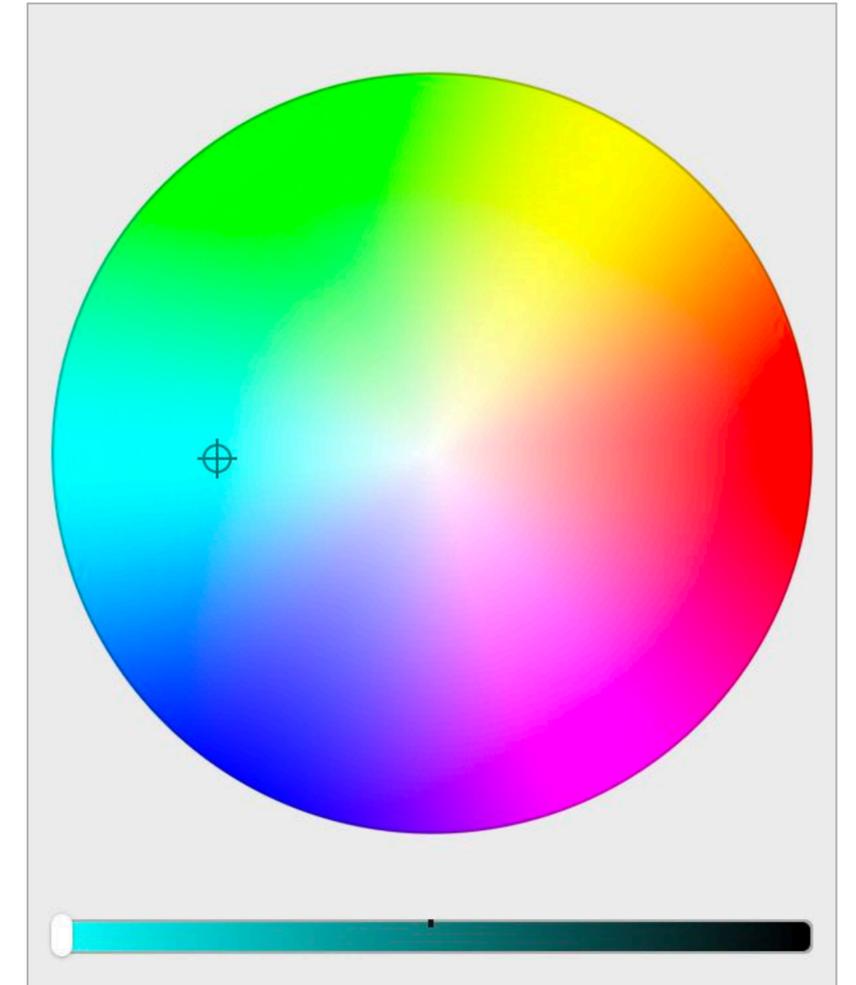
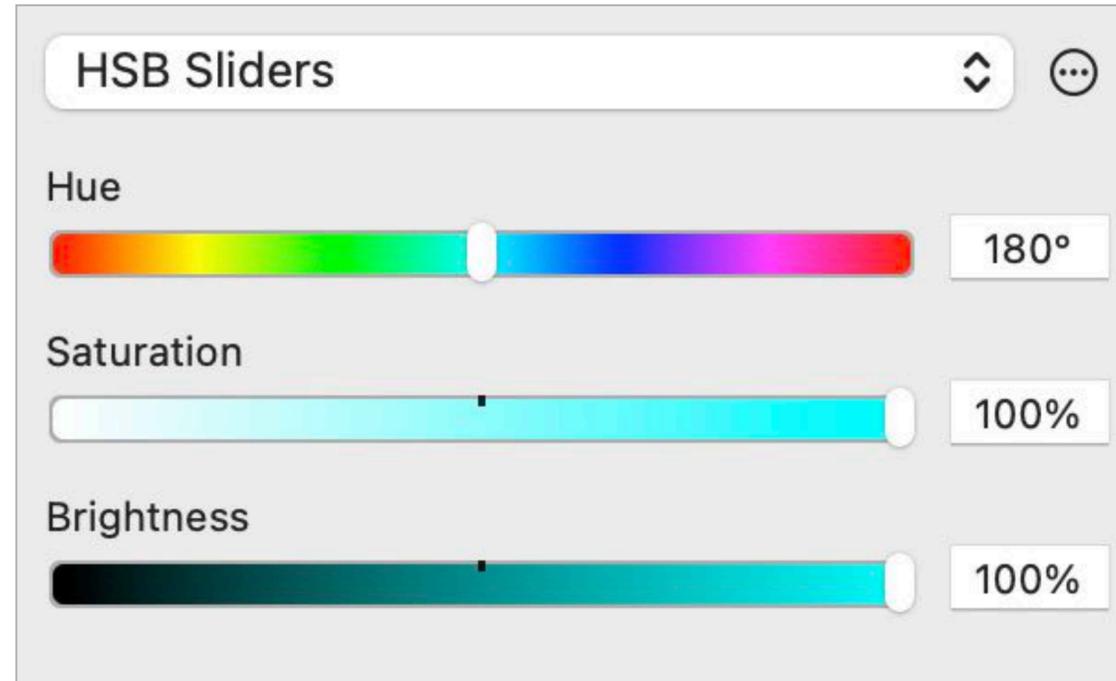
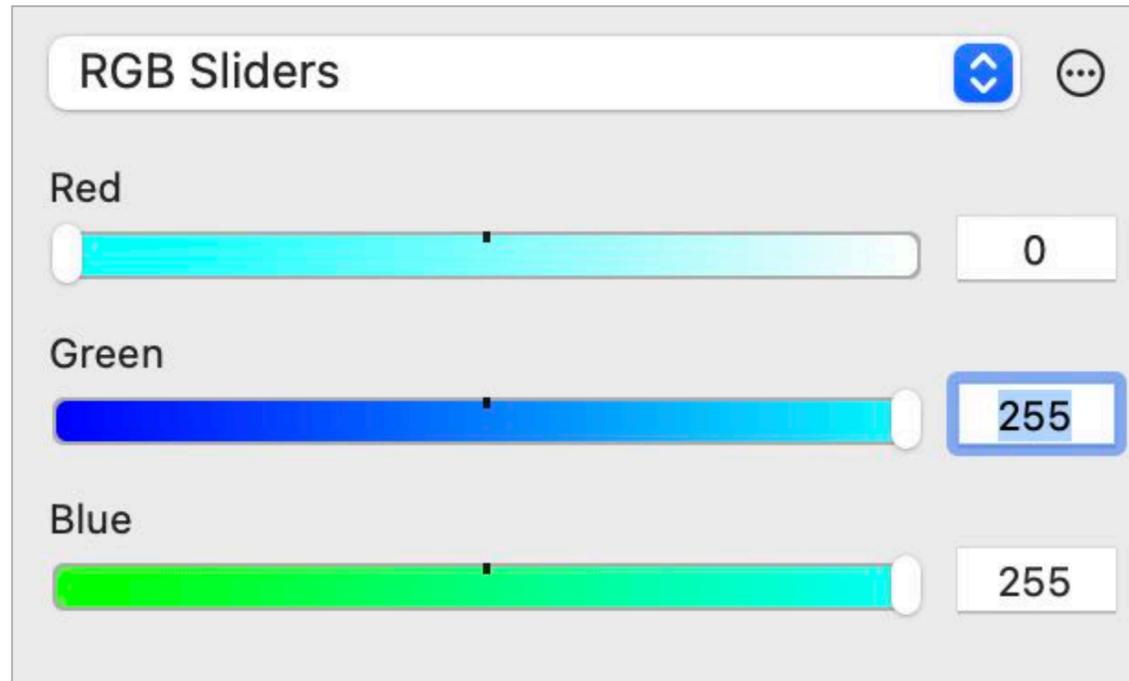


Lecture 14:

Color

Computer Graphics: Rendering, Geometry, and Image Manipulation
Stanford CS248A, Winter 2026

Let's choose a color...



Choosing the color Cyan in three different color pickers...



Color in the world is amazing!



Mission Peak, Bay Area



Meow Wolf, Santa Fe, NM



Image credit: Jabin Botsford/The Washington Post/Getty Images (taken from [CNN.com](https://www.cnn.com))



A Good Dinner (Redwood City)





Sydney Harbor, Australia



Starry Night, Van Gogh



Marilyn Monroe, Andy Warhol

Why do we need to be able to talk precisely about color?

Kayvon's walls circa 2022



Why do we need to be able to talk precisely about color?

Kayvon's wall last night





SAMSUNG

QLED 8K

SONY

LG

VIZIO

The Frame

SAMSUNG

Immer

SANUS

PREMIUM TV WALL MOUNT

SAMSUNG

4K Gaming



TV when it's on.
Art when it's off.



Art Store
Choose from a vast library of art



QLED
Quantum Dot color and clarity
Samsung Smart TV

The Frame

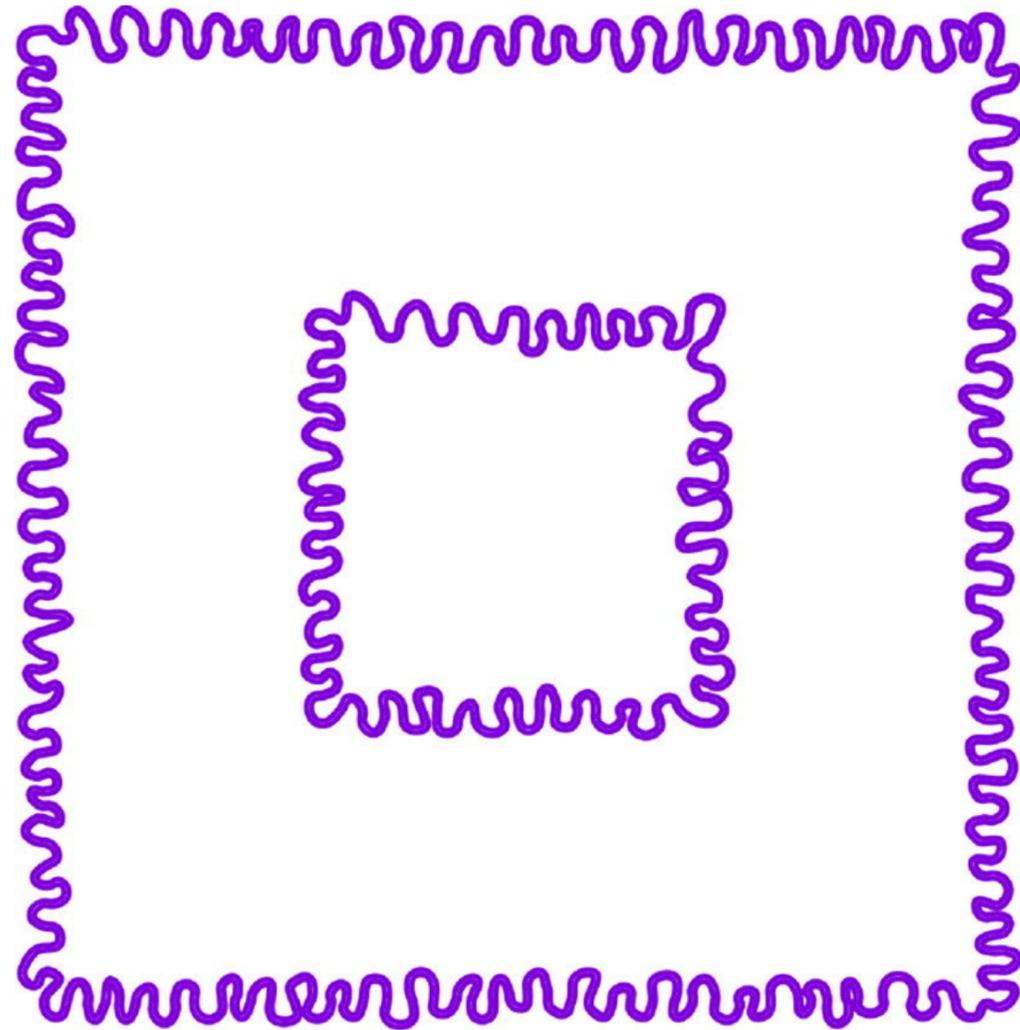


But... how we receive color can be surprising



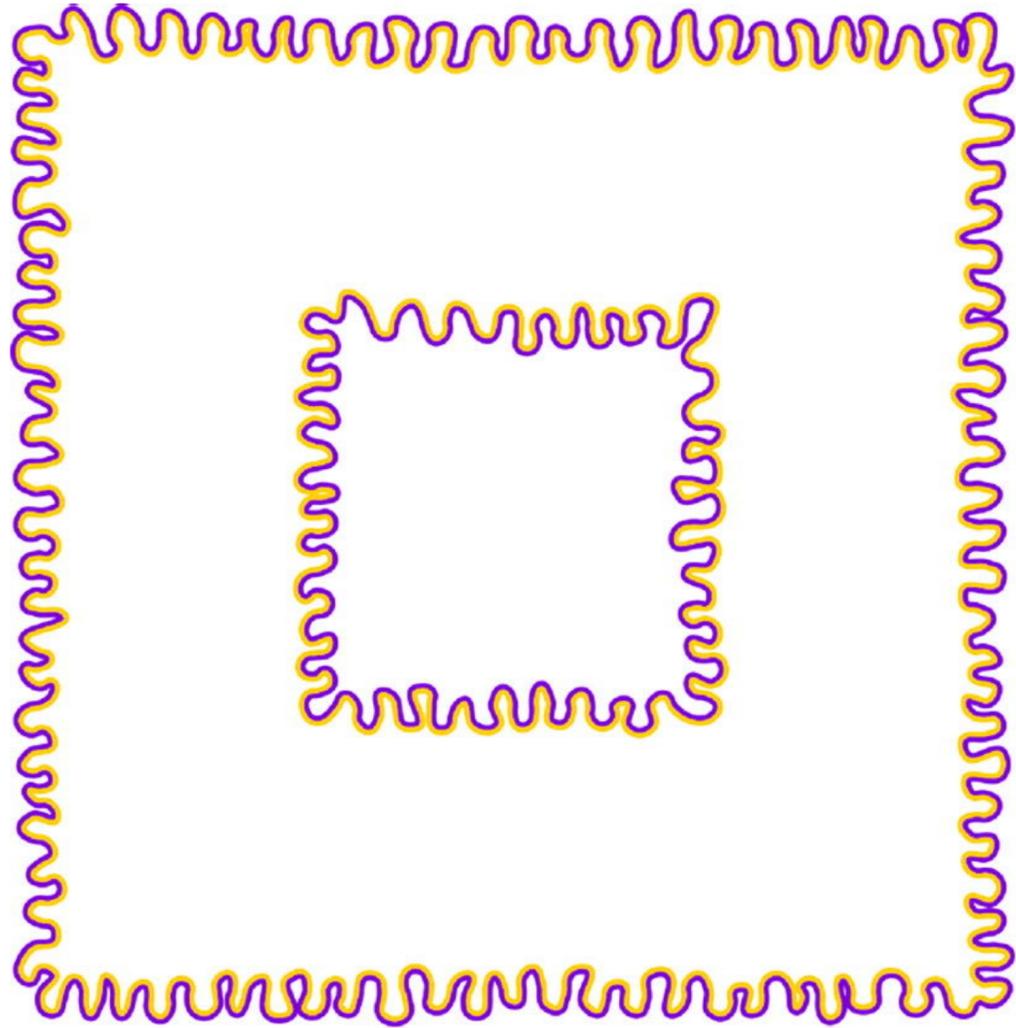


Watercolor illusion



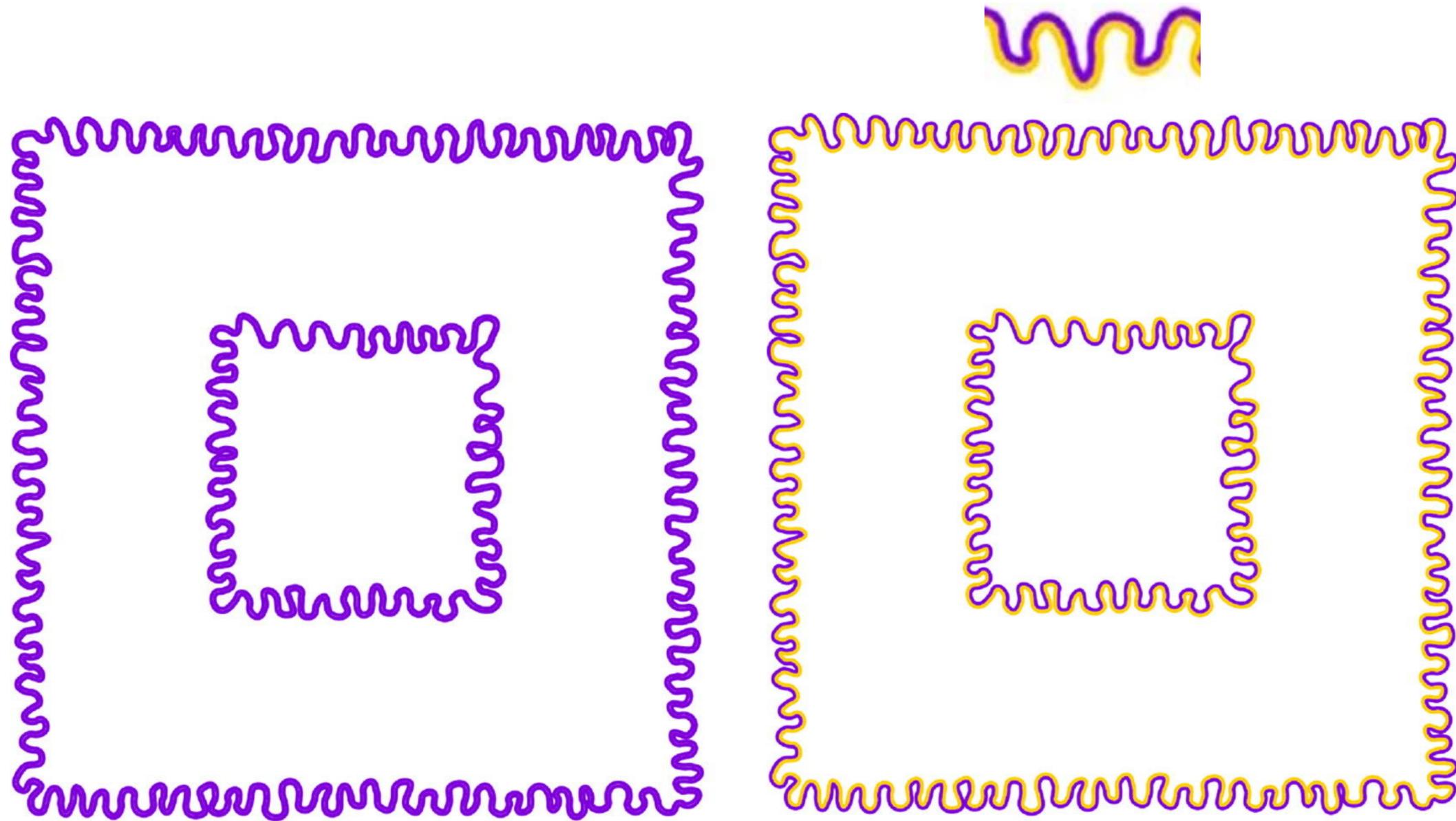
Pinna & Tanca, *JOV*, 2008

Watercolor illusion



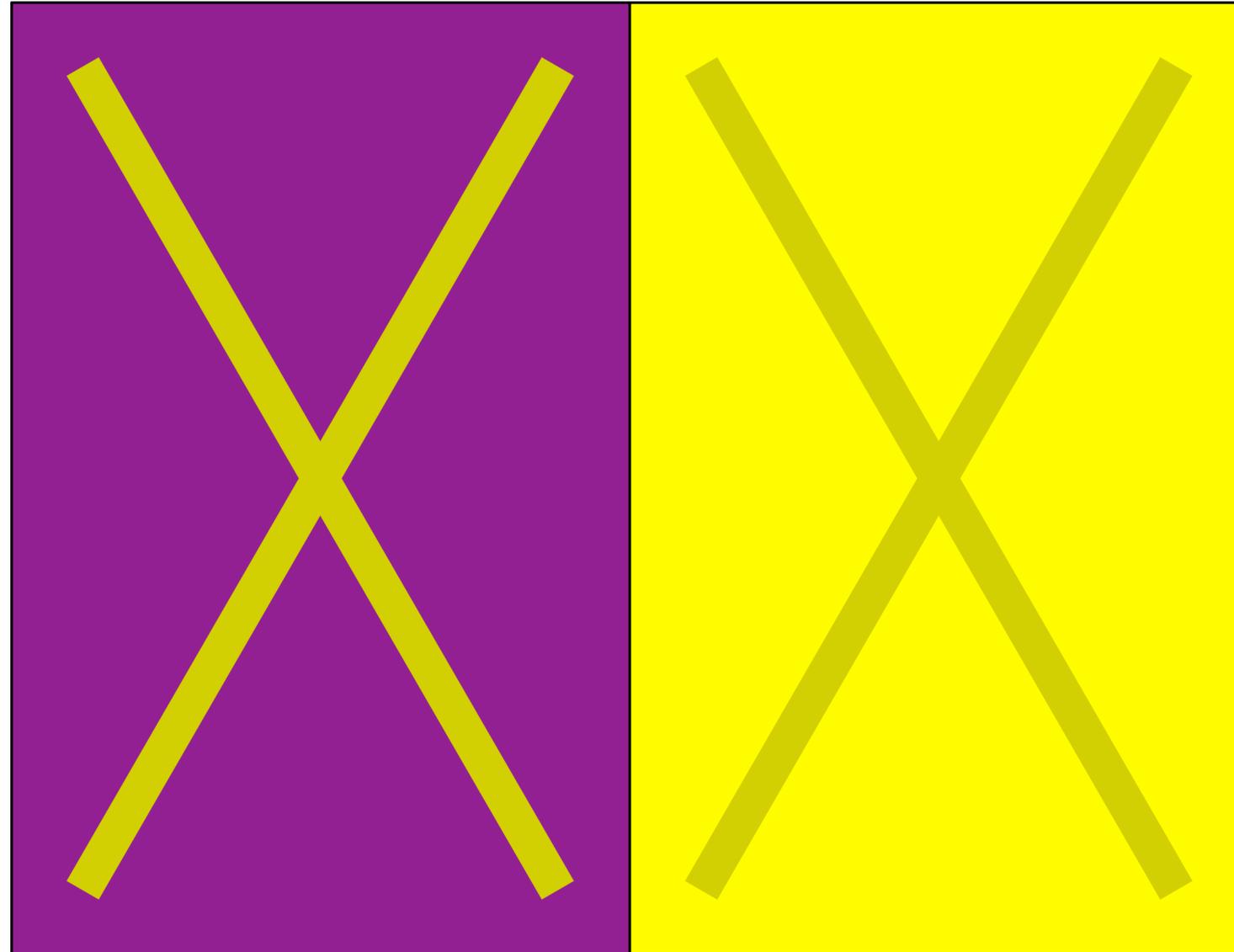
Pinna & Tanca, *JOV*, 2008

Watercolor illusion

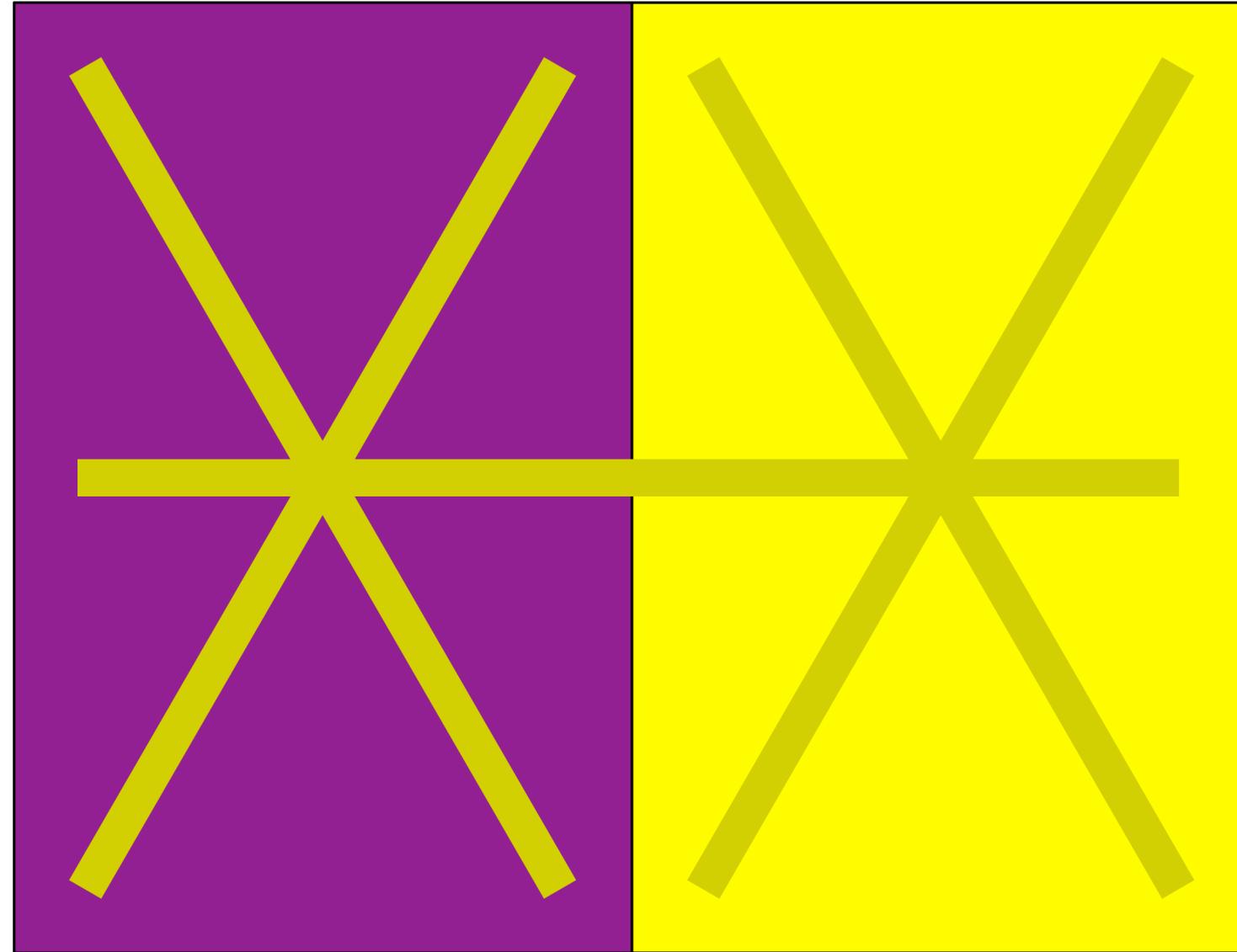


Pinna & Tanca, *JOV*, 2008

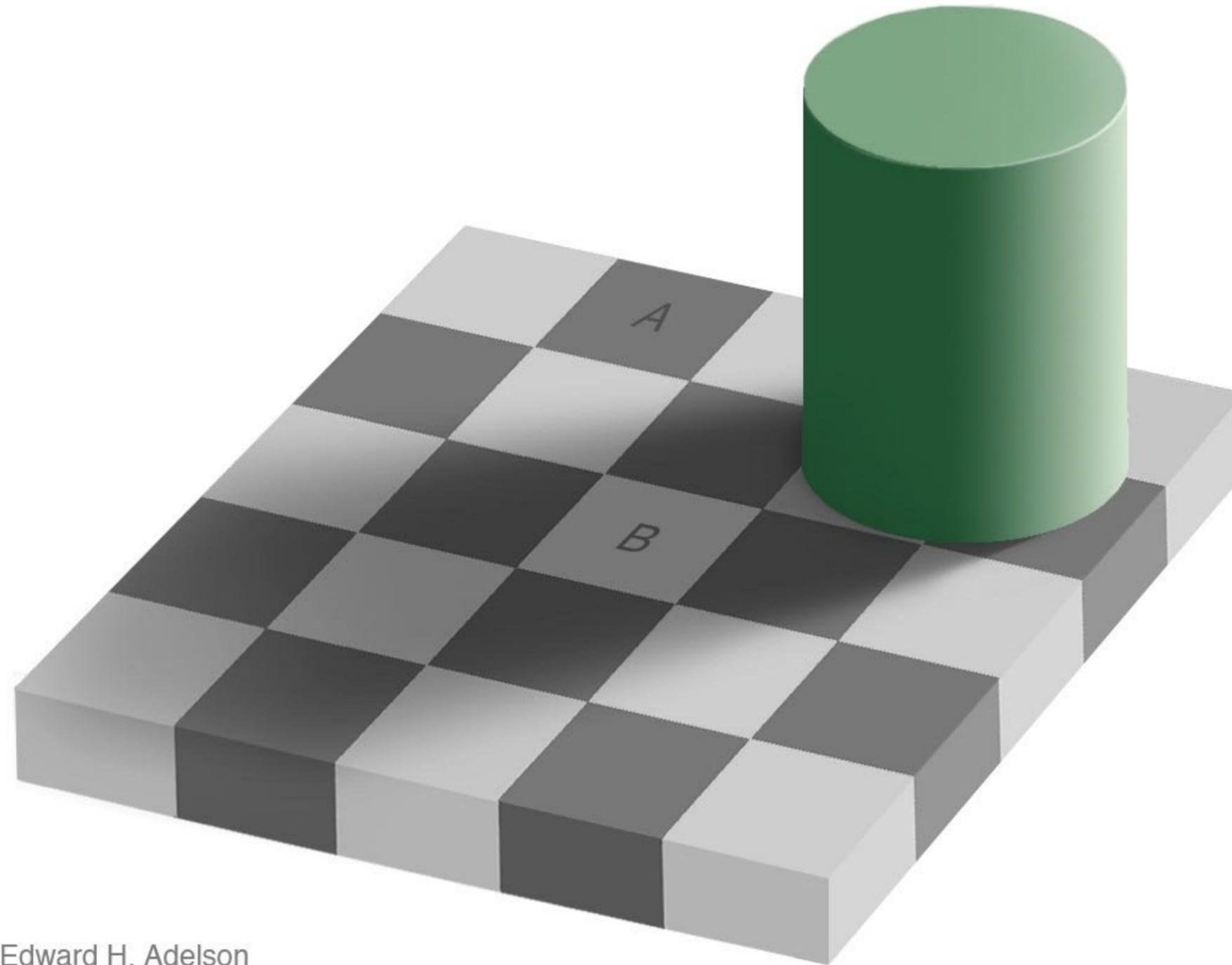
Which "X" is darker?



Which "X" is darker?

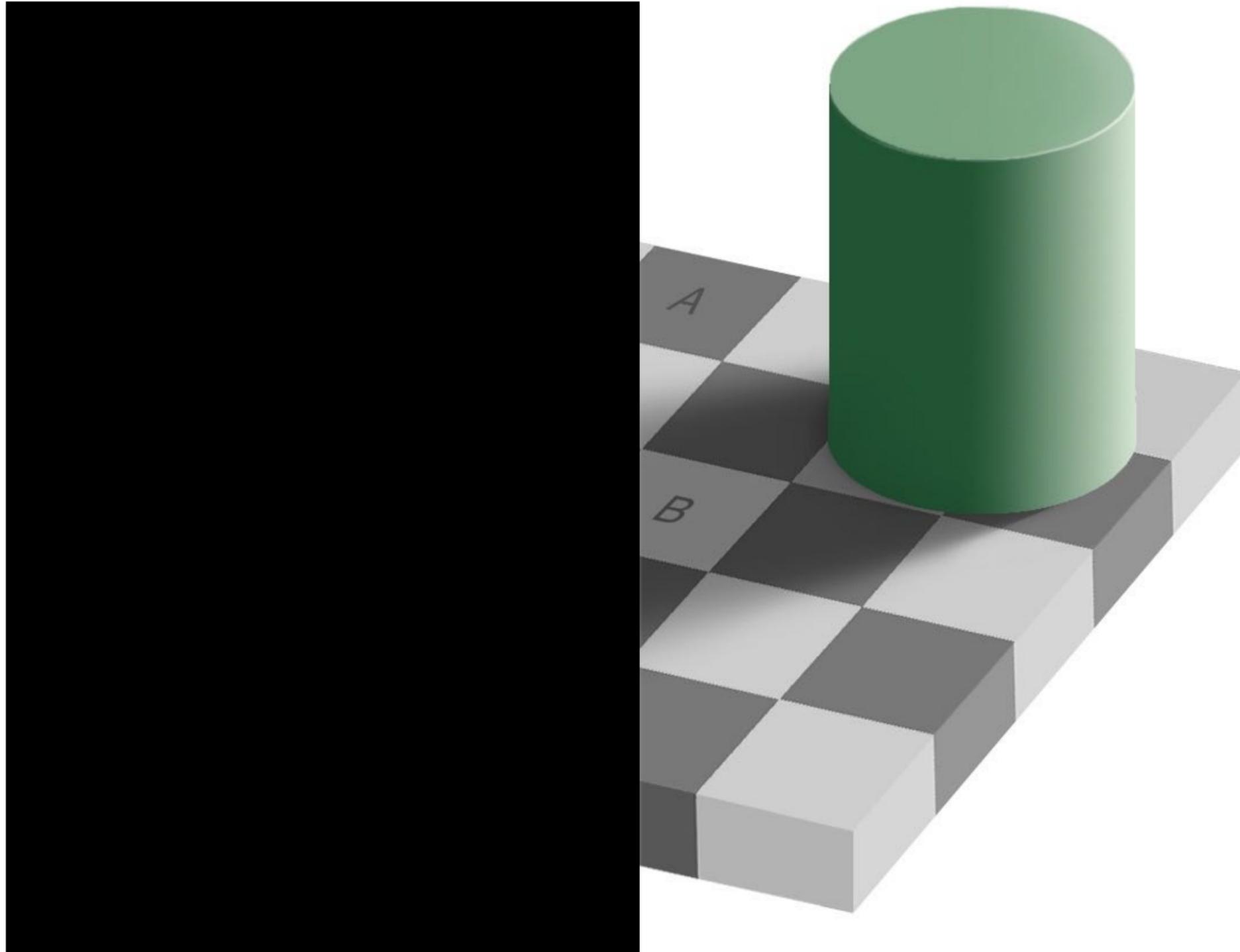


How we receive color can be surprising

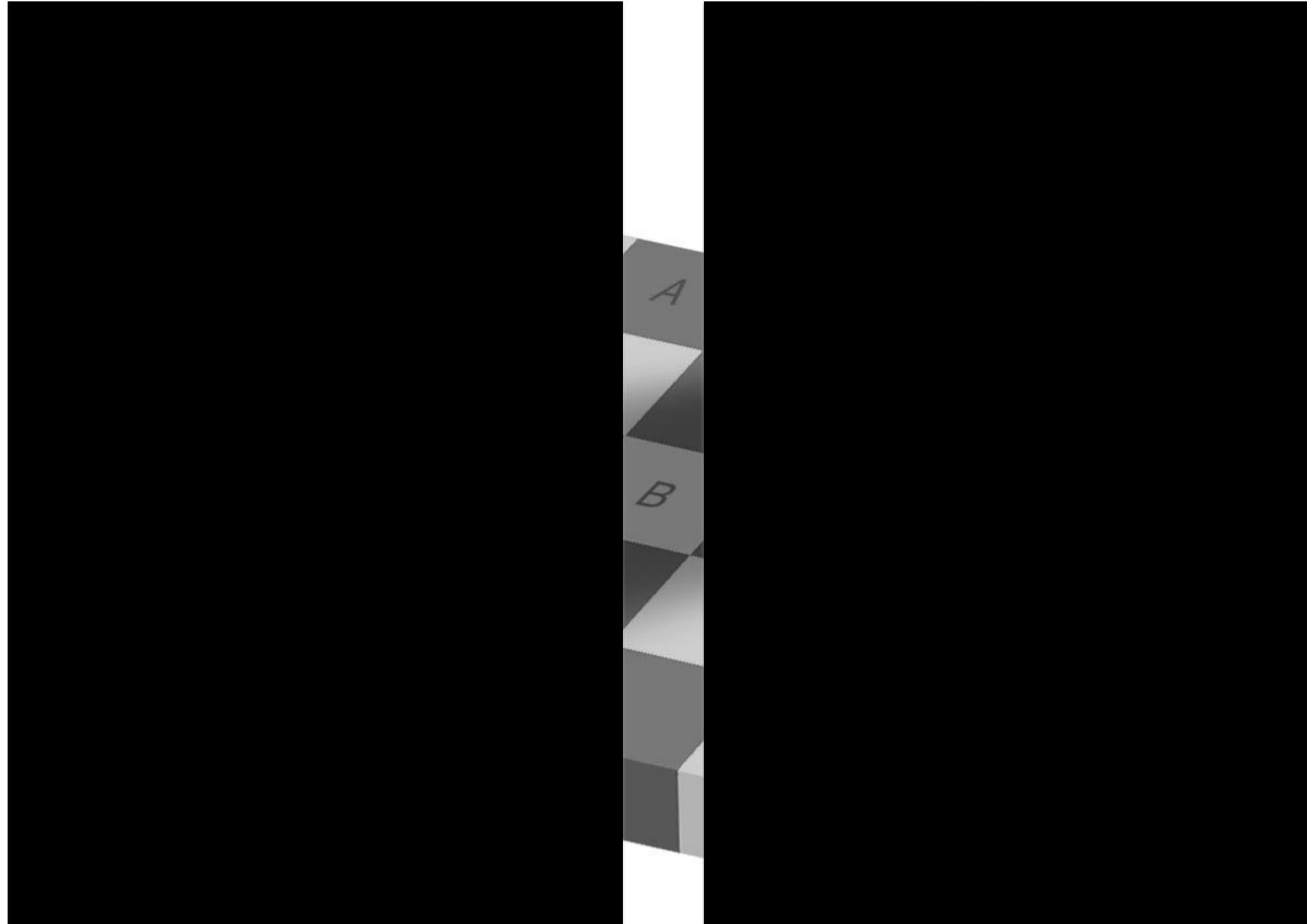


Edward H. Adelson

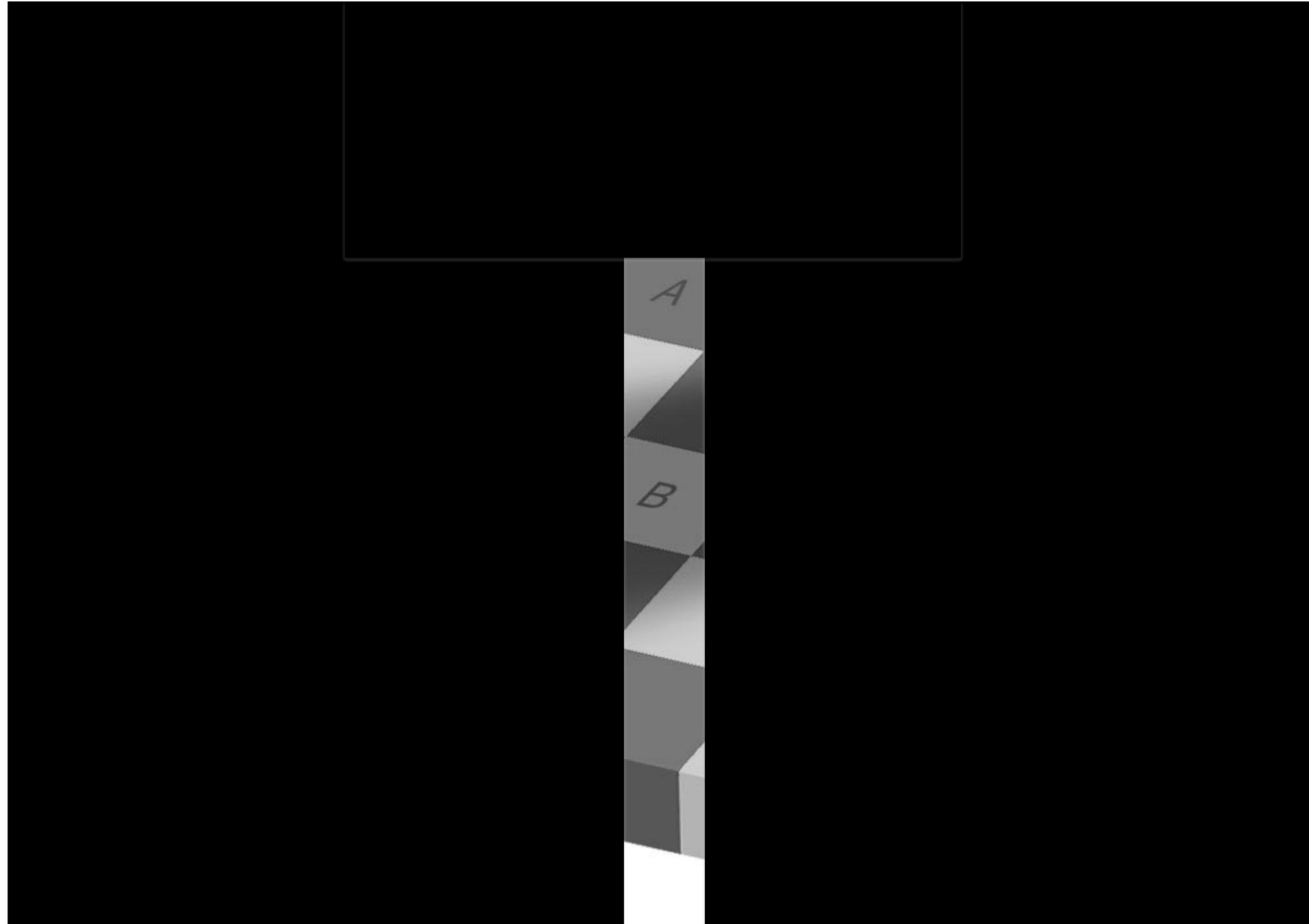
How we receive color can be surprising is relative



