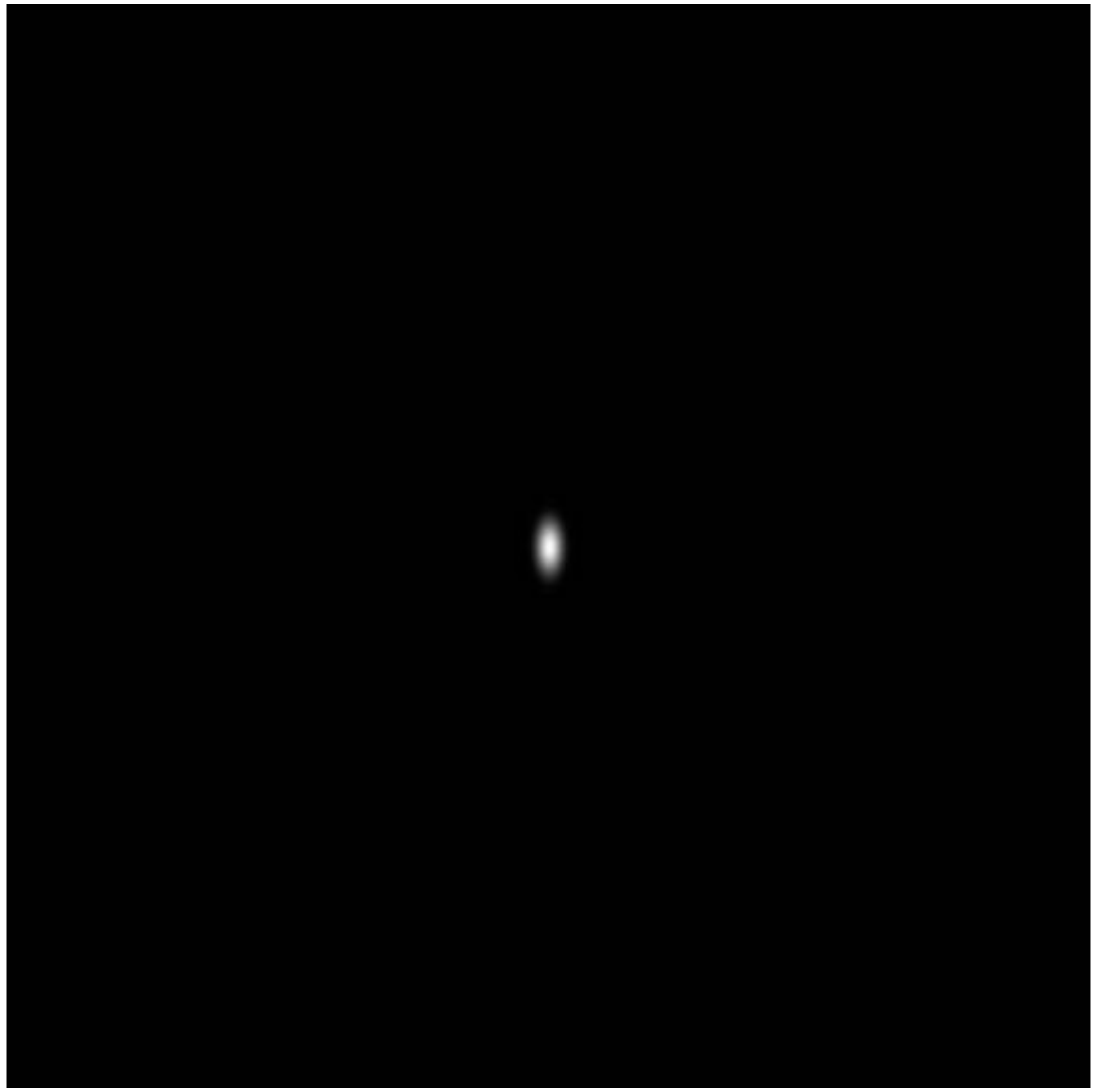
$$\exp(-r^2/32^2)$$



$$\exp(-x^2/32^2) \times \exp(-y^2/16^2)$$



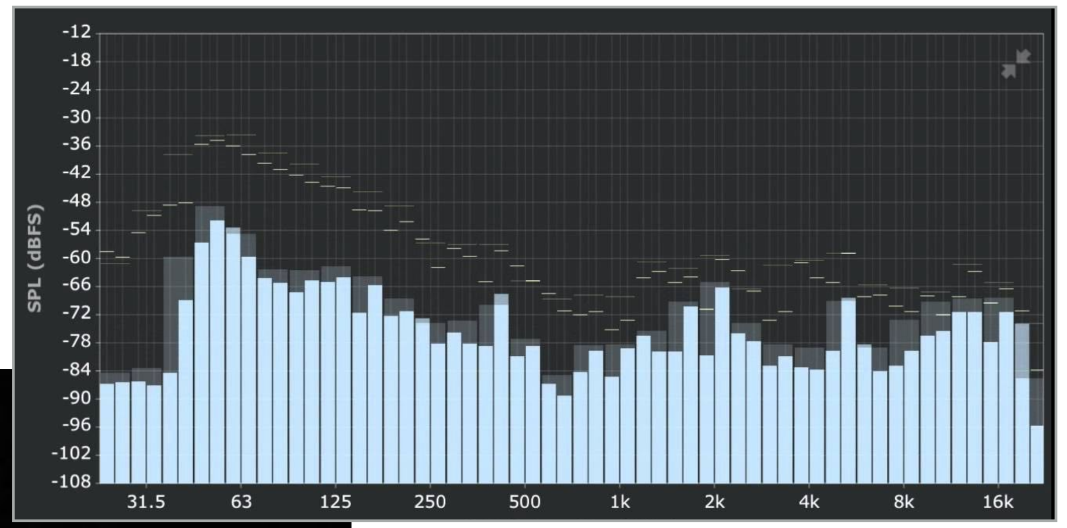
Spatial domain



Frequency domain

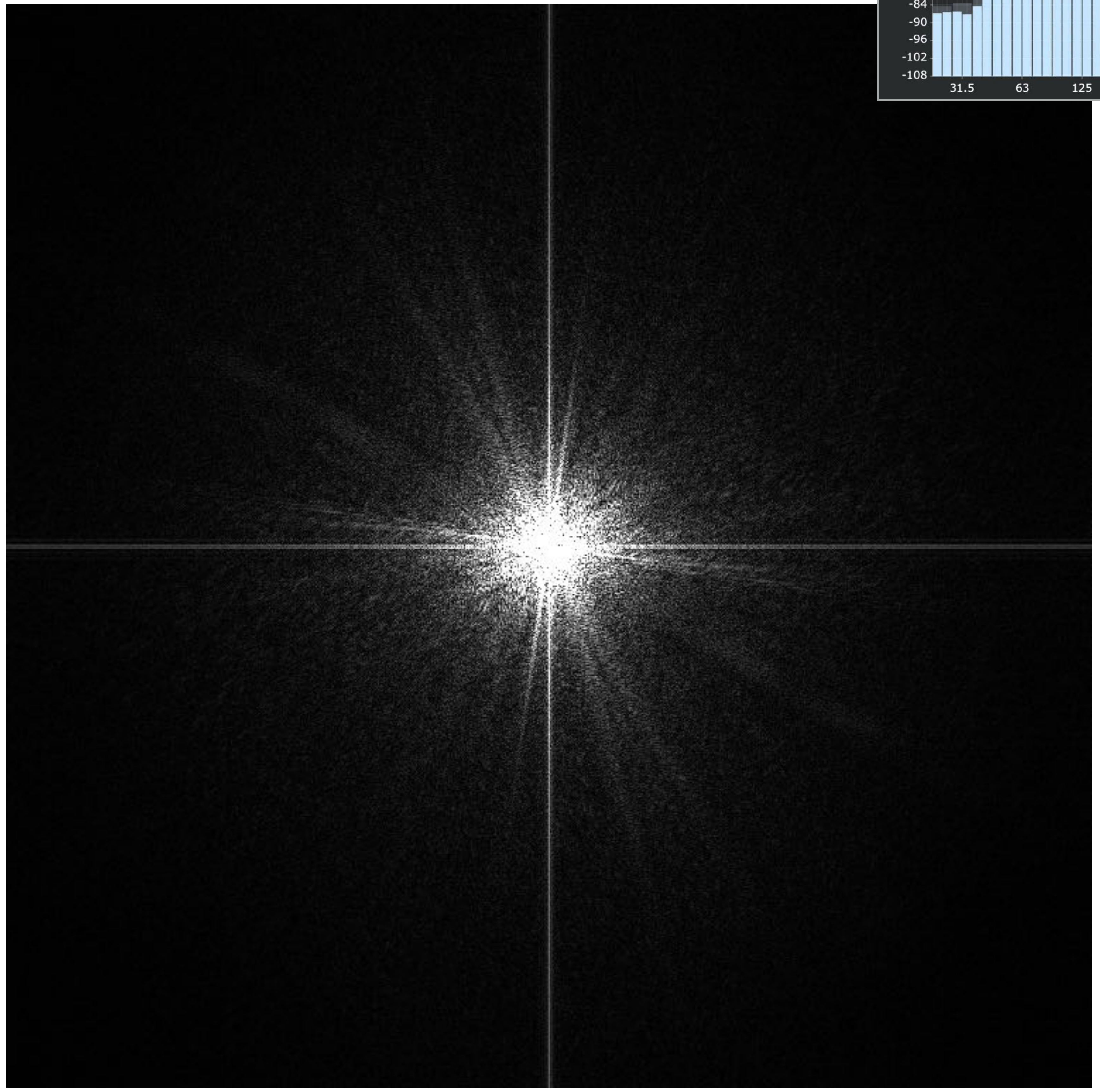
Image filtering
(in the frequency domain)

Manipulating the frequency content of images



The visualization below is the 2D frequency domain equivalent of the 1D audio spectrum I showed you earlier *

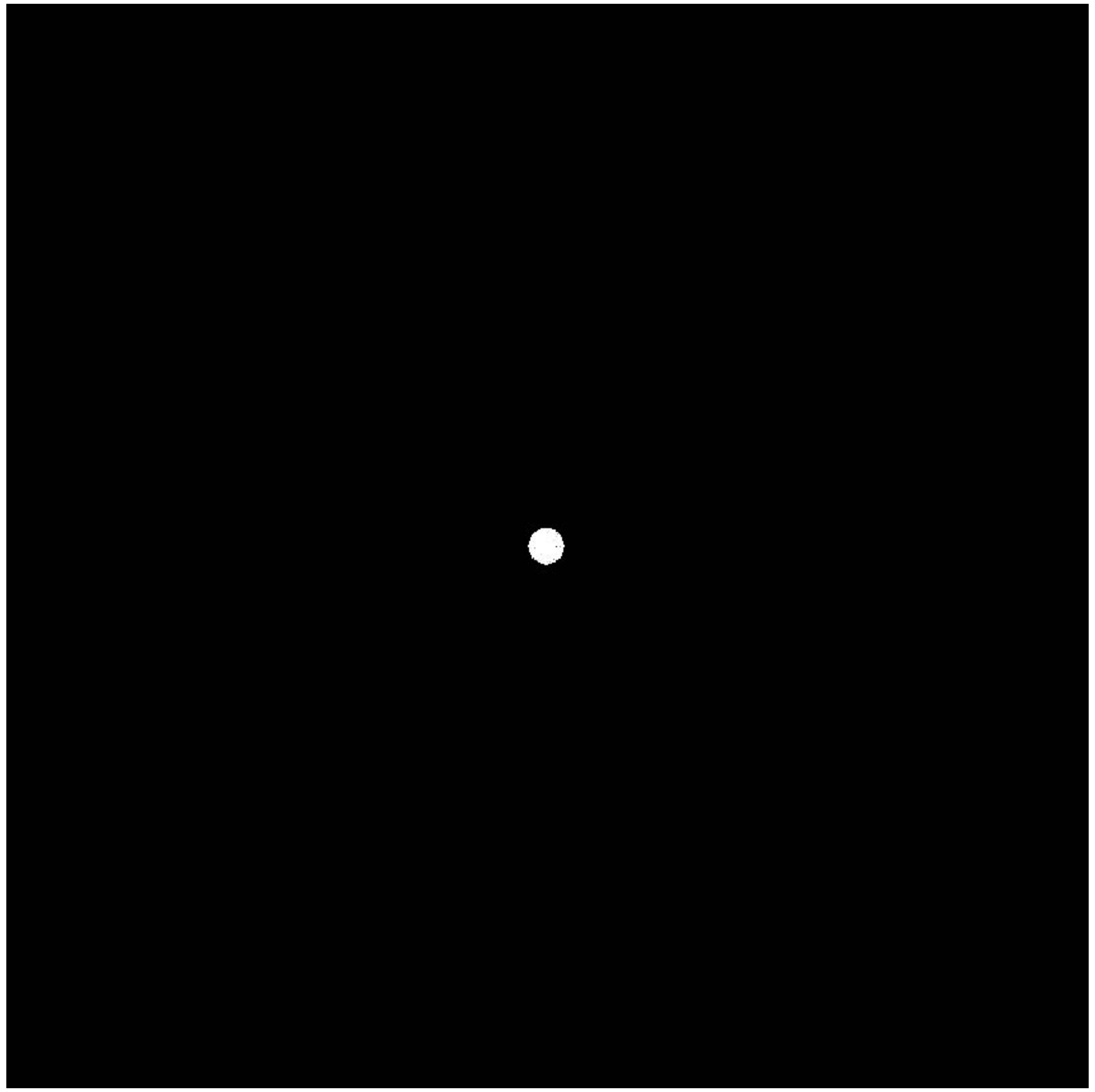
Spatial domain



Frequency domain

Low frequencies only (smooth gradients)

