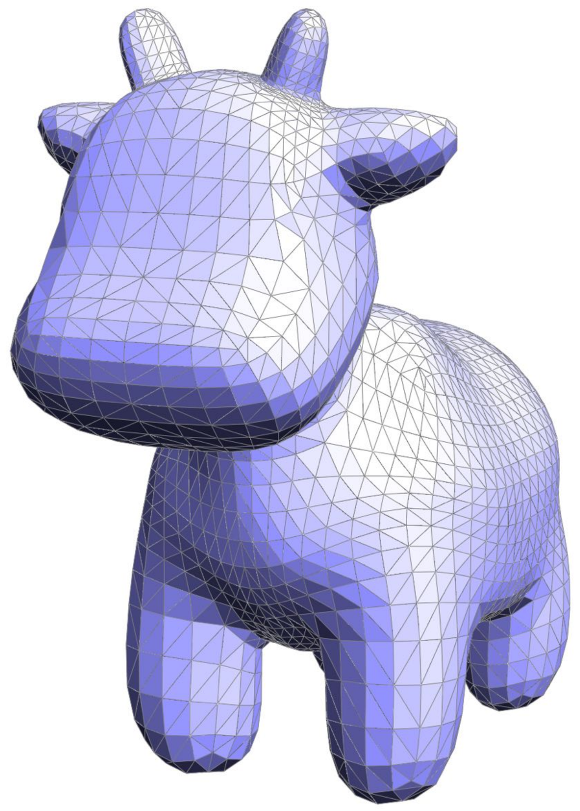
Lecture 10:

Radiometry, BRDFs, and the Reflection Equation

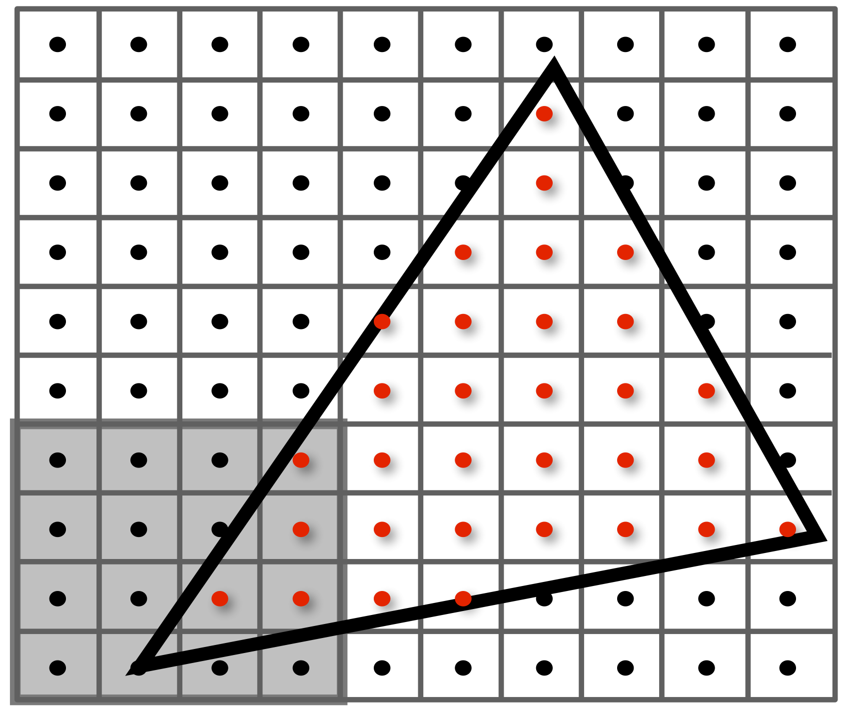
Interactive Computer Graphics
Stanford CS248A, Winter 2025

Things you know so far!

- Representing geometry
  - As triangle meshes, subdivision surfaces, implicit surfaces, etc.



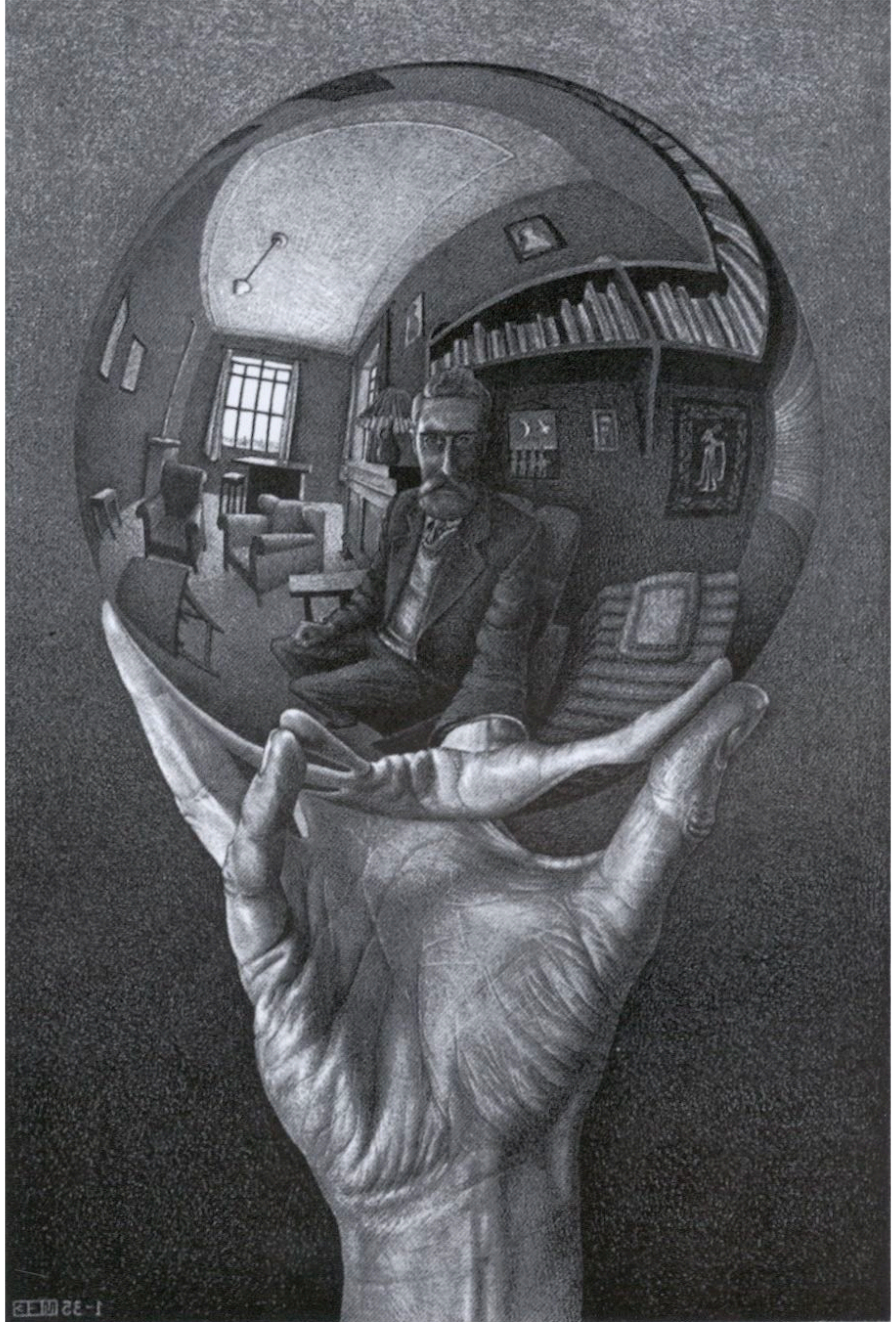
- Visibility and occlusion
  - Rasterization: determining what point on what triangle covers a sample
  - Ray tracing: determining what triangle a ray hits



- Today: basics of lights and materials
  - Computing the "appearance" of the surface at a point
  - Thinking about "appearance" in terms of reflected light (electromagnetic energy)

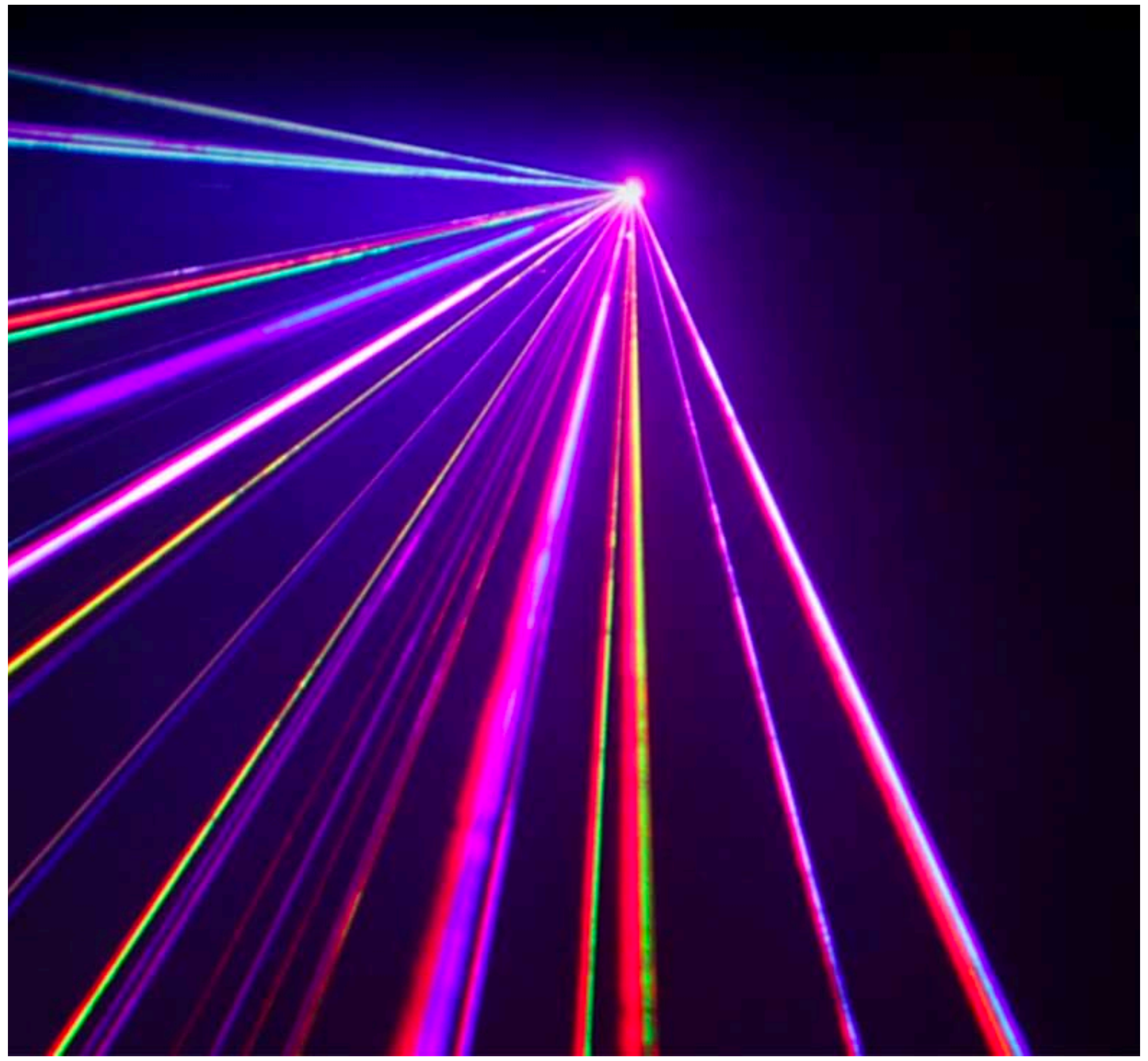
“Shading” in drawing

- Depicting the appearance of the surface
- Due to factors like surface material, lighting conditions



MC Escher pencil sketch

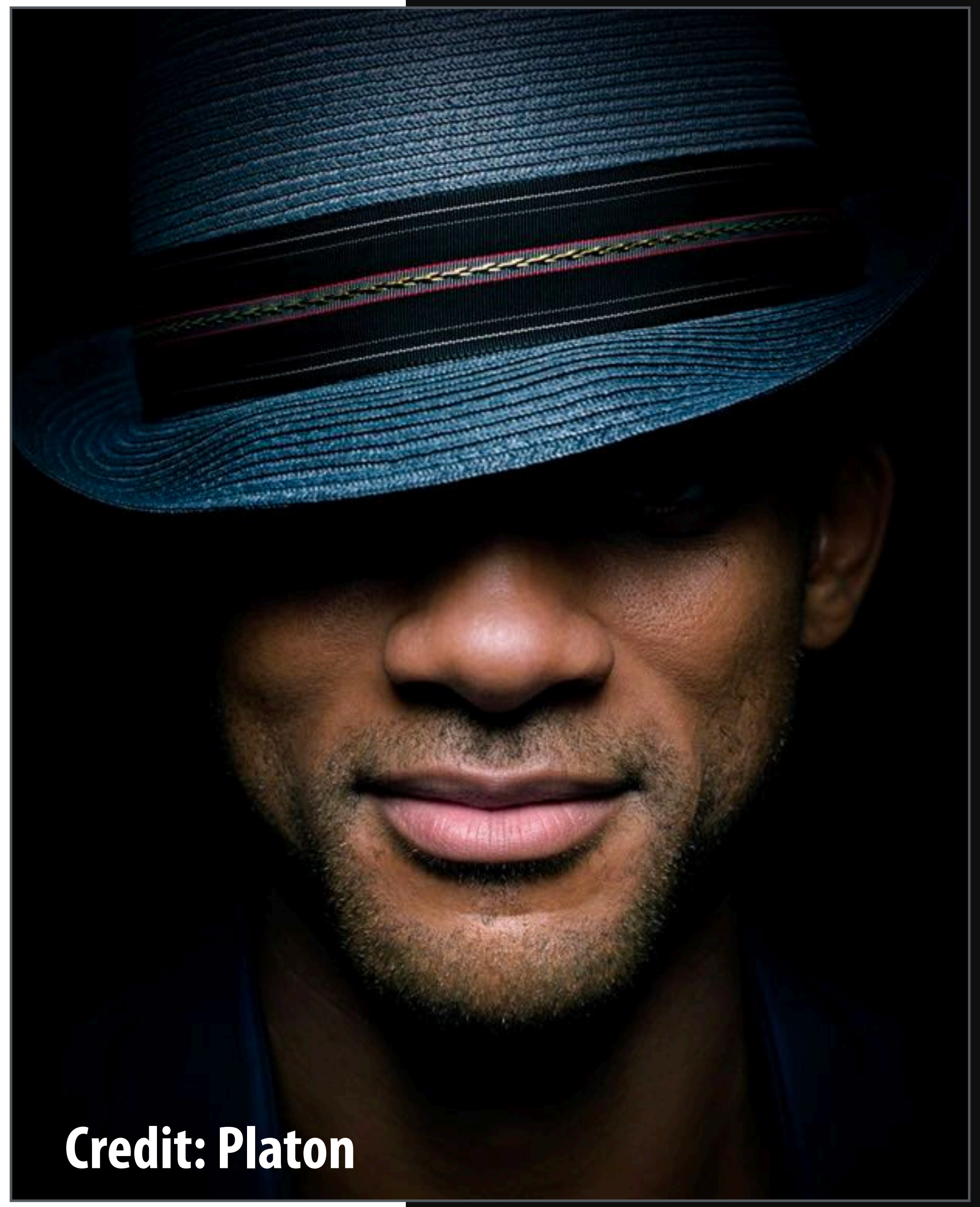
Lighting

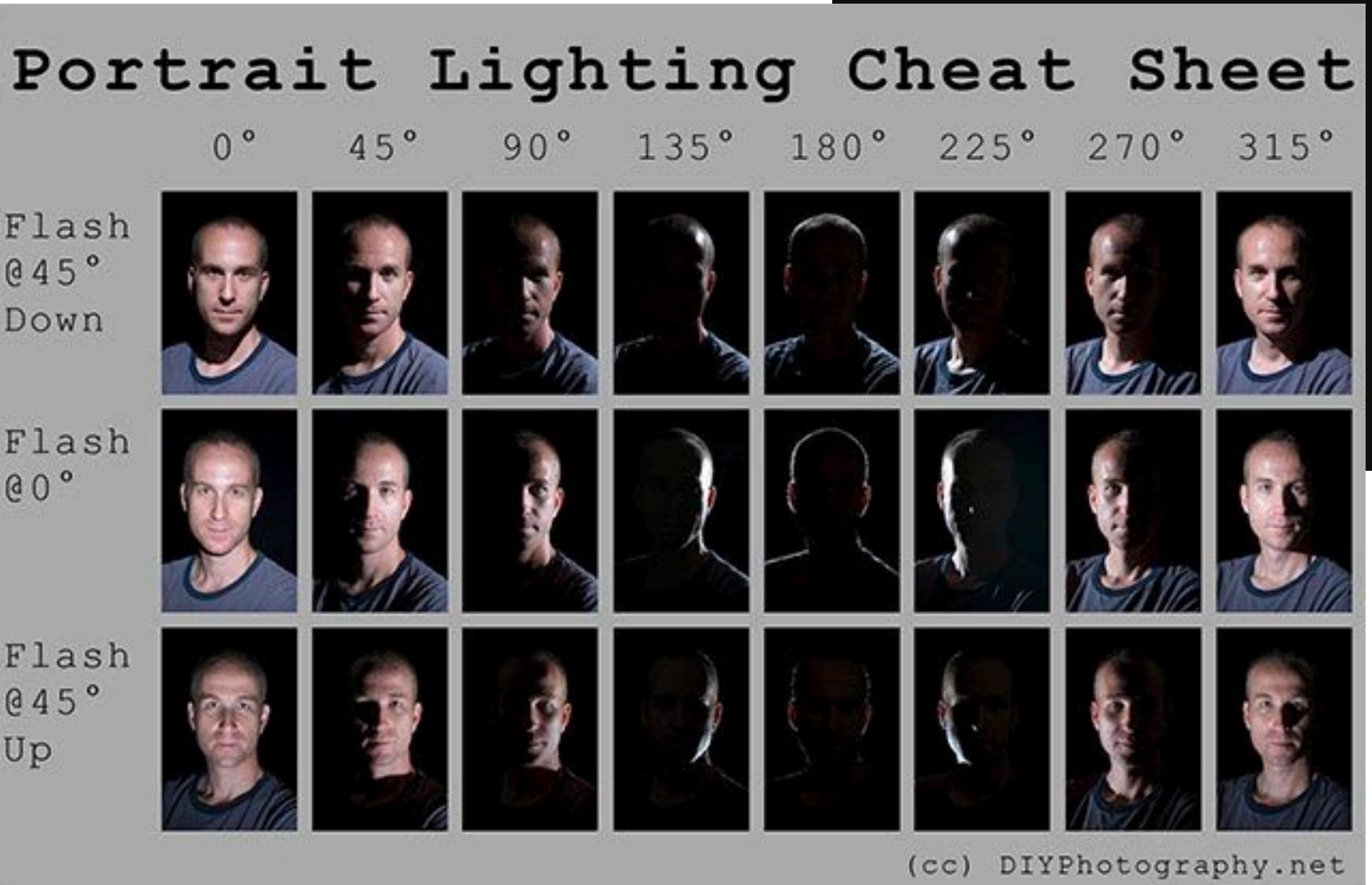


Lighting

Credit: Wikipedia
(Nasir ol Molk Mosque)

Lighting


Credit: Platon


Portrait Lighting Cheat Sheet
0° 45° 90° 135° 180° 225° 270° 315°
Flash @45° Down
Flash @0°
Flash @45° Up
(cc) DIYPhotography.net

Diffuse material

Plastic

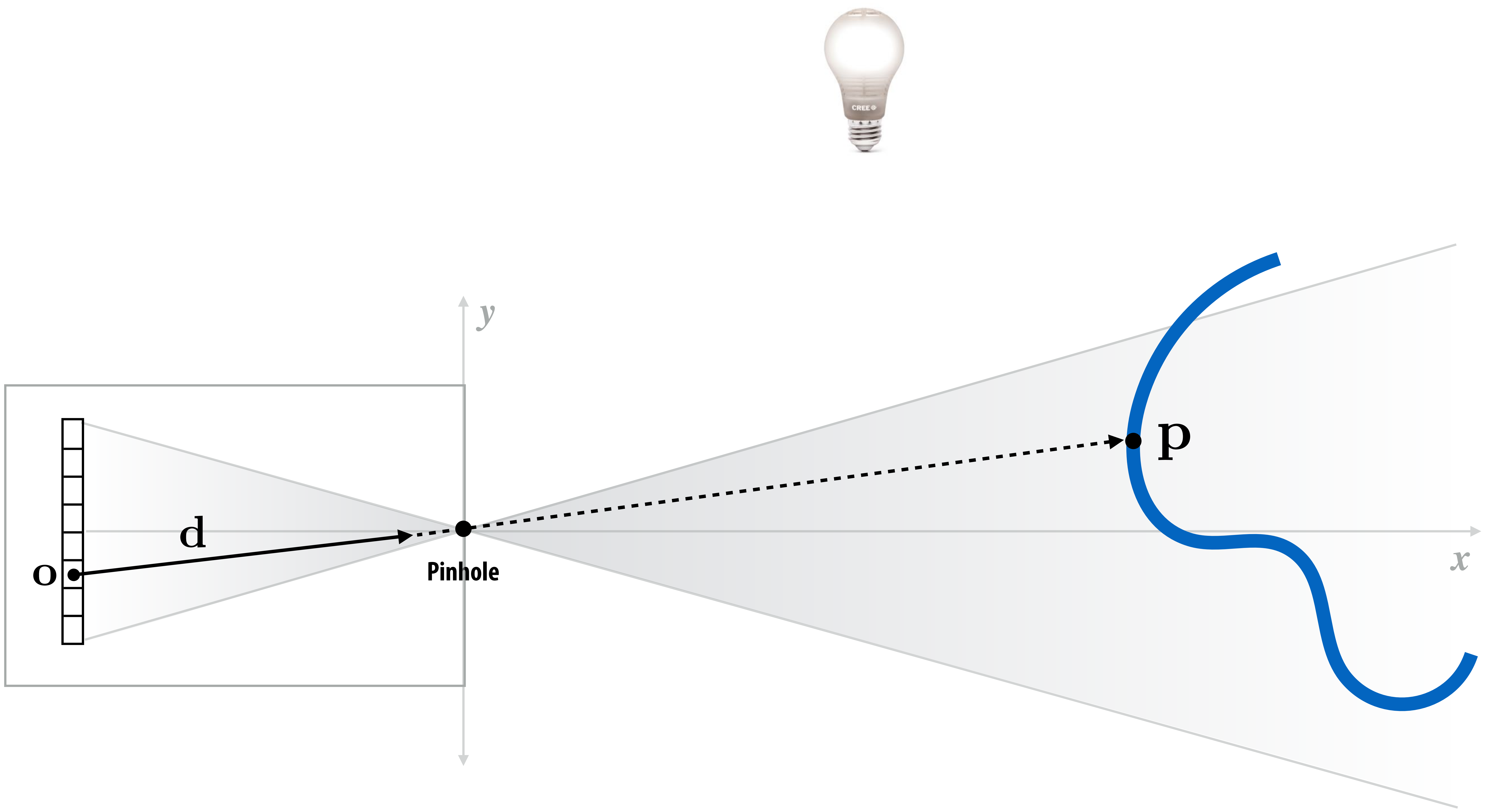
Red semi-gloss paint

Ford mystic lacquer paint

Mirror

Gold

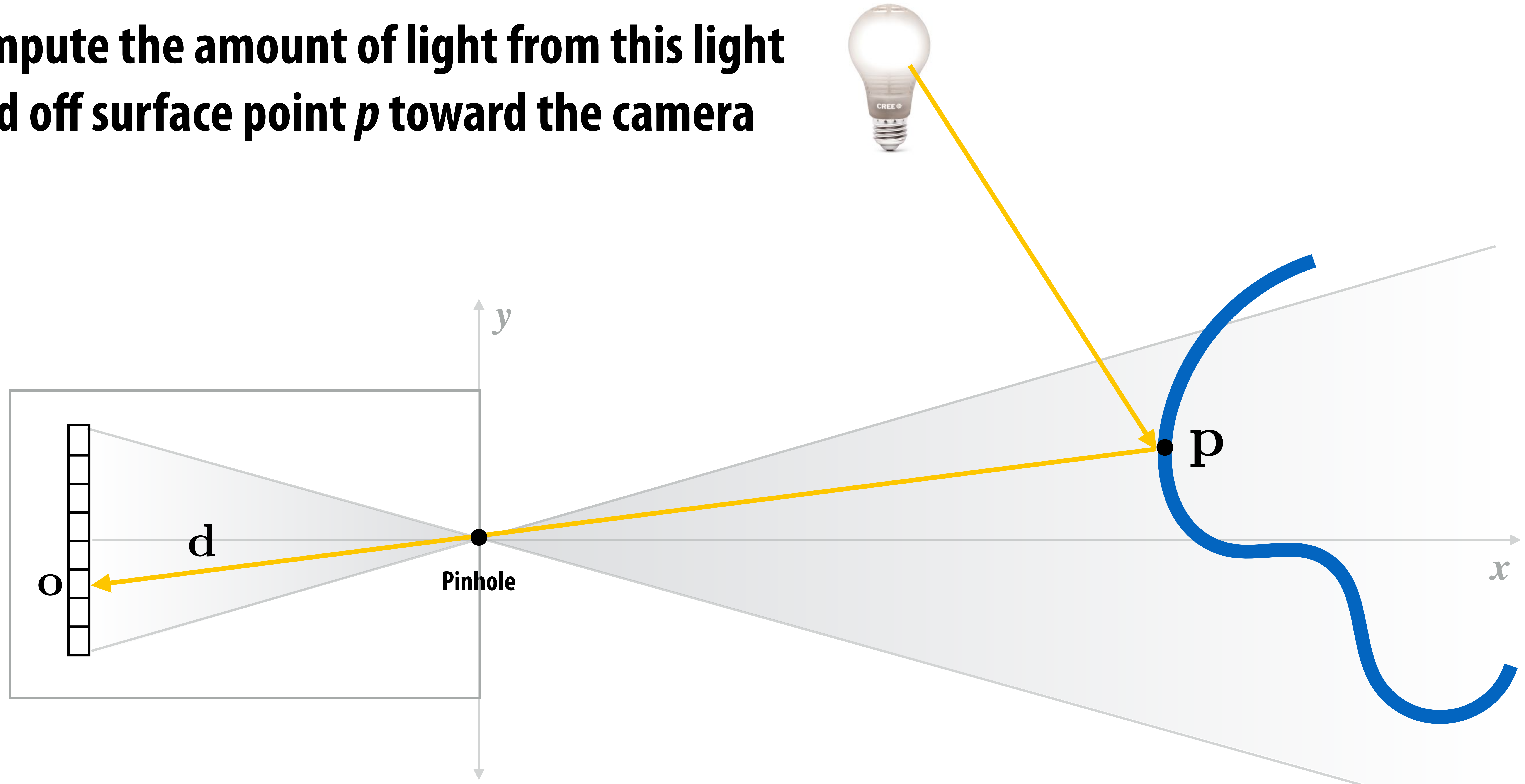
A renderer measures light energy along a ray