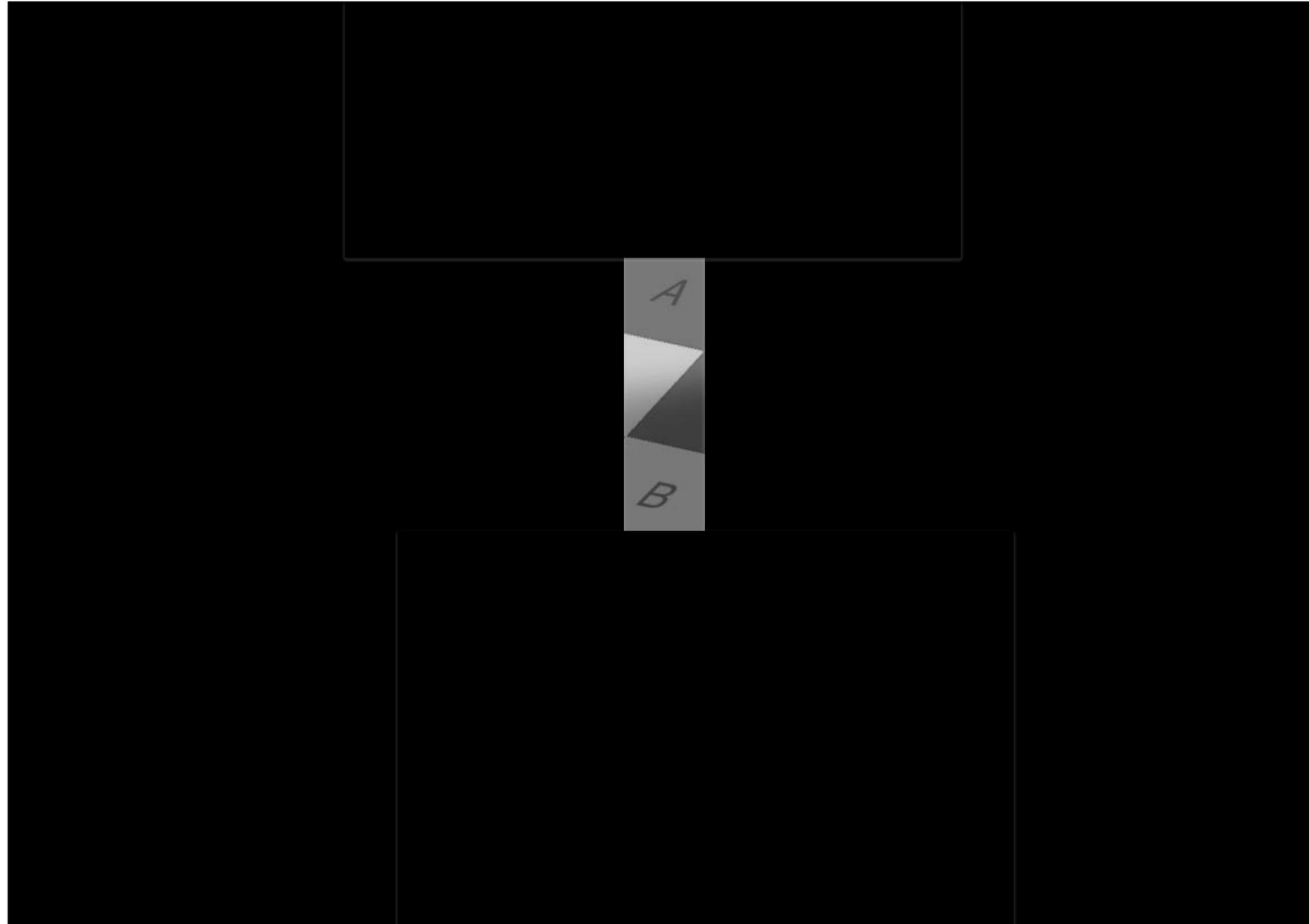
How we receive color can be surprising is relative



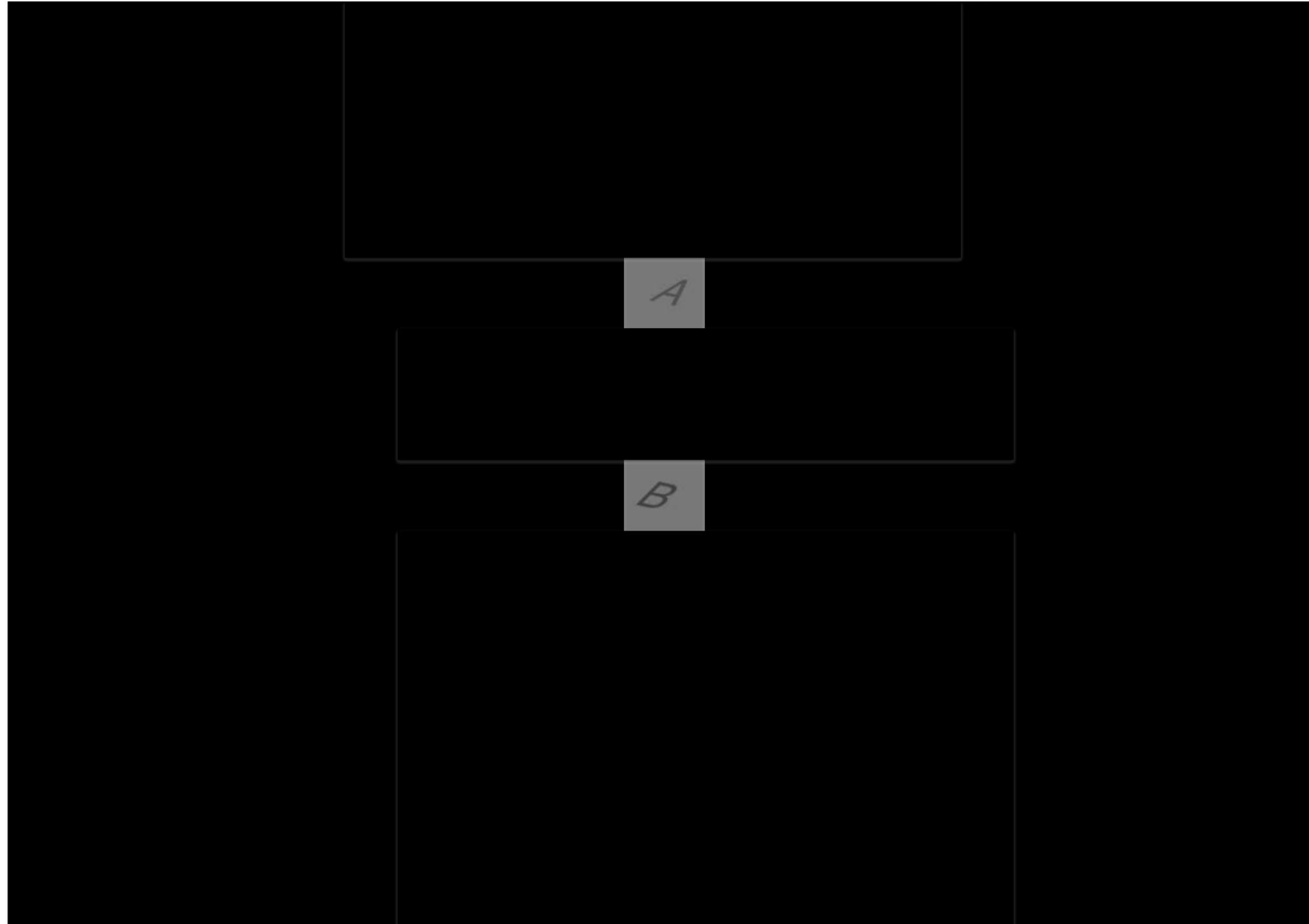
How we receive color can be surprising is relative



How we receive color can be surprising is relative



How we receive color can be surprising is relative



What is color?

- **Color is a phenomenon of human perception; it is not a universal property of light**
- **Colors are the perceptual sensations that arise from seeing light of different spectral power distributions**
- **As you will learn, different wavelengths of light are not “colors”**

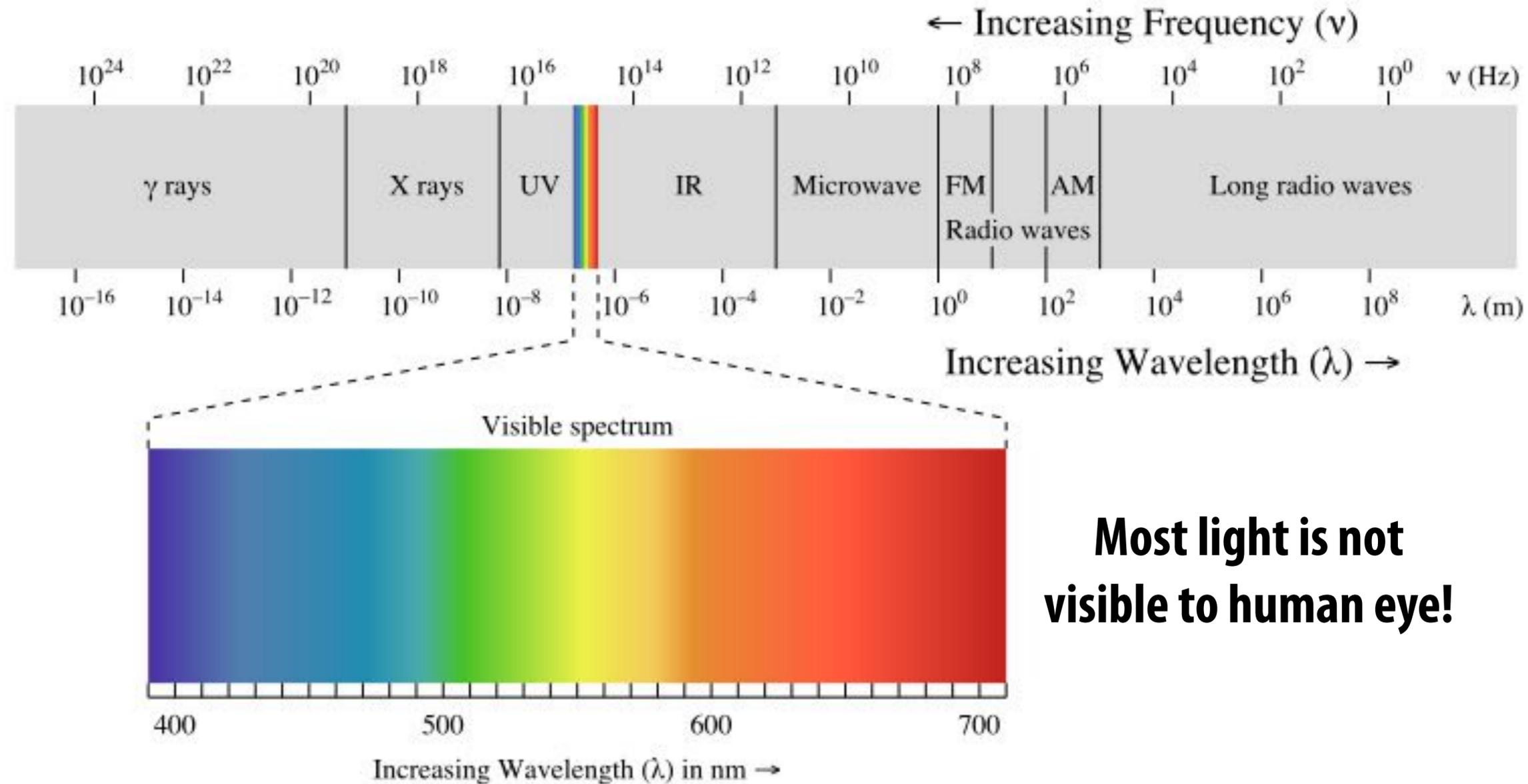
The physical basis of color

(Review: what is light?)

Electromagnetic radiation

Light is electromagnetic radiation (oscillating electromagnetic field)

Perceived color is *related to* frequency of oscillation



Most light is not visible to human eye!

Spectral power distribution (SPD)

- **The amount of light present at each wavelength**
- **Units:**
 - **Radiometric units / nanometer (e.g. watts / nm)**
 - **Can also be unit-less (see below)**
- **Note: SPD is often visualized in “relative units” scaled to maximum power wavelength when absolute units are not important (the diagrams in this lecture do this)**

Spectral power distribution of common light sources

Describes distribution of power (energy/time) by wavelength

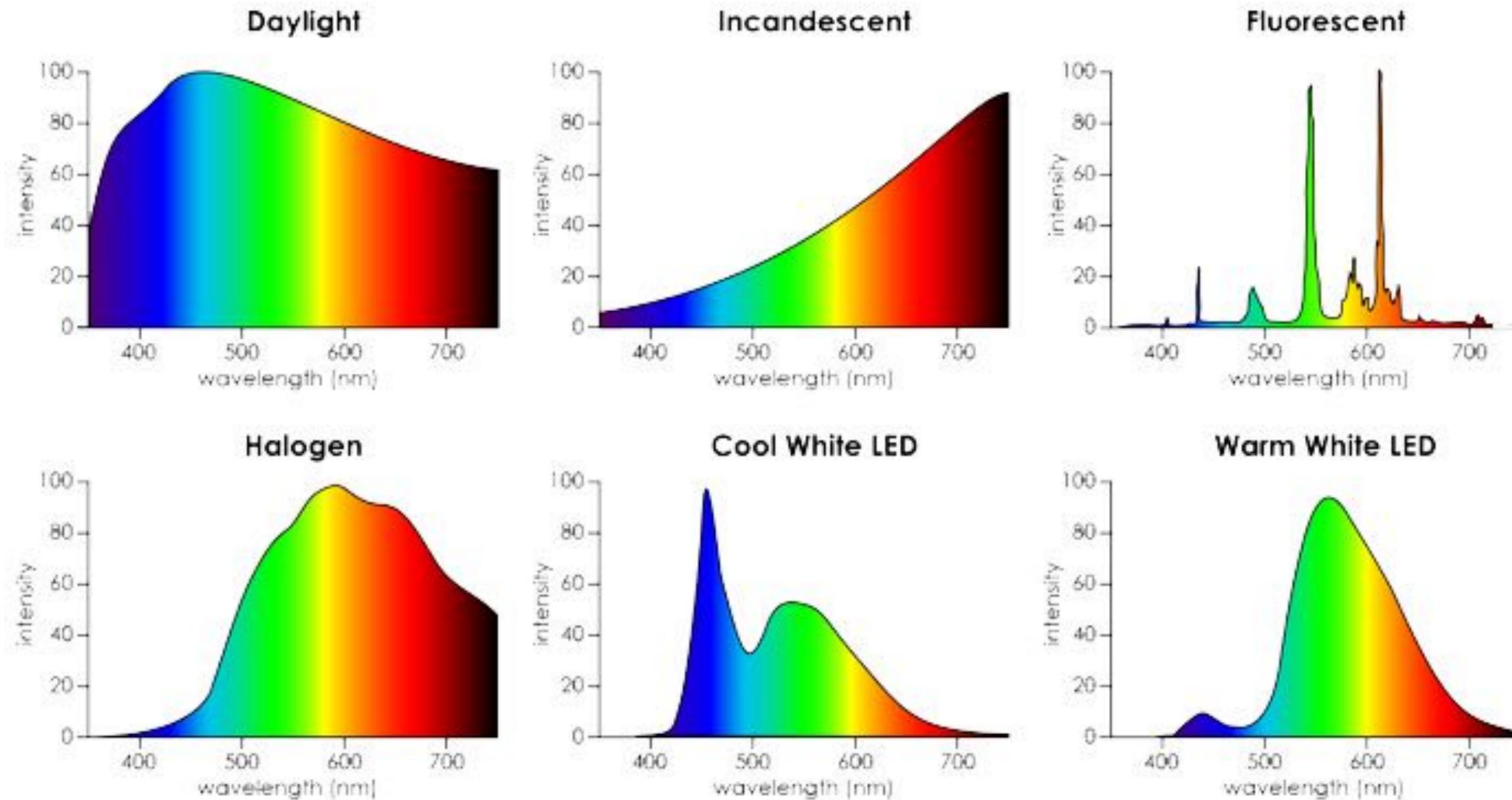
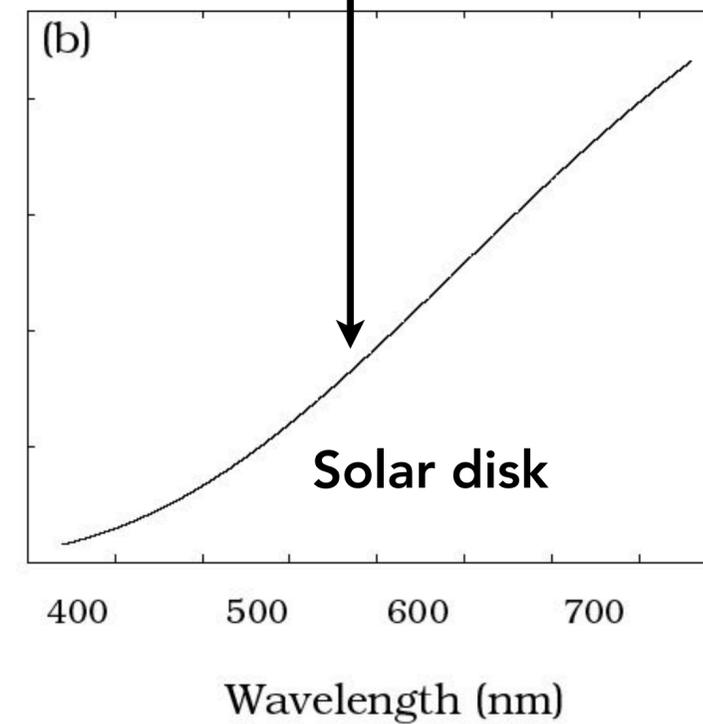
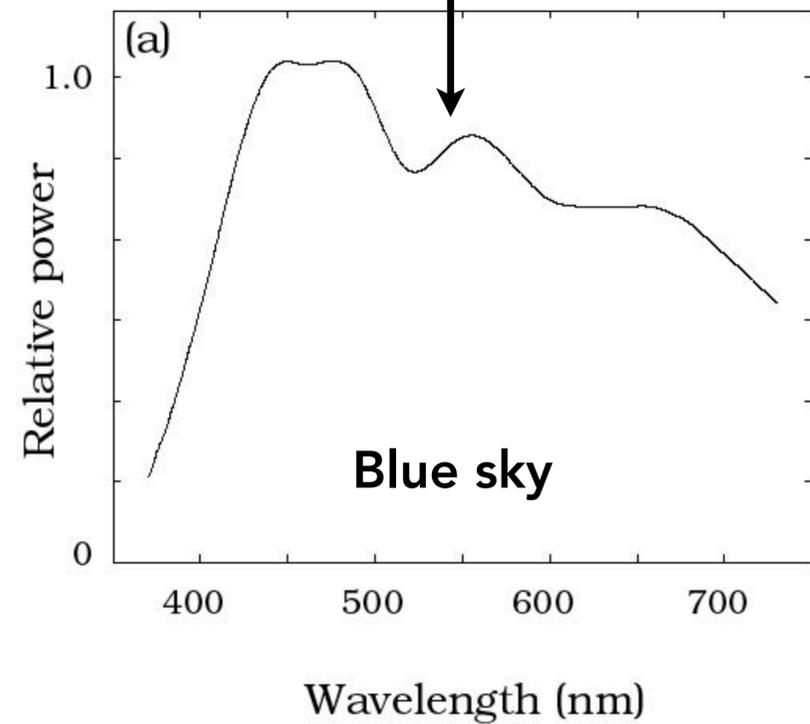


Figure credit:

Daylight spectral power distributions vary

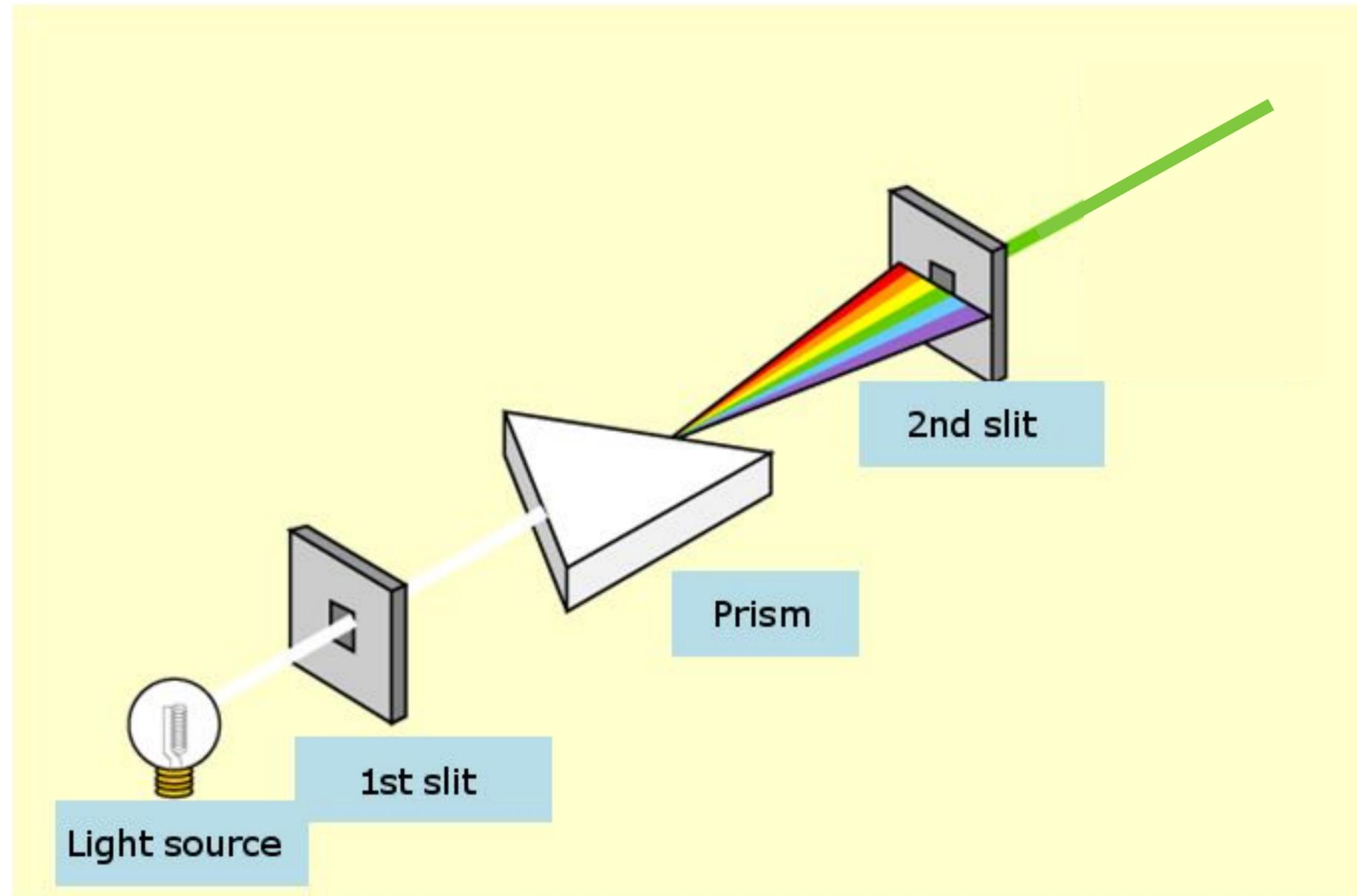


[Brian Wandell]

Measuring the spectral power distribution

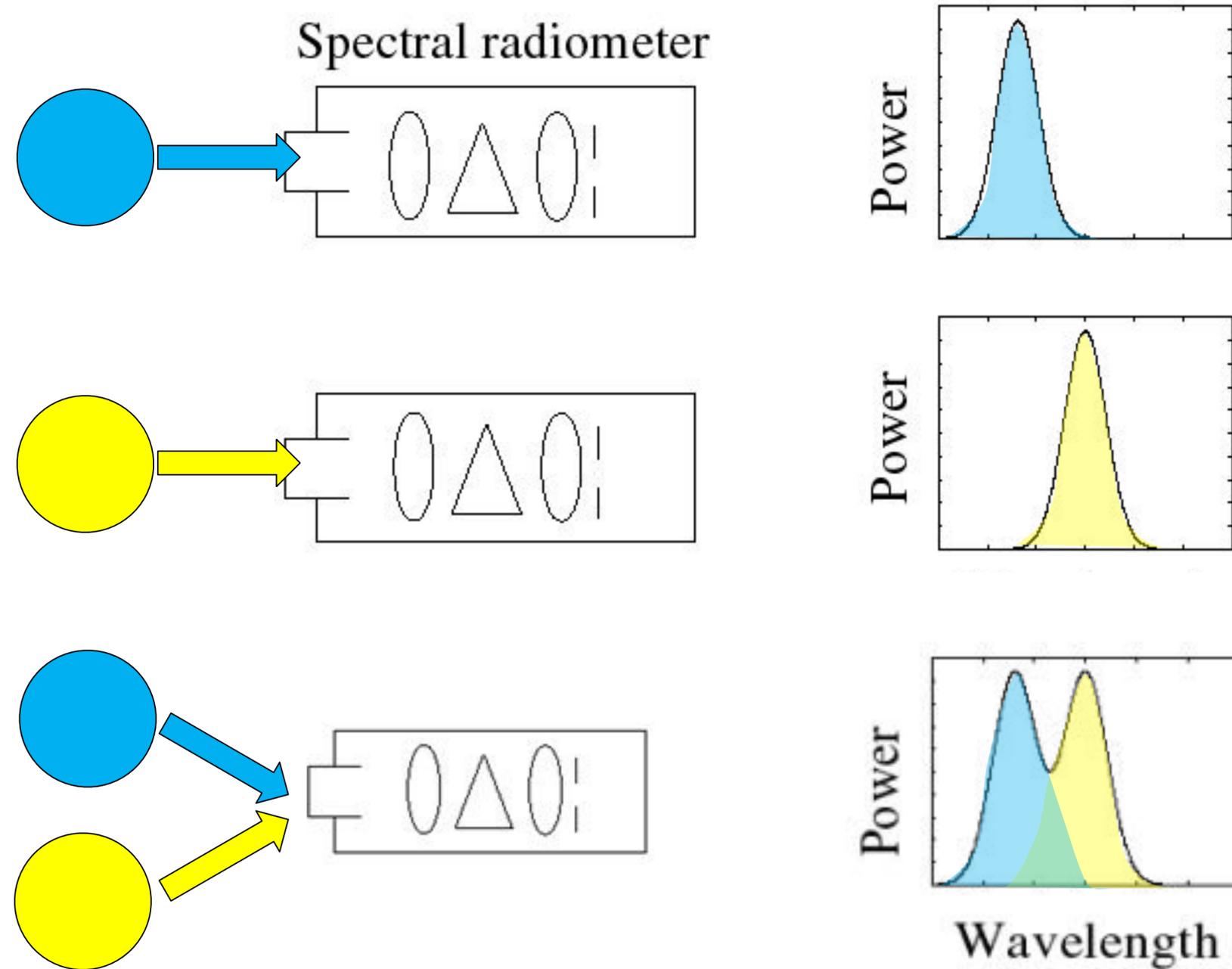


Monochromator



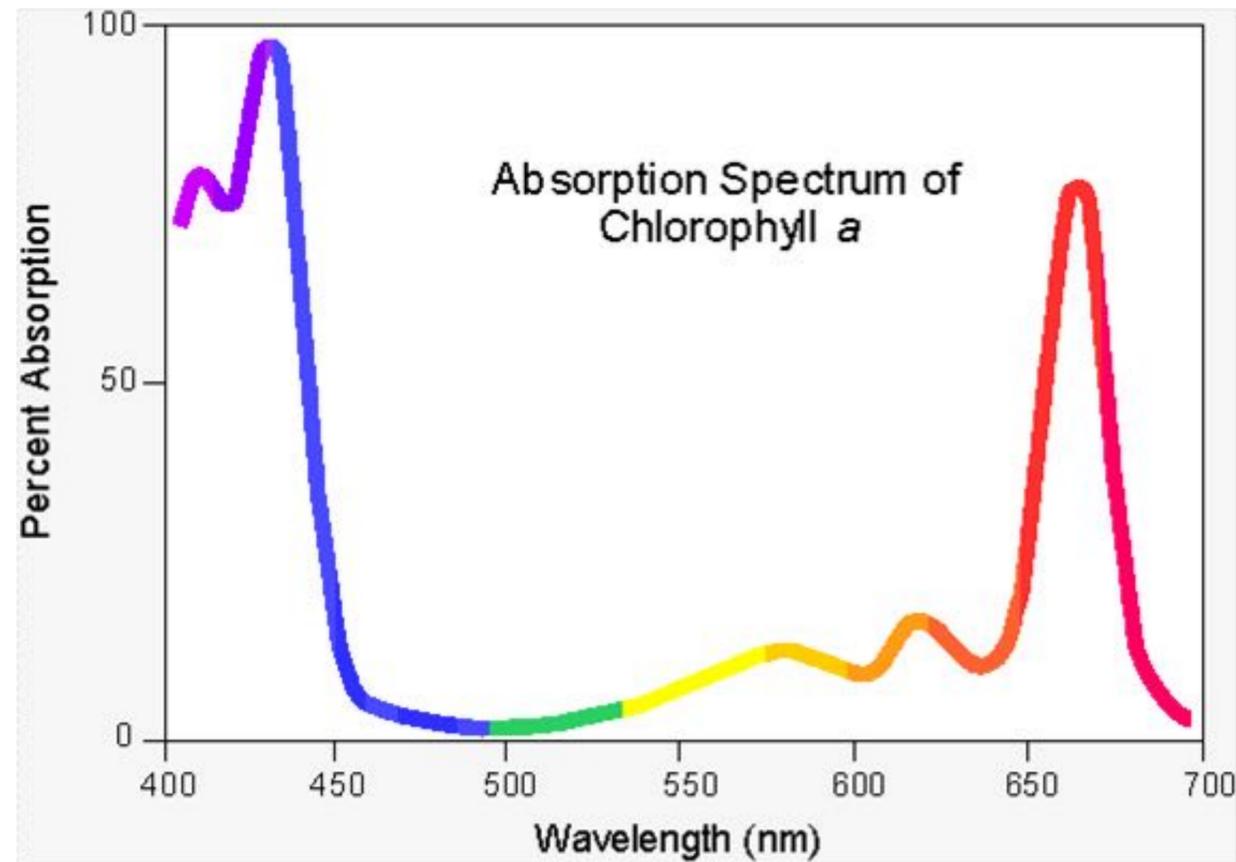
**A monochromator delivers light of a single wavelength from a light source with broad spectrum.
Control which wavelength by angle of prism.**

Superposition (linearity) of spectral power distributions



Absorption spectrum

- Emission spectrum is *intensity* as a function of frequency
- Absorption spectrum is *fraction absorbed* as function of frequency

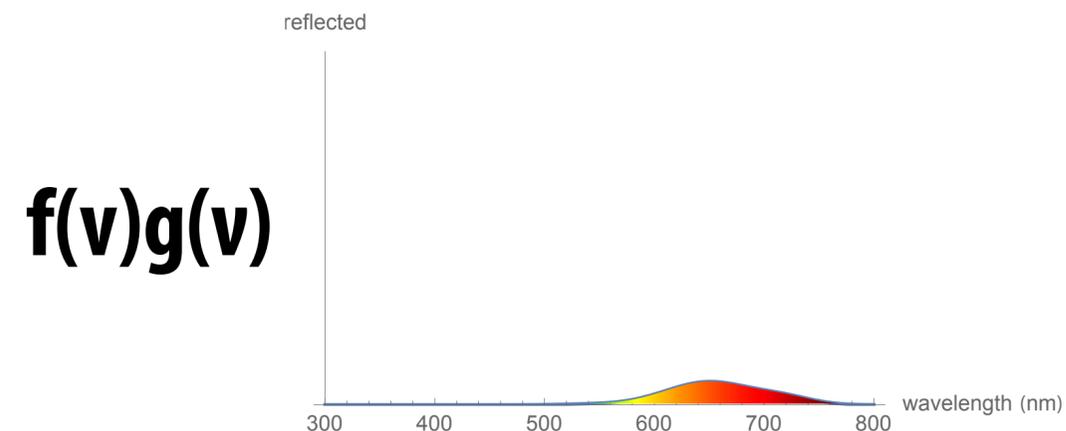
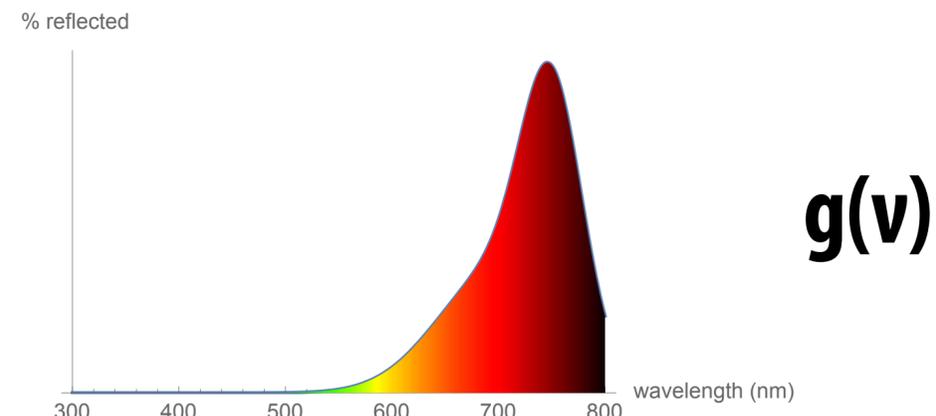
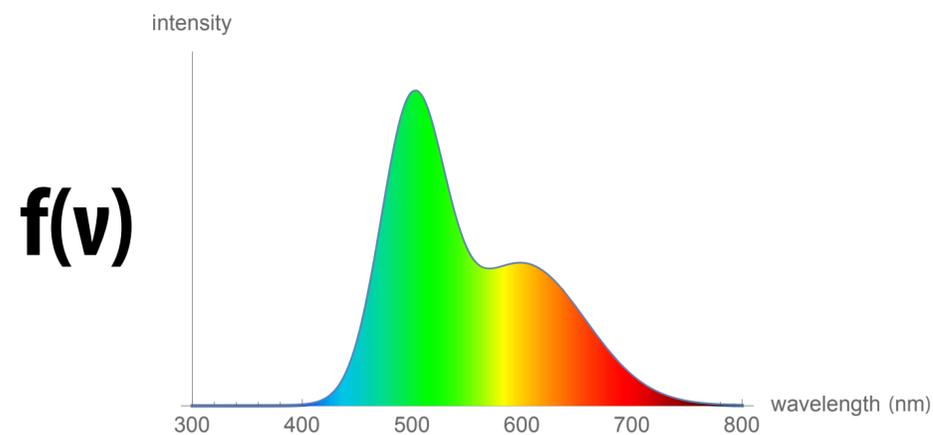
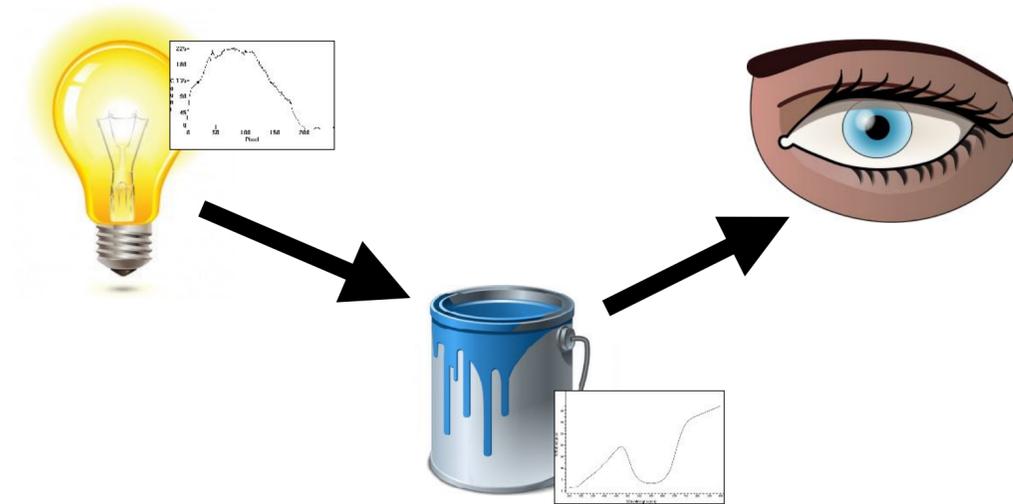


Q: What does an object with this absorption spectrum look like?