Spatial domain



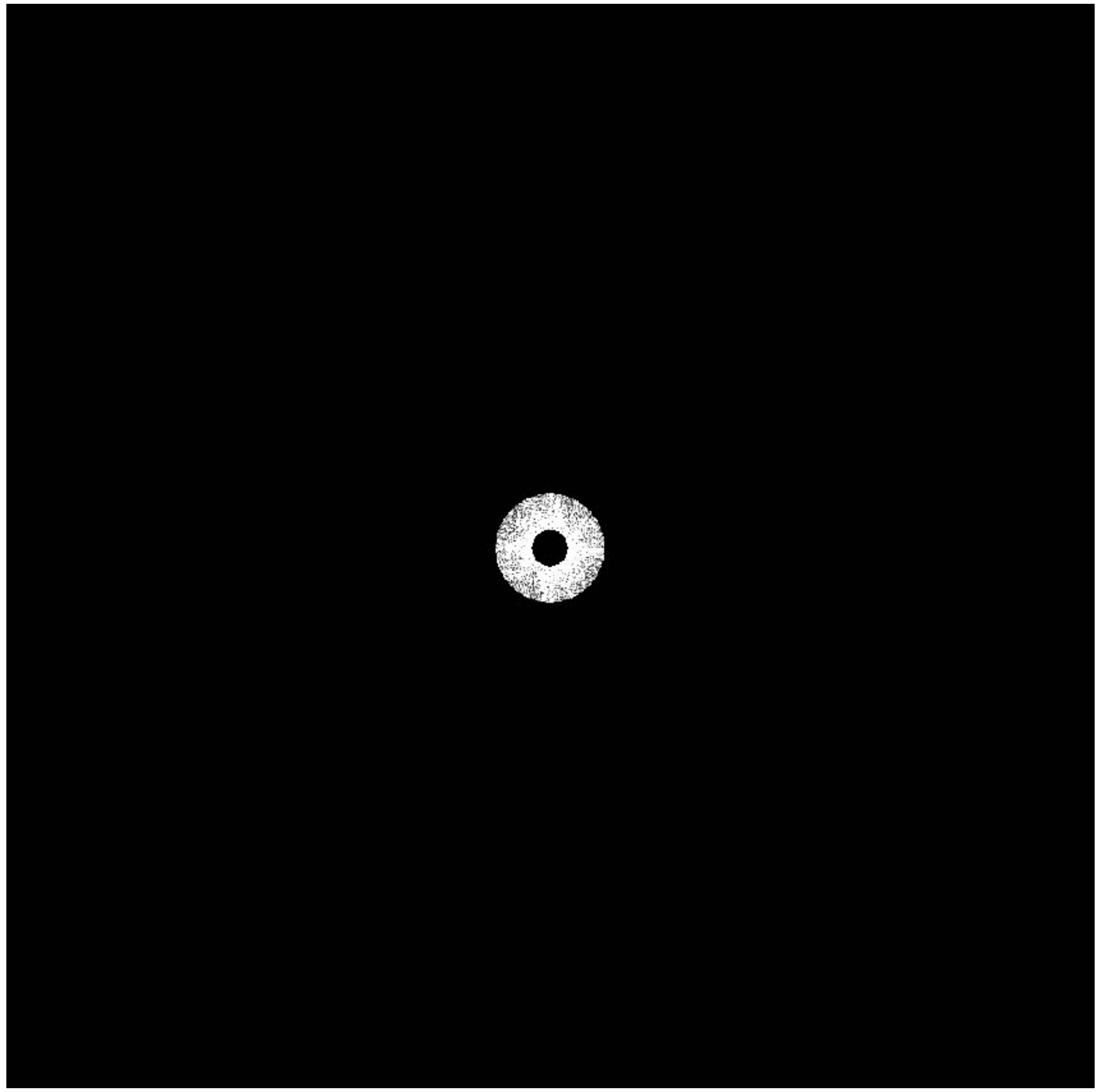
Frequency domain

(after low-pass filter)

All frequencies above cutoff have 0 magnitude

Mid-range frequencies

Spatial domain

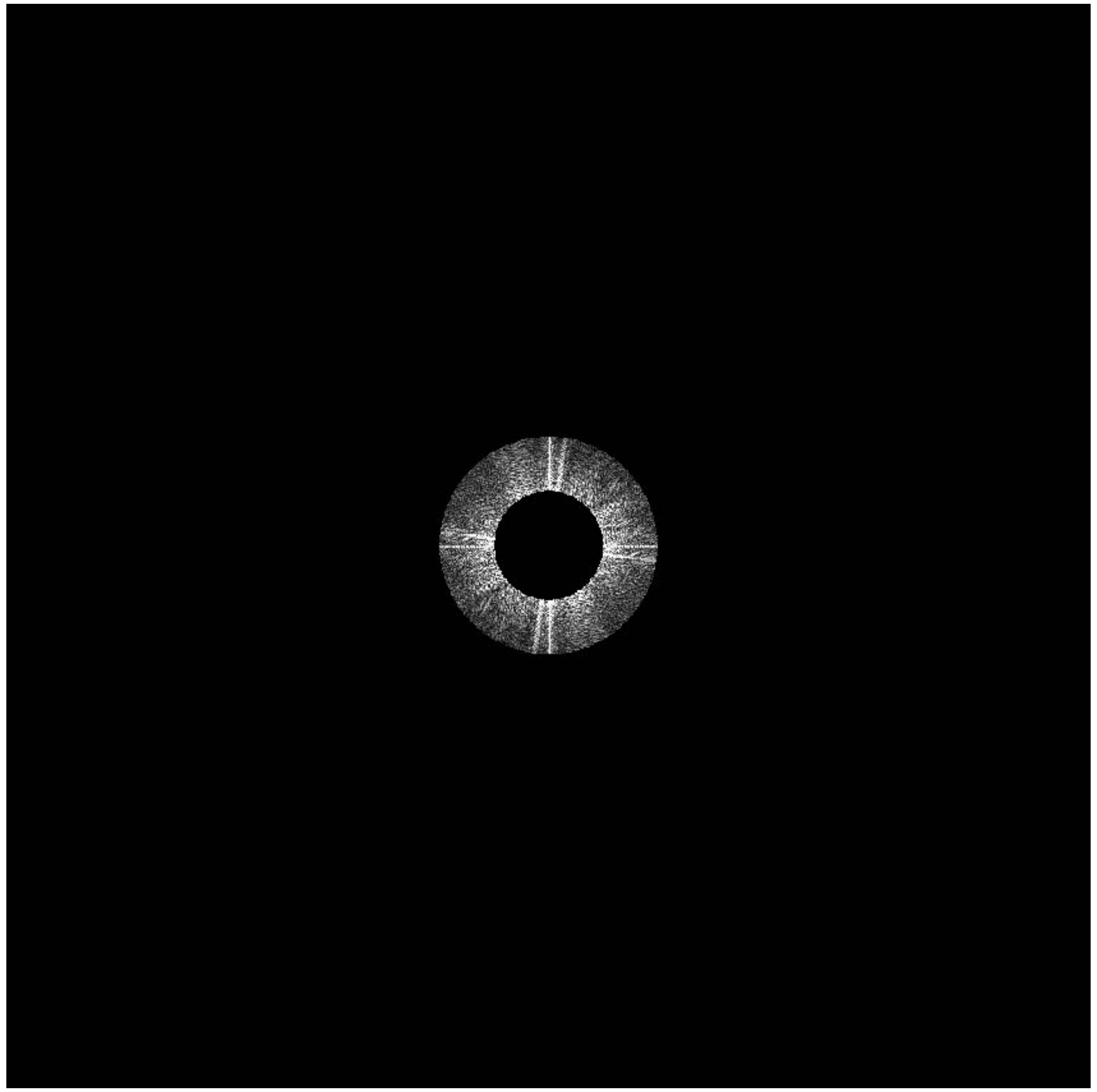


Frequency domain

(after band-pass filter)

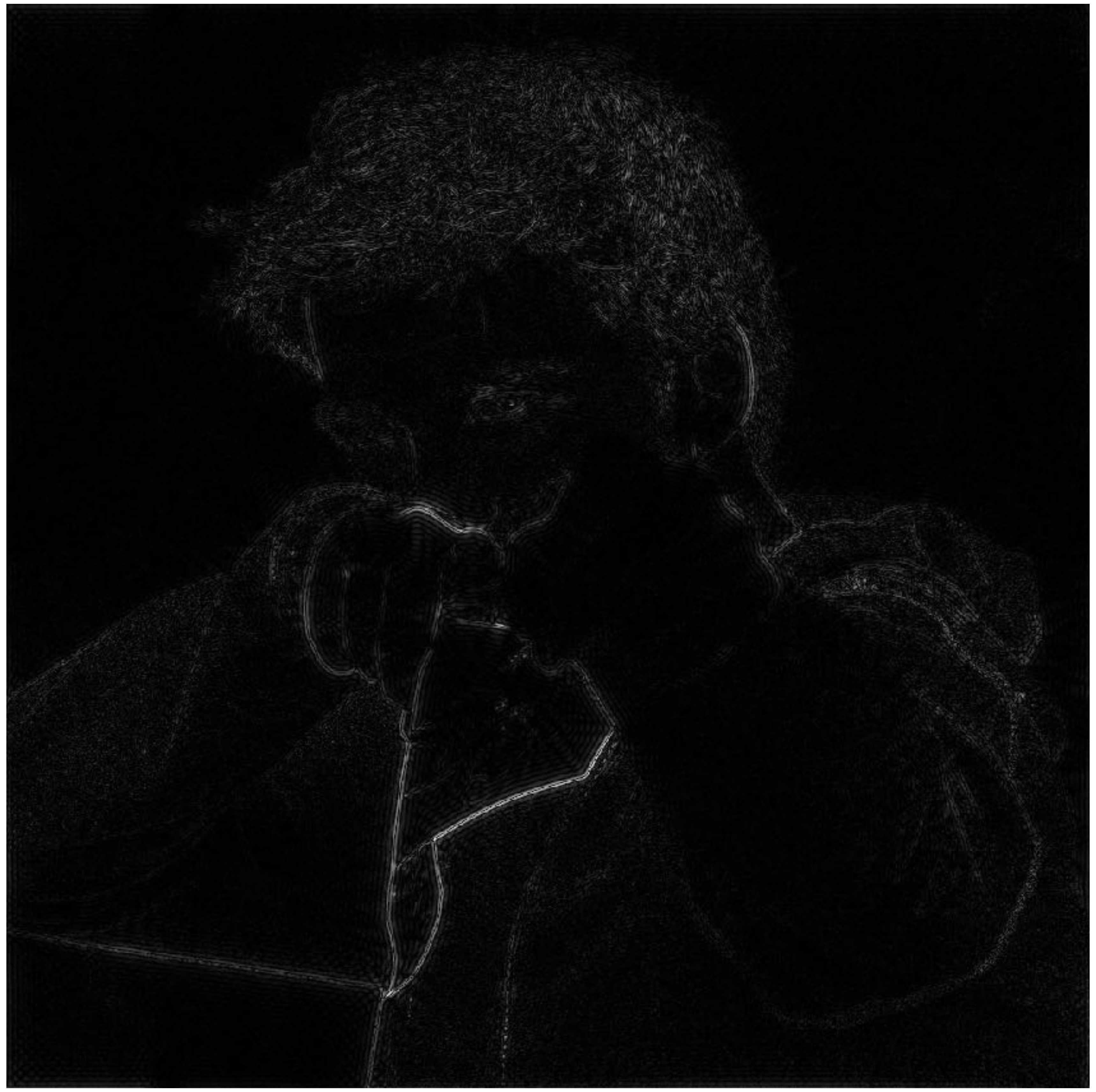
Mid-range frequencies

Spatial domain

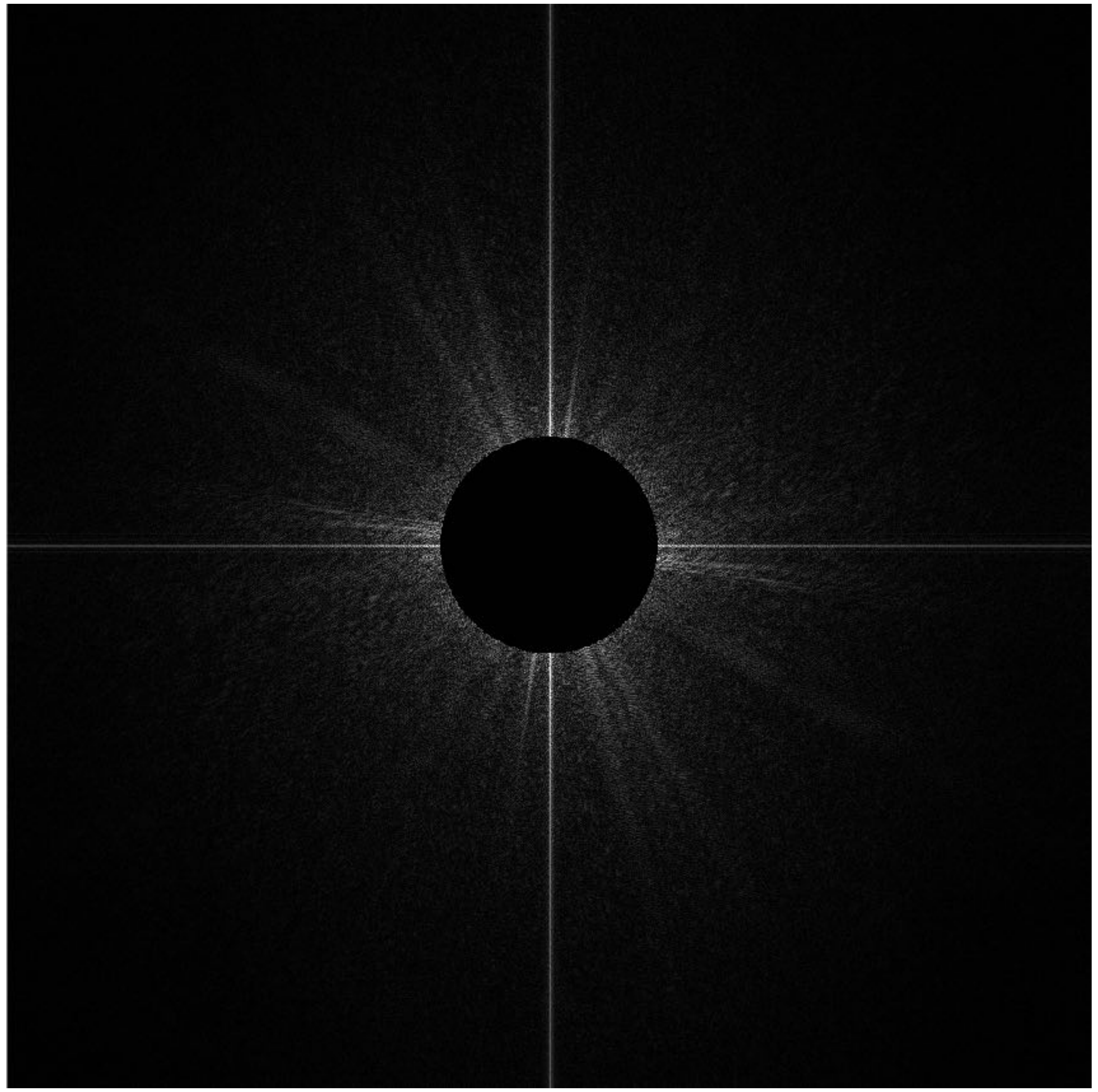


Frequency domain
(after band-pass filter)

High frequencies (edges)