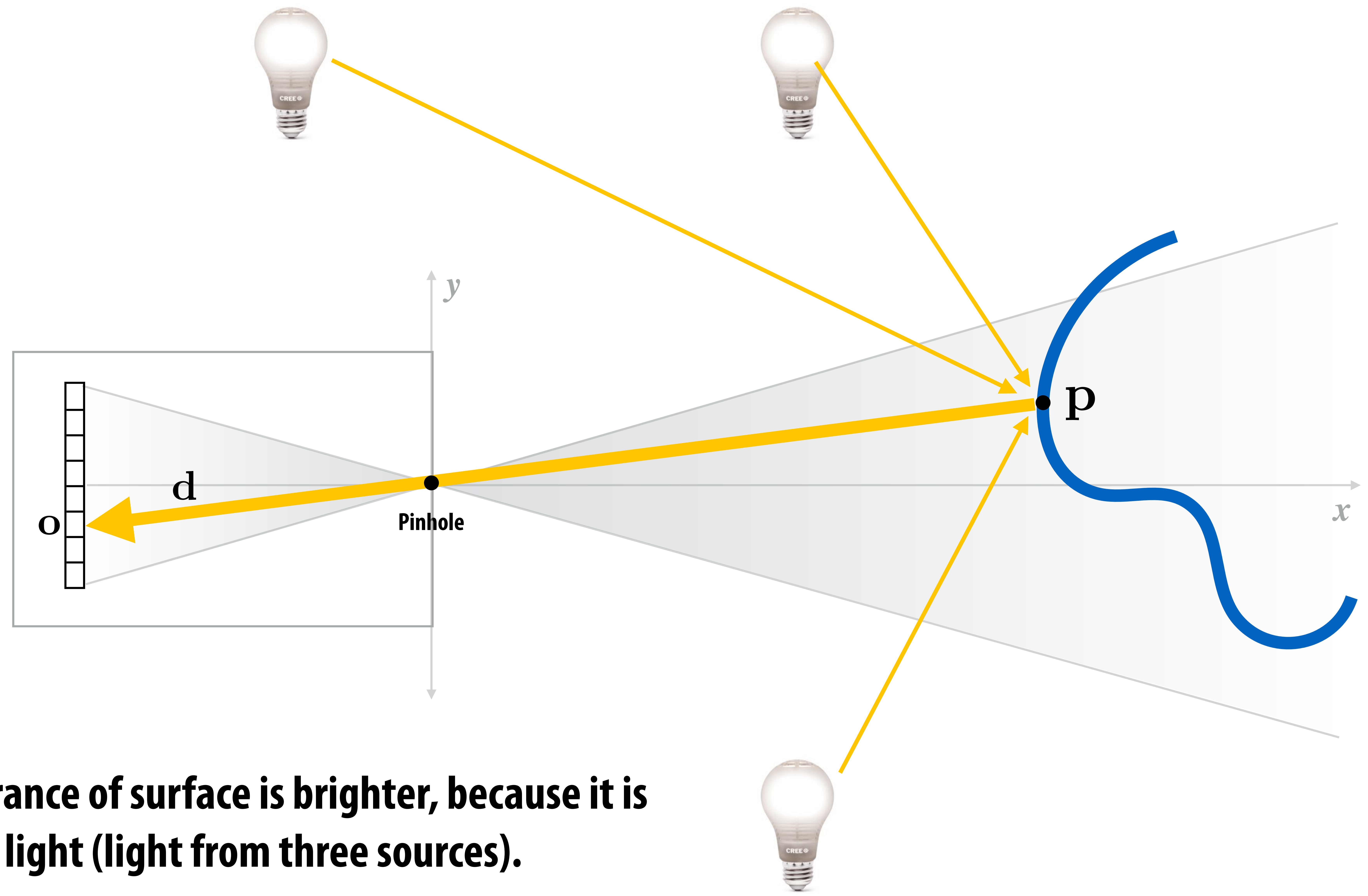
Up until now in the course I've said that we are sampling "the color of the surface" visible along a ray… but now let's make that more precise.

Renderer measures light energy along a ray

We want to compute the amount of light from this light source reflected off surface point *p* toward the camera



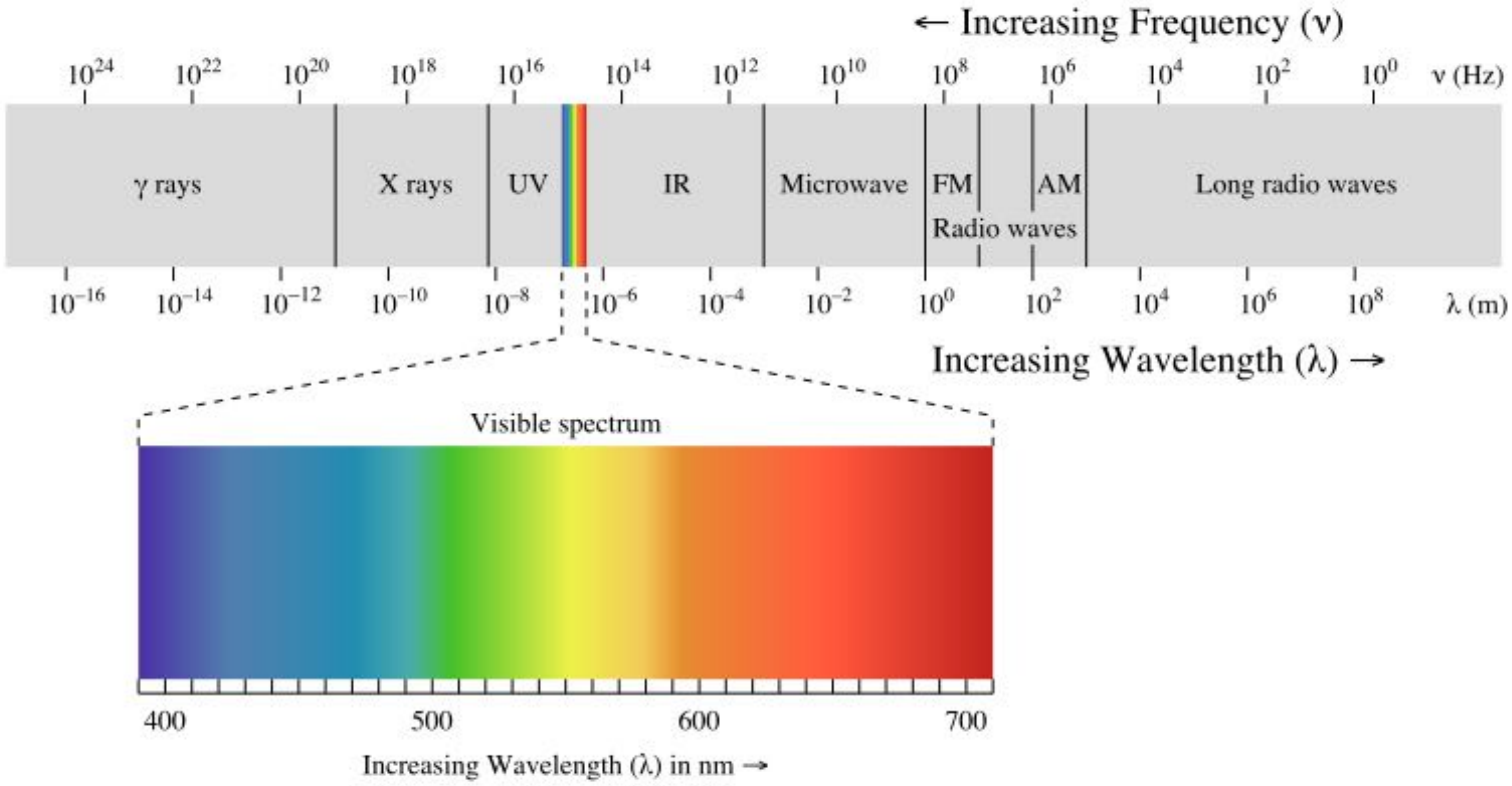
Multiple light sources



Now the appearance of surface is brighter, because it is reflecting more light (light from three sources).

What is light?

Light is electromagnetic radiation that is visible to the eye



What do lights do?

Cree 11 W LED light bulb
("60 Watt" incandescent replacement)

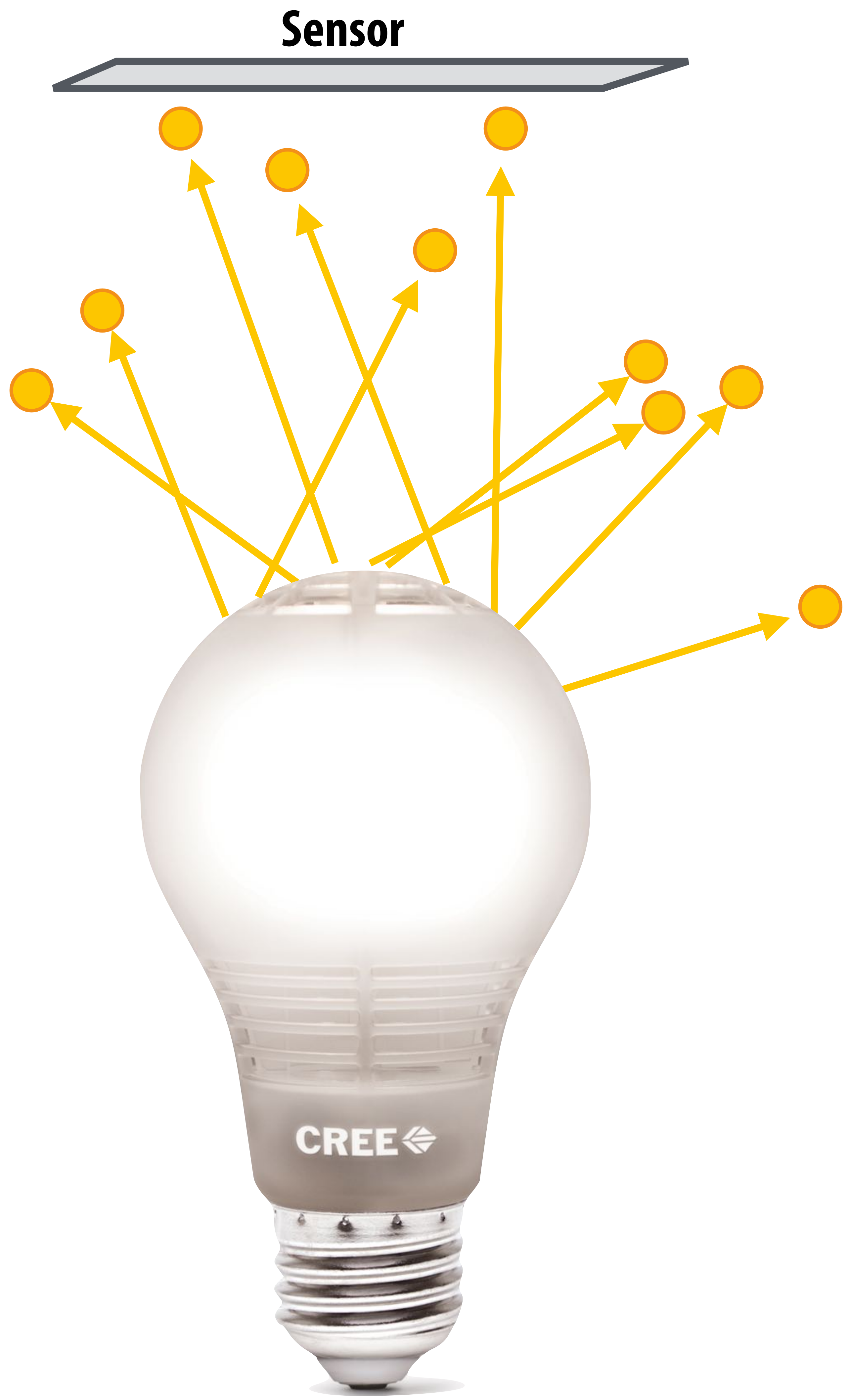
- Physical process converts input energy into photons
  - Each photon carries a small amount of energy
- Over some amount of time, light fixture consumes some amount of energy, Joules
  - Some input energy is turned into heat, some into photons
- Energy of photons hitting an object ~ exposure
  - Film, sensors, sunburn, solar panels, …
- In graphics we generally assume "*steady state*" process
  - Rate of energy consumption = power, Watts (Joules/second)

Measuring illumination: radiant flux (power)

- Given a sensor, we can count how many photons reach it
  - Over a period of time, gives the power received by the sensor
- Given a light, consider counting the number of photons emitted by it
  - Over a period of time, gives the power emitted by the light
- Energy carried by a photon:

$$Q = \frac{hc}{\lambda}$$

$$h \approx 6.626 \times 10^{-34}$$



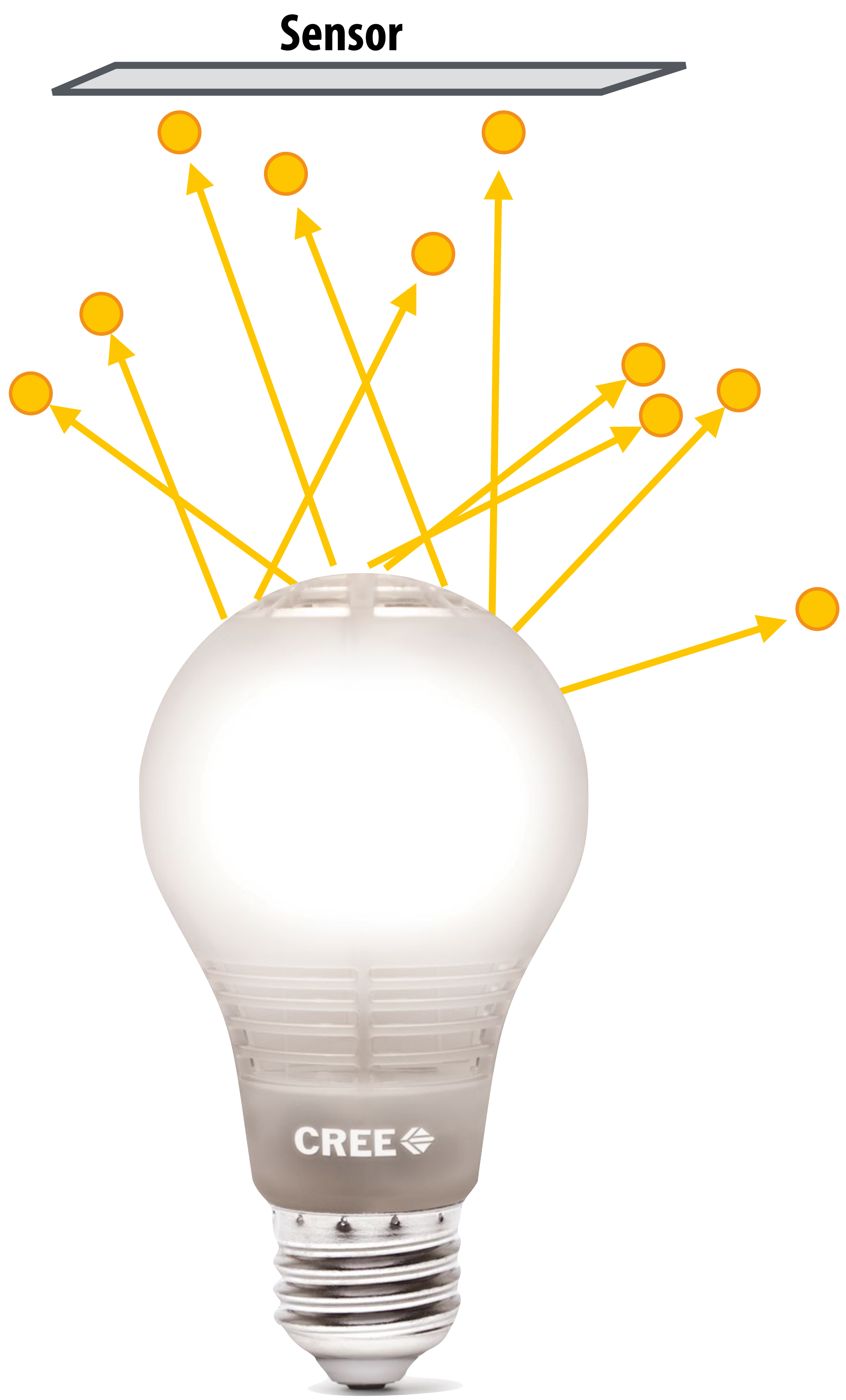
Measuring illumination: radiant flux (power)

- **Flux: energy per unit time (Watts) received by the sensor (or emitted by the light)**

$$\Phi = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t} \quad \left[\frac{\mathrm{J}}{\mathrm{s}}\right]$$

- **Time integral of flux is total radiant energy**

$$Q = \int_{t_0}^{t_1} \Phi(t)\,\mathrm{d}t$$



Spectral power distribution

Describes distribution of energy by wavelength

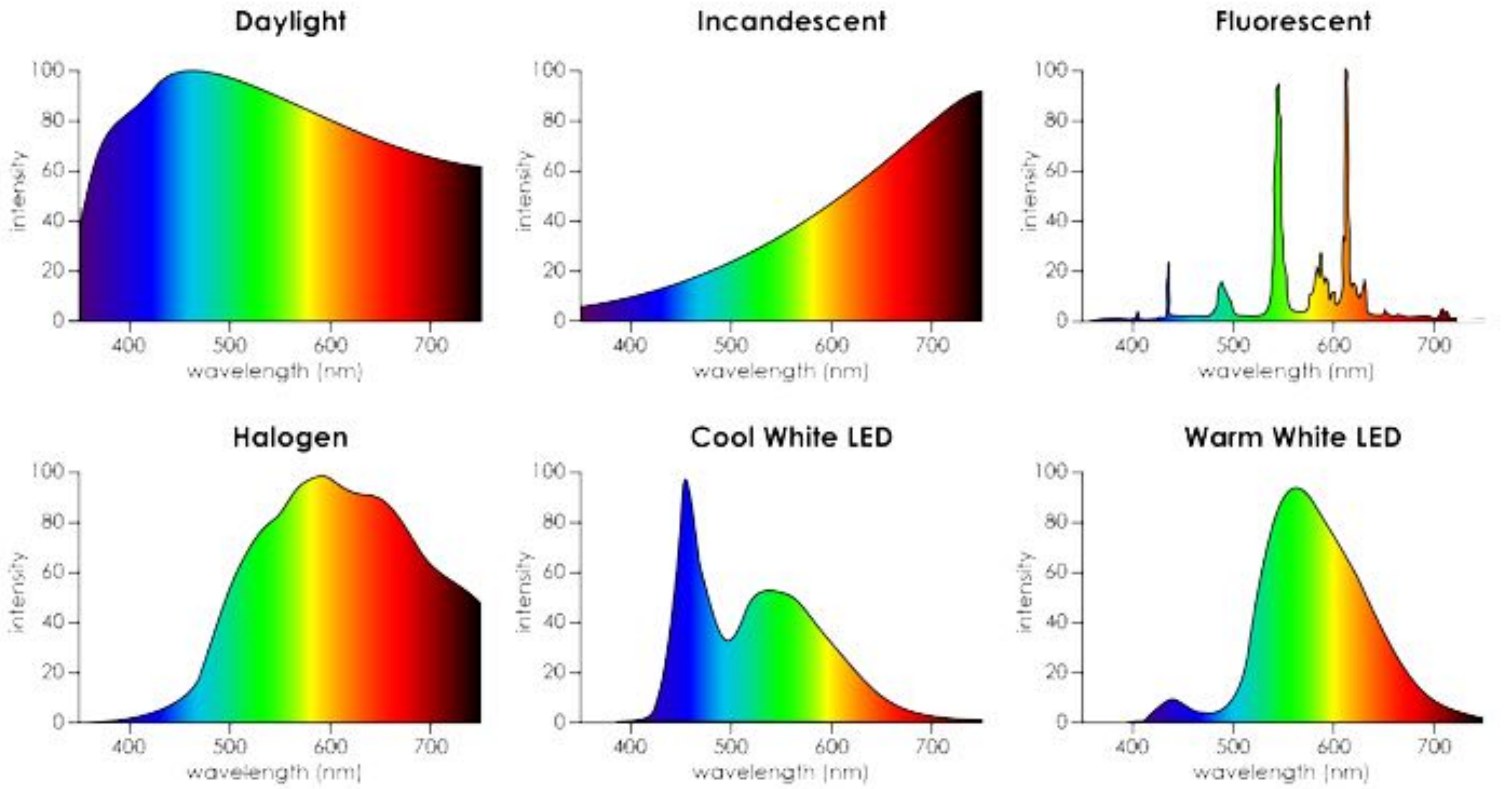


Figure credit:

