Next up!!!

Radiometry & Photometry

The Light Field
Radiometry and Photometry

Measure spatial properties of light

- Radiant power
- Radiant intensity
- Irradiance
  - Inverse square law and cosine law
- Radiance
- Radiant exitance (radiosity)

Goal is to perform lighting calculations in a physically correct way

Acknowledgement: Pat Hanrahan
Photon energy is directly proportional to frequency.

\[ E = hf \]

where

- \( E \) is energy (J)
- \( h \) is Planck's constant: \( 6.62607015 \times 10^{-34} \text{ (m}^2\text{kgs}^{-1}) \)
- \( f \) is frequency (Hz)

Additionally,

\[ E = \frac{hc}{\lambda} \]

<table>
<thead>
<tr>
<th>Color</th>
<th>Wavelength (nm)</th>
<th>Frequency (THz)</th>
<th>Photon energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>violet</td>
<td>380–450</td>
<td>670–790</td>
<td>2.75–3.26</td>
</tr>
<tr>
<td>blue</td>
<td>450–485</td>
<td>620–670</td>
<td>2.56–2.75</td>
</tr>
<tr>
<td>cyan</td>
<td>485–500</td>
<td>600–620</td>
<td>2.48–2.56</td>
</tr>
<tr>
<td>green</td>
<td>500–565</td>
<td>530–600</td>
<td>2.19–2.48</td>
</tr>
<tr>
<td>yellow</td>
<td>565–590</td>
<td>510–530</td>
<td>2.10–2.19</td>
</tr>
<tr>
<td>orange</td>
<td>590–625</td>
<td>480–510</td>
<td>1.98–2.10</td>
</tr>
<tr>
<td>red</td>
<td>625–750</td>
<td>400–480</td>
<td>1.65–1.98</td>
</tr>
</tbody>
</table>
KEY IDEA OF PHOTOMETRY:
Radiant power at each wavelength is weighted by a luminosity function that models human brightness sensitivity.

Rendering under low-light conditions

HUMAN
https://en.wikipedia.org/wiki/Scotopic_vision
Rendering under low-light conditions

CAT
https://en.wikipedia.org/wiki/Scotopic_vision
Radiant Energy and Power

Power (\(\Phi\)) units:
- Watts (radiometry)
- Lumens (photometry)

Related concepts:
- Spectral efficacy
- Energy efficiency

Energy: Joules vs. Talbot
- Exposure
  - Film response
  - Skin - sunburn

Conversion factor: 683 lumens/Watt @ 550nm

Photometric luminance:
\[ Y = \int V(\lambda)L(\lambda)\,d\lambda \]
Radiant Intensity
Radiant Intensity

Definition: The *radiant* (*luminous*) intensity, $I$, is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

$$\left[ \frac{W}{sr} \right] \left[ \frac{lm}{sr} \right] = cd = candela$$
Radiant Intensity

https://www.jensign.com/LEDIntensity
Angles and Solid Angles

Angle

\[ \theta = \frac{l}{r} \]

⇒ circle has 2 \( \pi \) radians

Solid angle

\[ \Omega = \frac{A}{r^2} \]

⇒ sphere has 4 \( \pi \) steradians
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin\theta \, d\phi) \]
\[ = r^2 \sin\theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ dA = (r \, d\theta)(r \, \sin \theta \, d\phi) \]
\[ = r^2 \, \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[
\Omega = \int \, d\omega \\
= \int \int \sin \theta \, d\theta \, d\phi \\
= \int \int d\cos \theta \, d\phi \\
= 4\pi
\]
Isotropic Point Source

\[ \Phi = \int_{S^2} I \, d\omega \]

\[ = 4\pi I \]

\[ I = \frac{\Phi}{4\pi} \]
Anisotropic Point Sources

http://www.photometrictesting.co.uk/File/understanding_photometric_data_files.php
\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]

where \( \theta \in [0, \frac{\pi}{2}] \)
\[\vec{\omega} \]

\[\theta \]

\[\hat{A} \]

\[I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s\]

where \( \theta \in [0, \frac{\pi}{2}] \)

\[\Phi = \int_{0}^{2\pi} \int_{0}^{1} I(\omega) \, d\cos \theta \, d\varphi\]