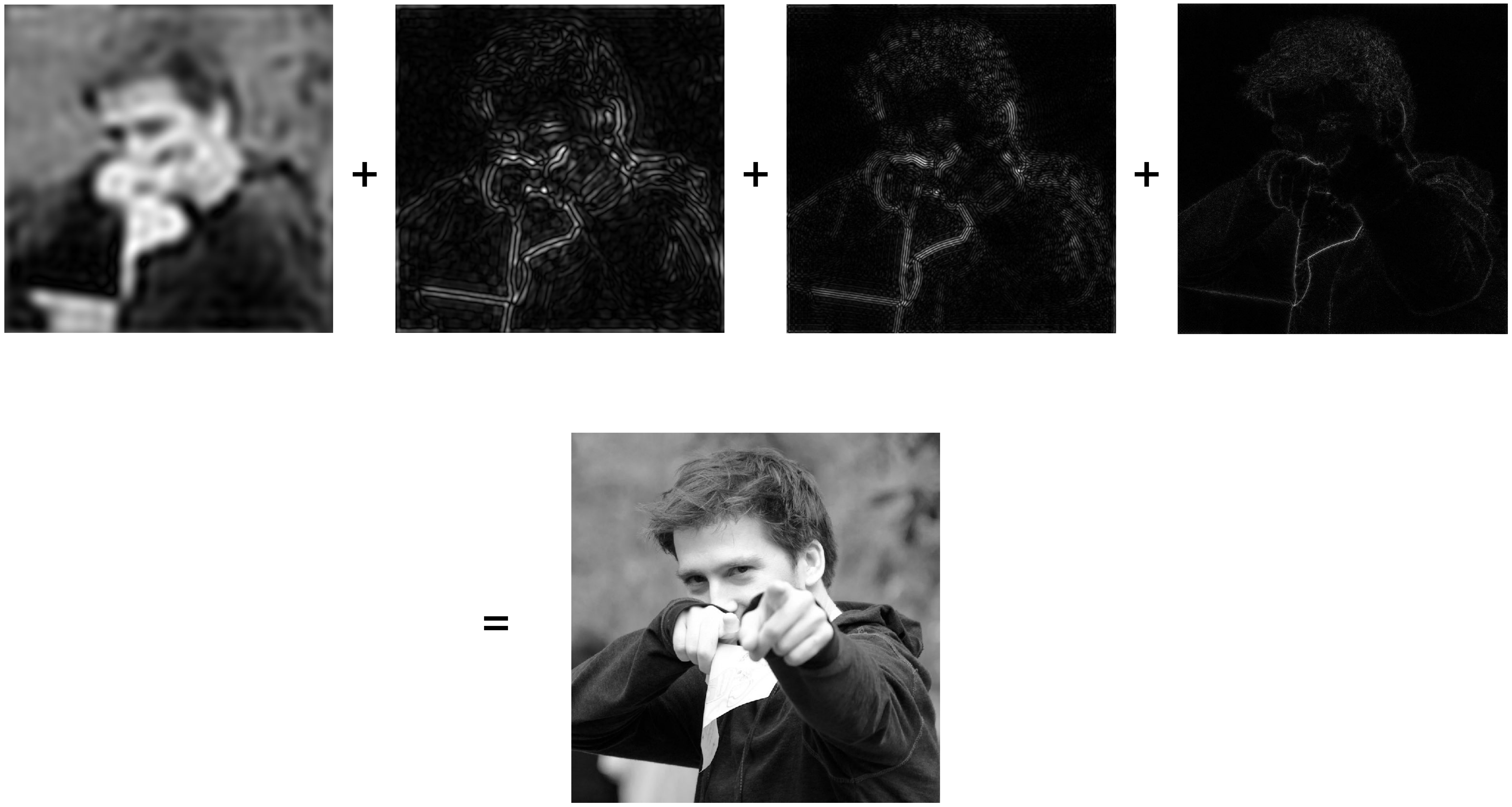


Spatial domain
(strongest edges)

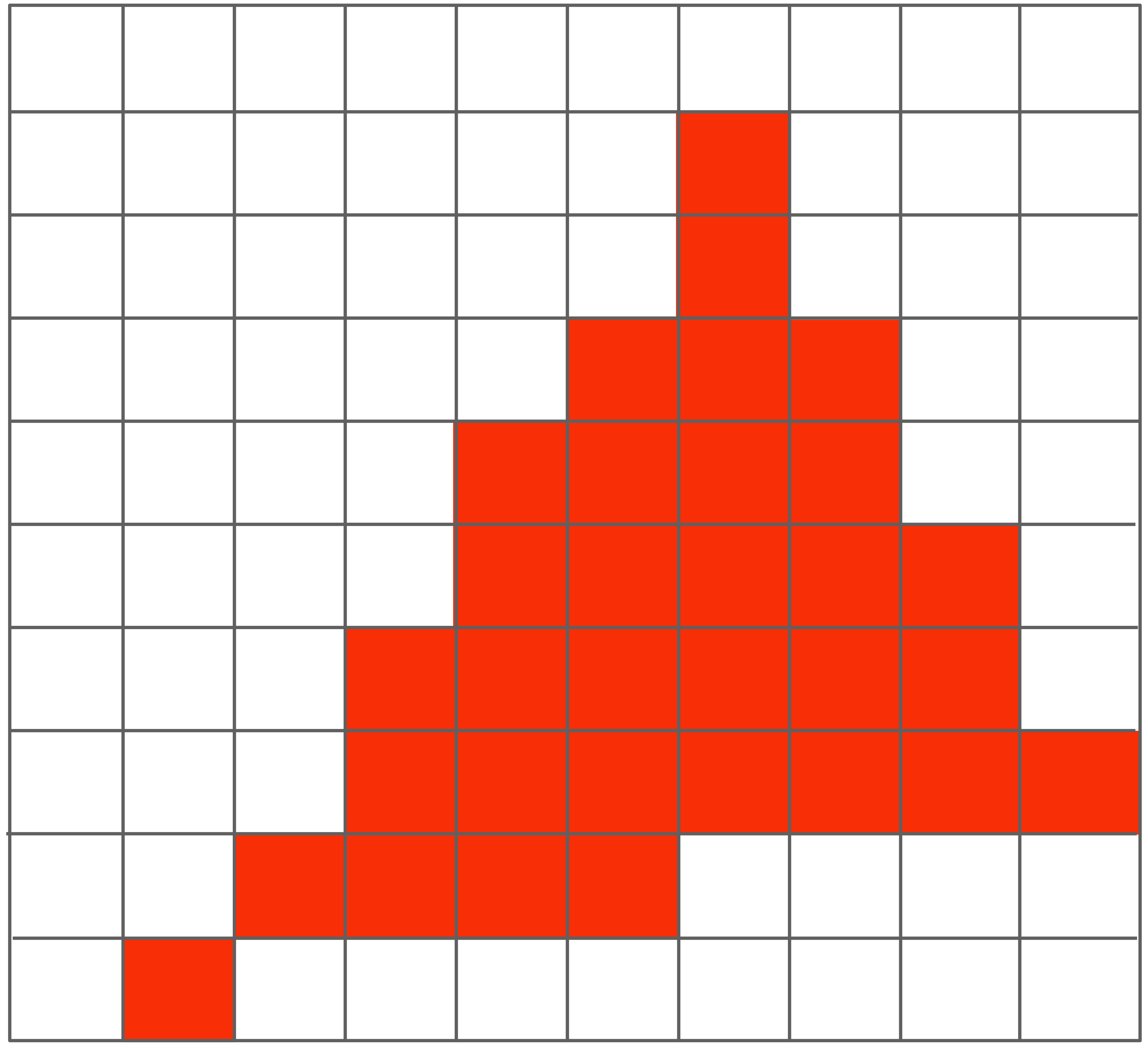


Frequency domain
(after high-pass filter)
All frequencies below threshold have 0 magnitude

An image as a sum of its frequency components

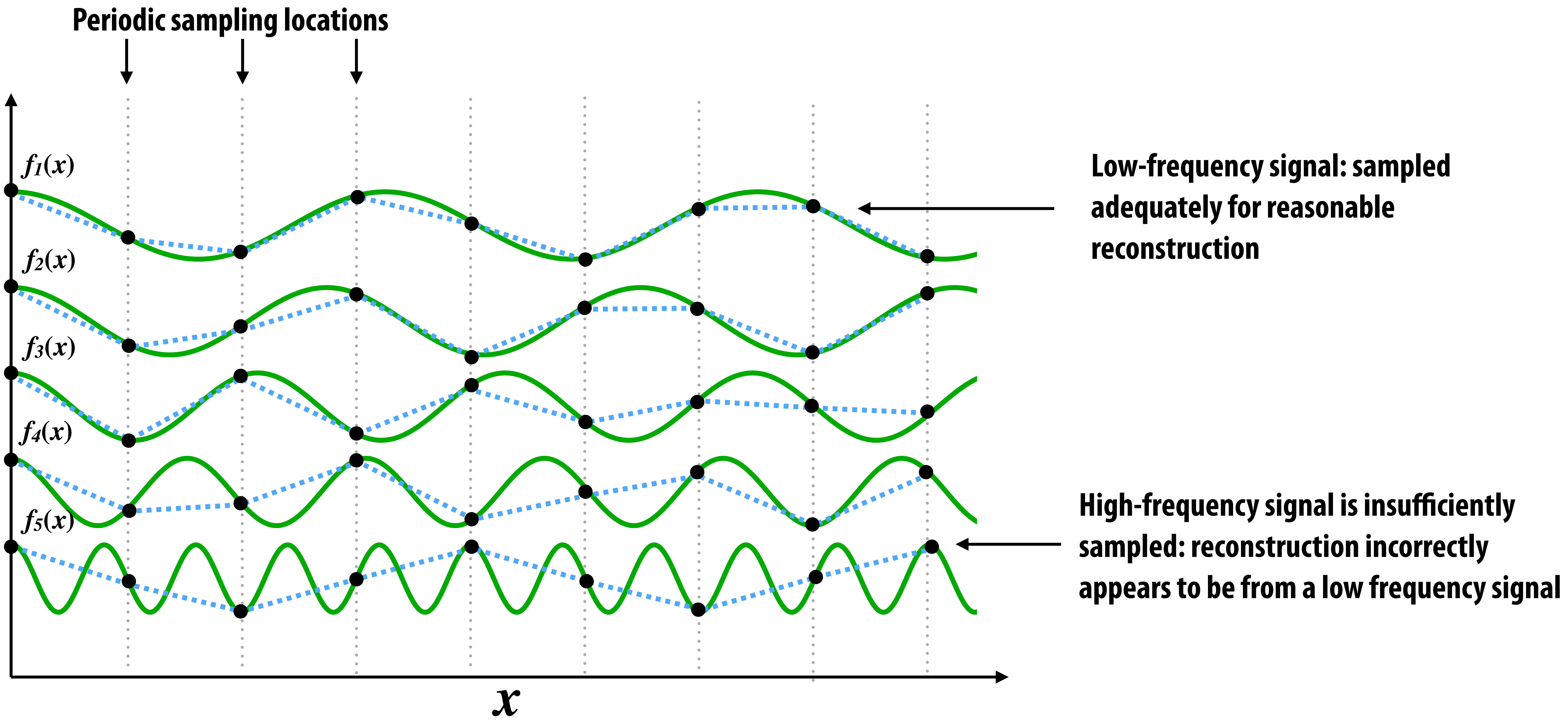


Back to our problem of artifacts in images

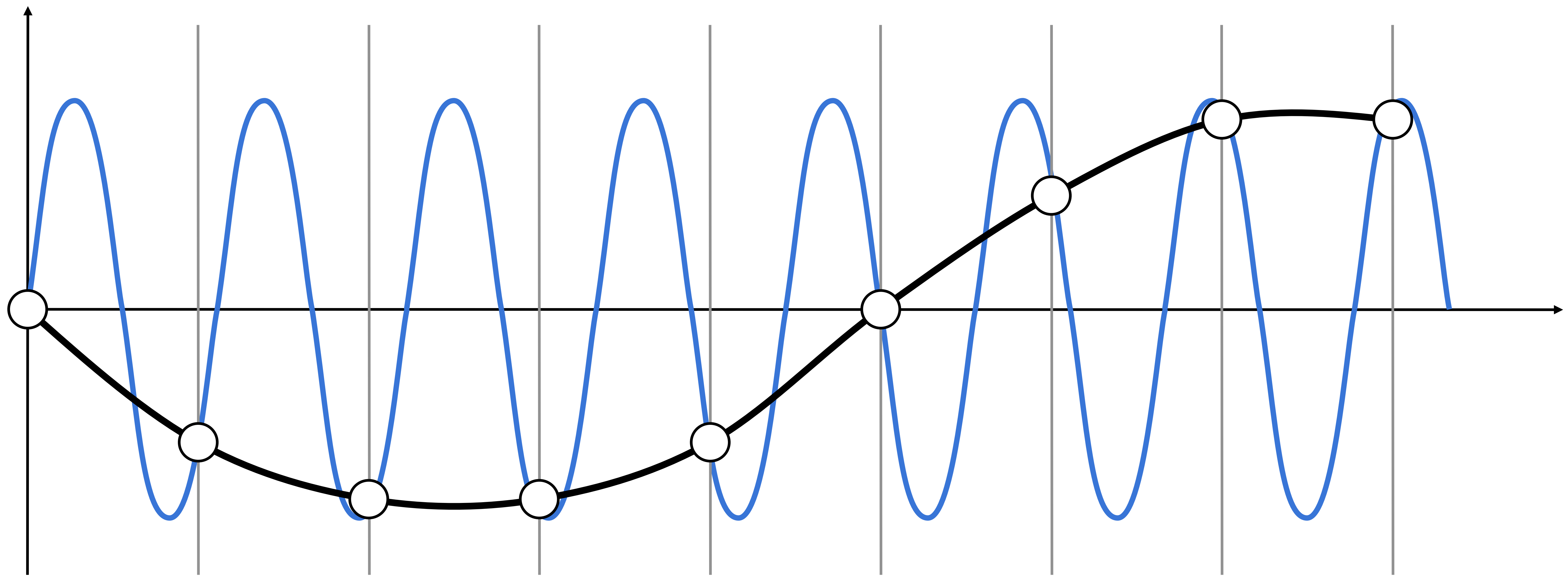


Jaggies!

