Next up!!!

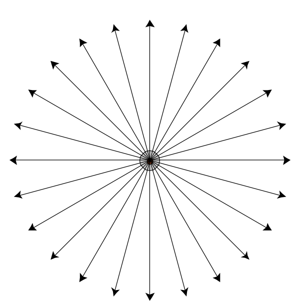
Radiometry & Photometry

The Light Field

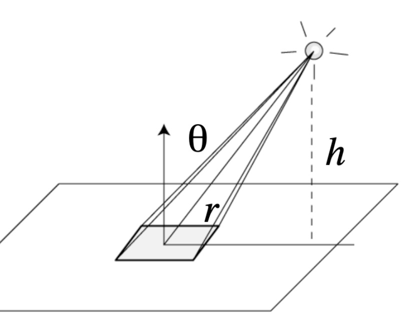
Radiometry & Photometry

Radiometry and Photometry

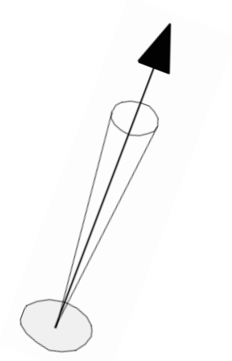
Measure spatial properties of light



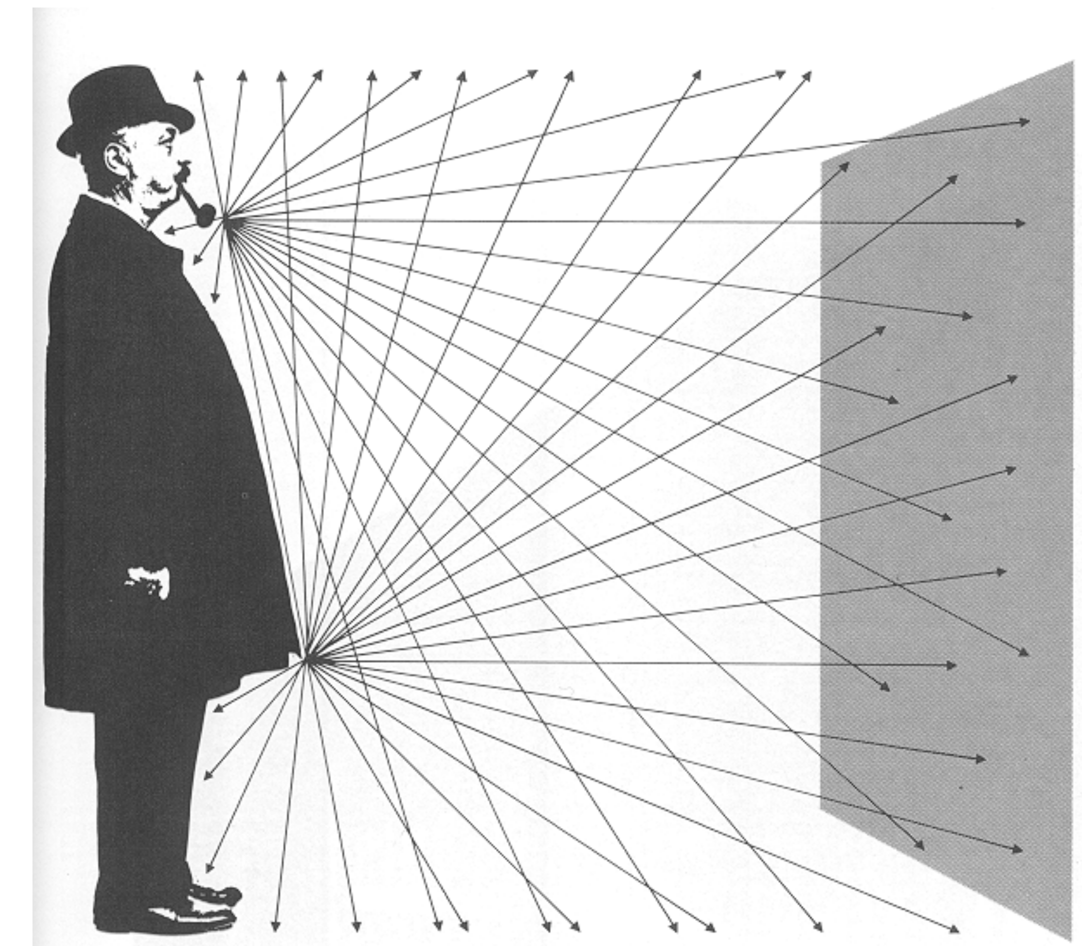
- Radiant power
- Radiant intensity



- Irradiance
  - Inverse square law and cosine law



- Radiance
- Radiant exitance (radiosity)



From London and Upton

Goal is to perform lighting calculations in a physically correct way

Acknowledgement: Pat Hanrahan

Photon energy is directly proportional to frequency.

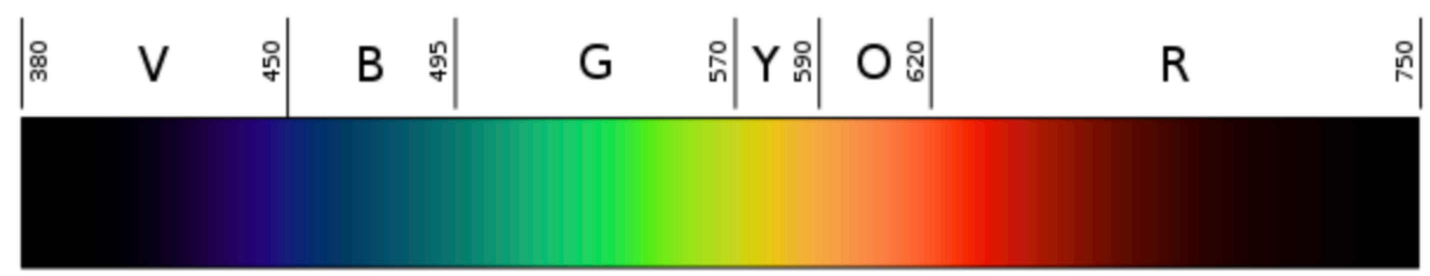
$$E = hf$$

where

- E is energy (J)
- h is Planck's constant: 6.62607015 × 10^{-34} (m^2kgs^{-1})
- f is frequency (Hz)

Additionally,

$$E = \frac{hc}{\lambda}$$

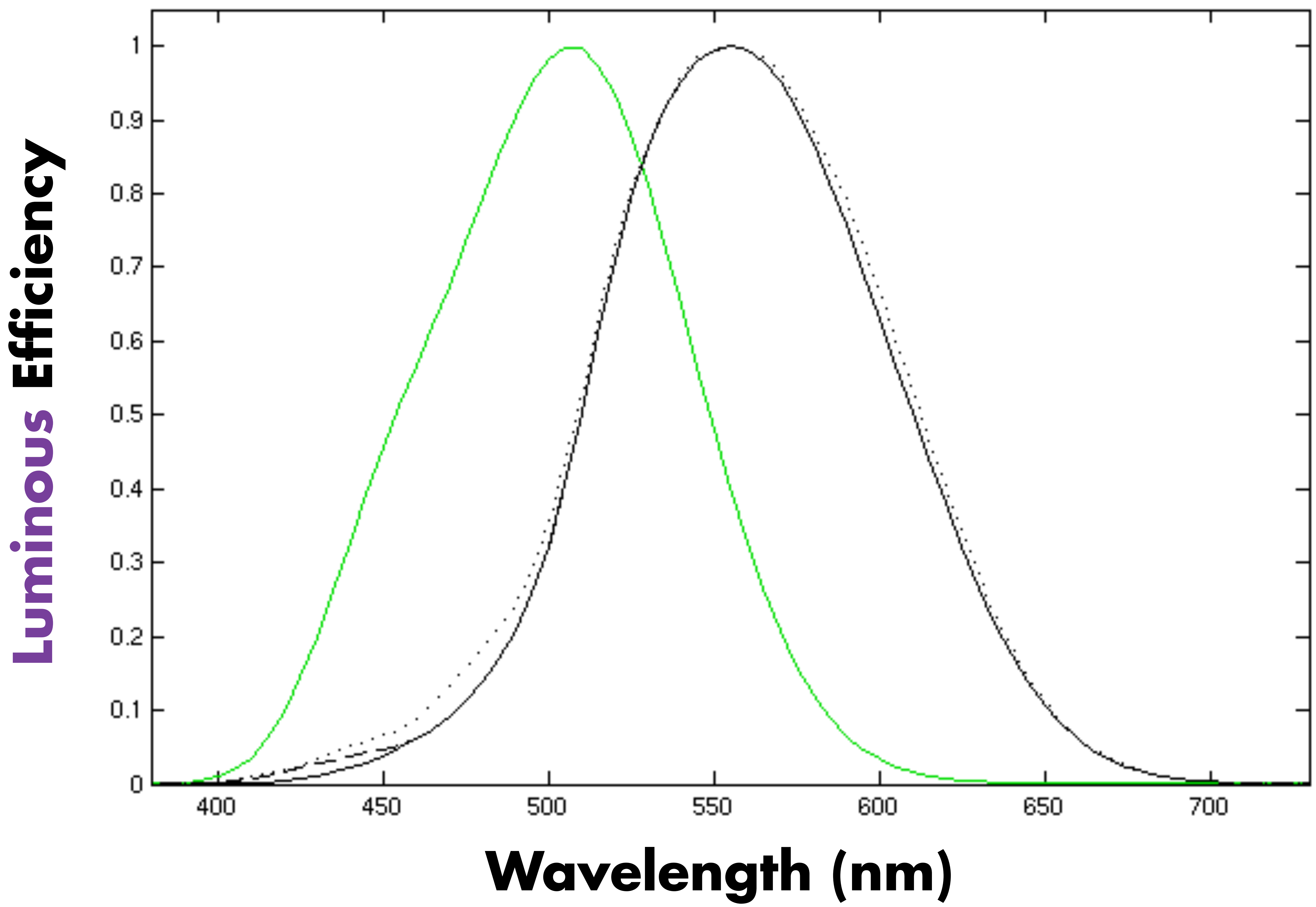


Color	Wavelength (nm)	Frequency (THz)	Photon energy (eV)
violet	380–450	670–790	2.75–3.26
blue	450–485	620–670	2.56–2.75
cyan	485–500	600–620	2.48–2.56
green	500–565	530–600	2.19–2.48
yellow	565–590	510–530	2.10–2.19
orange	590–625	480–510	1.98–2.10
red	625–750	400–480	1.65–1.98

https://en.wikipedia.org/wiki/Photon_energy

https://en.wikipedia.org/wiki/Visible_spectrum

KEY IDEA OF PHOTOMETRY:
Radiant power at each wavelength is weighted by a luminosity function that models human brightness sensitivity



"**Photopic** (**daytime-adapted**, black curve) and **scotopic** [1] (**darkness-adapted**, green curve) luminosity functions. The photopic includes the CIE 1931 standard [2] (solid), the Judd-Vos 1978 modified data [3] (dashed), and the Sharpe, Stockman, Jagla & Jägle 2005 data [4] (dotted). The horizontal axis is wavelength in nm." https://en.wikipedia.org/wiki/Photometry_(optics)

Rendering under low-light conditions

HUMAN

https://en.wikipedia.org/wiki/Scotopic_vision

Rendering under low-light conditions

CAT

https://en.wikipedia.org/wiki/Scotopic_vision

Radiant Energy and Power

Power (Φ) units:

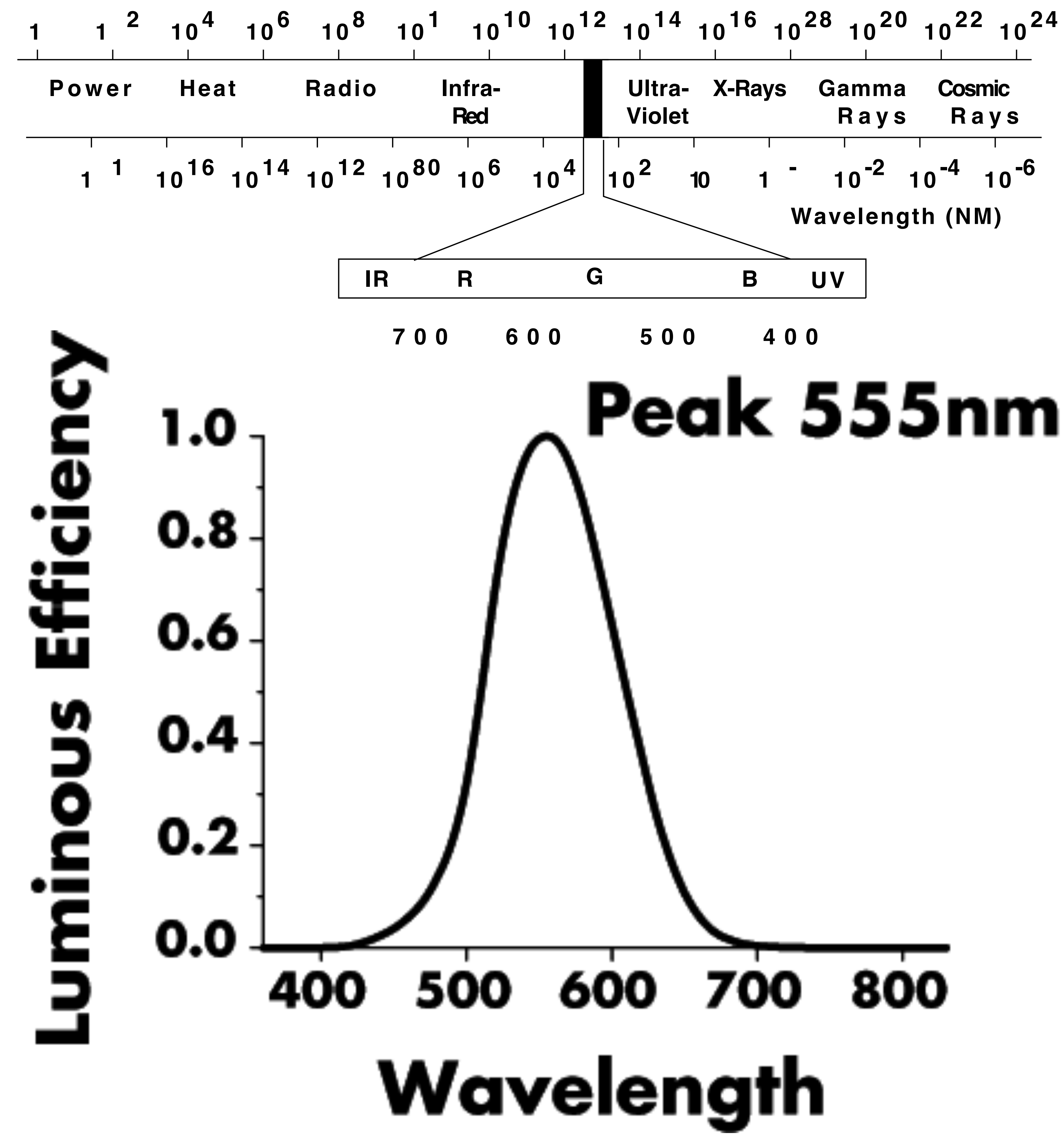
- Watts (radiometry)
- **Lumens** (**photometry**)

Related concepts:

- Spectral efficacy
- Energy efficiency

Energy: Joules vs. Talbot

- Exposure
  - Film response
  - Skin - sunburn



Photometric luminance:

$$Y = \int V(\lambda) L(\lambda) d\lambda$$

Conversion factor: 683 lumens/Watt @ 550nm

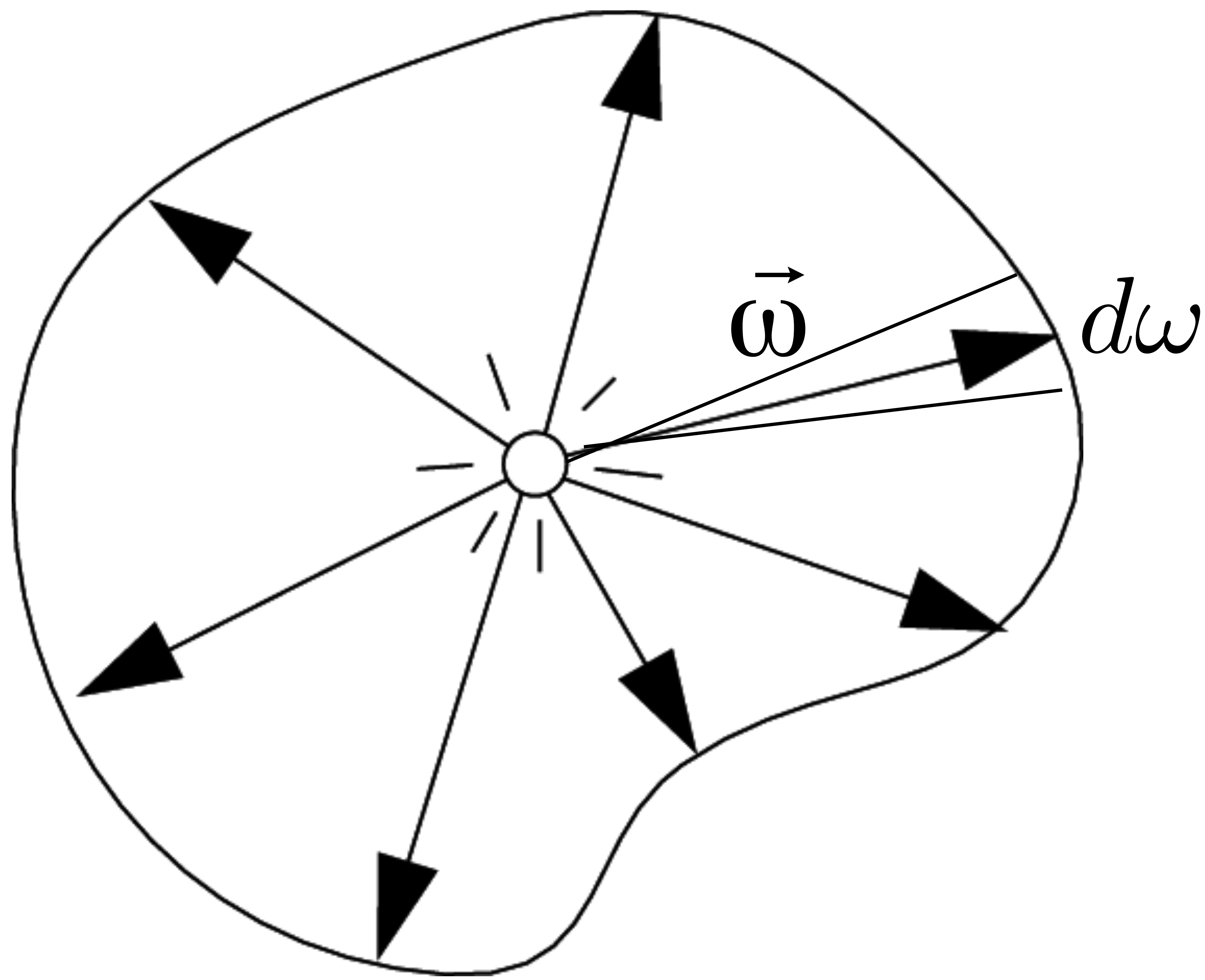
Radiant Intensity

Radiant Intensity

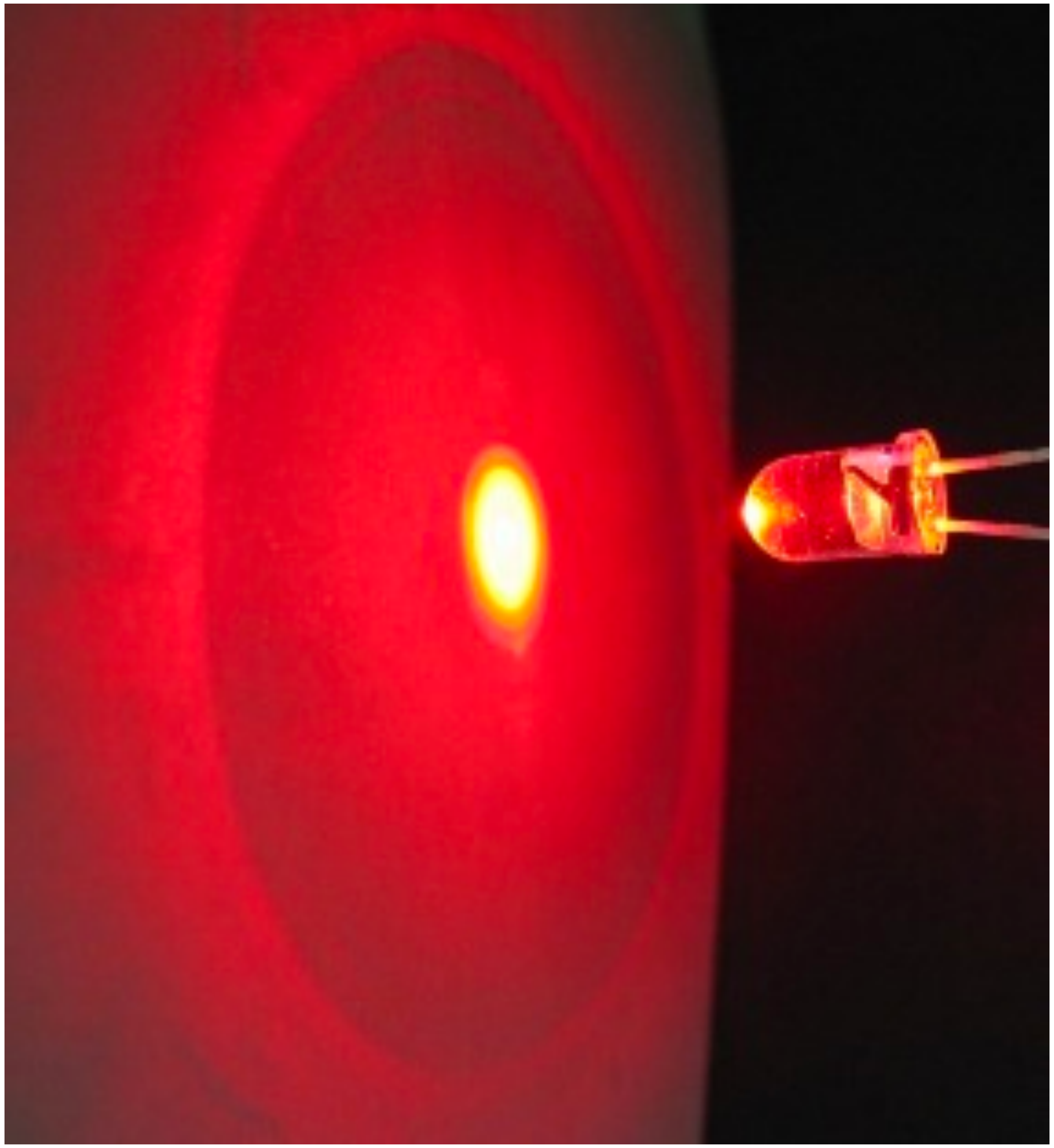
Definition: The *radiant* (luminous) *intensity*, I, is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