Interaction of emission and reflection

■ Consider what happens when light gets reflected from a surface

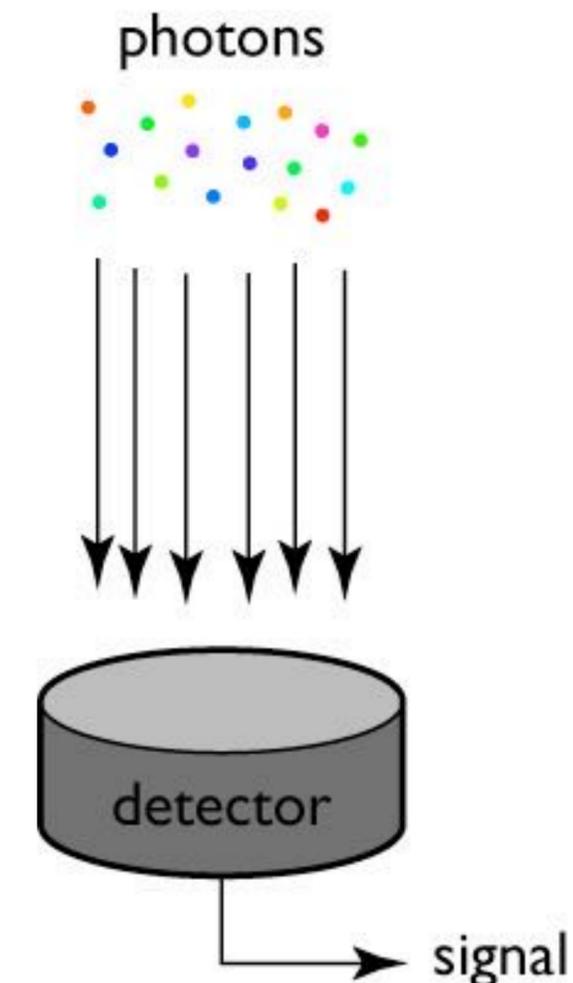
- ν — frequency of light (Greek “nu”)
- Light source has emission spectrum $f(\nu)$
- Surface has reflection spectrum $g(\nu)$
- Resulting intensity is the *product* $f(\nu)g(\nu)$



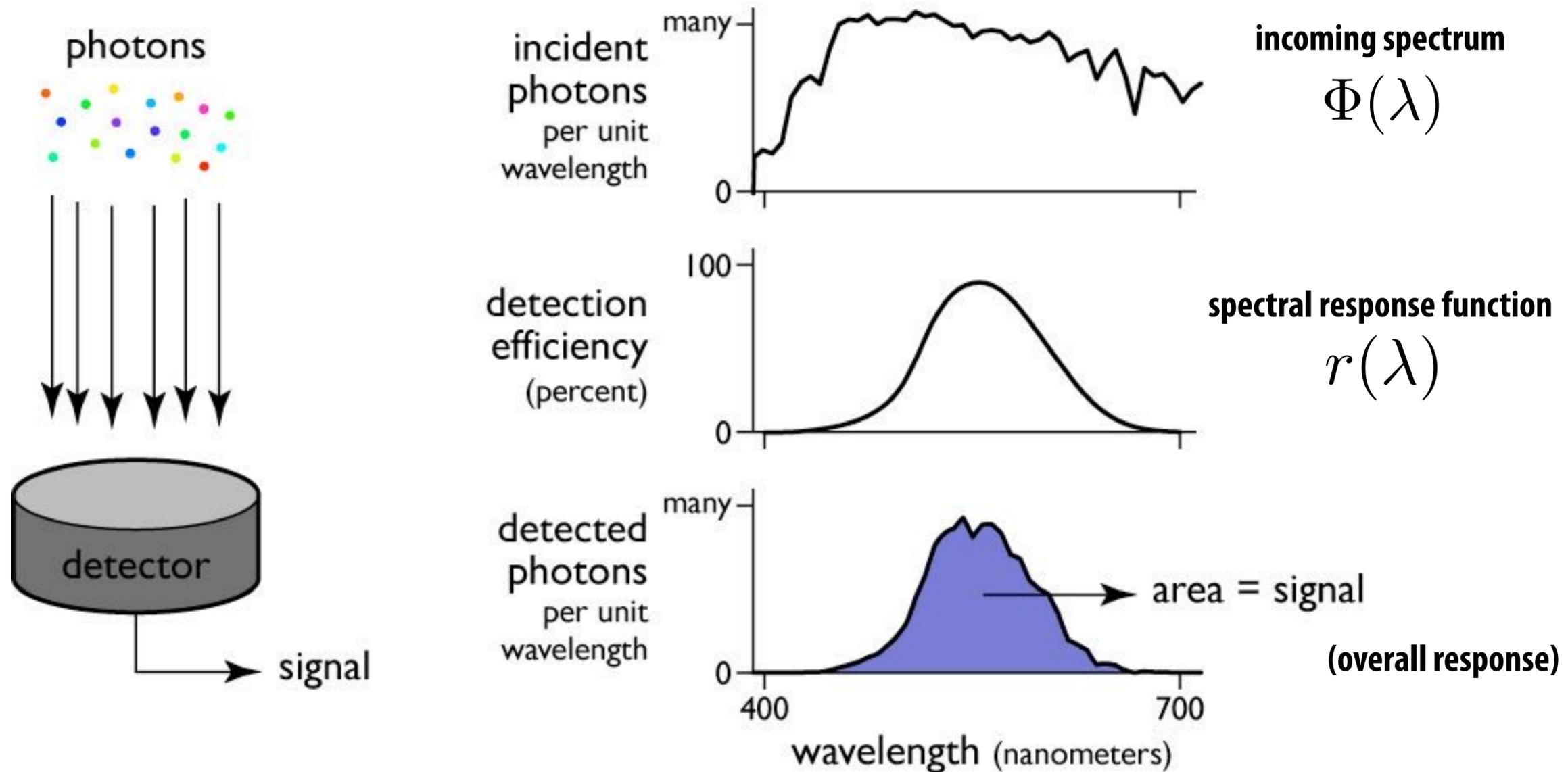
Measuring Light

A simple model of a light detector

- Produces a single value (a number) when photons land on it
 - Value depends only on number of photons detected
 - Each photon has a probability of being detected that depends on the wavelength
 - No way to distinguish between signals caused by light of different wavelengths: there is just a number
- This model holds for many detectors:
 - based on semiconductors (e.g., digital cameras)
 - based on visual photopigments (e.g., human eyes)



Simple model of a light detector



$$R = \int_{\lambda} \Phi(\lambda) r(\lambda) d\lambda$$

Dimensionality reduction from ∞ to 1

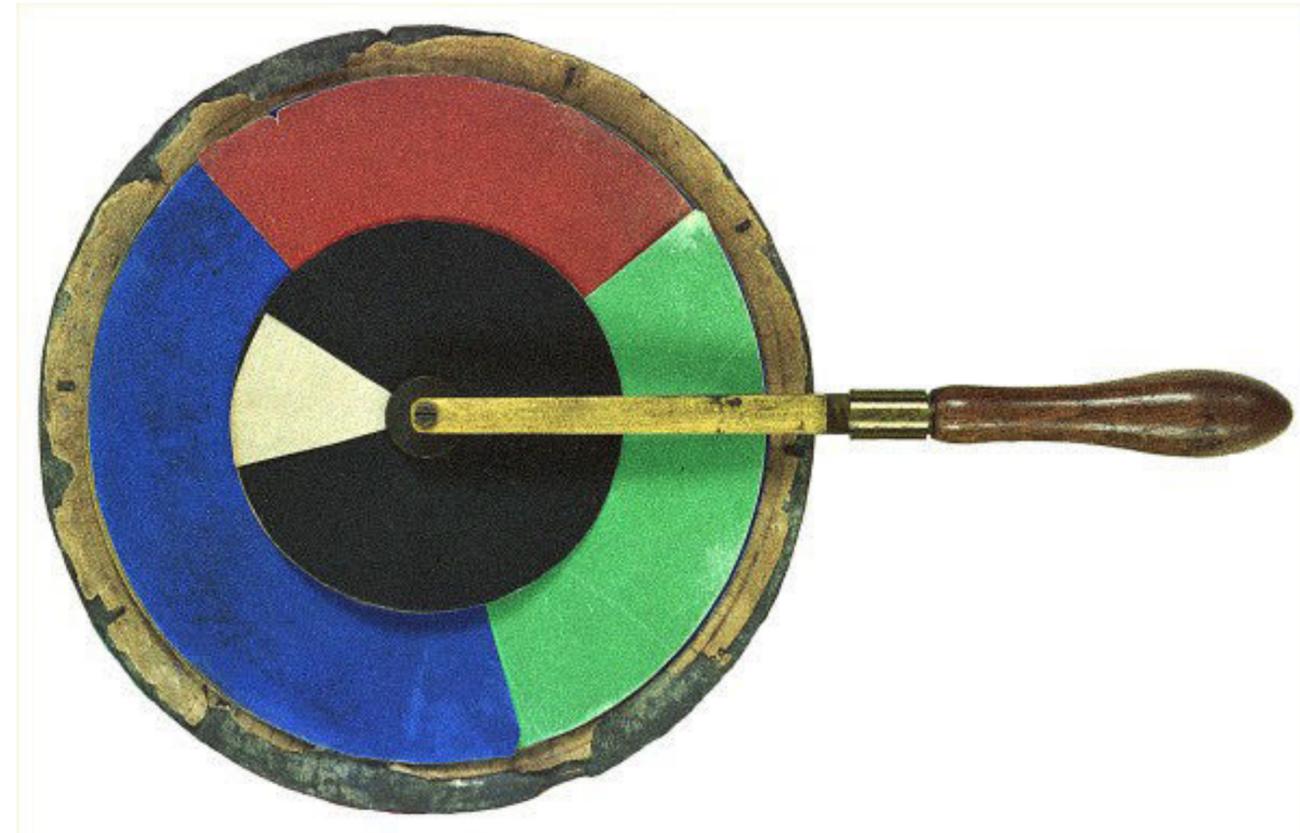
- **At the detector:**

- **SPD is a function of wavelength** (∞ - dimensional signal)
- **Detector output is a scalar value** (1 - dimensional signal)

Tristimulus Theory of Color

Searching for a Linear Systems Basis for Colors: The Color Matching Experiment

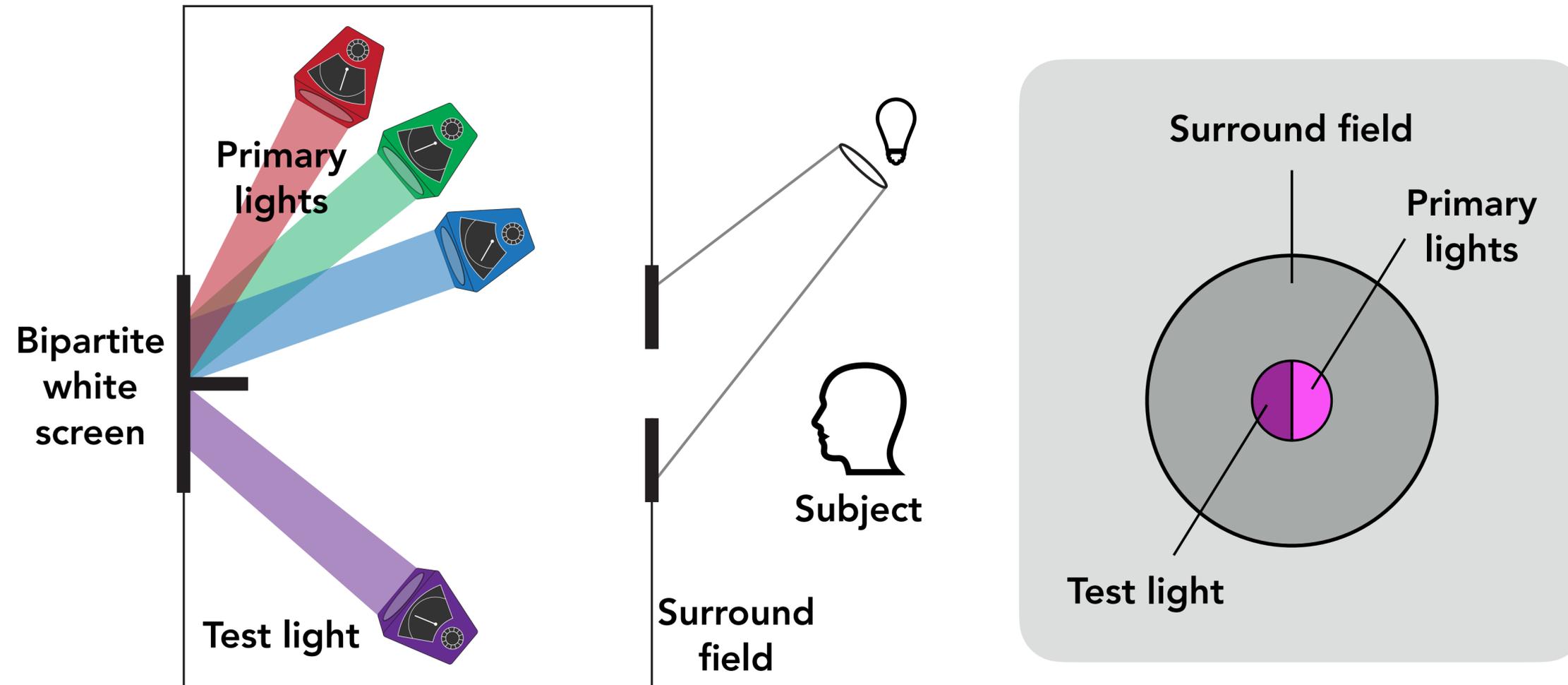
Maxwell's Crucial Color Matching Experiment



<http://designblog.rietveldacademie.nl/?p=68422>

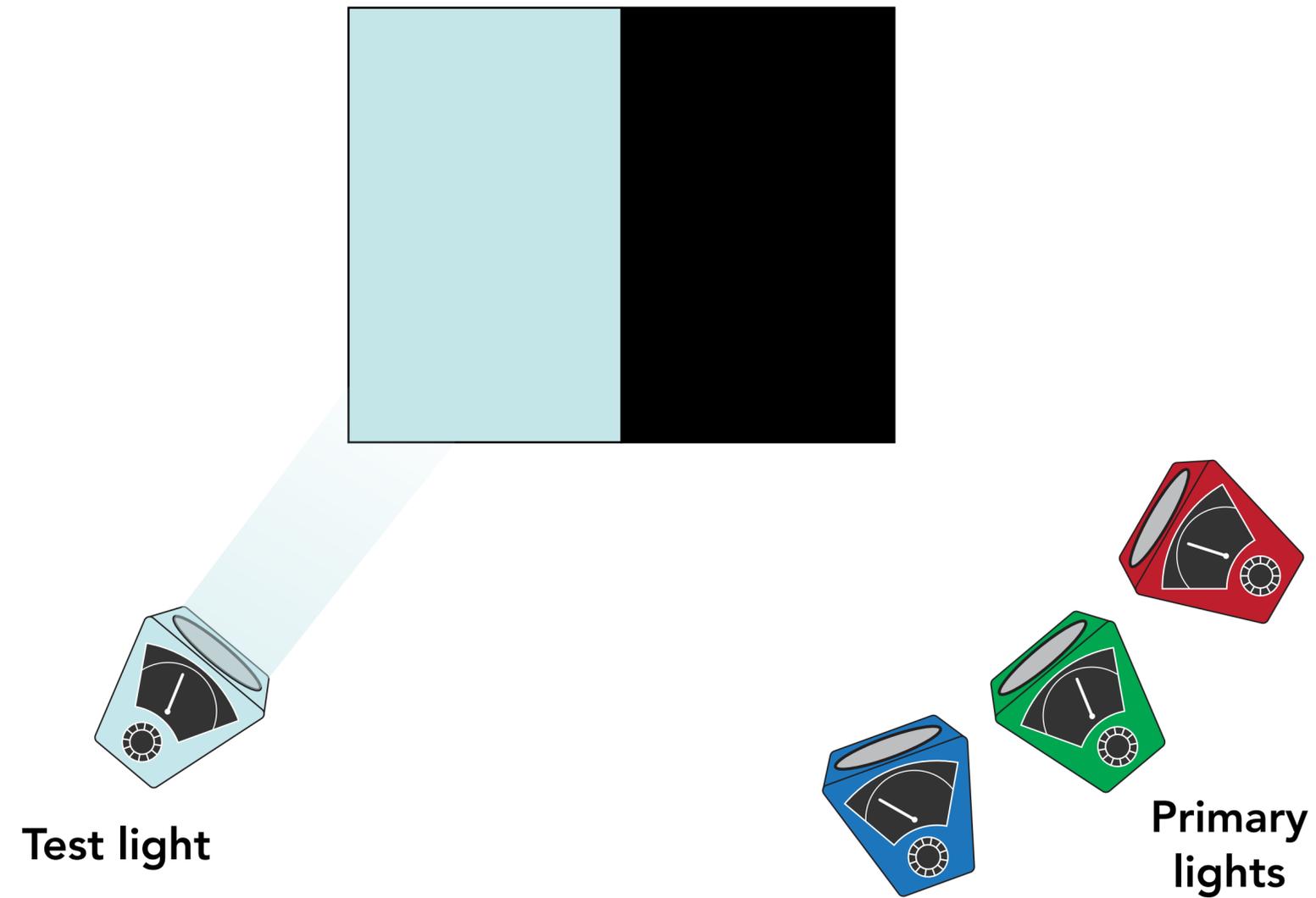
Portrait: <http://rsta.royalsocietypublishing.org/content/366/1871/1685>

Color Matching Experiment



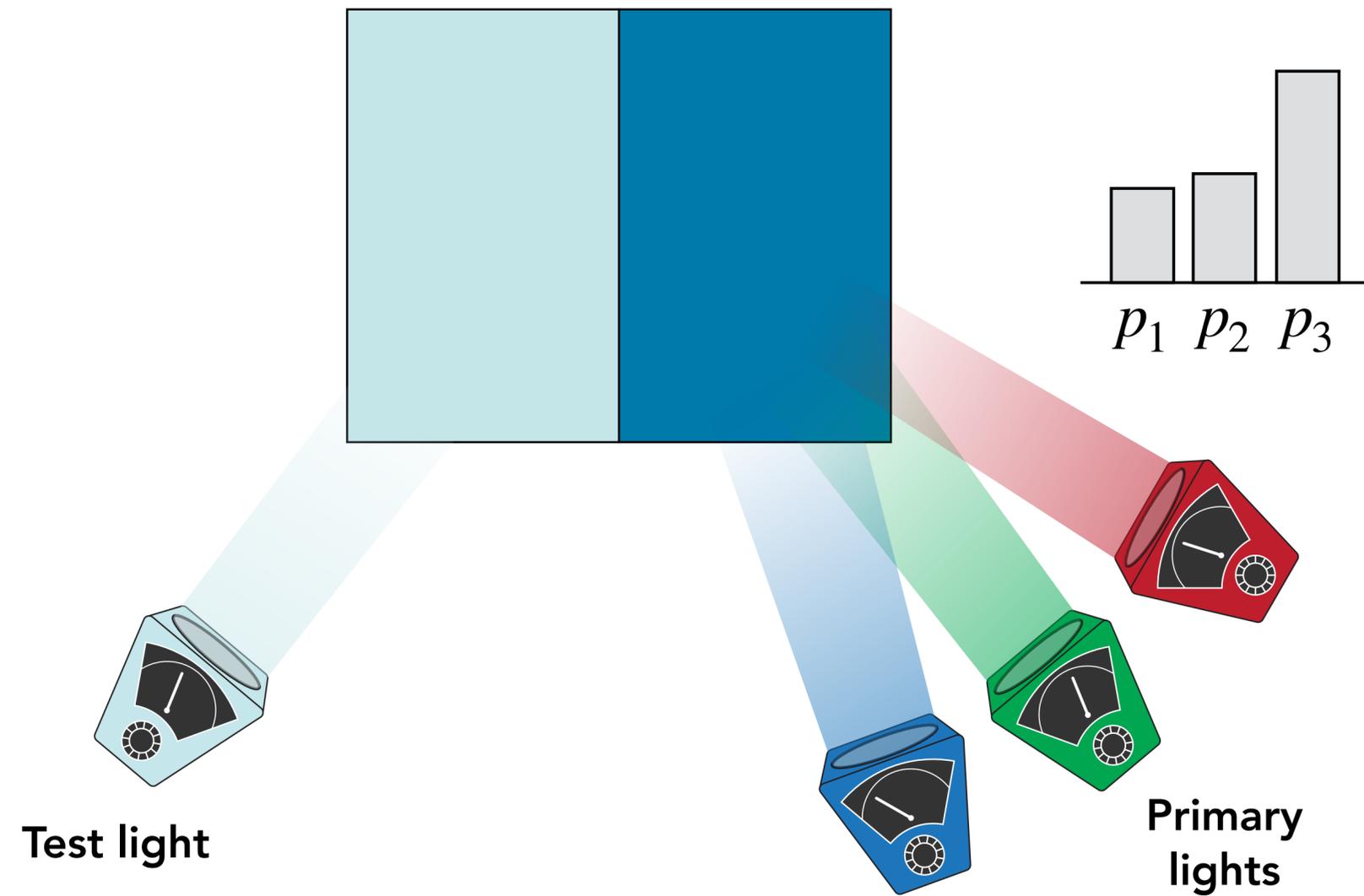
- Same idea as spinning top, fancier implementation (Maxwell did this too)
- Show test light spectrum on left
- Mix “primaries” on right until they match
- The primaries need not be RGB

Example Experiment



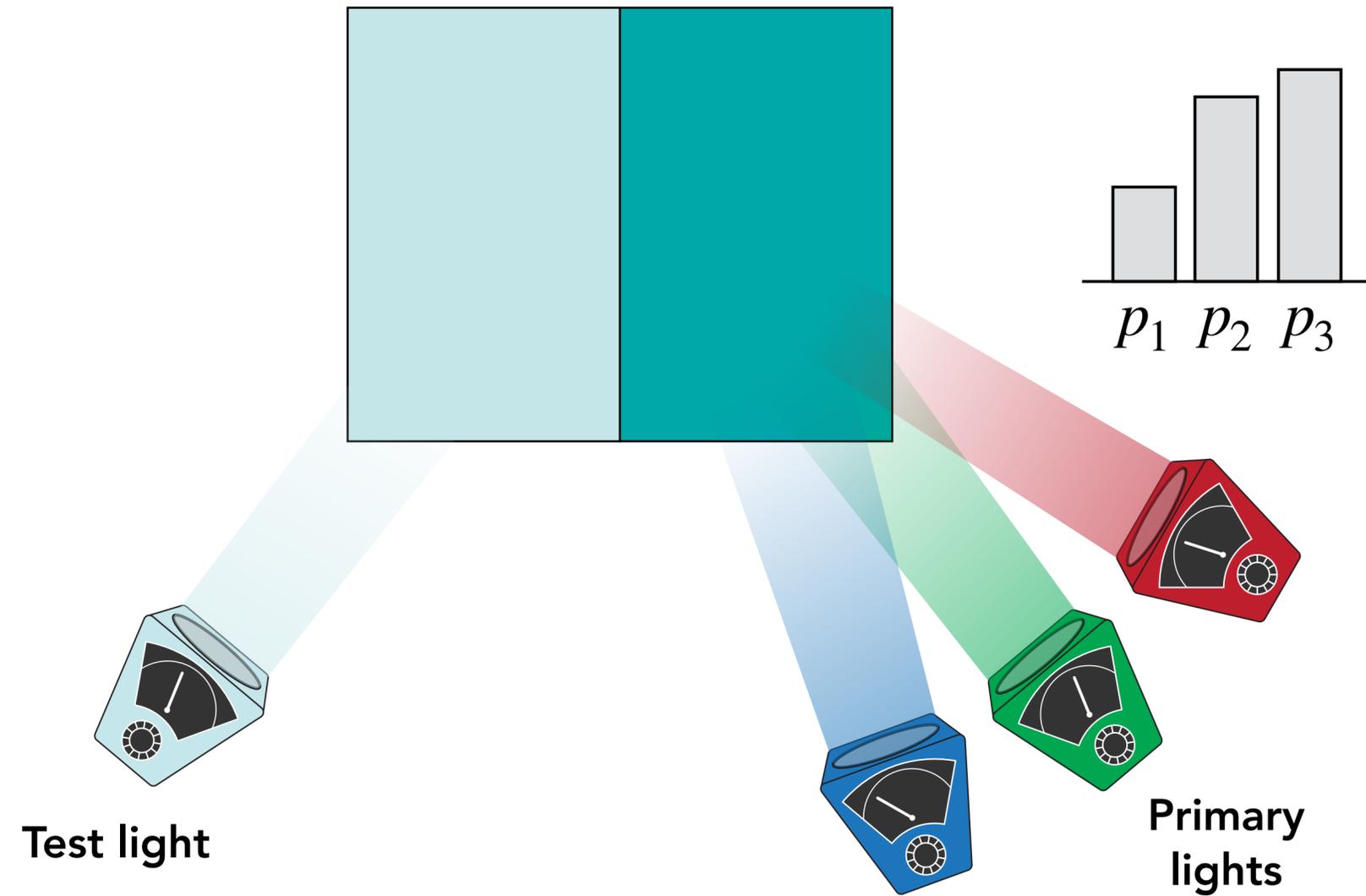
Slide credit: Kotani, Durand, Freeman, Ng

Example Experiment



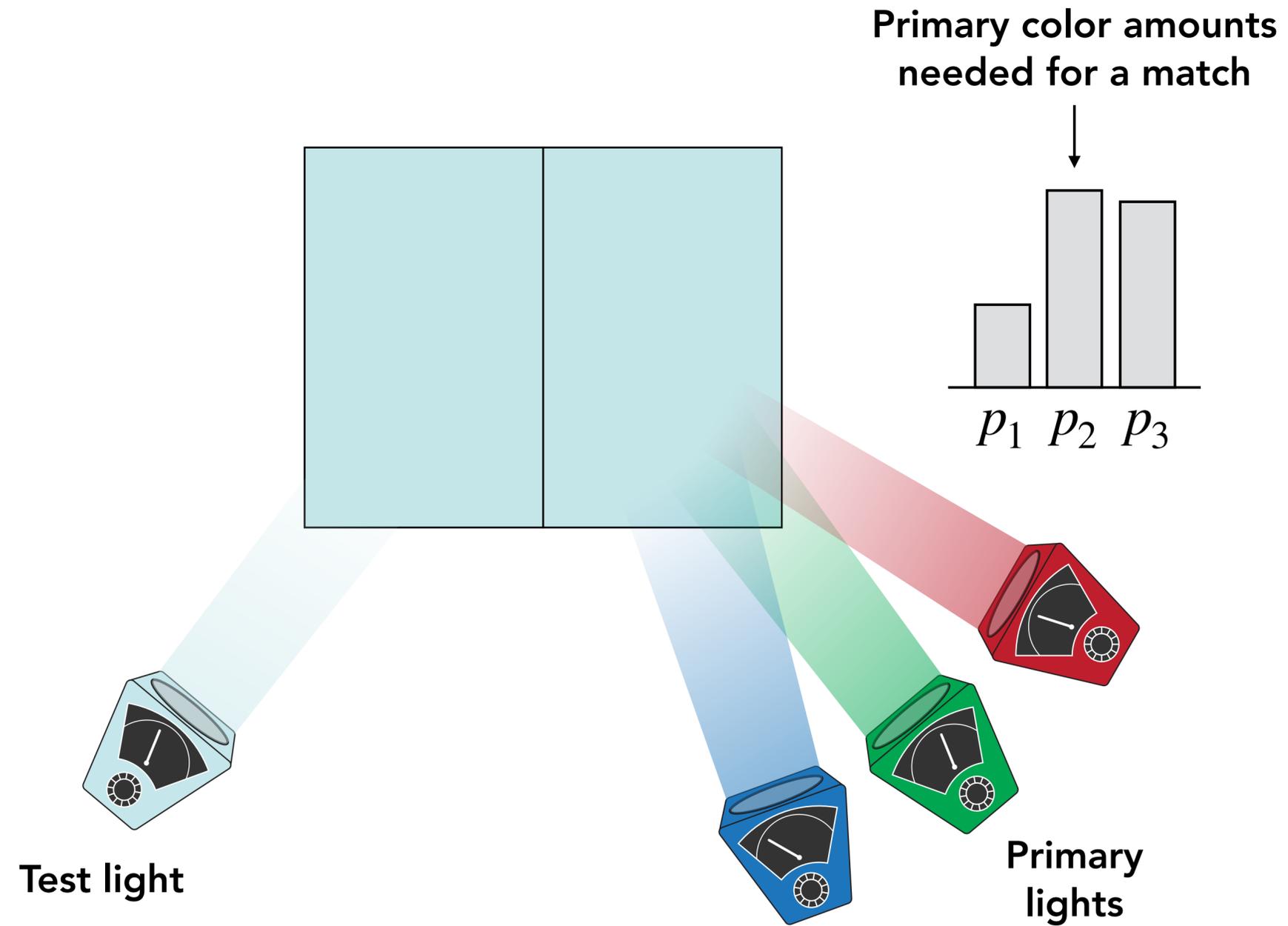
Slide credit: Kotani, Durand, Freeman, Ng

Example Experiment



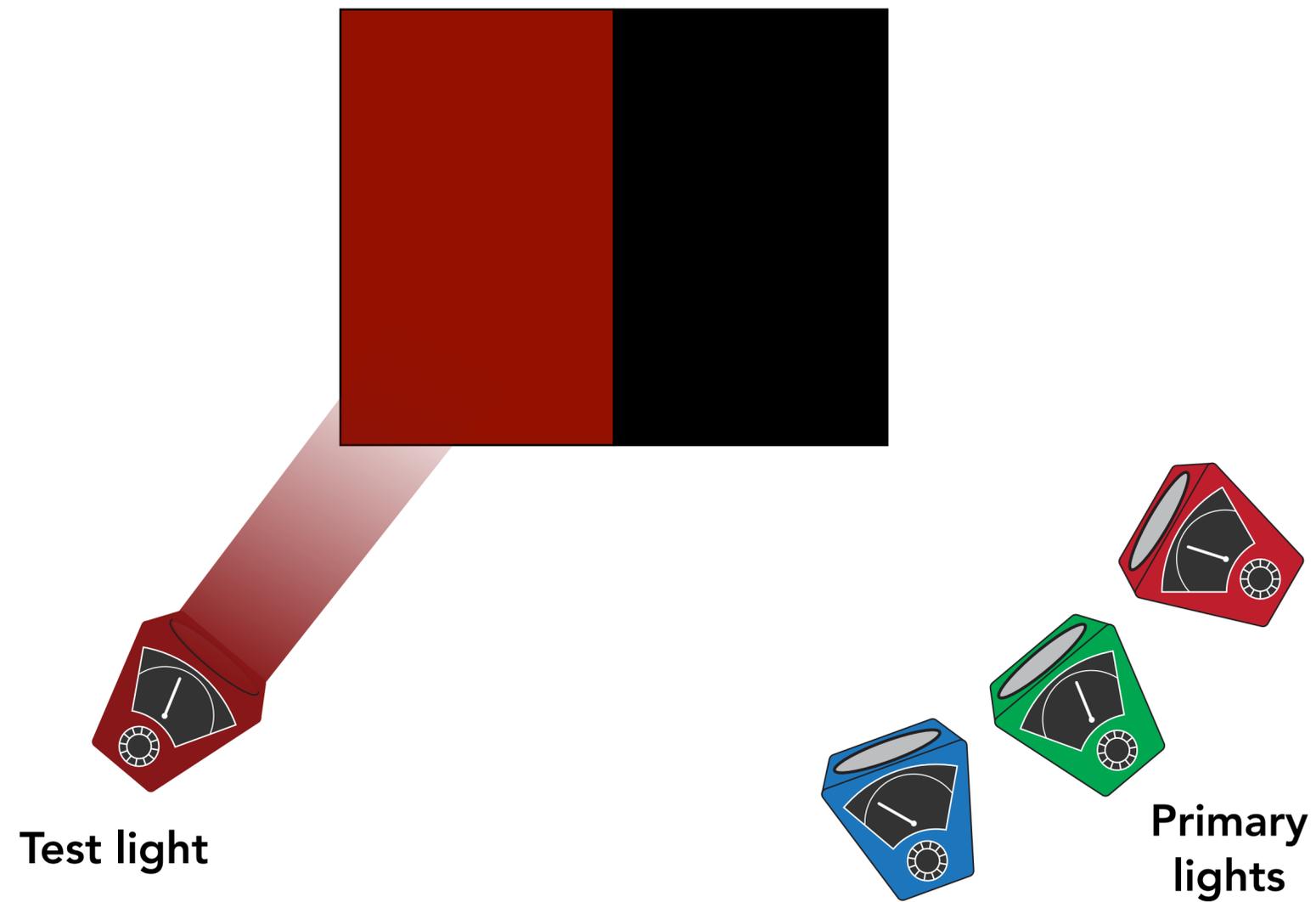
Slide credit: Kotani, Durand, Freeman, Ng

Example Experiment



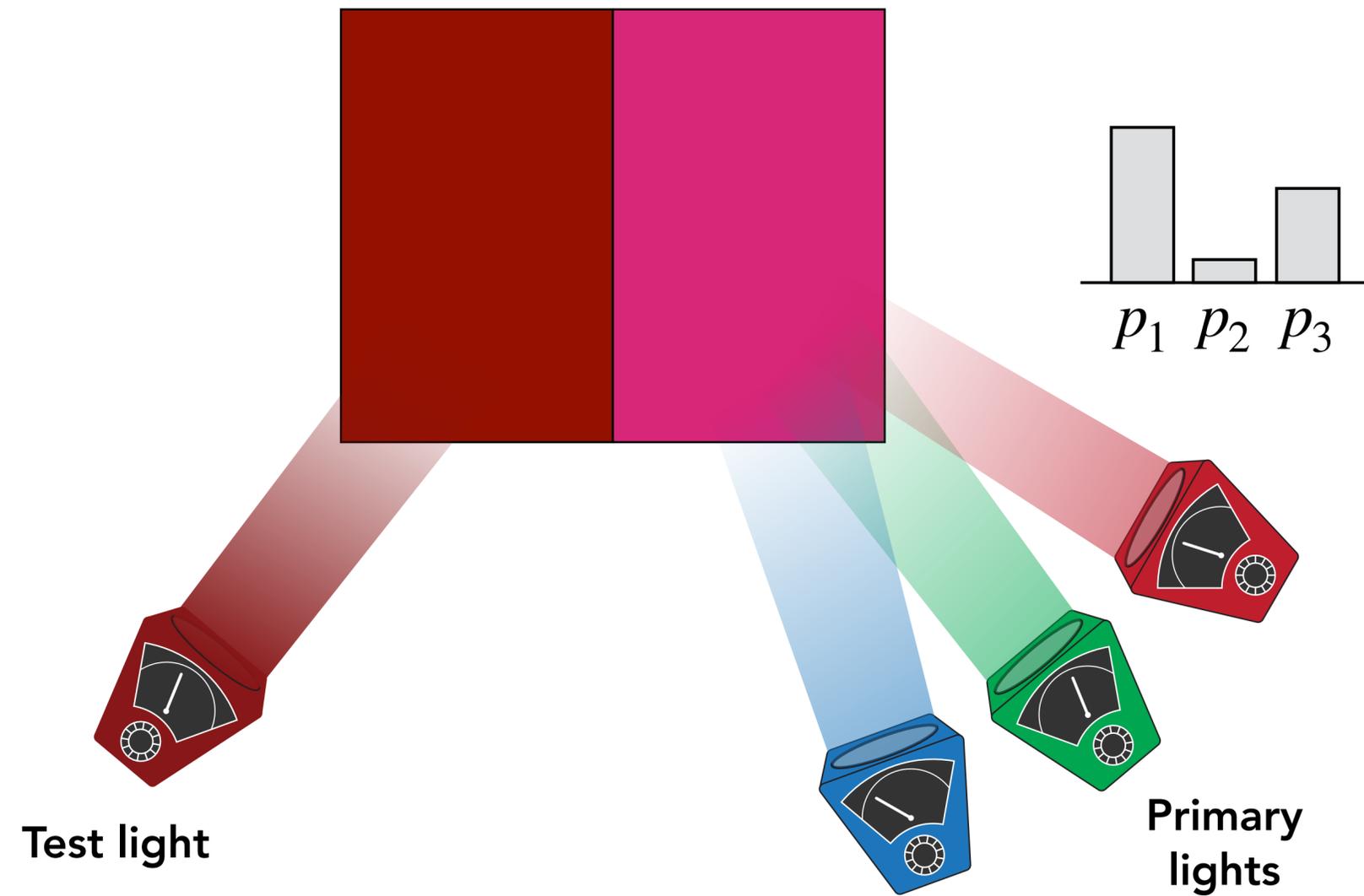
Slide credit: Kotani, Durand, Freeman, Ng

Experiment 2: Out of Gamut



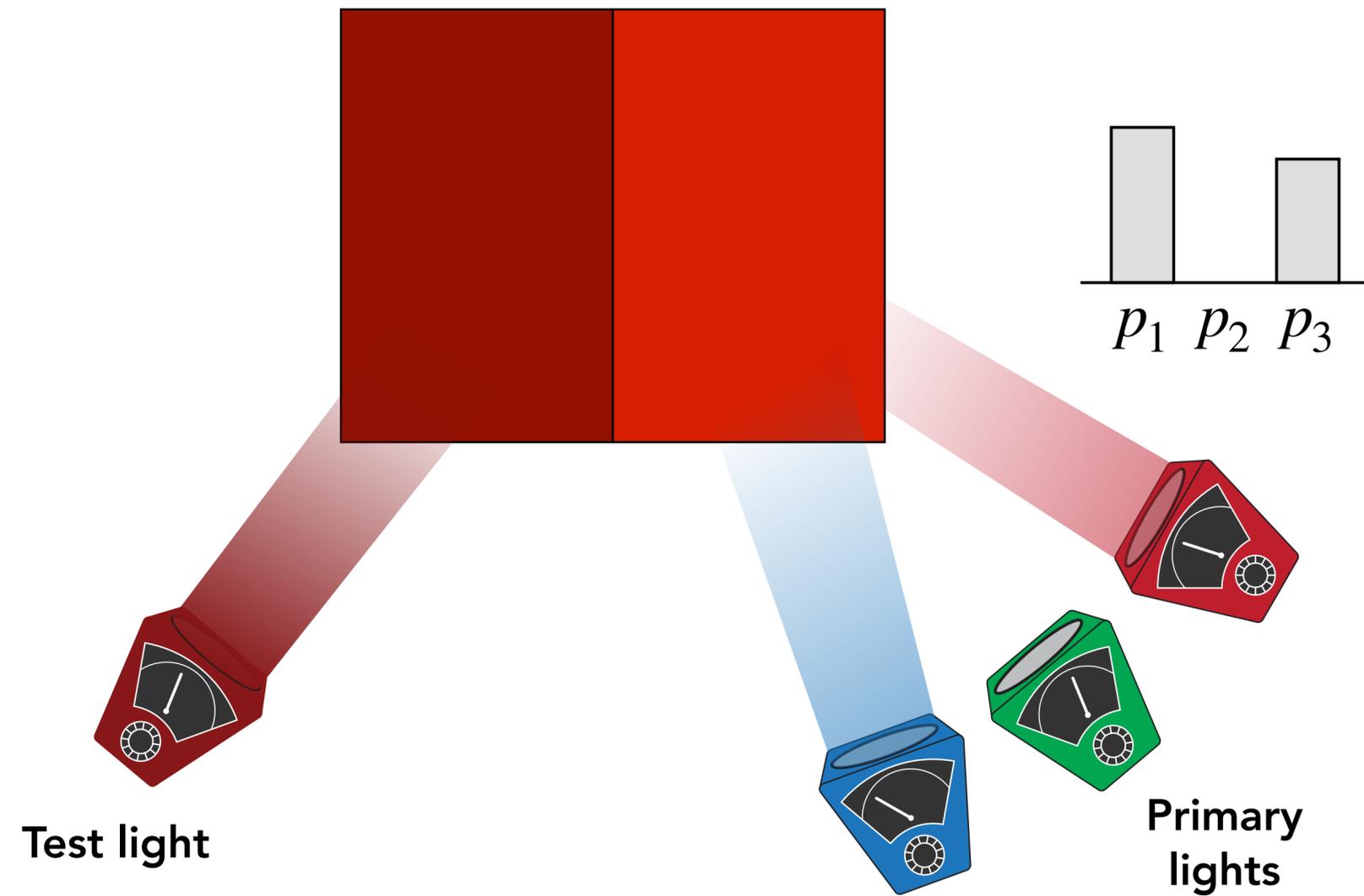
Slide credit: Kotani, Durand, Freeman, Ng

Experiment 2: Out of Gamut



Slide credit: Kotani, Durand, Freeman, Ng

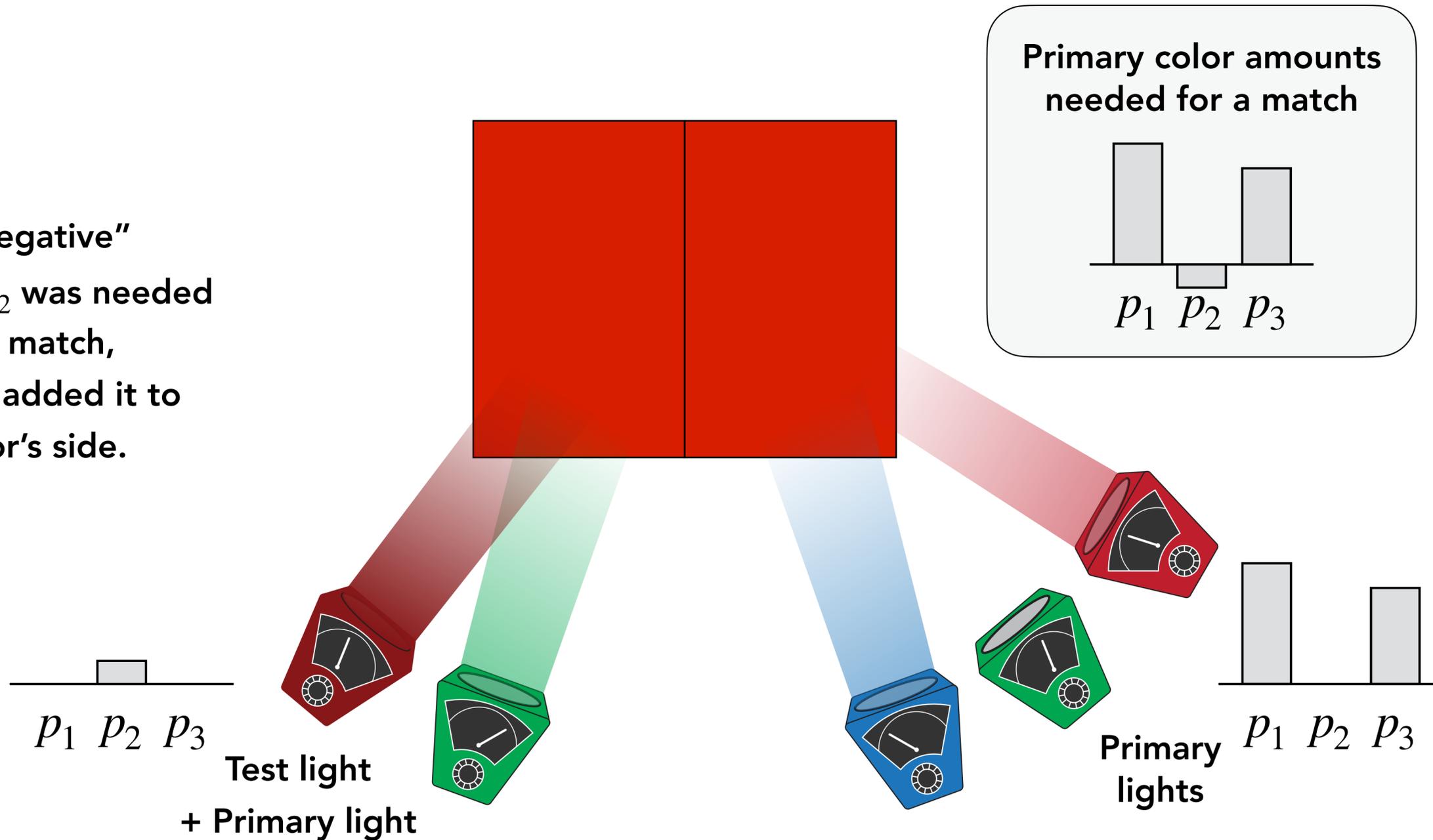
Experiment 2: Out of Gamut



Slide credit: Kotani, Durand, Freeman, Ng

Experiment 2: Out of Gamut

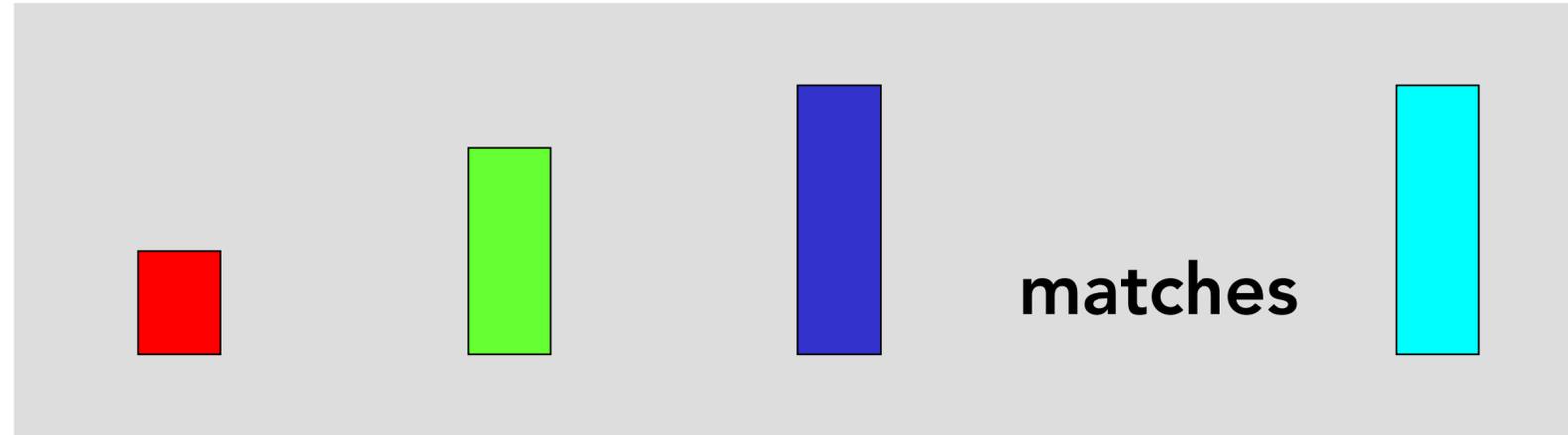
We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.