Higher frequencies need denser sampling



Undersampling creates frequency "aliases"

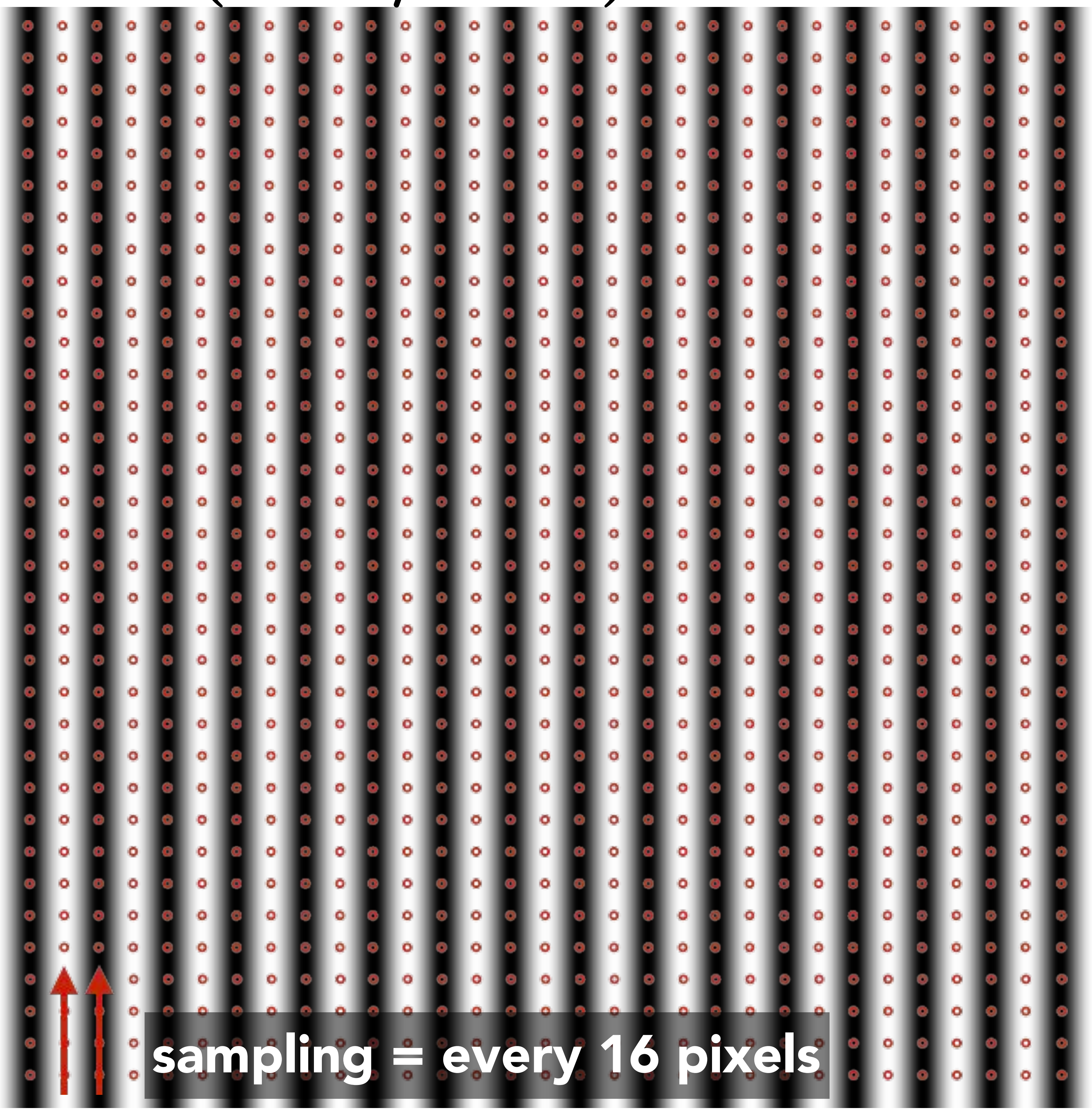


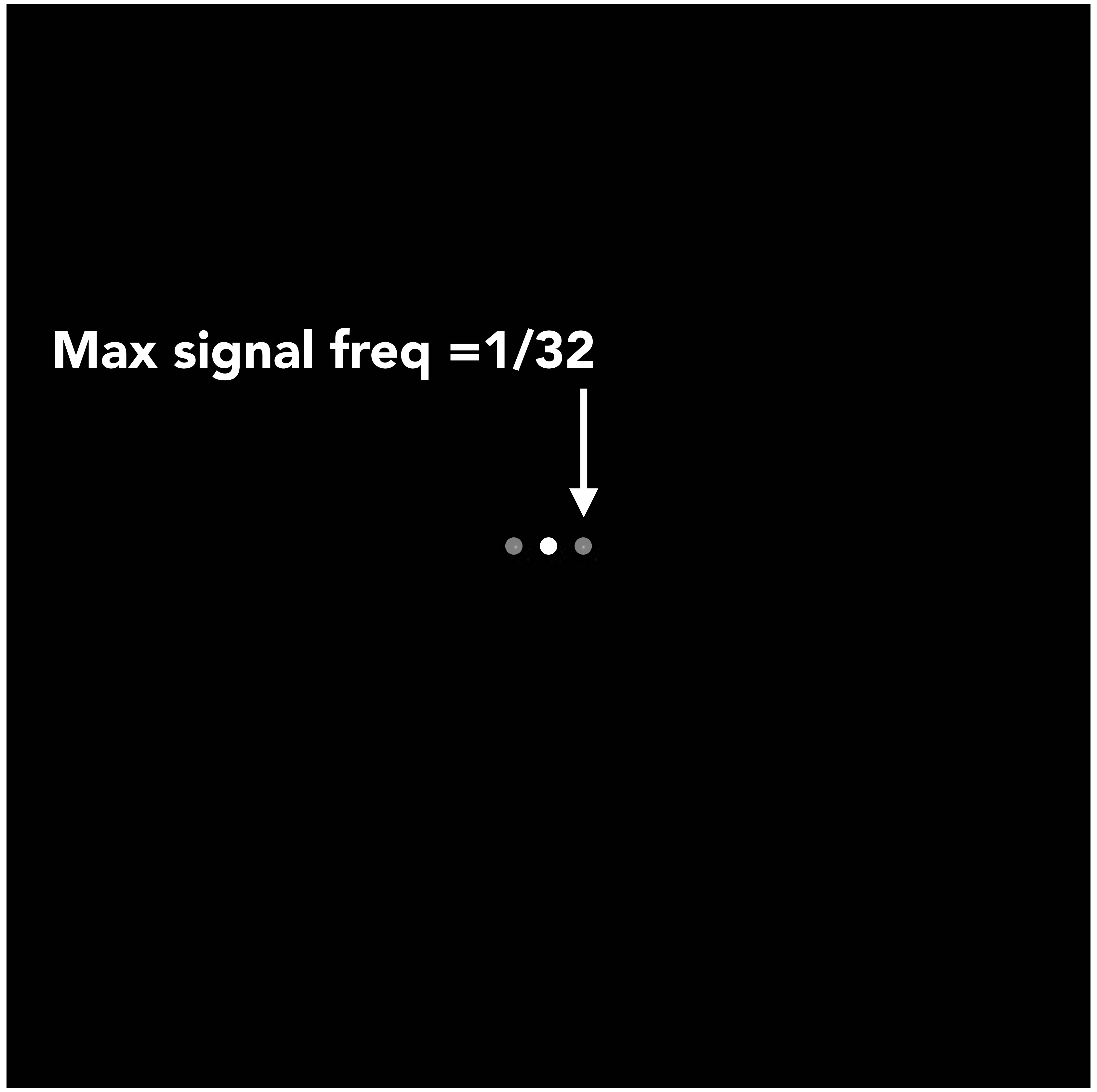
High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

Example: sampling rate vs signal frequency

$\sin(2\pi/32)x$ — **frequency 1/32; 32 pixels per cycle**





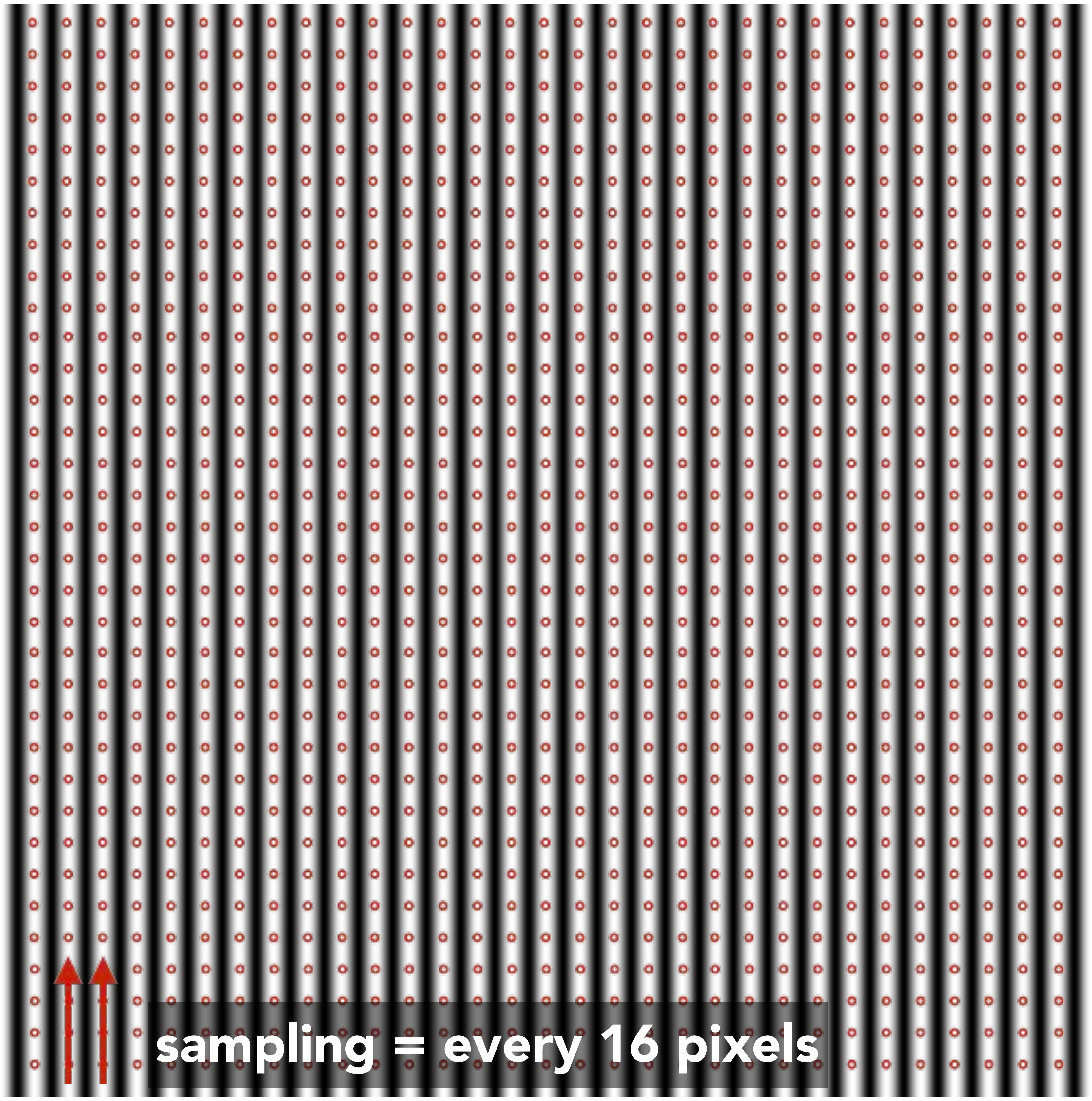
Spatial domain **Frequency domain**

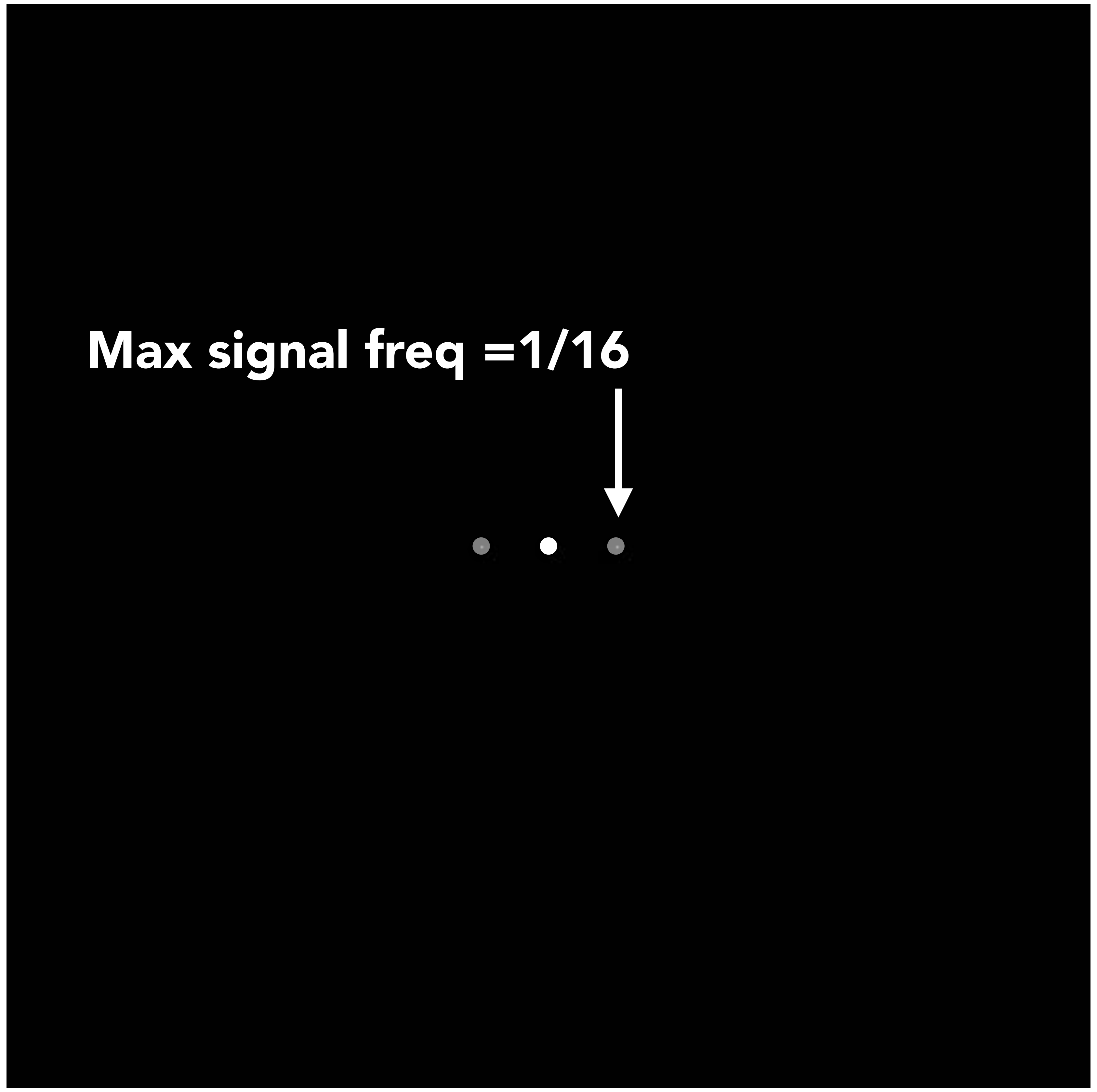
Sampling at twice the frequency of the signal: no aliasing! *

* Technically in this example there is no "pre-aliasing". There is "post-aliasing" if reconstruction from these measurements is not perfect