$$\left[\frac{W}{sr}\right]\left[\frac{lm}{sr} = cd = candela\right]$$



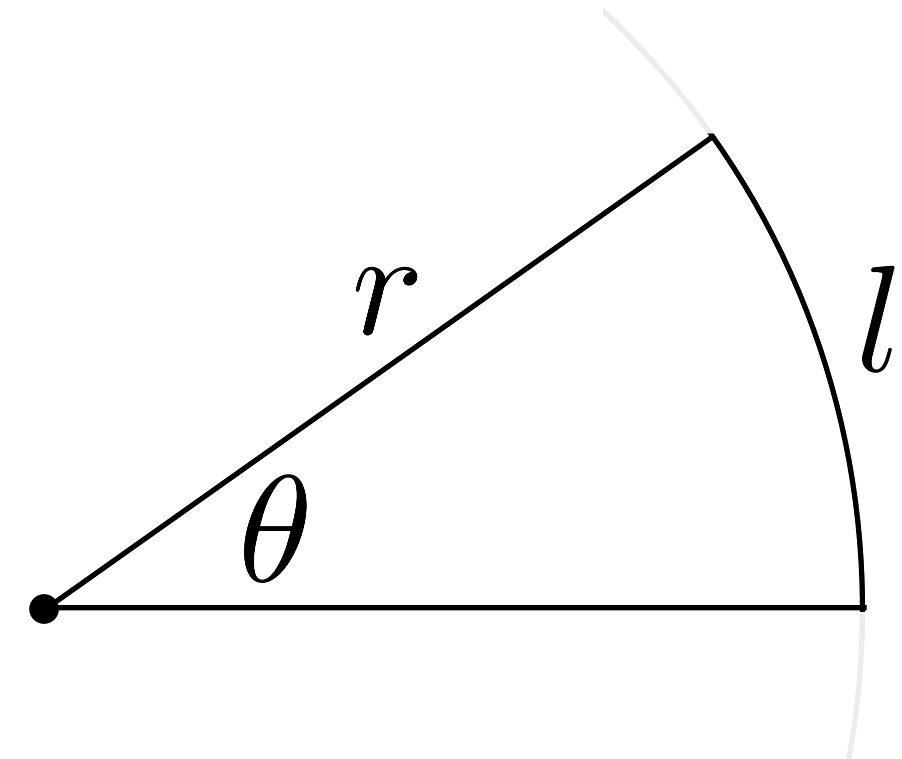
Radiant Intensity



https://www.jensign.com/LEDIntensity

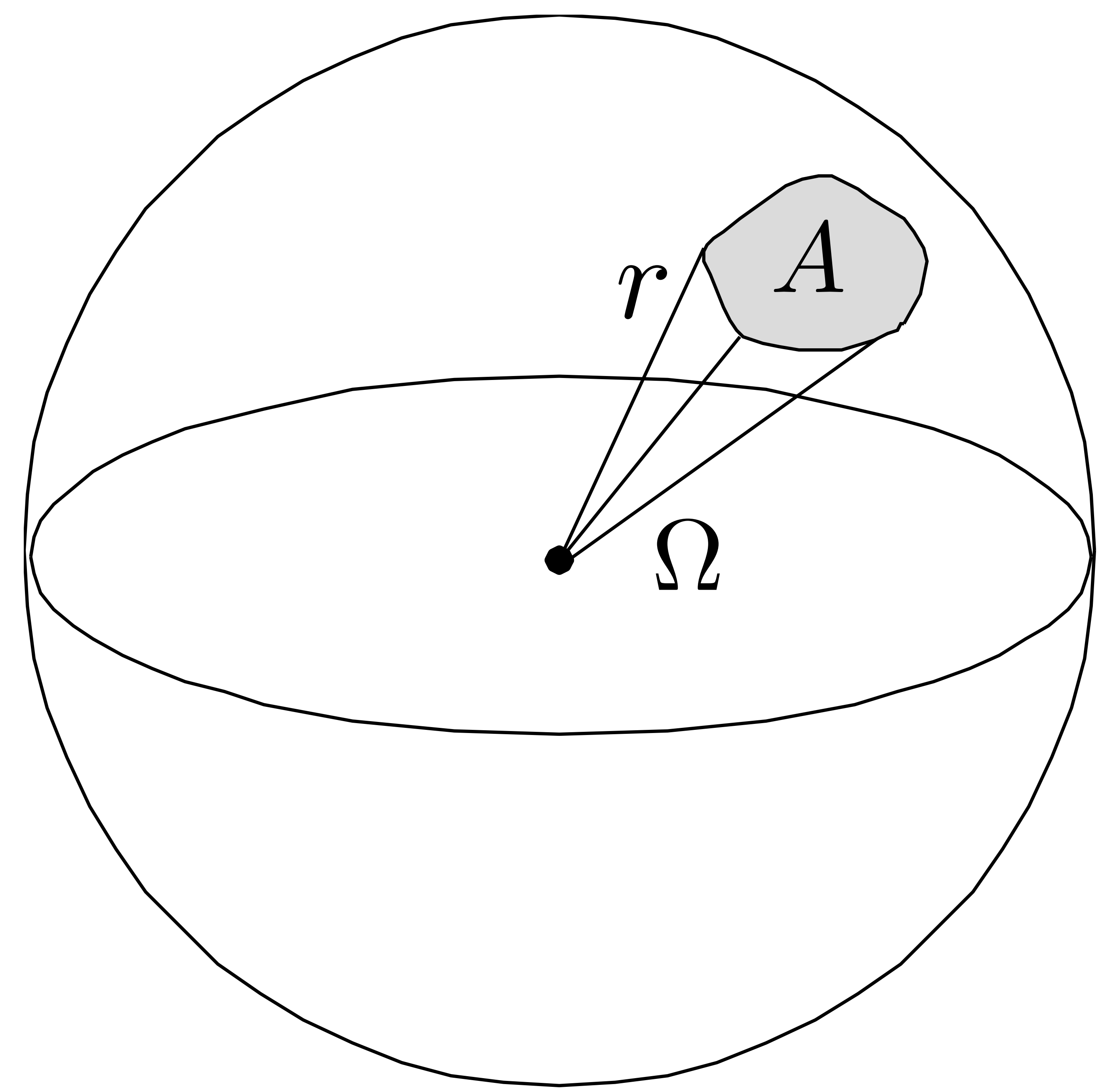
Angles and Solid Angles

Angle $$\theta = \frac{l}{r}$$



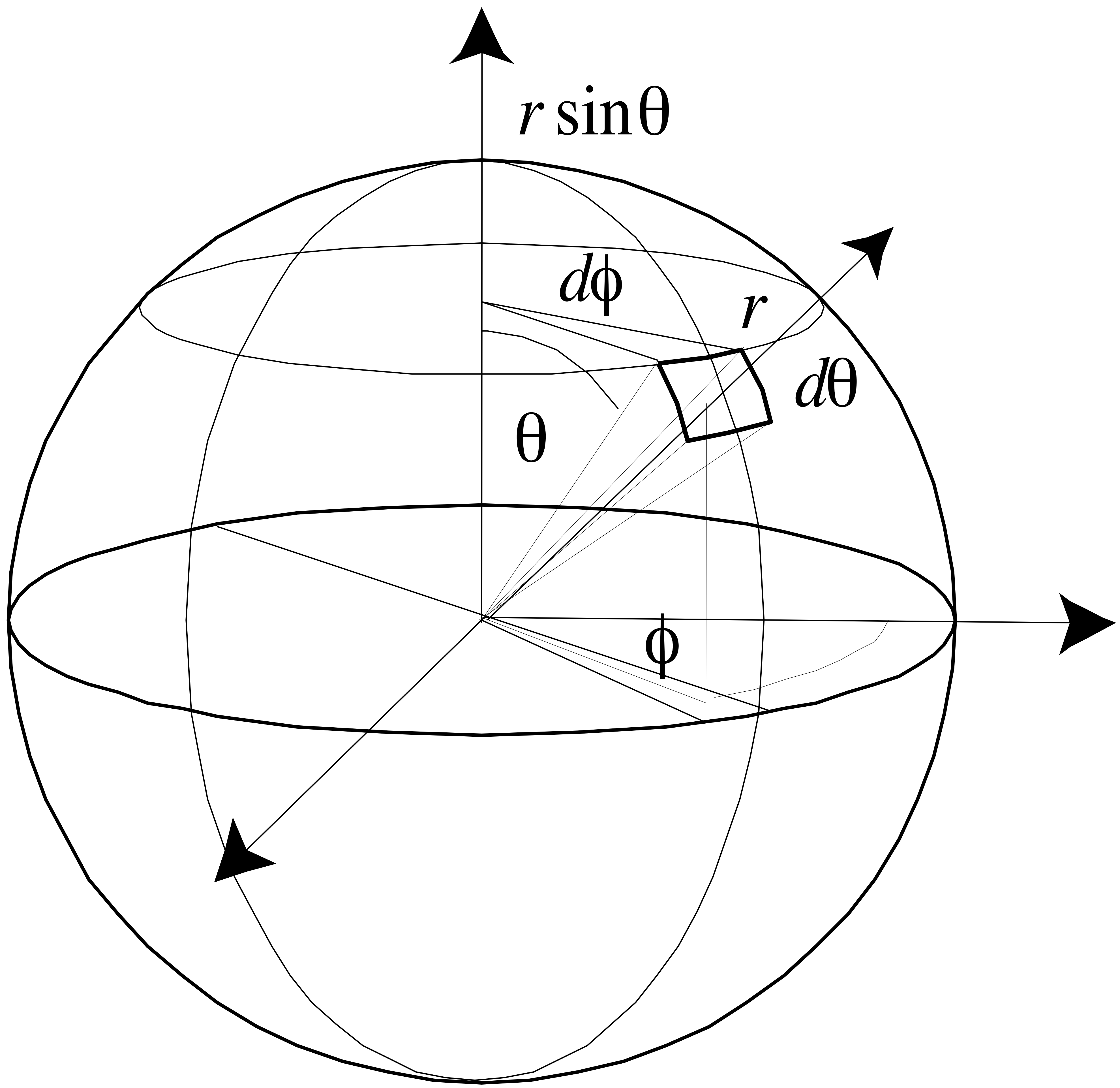
⇒ circle has 2 π radians

Solid angle $$\Omega = \frac{A}{r^2}$$



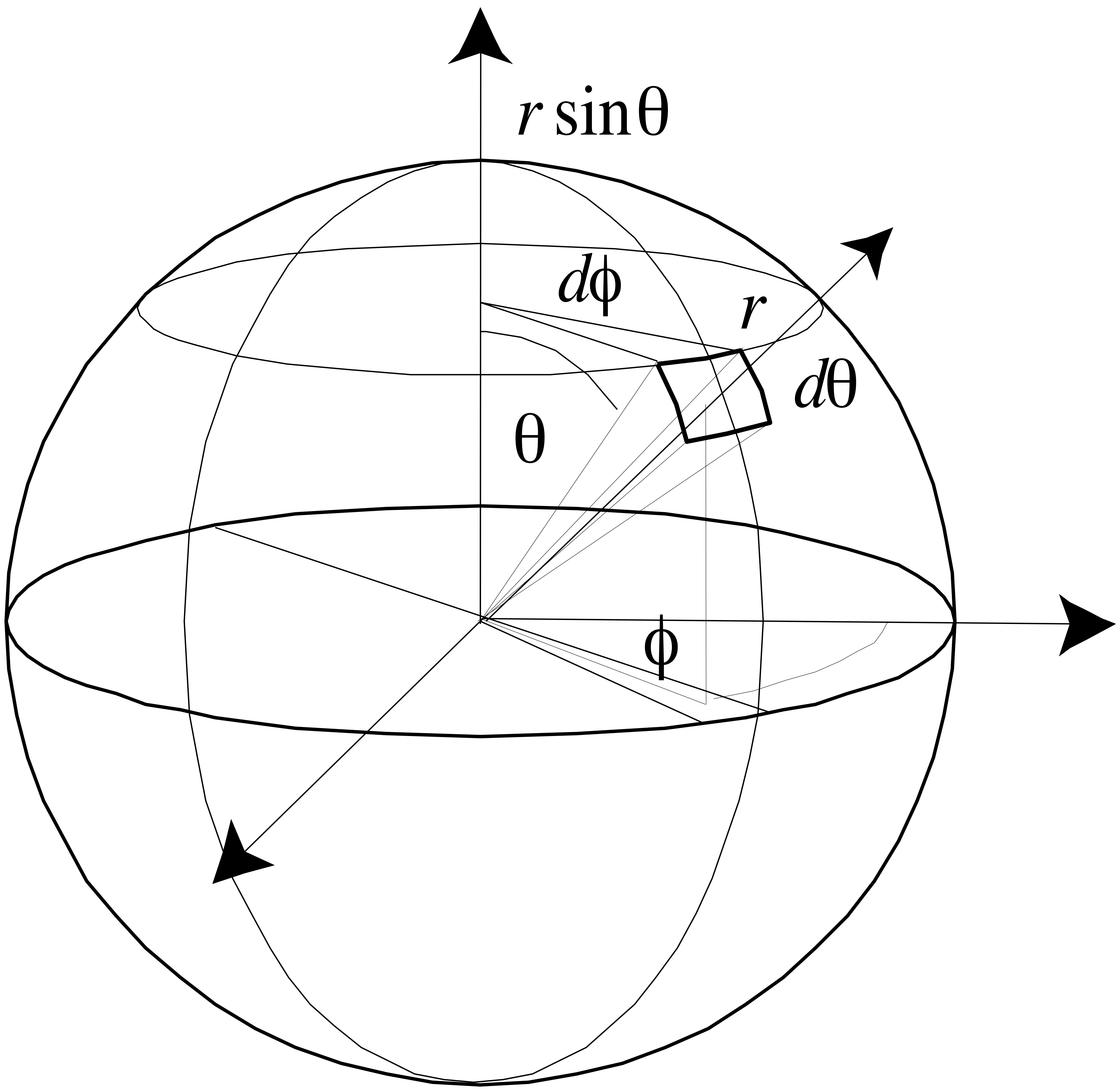
⇒ sphere has 4 π steradians

Differential Solid Angles



$$dA = (r\, d\theta)(r \sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

Differential Solid Angles

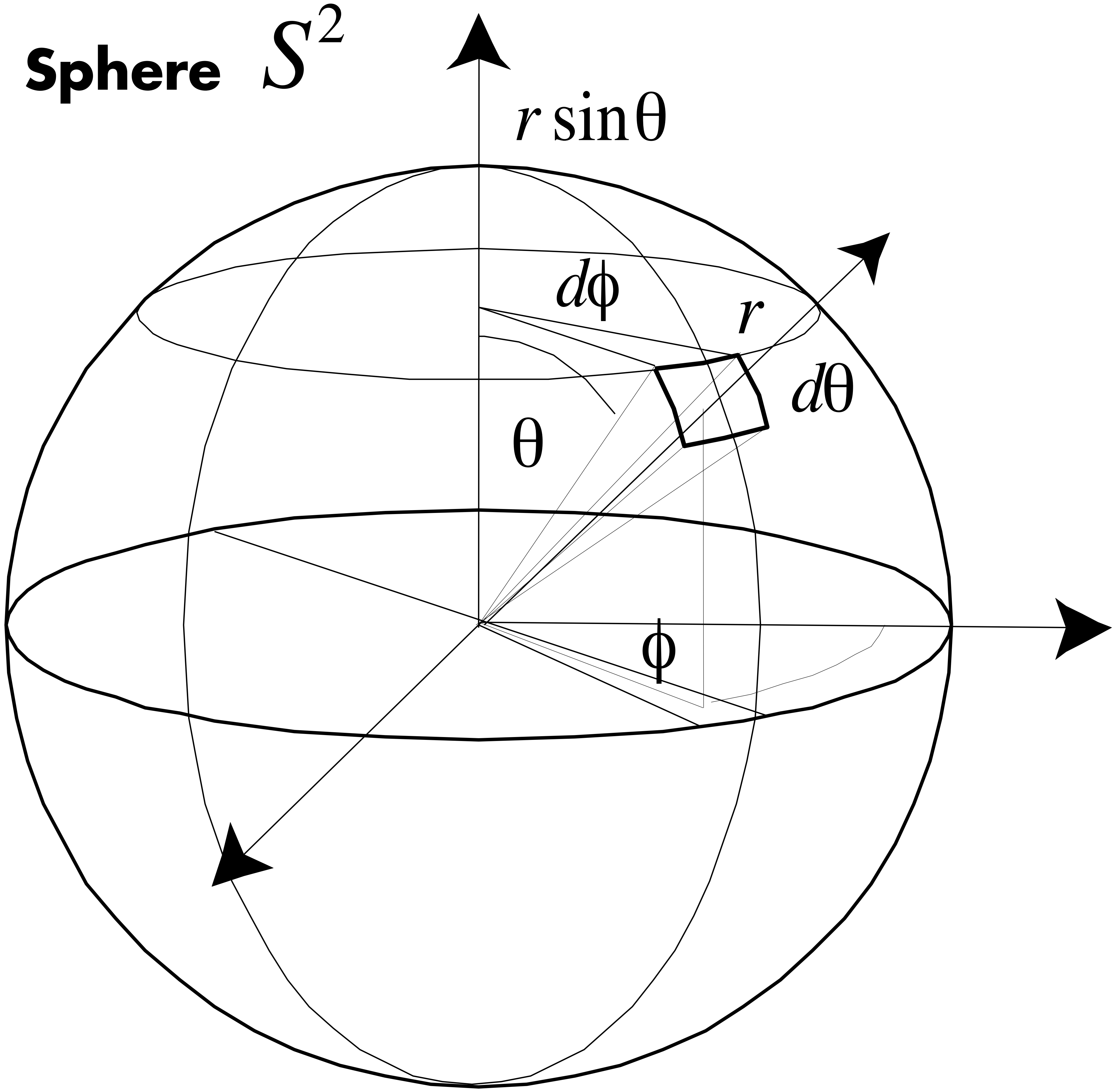


$$dA = (r\, d\theta)(r \sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

Differential Solid Angles

Sphere S^2



$$d\omega = \sin\theta \, d\theta \, d\phi$$