Note: \( d\cos \theta = -\sin \theta \, d\theta \)
$I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s$

where $\theta \in [0, \frac{\pi}{2}]$

$$\Phi = \int_0^{2\pi} \int_0^1 I(\omega) d\cos\theta \, d\phi = 2\pi \int_0^1 \cos^s \theta \, d\cos\theta = \frac{2\pi}{s+1}$$
Warn’s Spotlight

\[ \Phi = \int_0^{2\pi} \int_0^1 I(\omega) \, d\cos\theta \, d\varphi = 2\pi \int_0^1 \cos^s \theta \, d\cos\theta = \frac{2\pi}{s + 1} \]

\[ I(\omega) = \Phi \frac{s + 1}{2\pi} \cos^s \theta \]

\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]

where \( \theta \in [0, \frac{\pi}{2}] \)
Light Source Goniometric Diagrams

3. Porcelain-enameded ventilated standard dome with incandescent lamp
   - 0%
   - 83.5%

4. Pendant diffusing sphere with incandescent lamp
   - 35.5%
   - 45%

2. Concentric ring unit with incandescent silvered-bowl lamp
   - 83%
   - 31%

5. R-40 flood with specular anodized reflector skirt; 45° cutoff
   - 0%
   - 85%
IES Light Profiles

The Illuminating Engineering Society (IES) has defined a file format which describes a light's distribution from a light source using real world measured data. These IES Photometric files, or IES Profiles, are a lighting industry standard method of diagramming the brightness and falloff of light as it exists a particular real world light fixture. It enables them to account for reflective surfaces in the light fixture, the shape of the light bulb, and any lensing effects that happens. This type of photometric lighting is primarily used in Enterprise fields (such as Media and Entertainment or Architecture and Manufacturing), but is often used in games production to achieve realistic lighting effects, too.

Complex Light Sources

https://specialneedstoys.com/usa/h20-projector.html
Irradiance
Beautiful Irradiance: Caustics
Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface.

\[ E(x) \equiv \frac{d\Phi_i}{dA} \]

\[
\left[ \frac{W}{m^2} \right] \left[ \frac{lm}{m^2} = lux \right]
\]

Also referred to as the radiant (luminous) incidence.
## Typical Values of Illuminance [lm/m²]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight plus skylight</td>
<td>100,000 lux</td>
</tr>
<tr>
<td>Sunlight plus skylight (overcast)</td>
<td>10,000</td>
</tr>
<tr>
<td>Interior near window (daylight)</td>
<td>1,000</td>
</tr>
<tr>
<td>Artificial light (minimum)</td>
<td>100</td>
</tr>
<tr>
<td>Moonlight (full)</td>
<td>0.02</td>
</tr>
<tr>
<td>Starlight</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Beam Power in Terms of Irradiance

\[ \Phi = EA \]

\[ E = \frac{\Phi}{A} \]
Beam Power Falling on the Surface

\[ \Phi' = E' A' \]

\[ E' = \frac{\Phi'}{A'} \]
Projected Area

\[ A = A' \cos \theta \]

Project \( A' \) onto the surface perpendicular to the direction.
Lambert’s Cosine Law

\[ A = A' \cos \theta \]
\[ \Phi = \Phi' \]

\[
E' = \frac{\Phi'}{A'} = \frac{\Phi}{A} \cos \theta = E \cos \theta
\]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

Power in the beam

\[ d\Phi = I \, d\omega \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

Angle subtended by \( dA \)

\[ d\omega = \frac{\cos\theta}{r^2} \, dA \]
Irradiance: Isotropic Point Source

\[
I = \frac{\Phi}{4\pi}
\]

\[
I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} \, dA
\]
Irradiance: Isotropic Point Source

\[ I d\omega = \frac{\Phi \cos \theta}{4\pi r^2} dA = E dA \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ E = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} \]
The Invention of Photometry

Bouguer’s classic experiment
- Compare a light source and a candle
- Move until they both appear equally bright
- Intensity is proportional to ratio of distances squared

Definition of a candela
- Originally a “standard” candle
- Currently 550 nm laser with 1/683 W/sr
- 1 of 6 fundamental SI units
Radiance
Area Lights – Surface Radiance