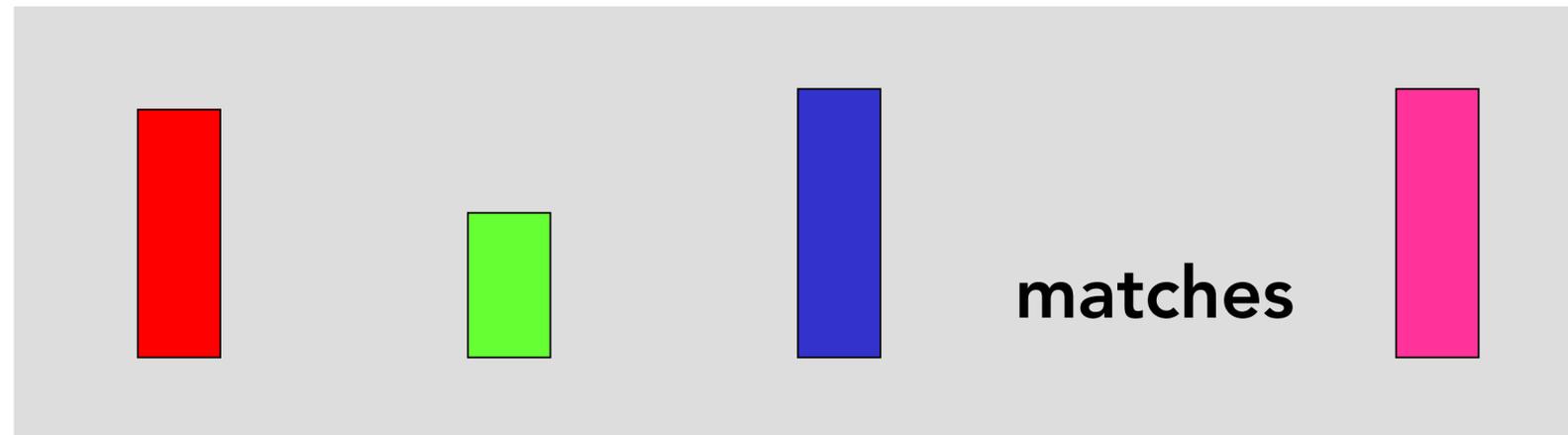
Slide credit: Kotani, Durand, Freeman, Ng

The color matching experiment is linear

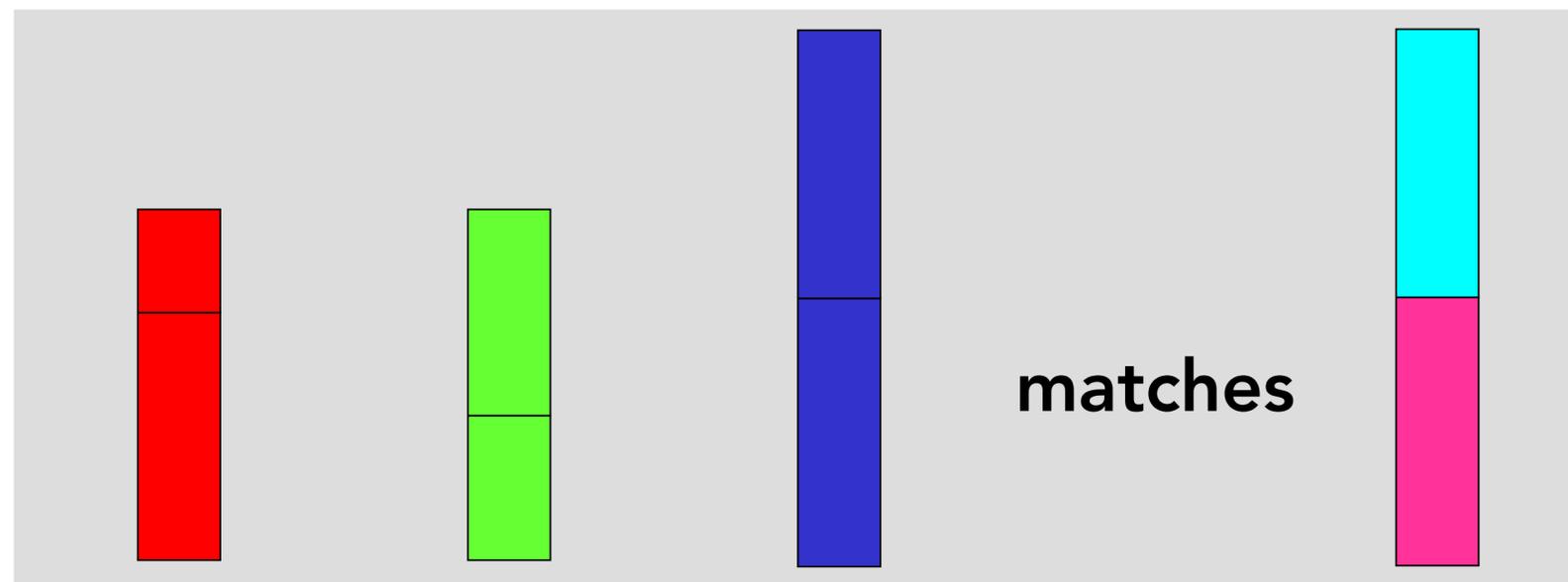
If



and



then



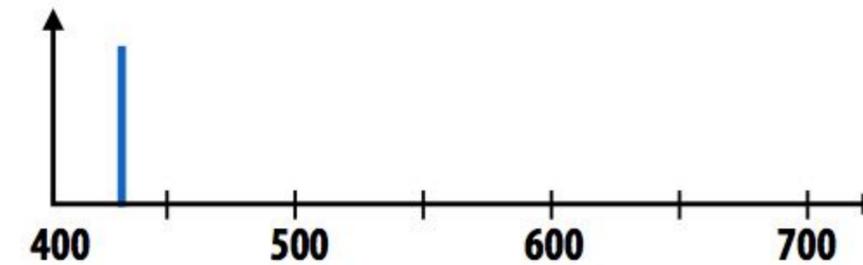
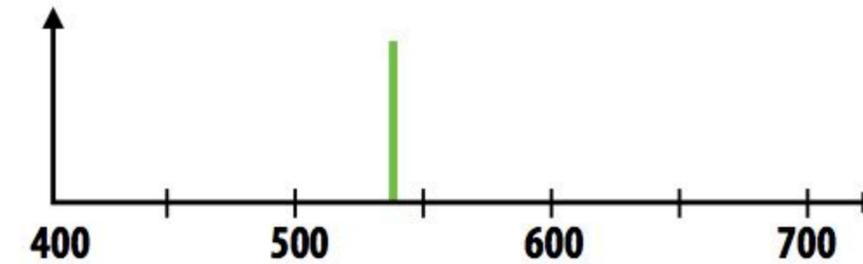
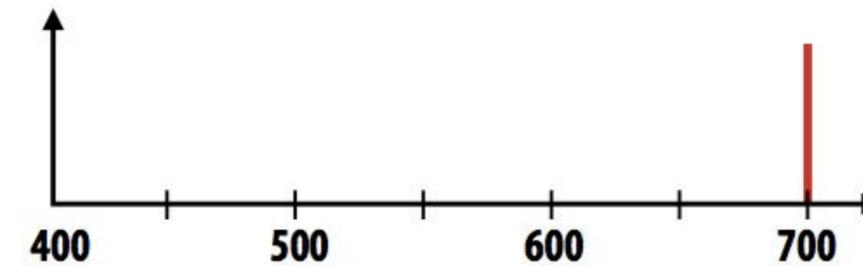
Brian Wandell

What is the Dimensionality of Human Color Perception?

- **And how do we know?**
- **What is the definition of “dimension” here?**
 - **We can appeal to linear systems theory, where “dimension” equals the rank of a basis for the linear space.**
- **In the color matching experiment, empirically one finds:**
 - **For subjects with “normal” color vision, three primary colors are necessary and sufficient to match any test color. Four primaries work but are unnecessary; two are insufficient.**
 - **For red-green color-blind subjects, only two primary colors are necessary and sufficient to match any test color.**

CIE RGB Color Matching Experiment

Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE RGB standard

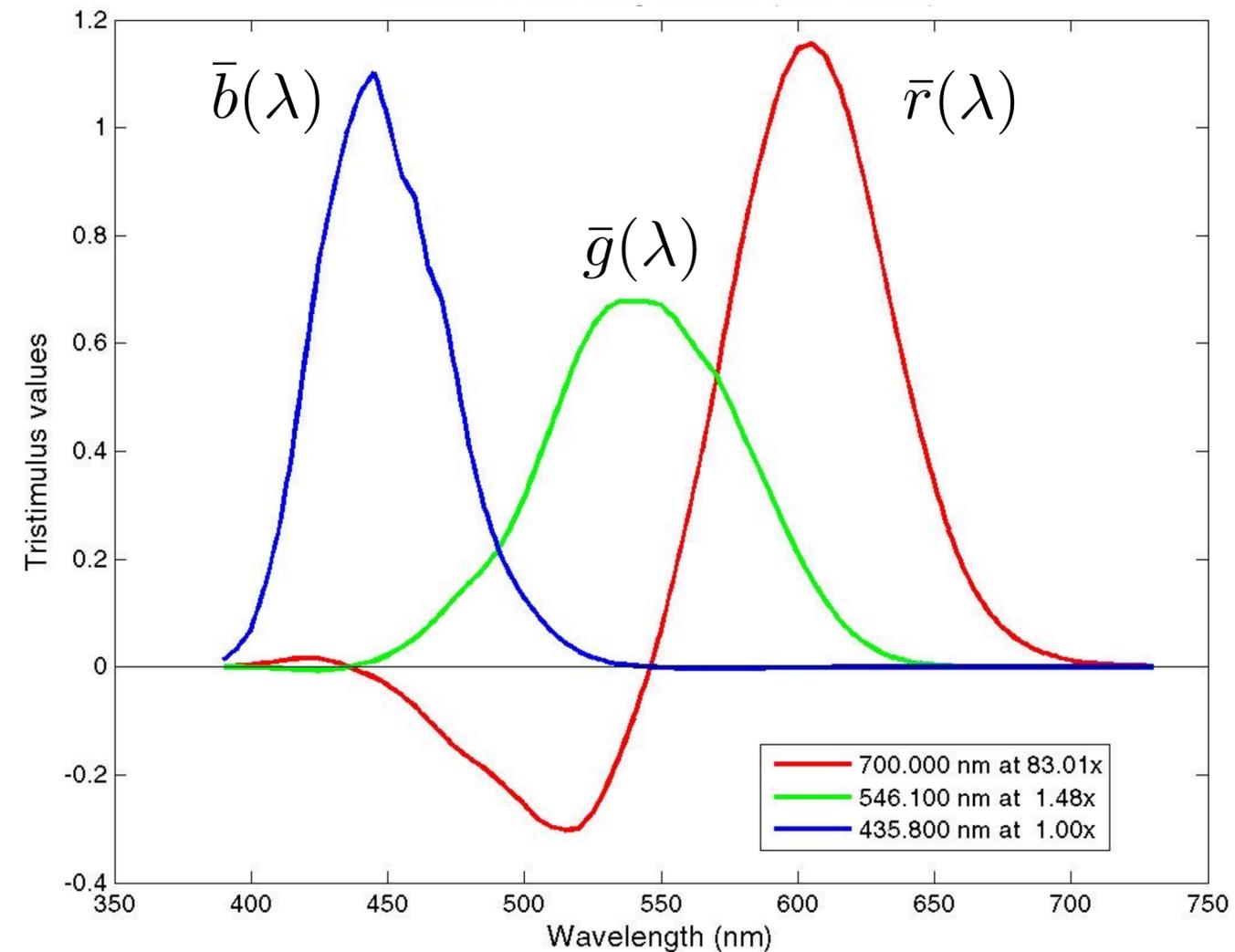


The test light is also a monochromatic light



CIE RGB Color Matching Functions

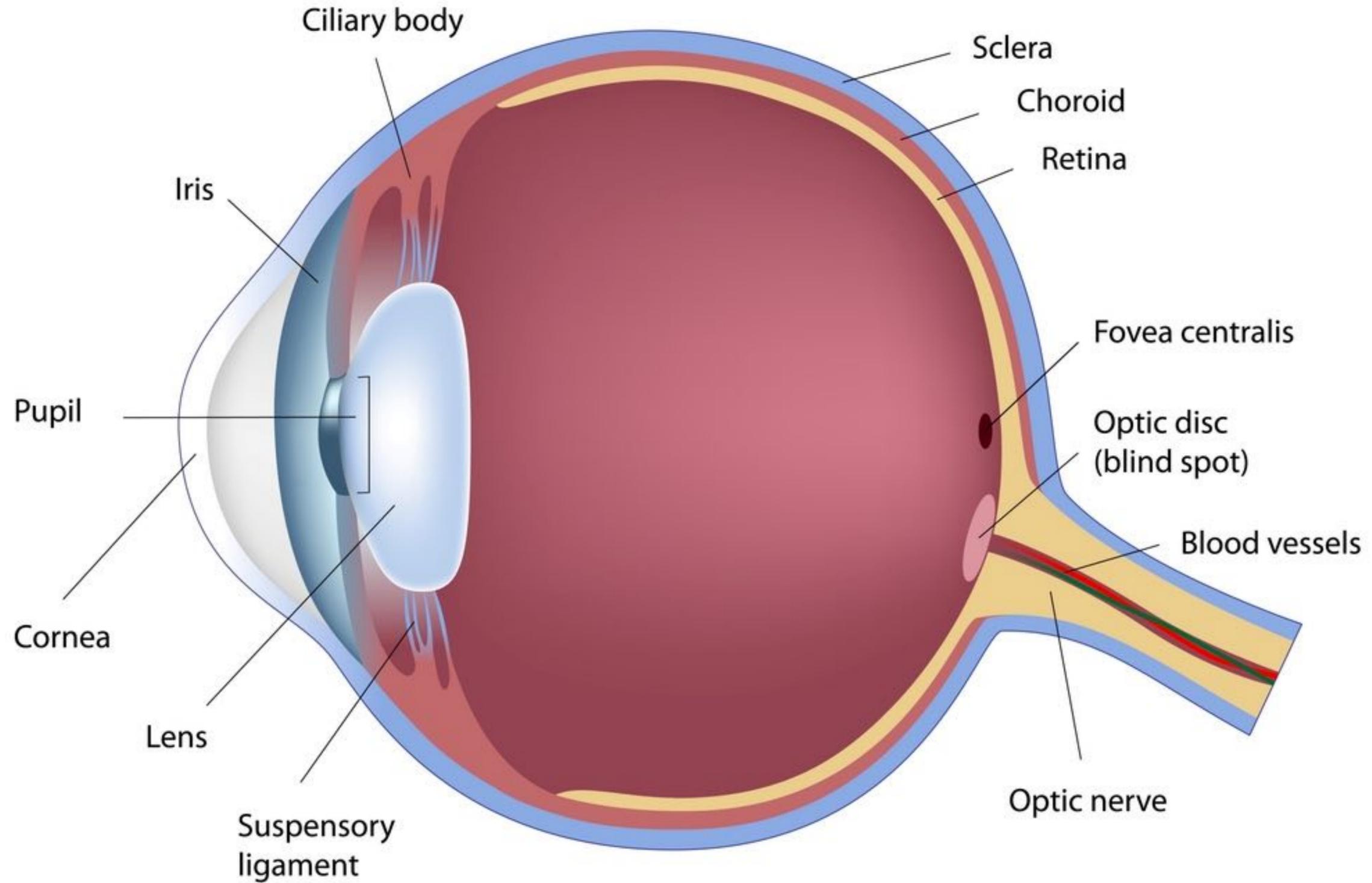
Graph plots how much of each CIE RGB primary light must be combined to match a monochromatic light of wavelength given on x-axis



Careful: these are not response curves or primary spectra!

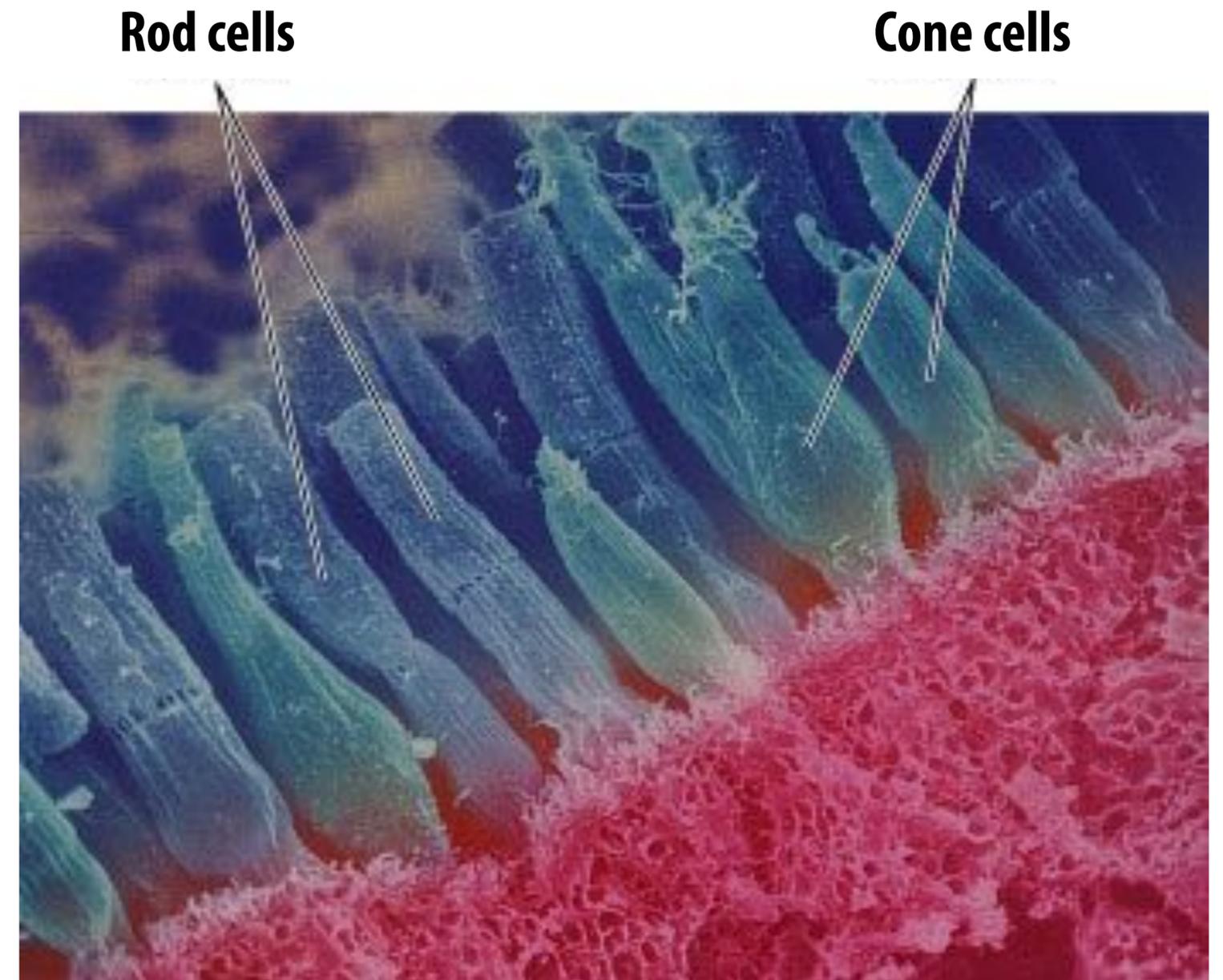
Biological basis of color

The eye

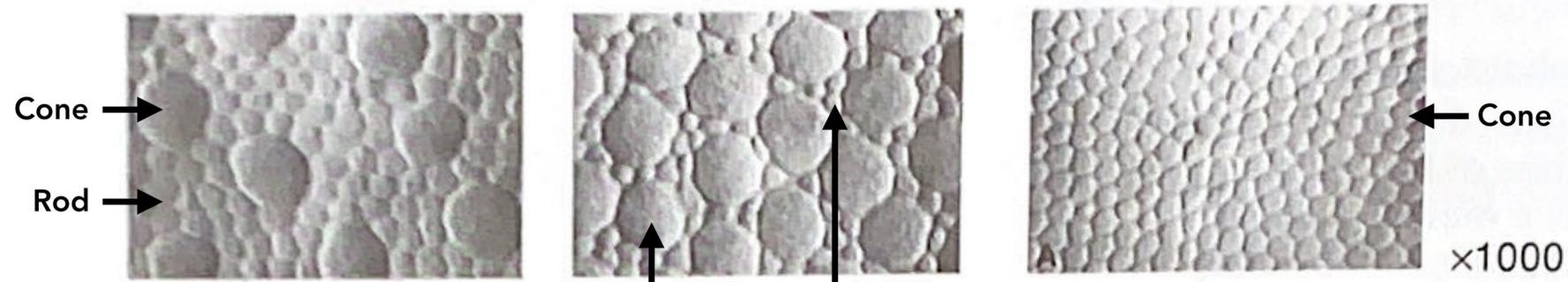


The eye's photoreceptor cells: rods and cones

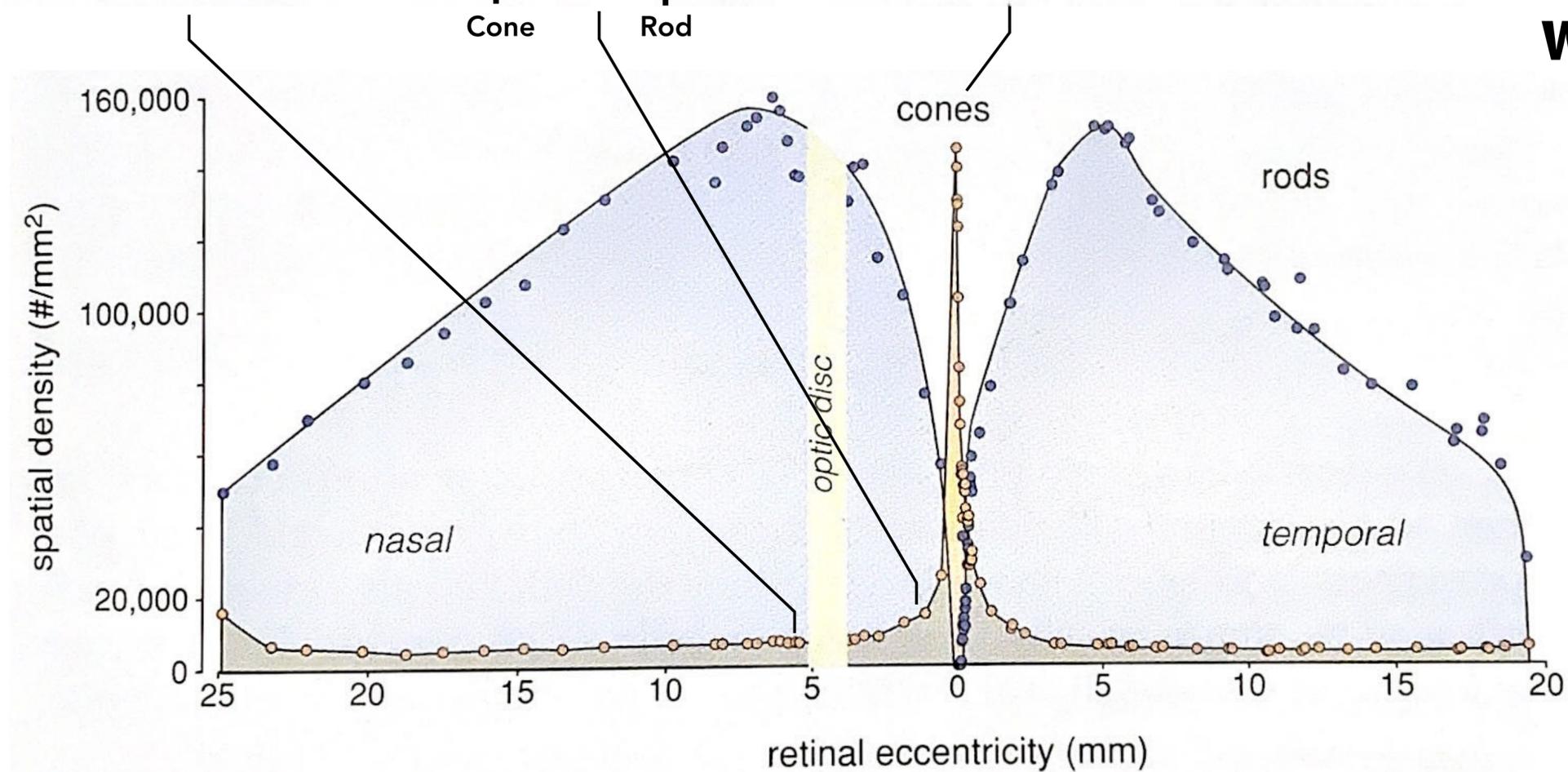
- **Rods are primary receptors under dark viewing conditions (scotopic conditions)**
 - Approx. 120 million rods in human eye
 - Sense light intensity (shades of gray, not color)
- **Cones are primary receptors under high-light viewing conditions (photopic conditions, e.g., daylight)**
 - Approx. 6-7 million cones in the human eye
 - There are three types of cones
 - Each of the three types of cone feature a different “spectral response”. This will be critical to color vision (more on this soon...)



Photoreceptor Size and Distribution Vary Across Retina



- No rods in fovea
- Cones increase in size and spacing with eccentricity



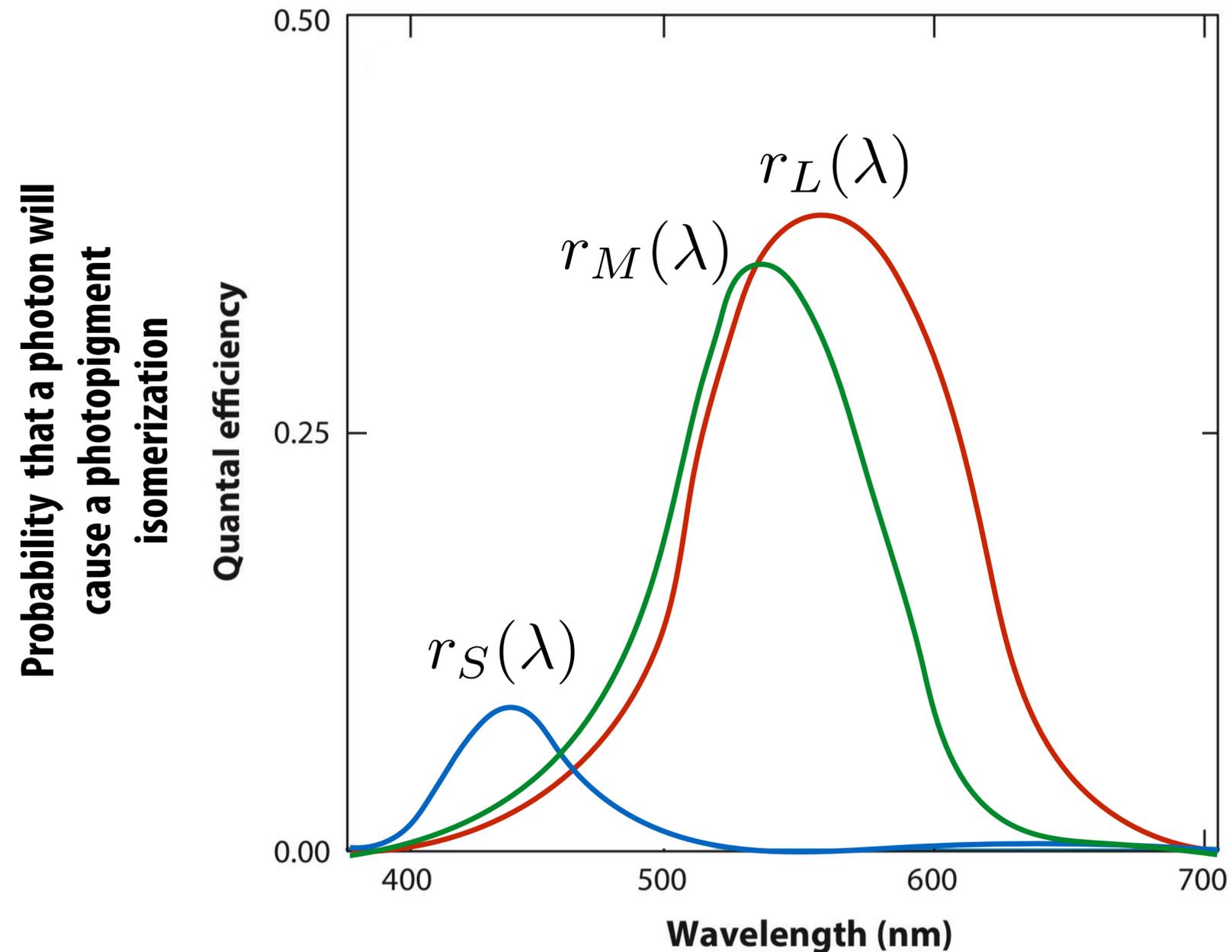
after Østerberg, 1935; as modified by Rodieck, 1988

Rodieck, p. 42

Human retinal cone cell response functions (L, M, S types)

Three types of cone cells: S, M, and L

(corresponding to peak response at short, medium, and long wavelengths)

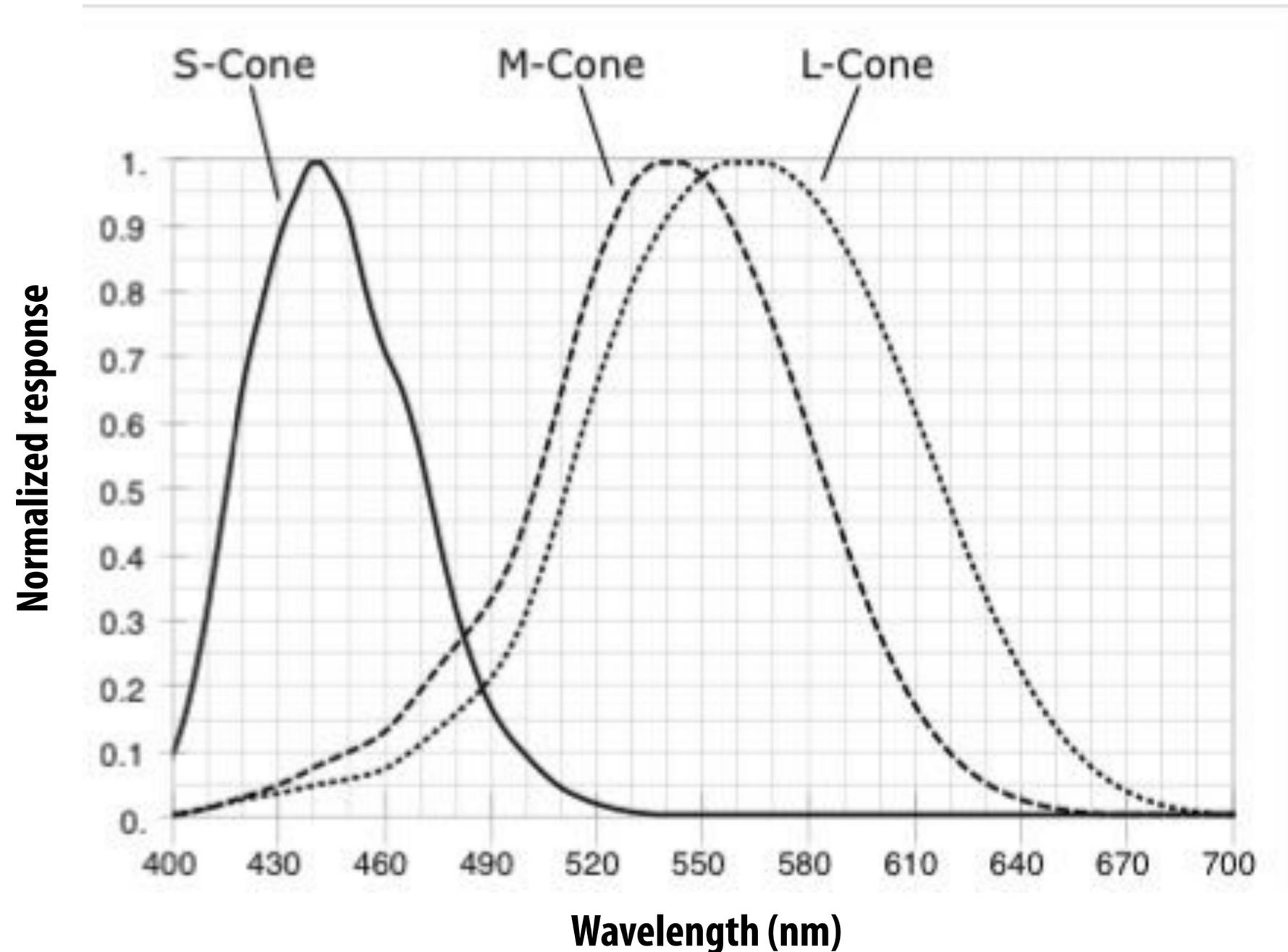


Brainard, Color and the Cone Mosaic, 2015.

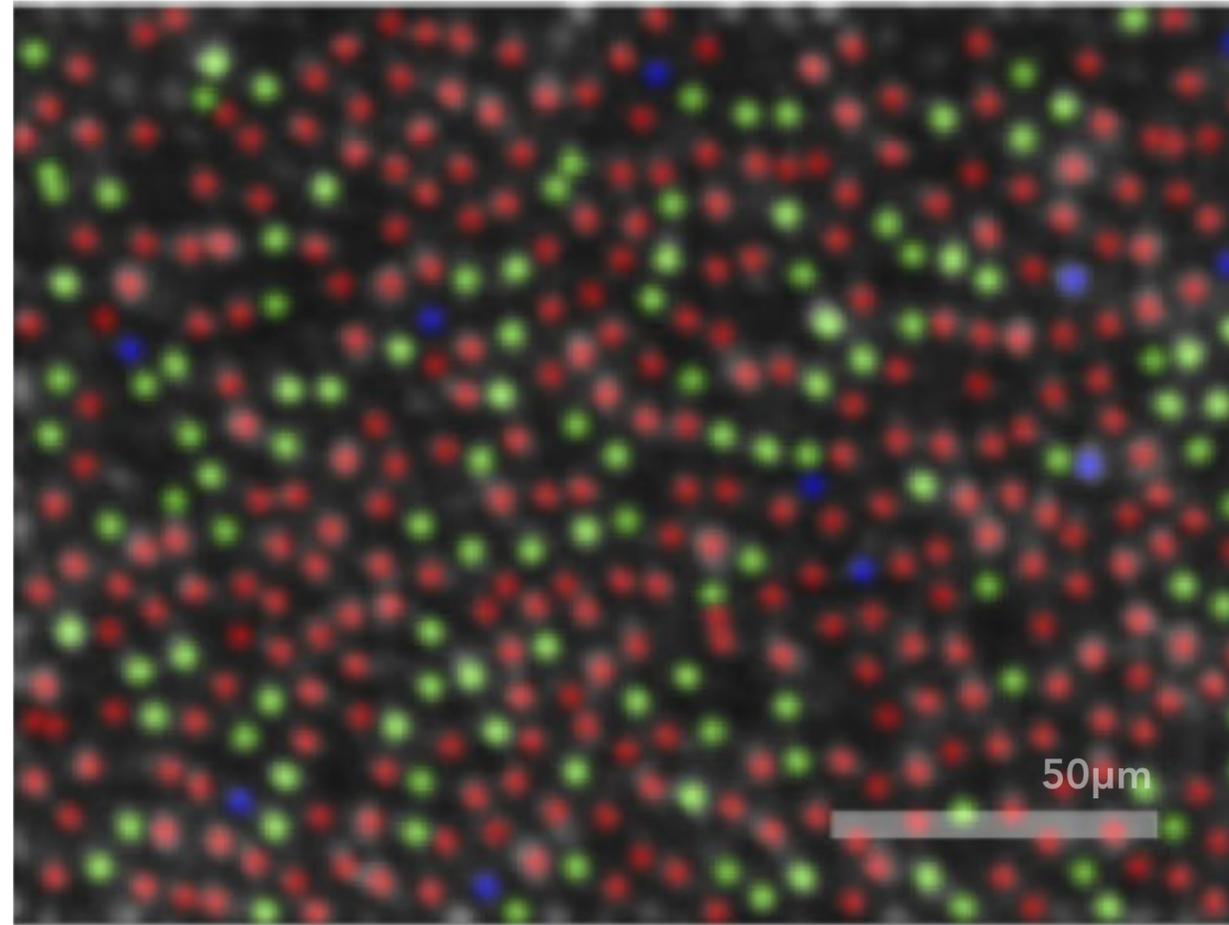
Human retinal cone cell normalized response functions (L, M, S types)

Three types of cone cells: S, M, and L

(corresponding to peak response at short, medium, and long wavelengths)



On the Retina, Three Types of Cone Cells



Sabesan Lab, UW. Pandiyan et al. 2020.