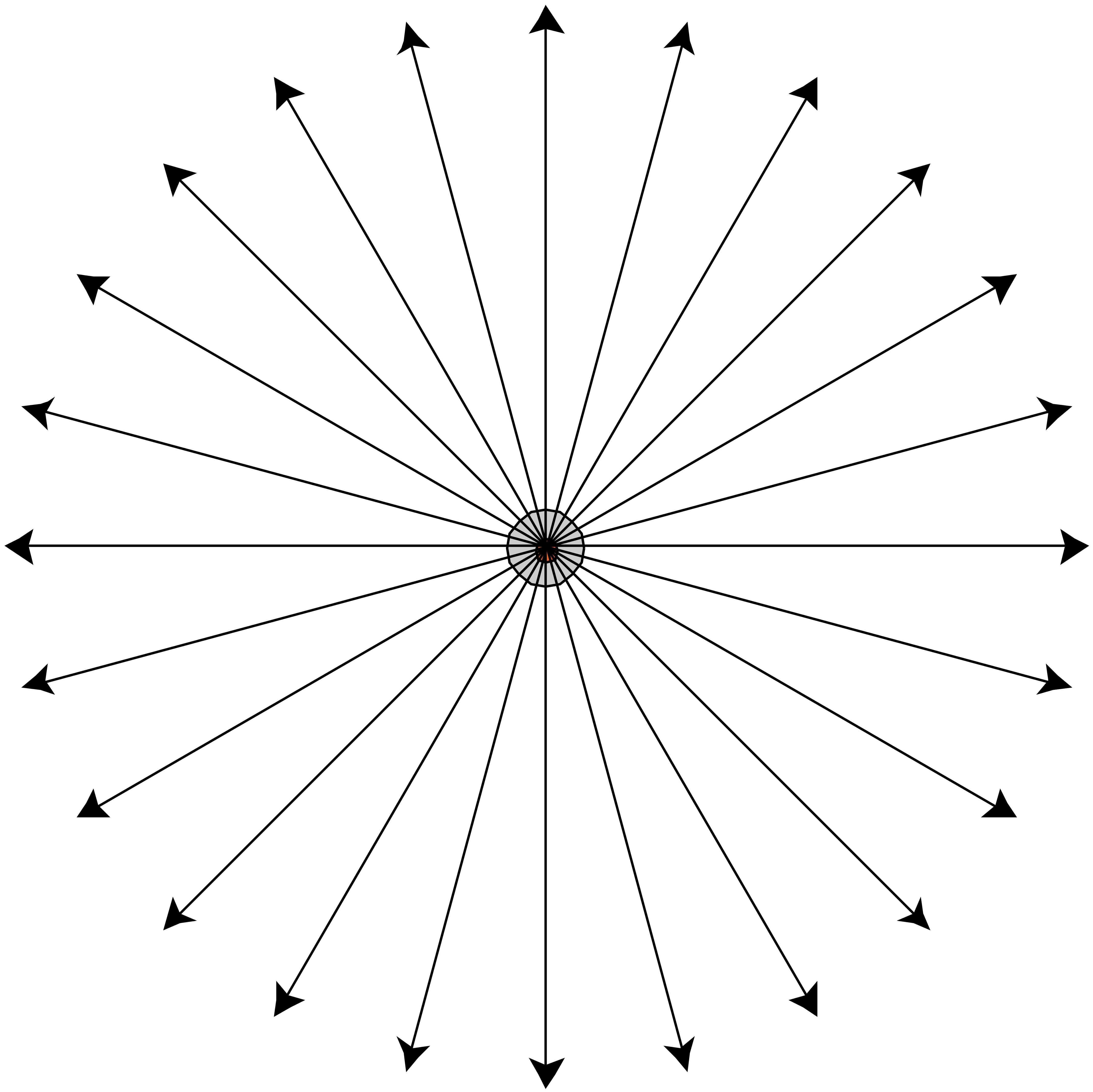
$$\Omega = \int_{S^2} d\omega$$

$$= \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi$$

$$= \int_{-1}^{1} \int_0^{2\pi} d\cos\theta \; d\phi$$

$$= 4\pi$$

Isotropic Point Source

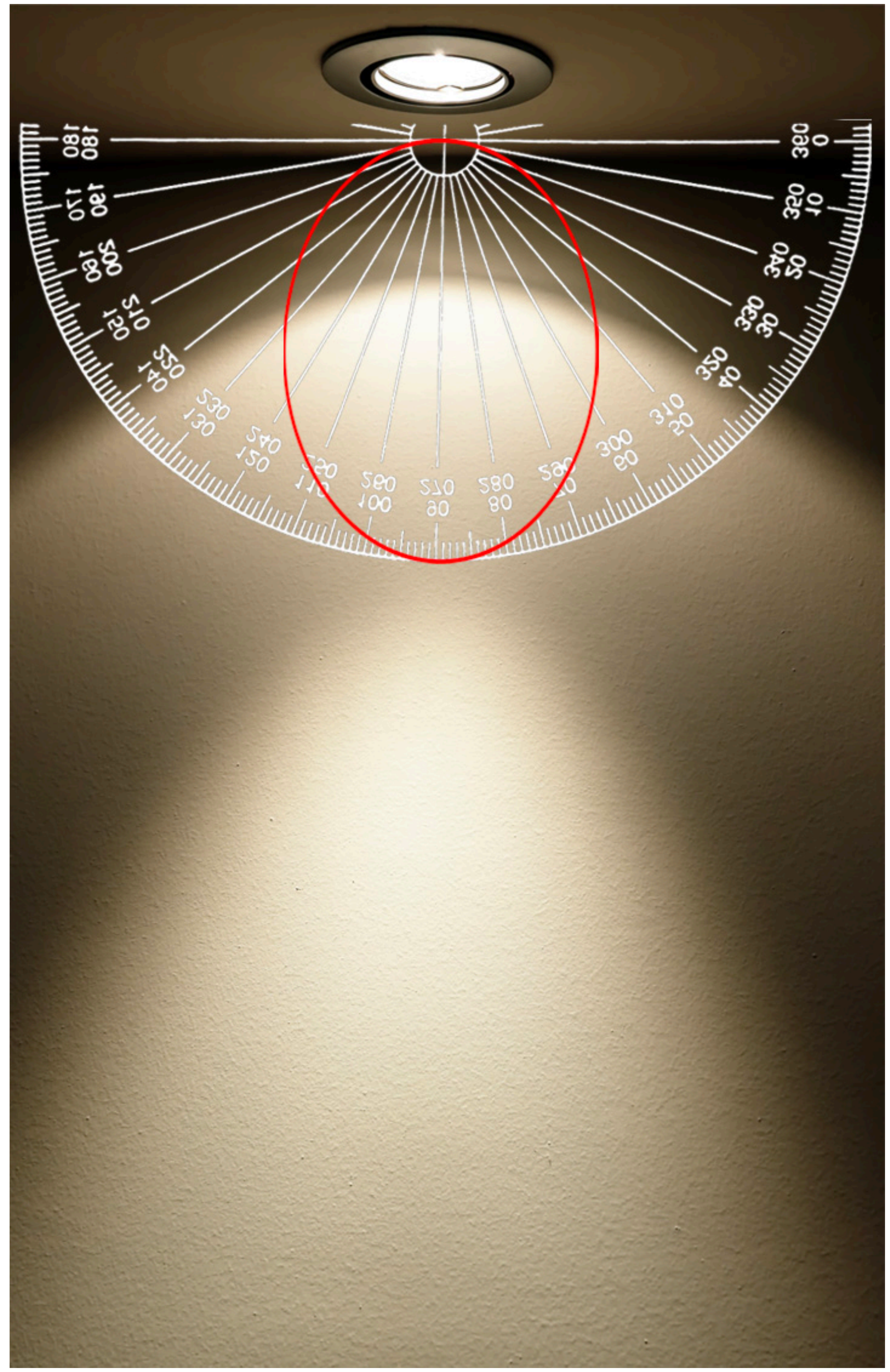


$$\Phi = \int_{S^2} I \, d\omega$$

$$= 4\pi I$$

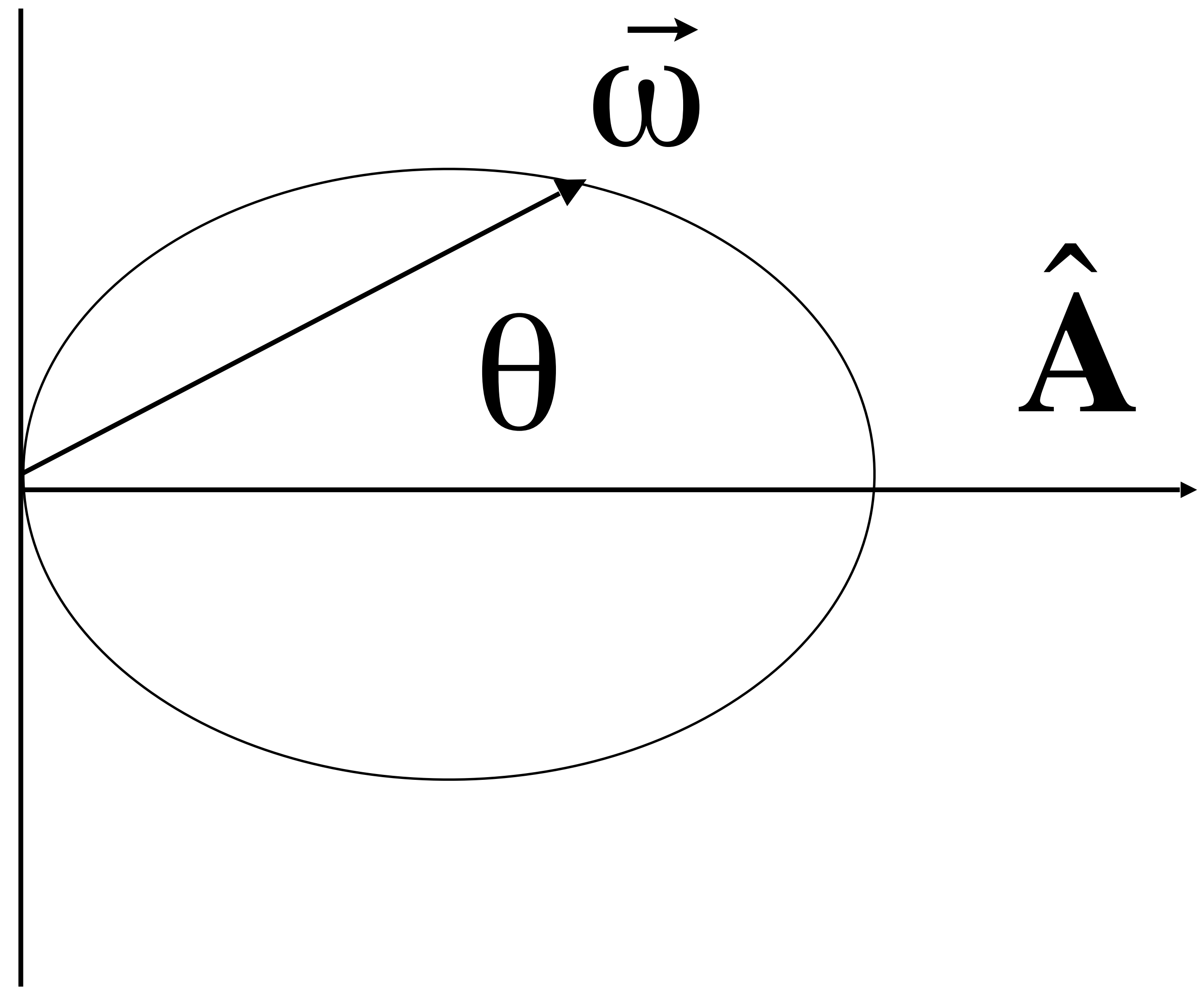
$$I = \frac{\Phi}{4\pi}$$

Anisotropic Point Sources



http://www.photometrictesting.co.uk/File/understanding_photometric_data_files.php

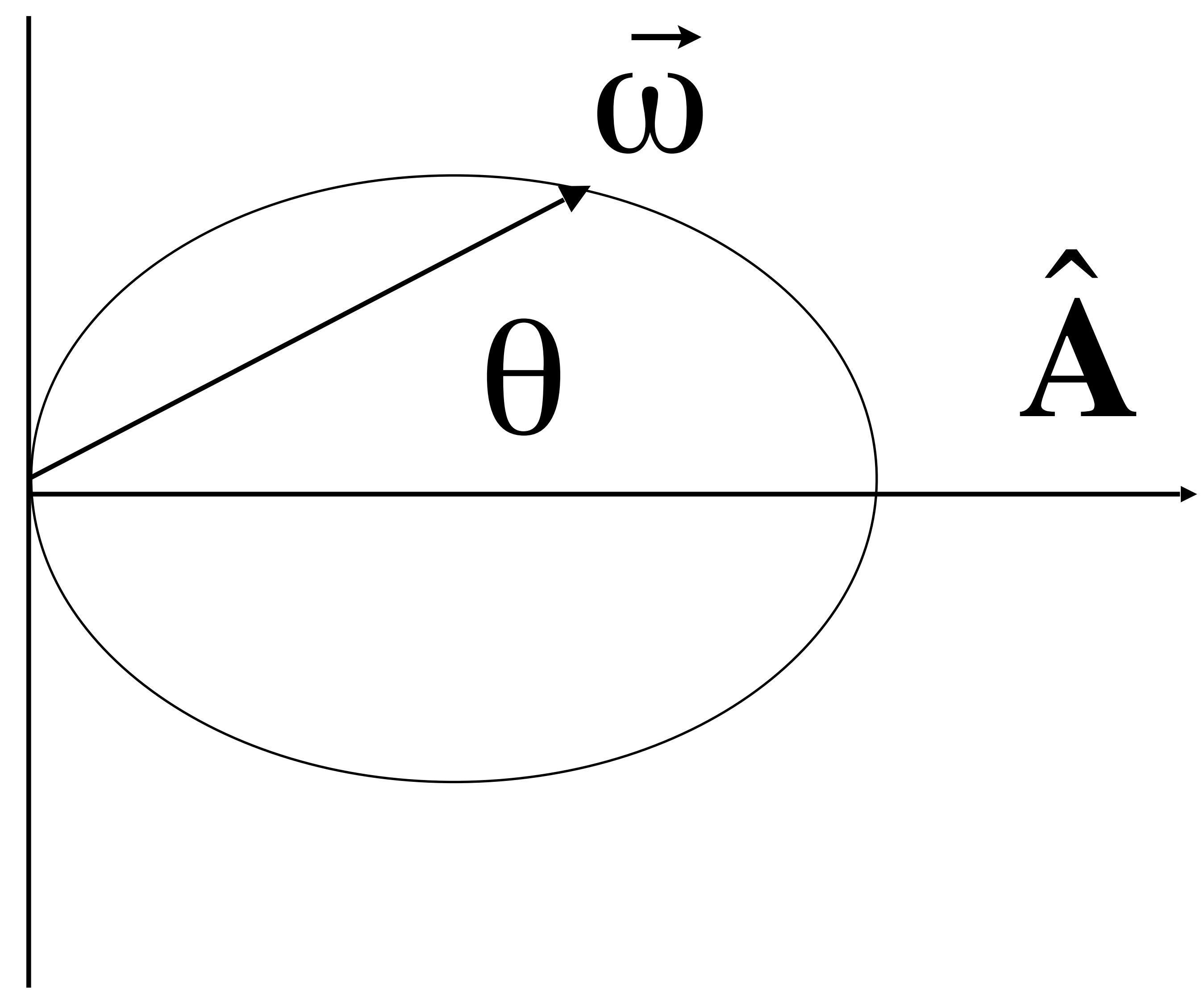
Warn's Spotlight



$$I(\omega) = \cos^s \theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

where $\theta \in [0, \frac{\pi}{2}]$

Warn's Spotlight



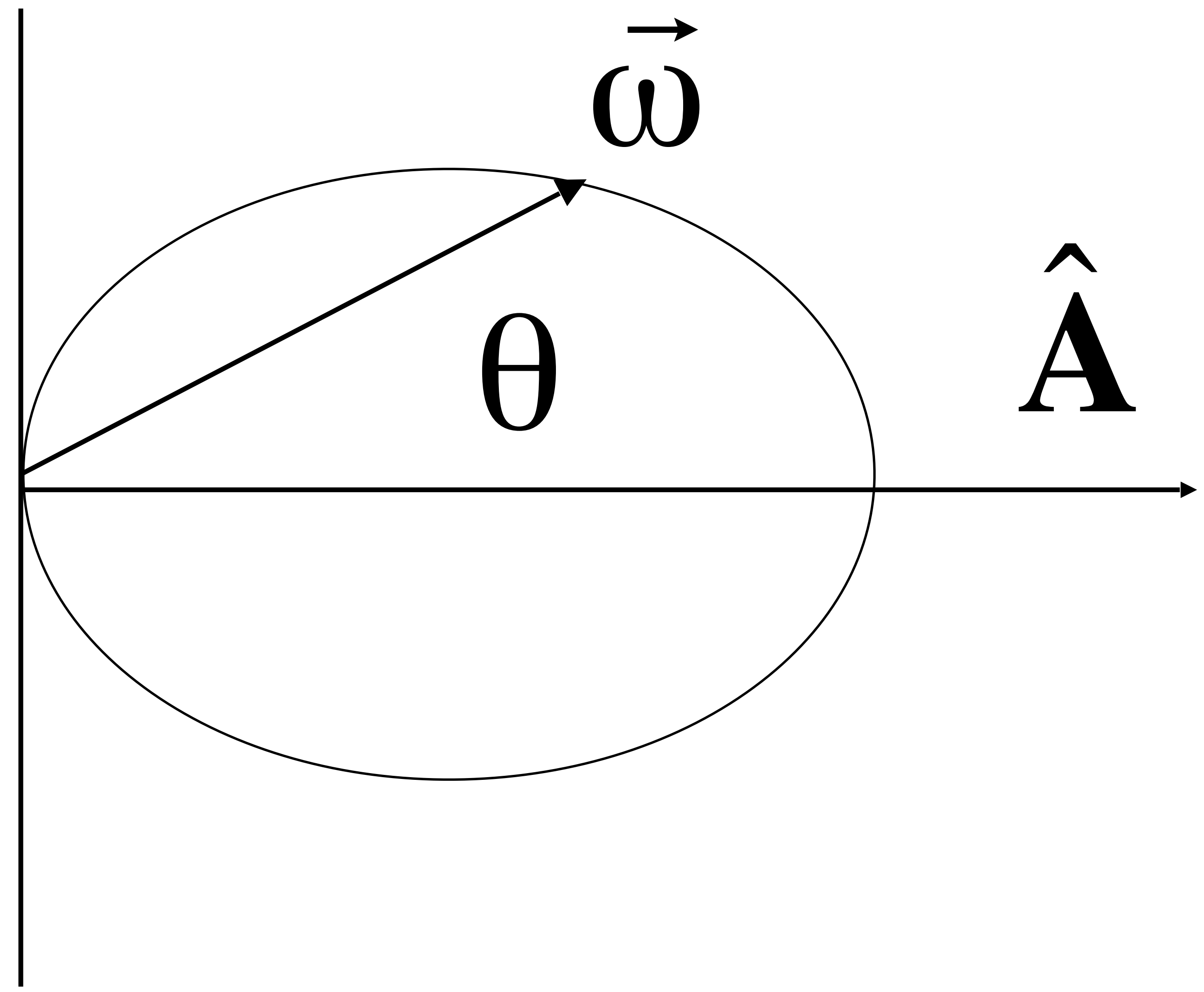
$$I(\omega) = \cos^s \theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