Definition: The surface radiance (luminance) is the intensity per unit area leaving a surface

\[ L(x, \omega) \equiv \frac{dI(x, \omega)}{dA} = \frac{d^2 \Phi(x, \omega)}{d\omega dA} \]

\[
\begin{bmatrix}
\frac{W}{sr \ m^2} \\
\frac{cd}{m^2}
\end{bmatrix}
\begin{bmatrix}
\frac{lm}{sr \ m^2}
\end{bmatrix}
= nit
\]
## Typical Values of Luminance [cd/m^2]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Luminance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface of the sun</td>
<td>2,000,000,000 nit</td>
</tr>
<tr>
<td>Sunlight clouds</td>
<td>30,000</td>
</tr>
<tr>
<td>Clear sky</td>
<td>3,000</td>
</tr>
<tr>
<td>Overcast sky</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Directional Power Leaving a Surface

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta \, dA \, d\omega \]

Same \( dA \) for all directions introduces cosine
Radiant Exitance
(Radiosity)
Radiant Exitance

Definition: The radiant *(luminous)* exitance is the power per unit area leaving a surface.

\[
M(x) \equiv \frac{d\Phi_o}{dA}
\]

\[
\begin{bmatrix}
\frac{W}{m^2} \\
\frac{lm}{m^2}
\end{bmatrix} = lux
\]

In computer graphics, this quantity is usually referred to as the *radiosity* \((B)\)
Area Light Source

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta \, dA \, d\omega \]
Area Light Source

\[ dM(x, \omega) = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta \, d\omega \]

\[ \theta \]

\[ L_o(x, \omega) \]

\[ dA \]
Area Light Source

\[ M = \int_{H^2} dM(x, \omega) = \int_{H^2} L_o(x, \omega) \cos \theta \, d\omega \]

\( \int_{H^2} \)

\( L_o(x, \omega) \)

\( d\omega \)

\( H^2 \)

Hemisphere
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

Uniform means \( L_o \) is not a function of direction

\[ d\omega \]

\[ \theta \]

\[ dA \]

\[ L_o(x, \omega) = L_o \]
Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \ d\omega \]
Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int_{H^2} \cos \theta \, d\omega = \pi \]
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

\[ = \pi L_o \]

\[ L_o = \frac{M}{\pi} \]
Radiometry and Photometry

Summary
# Radiometric and Photometric Terms

<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance</td>
<td>Illuminance</td>
</tr>
<tr>
<td></td>
<td>Radiosity</td>
<td>Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
<td>Luminous Intensity</td>
</tr>
</tbody>
</table>
## Photometric Units

<table>
<thead>
<tr>
<th>Photometry</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MKS</td>
</tr>
<tr>
<td></td>
<td>CGS</td>
</tr>
<tr>
<td></td>
<td>British</td>
</tr>
<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
</tr>
<tr>
<td>Luminous Power</td>
<td>Lumen</td>
</tr>
<tr>
<td>Illuminance</td>
<td>Lux</td>
</tr>
<tr>
<td></td>
<td>Phot</td>
</tr>
<tr>
<td></td>
<td>Footcandle</td>
</tr>
<tr>
<td>Luminosity</td>
<td></td>
</tr>
<tr>
<td>Luminance</td>
<td>Nit</td>
</tr>
<tr>
<td></td>
<td>Apostilb, Blondel</td>
</tr>
<tr>
<td></td>
<td>Stilb</td>
</tr>
<tr>
<td></td>
<td>Lambert</td>
</tr>
<tr>
<td></td>
<td>Footlambert</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela (Candle, Candlepower, Carcel, Hefner)</td>
</tr>
</tbody>
</table>

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?"  
-- James Kajiya
Next up!!!

The Light Field

BONUS
The Light Field

Last lecture: Radiometry and photometry

This lecture: Light field = radiance function on rays
- Conservation of radiance
- Measurement equation
- Throughput and counting rays
- Irradiance calculations

Subtitle: “Lines and Light”

Acknowledgement: Pat Hanrahan

From London and Upton
Field Radiance or Light Field

Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.

\[ L(x, \omega) \equiv \frac{d^2 \Phi(x, \omega)}{d\omega \ dA} \]
Capture Light Field

$L(x, y, \theta, \varphi)$

Captures all the light leaving an object - like a hologram
Multi-Camera Array
Two-Plane 4D Light Field

2D Array of Cameras

2D Array of Images

$L(u,v,s,t)$
Lenslet Arrays

The micro lens array is placed in front of the image sensor.

http://cameramaker.se/Lightfield.htm
Lenslet Arrays

The unprocessed light field image

Detail of the light field image

http://cameramaker.se/Lightfield.htm
Ren Ng
Light Field Camera

Lens
The Lytro Light Field Camera starts with an 8X optical zoom, 1/2 aperture lens. The aperture is constant across the zoom range allowing for unheard of light capture.