- **Three types of cone cells: S, M, and L (corresponding to peak response at short, medium, and long wavelengths)**

Spectral response of cones

Three types of cones: S, M, and L cones

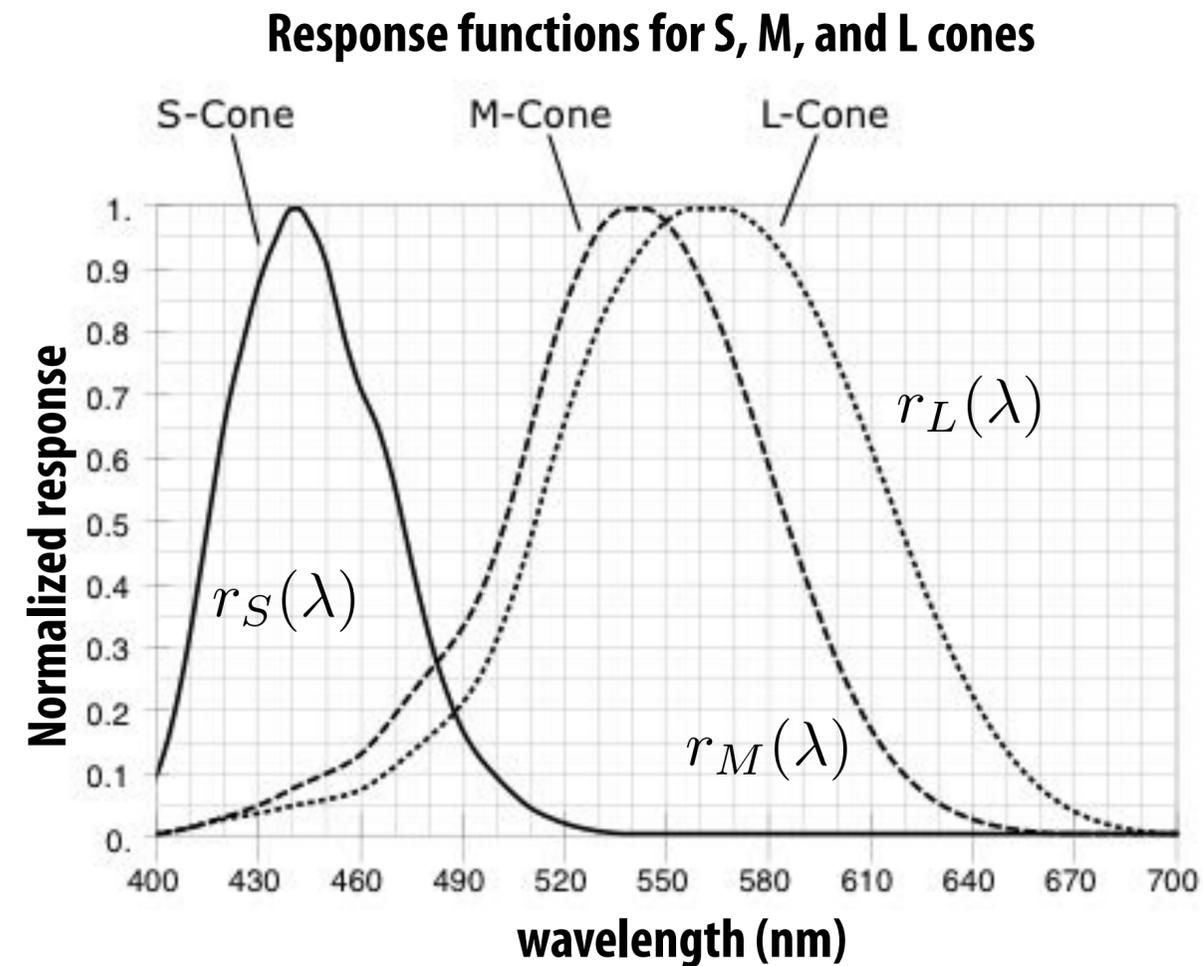
(corresponding to peak response at short, medium, and long wavelengths)

$$S = \int_{\lambda} \Phi(\lambda) r_S(\lambda) d\lambda$$

$$M = \int_{\lambda} \Phi(\lambda) r_M(\lambda) d\lambda$$

$$L = \int_{\lambda} \Phi(\lambda) r_L(\lambda) d\lambda$$

Each cone has a different response curve



Spectral response of cones (discrete form)

Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

Discrete form: written as vector dot products:
(now using vector s to denote discrete representation of SPD $\Phi(\lambda)$)

$$S = r_S \cdot s$$

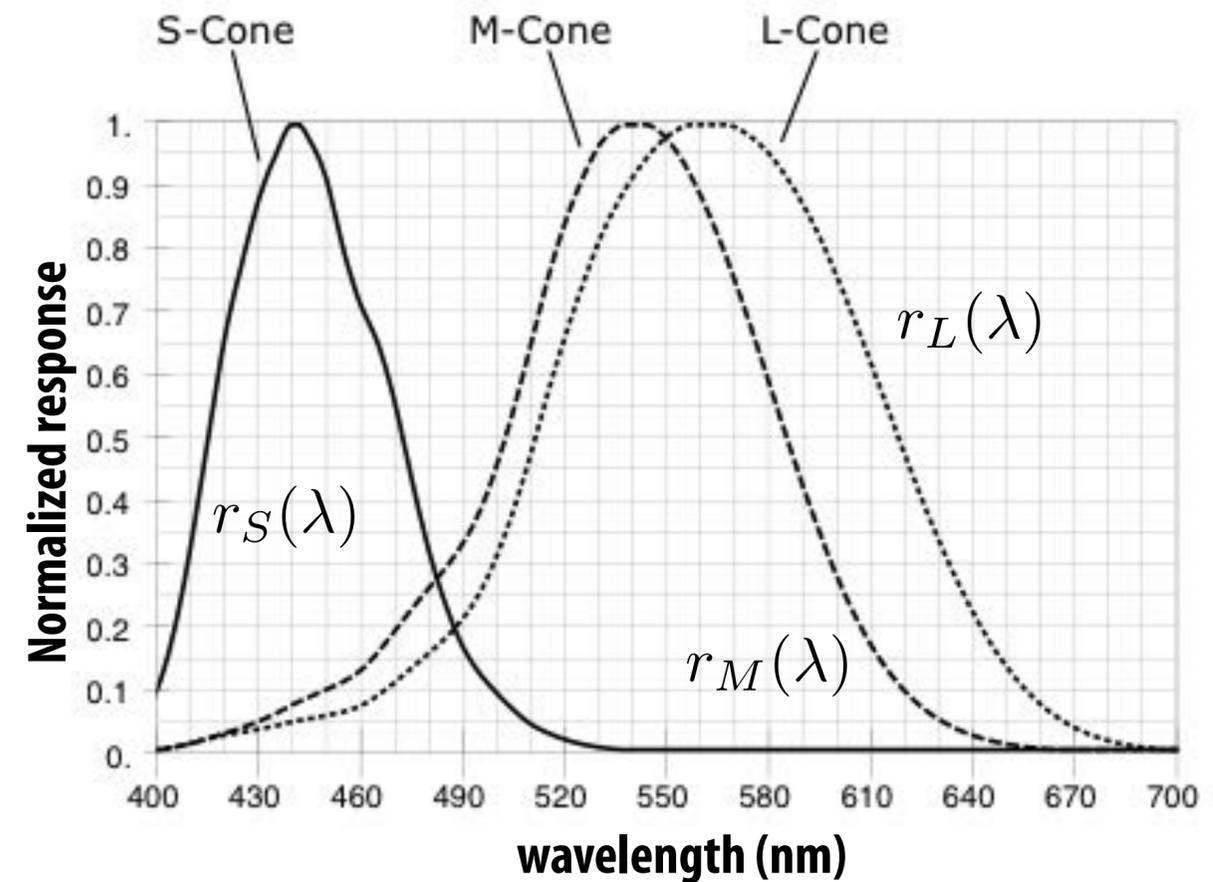
$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

Matrix formulation:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Response functions for S, M, and L cones



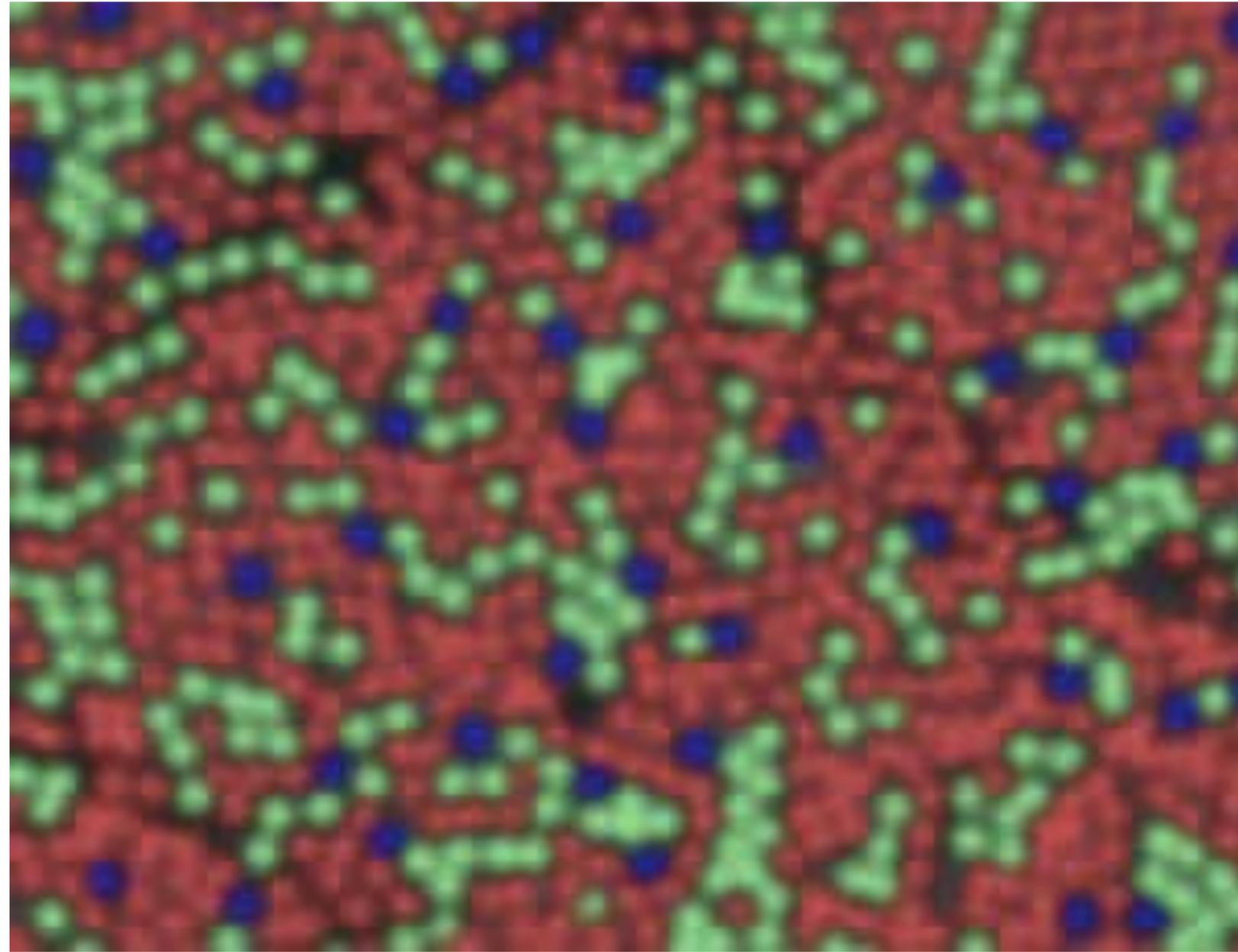
Example: spectral response of human cone cells

Goal: at each pixel, choose scalar R, G, B values for display to perceptually match the real world color



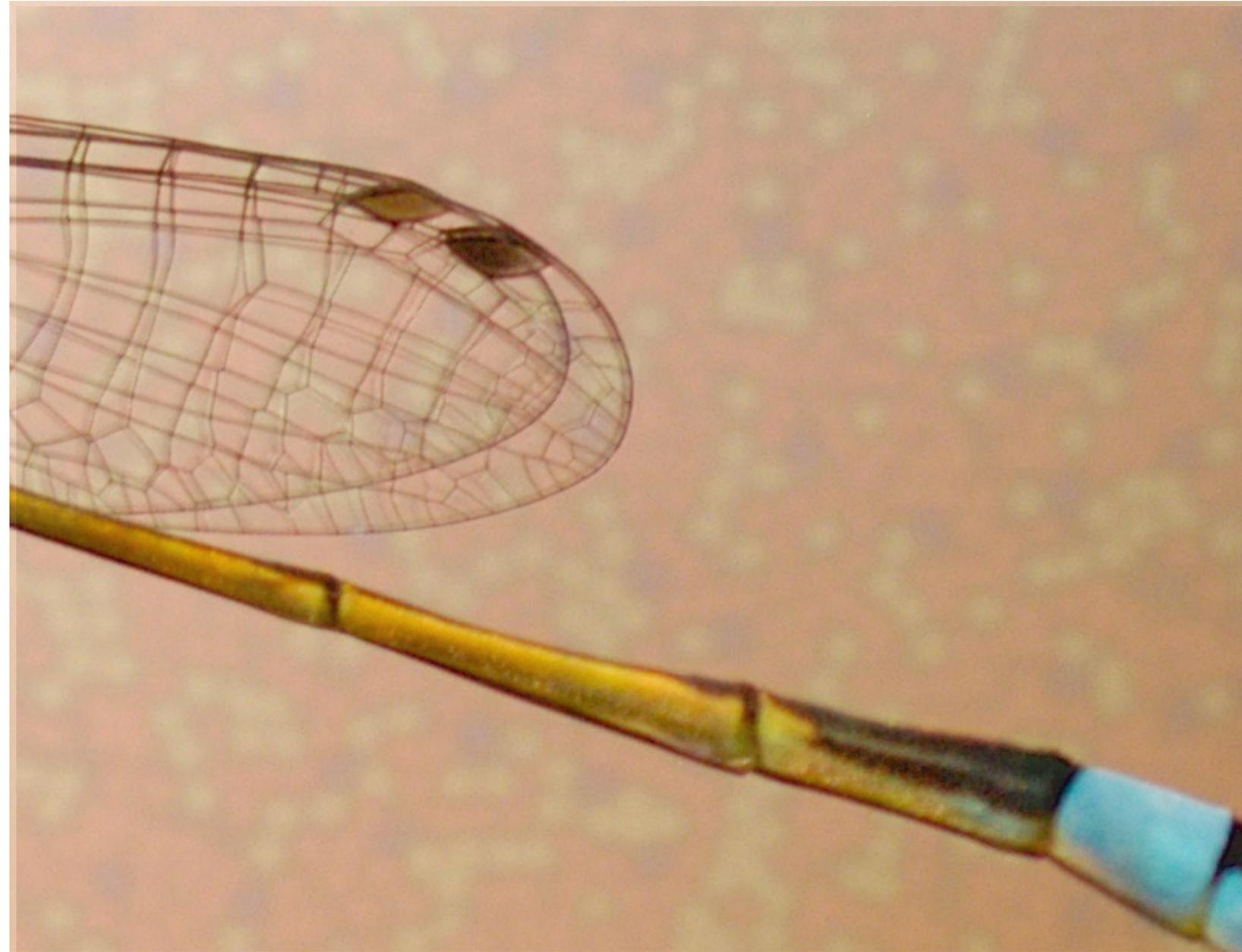
The scene

Example: spectral response of human cone cells



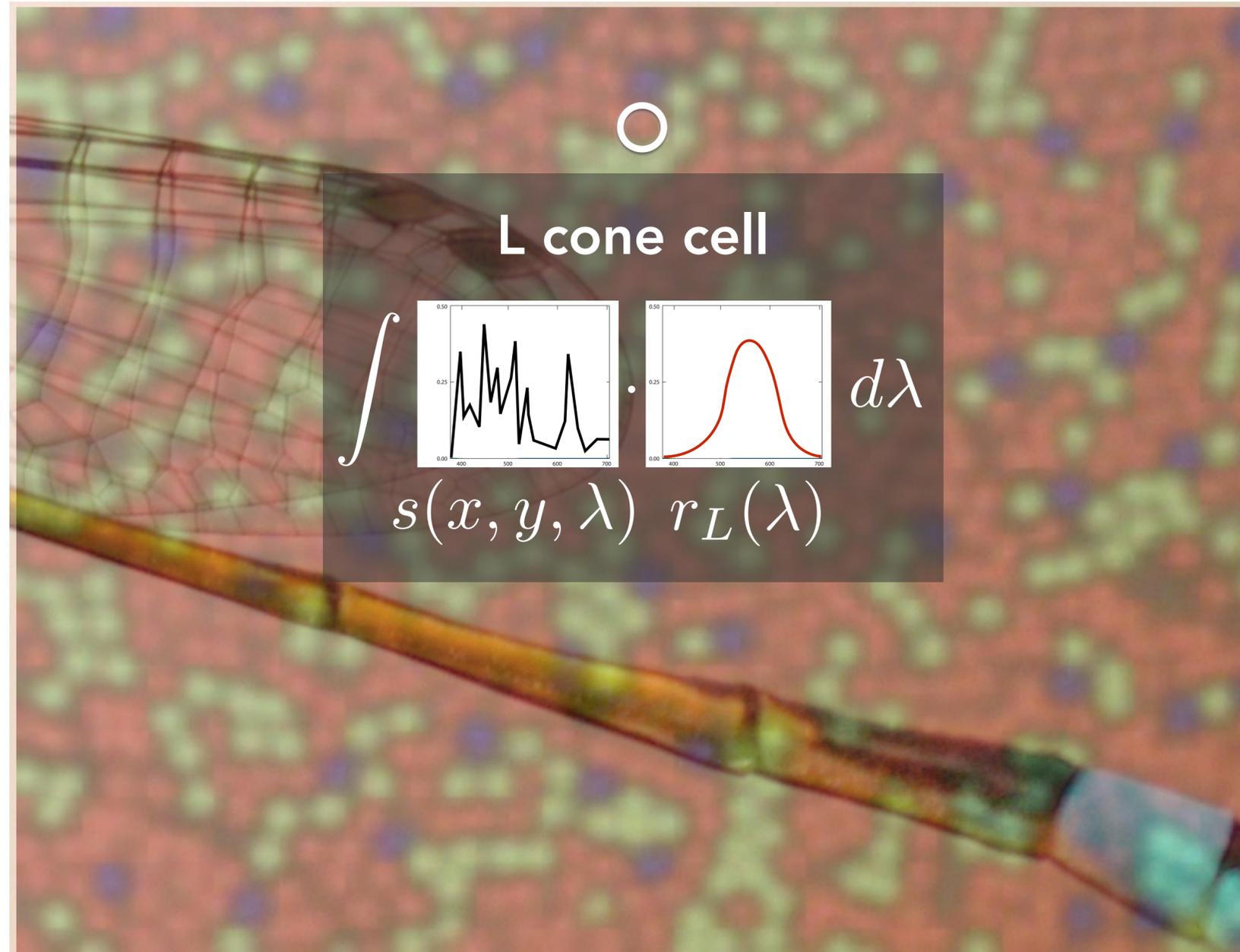
Scene projected onto retina

Example: spectral response of human cone cells

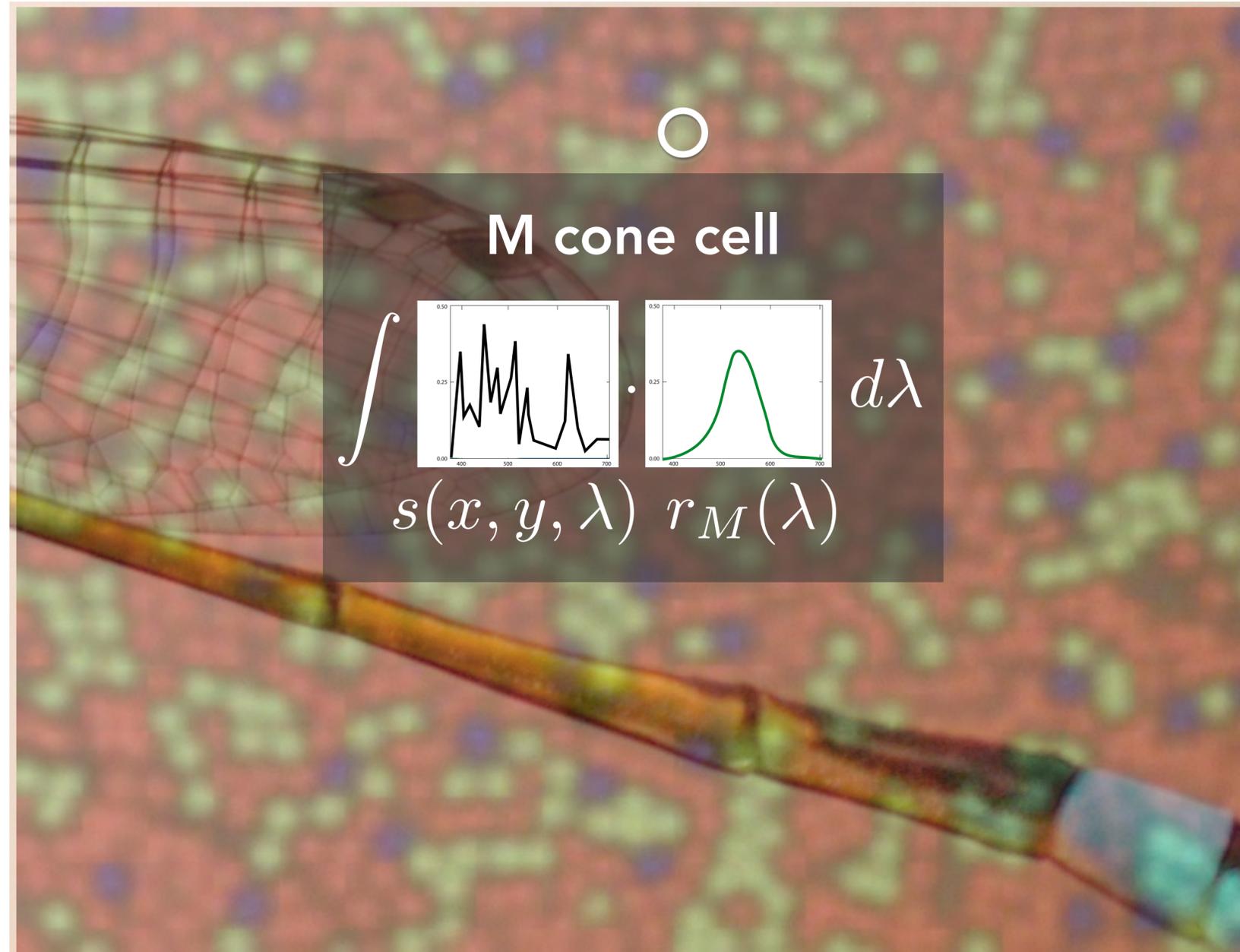


Scene projected onto retina

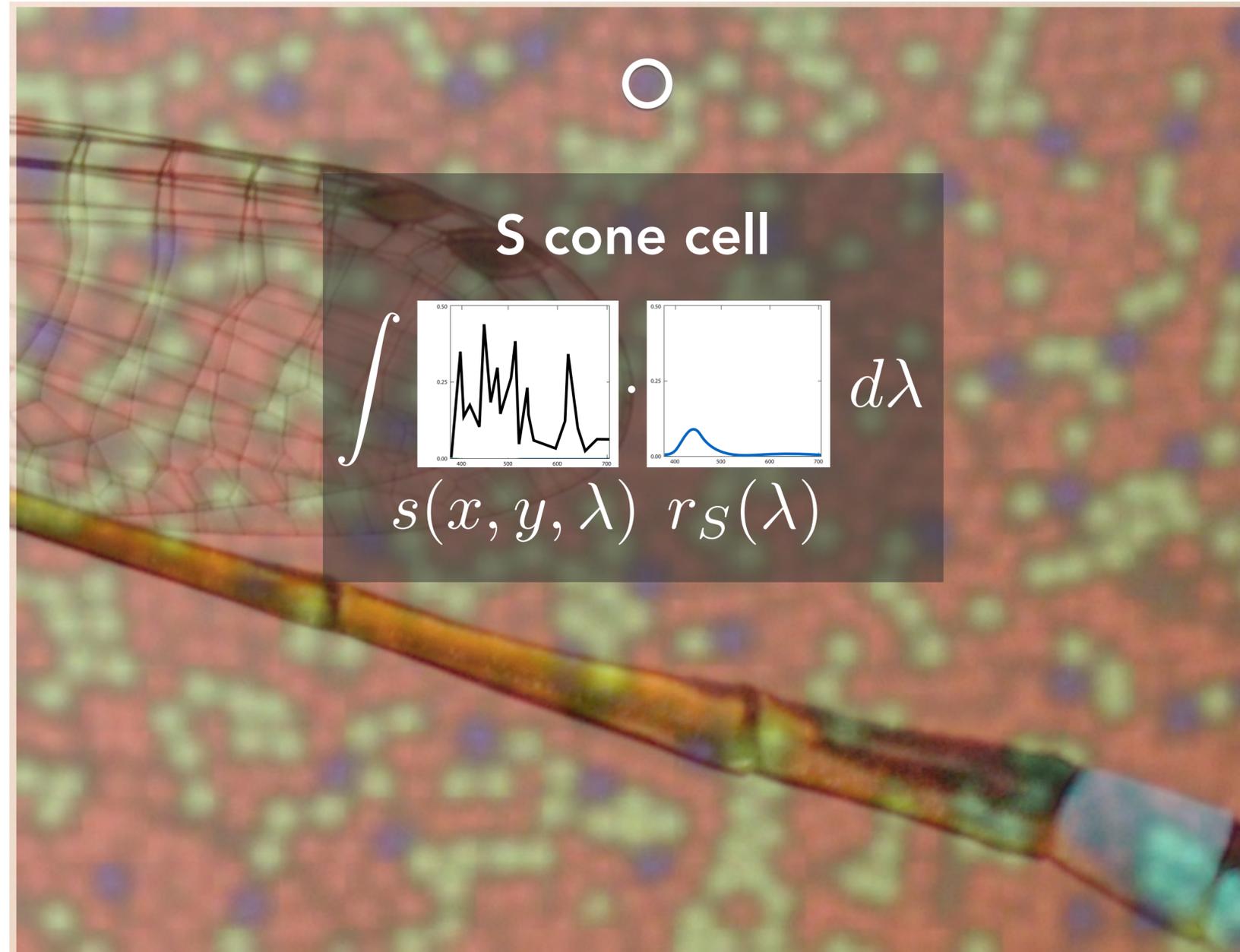
Example: spectral response of human cone cells



Example: spectral response of human cone cells



Example: spectral response of human cone cells



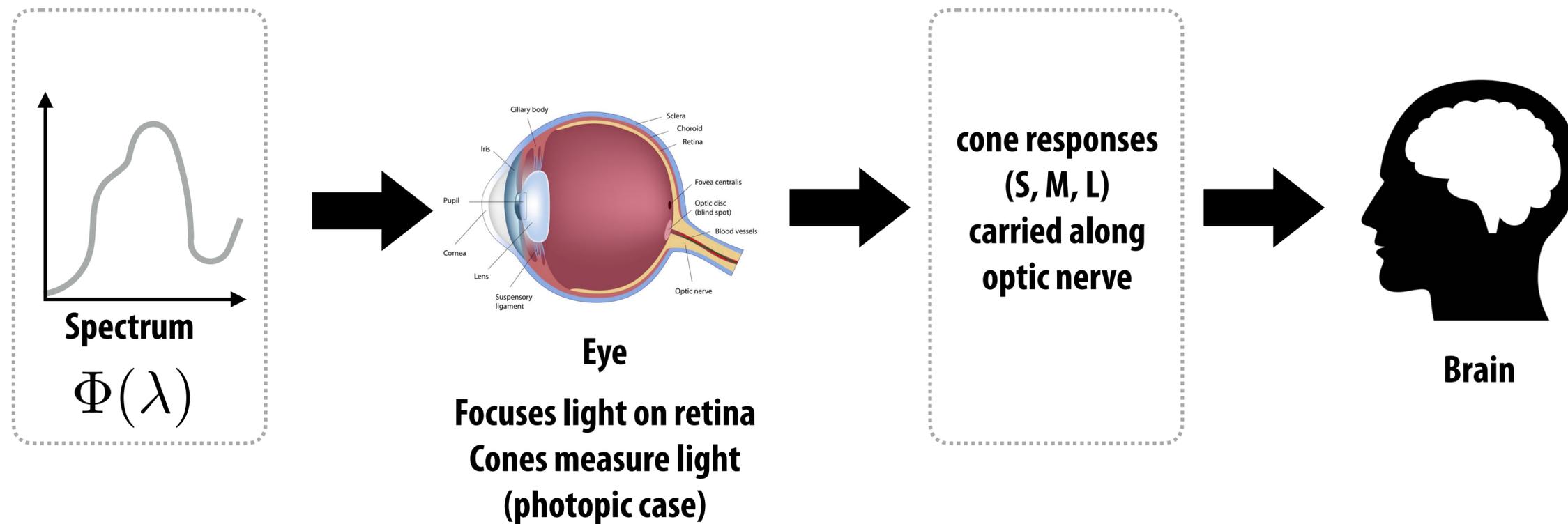
Dimensionality reduction from ∞ to 3

- **At each point on the human retina:**
 - **Spectral power distribution is a function of wavelength (∞ - dimensional signal)**
 - **3 types of cones near that position produce three scalar values (3 - dimensional signal)**

- **What about 2D images?**
 - **The dimensionality reduction described above is happening at every 2D position in our visual field**

The human visual system

- Human eye does not directly measure the spectrum of incoming light
 - a.k.a. the brain does not receive “a spectrum” from the eye
- The eye measures three response values = (S, M, L). The result of integrating the incoming spectrum against response functions of S, M, L-cones



Metamerism

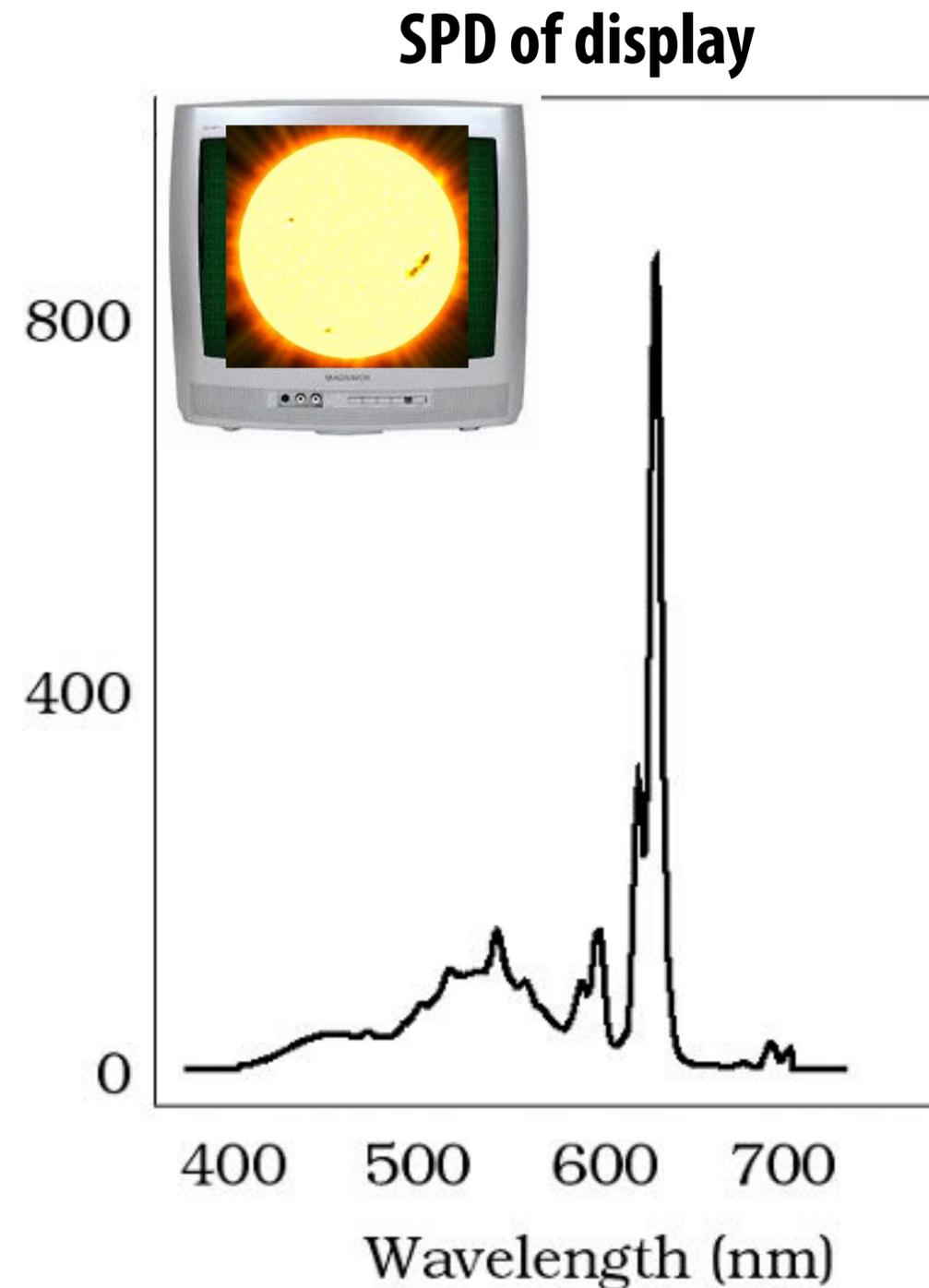
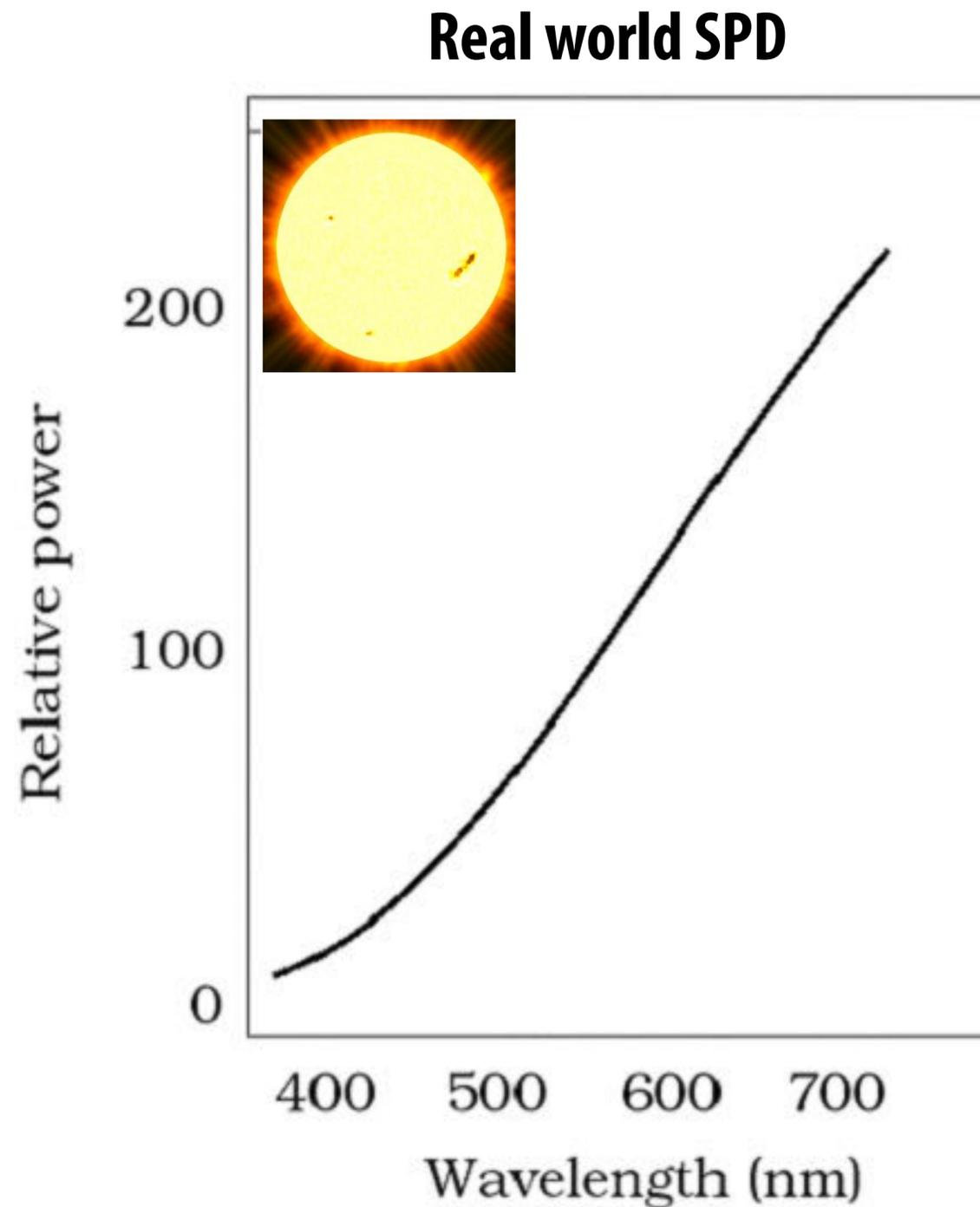
Metamers

- **Metameters are two different spectra (∞ -dim) that project to the same (S,M,L) (3-dim) response.**
 - **These will appear to have the same color to a human**

- **The existence of metamers is critical to color reproduction**
 - **It means a computer display does not have to reproduce the full spectrum of a real world scene for it to be perceived to look like the scene**
 - **It just needs to produce a metamer of the real spectrum**
 - **Example: display with pixels of only three colors can produce a metamer for the perceived color of a real-world scene**

Metamerism

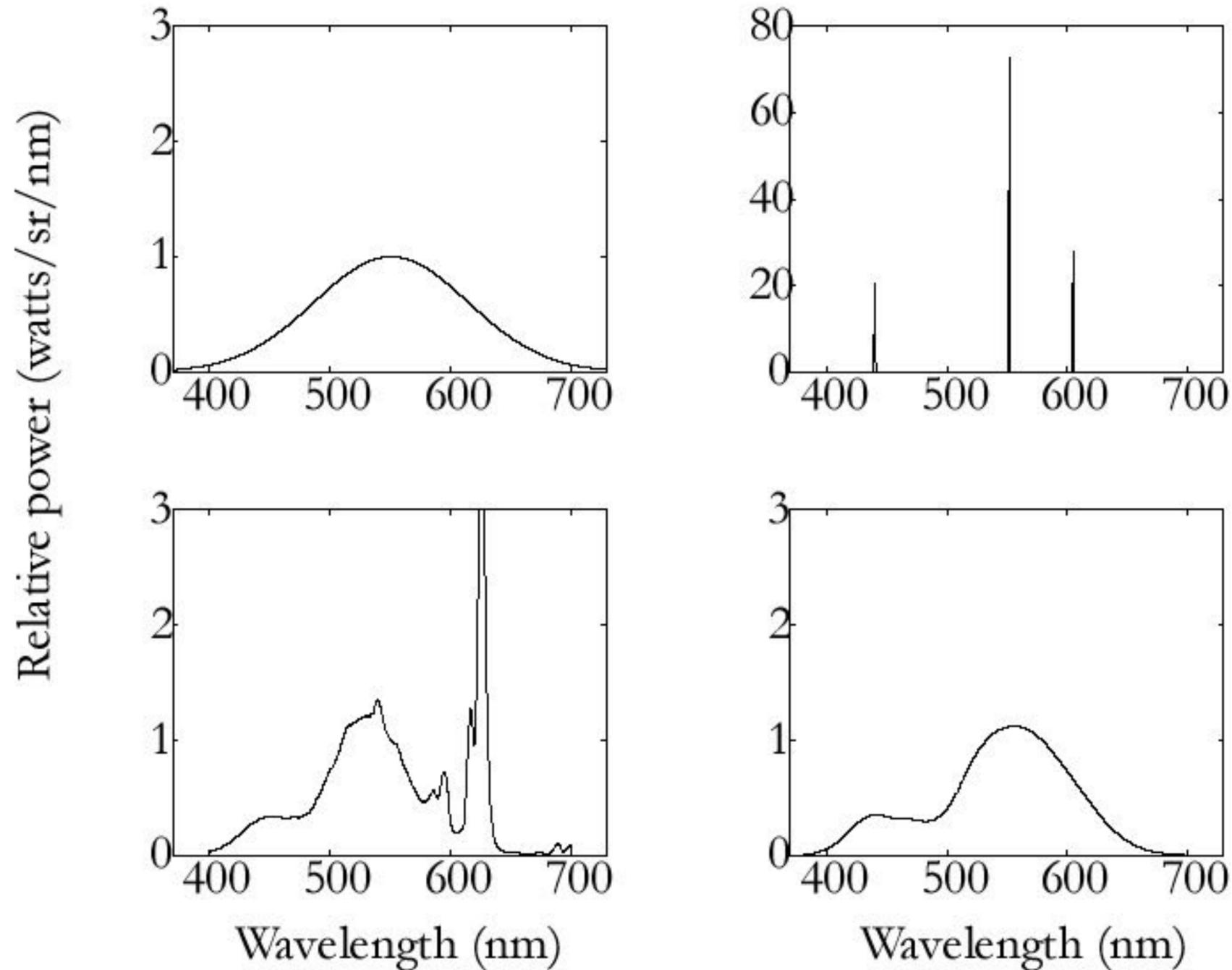
Color matching is an important illusion that is understood quantitatively



Brian Wandell

Metamerism is a big effect

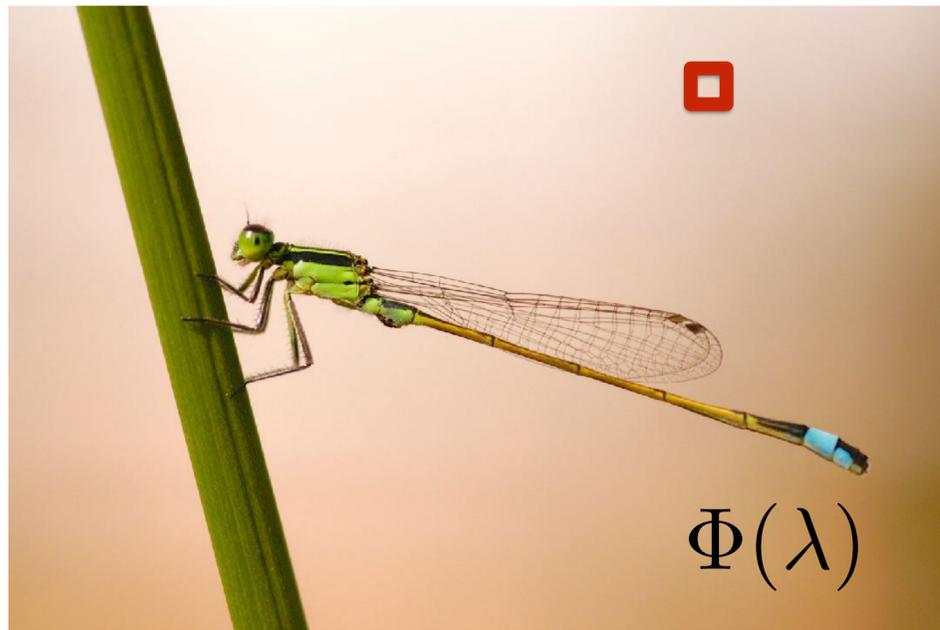
These four different spectrum are metamers (they produce the same response)



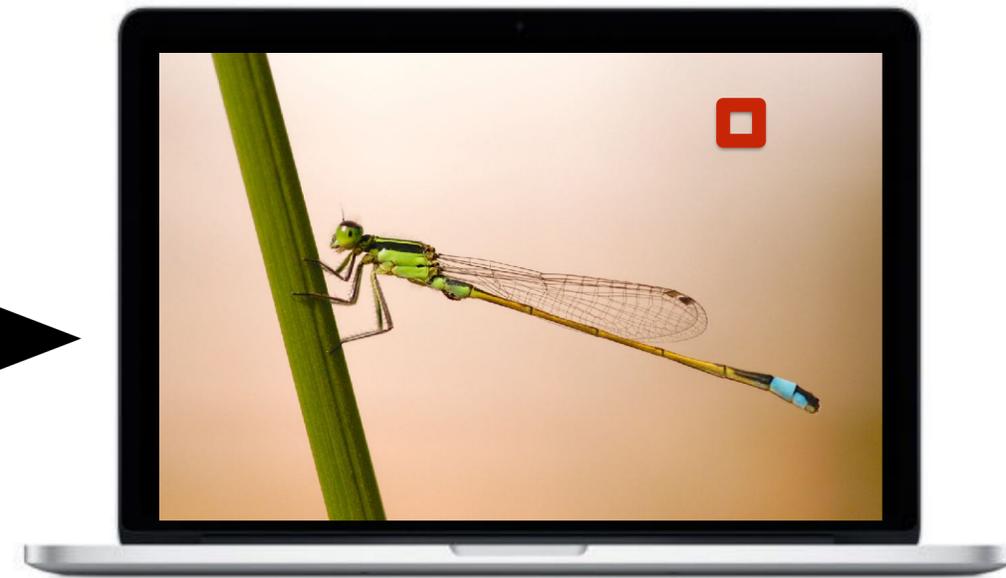
Color Reproduction

Color reproduction problem

- Goal: at each pixel, choose R , G , B values for display so that the output color matches the appearance of the target color in the real world.