where $\theta \in [0, \frac{\pi}{2}]$

$$\Phi = \int_0^{2\pi} \int_0^1 I(\omega)\, d\cos\theta \, d\varphi$$

Note: $d\cos\theta = -\sin\theta\, d\theta$

Warn's Spotlight

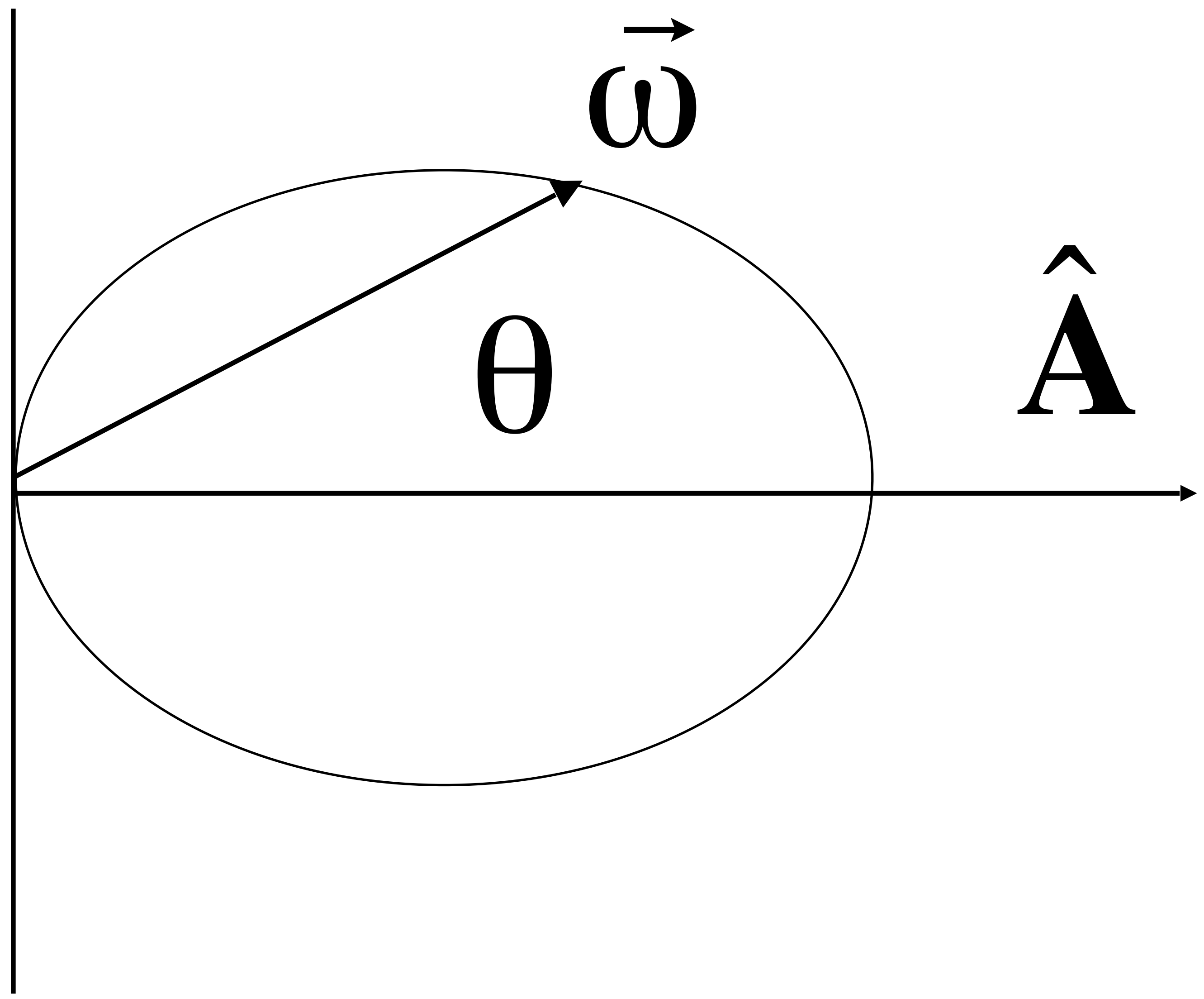


$$I(\omega) = \cos^s\theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

where $\theta \in [0, \frac{\pi}{2}]$

$$\Phi = \int_0^{2\pi}\int_0^1 I(\omega)\, d\cos\theta\, d\varphi = 2\pi\int_0^1 \cos^s\theta\, d\cos\theta = \frac{2\pi}{s+1}$$

Warn's Spotlight



$$I(\omega) = \cos^s \theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

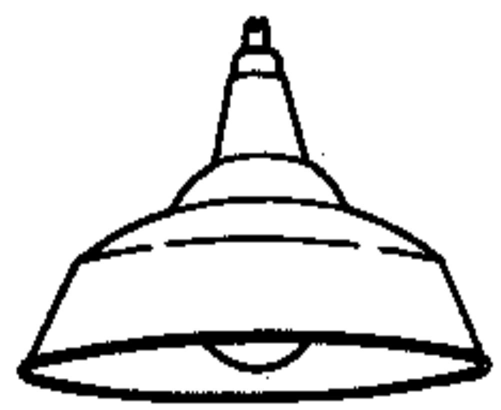
where $\theta \in [0, \frac{\pi}{2}]$

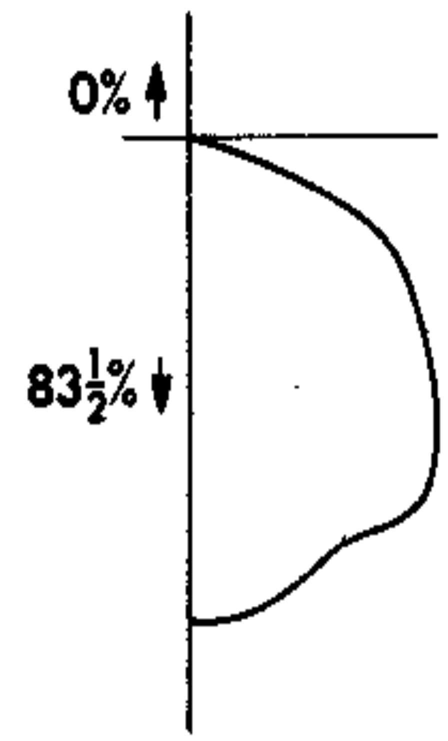
$$\Phi = \int_0^{2\pi} \int_0^1 I(\omega)\, d\cos\theta\, d\varphi = 2\pi \int_0^1 \cos^s \theta\, d\cos\theta = \frac{2\pi}{s+1}$$

$$I(\omega) = \Phi \frac{s+1}{2\pi} \cos^s \theta$$

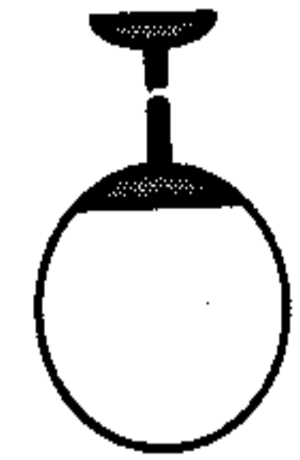
Light Source Goniometric Diagrams

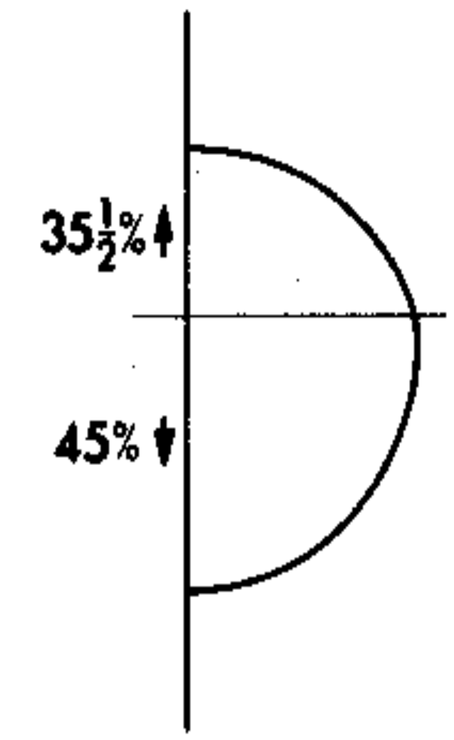
3





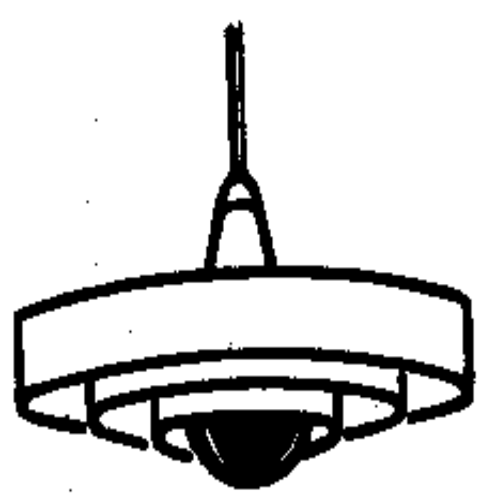
Porcelain-enameled ventilated standard dome with incandescent lamp

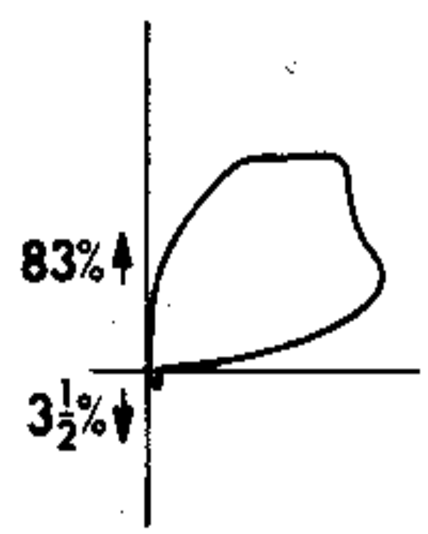




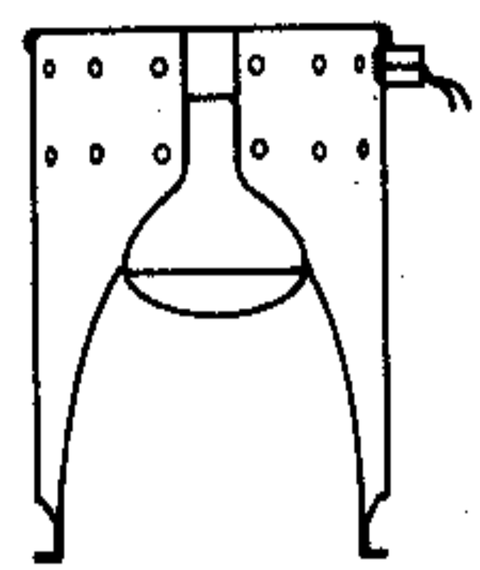
Pendant diffusing sphere with incandescent lamp

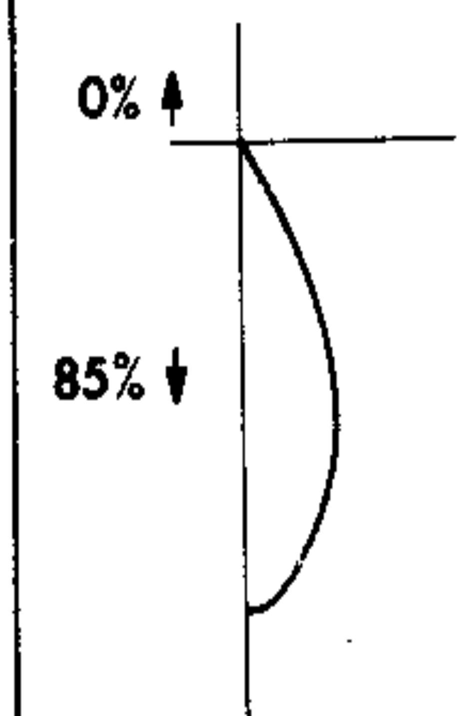
2





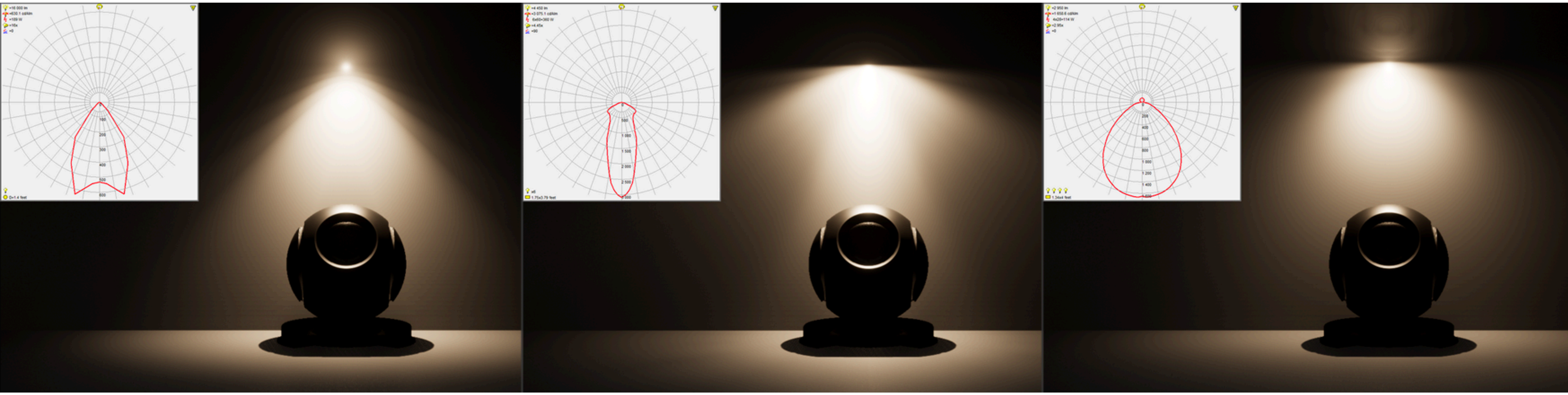
Concentric ring unit with incandescent silvered-bowl lamp