Example: sampling rate vs signal frequency

$\sin(2\pi/16)x$ — **frequency 1/16; 16 pixels per cycle**



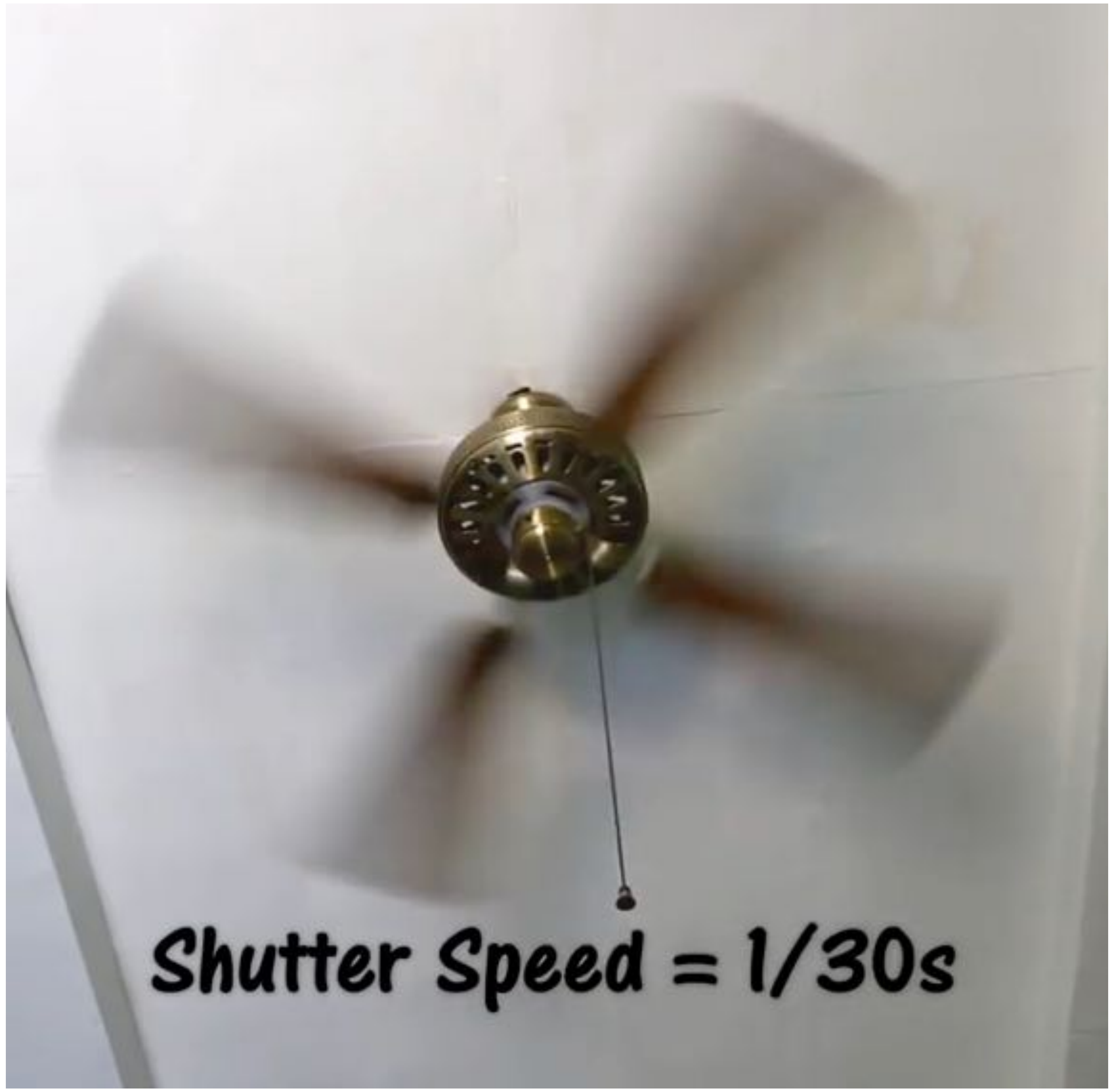


Sampling at same frequency as signal: dramatic aliasing! (due to undersampling)

Anti-aliasing idea:
remove high frequency information from a signal before sampling it

Video: point vs antialiased sampling

Single point in time



Motion blurred

Video: point sampling in time

Credit: Aris & cams youtube, https://youtu.be/NoWwxTktoFs

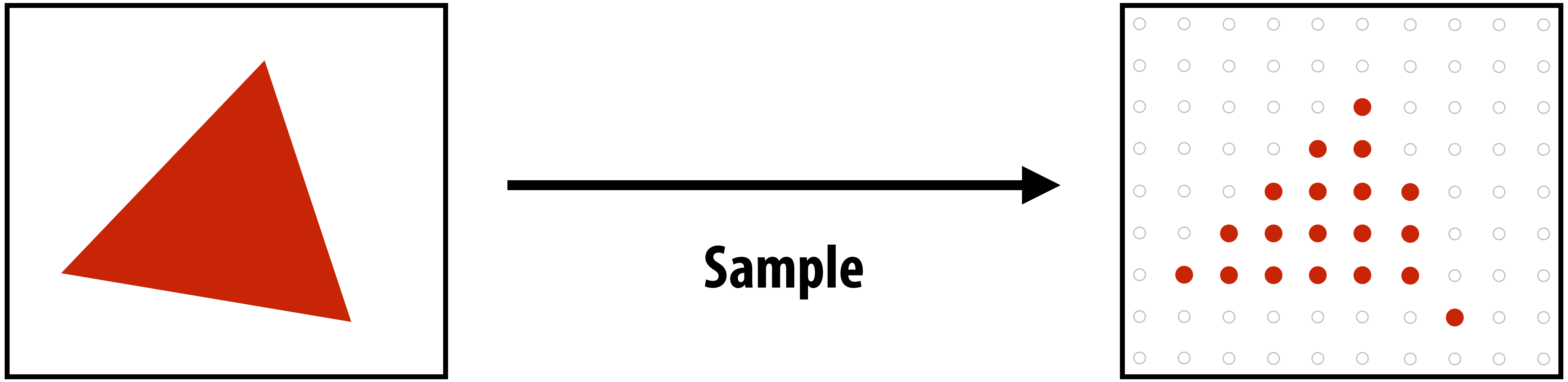
30 fps video. 1/800 second exposure is sharp in time, causes time aliasing.

Video: motion-blurred sampling

Credit: Aris & cams youtube, https://youtu.be/NoWwxTktoFs

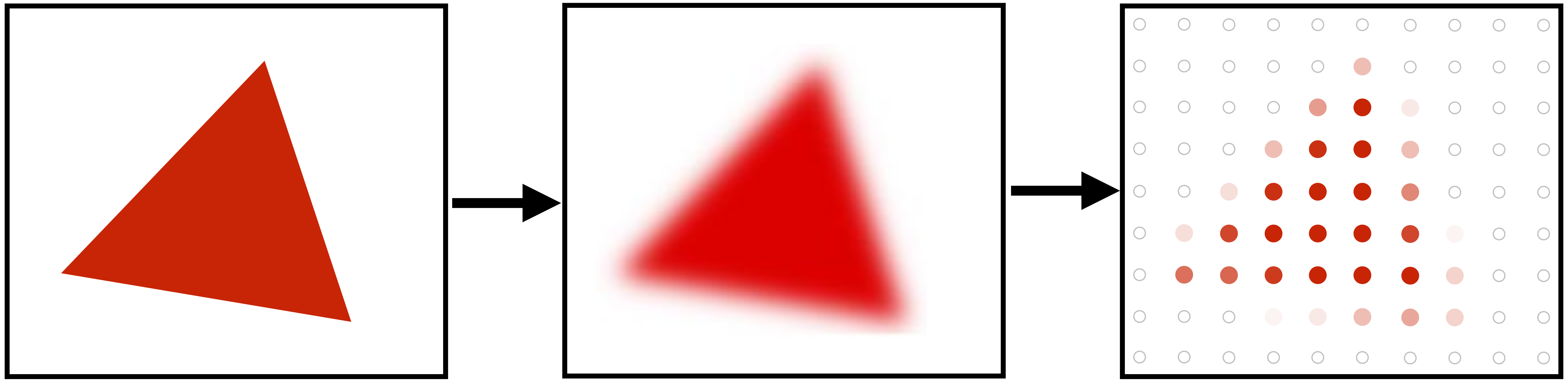
30 fps video. 1/30 second exposure is motion-blurred in time, reduces aliasing.

Rasterization is sampling in 2D space



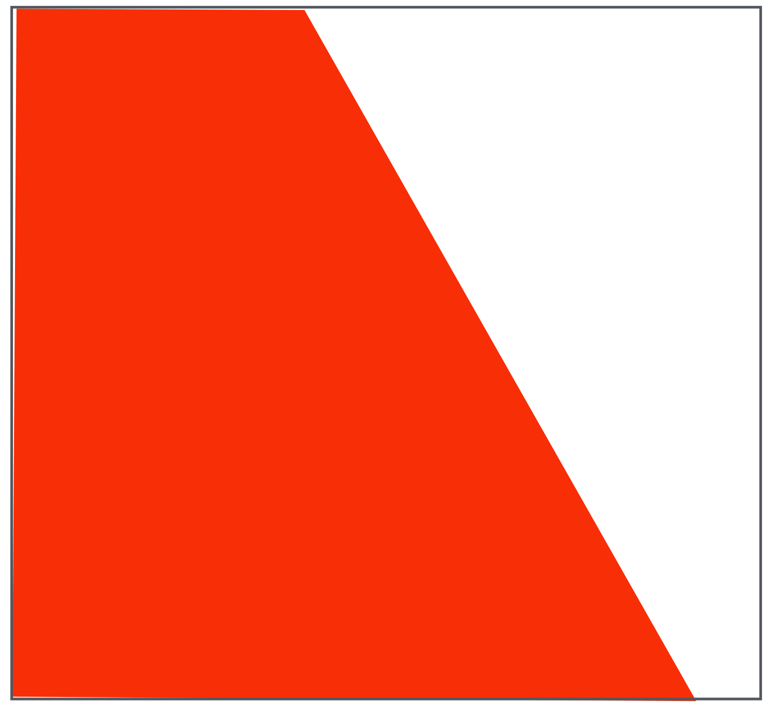
Note jaggies in rasterized triangle
(pixel values are either red or white: sample is in or out of triangle)

Anti-aliasing by pre-filtering the signal

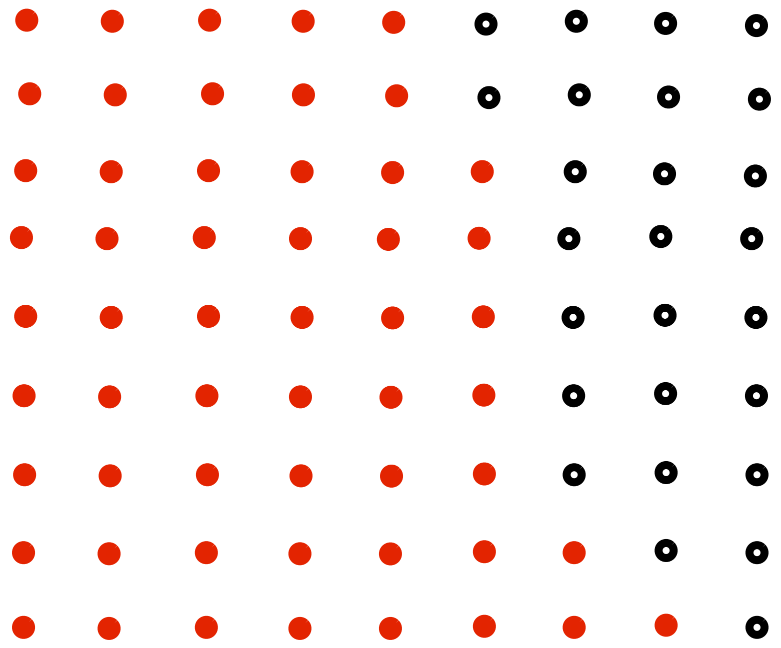


Note anti-aliased edges of rasterized triangle:
pixel values take intermediate values

Pre-filtering by "supersampling" then "blurring" (averaging)

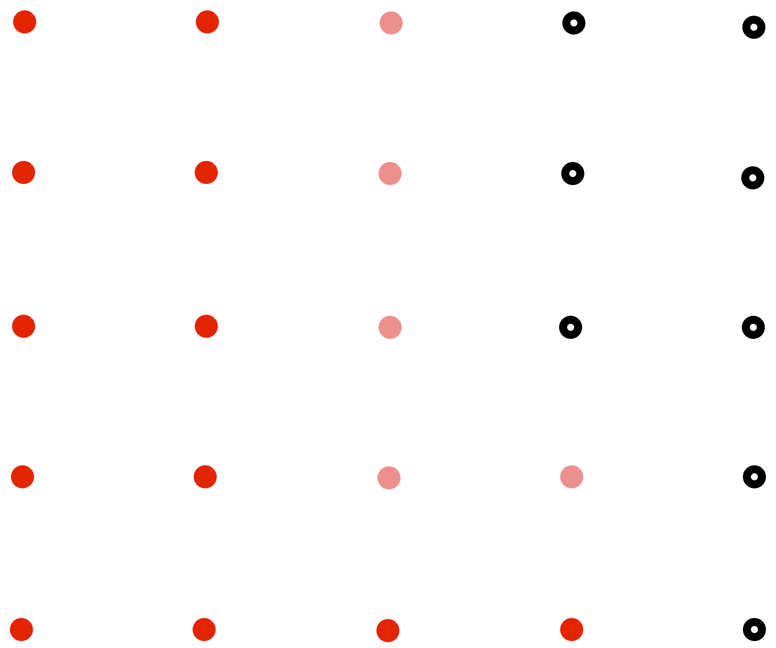


Original signal
(with high frequency edge)

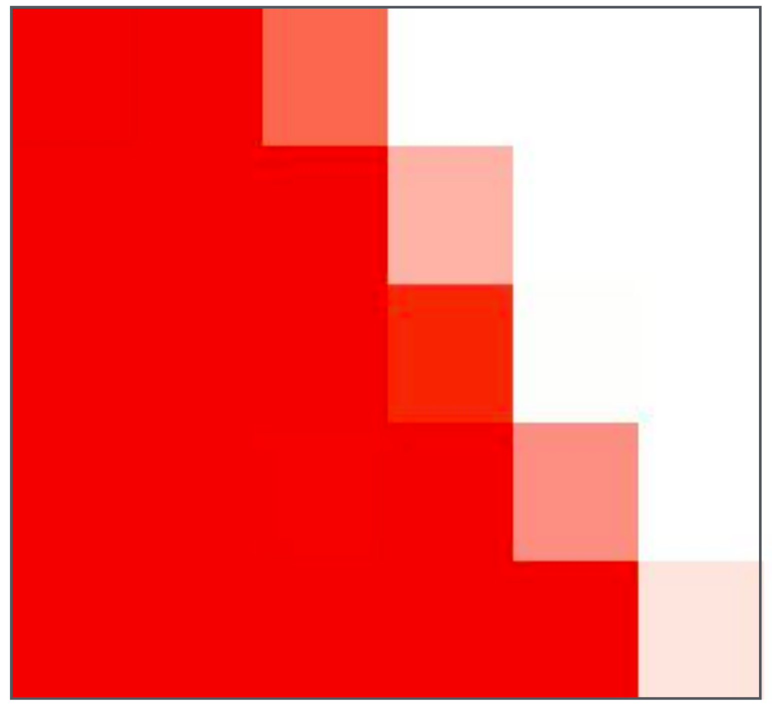


Dense sampling of signal
(supersampling)

Reconstructed signal with high frequencies reduced
(Blurring via averaging over pixel, etc)