Light Field Engine 1.0
The Light Field Engine replaces the supercomputer from the lab and processes the light ray data captured by the sensor.

The Light Field Engine travels with every living picture as it is shared, letting you refocus pictures right on the camera, on your desktop and online.

Light Field Sensor
From a roomful of cameras to a micro-lens array specially adhered to a standard sensor, the Lytro's Light Field Sensor captures 11 million light rays.

Lytro Camera
Light Field Inside the Camera Body

Ray carrying $L(u, v, x, y)$
Lytro Immerge 2.0

Google VR180

Left: A time lapse video of recording a spherical light field on the flight deck of Space Shuttle Discovery.
Right: Light field rendering allows us to synthesize new views of the scene anywhere within the spherical volume by sampling and interpolating the rays of light recorded by the cameras on the rig.

https://www.blog.google/products/google-vr/experimenting-light-fields
Light Field Video

Fig. 1. Our light field capture rig and scene representations. (a) We record immersive light field video using 46 action sports cameras mounted to an acrylic dome. (b) Using deep view synthesis we infer an RGBA multi-sphere image (MSI) from the light field views. Every 10th spherical shell is highlighted. (c) We convert groups of MSI layers into Layered Meshes (each shown as a different color), which are texture atlassed and compressed into light field video.

https://augmentedperception.github.io/deepviewvideo

Fig. 7. An example texture atlas from the Dog scene in Figure 8.
Estimating Light Fields from Images

NeRF
Representing Scenes as Neural Radiance Fields for View Synthesis

ECCV 2020

https://www.matthewtancik.com/nerf
Estimating Light Fields from Images

Plenoxels
Radiance Fields without Neural Networks
CVPR 2022

https://alexyu.net/plenoxels
Properties of Radiance
Properties of Radiance

Fundamental field quantity that characterizes the distribution of light in an environment

- Radiance is the quantity associated with a ray
- Ray tracers compute radiance

Radiance $L(x, \omega)$ is invariant along a ray

- Reduces parameters from 5D to 4D

Response of a sensor proportional to the radiance

- Cameras measure radiance
1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates.

\[ d^2\Phi_1 = d^2\Phi_2 \]

\[ d^2\Phi_1 = L_1 d\omega_1 dA_1 \]

\[ d^2\Phi_2 = L_2 d\omega_2 dA_2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]
Quiz

Does radiance increase under a magnifying glass?

No!!
The Measurement Equation
2nd Law: Sensors measure radiance

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[ R = \int \int L d\omega \, dA = \bar{L}T \]

\[ T = \int \int d\omega \, dA \]

\( T \) (throughput) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered

\( T \) is constant, sensor response \( R \) proportional to average radiance \( \bar{L} \)
Quiz

Assume a wall has constant radiance

Does the response of a sensor when pointed a wall depend on the distance to the wall?

No!!
Supplemental Material
Throughput

Measuring the Number of Rays
Beam of Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \quad \quad \quad \quad dA_2(u_2, v_2) \]

Definition:
The throughput is the number of rays in a beam.
Number of Rays in a Beam

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \text{---} \quad dA_2(u_2, v_2) \]

How many rays are in this beam?

This many: \[ d^2T = \frac{dA_1 dA_2}{\|x_1 - x_2\|^2} \]
Why is this the Right Measure?

\[ dA_1(u_1, v_1) \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]

Suppose we double the area \( dA_1 \) (or \( dA_2 \)), then the number of rays in the beam should double.