R-40 flood with specular anodized reflector skirt; 45° cutoff

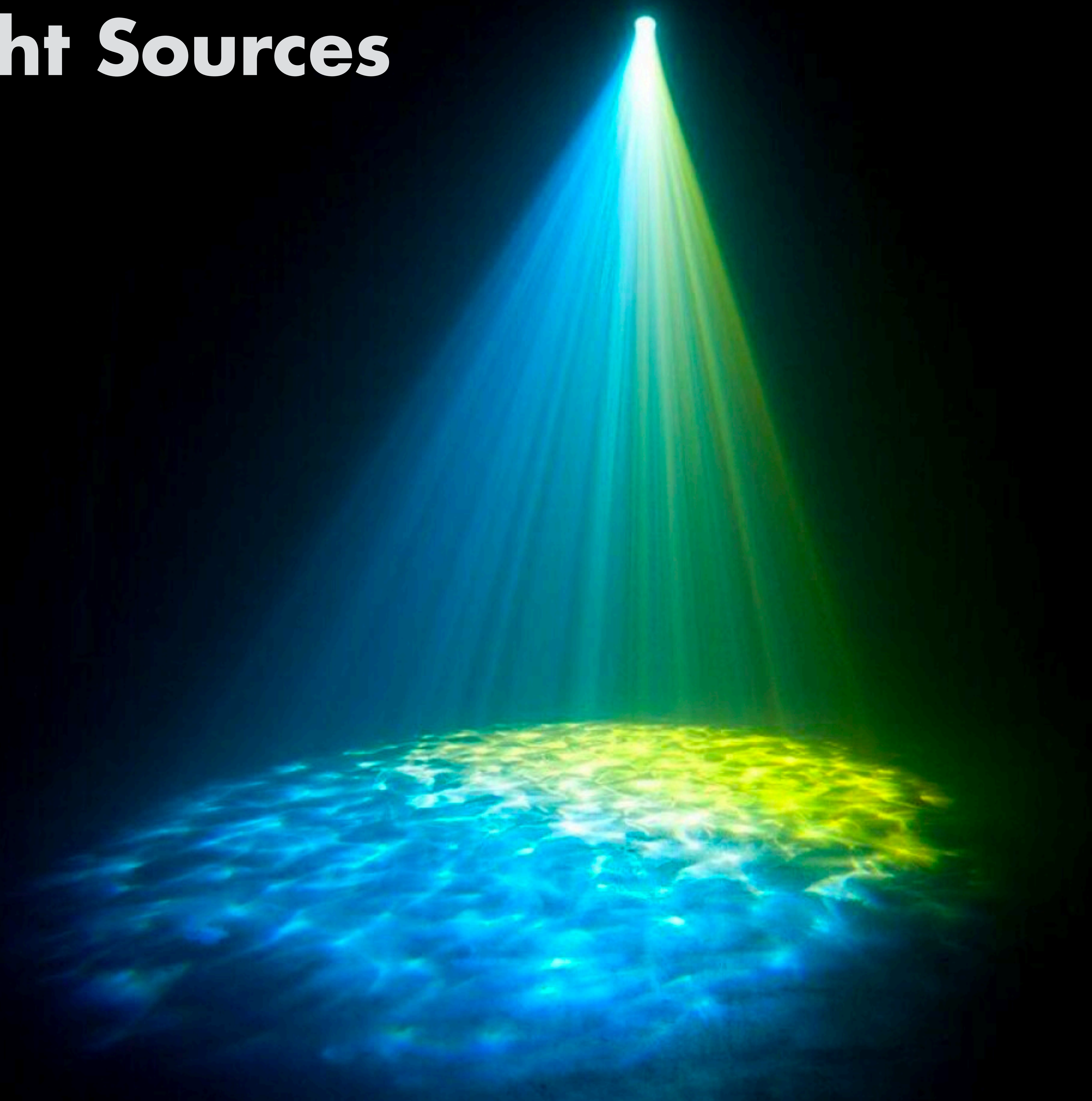
IES Light Profiles



https://docs.unrealengine.com/4.26/en-US/BuildingWorlds/LightingAndShadows/IESLightProfiles

The Illuminating Engineering Society (IES) has defined a file format which describes a light's distribution from a light source using real world measured data. These IES Photometric files, or IES Profiles, are a lighting industry standard method of diagramming the brightness and falloff of light as it exists a particular real world light fixture. It enables them to account for reflective surfaces in the light fixture, the shape of the light bulb, and any lensing effects that happens. This type of photometric lighting is primarily used in Enterprise fields (such as Media and Entertainment or Architecture and Manufacturing), but is often used in games production to achieve realistic lighting effects, too.

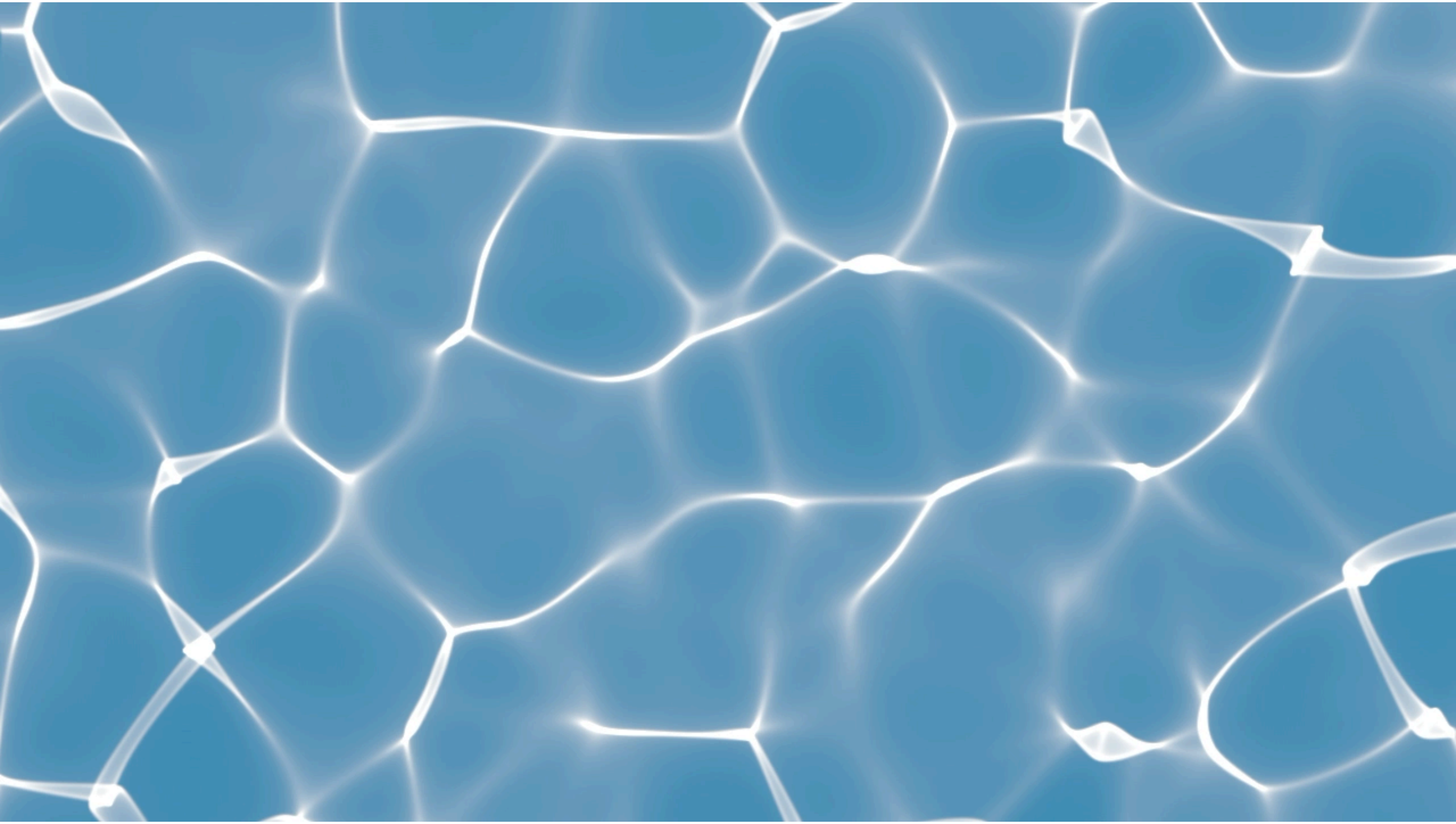
Complex Light Sources



https://specialneedstoys.com/usa/h20-projector.html

Irradiance

Beautiful Irradiance: Caustics



Irradiance

Definition: The *irradiance* (*illuminance*) is the power per unit area *incident* on a surface.

$$E(x) \equiv \frac{d\Phi_i}{dA}$$

$$\left[\frac{W}{m^2}\right] \quad \left[\frac{lm}{m^2} = lux\right]$$

Also referred to as the *radiant* (*luminous*) *incidence*.

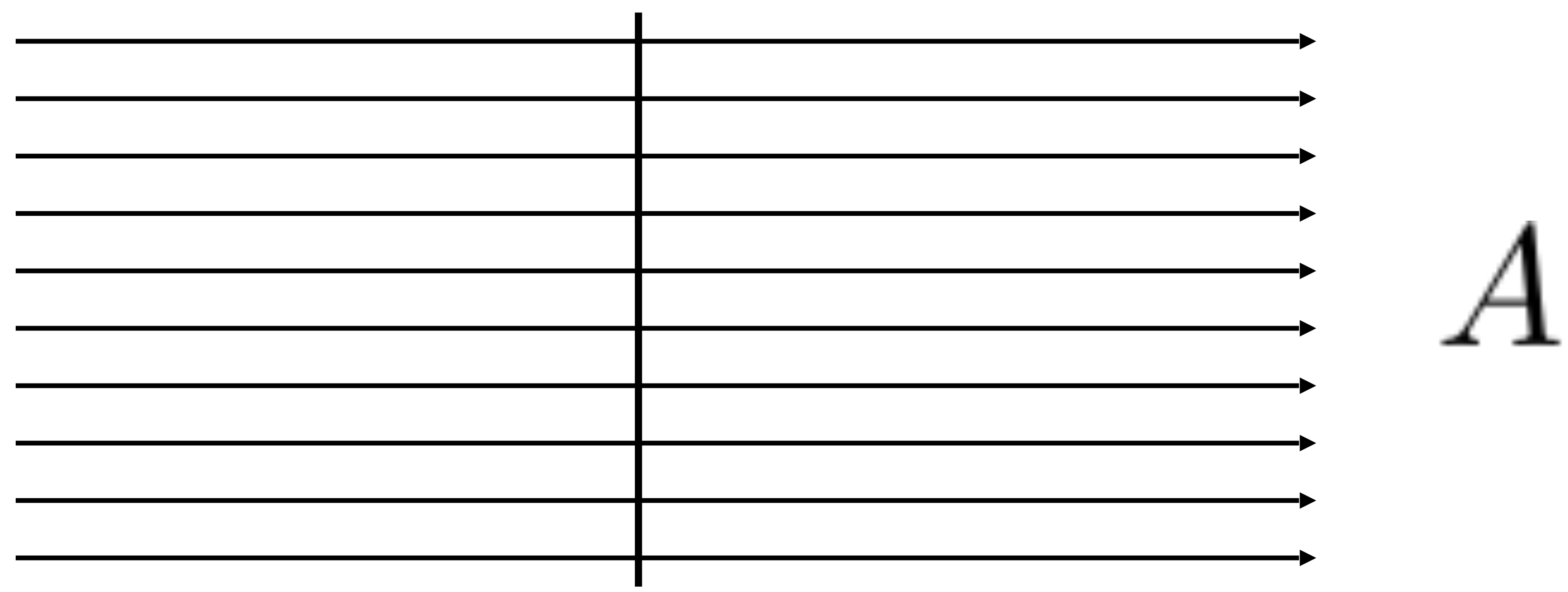
Typical Values of Illuminance [lm/m^2]

Sunlight plus skylight	**100,000 lux**
Sunlight plus skylight (overcast)	**10,000**
Interior near window (daylight)	**1,000**
Artificial light (minimum)	**100**
Moonlight (full)	**0.02**
Starlight	**0.0003**