admesy
ADVANCED MEASUREMENT SYSTEMS

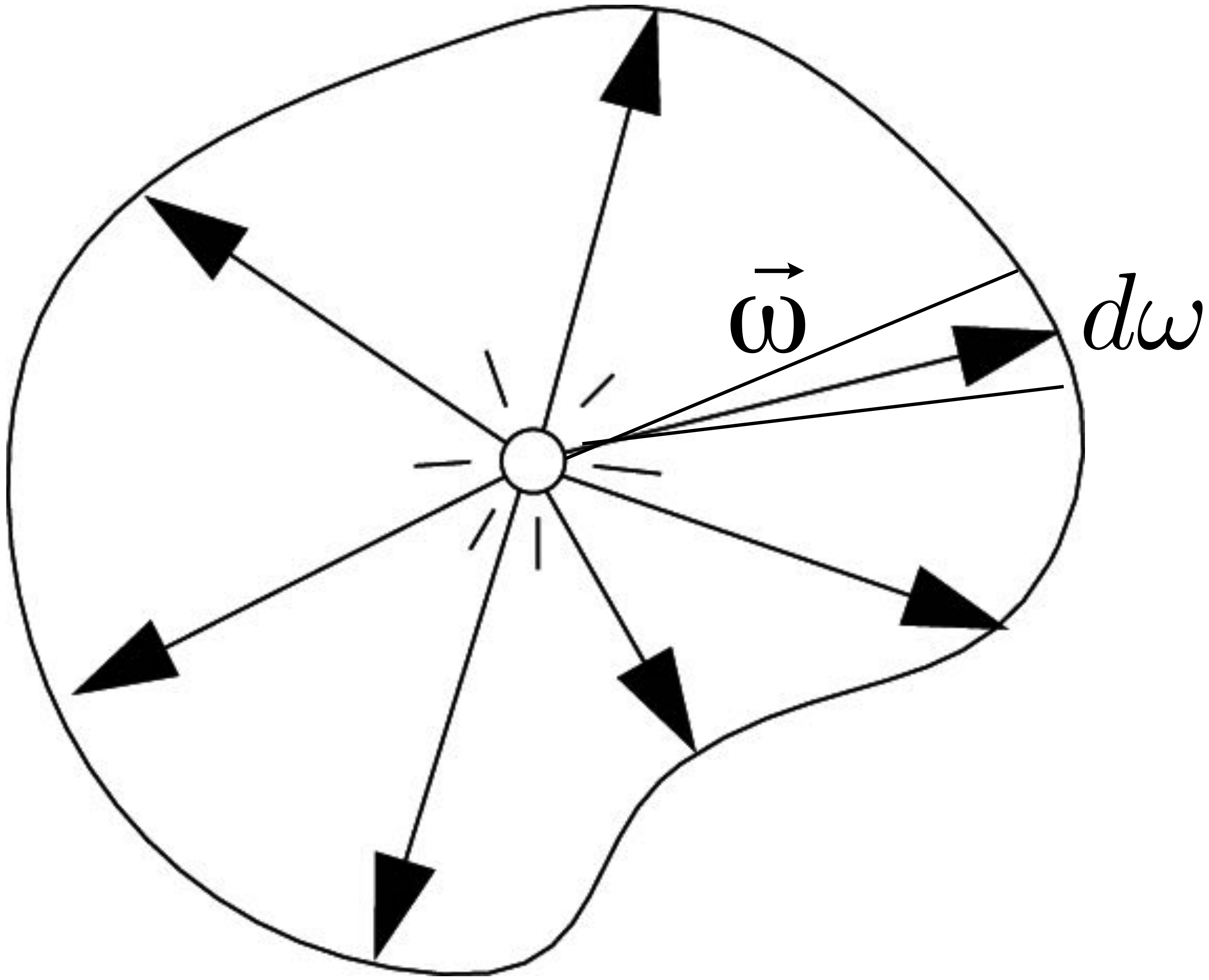
"Warm" vs. "cool" white light LED

Credit: https://www.ledholidaylighting.com/LED-faq.aspx

Radiant intensity

The *radiant intensity* is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$



$$\left[\frac{W}{sr}\right]$$

Units = Watts per steradian

Angles and solid angles

Angle

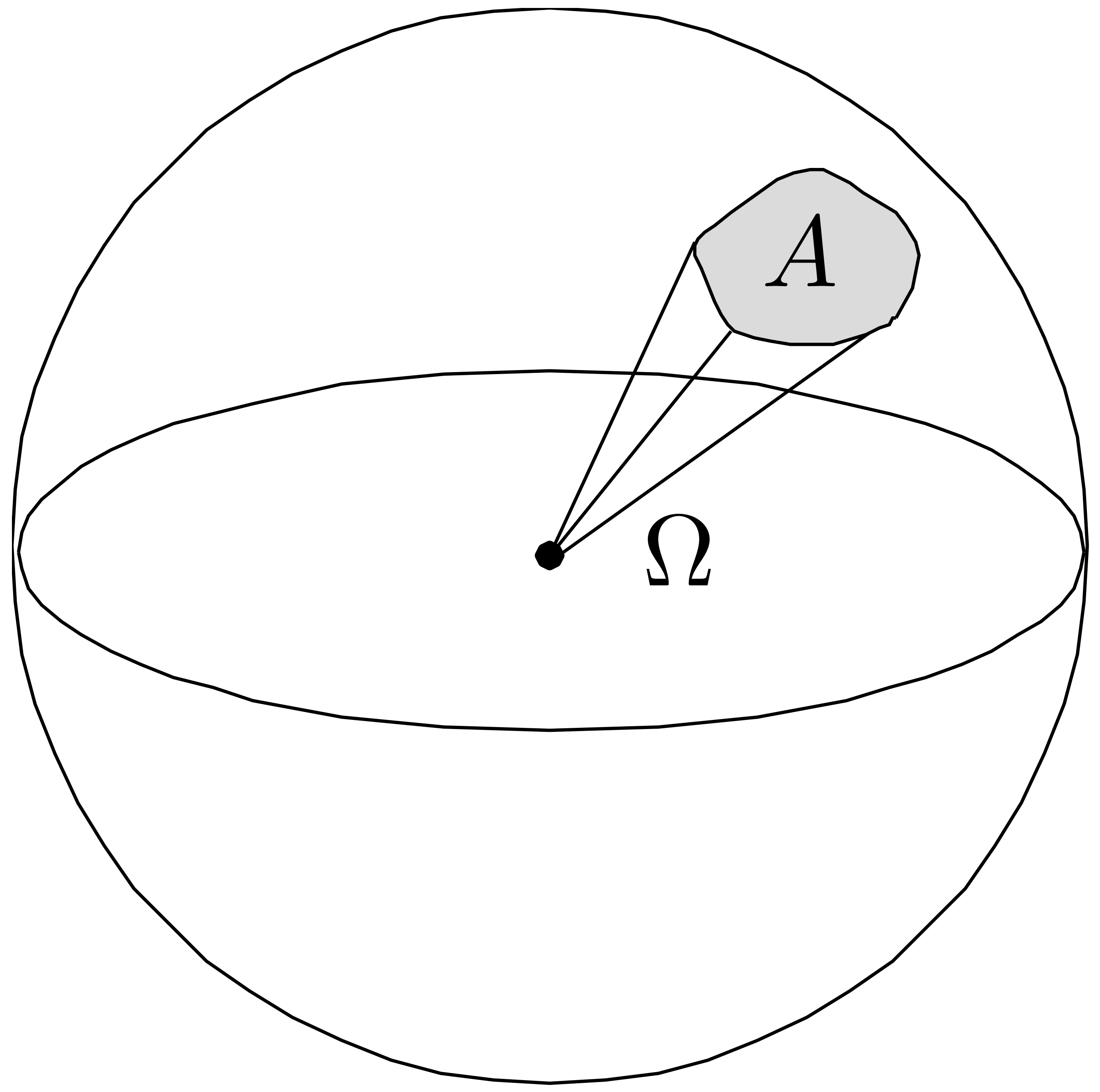
$$\theta = \frac{l}{r}$$

⇒ circle has 2 π radians

Solid angle

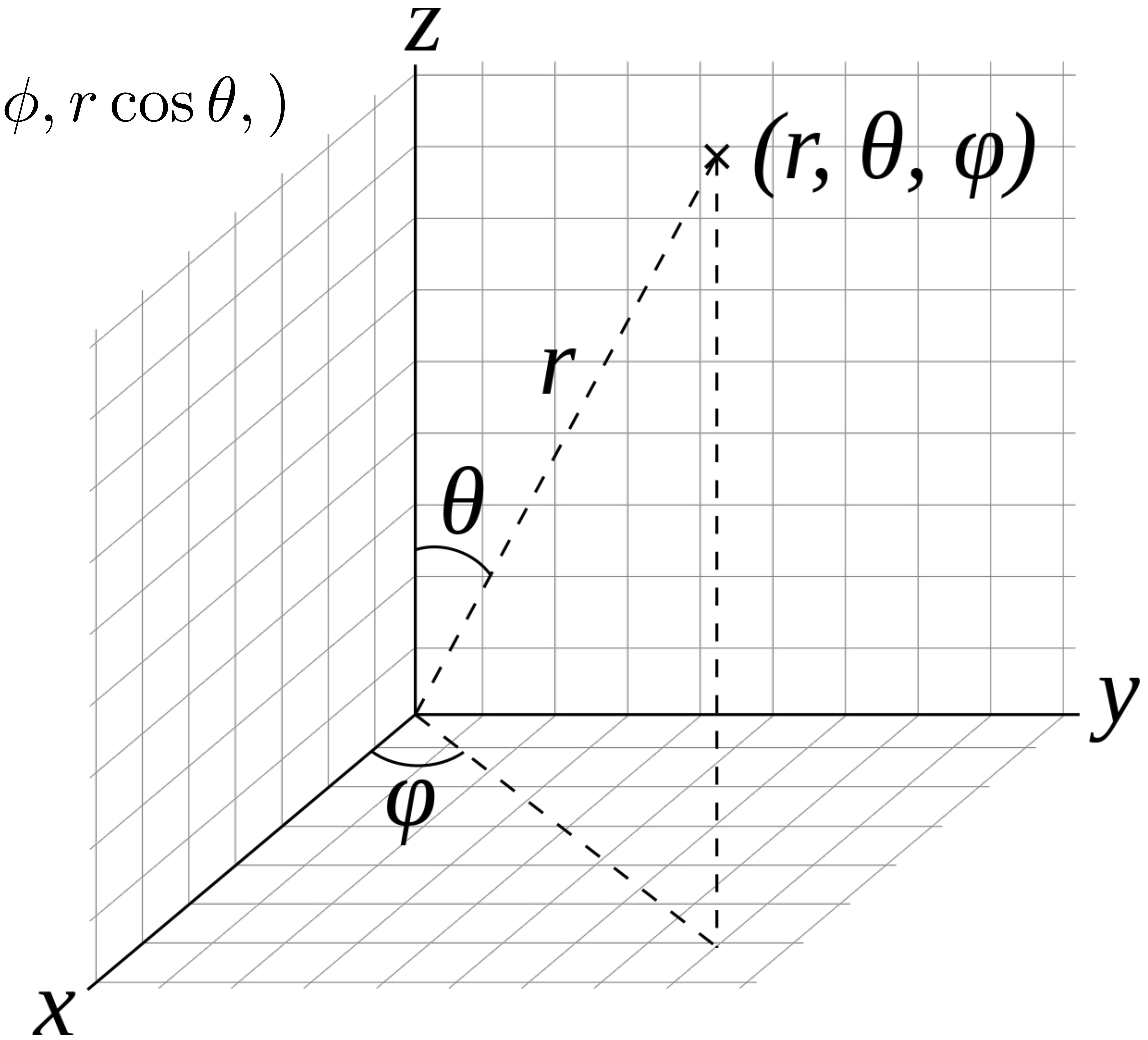
$$\Omega = \frac{A}{R^2}$$

⇒ sphere has 4 π steradians



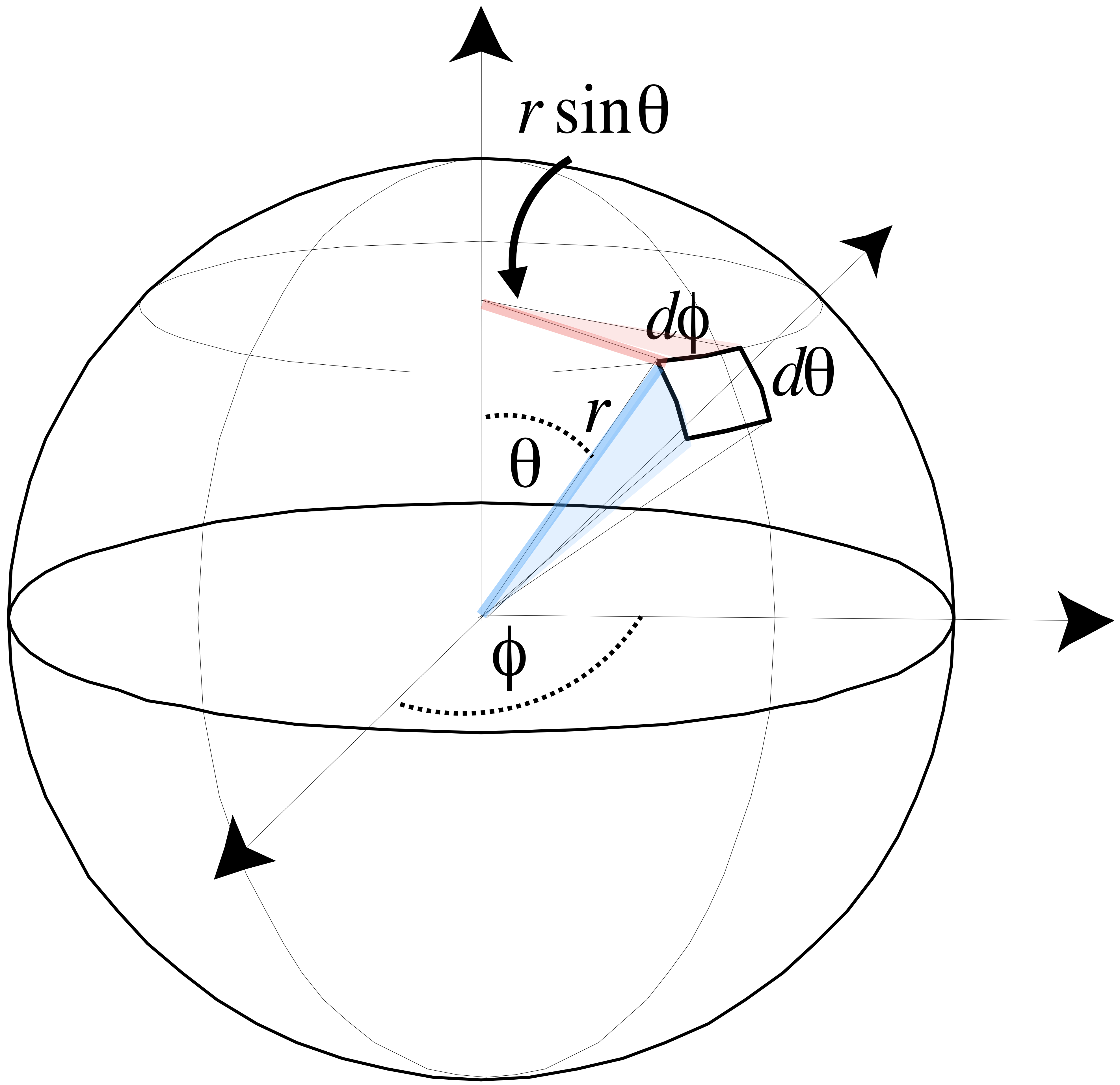
Review of spherical coordinates

$$(x, y, z) = (r \sin\theta \cos\phi, r \sin\theta \sin\phi, r \cos\theta,)$$



Differential solid angles

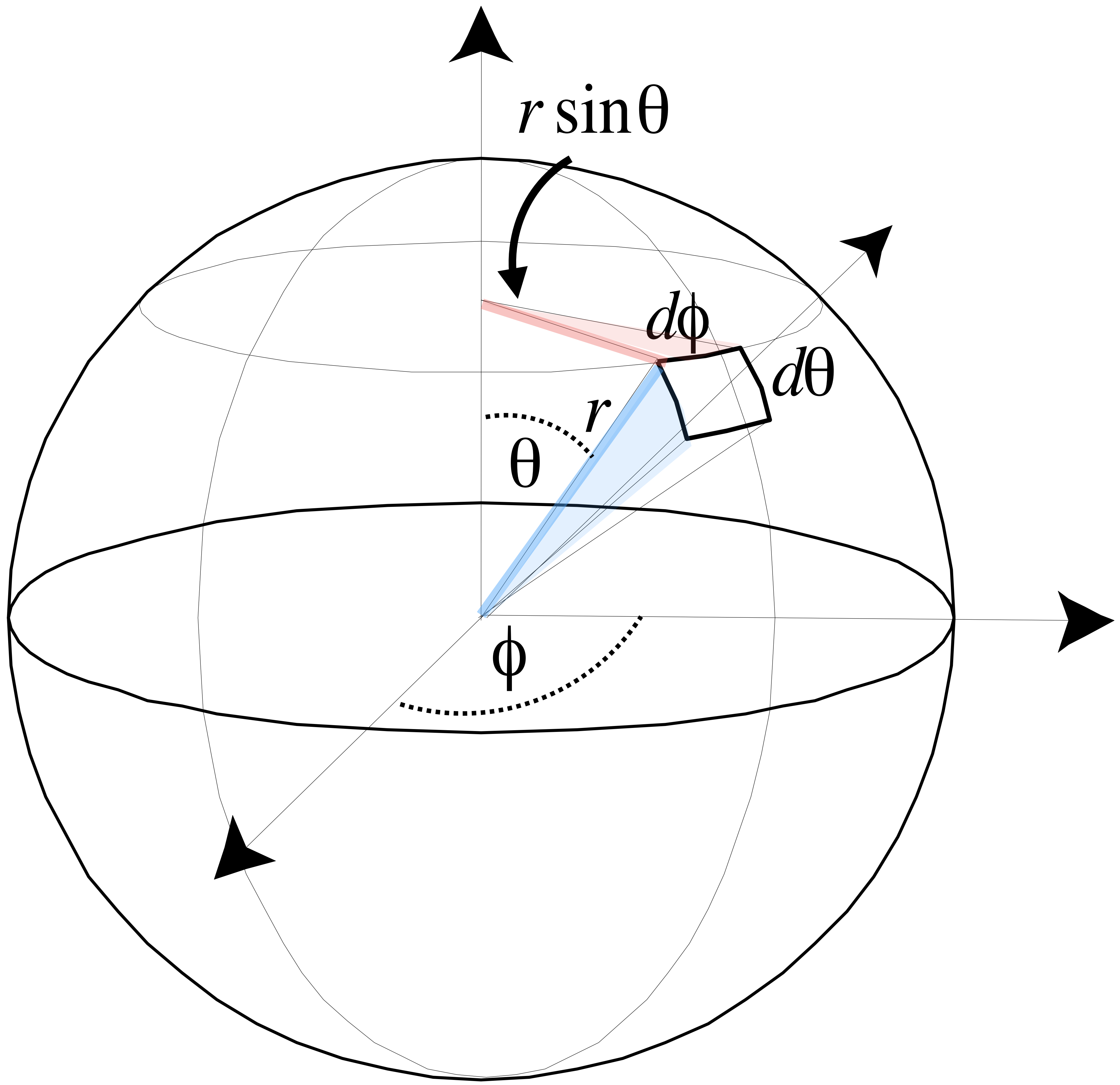
Sphere with radius r



$$dA = (r\, d\theta)(r \sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

Differential solid angles

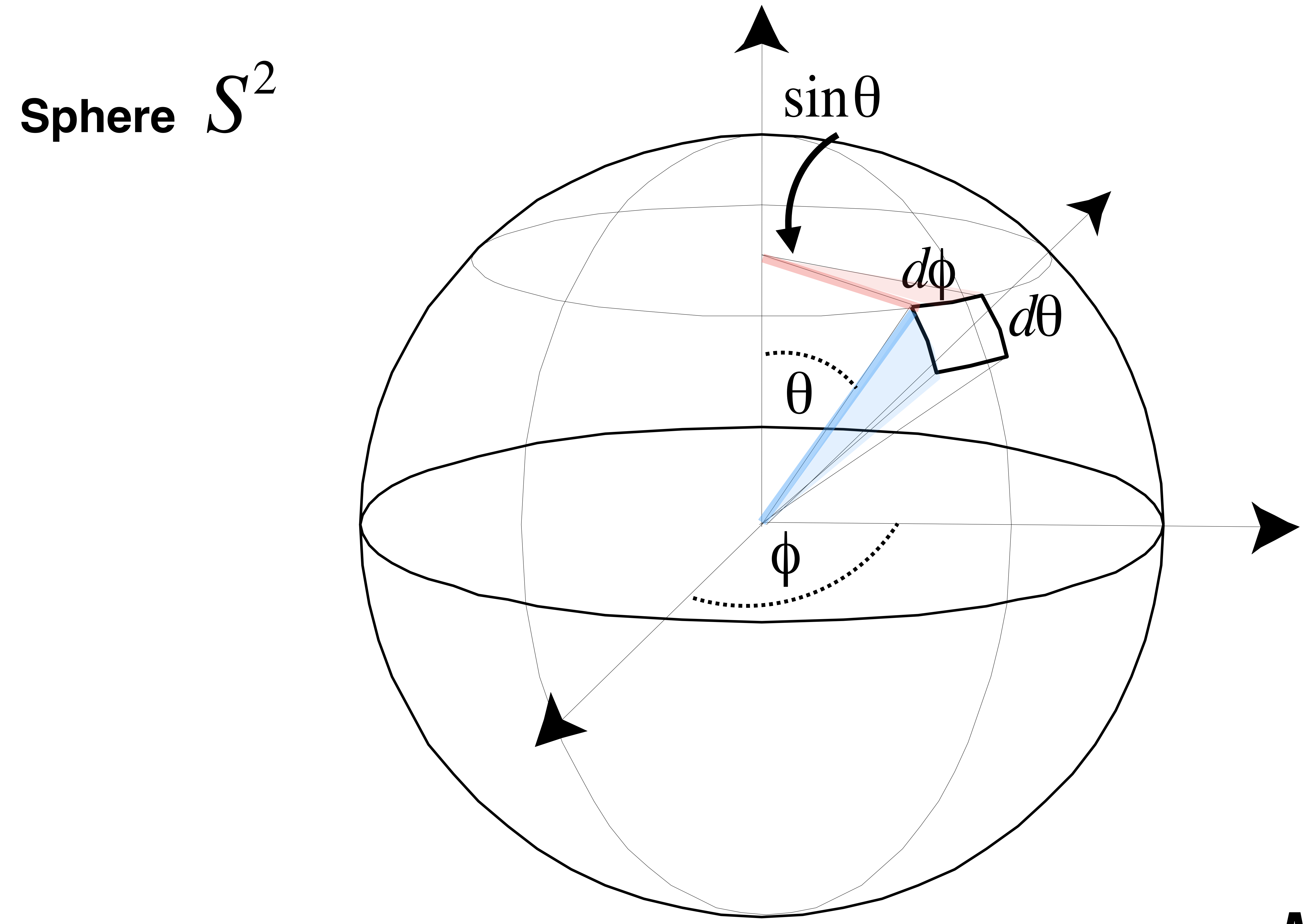
Sphere with radius r



$$dA = (r\, d\theta)(r \sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

Integrating solid angle over the unit sphere



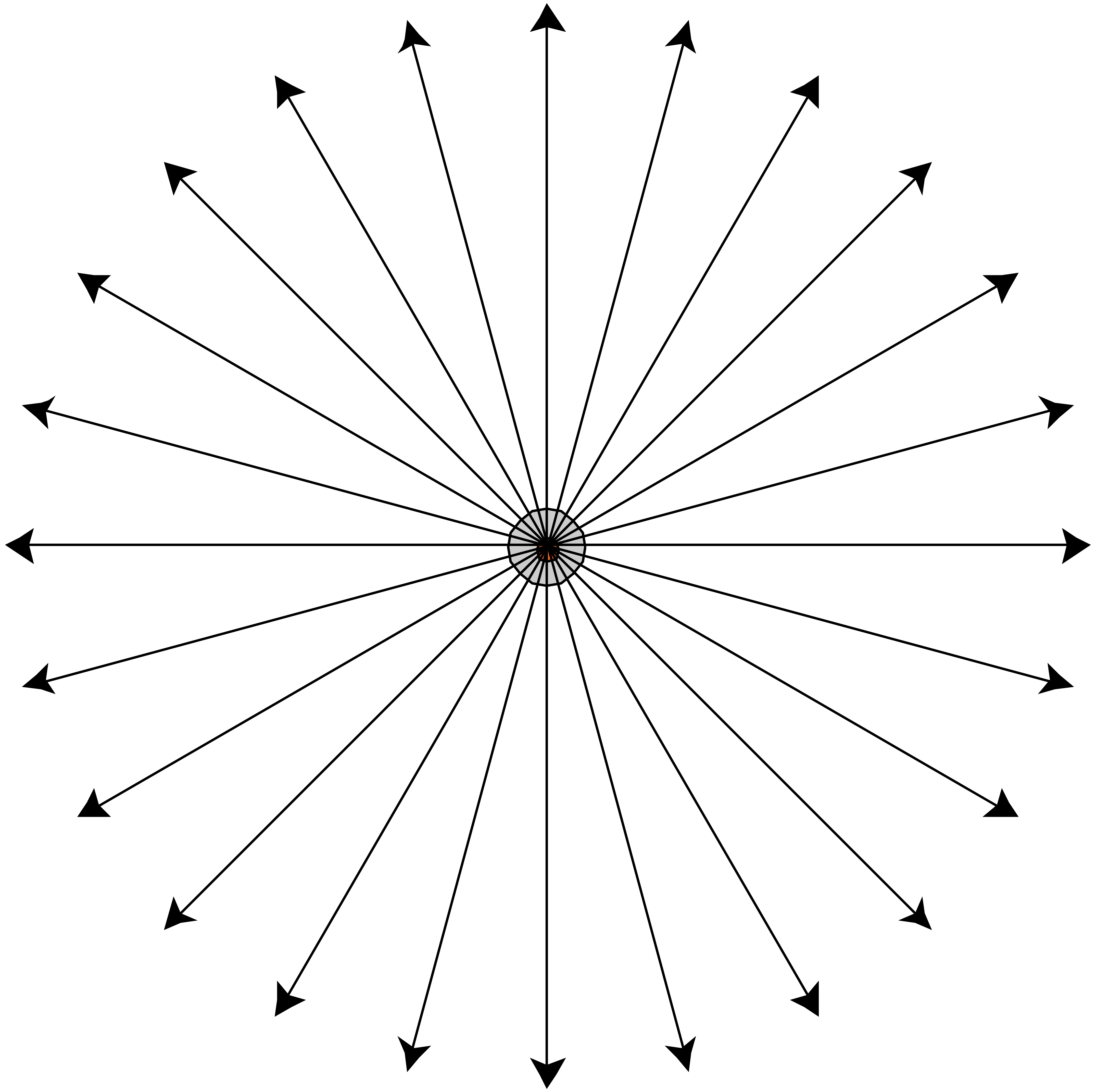
$$\mathrm{d}\omega = \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi$$

$$\begin{aligned} \Omega &= \int_{S^2} \mathrm{d}\omega \\ &= \int_0^{2\pi} \int_0^{\pi} \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi \\ &= 4\pi \end{aligned}$$

A sphere subtends 4π steradians.