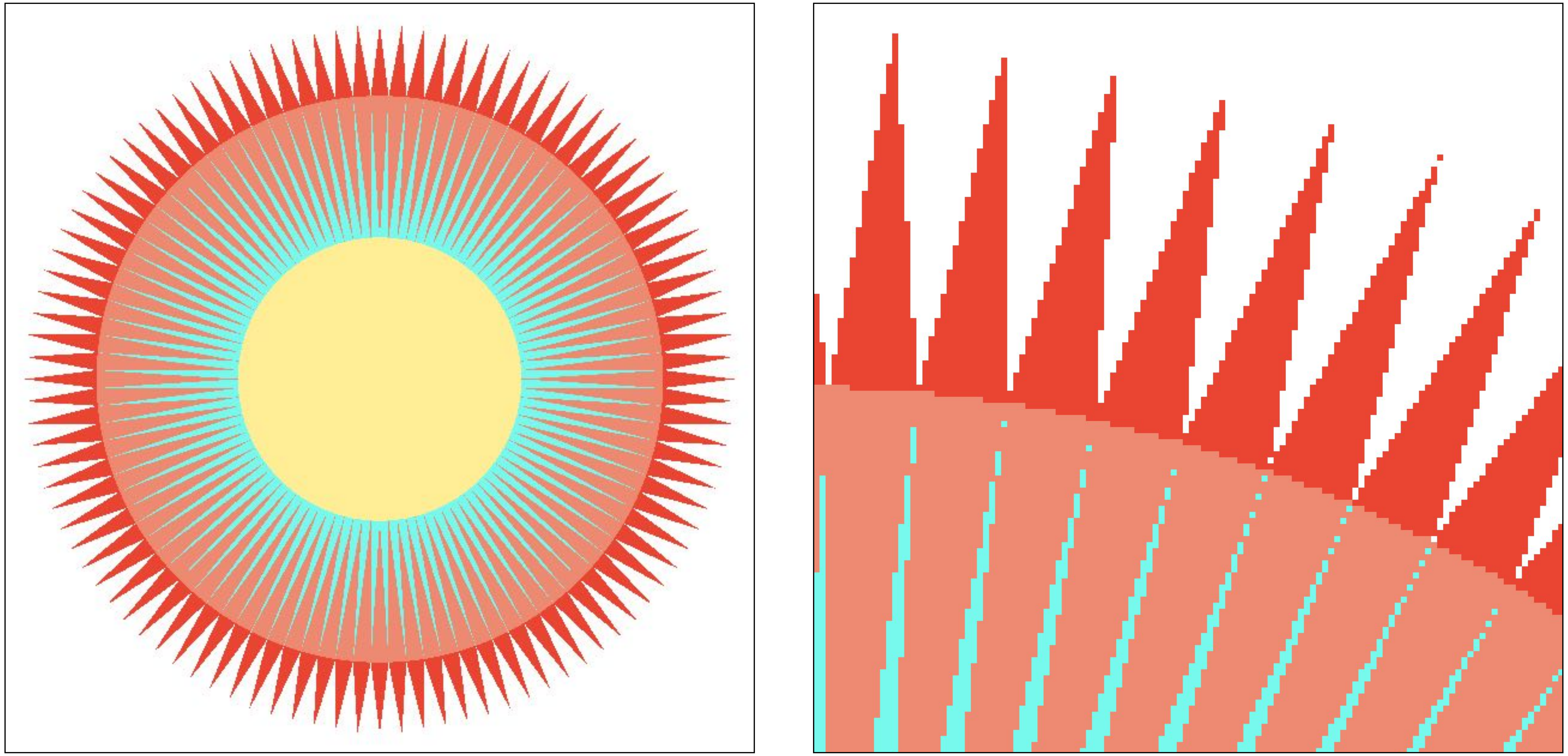


Coarsely sampled signal
(to store in image, or send to display)