Suppose we half the distance \( |x_1 - x_2|^2 \), then the number of rays also doubles.
Alternative Definition of Radiance

Two physical laws

- Energy is conserved $\Delta \Phi$
- The size of a beam $\Delta T$ does not change as rays propagate

Radiance is the ratio of 2 conserved quantities, therefore, the radiance is also conserved

$$L(r) = \lim_{\Delta T \to r} \frac{\Delta \Phi}{\Delta T}$$
Changing Ray Coordinates

Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

$$d\omega_2(\theta_2, \phi_2) \quad \begin{array}{c} \infty \quad \infty \end{array} \quad dA_2(u_2, v_2)$$

$$d^2T = \frac{dA_1}{\left| x_1 - x_2 \right|^2} dA_2 = d\omega_2 dA_2$$

Same value for the throughput, different formula
Changing Ray Coordinates

Parameterize rays wrt to source \( r(u_1, v_1, \theta_1, \phi_1) \)

\[
dA_1(u_1, v_1) \rightarrow \theta_1 \rightarrow \phi_1 \rightarrow d\omega_1(\theta_1, \phi_1)
\]

\[
d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1
\]
Changing Ray Coordinates Again

Tilting the surfaces re-parameterizes the rays!

\[ dA_1(u_1, v_1) \quad \rightarrow \quad r(u_1, v_1, u_2, v_2) \quad \leftarrow \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{\cos \theta_1 \cos \theta_2}{\left| x_1 - x_2 \right|^2} \quad dA_1 dA_2 \]
Number of Rays Hitting a Shape

Parameterize rays by \( r(x, y, \theta, \phi) \)

Projected area \( \tilde{A}(\vec{\omega}) \)

Measuring the number or rays that hit a shape

\[
T = \int d\omega(\theta, \varphi) dA(x, y) = \int d\omega(\theta, \varphi) \int dA(x, y)
\]

\[
= \int \tilde{A}(\theta, \varphi) d\omega(\theta, \varphi)
\]

Sphere: \( T = 4\pi \tilde{A} = 4\pi^2 R^2 \)
Calculated Another Way

Parameterize rays by \( r(u,v,\theta,\phi) \)

\[
T = \left[ \int_{\mathcal{H}^2(\mathbf{N})} \cos \theta \, d\omega(\theta, \phi) \right] \left[ \int_{\mathcal{M}^2} dA(u,v) \right]
\]

**Sphere:** \( T = \pi S = 4\pi^2 R^2 \)

**Crofton’s Theorem:** \( 4\pi \overline{A} = \pi S \Rightarrow \overline{A} = \frac{S}{4} \)
Irradiance from a Uniform Area Light Source
Irradiance from the Environment

\[ d^2 \Phi_i (x, \omega) = L_i (x, \omega) \cos \theta \, dA \, d\omega \]

\[ dE(x, \omega) = L_i (x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{H^2} L_i (x, \omega) \cos \theta \, d\omega \]
Irradiance from a Uniform Area Source

$$E(x) = \int_{H^2} L \cos \theta \, d\omega$$

$$= L \int_{\Omega} \cos \theta \, d\omega$$

$$= L \tilde{\Omega}$$

Direct Illumination
Uniform Disk Source

Geometric Derivation

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

Algebraic Derivation

\[
\tilde{\Omega} = \int_0^{2\pi} \int_0^\pi \cos \theta \, d\phi \, d\cos \theta \\
= 2\pi \left[ \frac{\cos^2 \theta}{2} \right]_1^{\cos \alpha} \\
= \pi \sin^2 \alpha \\
= \pi \frac{r^2}{r^2 + h^2}
\]
Spherical Source

Geometric Derivation

Algebraic Derivation

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

\[ \tilde{\Omega} = \int \cos \theta \, d\omega \]

\[ = \pi \sin^2 \alpha \]

\[ = \pi \frac{r^2}{R^2} \]
The Sun

Solar constant (normal incidence at zenith)

Irradiance 1353 W/m²
Illuminance 127,500 lm/m² = 127.5 kilolux

Solar angle

α = .25 degrees = .004 radians (half angle)

\[ \tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \alpha^2 = 6 \times 10^{-5} \text{ steradians} \]

Solar radiance

\[ L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \text{ W/m}^2}{6 \times 10^{-5} \text{ sr}} = 2.25 \times 10^7 \frac{\text{W}}{\text{m}^2 \cdot \text{sr}} \]
Polygonal Source
Polygonal Source
Polygonal Source
Consider 1 Edge

Area of sector $\gamma$

Projected area of sector

$$A = \gamma \cos \theta = \gamma \hat{N}_E \cdot \hat{N}$$
Lambert’s Formula

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]
Penumbras and Umbras