Isotropic point source

Radiating total power Φ. Radiating with same intensity *I* in all directions.

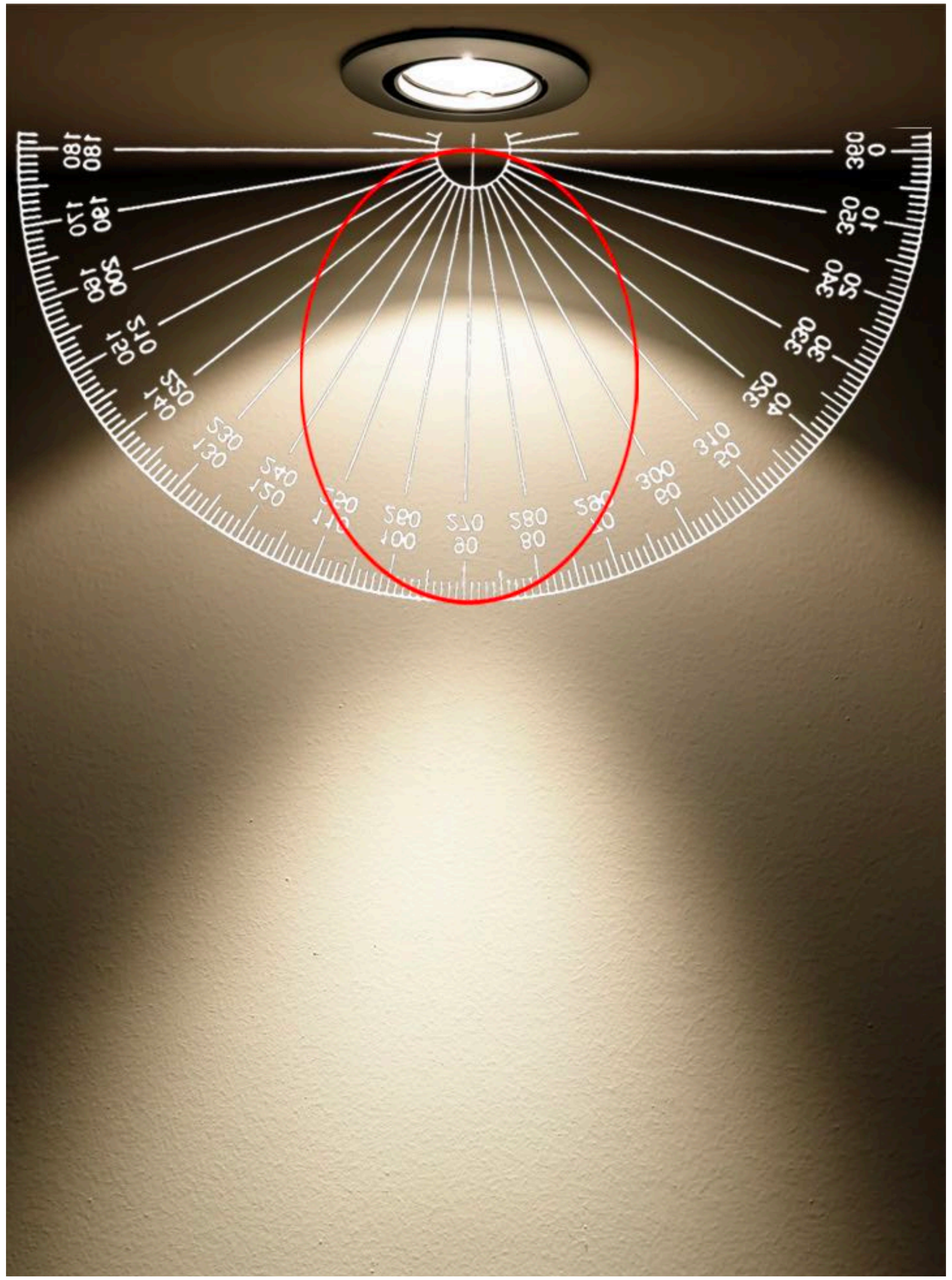


$$\Phi = \int_{S^2} I \, d\omega$$

$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

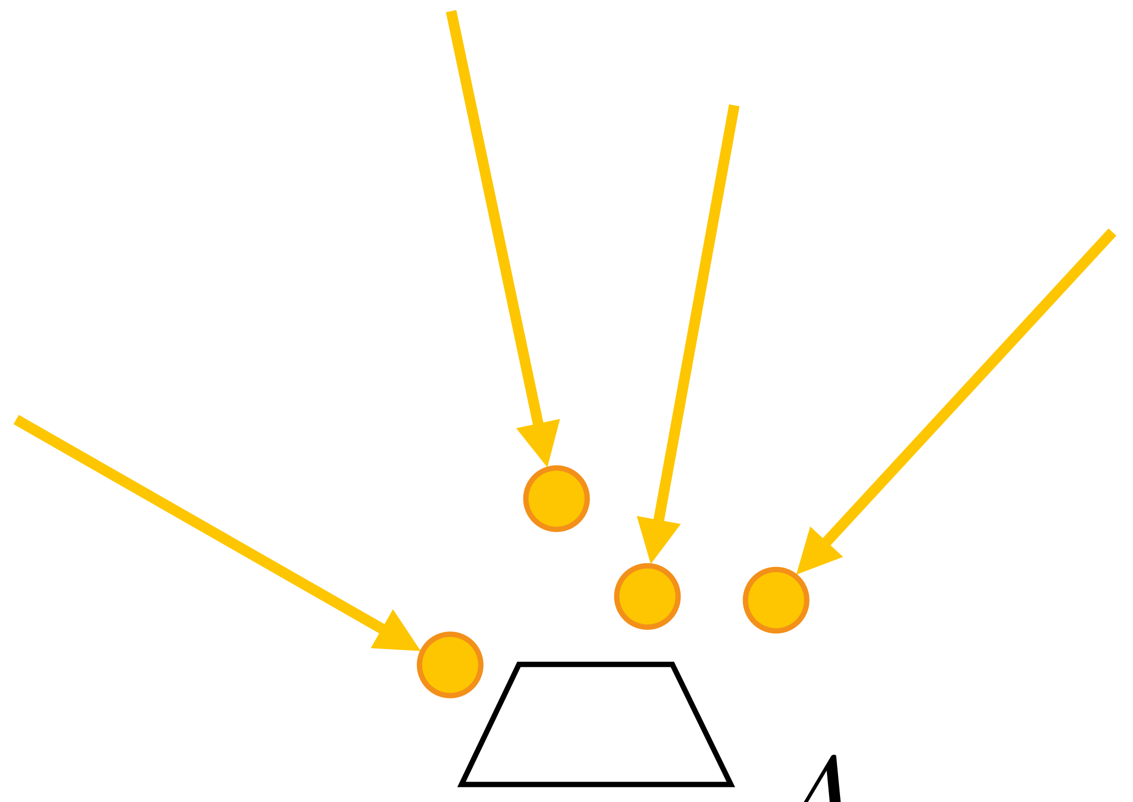
Anisotropic intensity distributions



http://www.photometrictesting.co.uk/File/understanding_photometric_data_files.php

Measuring illumination: irradiance

- **Flux: time density of energy**
- **Irradiance: area density of flux**



Given a sensor of with area *A*, we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

Irradiance (E) is given by taking the limit of area at a single point on the sensor:

$$E(\mathrm{p}) = \lim_{\Delta \to 0} \frac{\Delta\Phi(\mathrm{p})}{\Delta A} = \frac{\mathrm{d}\Phi(\mathrm{p})}{\mathrm{d}A} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Units = Watts per area

Beam power in terms of irradiance

Consider beam with flux Φ incident on surface with area A

$$E = \frac{\Phi}{A}$$

$$\Phi = EA$$

Projected area

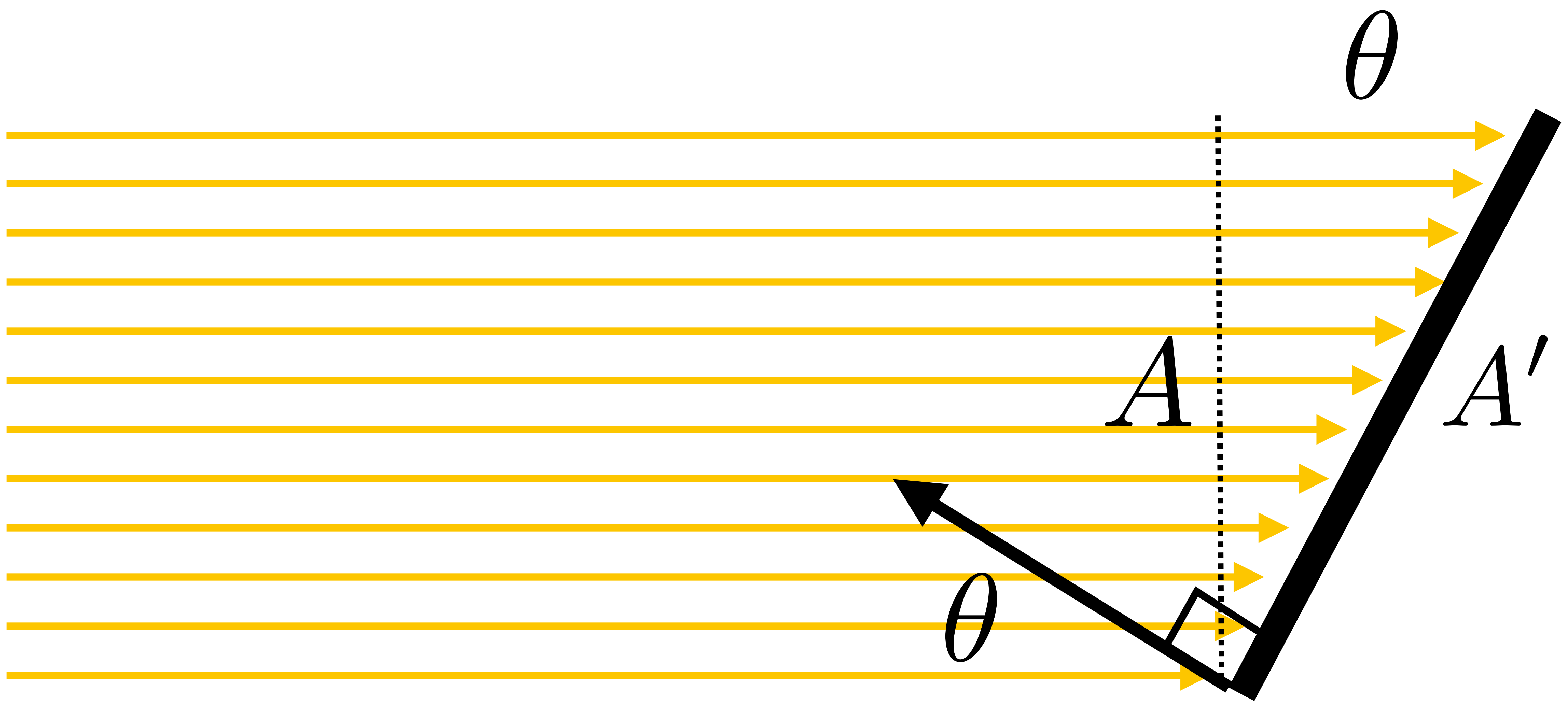
Consider beam with flux Φ incident on angled surface with area *A'*

$$A = A' \cos \theta$$

A = projected area of surface relative to direction of beam

Lambert's Law

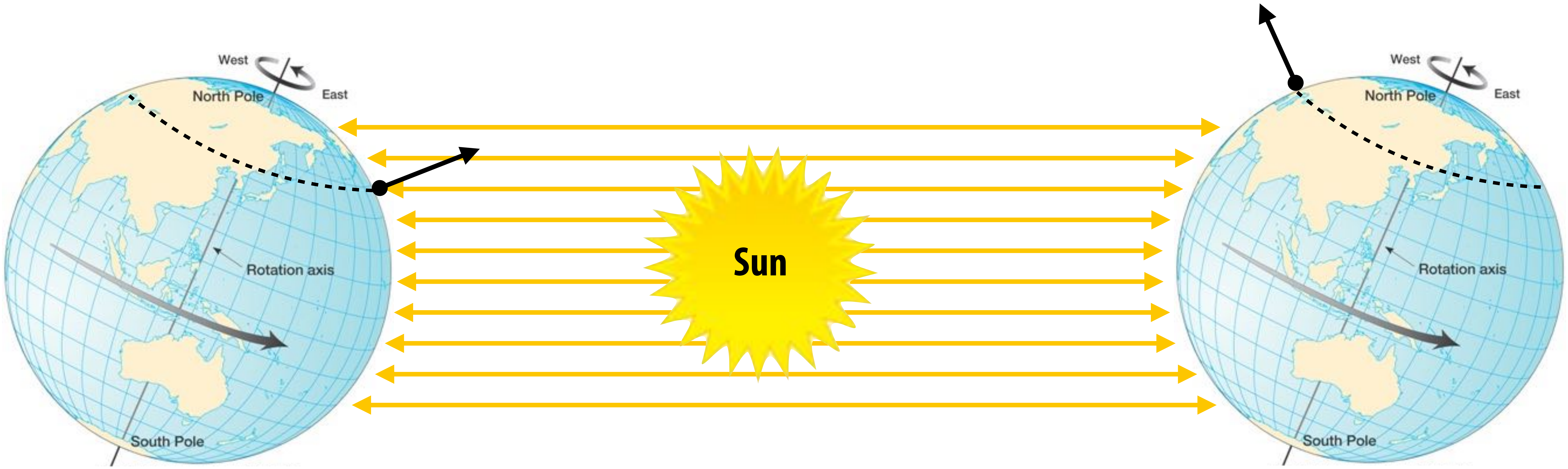
Irradiance at surface is proportional to cosine of angle between light direction and surface normal.



$$A = A' \cos\theta$$

$$E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$$

Why do we have seasons?



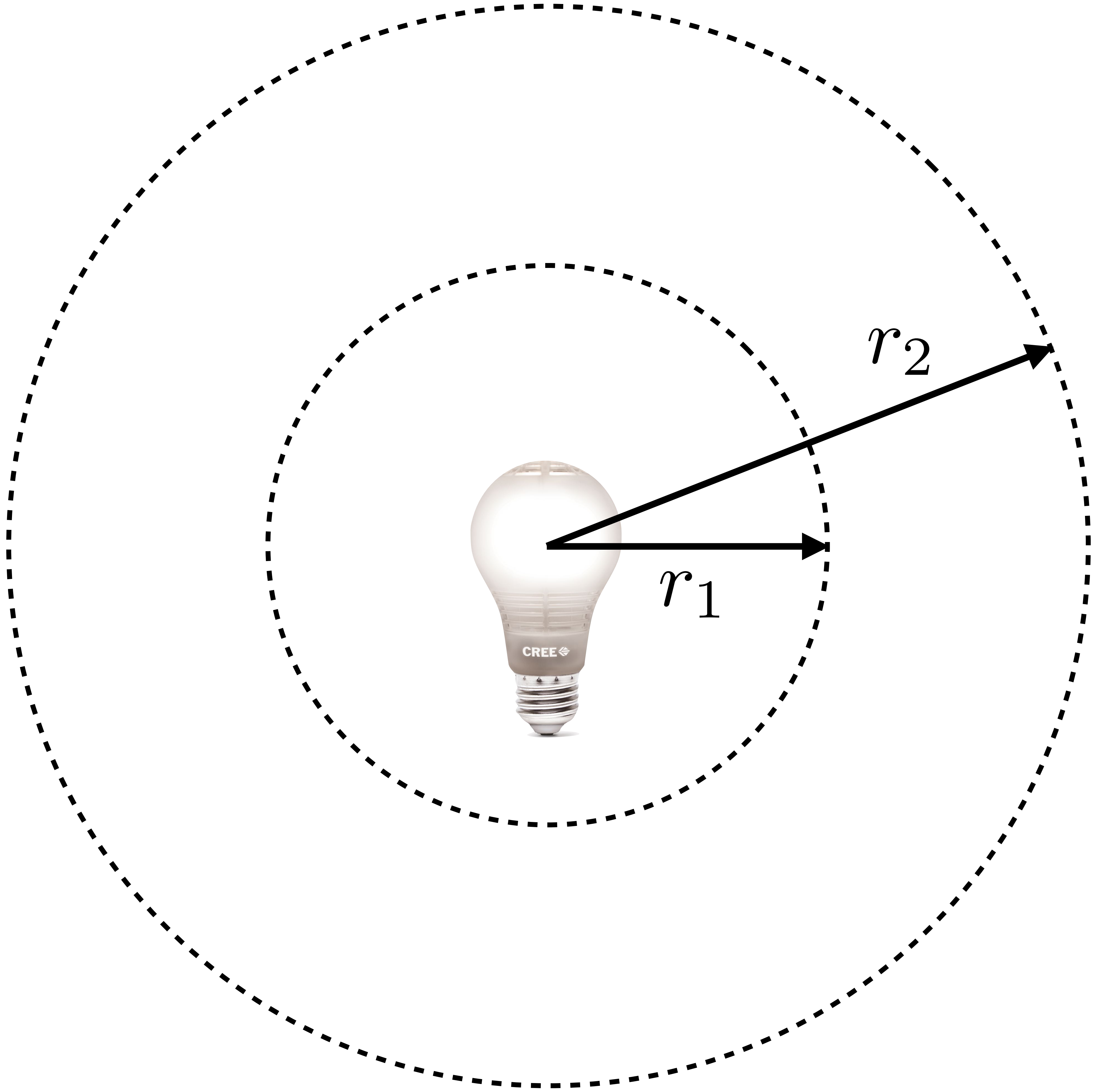
Summer
(Northern hemisphere)

Winter
(Northern hemisphere)

Earth's axis of rotation: ~23.5° off axis

[Image credit: Pearson Prentice Hall]

Irradiance falloff with distance



Assume light is emitting flux Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:

$$E_1 = \frac{\Phi}{4\pi r_1^2}$$

$$E_2 = \frac{\Phi}{4\pi r_2^2}$$

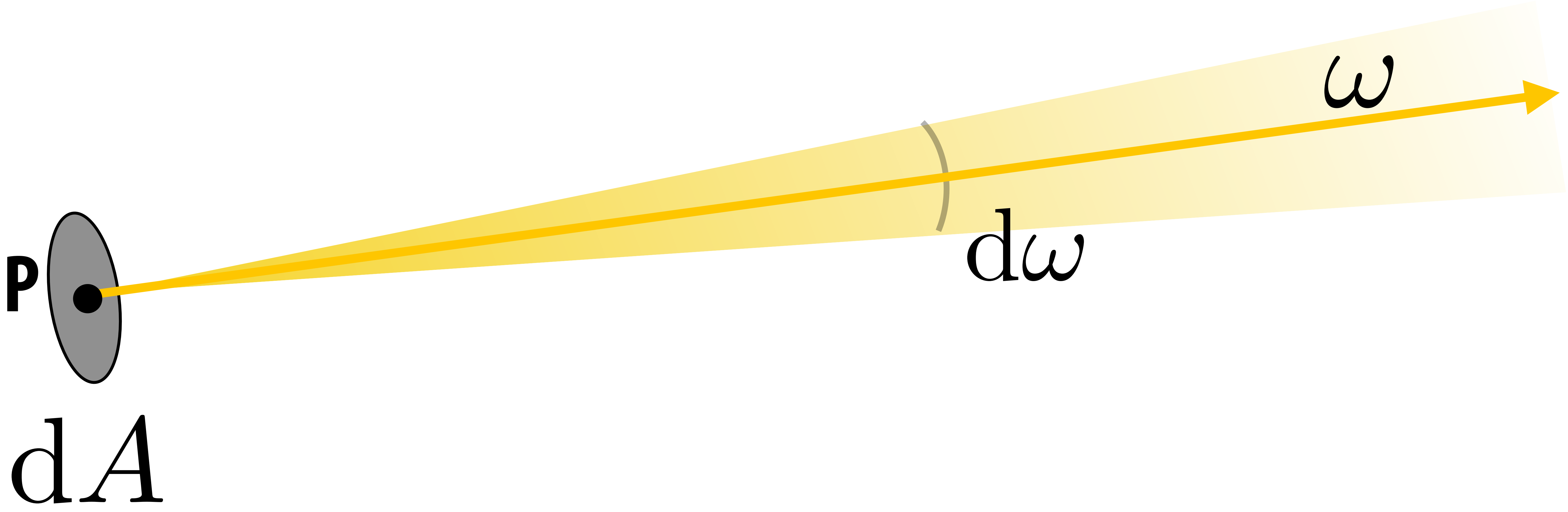
$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$

Why does a room get darker farther from a light source?

Image credit: LeRamz on Flickr

Measuring illumination: radiance

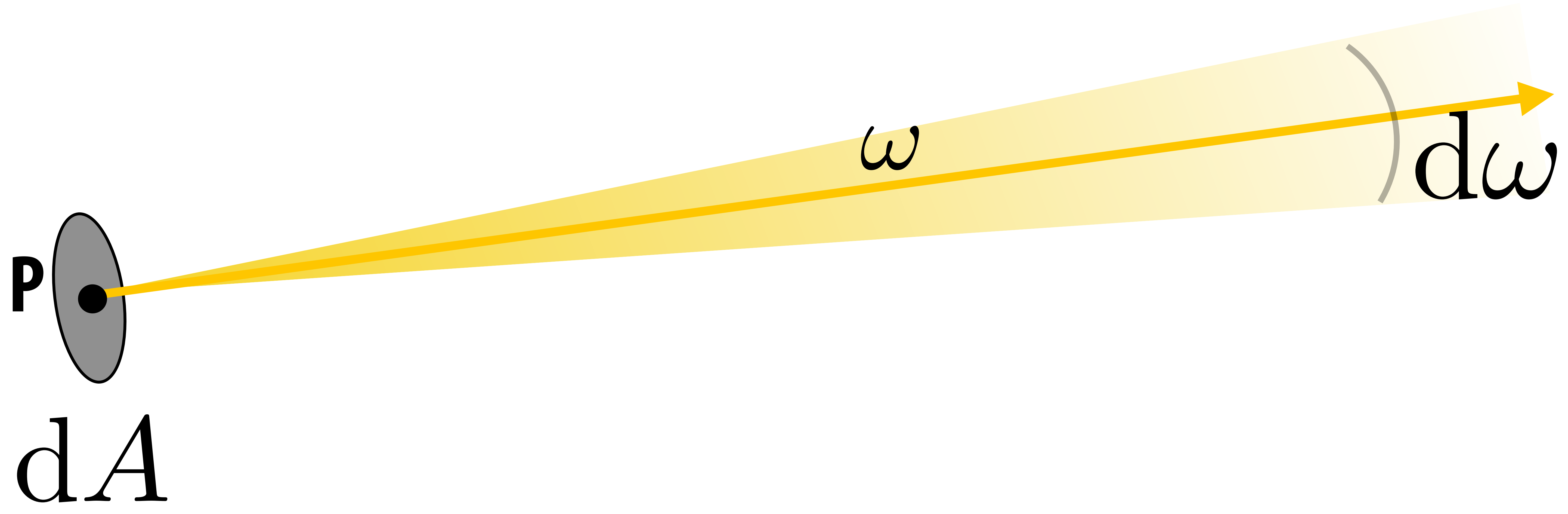
- **Radiance (L) is the solid angle density of irradiance (irradiance per unit direction) where the differential surface area is oriented to face in the direction ω**



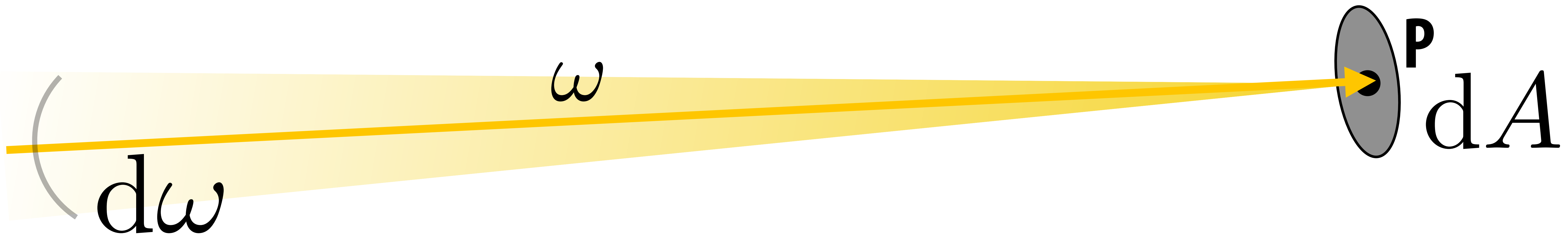
In other words, radiance is energy along a ray defined by origin point *p* and direction ω

$$L(\mathrm{p}, \omega) = \lim_{\Delta \to 0} \frac{\Delta \Phi(\mathrm{p}, \omega)}{\Delta A \Delta \omega} = \frac{\mathrm{d}^2 \Phi(\mathrm{p}, \omega)}{\mathrm{d}A \, \mathrm{d}\omega}$$

Radiance as energy in an infinitesimal small beam



Outgoing from a region: energy leaving an tiny patch of area leaving in the direction of a tiny solid angle



Incoming to a region: Energy arriving at a tiny patch of area from a tiny solid angle.