Target spectrum
(what is seen in real world)



Display outputs spectrum
 $R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda)$

Additive Color

- Given a set of primary lights, each with its own spectral distribution (e.g. R,G,B display pixels):

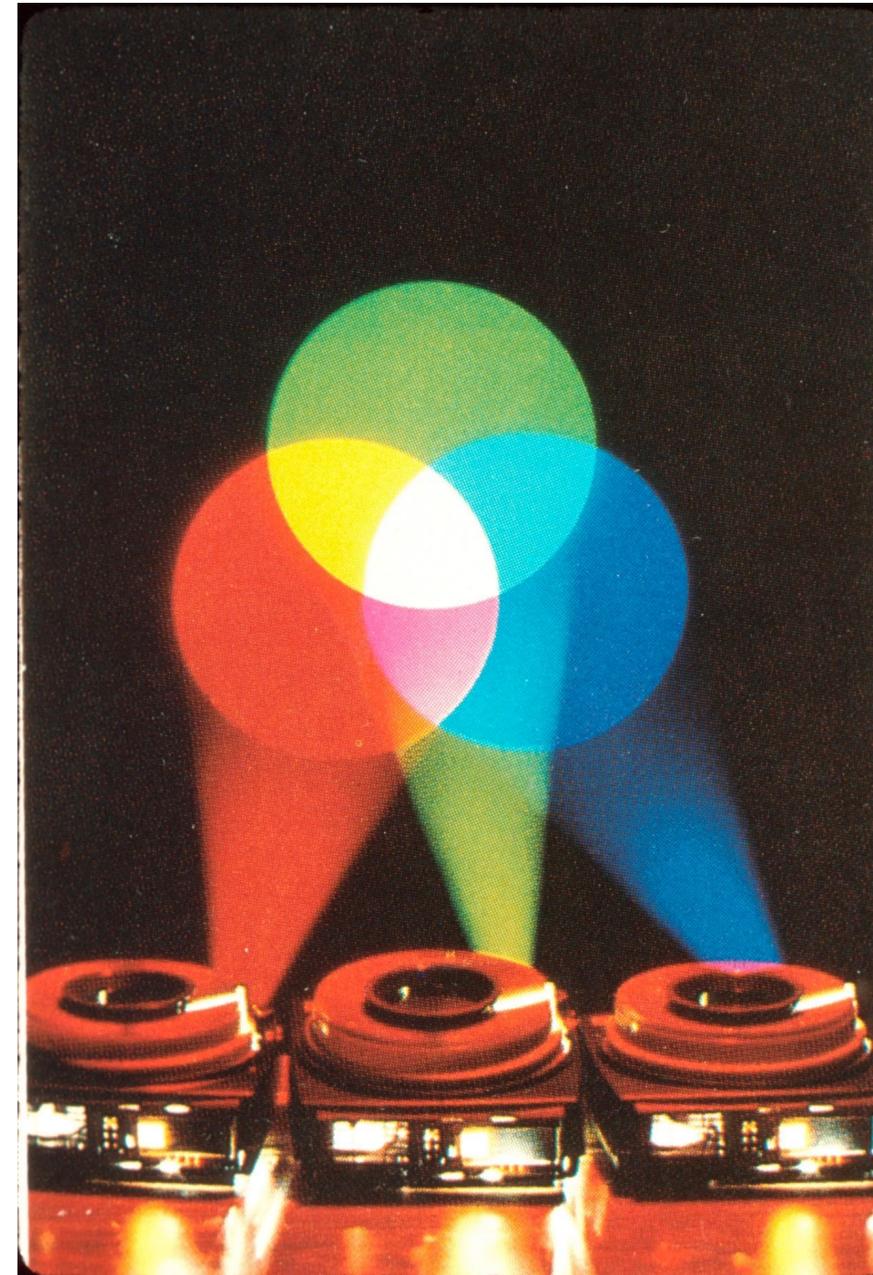
$$s_R(\lambda), s_G(\lambda), s_B(\lambda)$$

- We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:

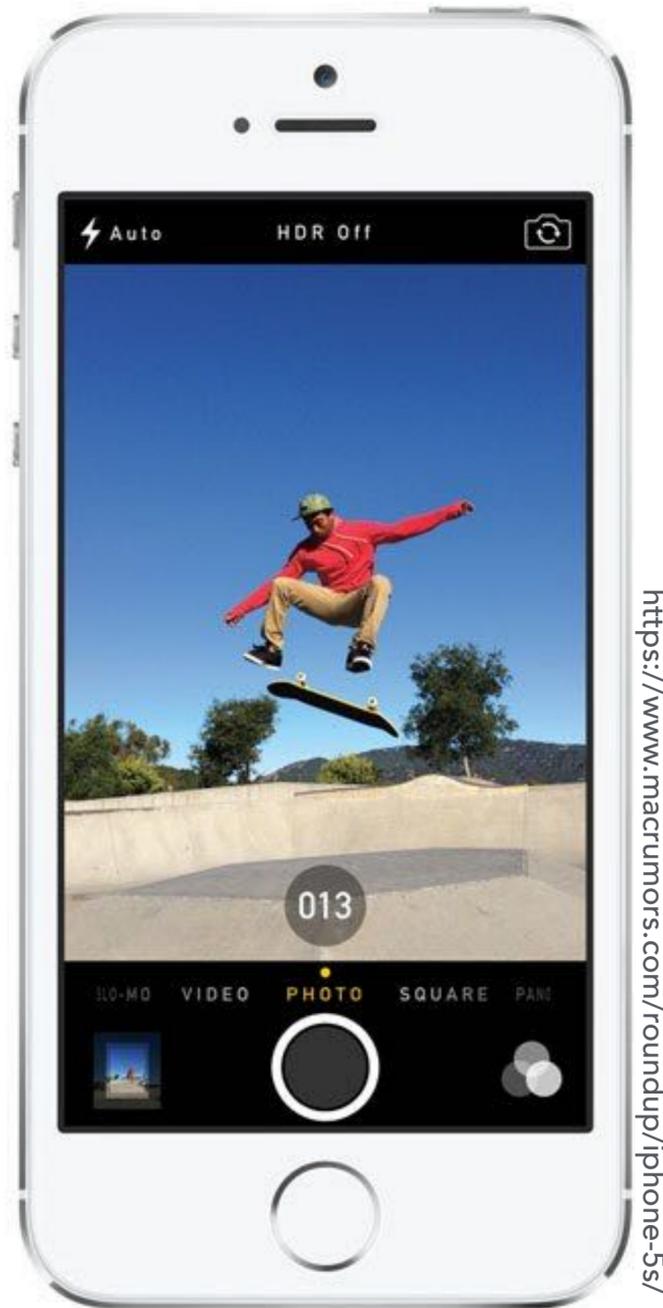
$$R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda)$$

- The color is now described by the scalar values:

$$R, G, B$$

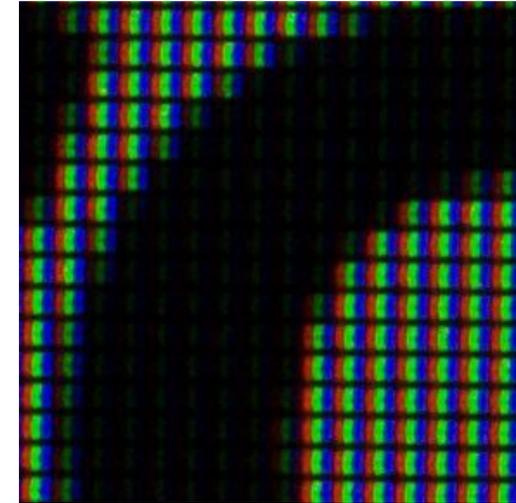
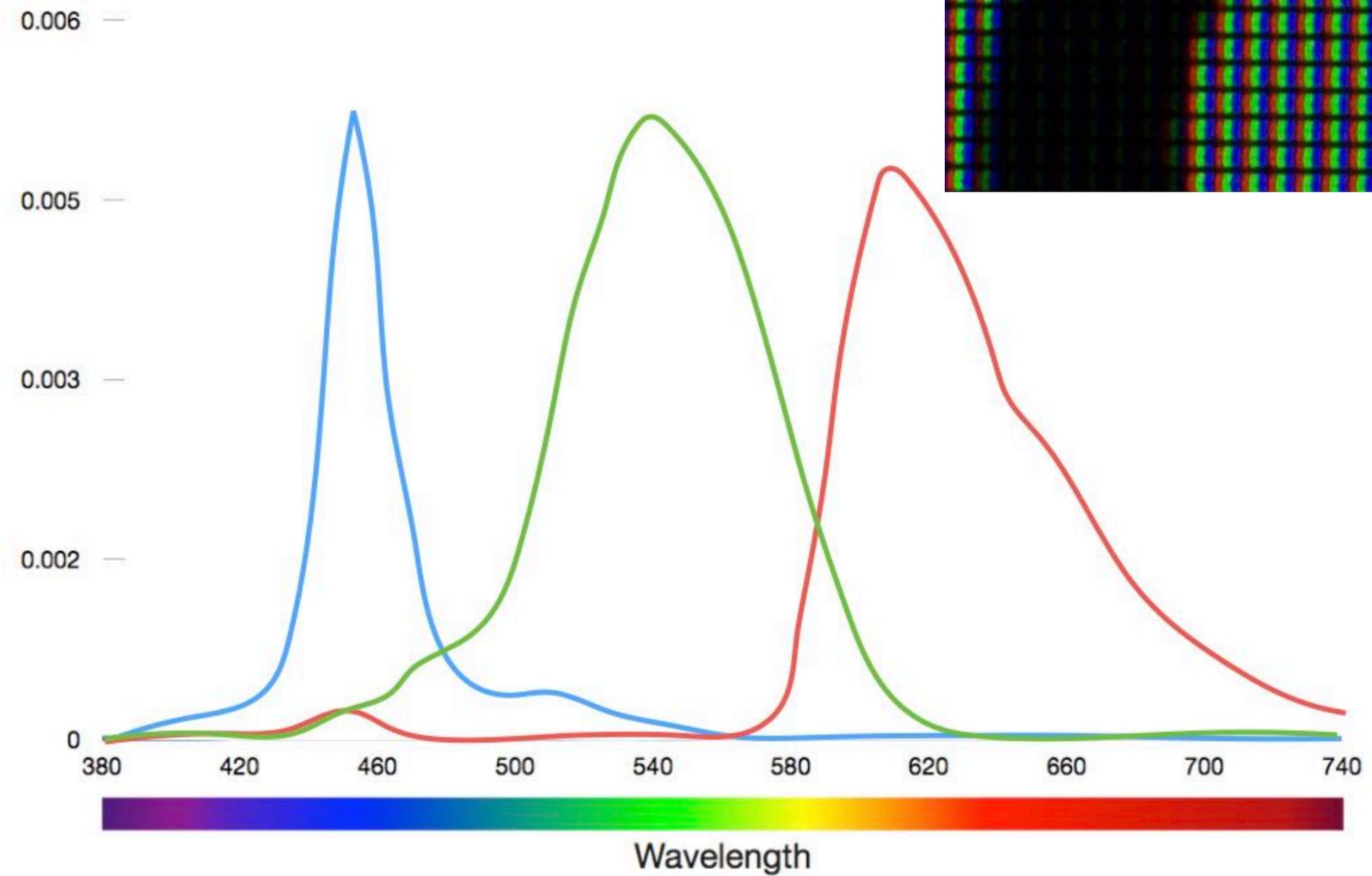


Example RGB Emission Spectra ("Color Primaries") for Phone Display



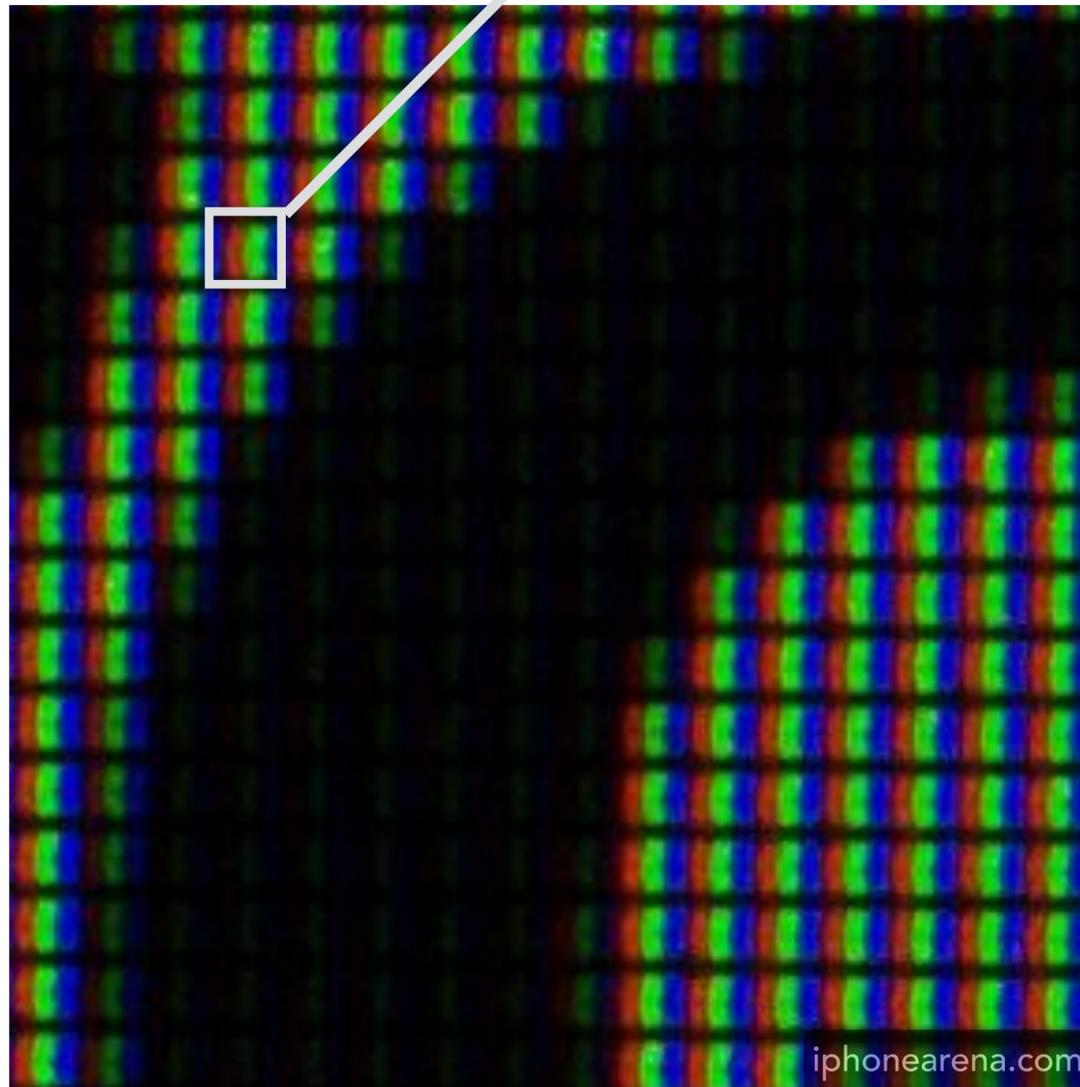
RGB pixel spectra (iPhone 5)

Credit: Yurek, <https://dot-color.com/tag/color-2/page/2/>

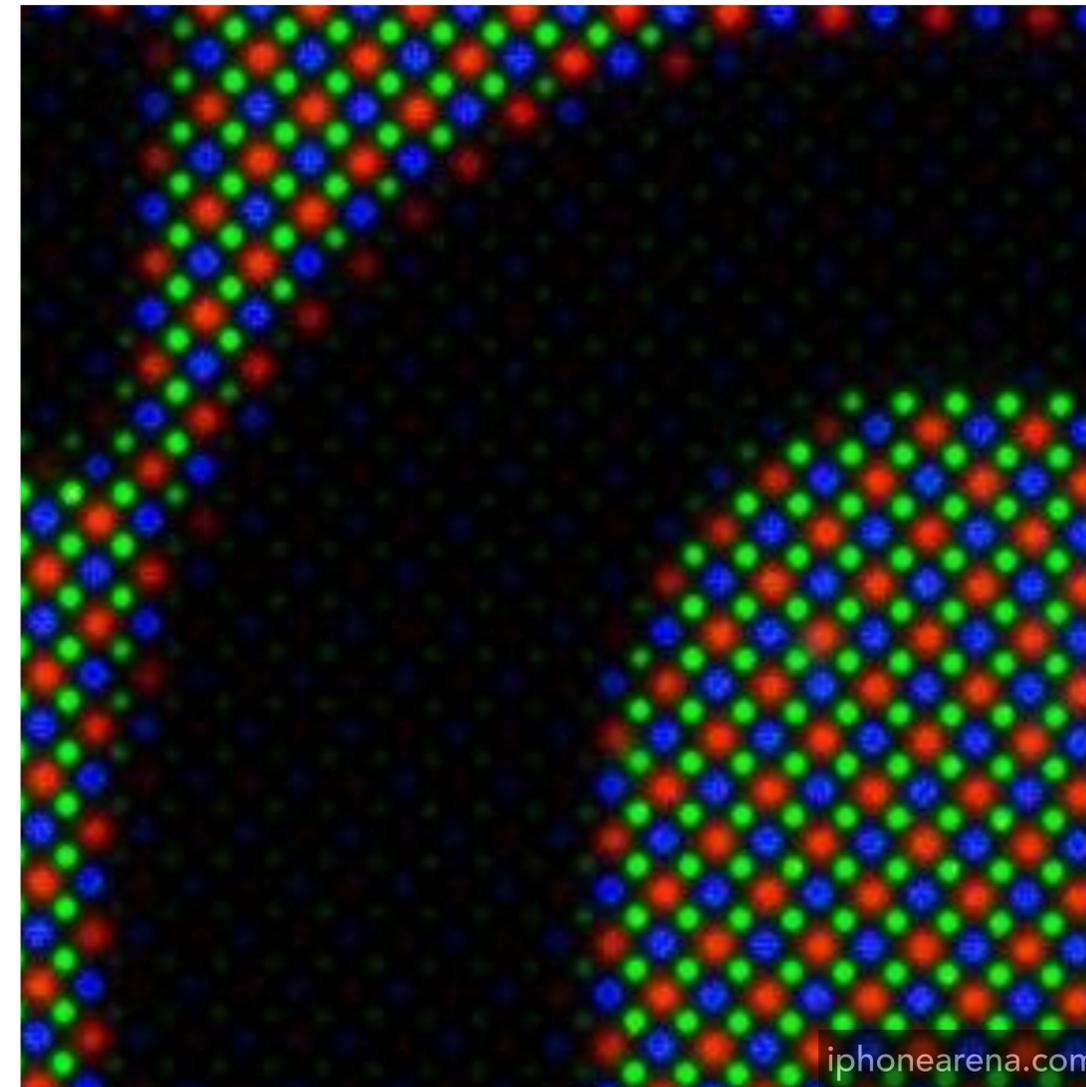


Recall: real LCD screen pixels (closeup)

One pixel



iPhone 6S



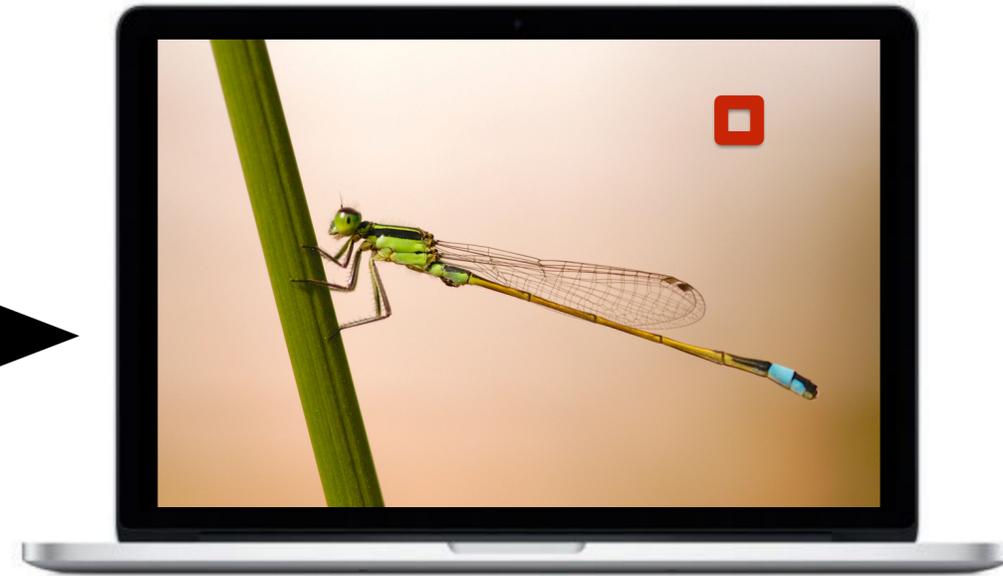
Galaxy S5

Notice R, G, B sub-pixel geometry.
Effectively three lights at each (x,y) location.

Color Reproduction Problem



Target real spectrum $s(\lambda)$



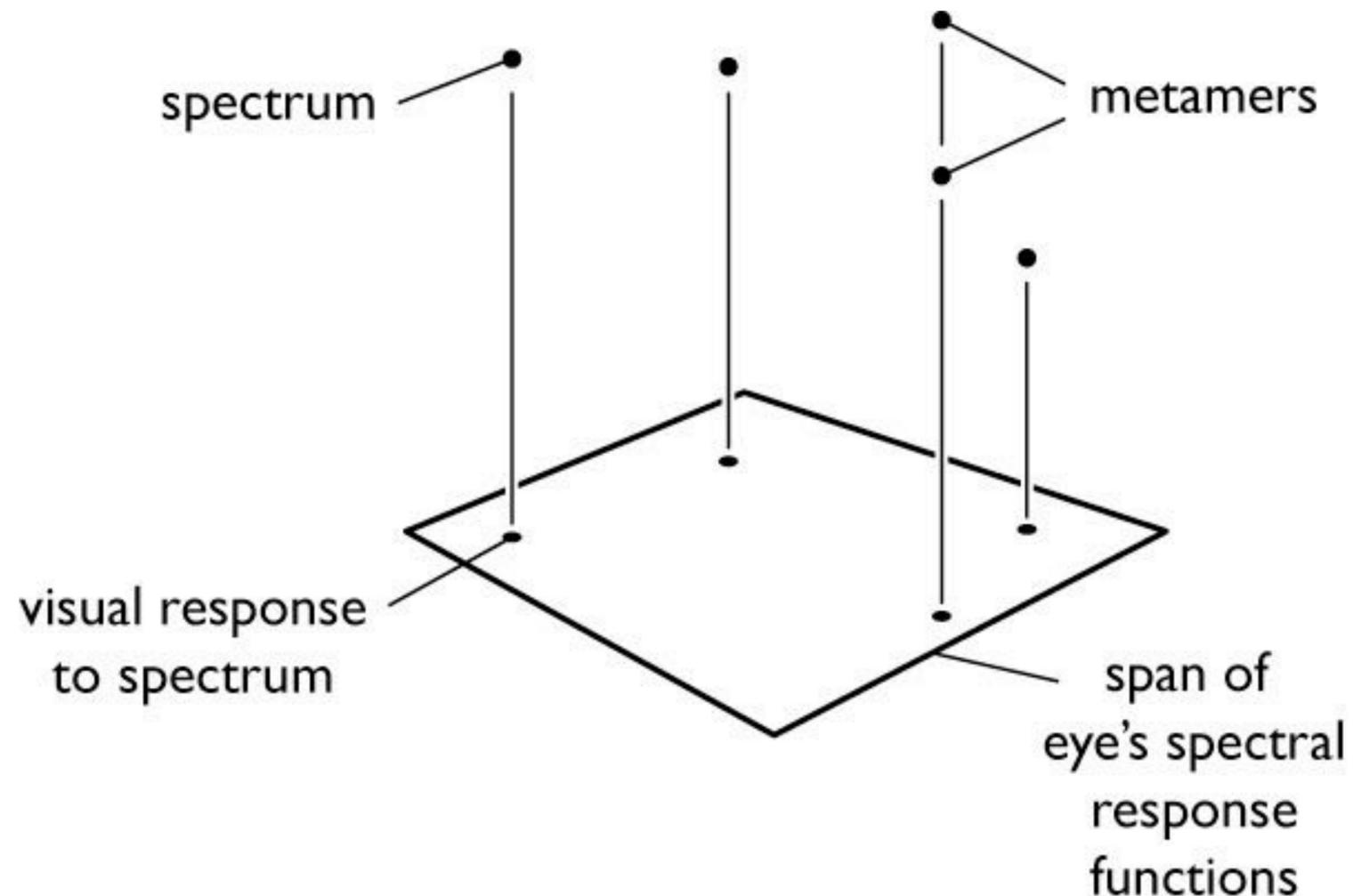
Display outputs spectrum

$$R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda)$$

- **Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.**

Pseudo-Geometric Interpretation

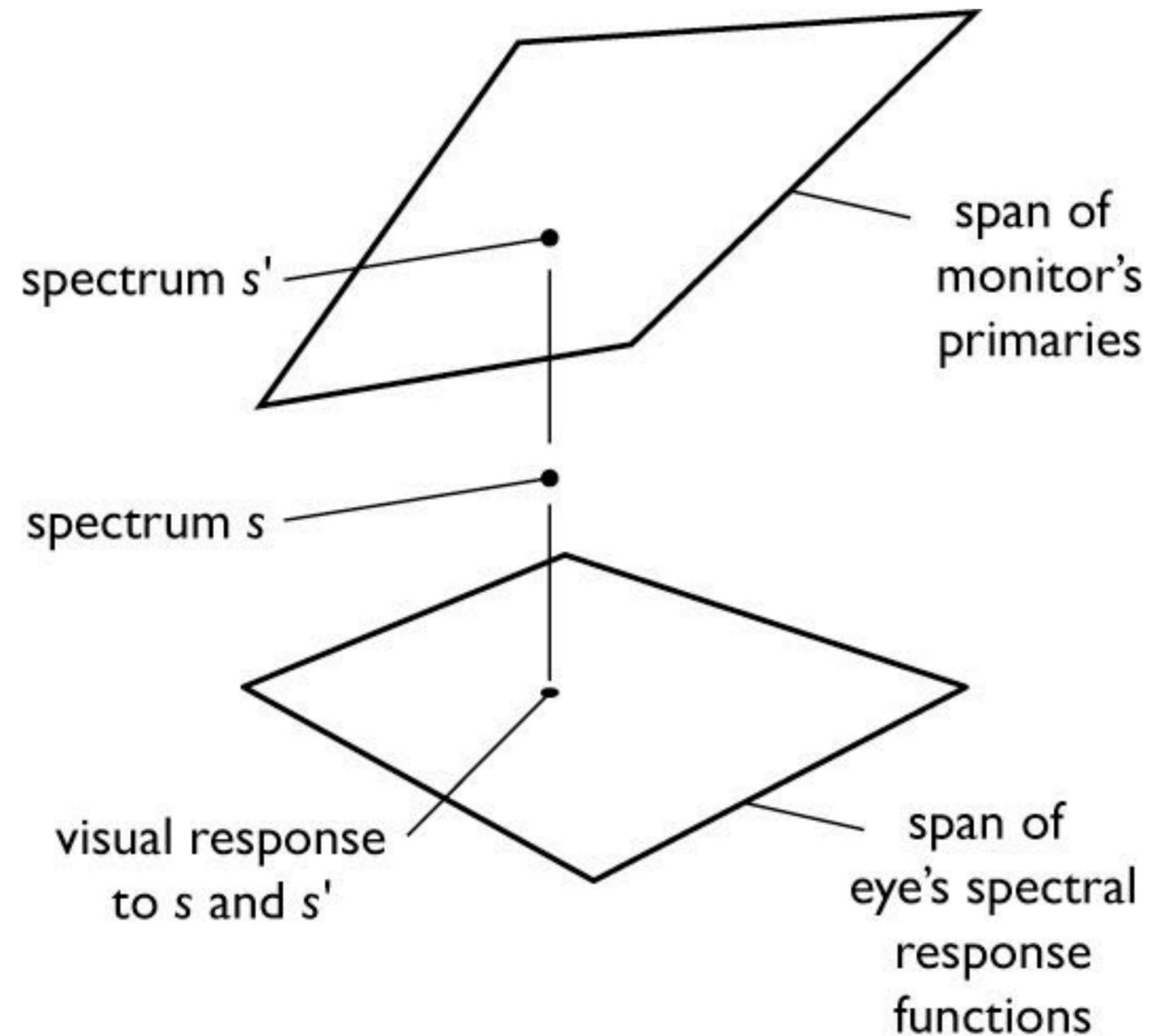
- We are projecting a high dimensional vector (wavelength spectrum function) onto a low-dimensional subspace (SML visual response)
 - Differences that are perpendicular to the basis vectors of the low-dimensional space are not detectable



Slide credit: Steve Marschner

Pseudo-Geometric Interpretation of Color Reproduction

- The display can only produce a low-dimensional subspace of all possible spectra (linear combinations of display primaries)
- In color reproduction, for a given spectrum s (high dimensional), we want to choose a spectrum s' in the display's low-dimensional subspace, such that s' and s project to the same response in the low-dimensional subspace of the eye's SML response



Slide credit: Steve Marschner

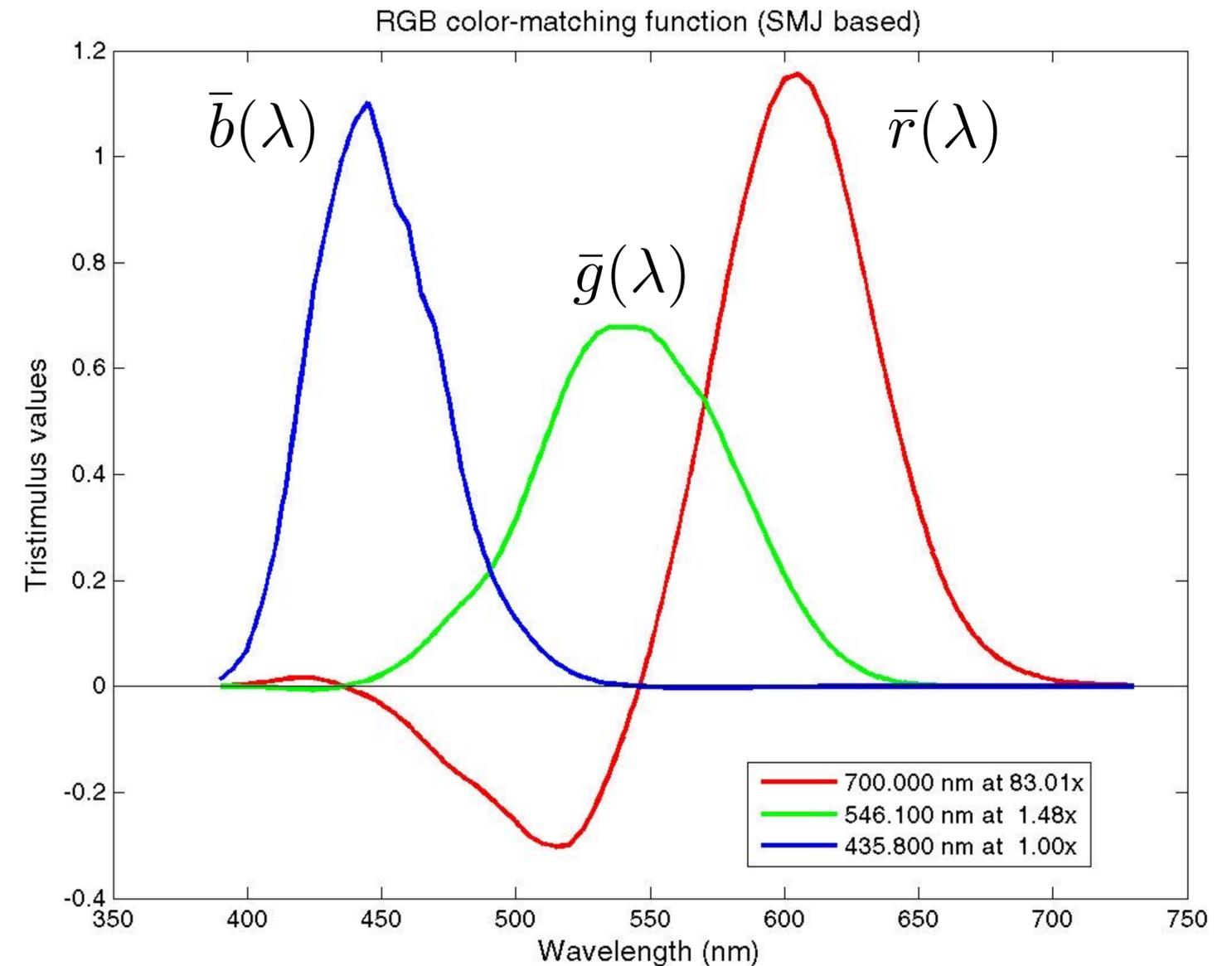
Color reproduction with matching functions

For any spectrum s , the perceived color is matched by the following formulas for scaling the CIE RGB primaries

$$R_{\text{CIE RGB}} = \int_{\lambda} s(\lambda) \bar{r}(\lambda) d\lambda$$

$$G_{\text{CIE RGB}} = \int_{\lambda} s(\lambda) \bar{g}(\lambda) d\lambda$$

$$B_{\text{CIE RGB}} = \int_{\lambda} s(\lambda) \bar{b}(\lambda) d\lambda$$



**Careful: these graphs are color matching curves:
they are not response curves or primary spectra!**

Color reproduction with matching functions

For any spectrum s , the perceived color is matched by the following formulas for scaling the CIE RGB primaries

Written as vector dot products:

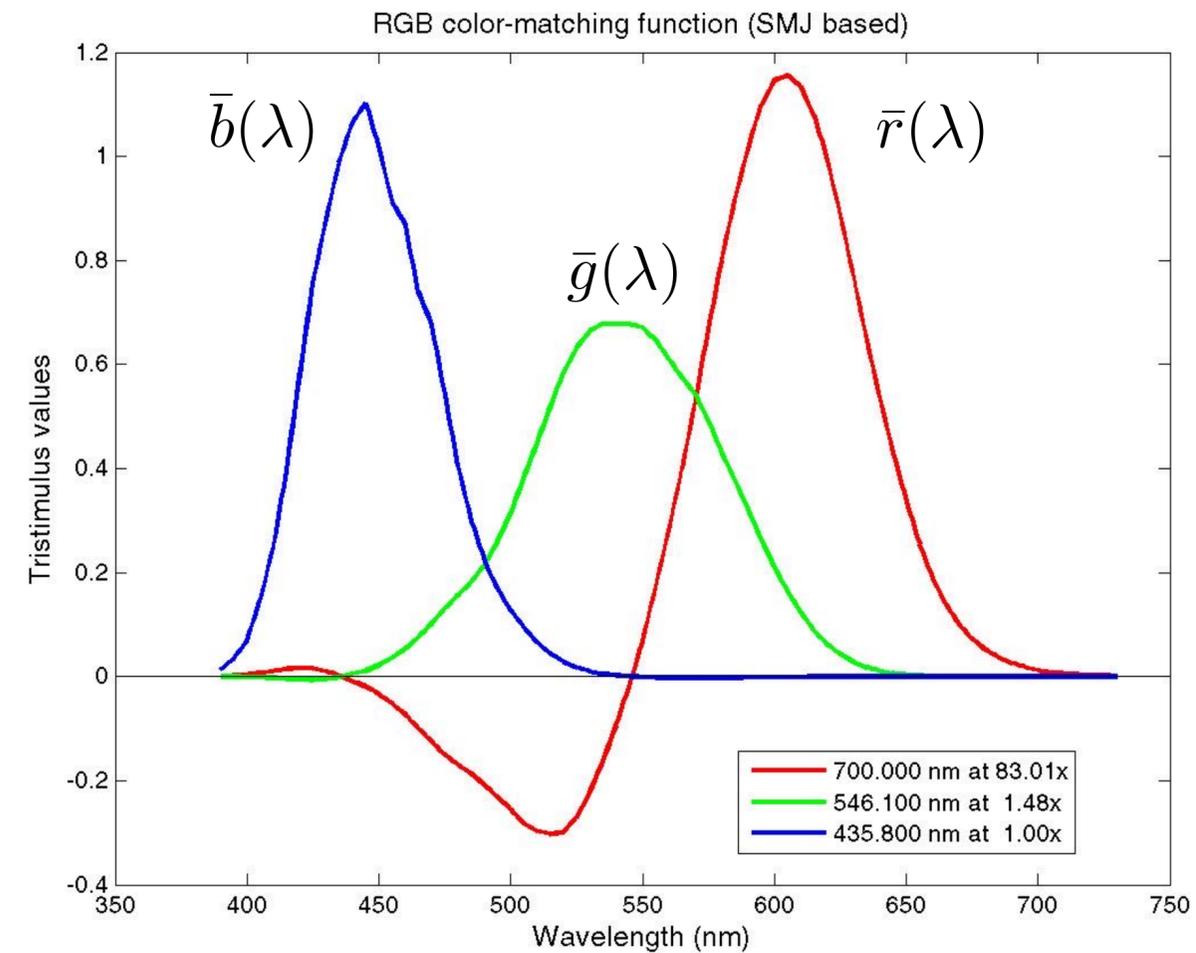
$$R_{\text{CIE RGB}} = s \cdot \bar{r}$$

$$G_{\text{CIE RGB}} = s \cdot \bar{g}$$

$$B_{\text{CIE RGB}} = s \cdot \bar{b}$$

Matrix formulation:

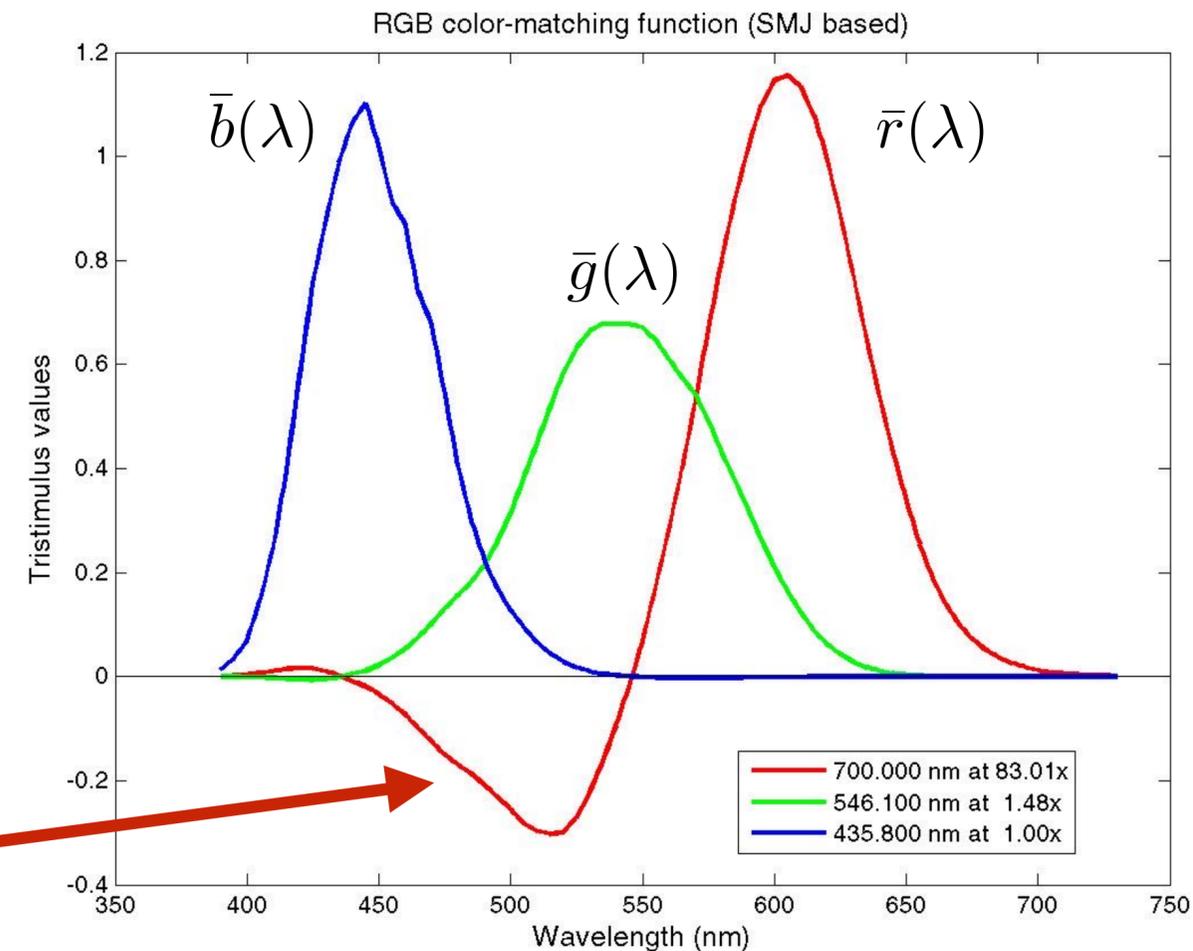
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{\text{CIE RGB}} = \begin{bmatrix} \text{---} & \bar{r} & \text{---} \\ \text{---} & \bar{g} & \text{---} \\ \text{---} & \bar{b} & \text{---} \end{bmatrix} \begin{bmatrix} | \\ | \\ | \end{bmatrix} s$$



Careful: these graphs are color matching curves:
they are not response curves or primary spectra!