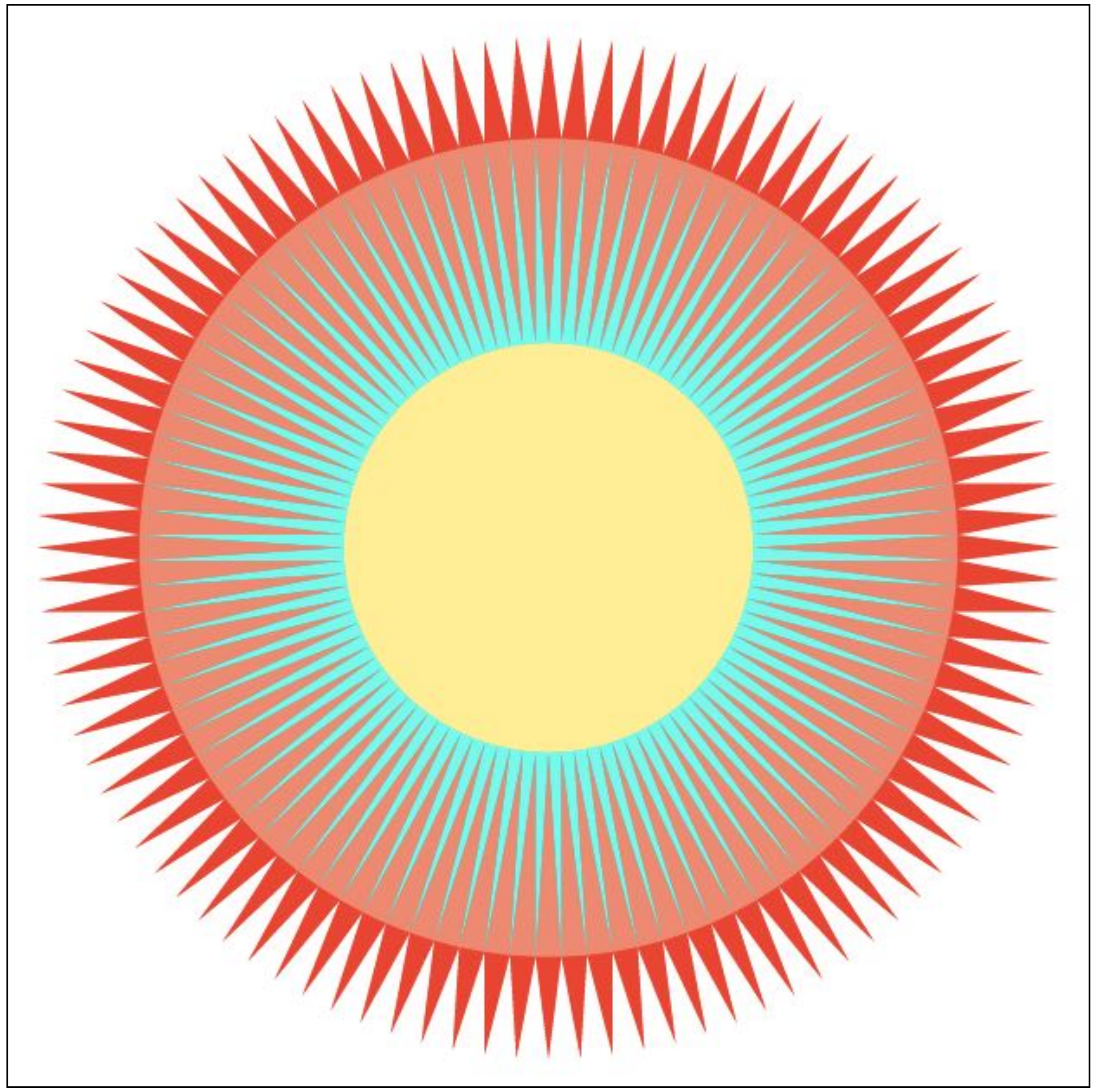


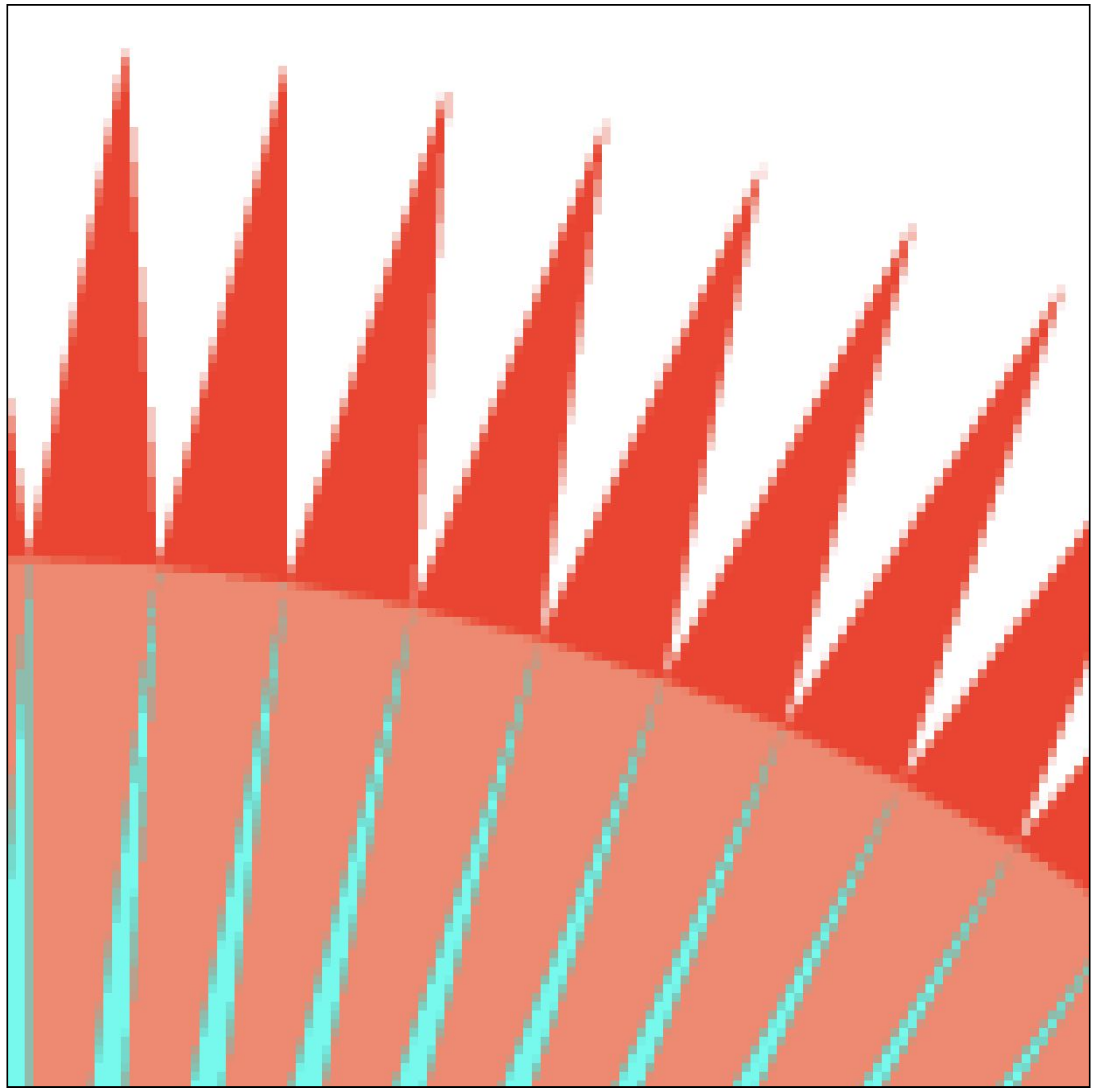
Reconstruction on display

Images rendered using one sample per pixel

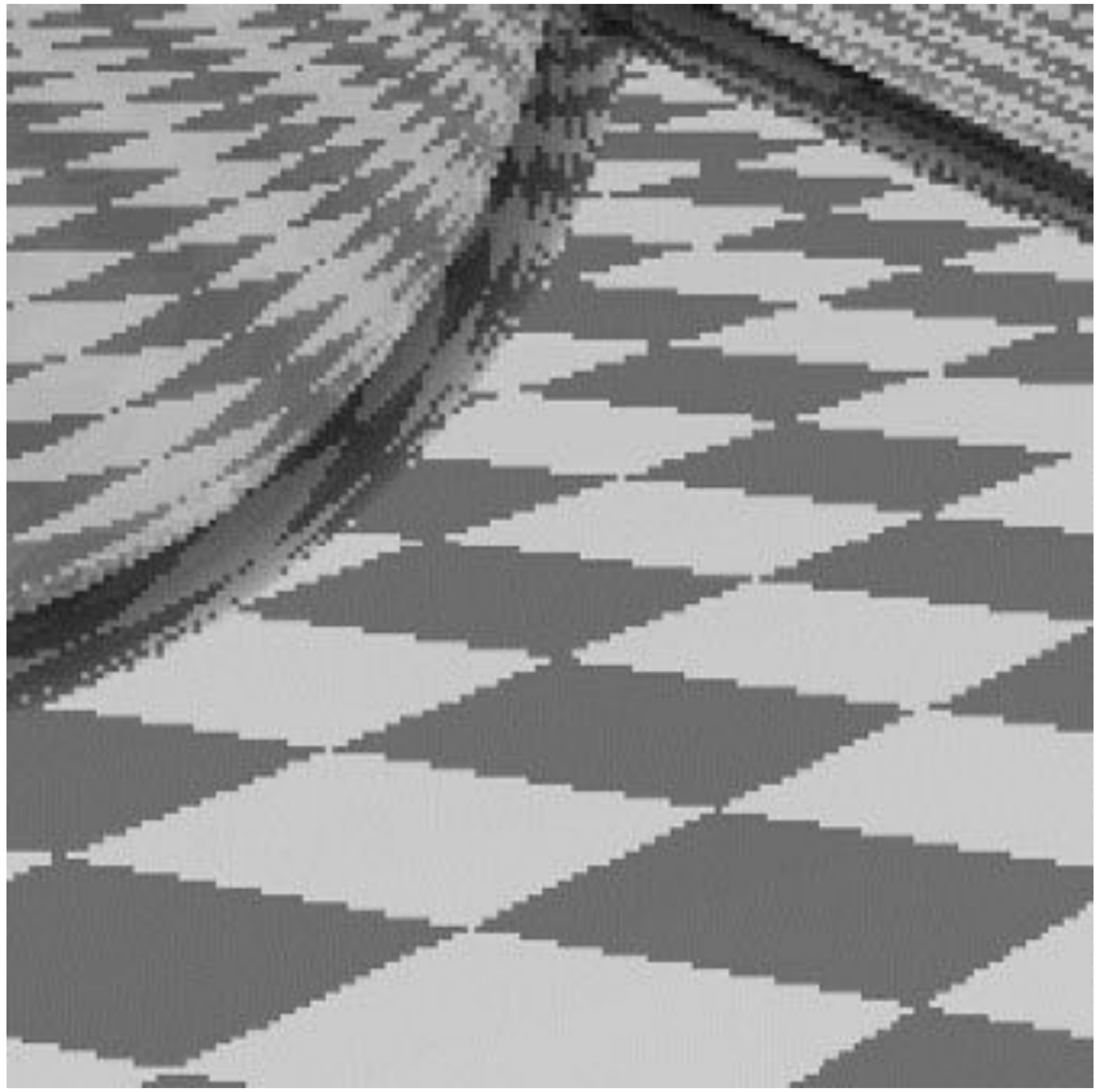


Anti-aliased results

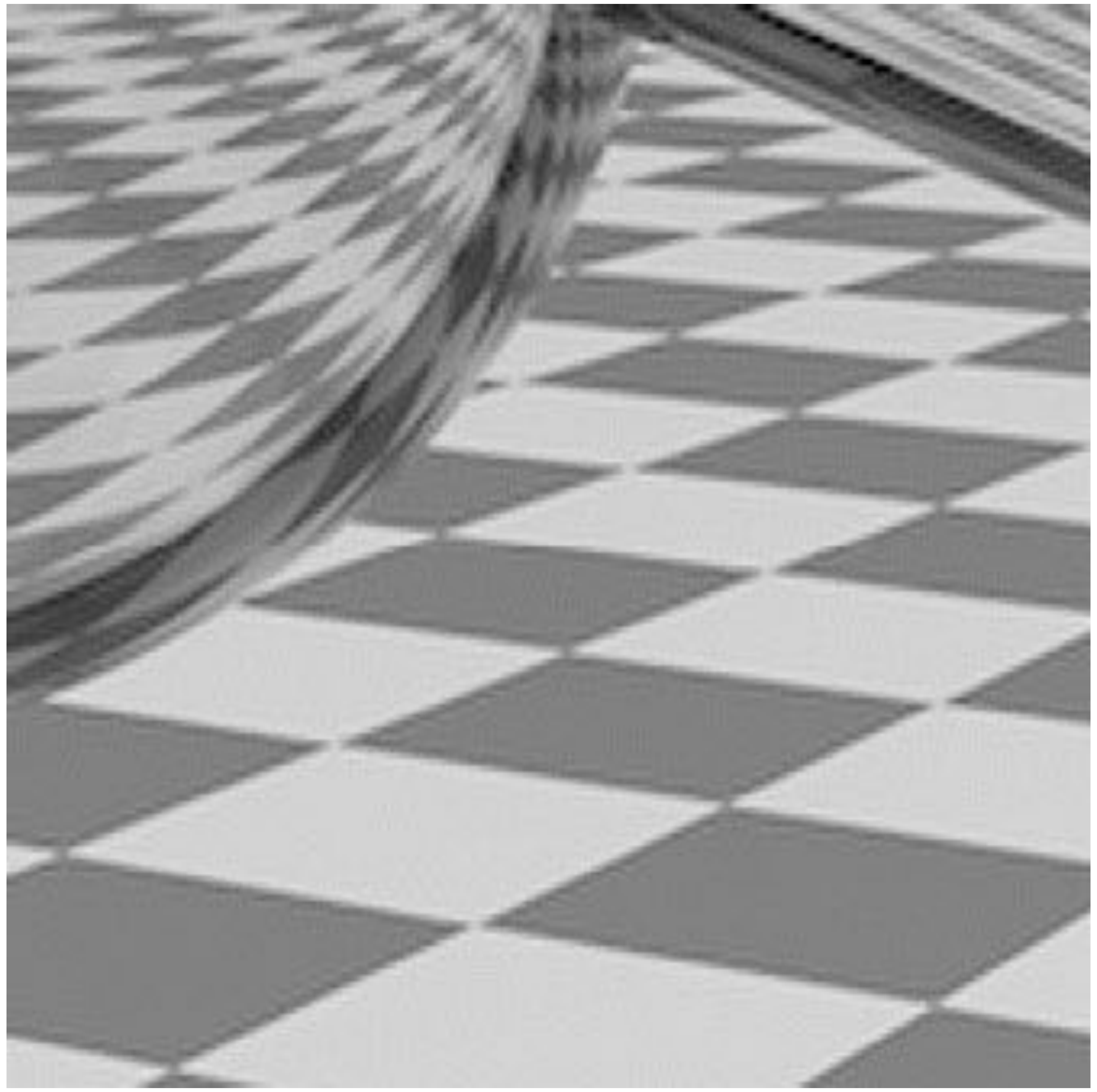




Benefits of anti-aliasing



Jaggies



Pre-filtered

Filtering = convolution

1D convolution

Signal	1	3	5	3	7	1	3	8	6	4

Filter	1	2	1

1D convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1	2	1

1x1 + 3x2 + 5x1 = 12

Result

12									

1D convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1	2	1

3x1 + 5x2 + 3x1 = 16

Result

12	16								

1D convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1	2	1

5x1 + 3x2 + 7x1 = 18

Result

12	16	18							

Box filter (used in a 2D convolution)

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Example: 3x3 box filter

2D convolution with box filter blurs the image

Original image

→

Blurred
(convolve with box filter)

Hmm… this reminds me of a low-pass filter…

Discrete 2D convolution

$$(f * g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j) I(x - i, y - j)$$

output image — $(f * g)(x, y)$

filter — $f(i, j)$

input image — $I(x - i, y - j)$

Consider $f(i, j)$ **that is nonzero only when:** $-1 \le i, j \le 1$

Then:

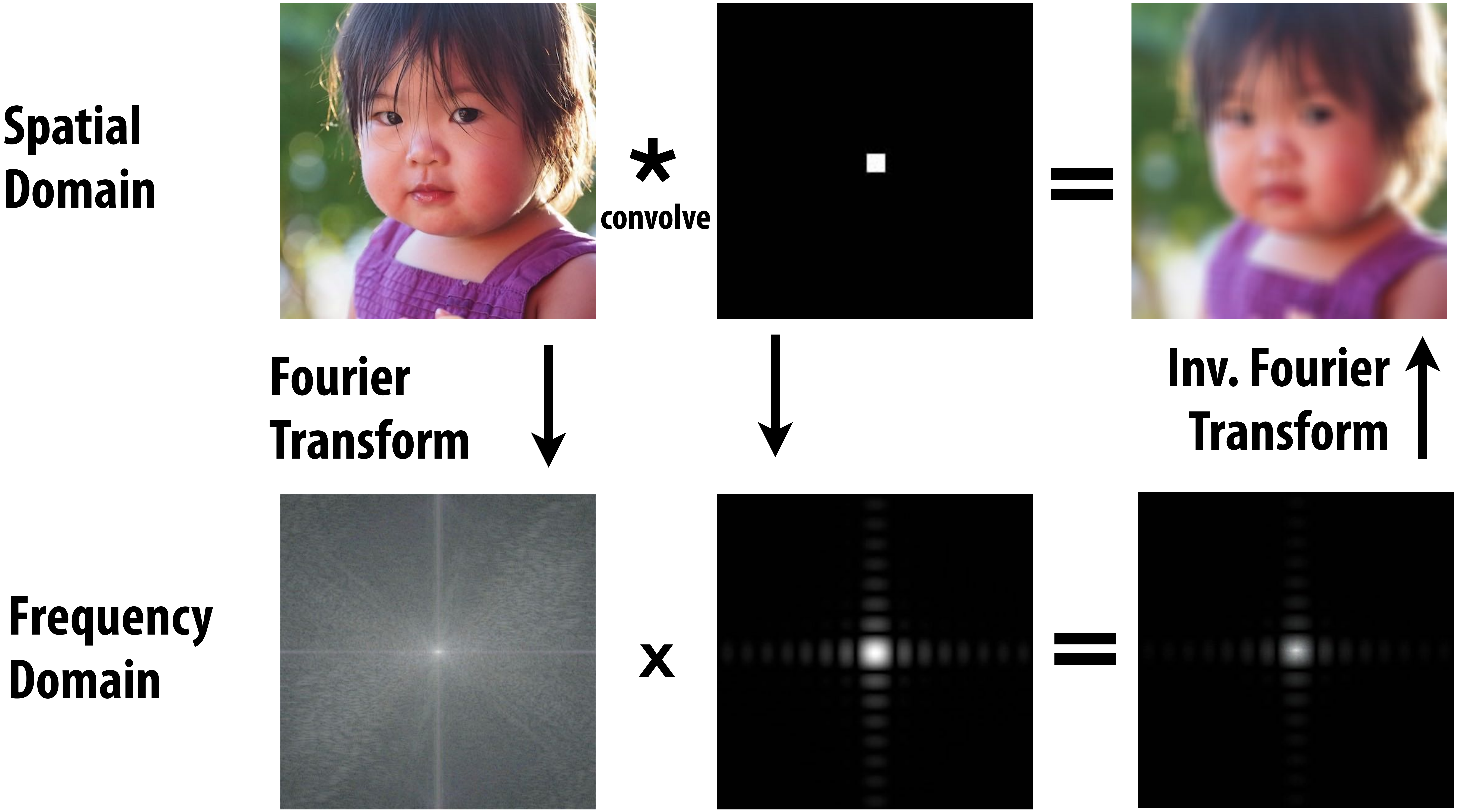
$$(f * g)(x, y) = \sum_{i,j=-1}^{1} f(i, j) I(x - i, y - j)$$

And we can represent f(i,j) as a 3x3 matrix of values where:

$f(i, j) = \mathbf{F}_{i,j}$ **(often called: "filter weights", "filter kernel")**

Convolution theorem

Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa



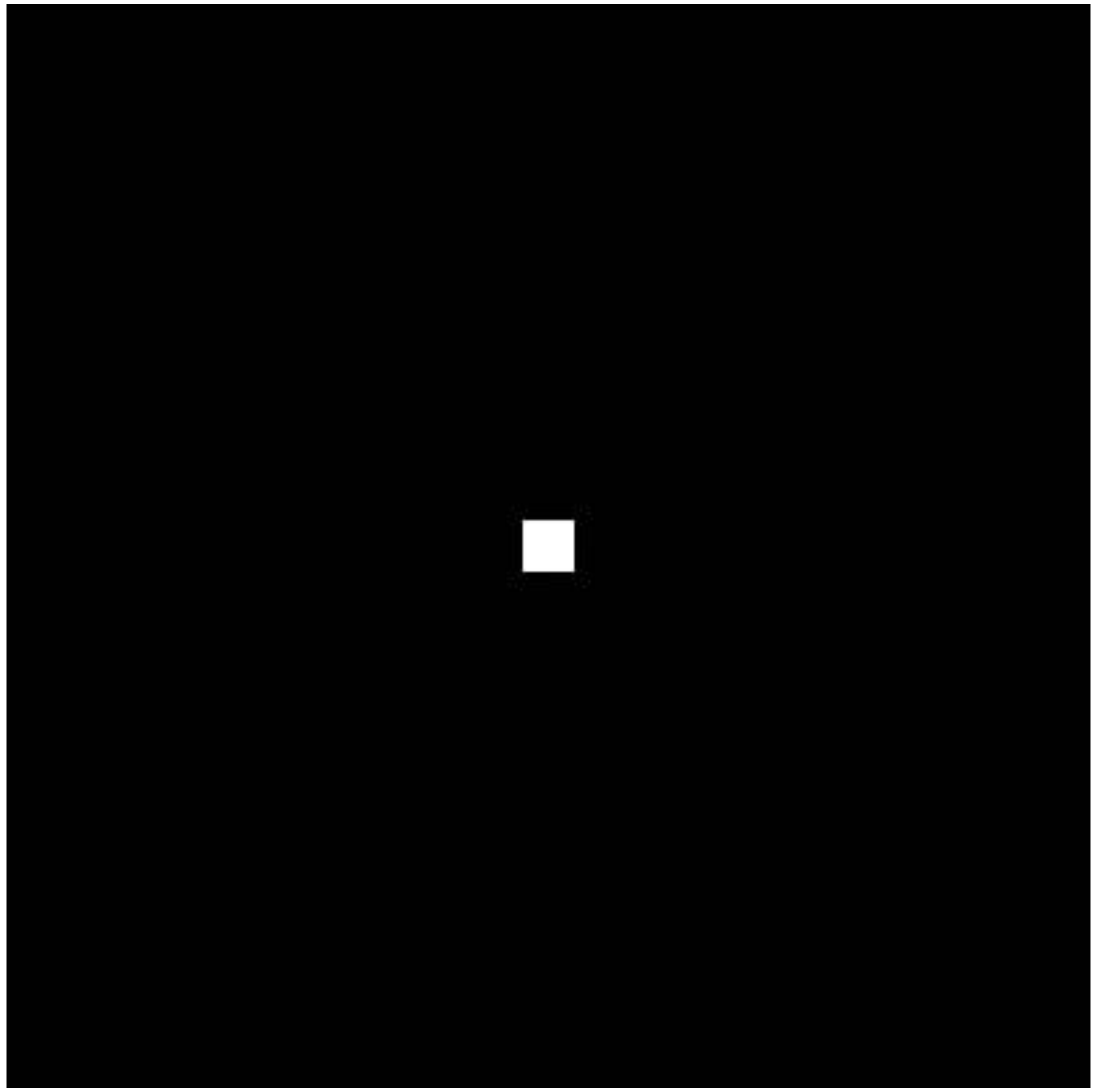
Convolution theorem

- Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa

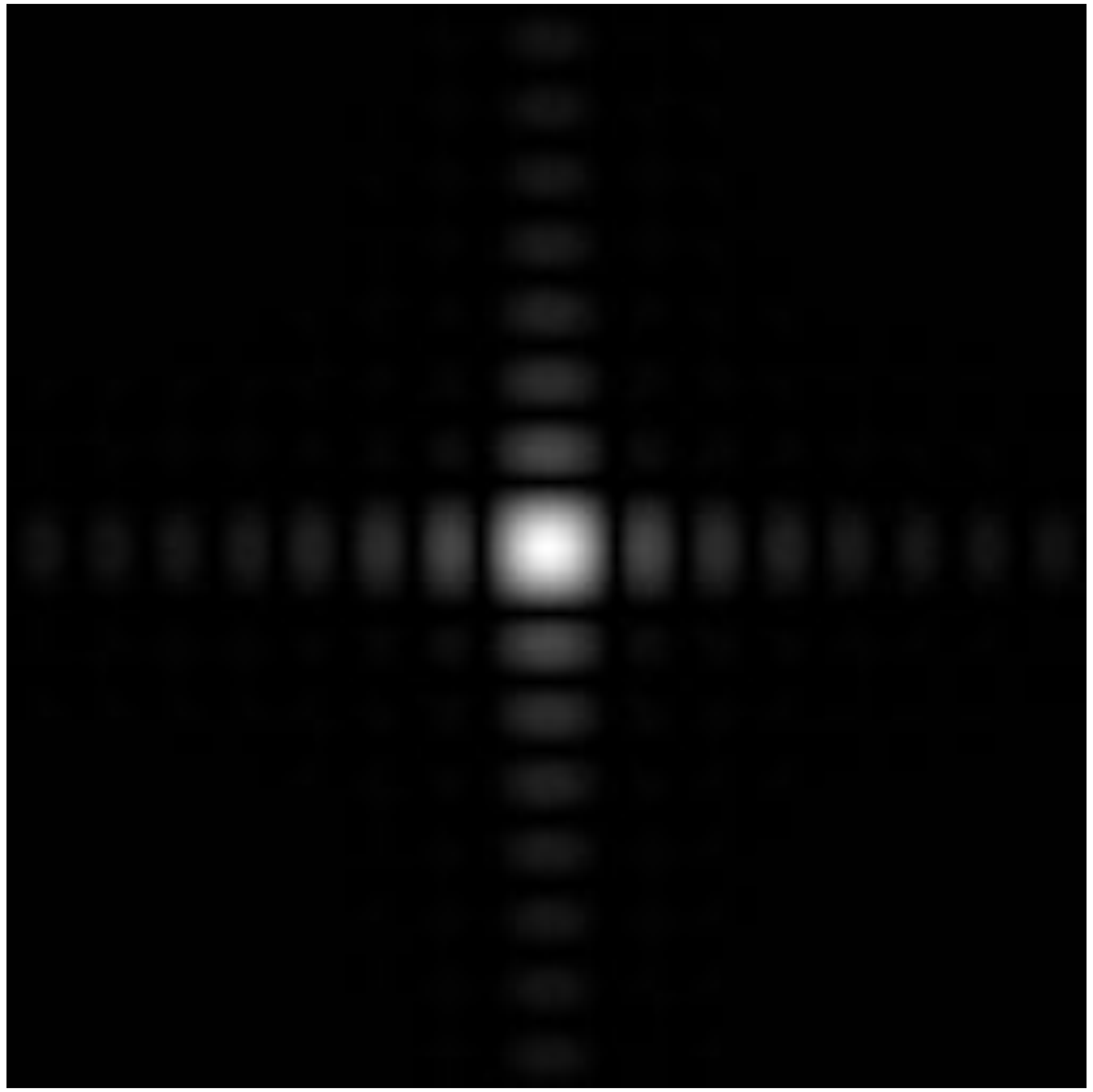
- Pre-filtering option 1:
  - Filter by convolution in the spatial domain