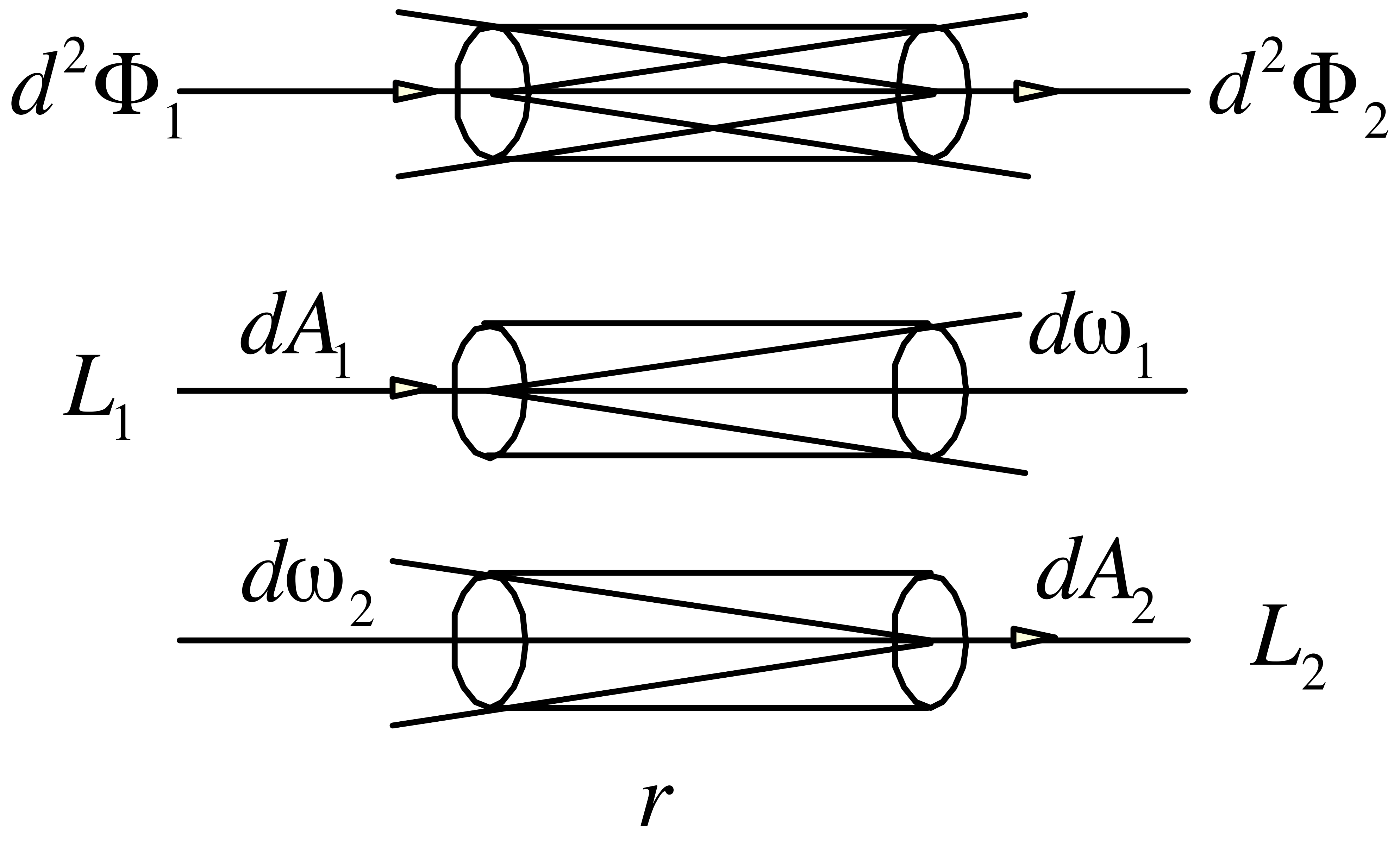
Properties of radiance

- Fundamental field quantity that characterizes the distribution of light in an environment
    - Radiance is the quantity associated with a ray
    - Ray tracers compute the radiance along a ray

- If we assume ray travel through a vacuum, radiance is invariant along a ray
    - For now, we won't consider "participating media" like fog, smoke, clouds, dust, etc.

1st Law: conservation of radiance

The radiance in the direction of a light ray remains constant as the ray propagates through empty space



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

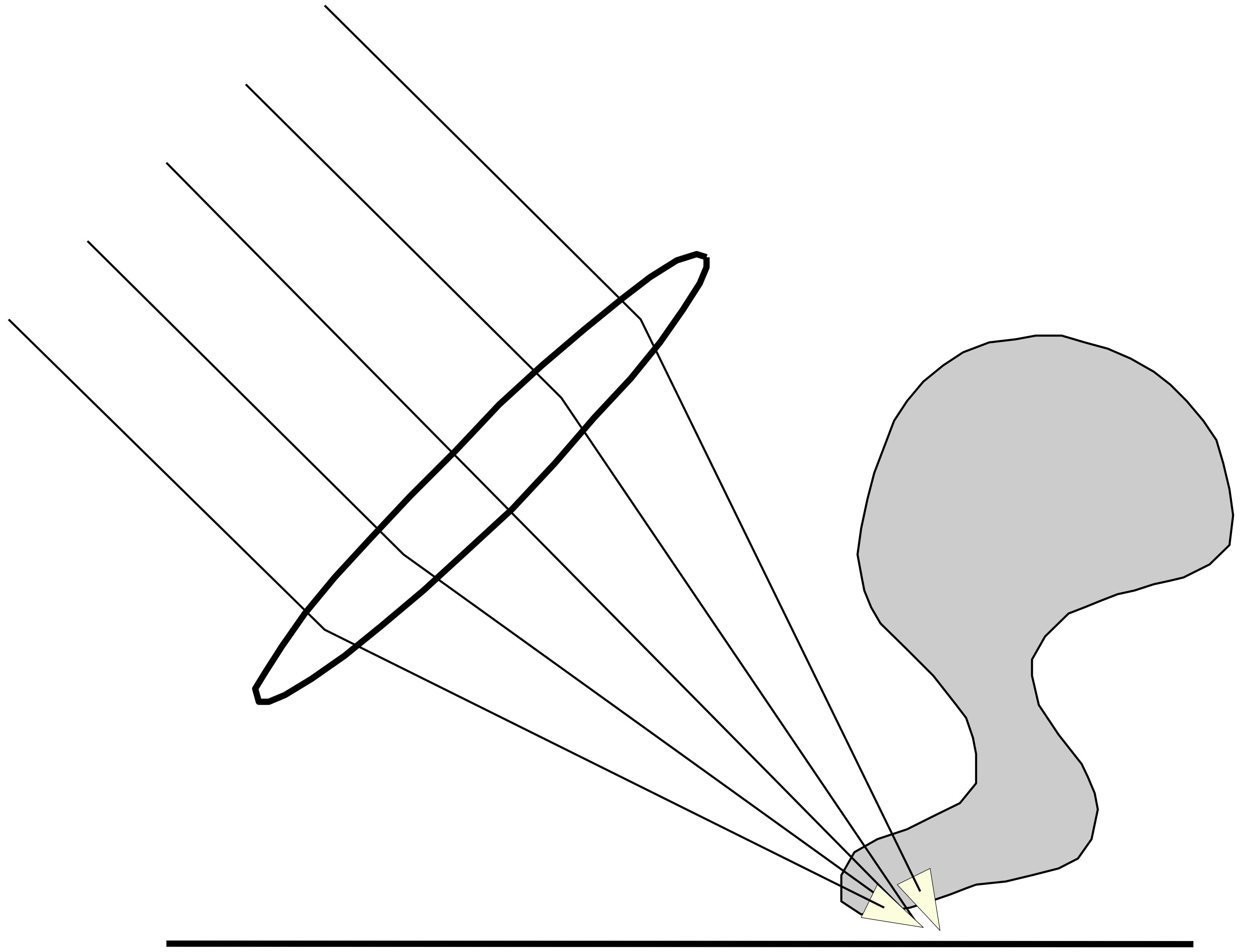
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

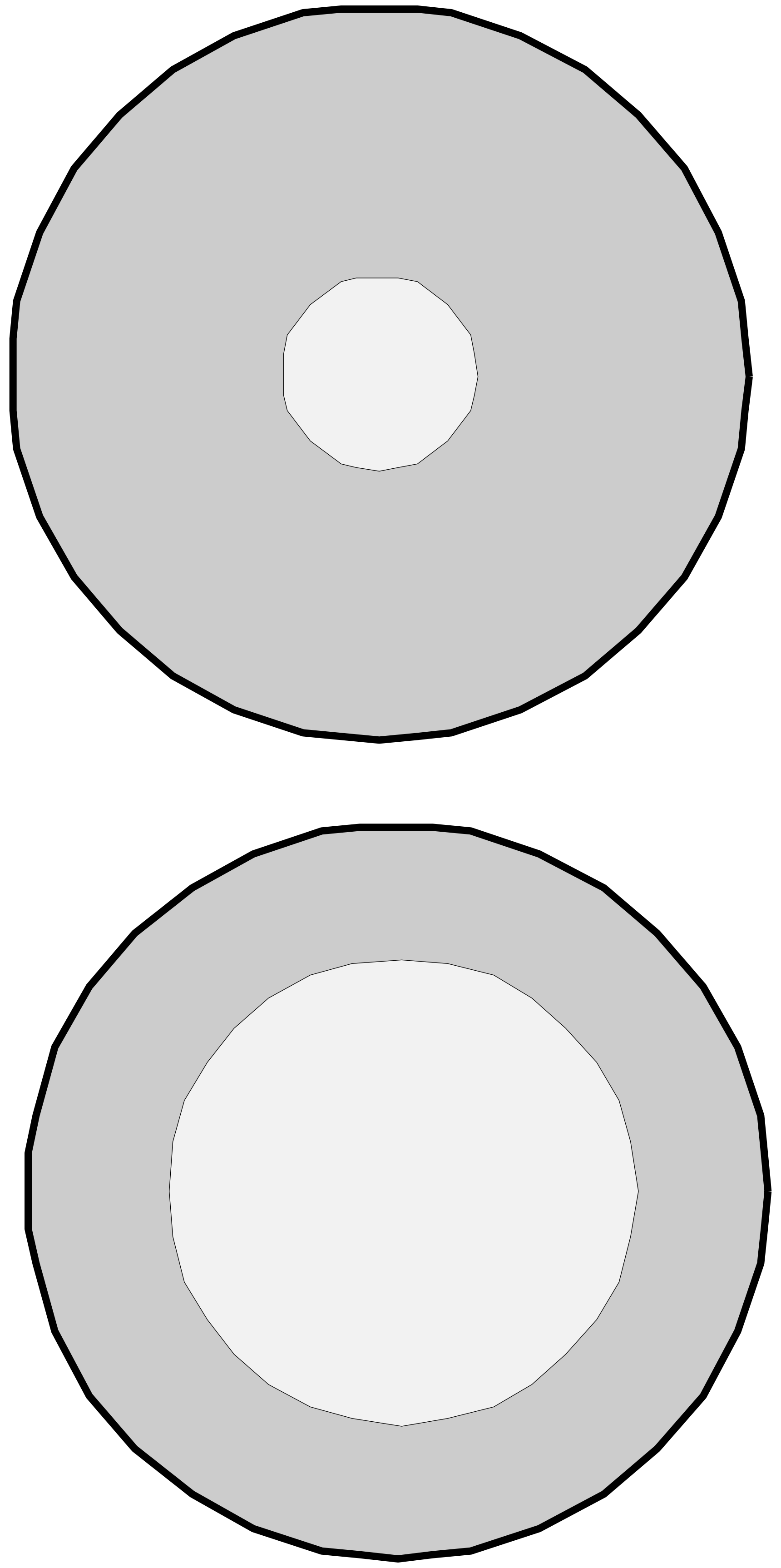
$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



No!!



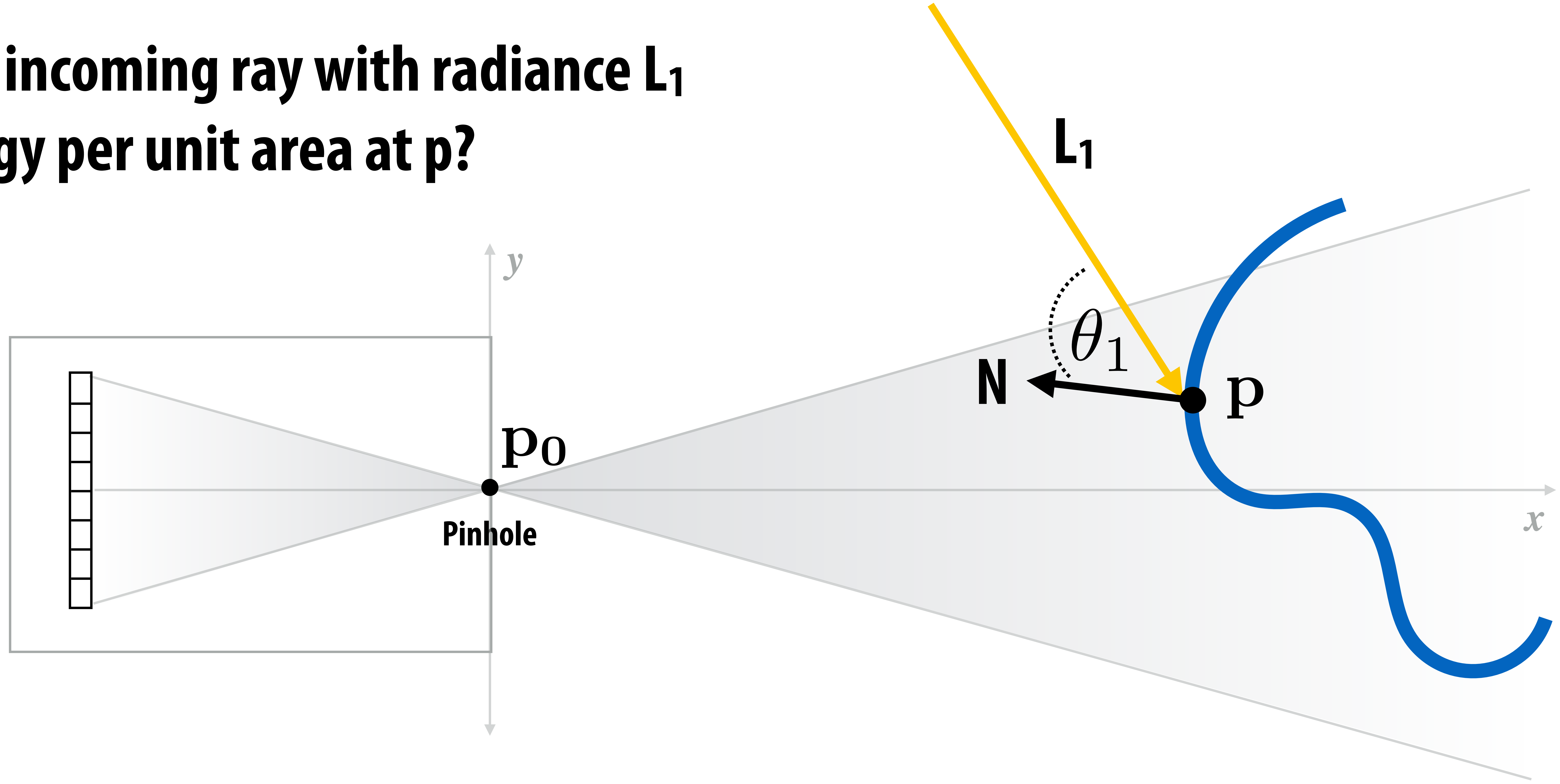
How much light hits the surface at point p?

(What is irradiance at point p?)

Imagine one incoming ray with radiance L_1

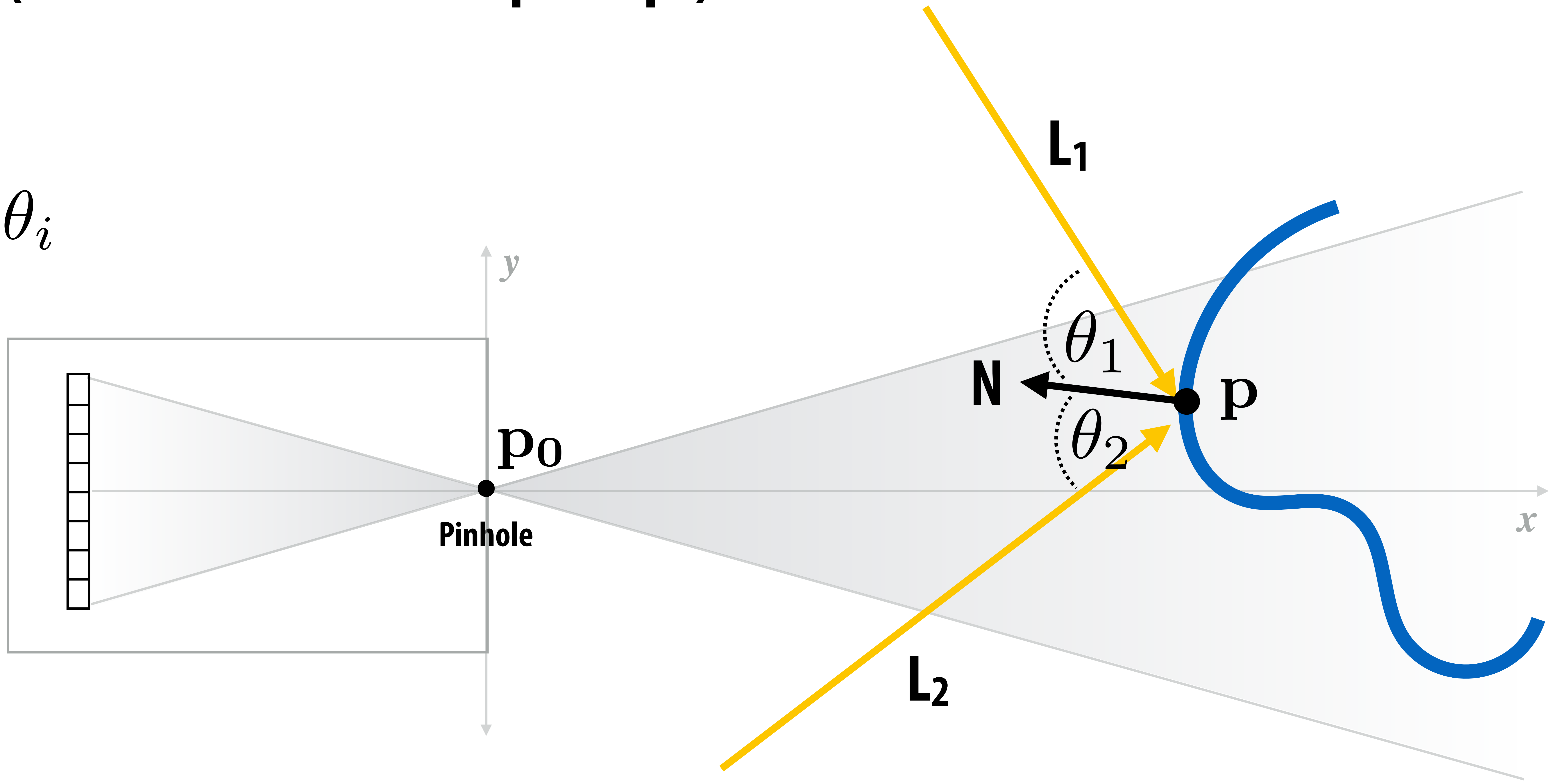
What is energy per unit area at p?

$$L_1 \cos\theta_1$$



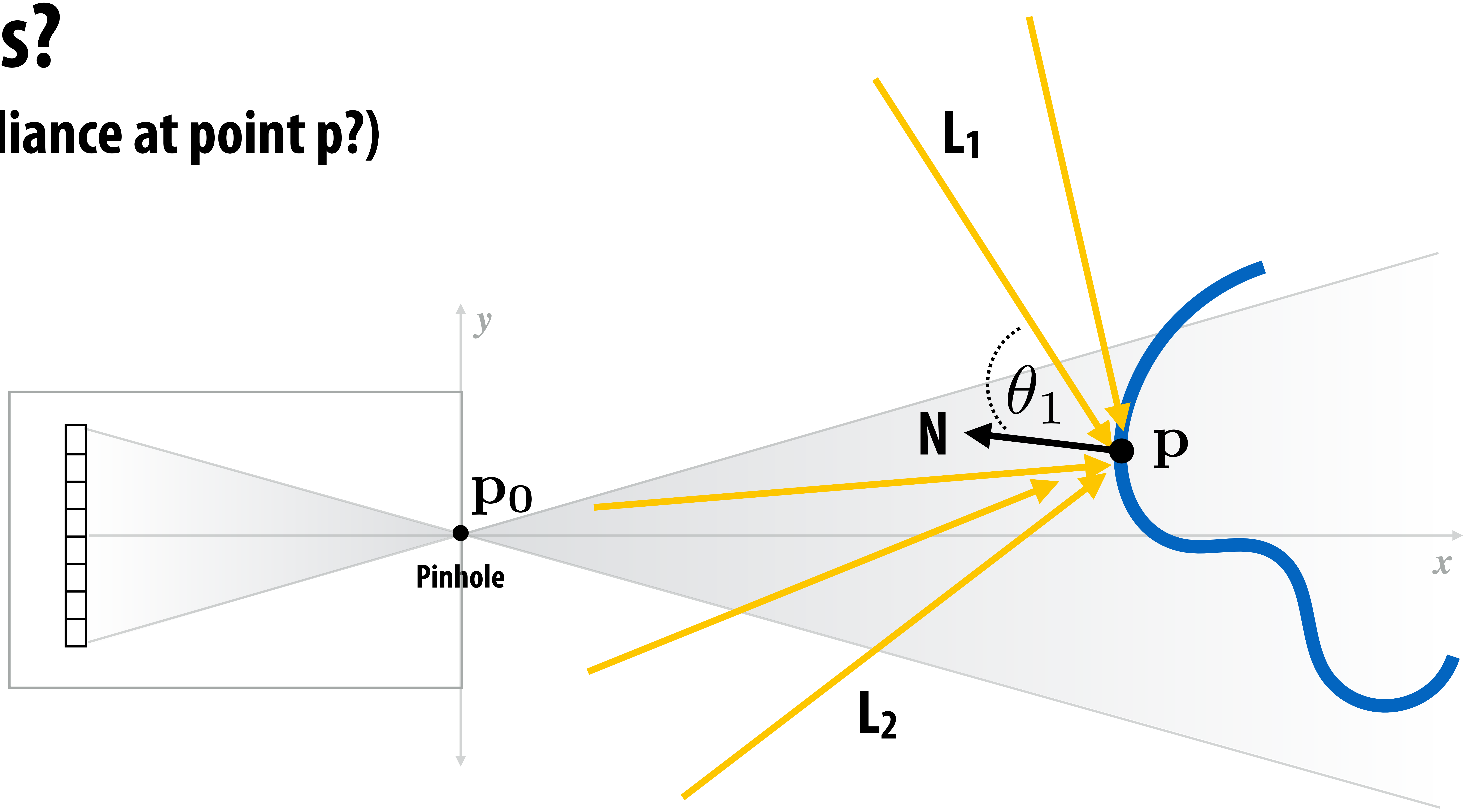
How much light hits the surface at point p from multiple light sources? (What is irradiance at point p?)

$$\sum_i L_i \cos\theta_i$$



How much light hits the surface at point p from light from all directions?