Negative red primary?

- There is no positive combination of red, blue, green lasers that yields color that appears the same to a human as monochromatic light of 500 nm (“blue-greenish” light)
- But adding red primary to 500 nm target light yields light whose color can be matched by a combination of blue and green primaries.



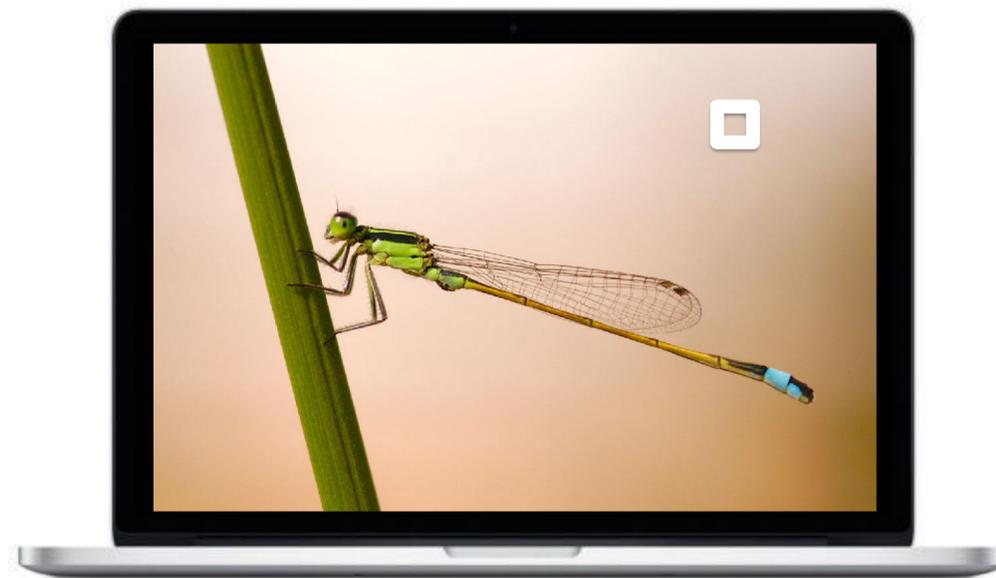
Wait a minute:
negative red?

Color reproduction as linear algebra

Spectrum produced by display given values R,G,B:

$$s_{\text{disp}}(\lambda) = R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda)$$

$$\Rightarrow \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Color reproduction as linear algebra

- What color do we perceive when we look at the display?

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix}$$
$$= \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

We want this displayed spectrum to be a metamer for the real-world target spectrum.

Color reproduction as linear algebra

Color perceived for display spectra with values R,G,B

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color perceived for real scene spectra, s

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

How do we reproduce the color of s ?

Set these lines equal and solve for R,G,B as a function of s !

Color reproduction as linear algebra

Solution:

$$\begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color reproduction as linear algebra

Solution (form #1):

$$\begin{array}{c}
 \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{array}{c} \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \end{array} \right)^{-1} \begin{array}{c} \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix} \end{array} \\
 \begin{array}{c} 1 \times 3 \\ \underbrace{\begin{array}{c} N \times 3 \quad 3 \times N \\ \hline 3 \times 3 \end{array}} \\ \end{array}
 \end{array} \quad \begin{array}{c} \underbrace{\begin{array}{c} N \times 3 \quad 1 \times N \\ \hline 1 \times 3 \end{array}}
 \end{array}$$

Solution (form #2):

$$\begin{array}{c}
 RGB = (\mathbf{M}_{SML} \mathbf{M}_{RGB})^{-1} \mathbf{M}_{SML} s \\
 \begin{array}{c} 1 \times 3 \\ \underbrace{\begin{array}{c} N \times 3 \quad 3 \times N \quad N \times 3 \\ \hline N \times 3 \end{array}} \quad 1 \times N \end{array}
 \end{array}$$

Color reproduction as linear algebra

Solution (form #3):

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \underbrace{\begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1}}_{\text{Nx3}} \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

**This Nx3 matrix contains, as row vectors,
“color matching functions”
associated with the primary lights s_R, s_G, s_B .**

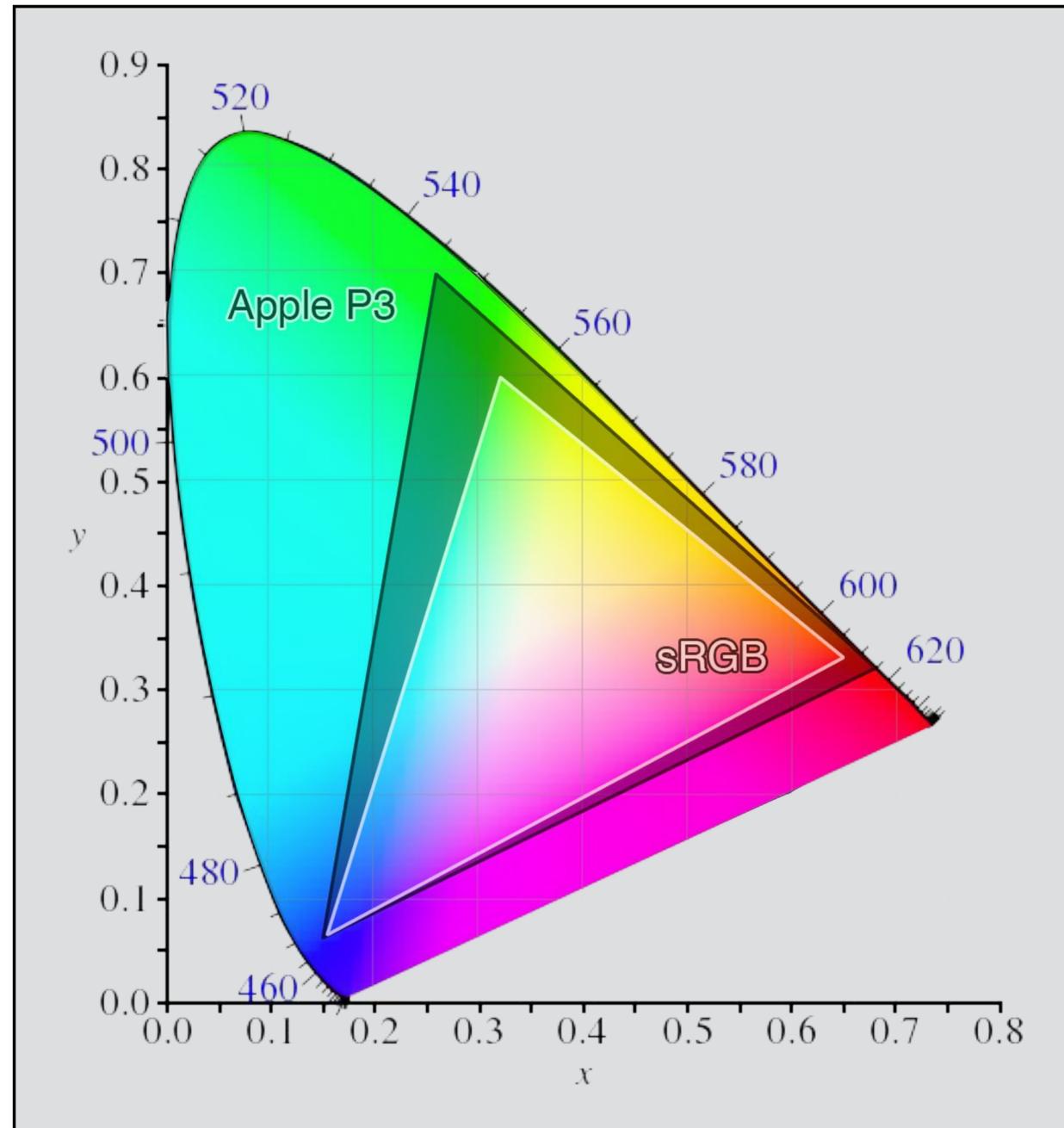
Color reproduction issue: no negative light

- **R,G,B values must be positive**
 - **Display primaries can't emit negative light**
 - **But solution formulas can certainly produce negative R,G,B values**

- **What do negative R,G,B values mean?**
 - **Display can't physically reproduce the desired color**
 - **Desired color is outside the display's "color gamut" (more on this later)**

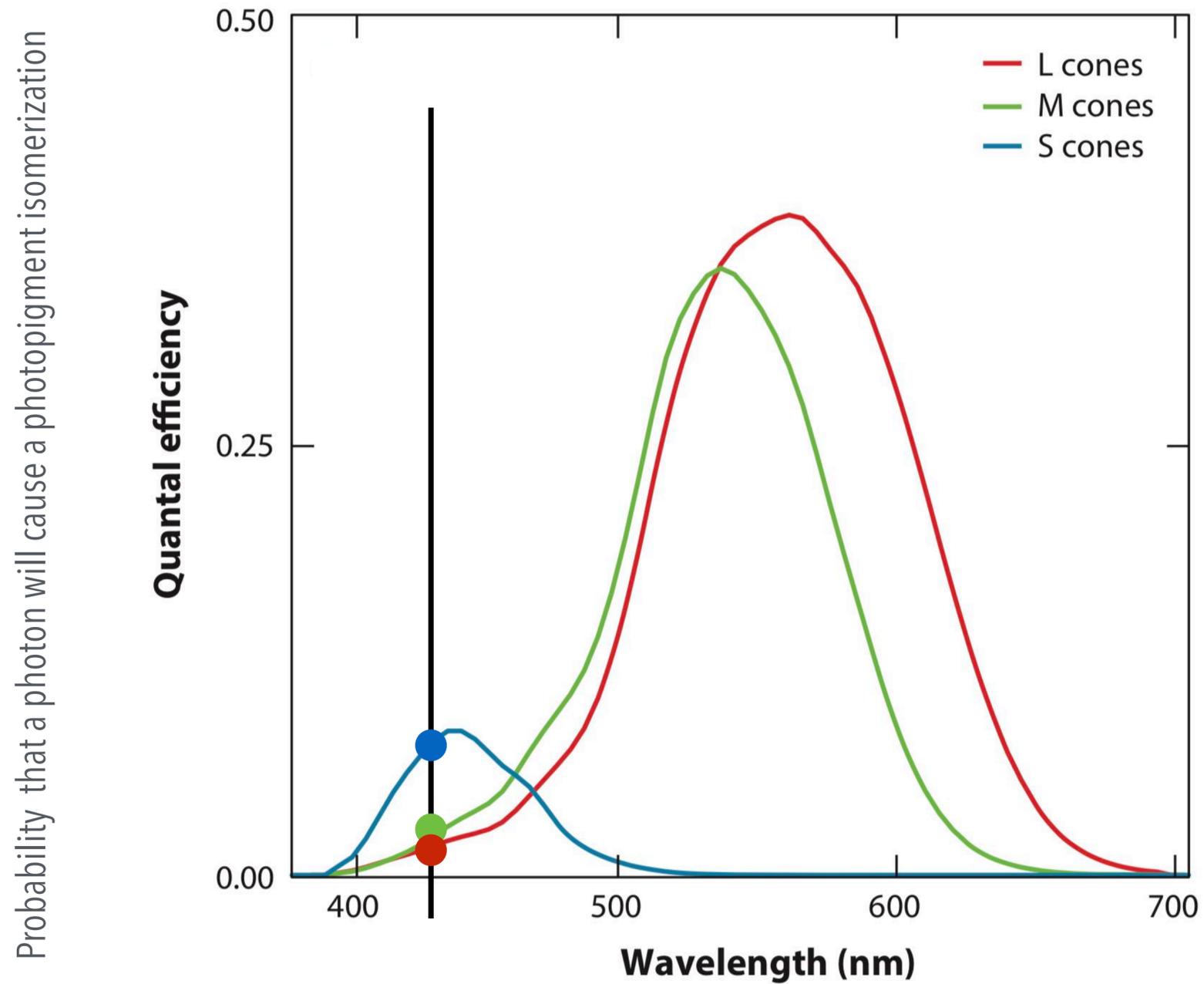
Color Representation / Color Gamut

Example: Color Gamut for sRGB and Apple P3



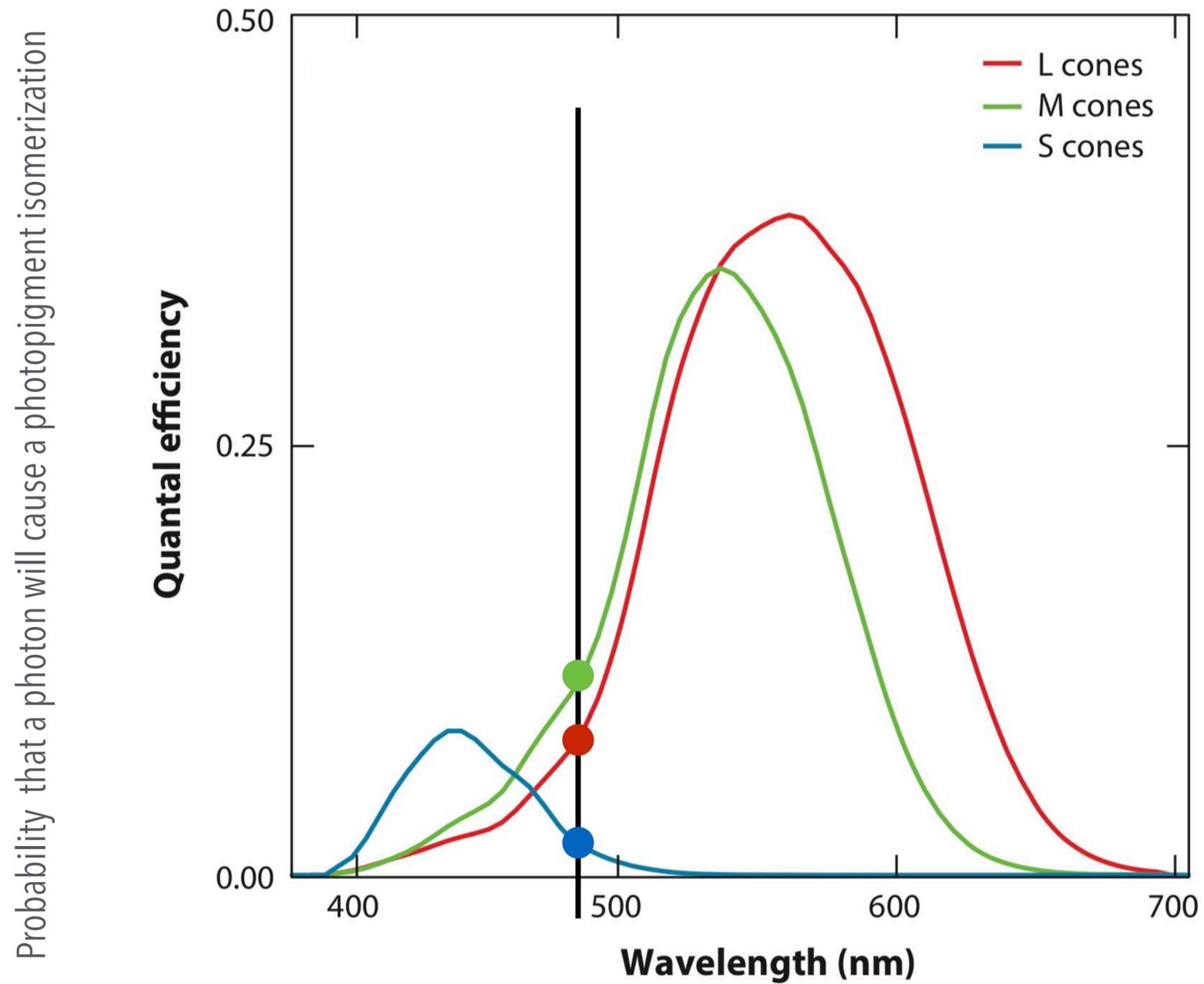
Wikipedia

LMS Response Values for Each Wavelength



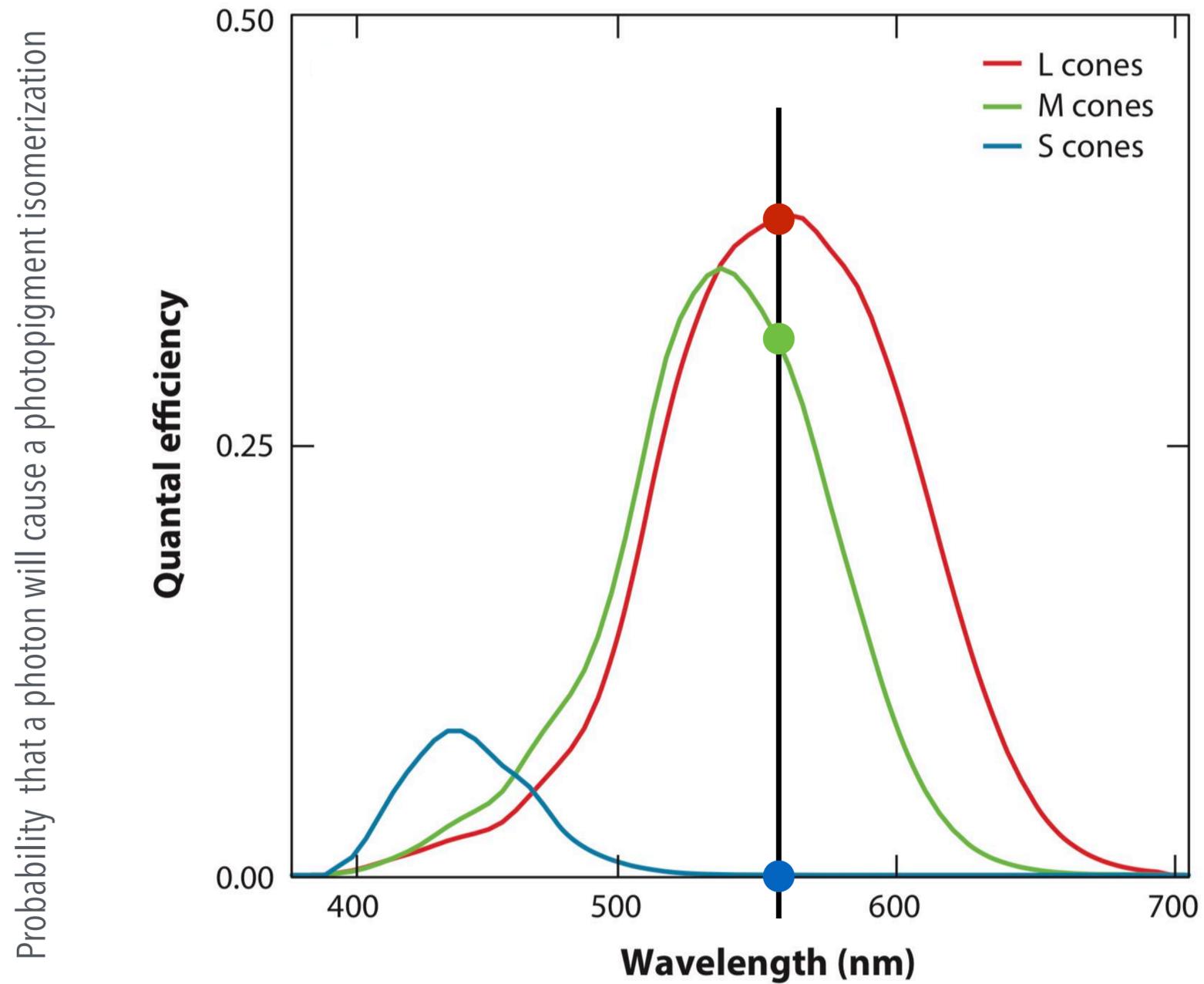
Brainard, Color and the Cone Mosaic, 2015.

LMS Response Values for Each Wavelength



Brainard, Color and the Cone Mosaic, 2015.

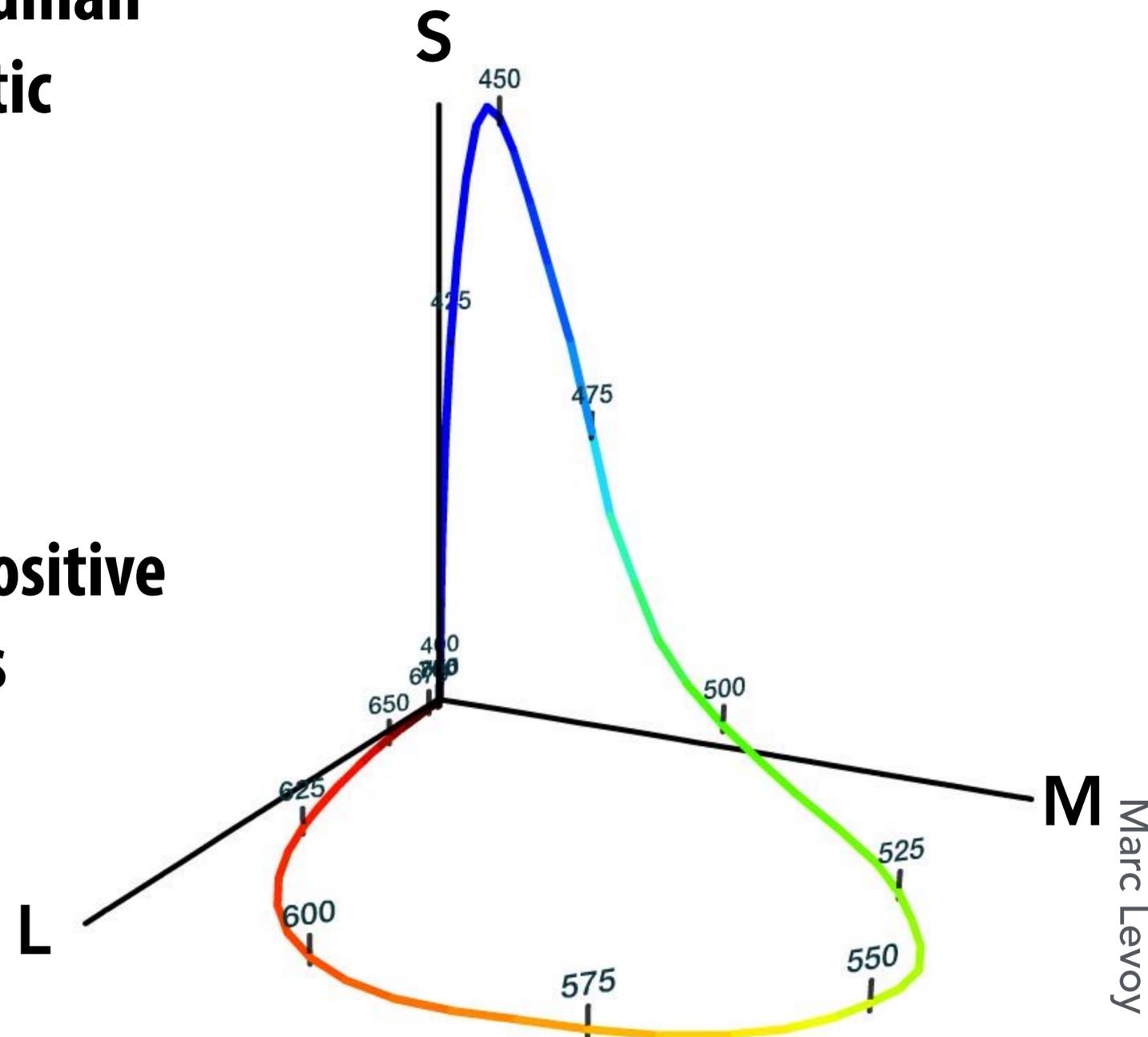
LMS Response Values for Each Wavelength



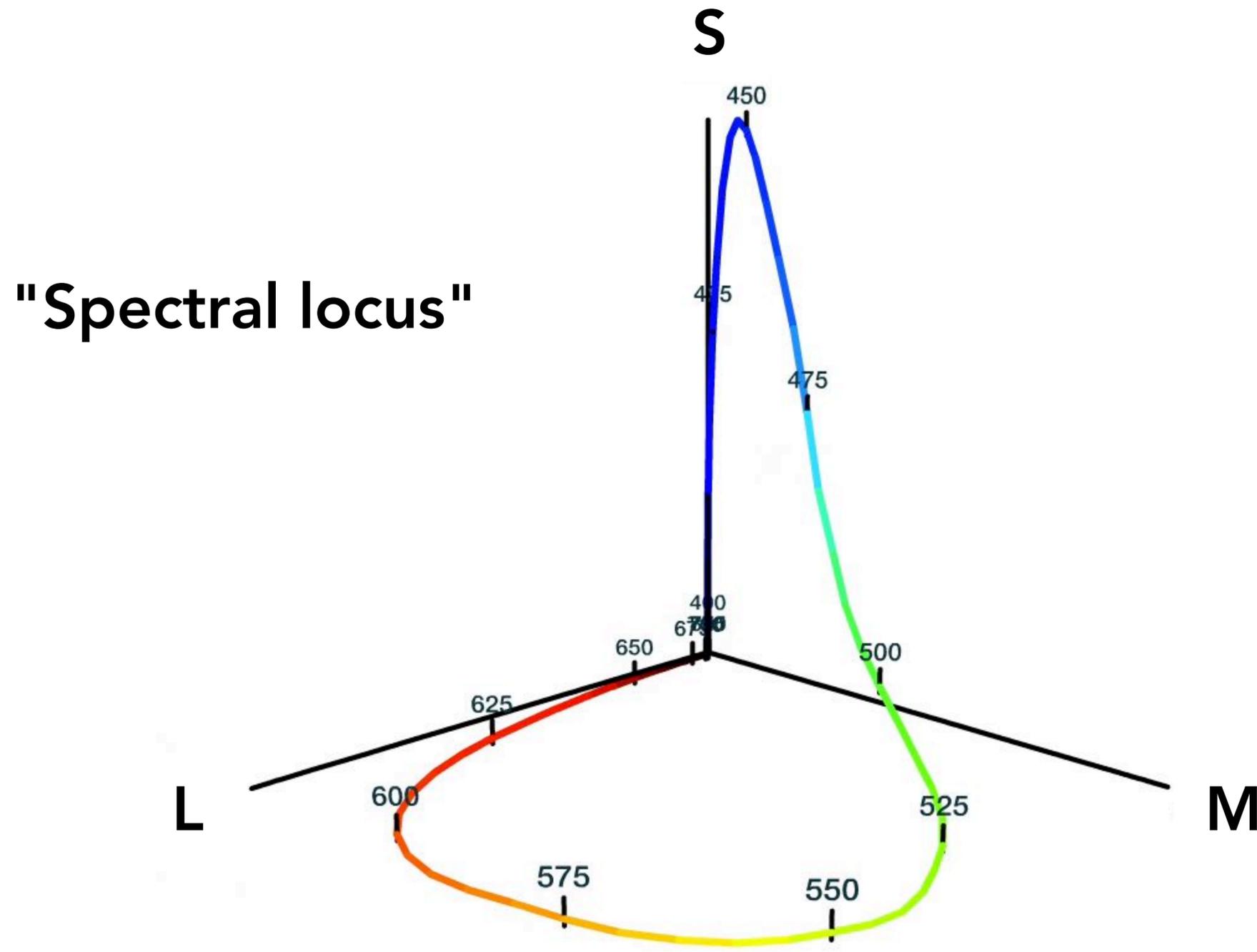
Brainard, Color and the Cone Mosaic, 2015.

LMS Responses Plotted as 3D Color Space

- Visualization of "spectral locus" of human cone cells' response to monochromatic light (light with energy in a single wavelength) as points in 3D space.
- This is a plot of the S, M, L response functions as a point in 3D space.
- Space of all possible responses are positive linear combinations of points on this curve.



LMS Responses Plotted as 3D Color Space

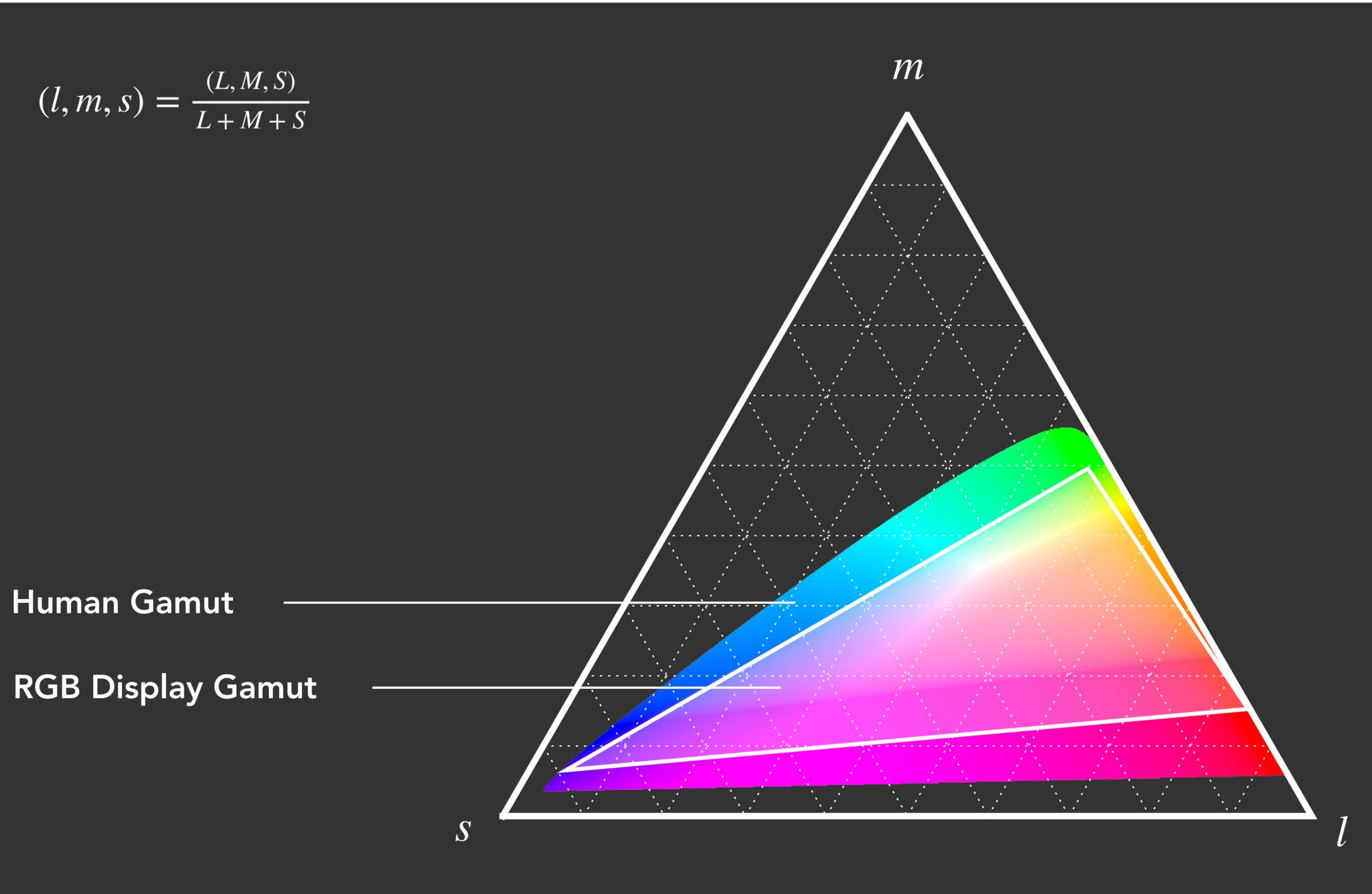


<https://graphics.stanford.edu/courses/cs178-10/applets/locus.html>

Dektar, Adams, Levoy

Color Gamut on Chromaticity Diagram (Maxwellian)

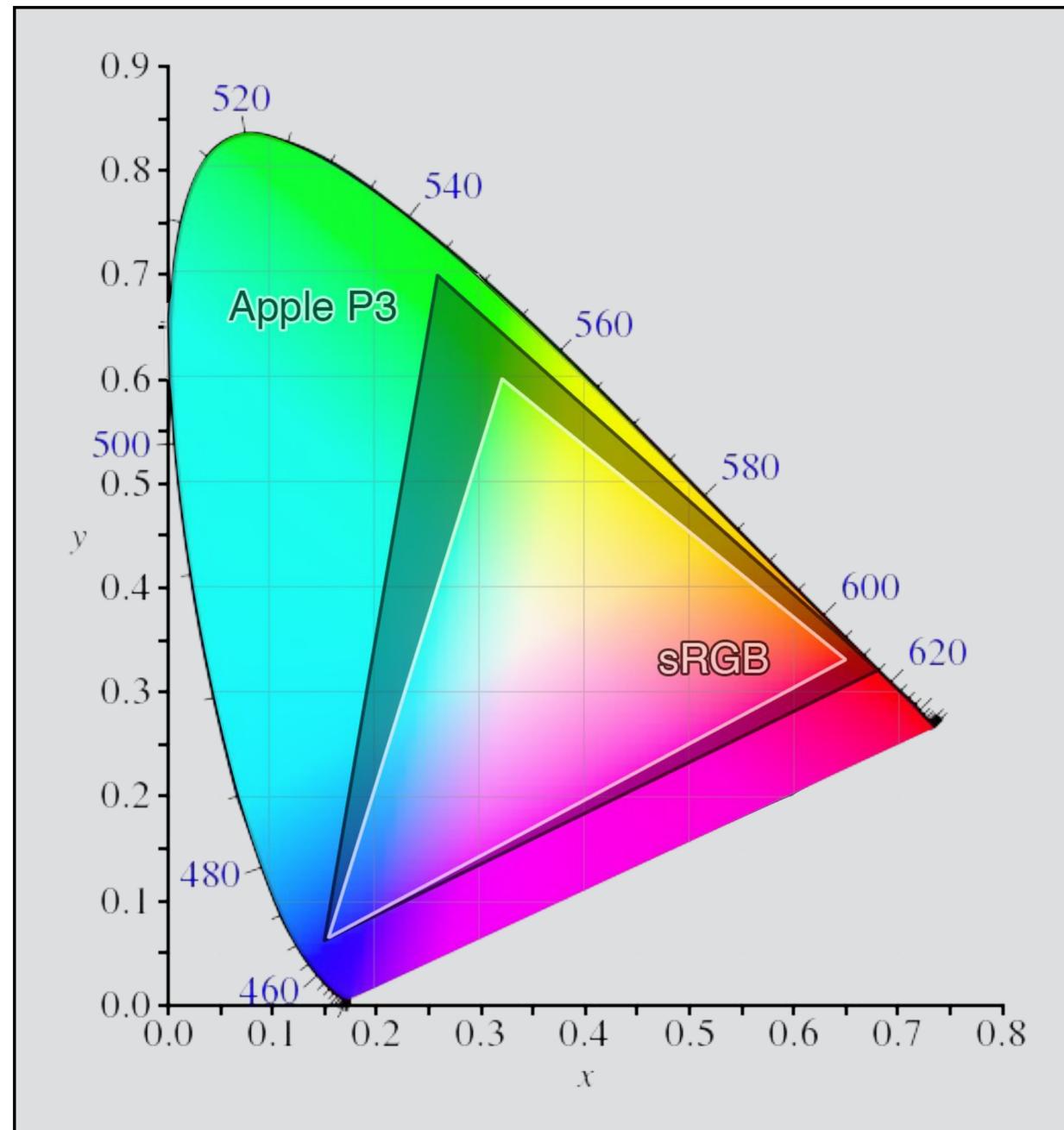
$$(l, m, s) = \frac{(L, M, S)}{L + M + S}$$



Color Gamut on Chromaticity Diagram (CIE 1931 xy)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9121 & -1.1121 & 0.2019 \\ 0.3709 & 0.6291 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

$$(x, y) = \frac{(X, Y)}{X + Y + Z}$$

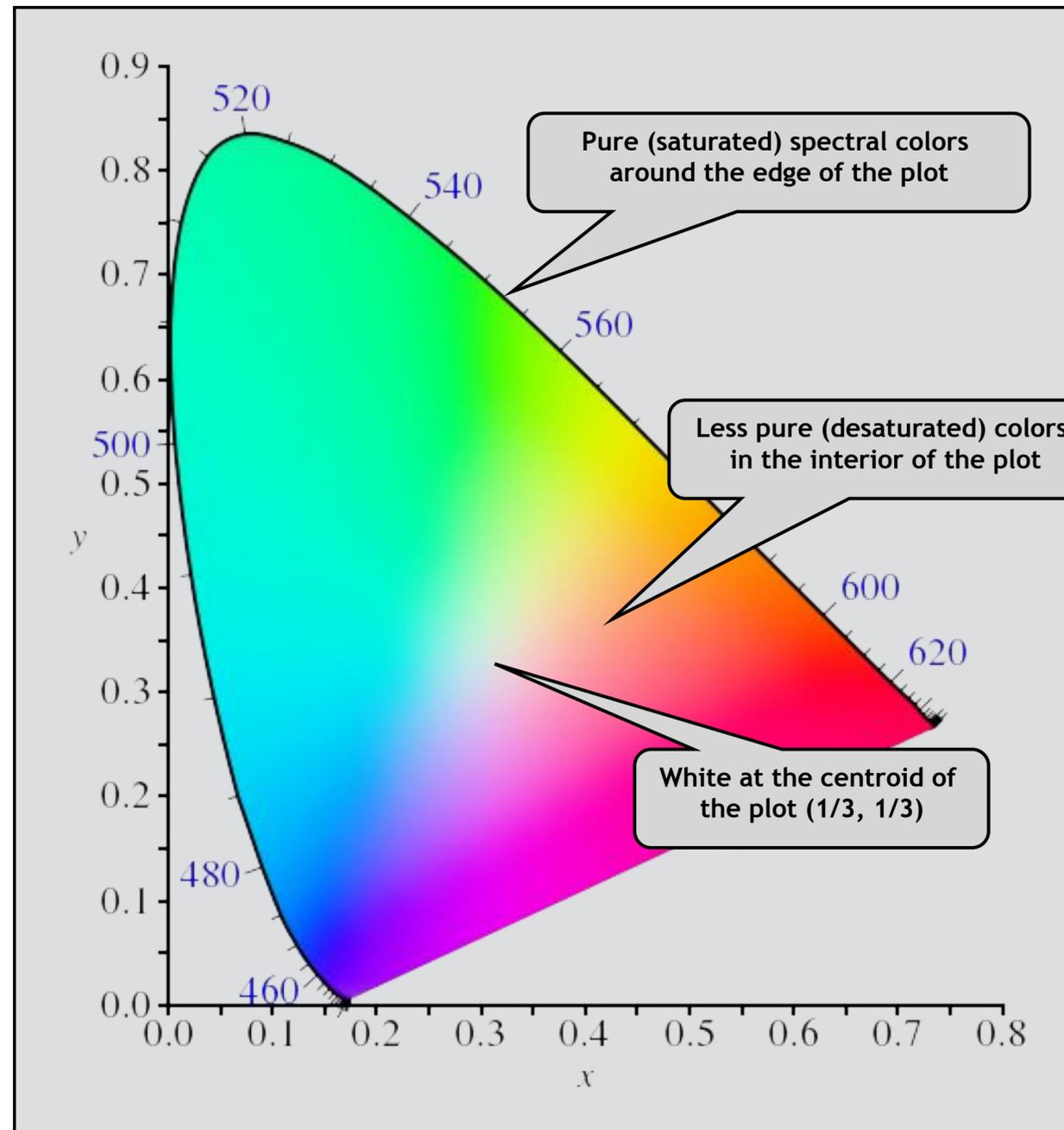


Wikipedia

Color Gamut on Chromaticity Diagram (CIE 1931 xy)