- Pre-filtering option 2:
  - Transform to frequency domain (Fourier transform)
  - Multiply by Fourier transform of convolution kernel
  - Transform back to spatial domain (inverse Fourier)

Box function = “low pass” filter

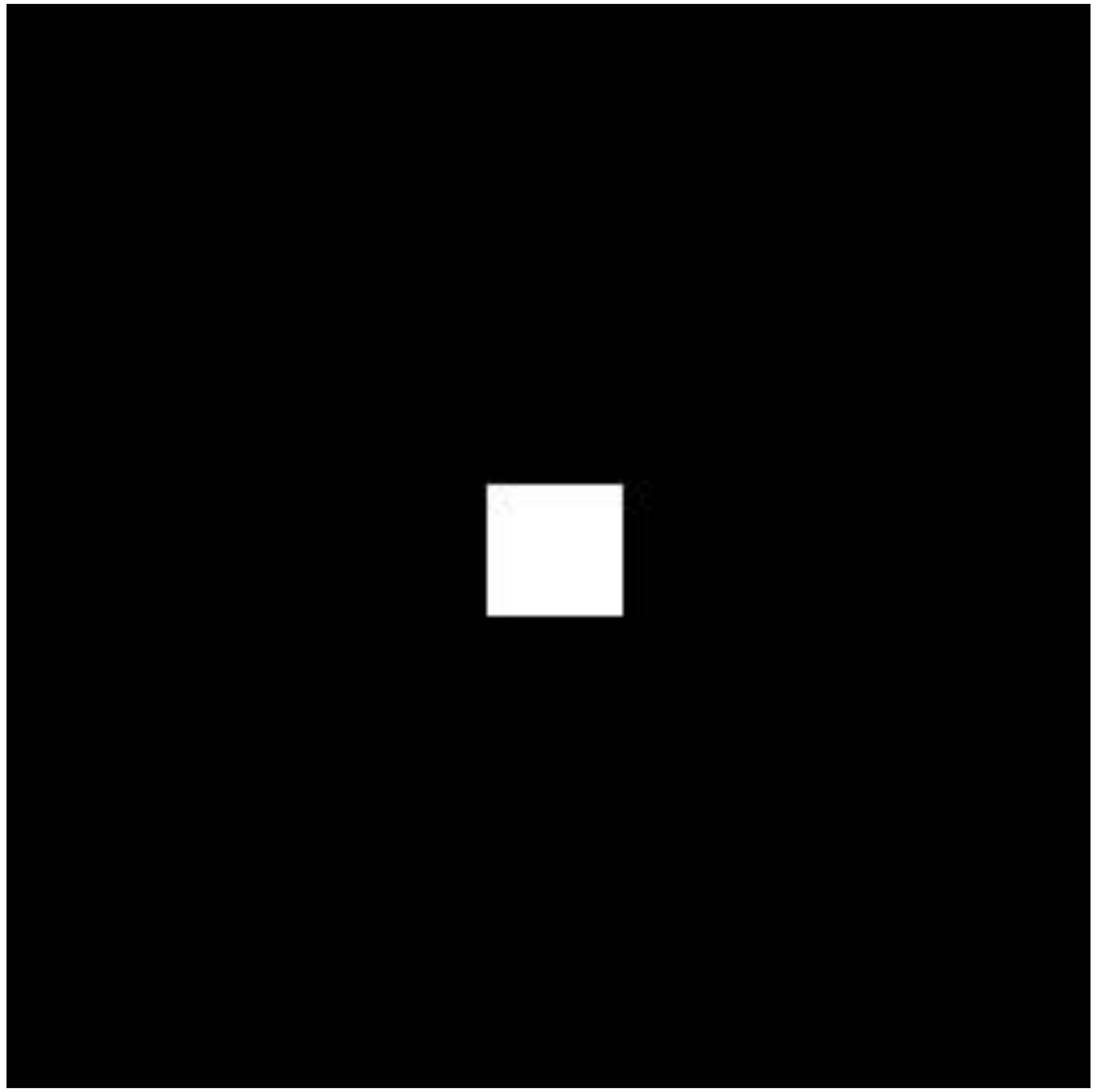


Spatial domain

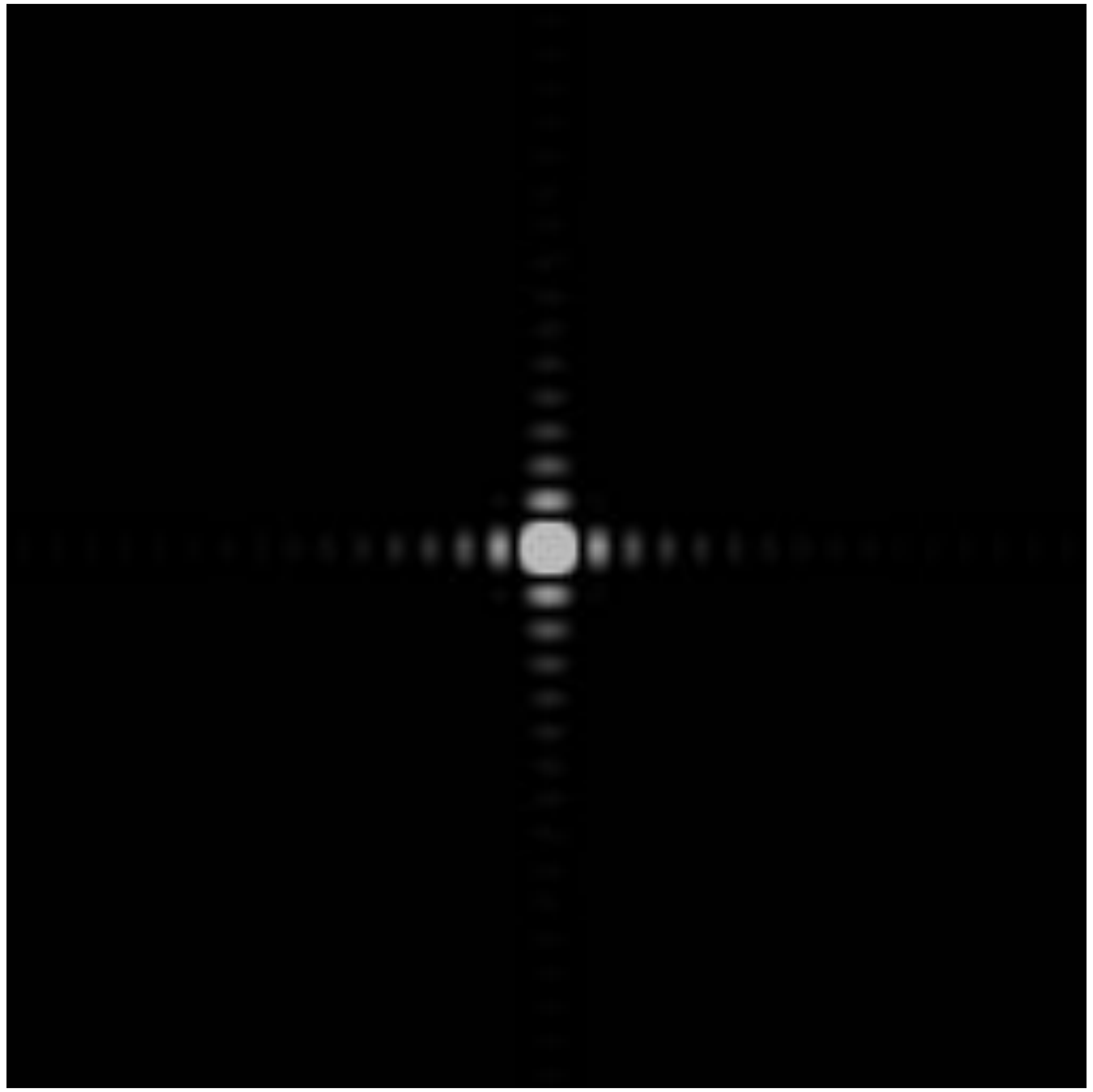


Frequency domain

Wider filter kernel = retain only lower frequencies



Spatial domain



Frequency domain

Wider filter kernel = lower frequencies

- As a filter is localized in the spatial domain,
  it spreads out in frequency domain

- Conversely, as a filter is localized in frequency domain, it spreads out in the spatial domain

How can we reduce aliasing error?

- Increase sampling rate
  - Higher resolution displays, sensors, framebuffers…
  - But: costly and may need very high resolution to sufficiently reduce aliasing

- Anti-aliasing
  - Simple idea: remove (or reduce) high frequencies before sampling
  - How to filter out high frequencies before sampling?

Anti-aliasing by averaging values in pixel area

- Convince yourself the following are the same:

- Option 1:
  - Convolve f(x,y) by a 1-pixel box-blur
  - Then sample the resulting signal at the center of every pixel

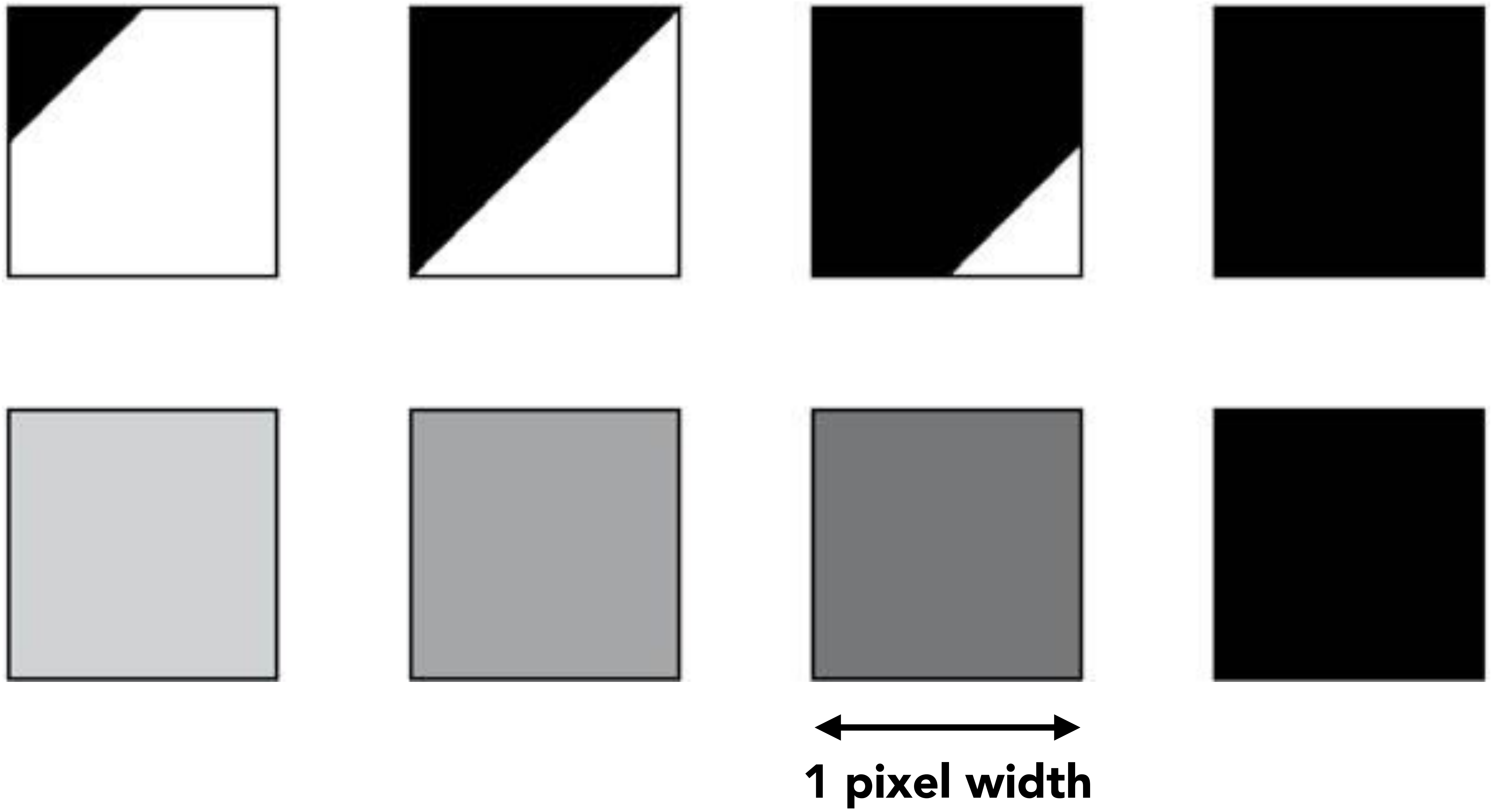
- Option 2:
  - Compute the average value of f(x,y) in the pixel

Anti-aliasing by computing average pixel value

When rasterizing one triangle, the value of f(x,y) = inside(tri,x,y) averaged over the area of a pixel is equal to the amount of the pixel covered by the triangle.

Original

Filtered



Today's summary

- Drawing a triangle = sampling triangle/screen coverage
- Pitfall of sampling: aliasing
- Reduce aliasing by prefiltering signal
  - Supersample
  - Reconstruct via convolution (average coverage over pixel)
    - Higher frequencies removed
  - Sample reconstructed signal once per pixel

- There is much, much more to sampling theory and practice...
  - If interested see: Stanford EE261 - The Fourier Transform and its Applications

Acknowledgements

- Thanks to Ren Ng, Pat Hanrahan, Keenan Crane for slide materials