(What is irradiance at point p?)

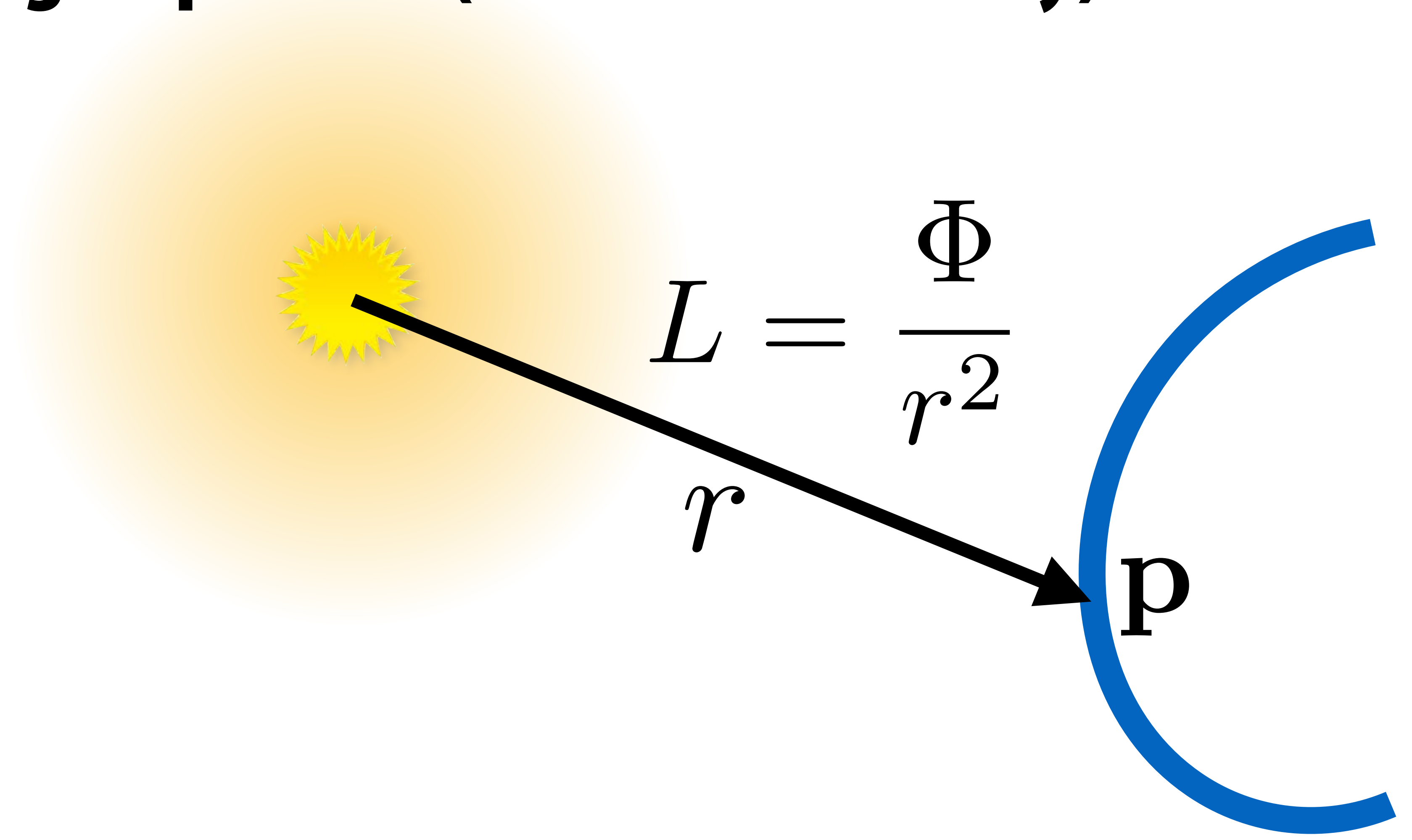


$$\int_{S^2} L(\omega_i)\cos\theta_i \, d\omega_i = \int_0^{2\pi}\int_0^{\pi} L(\theta_i,\phi_i)\cos\theta_i \sin\theta_i \, d\theta_i d\phi_i$$

Types of lights

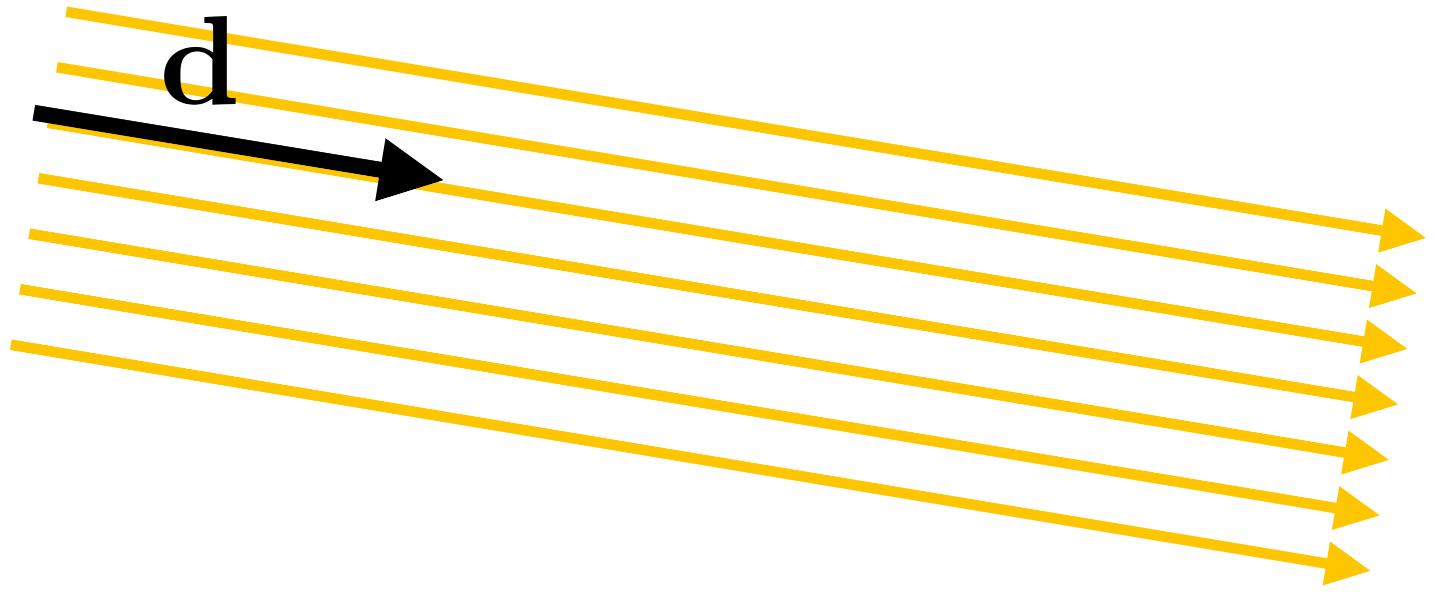
- **Attenuated omnidirectional point light**

  (emits equally in all directions, energy arriving at point P (radiant intensity) falls off with distance: $1/R^2$ falloff)



- **Infinite directional light in direction *d***

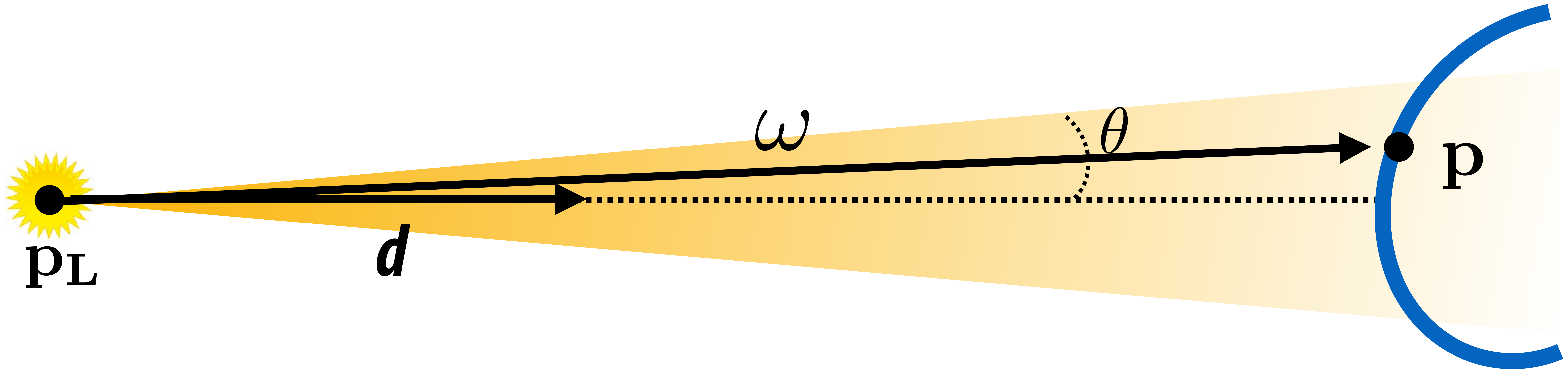
  (infinitely far away, all points in scene receive light with radiance L from direction *d*



Spot light

Does not emit equally in all directions…

intensity falls off in directions away from main spotlight direction *d*



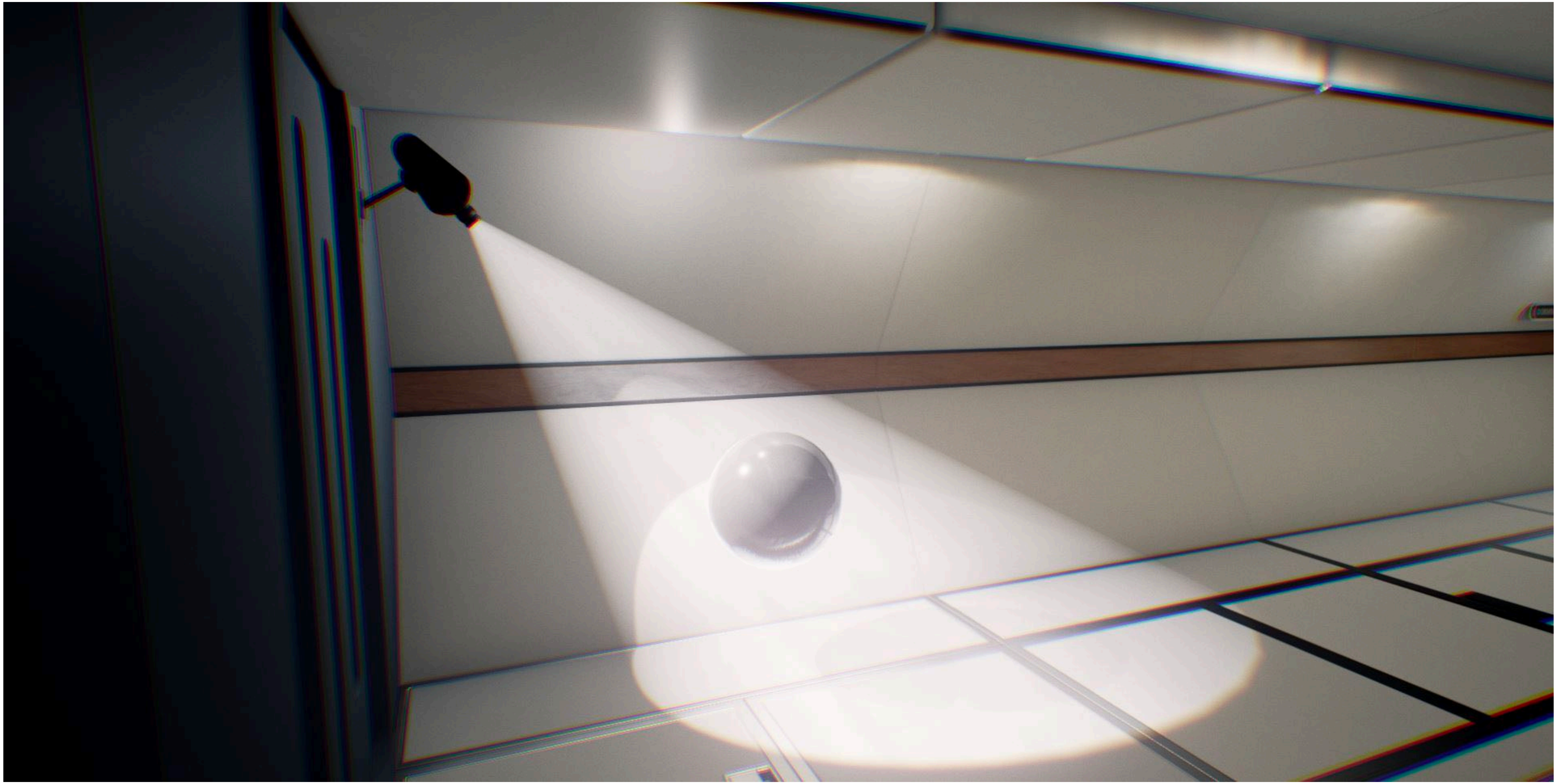
$$\omega = \text{normalize}(\mathbf{p} - \mathbf{p_L})$$

$$L(\omega) = 0 \quad \textbf{if } \omega \cdot \mathbf{d} > \cos\theta$$
$$= L_0 \quad \textbf{otherwise}$$

Or, if spotlight intensity falls off from direction d

$$L(\omega) \propto \omega \cdot \mathbf{d}$$

Spot light

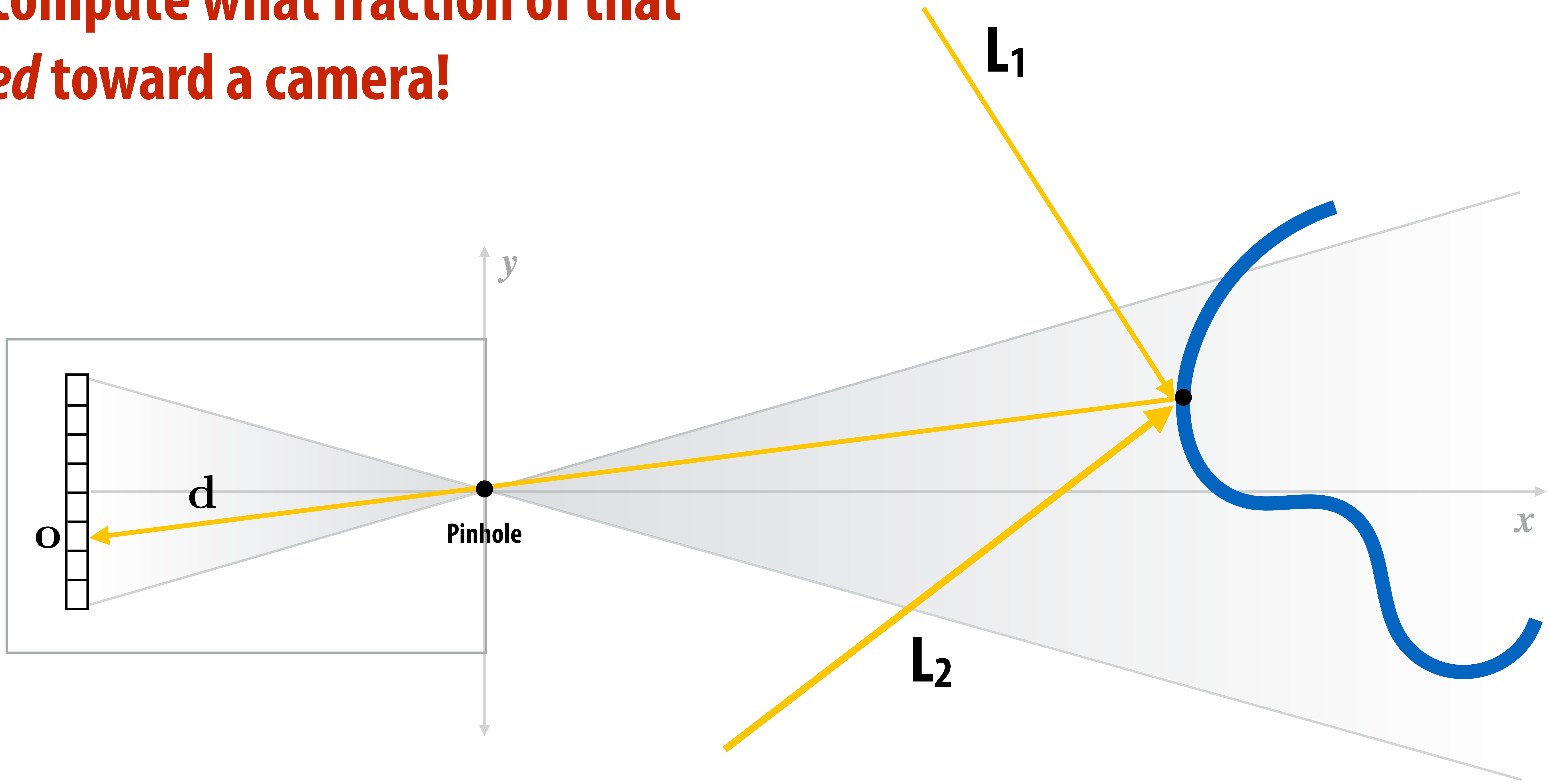


Environment light (represented by a texture map)

Pixel (x,y) stores radiance L from direction (ϕ, θ)

So far… we've discussed how to compute the light arriving at a surface point (radiance along incoming ray)

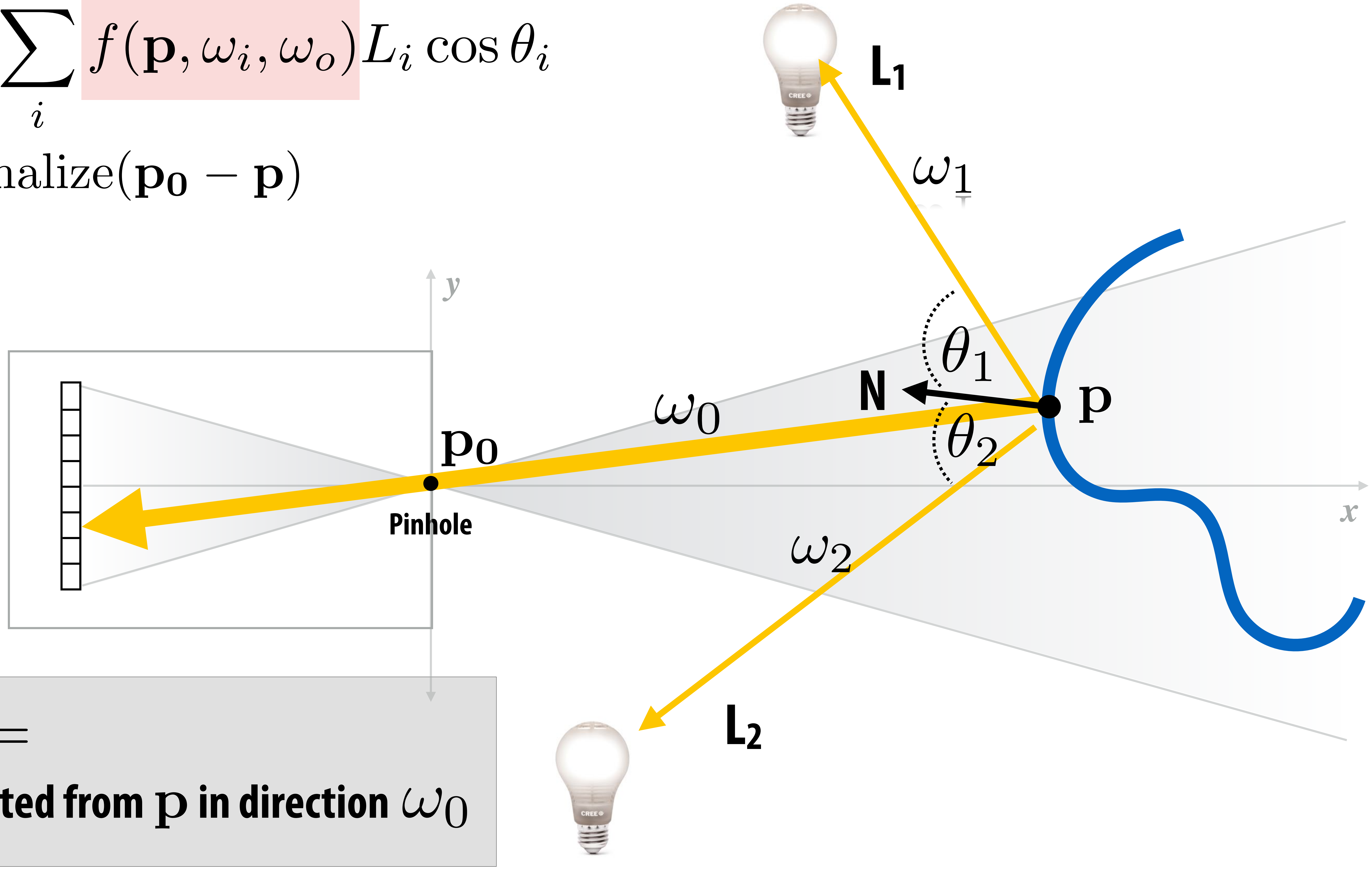
But goal is to compute what fraction of that light is *reflected* toward a camera!