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$$(x, y) = \frac{(X, Y)}{X + Y + Z}$$



Wikipedia

Color Spaces

Color spaces

- **Need three numbers to specify a color**
 - **But what three numbers?**
 - **A color space is an answer to this question**
 - **Same color has different coordinates in different color spaces. E.g. RGB, XYZ, Lab, HSV, ...**

- **Common example: color space defined by a display**
 - **Define colors by what R, G, B scalar values will produce them on your monitor**
 - **Output spectra $s = rR + gG + bB$ for some display primary spectra r, g, b**
 - **This a device dependent representation of color: if I choose R,G,B by looking at my display and send those values to you, you may not see the same color on your display (which might have different primaries, etc.)**
 - **Also leaves out some colors (limited gamut), e.g. vivid yellow**
 - **Because in file formats R, G, B usually constrained to be non-negative**

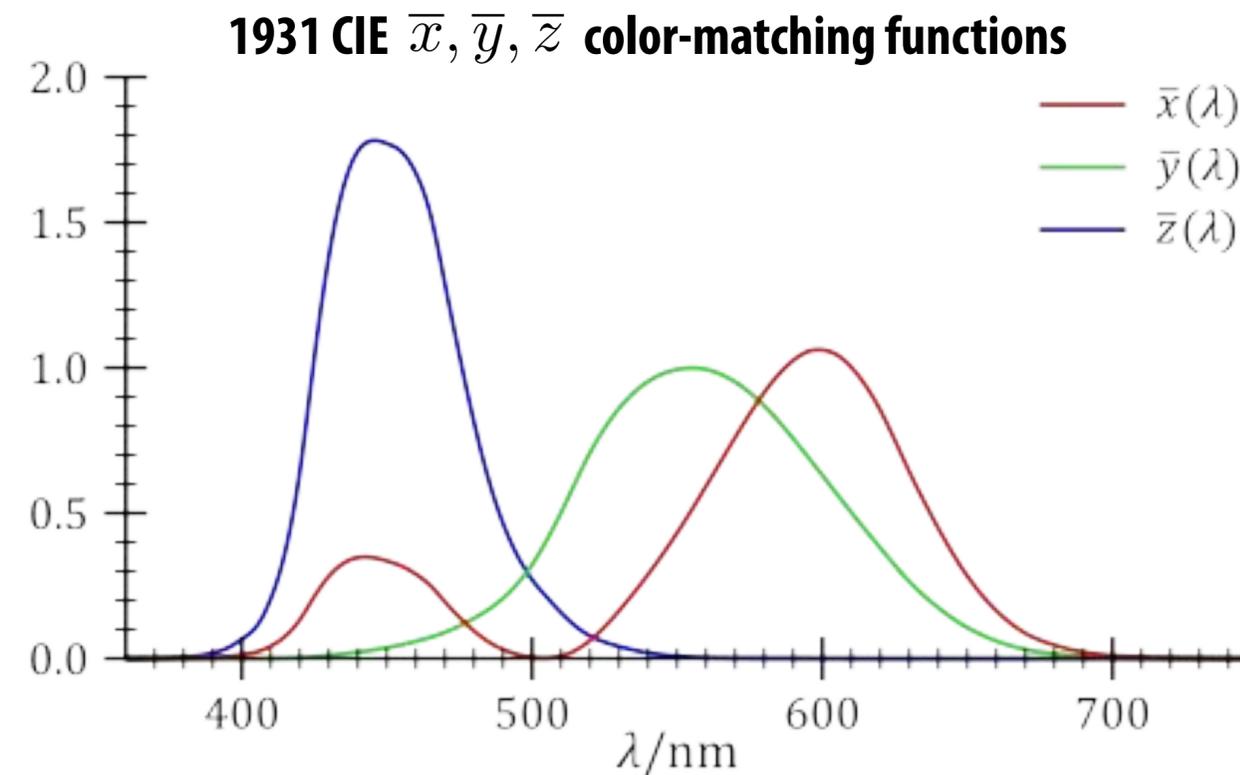
Standard color spaces

■ Standardized RGB (sRGB)

- **Makes a particular monitor's primaries the RGB standard**
- **Other color devices simulate that monitor by calibration**
- **sRGB is usable as an interchange color space; widely adopted today**

The historical “standard” color space: CIE XYZ

- Imaginary set of standard color primaries X, Y, Z
- Designed such that
 - X, Y, Z span all observable colors
 - Matching functions are strictly positive
 - Y is luminance (brightness absent color)
- “Imaginary” because the spectrum of the X,Y,Z primaries corresponding to these color matching functions are negative at some wavelengths



Careful: these graphs are color matching curves, not spectra!

For any spectrum $\Phi(\lambda)$, can express spectrum as weighted combination of primaries. Weights (X,Y,Z) given by:

$$X = k \int_{\lambda} \Phi(\lambda) \bar{x}(\lambda) d\lambda$$

$$Y = k \int_{\lambda} \Phi(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = k \int_{\lambda} \Phi(\lambda) \bar{z}(\lambda) d\lambda$$

Mathematically: just a change of basis

- By definition, all observable monochromatic spectra are positive points in XYZ space, so can convert a color's representation (in space defined by realizable primaries like RGB) to XYZ via a linear transform:
 - Consider display with 3 primaries (primaries need not be monochromatic light)
 - Compute XYZ coords of light emitted by display when providing it (1,0,0), (0,1,0), (0,0,1)
 - Light generated by display is linear combination of these vectors (non-negative weights)

$$\begin{aligned}
 \text{color of R primary ([1,0,0] on display)} &= R_x \mathbf{X} + R_y \mathbf{Y} + R_z \mathbf{Z} \\
 \text{color of G primary ([0,1,0] on display)} &= G_x \mathbf{X} + G_y \mathbf{Y} + G_z \mathbf{Z} \\
 \text{color of B primary ([0,0,1] on display)} &= B_x \mathbf{X} + B_y \mathbf{Y} + B_z \mathbf{Z}
 \end{aligned}
 \rightarrow
 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
 =
 \begin{bmatrix} R_x & G_x & B_x \\ R_y & G_y & B_y \\ R_z & G_z & B_z \end{bmatrix}
 \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

↑
XYZ representation

↑
color in space
of display primaries

- Example: Converting from CIE RGB to CIE XYZ:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
 =
 \frac{1}{0.17697}
 \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17687 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix}
 \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Brightness

- The color matching experiments measure how a human observer perceives color. The goal was to match the perceived color of one spectrum with a new spectrum (a metamer) formed via the combination of three primaries.
- We can also ask the question, given lights with two different colors but equal power, how bright do the lights look?



Luminance (brightness)

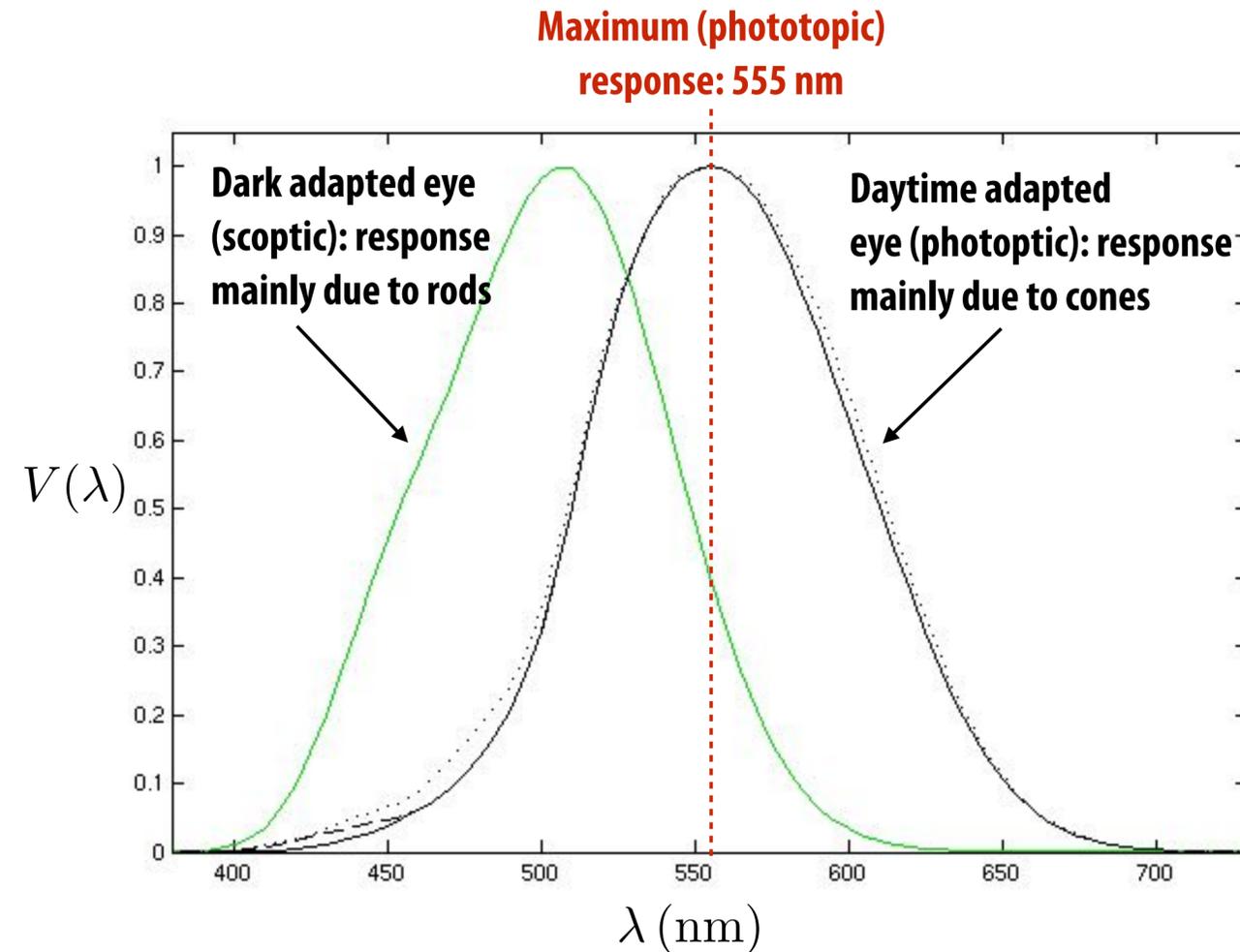
- Product of radiance and the eye's luminous efficiency

$$Y = \int \Phi(\lambda) V(\lambda) d\lambda$$

- Luminous efficiency is measure of how bright light at a given wavelength is perceived by a human (due to the eye's response to light at that wavelength)

- How to measure the eye's response curve $V(\lambda)$?

- Adjust power of monochromatic light source of wavelength λ until it matches the brightness of reference 555 nm source (photopic case)
- Notice: the sensitivity of photopic eye is maximized at ~ 555 nm



LMS responses plotted as 3D color space

- Visualization of "spectral locus" of human cone cells' response to monochromatic light (light with energy in a single wavelength) as points in 3D space.
- This is a plot of the S, M, L response functions as a point in 3D space.

Now consider XYZ responses as curve in 3D space

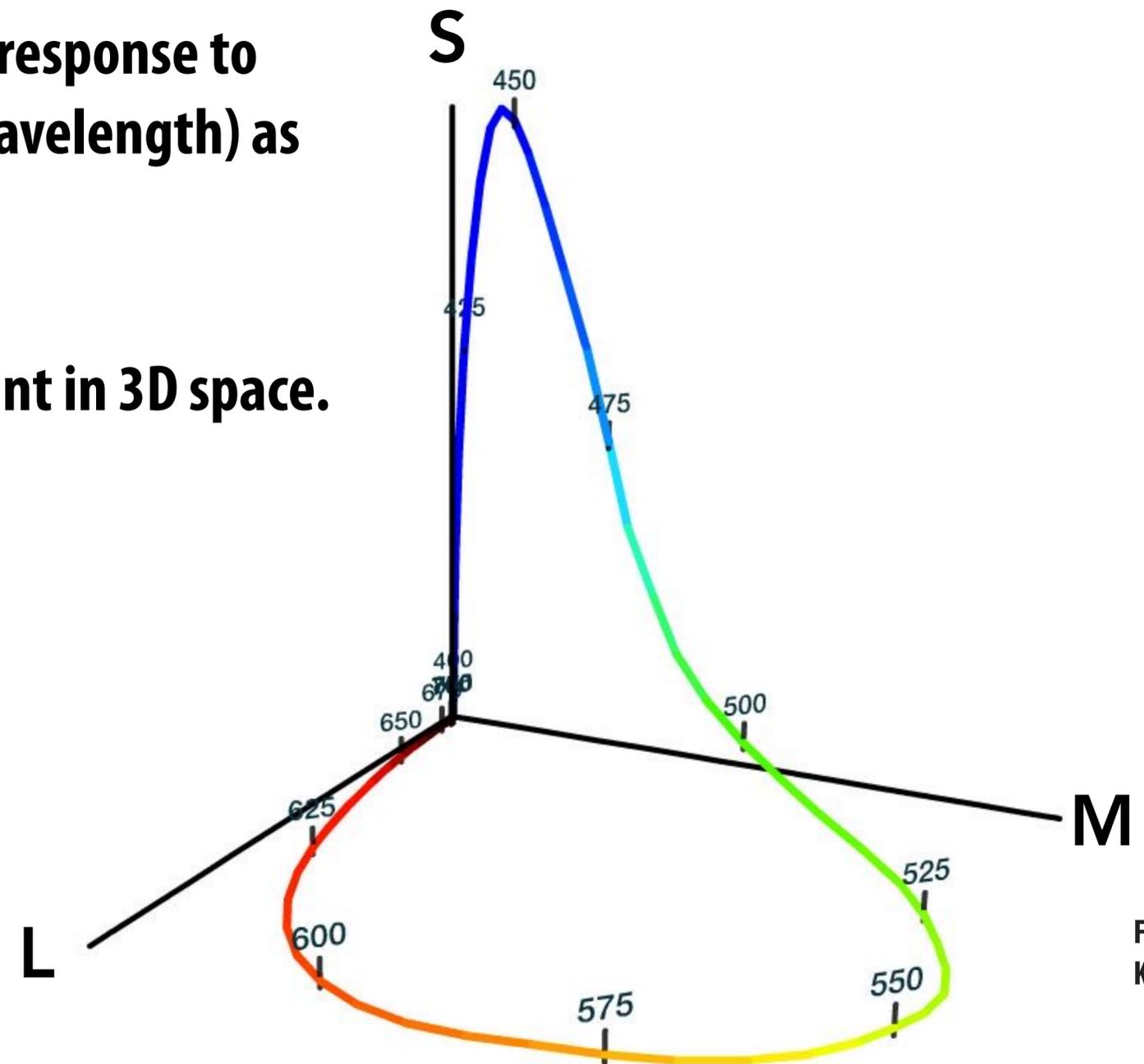
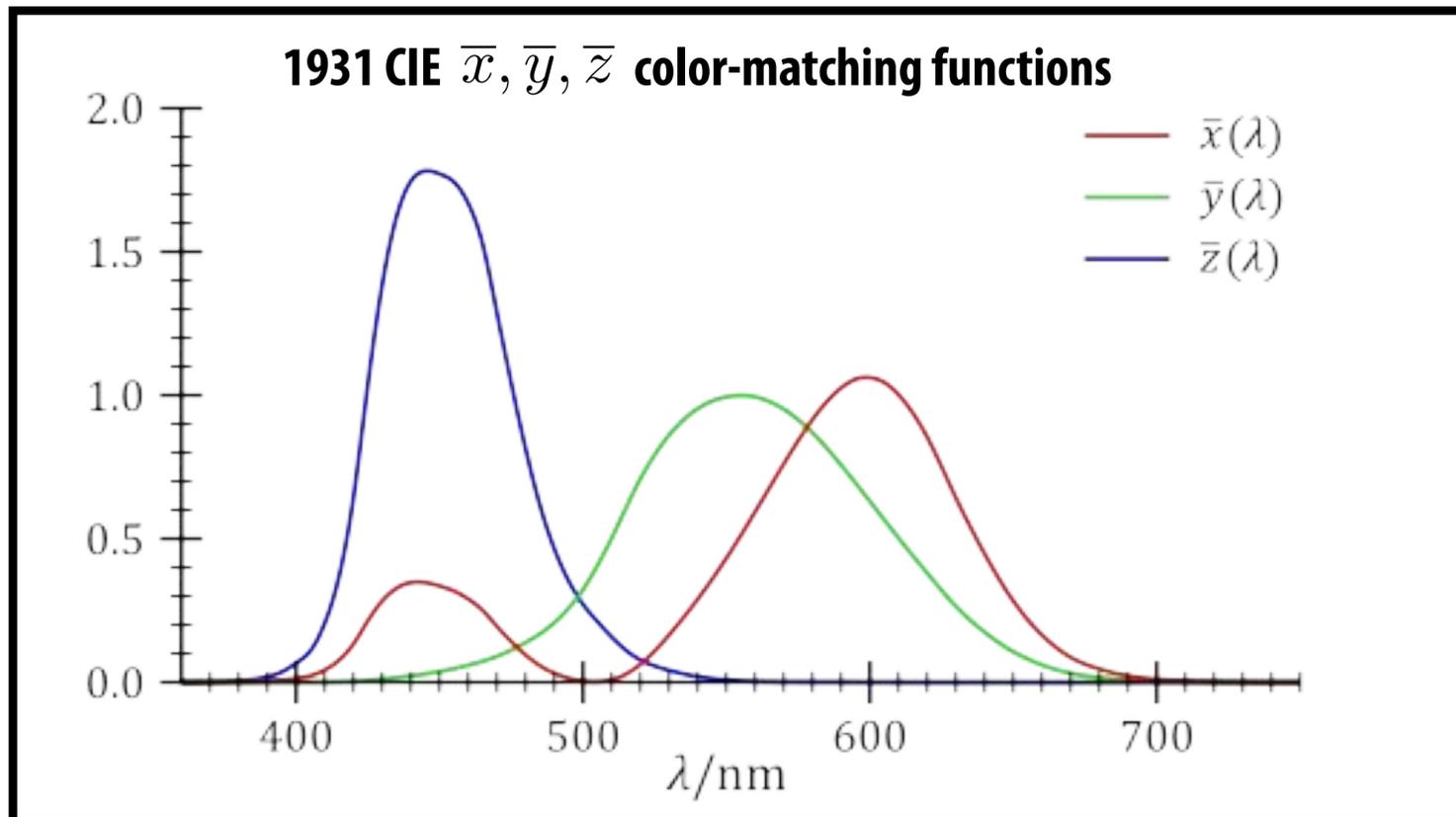


Figure Credit: Katie Dektar

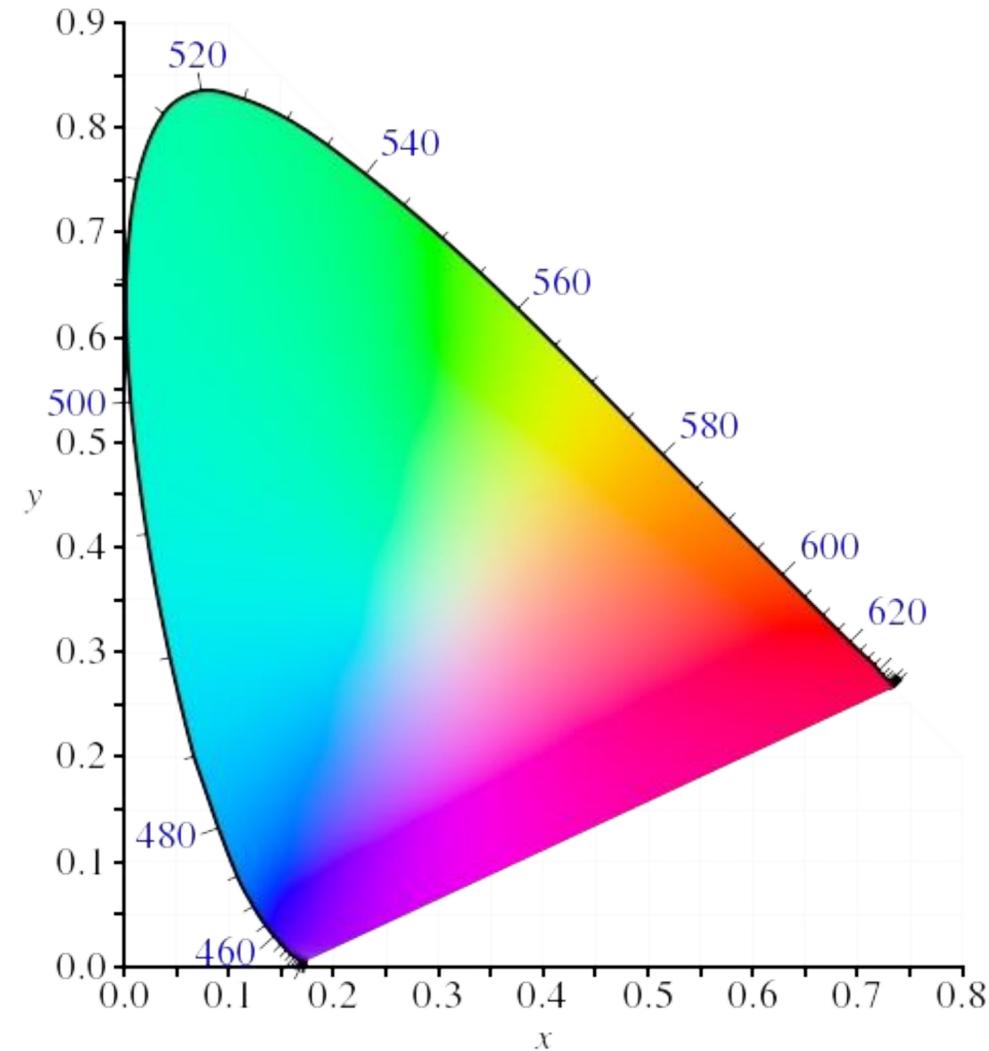
Separating luminance, chromaticity

- Luminance: Y
- Chromaticity: x, y, z , defined as

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

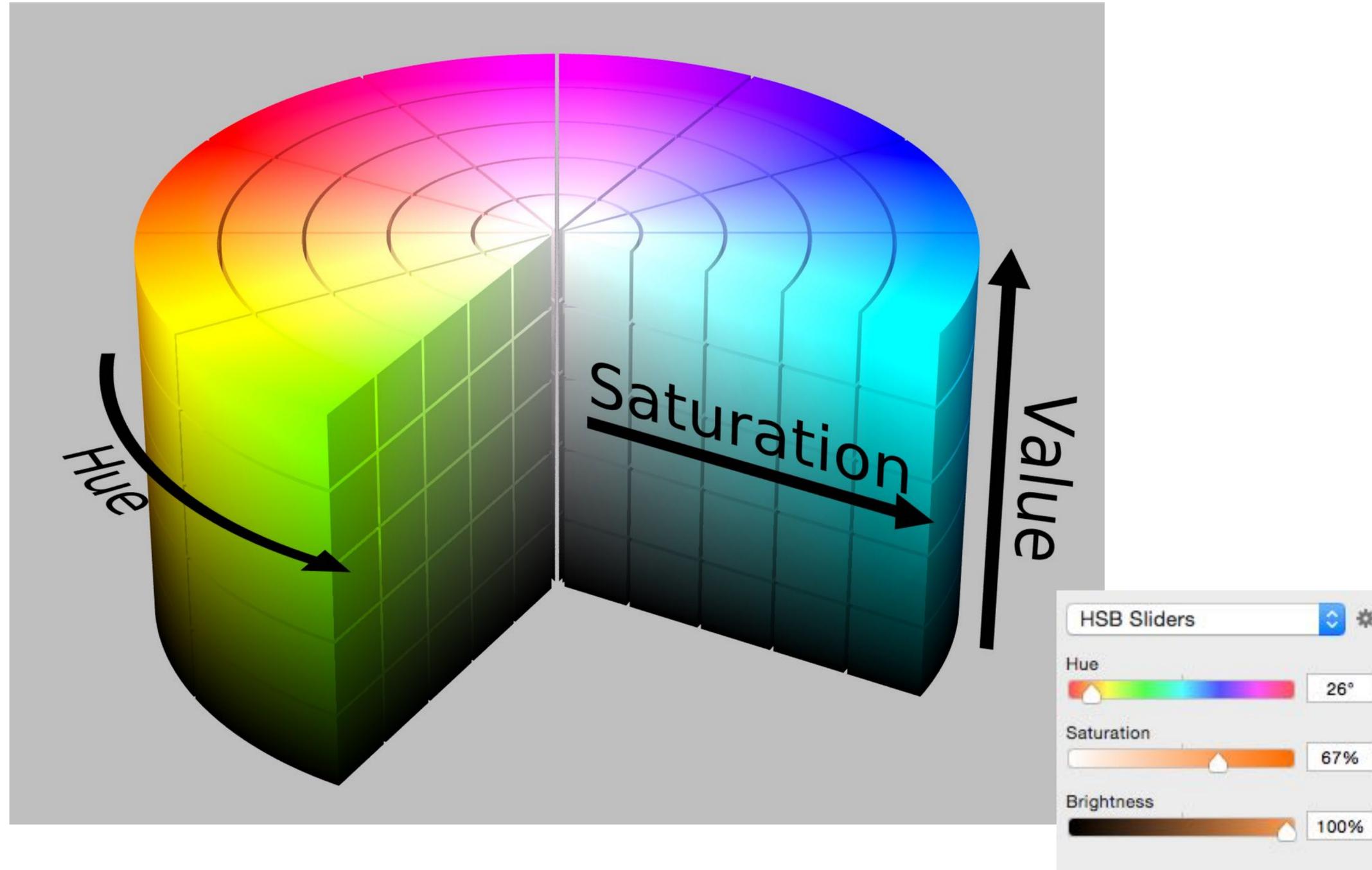


- since $x + y + z = 1$, we only need to record two of the three
- usually choose x and y , leading to (x, y, Y) coords

Perceptually Organized Color Spaces

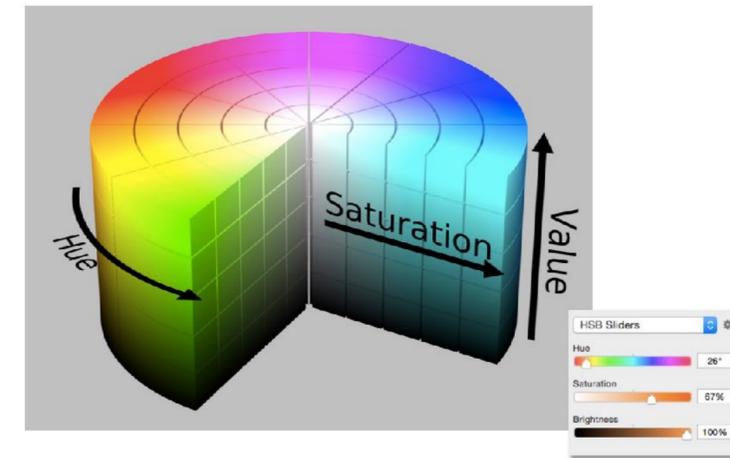
HSV (hue-saturation-value)

Axes of space correspond to natural notions of “characteristics” of color

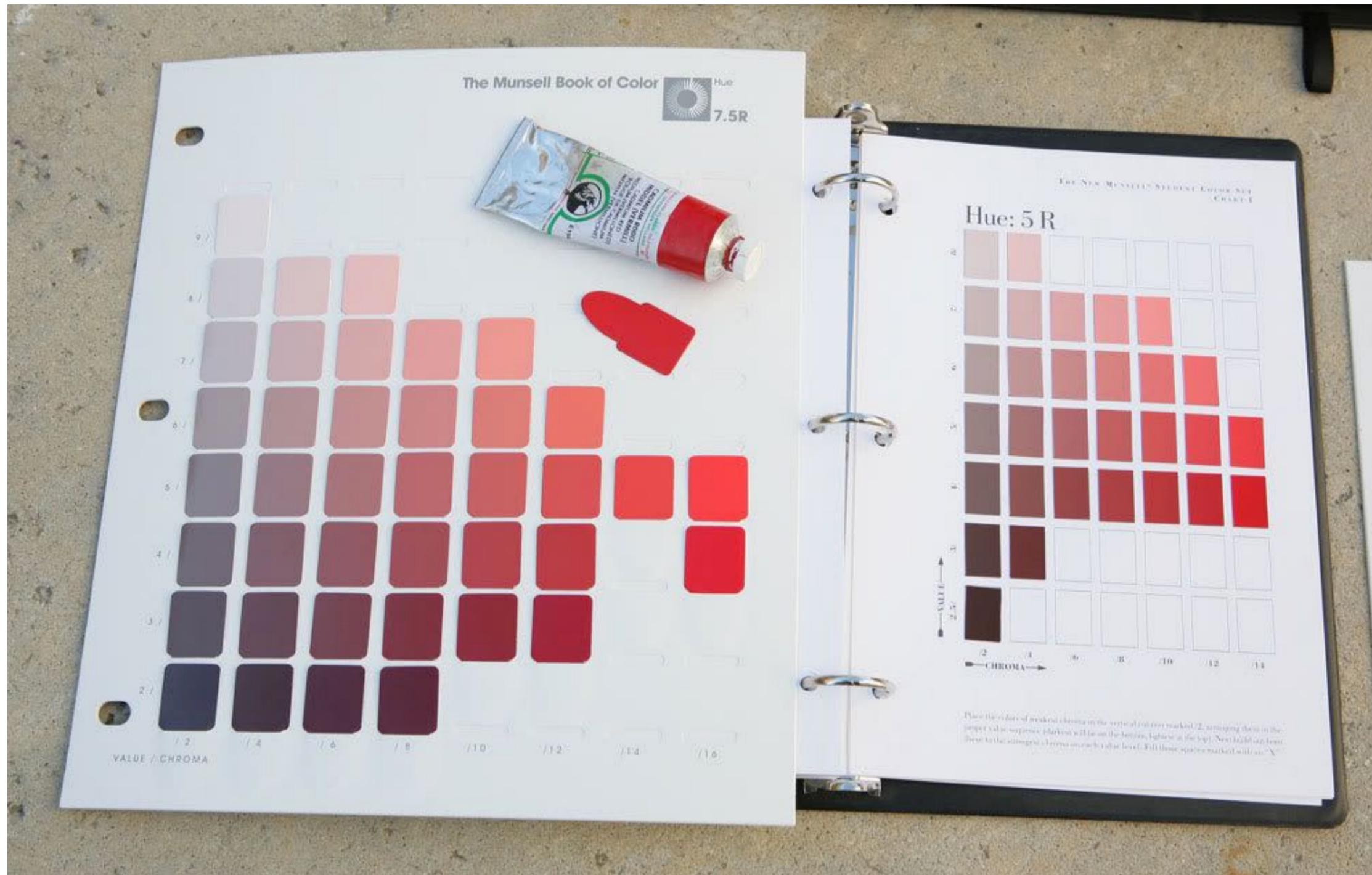


Perceptual dimensions of color

- **Hue**
 - the “kind” of color, regardless of attributes
 - colorimetric correlate: dominant wavelength
 - artist’s correlate: the chosen pigment color
- **Saturation**
 - the “colorfulness”
 - colorimetric correlate: purity
 - artist’s correlate: fraction of paint from the colored tube
- **Lightness (or value)**
 - the overall amount of light
 - colorimetric correlate: luminance
 - artist’s correlate: tints are lighter, shades are darker

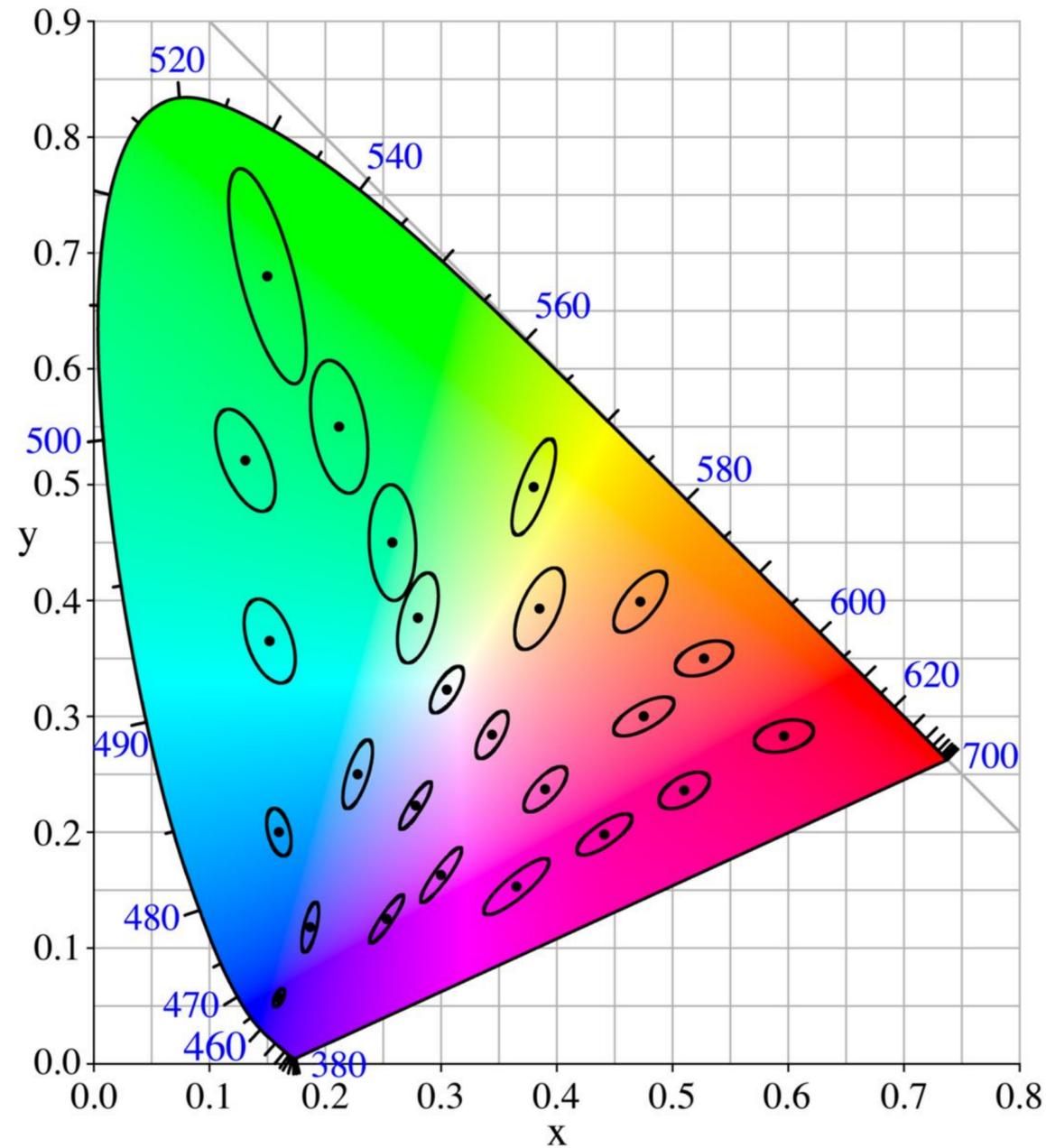


Munsell book of color



Swatch identified by three numbers: hue, value (lightness), and chroma (color purity)

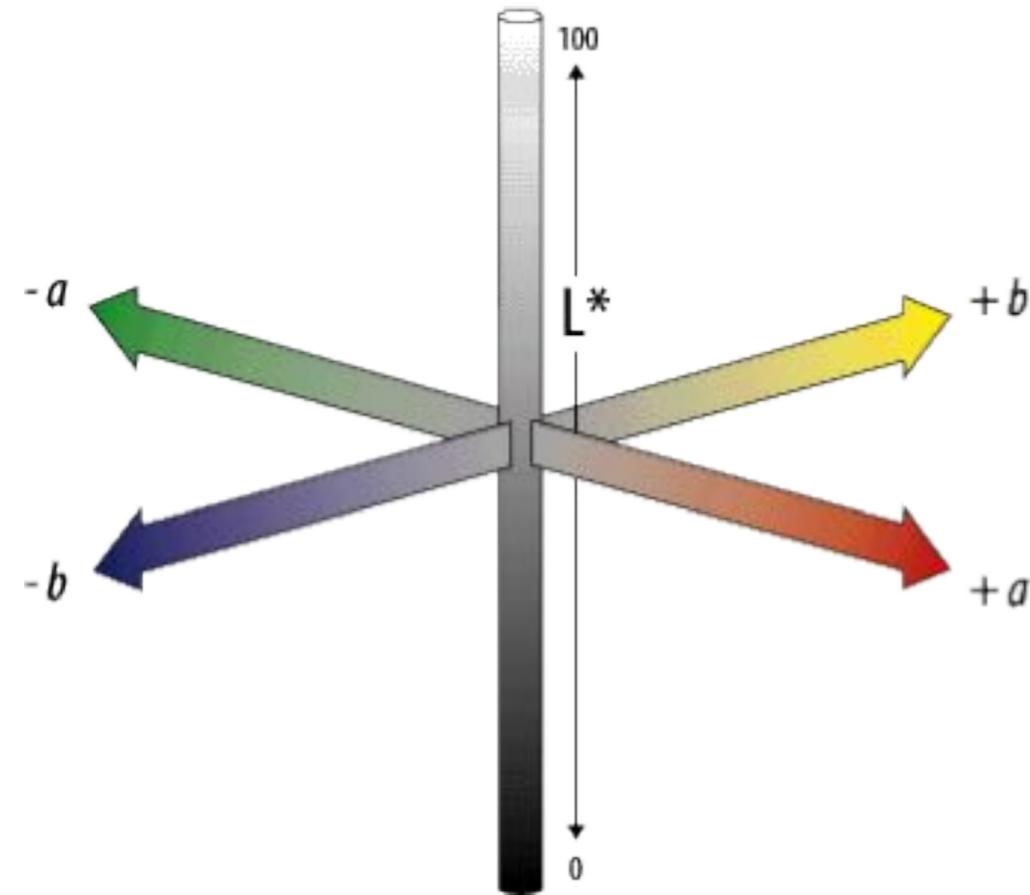
Perceptual non-uniformity



- In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x)
- Must non-linearly warp the diagram to achieve uniform perceptual distances

CIELAB Space ($L^*a^*b^*$)

- A commonly used color space that strives for perceptual uniformity
 - L^* is lightness
 - a^* and b^* are color-opponent pairs
 - a^* is red-green, and b^* is blue-yellow
 - A gamma transform is used for warping because perceived brightness is proportional to scene intensity $^\gamma$, where $\gamma \approx 1/3$



Things to remember

- **Physics of Light**
 - **Spectral power distribution (SPD)**
 - **Superposition (linearity)**
- **Tristimulus theory of color**
 - **Spectral response of human cone cells (S, M, L)**
 - **Metamers - different SPDs with the same perceived color**
 - **Color reproduction mathematics**
 - **Color matching experiment, per-wavelength matching functions**
- **Color spaces**
 - **CIE RGB, XYZ, xy chromaticity, LAB, HSV**
 - **Gamut**

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