How much light is REFLECTED from p toward p_0?

$$L(\mathbf{p}, \omega_o) = \sum_i f(\mathbf{p}, \omega_i, \omega_o) L_i \cos\theta_i$$

$$\omega_o = \text{normalize}(\mathbf{p_0} - \mathbf{p})$$



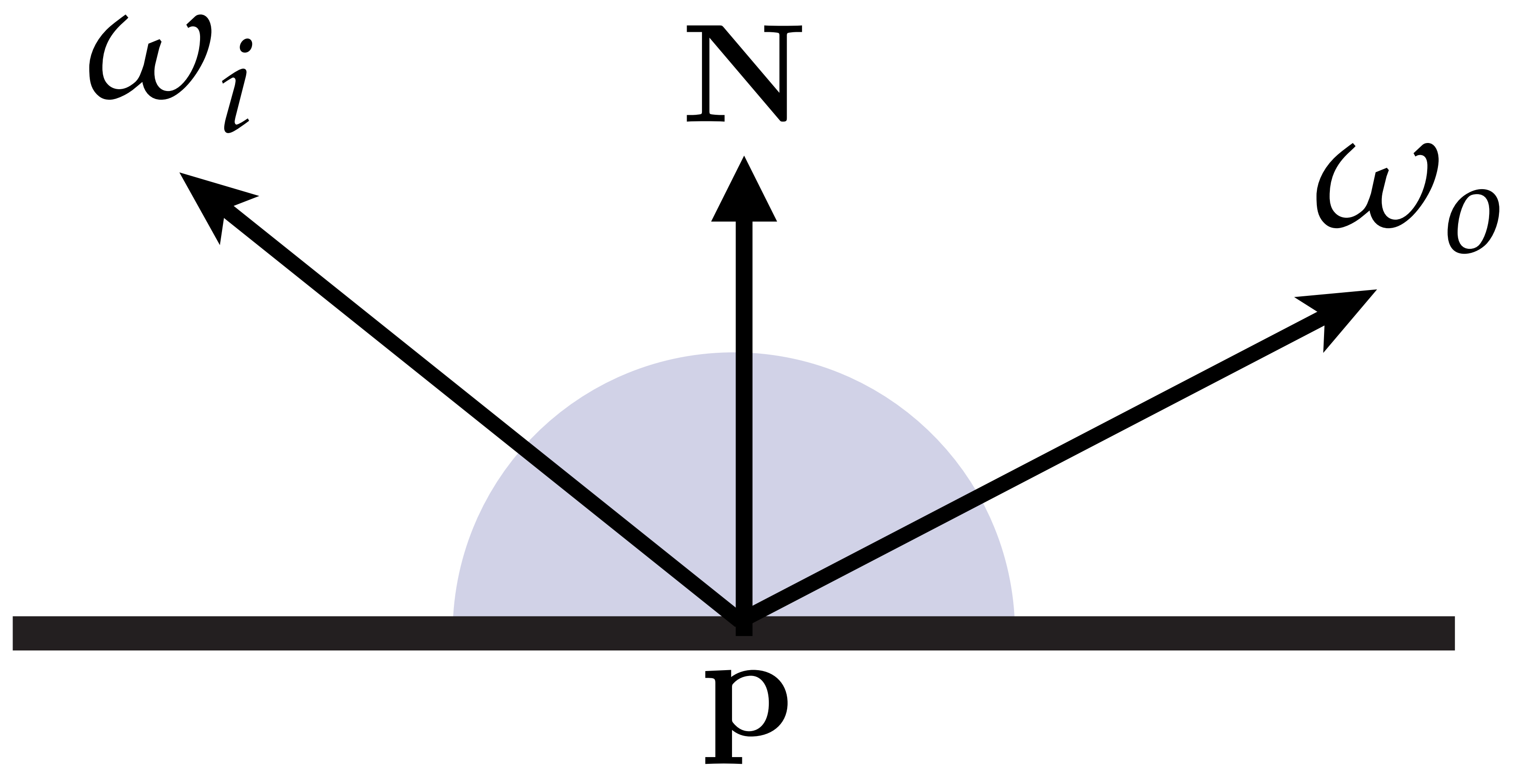
$L(\mathbf{p}, \omega_o) =$

Radiance reflected from $\mathbf{p}$ in direction ω_0

Bidirectional reflectance distribution function (BRDF)

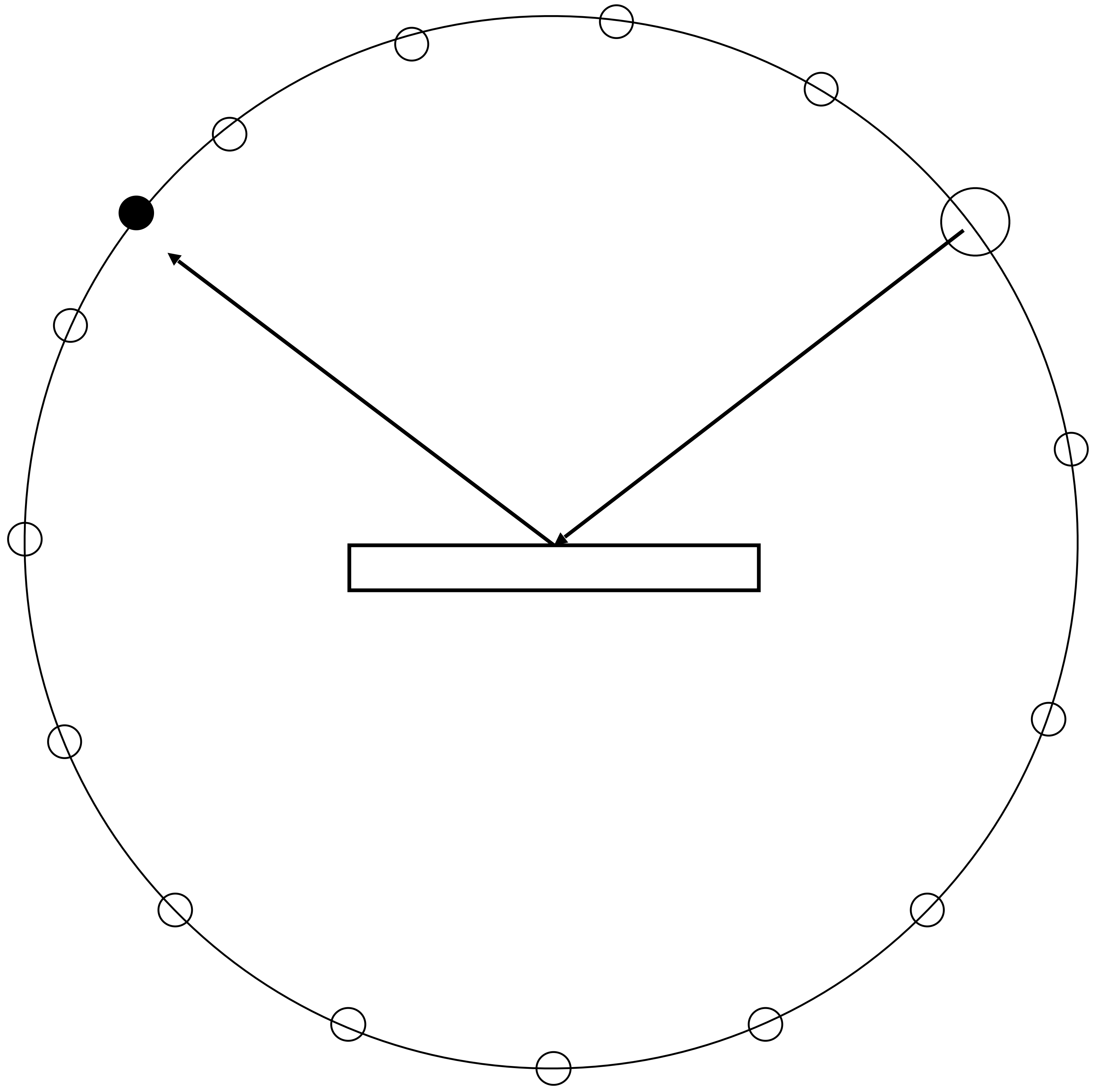
Gives fraction of light arriving at surface point p from incoming direction* ω_i is reflected in the direction ω_0 (outgoing direction)

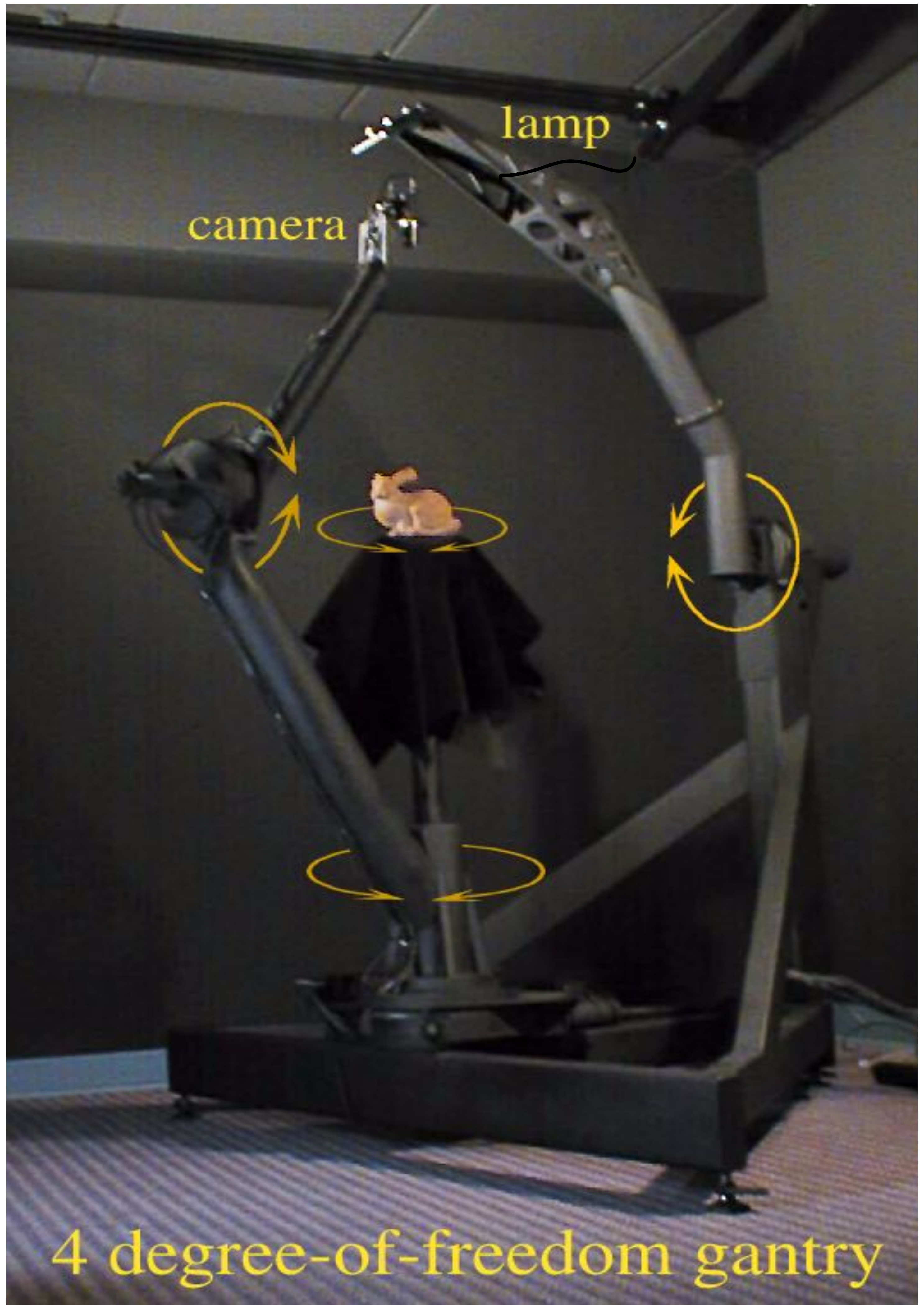
$$f(\mathbf{p}, \omega_i, \omega_o)$$



*** (Convention: ω_i is oriented out from the surface "towards the incoming direction")**

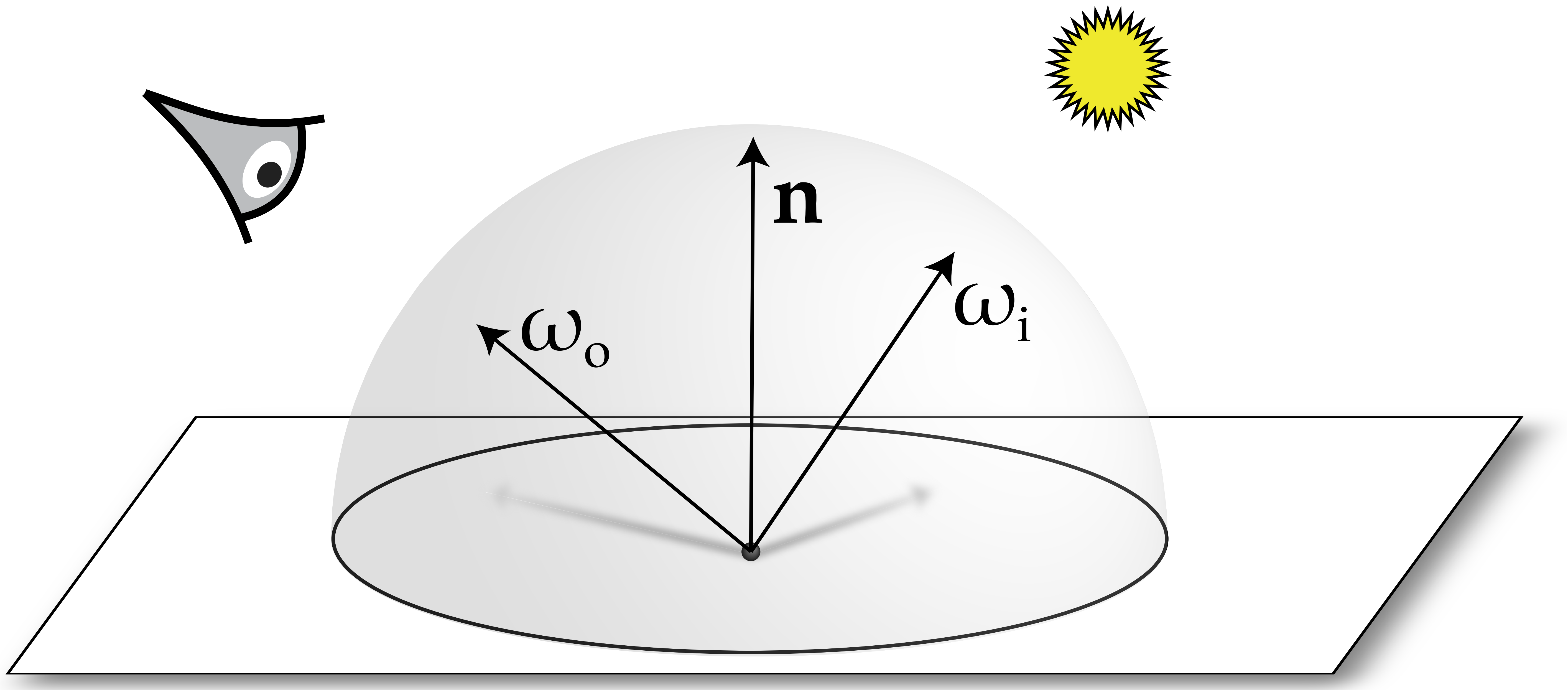
Measuring BRDFs: the Stanford Gonioreflectometer





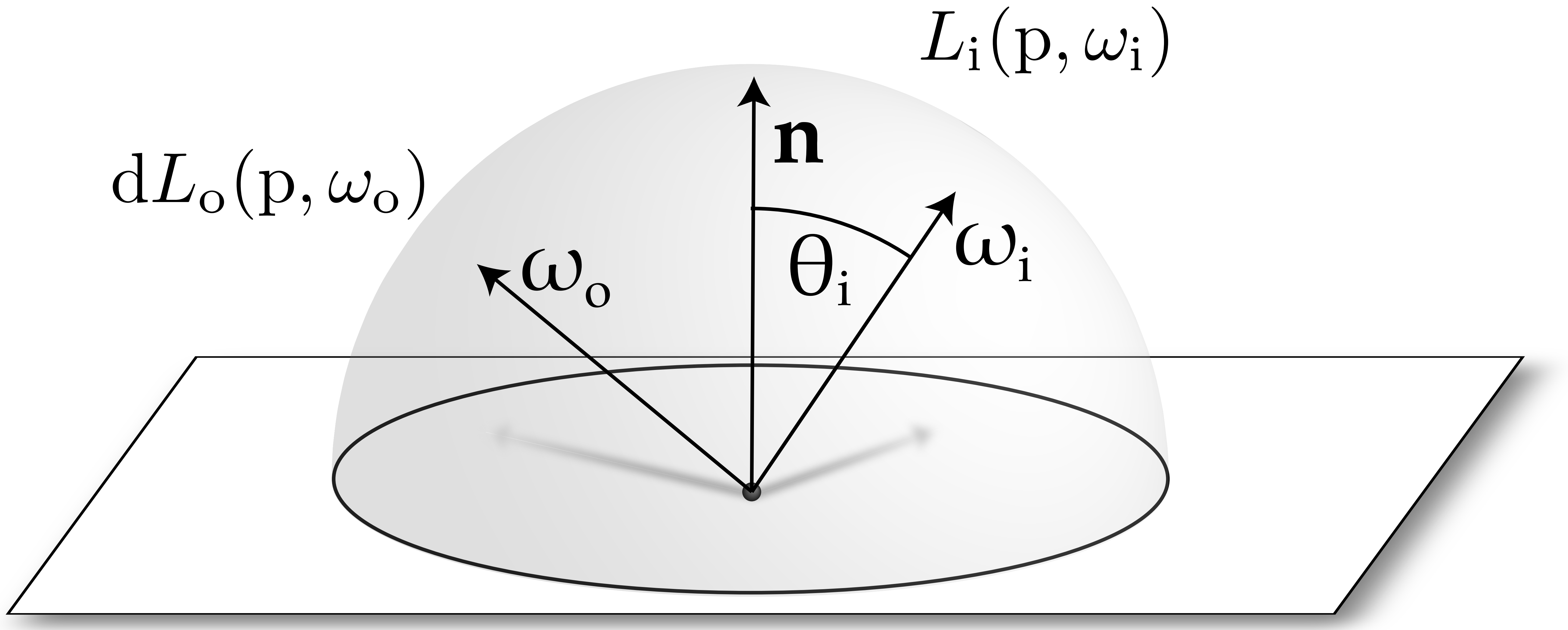
The reflection equation

Gives radiance reflected from point p in direction direction ω_0 due to light incident on the surface at p.



$$L_o(p, \omega_o) = \int_{\Omega^2} \underbrace{f_r(p, \omega_i \rightarrow \omega_o)}_{\textbf{BRDF}} \underbrace{L_i(p, \omega_i) \, \cos\theta_i}_{\textbf{Illumination}} \, d\omega_i$$

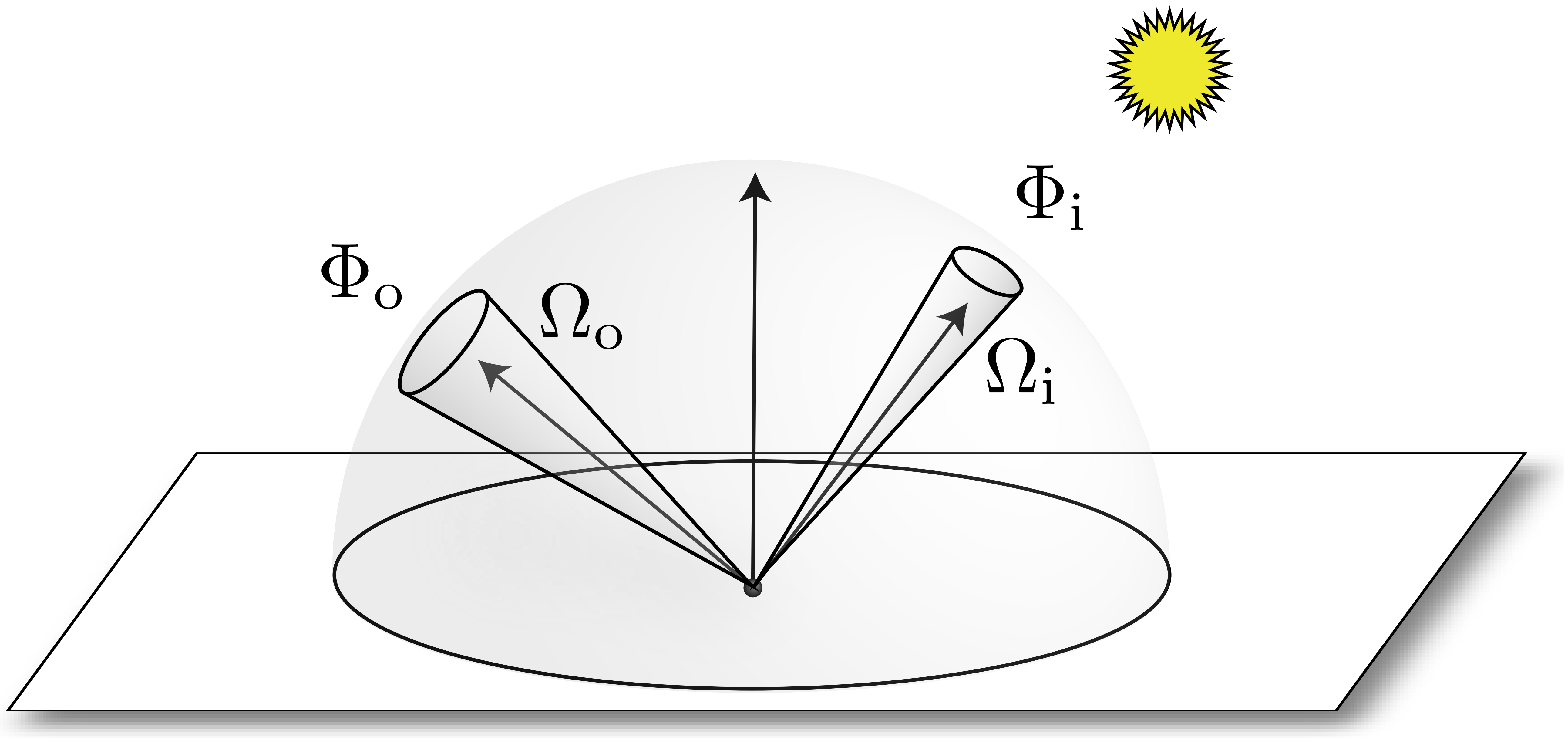
Units of the BRDF



$$f_{\mathrm{r}}(\omega_{\mathrm{i}} \rightarrow \omega_{\mathrm{o}}) \equiv \frac{\mathrm{d}L_{\mathrm{o}}(\omega_{\mathrm{o}})}{\mathrm{d}E_{\mathrm{i}}(\omega_{\mathrm{i}})} \quad \left[\frac{1}{sr}\right]$$

“For a given change in incident irradiance, how much does exit radiance change”

BRDF energy conservation



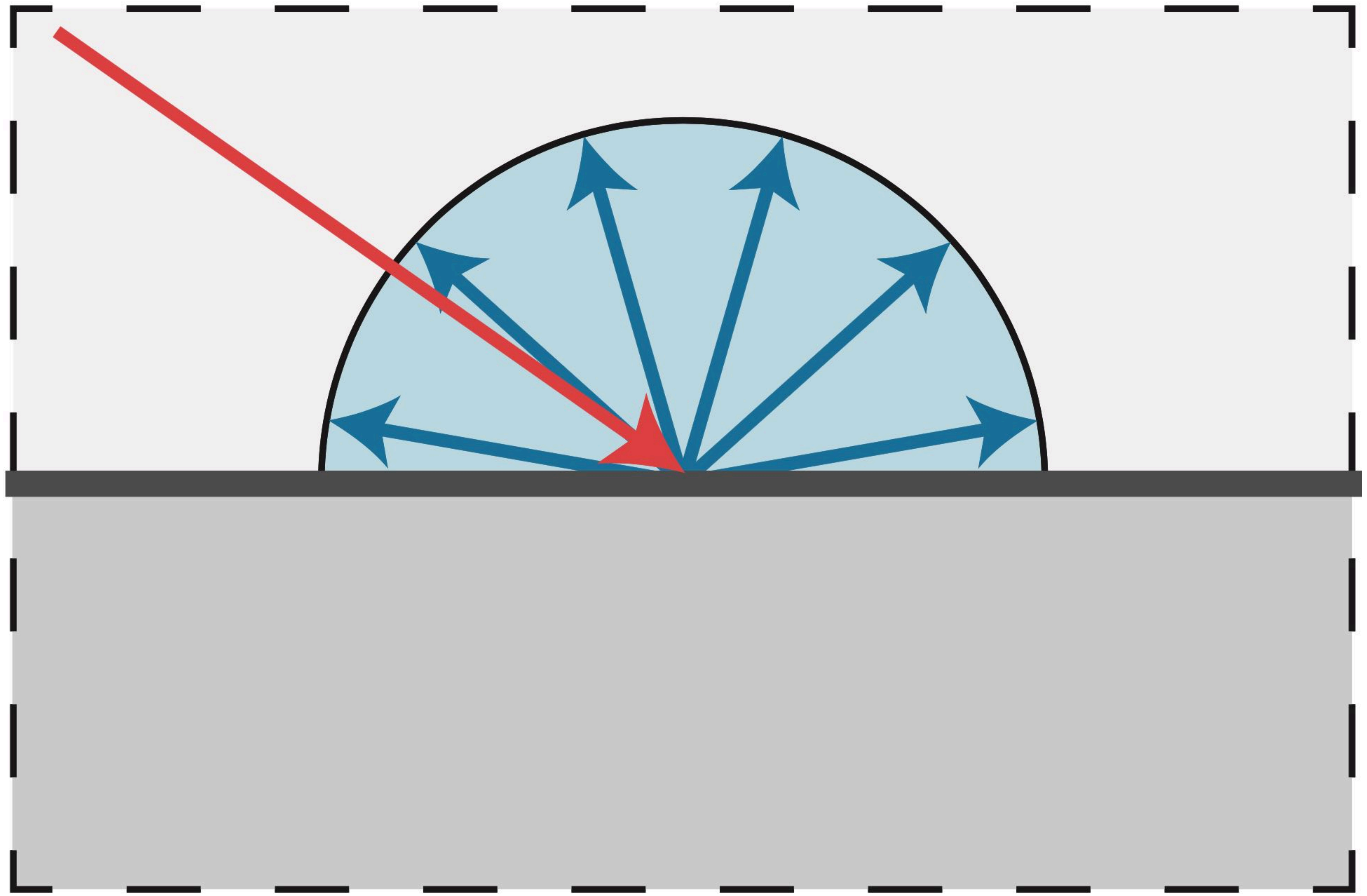
Reflectance $$\rho = \frac{\Phi_o}{\Phi_i} = \frac{\int_{\Omega_o} L_o(\omega_o)\cos\theta_o\,d\omega_o}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\,d\omega_i}$$

$$0 \le \rho \le 1$$

Reflection models

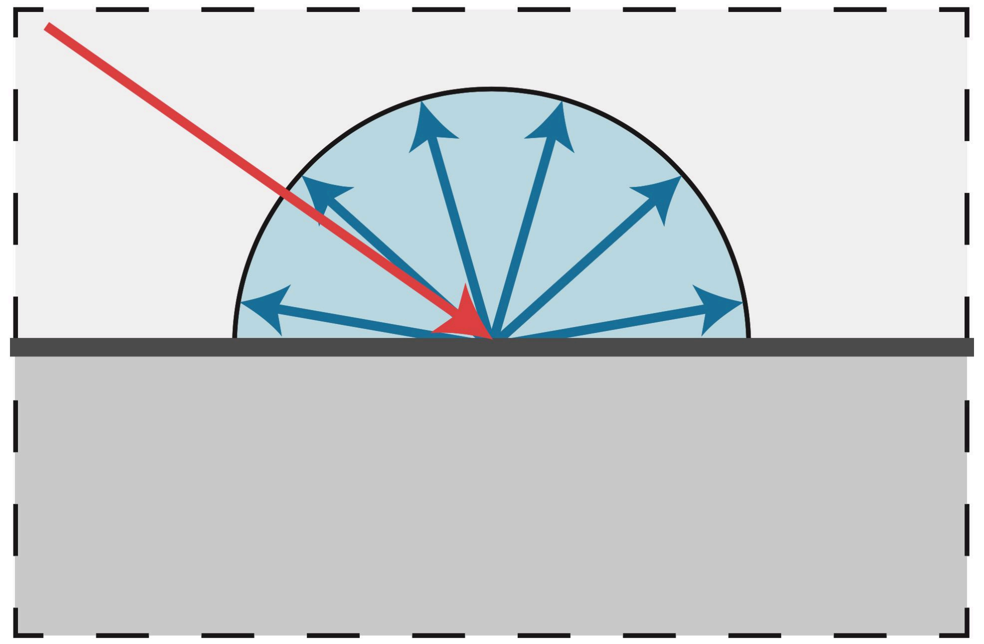
- *Reflection* is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency
- Choice of reflection function determines surface appearance

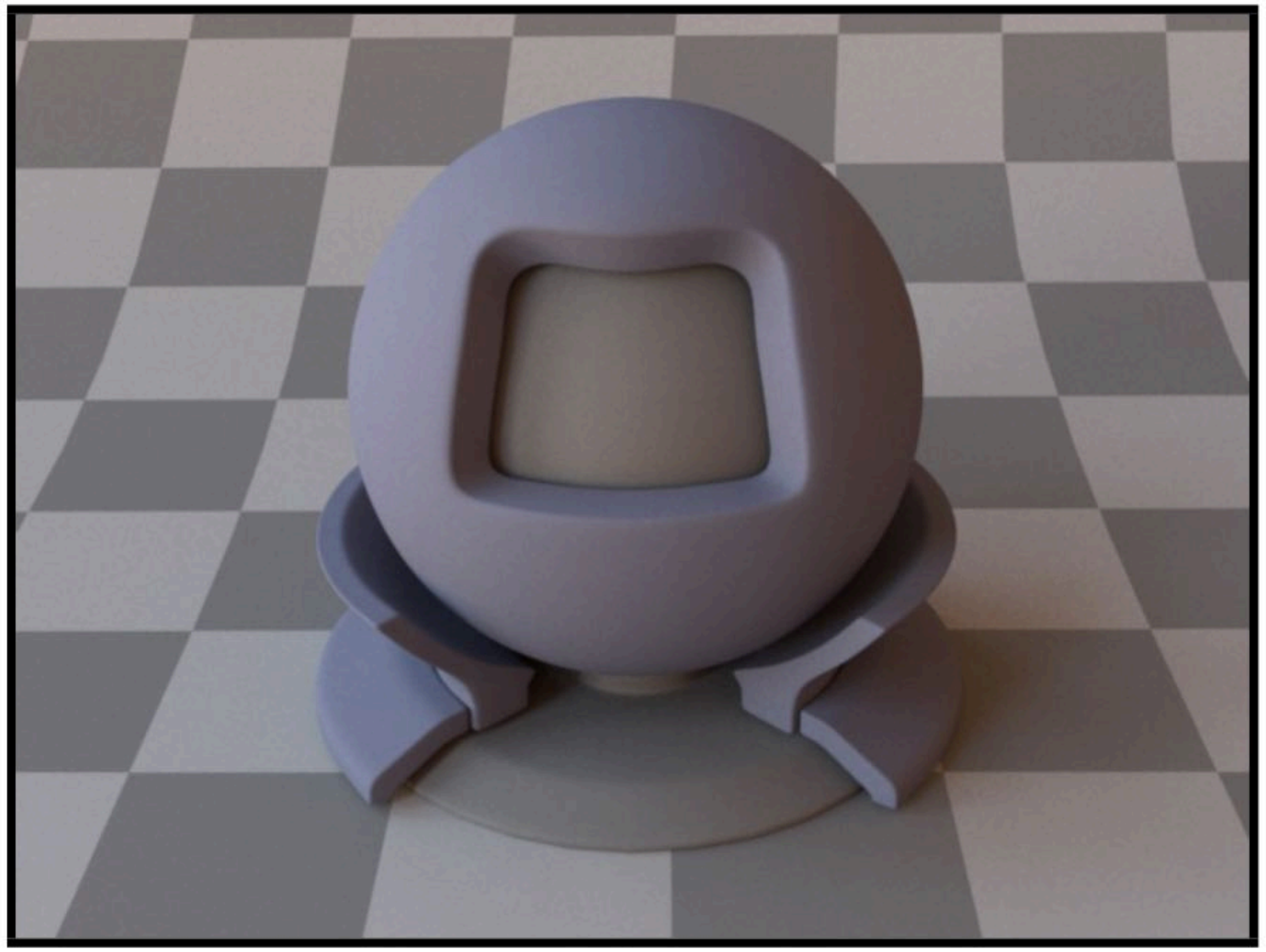
What is this material?



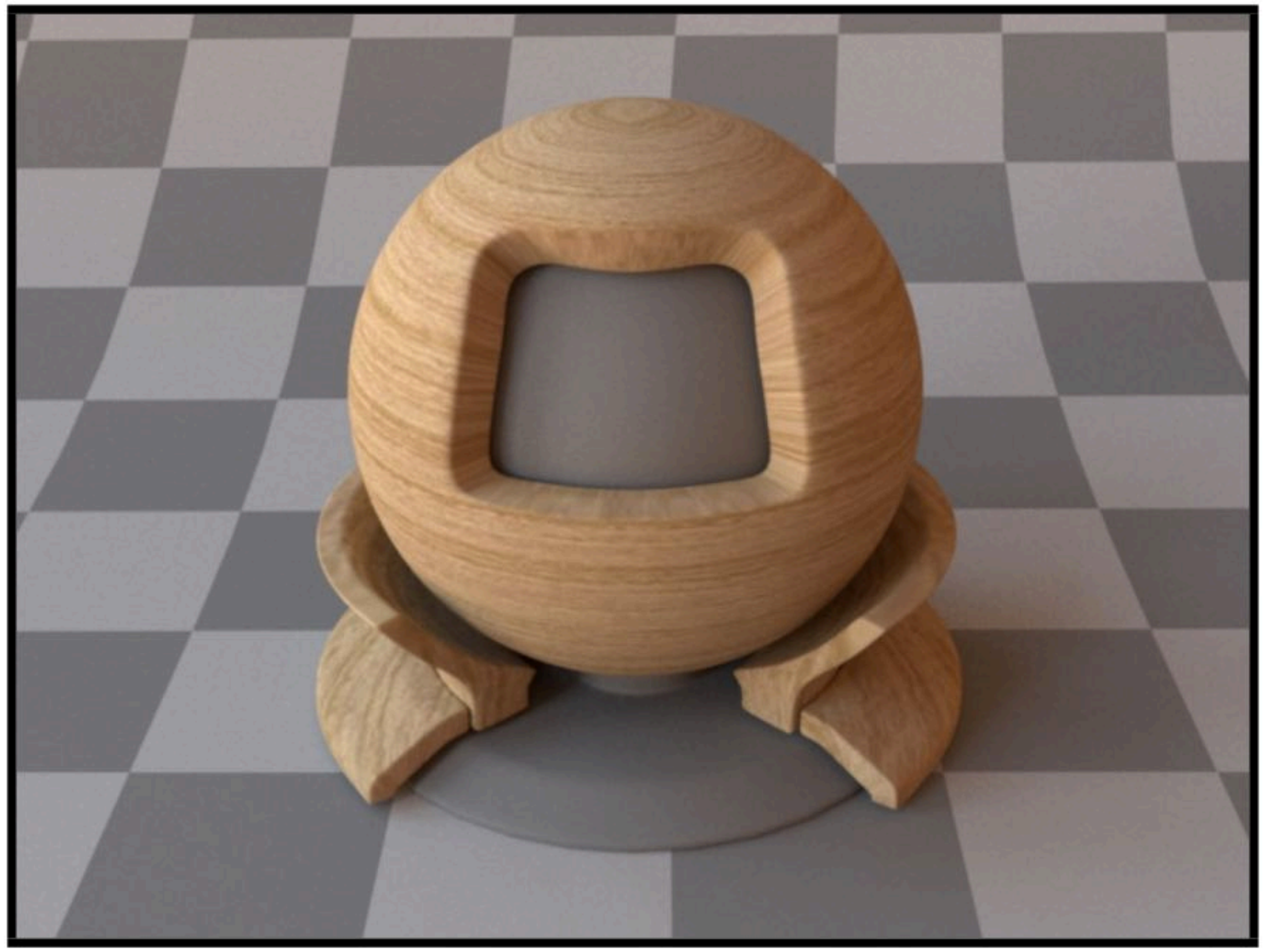
Light is scattered equally in all directions

Diffuse / Lambertian material





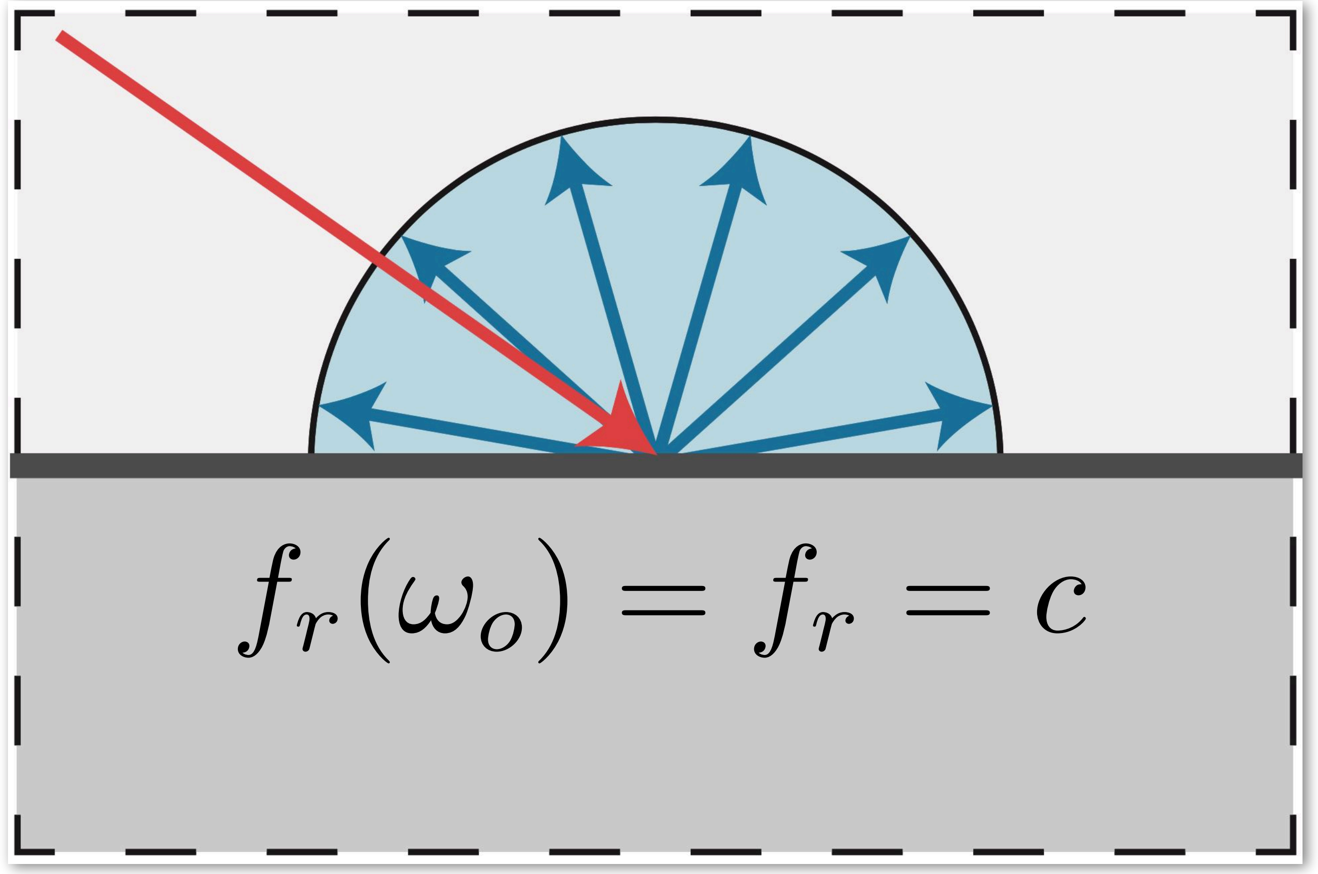
Uniform colored diffuse BRDF
Albedo (fraction of light reflected) is same
for all surface points p



Textured diffuse BRDF
Albedo is spatially varying,
and is encoded in texture map.

[Mitsuba renderer, Wenzel Jakob, 2010]

BRDF for diffuse surface with albedo ρ



$$L_o(w_o) = \int_{H^2} f_r L_i(w_i) \cos\theta_i \, d\omega_i$$

$$= f_r \int_{H^2} L_i(w_i) \cos\theta_i \, d\omega_i$$

$$= f_r E$$

Let E = total incoming irradiance

Let's call the overall reflectance (albedo) of the surface ρ

Total outgoing surface irradiance

$$\rho E = \int_{H^2} f_r E \cos\theta_o \, d\omega_o$$

($f_r E$ = $L_o(\omega_o)$)

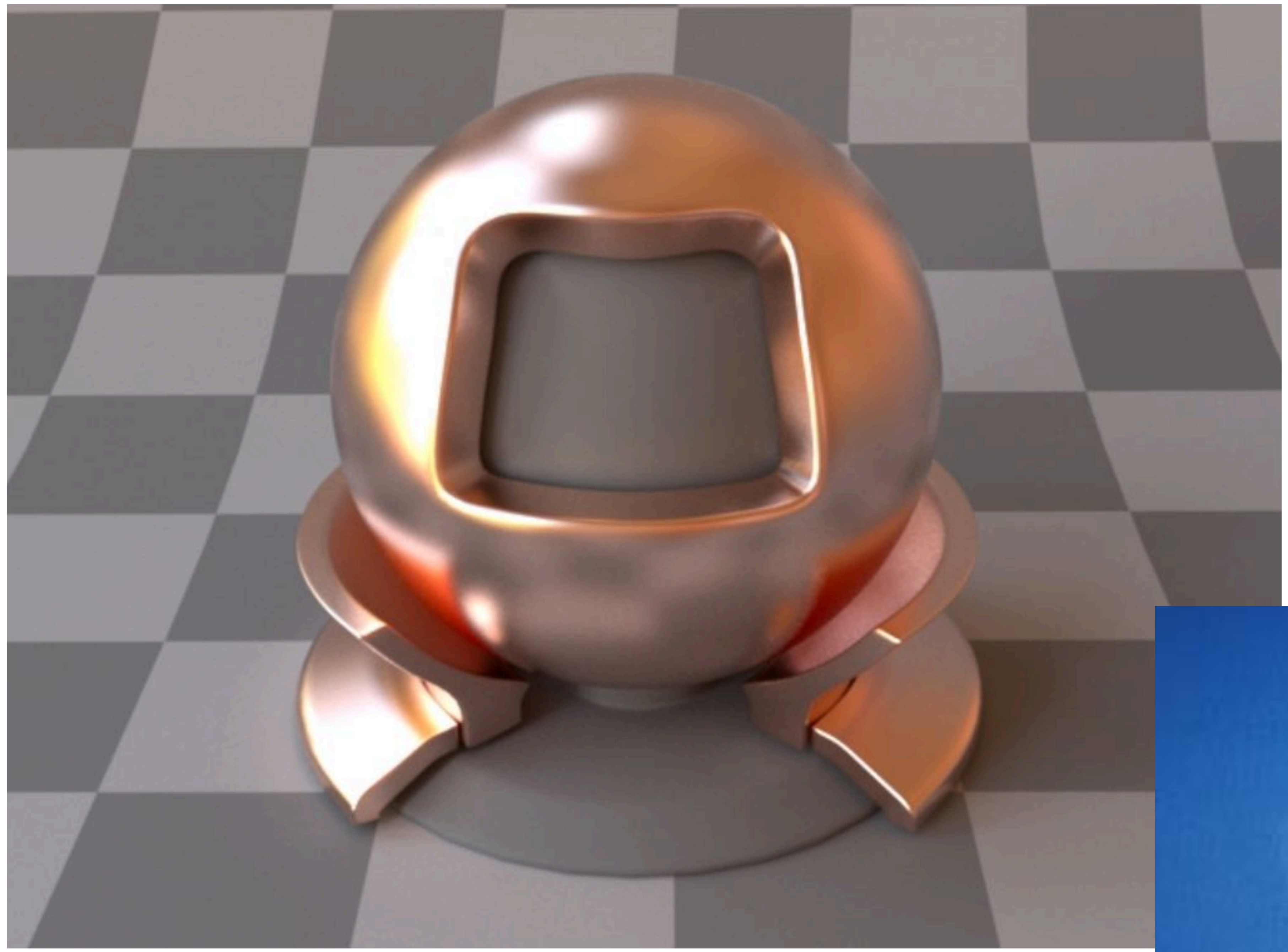
$$\rho = f_r \int_{H^2} \cos\theta_o \, d\omega_o$$

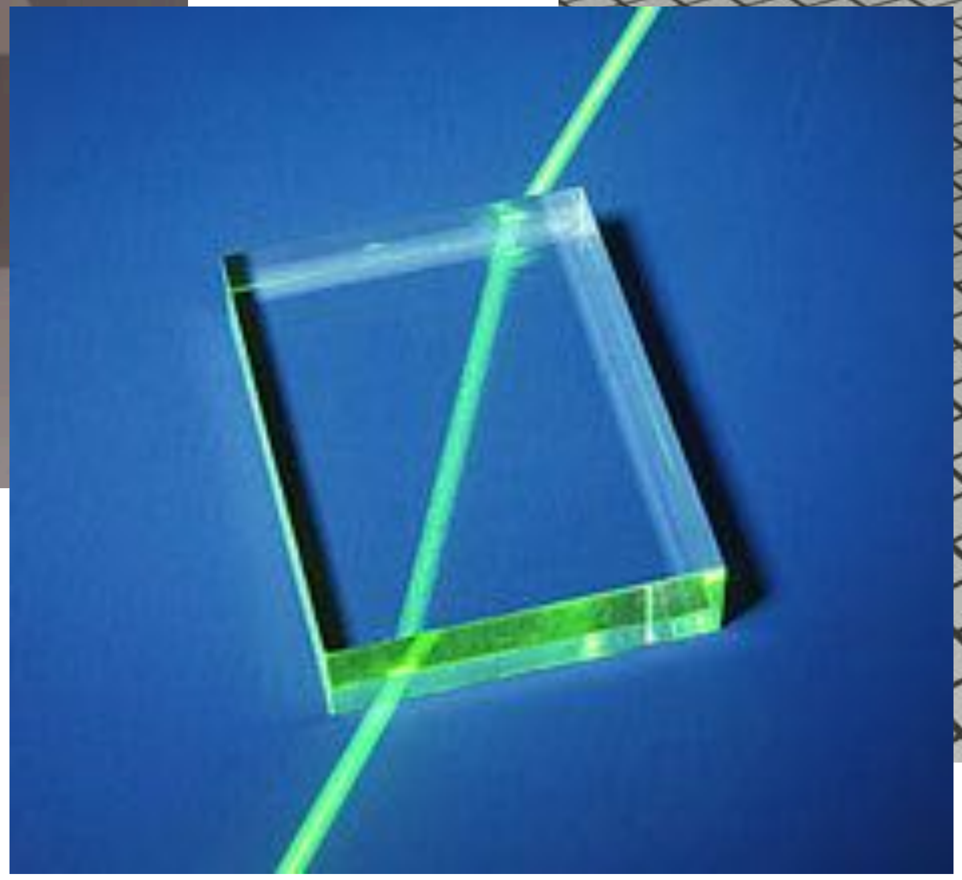
$$\rho = f_r \pi$$

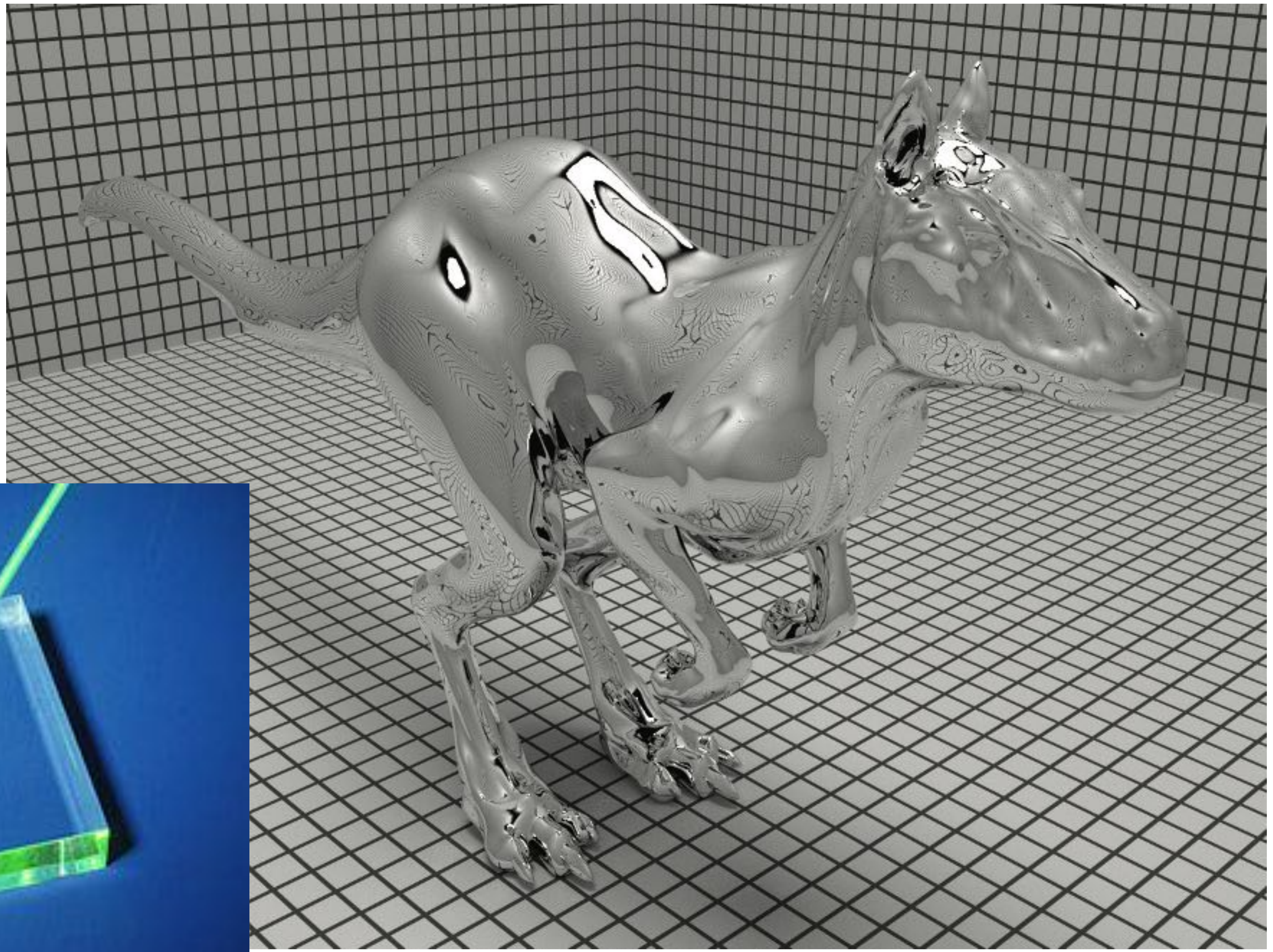
$$f_r = \frac{\rho}{\pi}$$

So given a desired ρ, the BRDF should be the constant $\frac{\rho}{\pi}$

Next time we'll talk about more types of materials:
Glossy materials, mirrors, glass, etc







Summary

- Appearance of a surface is determined by:
  - The amount of light reaching the surface from different directions
    - Surface irradiance: the amount of light arriving at a surface point
    - Radiance: the amount of light arriving at a surface point from a given direction
  - The reflectance properties of the surface:
    - BRDF(w_i,w_o): the fraction of energy from direction w_i reflected in direction w_o

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