Beam Power in Terms of Irradiance

$$\Phi = EA$$

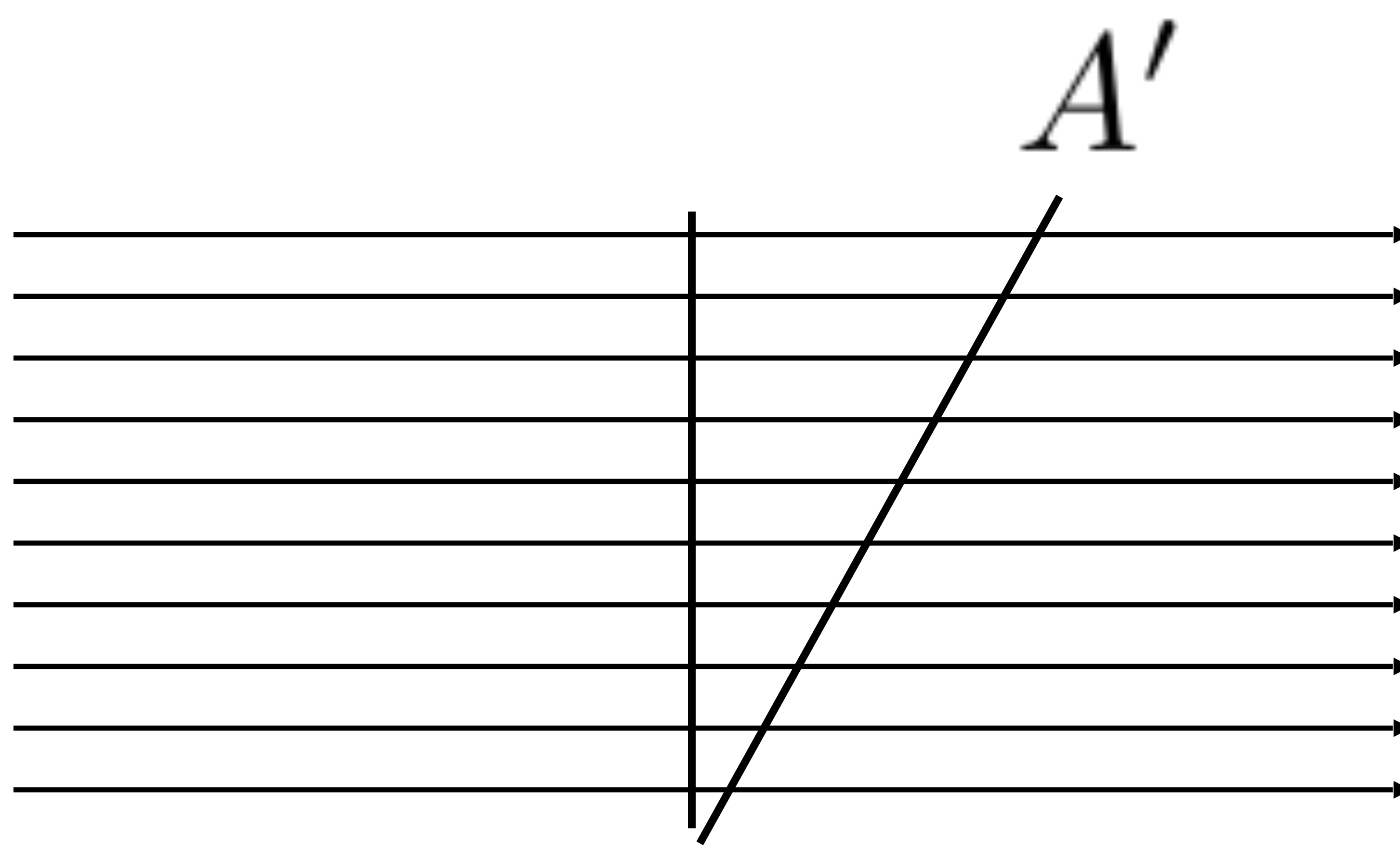
$$E = \frac{\Phi}{A}$$



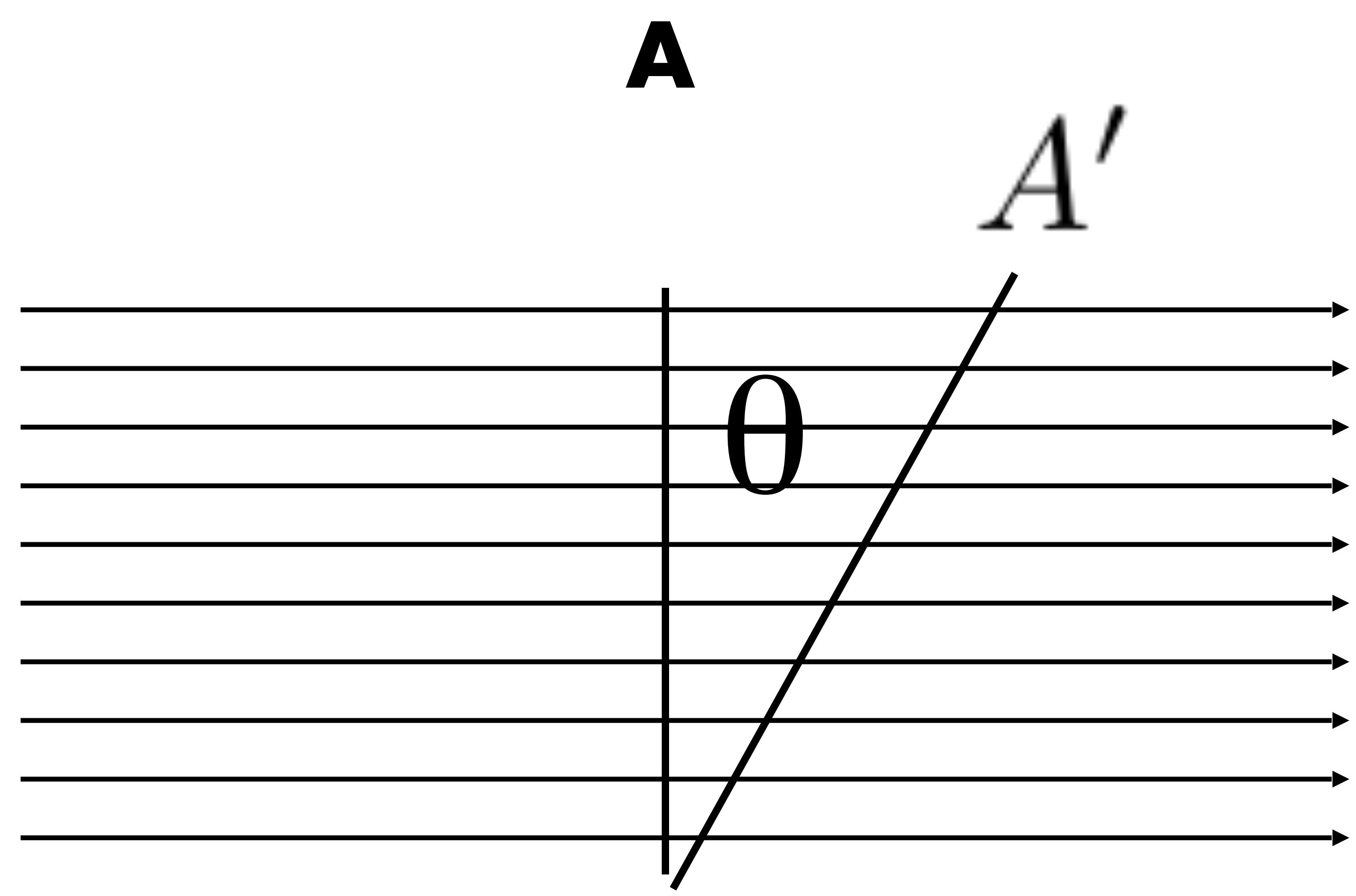
Beam Power Falling on the Surface

$$\Phi' = E' A'$$

$$E' = \frac{\Phi'}{A'}$$



Projected Area



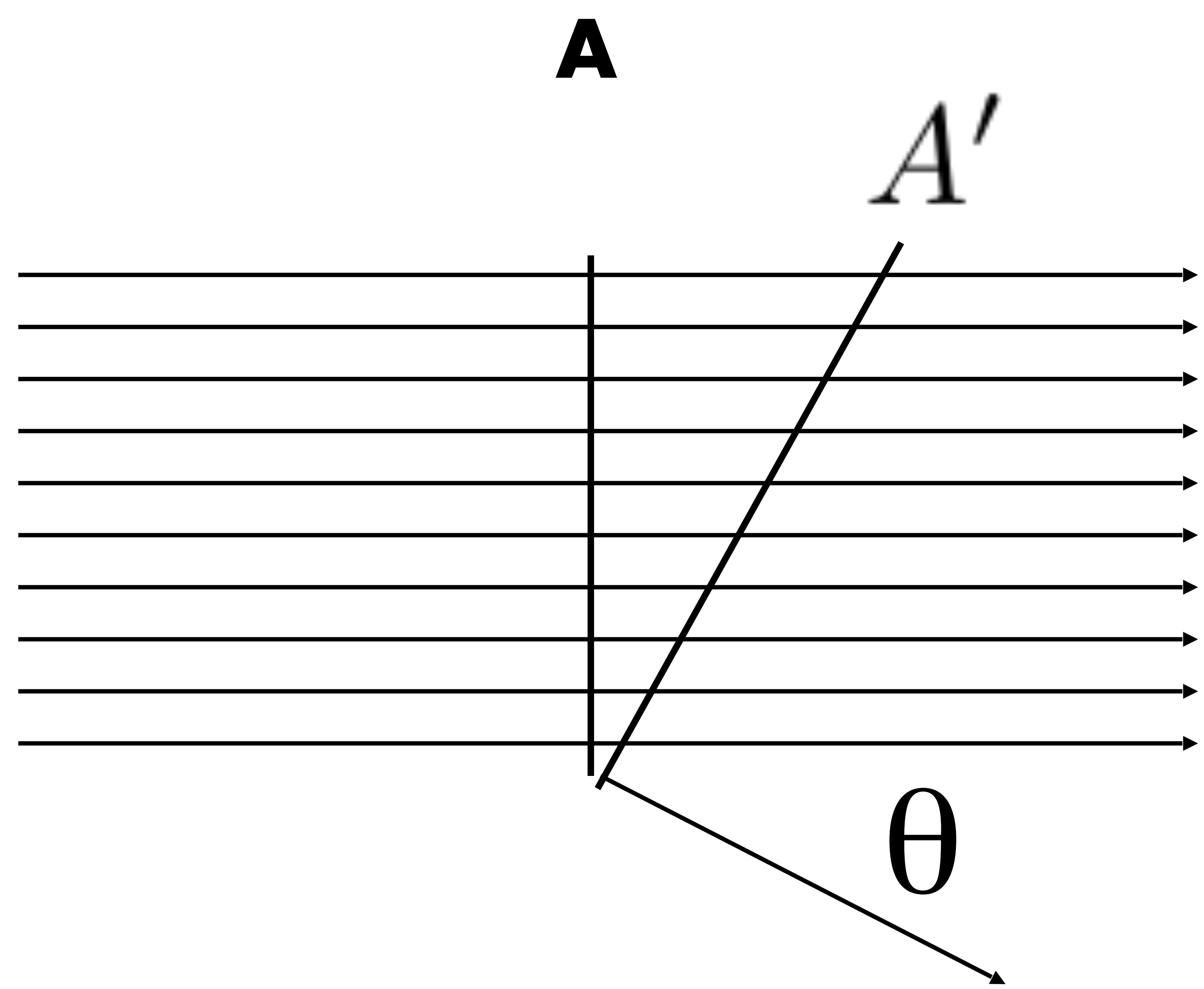
$$A = A' \cos\theta$$

Project *A'* onto the surface perpendicular to the direction

Lambert's Cosine Law

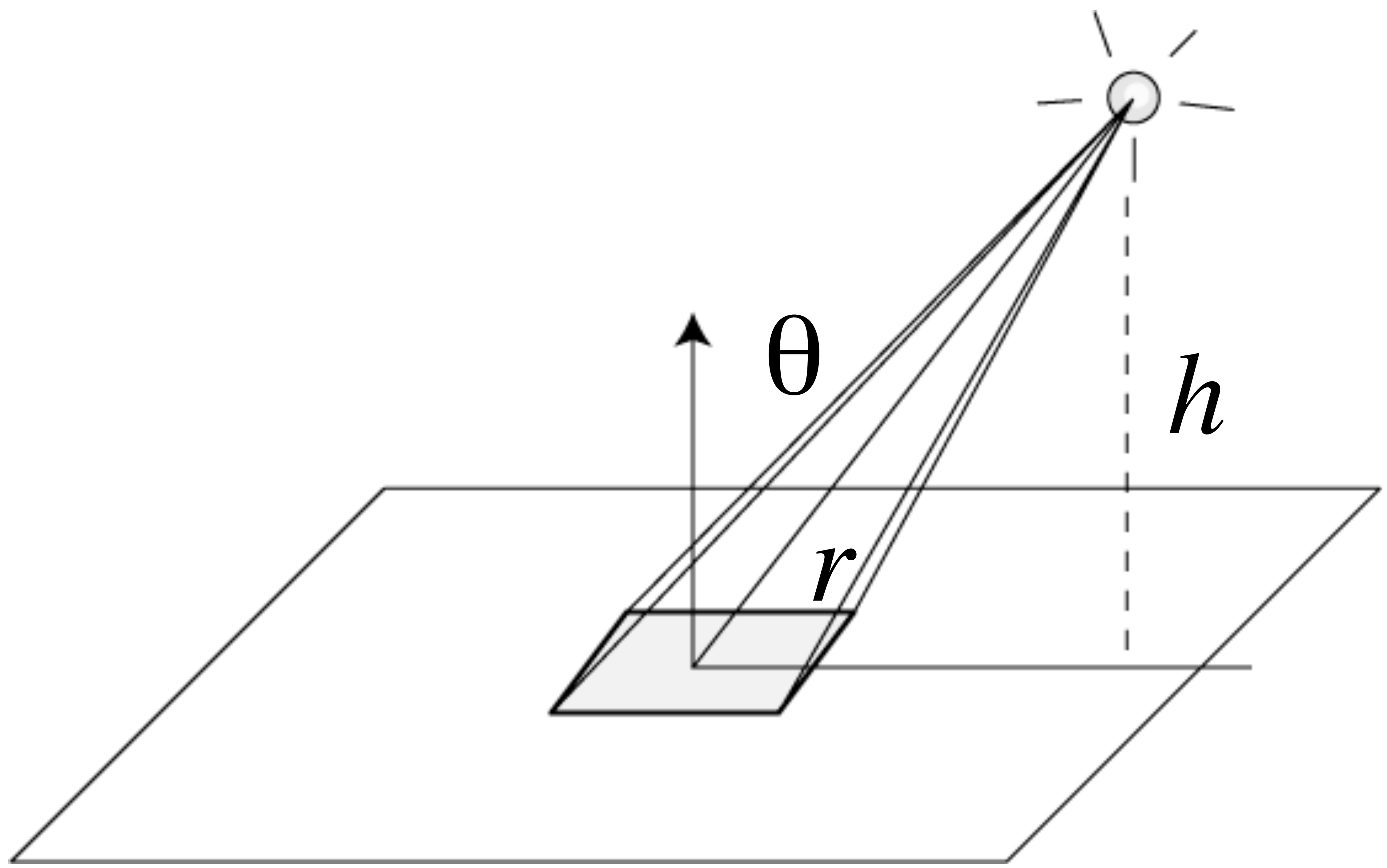
$$A = A' \cos\theta$$

$$\Phi = \Phi'$$



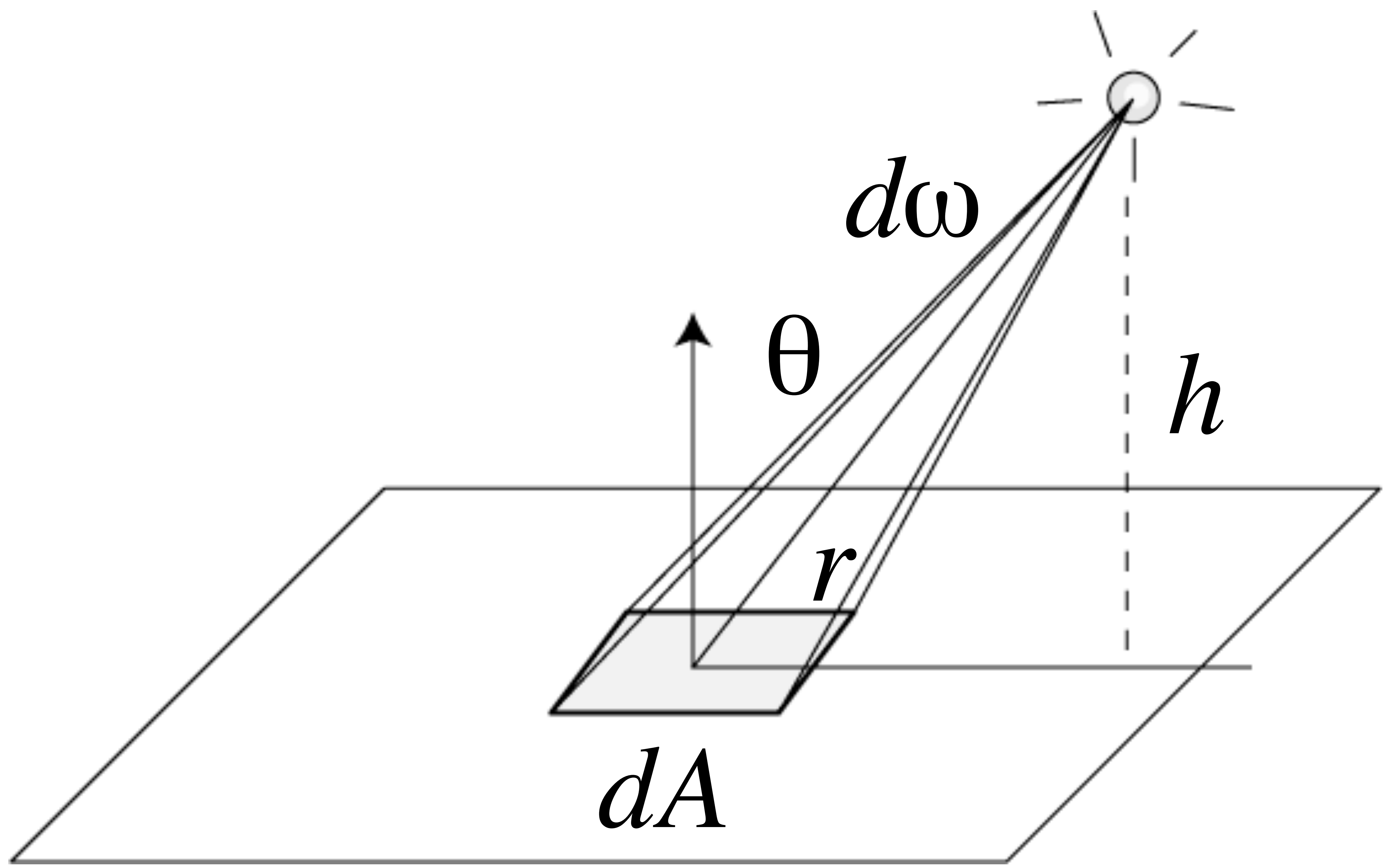
$$E' = \frac{\Phi'}{A'} = \frac{\Phi}{A}\cos\theta = E\cos\theta$$

Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

Irradiance: Isotropic Point Source

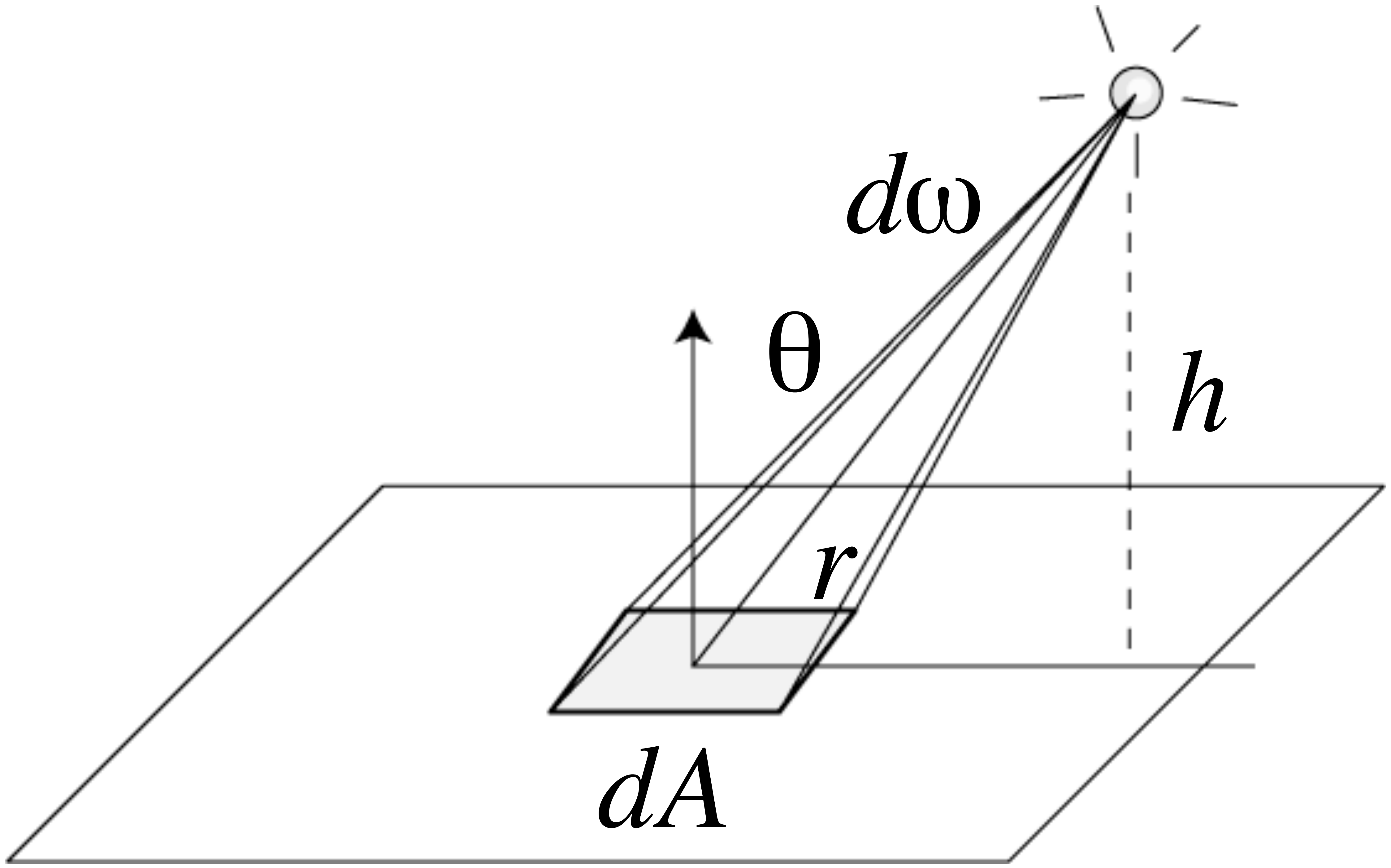


$$I = \frac{\Phi}{4\pi}$$

Power in the beam

$$d\Phi = I\, d\omega$$

Irradiance: Isotropic Point Source

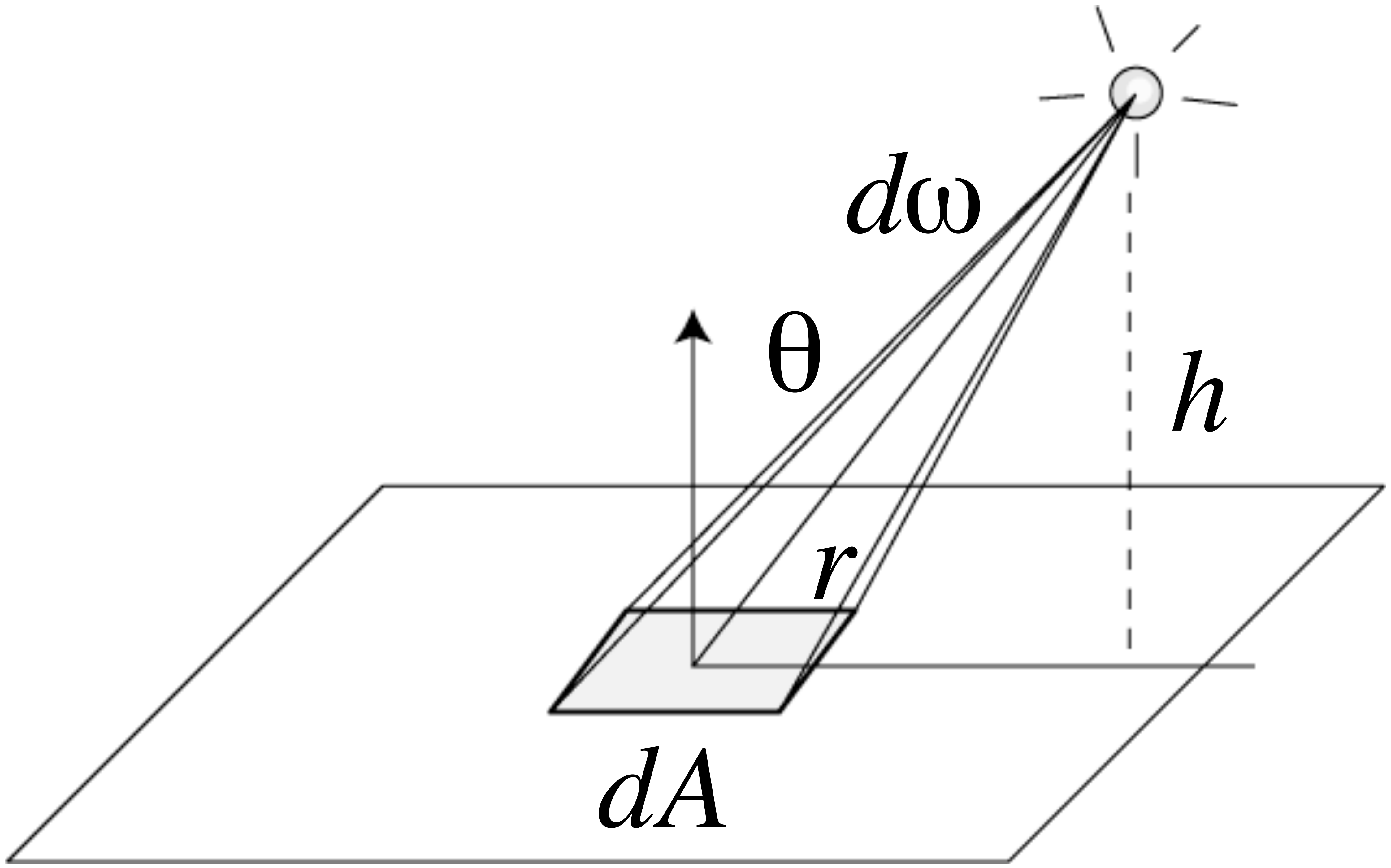


$$I = \frac{\Phi}{4\pi}$$

Angle subtended by *dA*

$$d\omega = \frac{\cos\theta}{r^2} dA$$

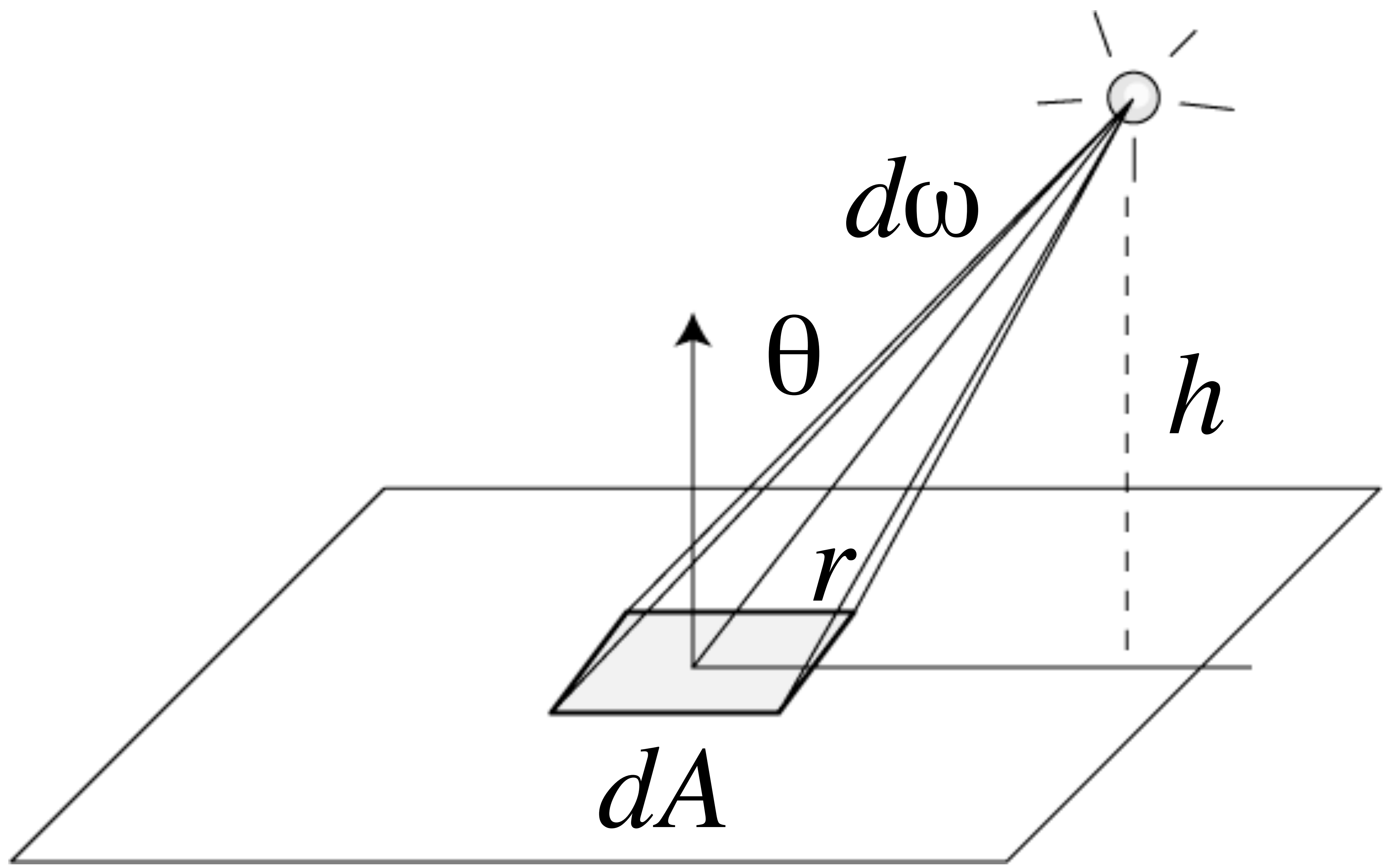
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$I\, d\omega = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} dA$$

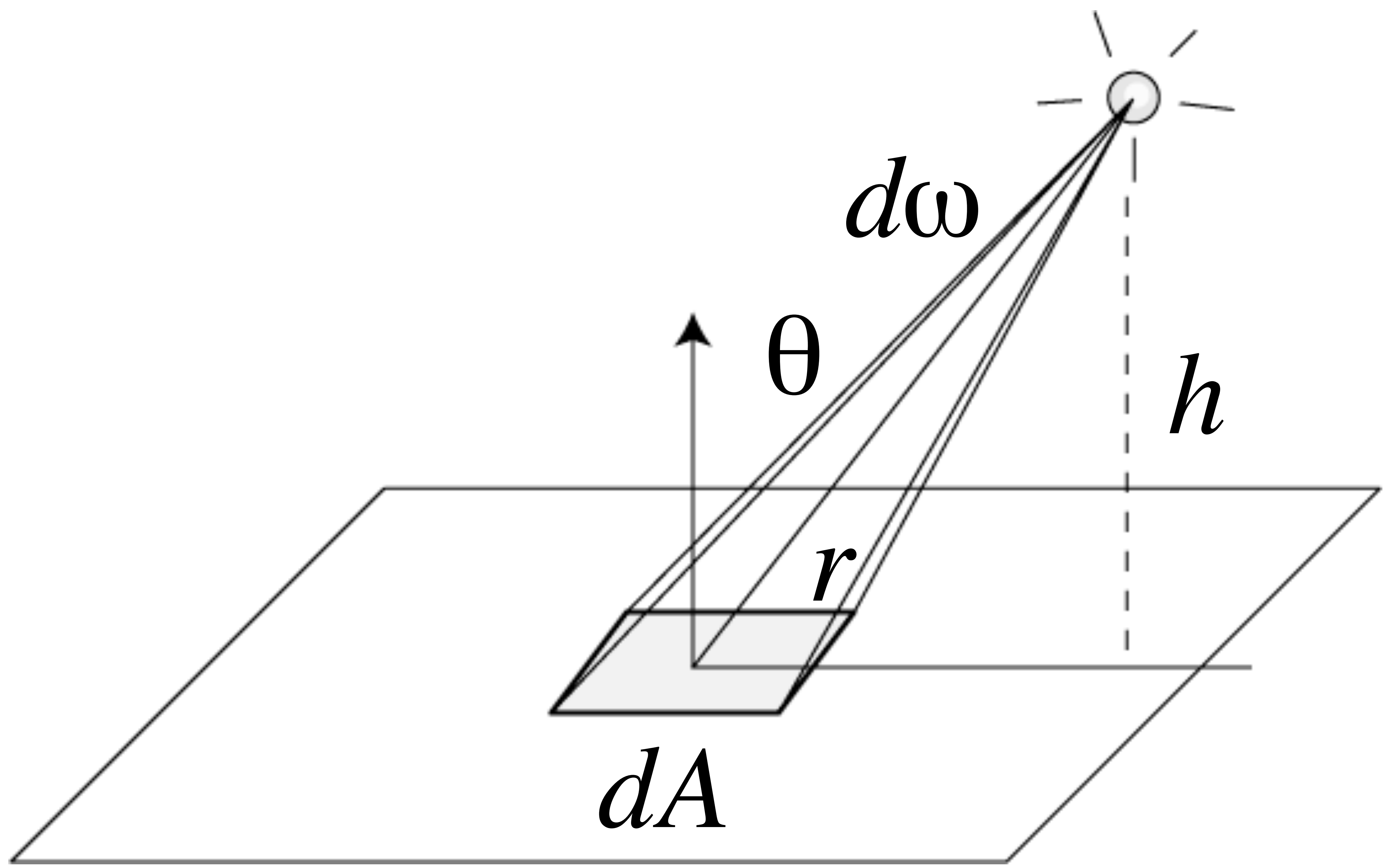
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$I\, d\omega = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} dA = E\, dA$$

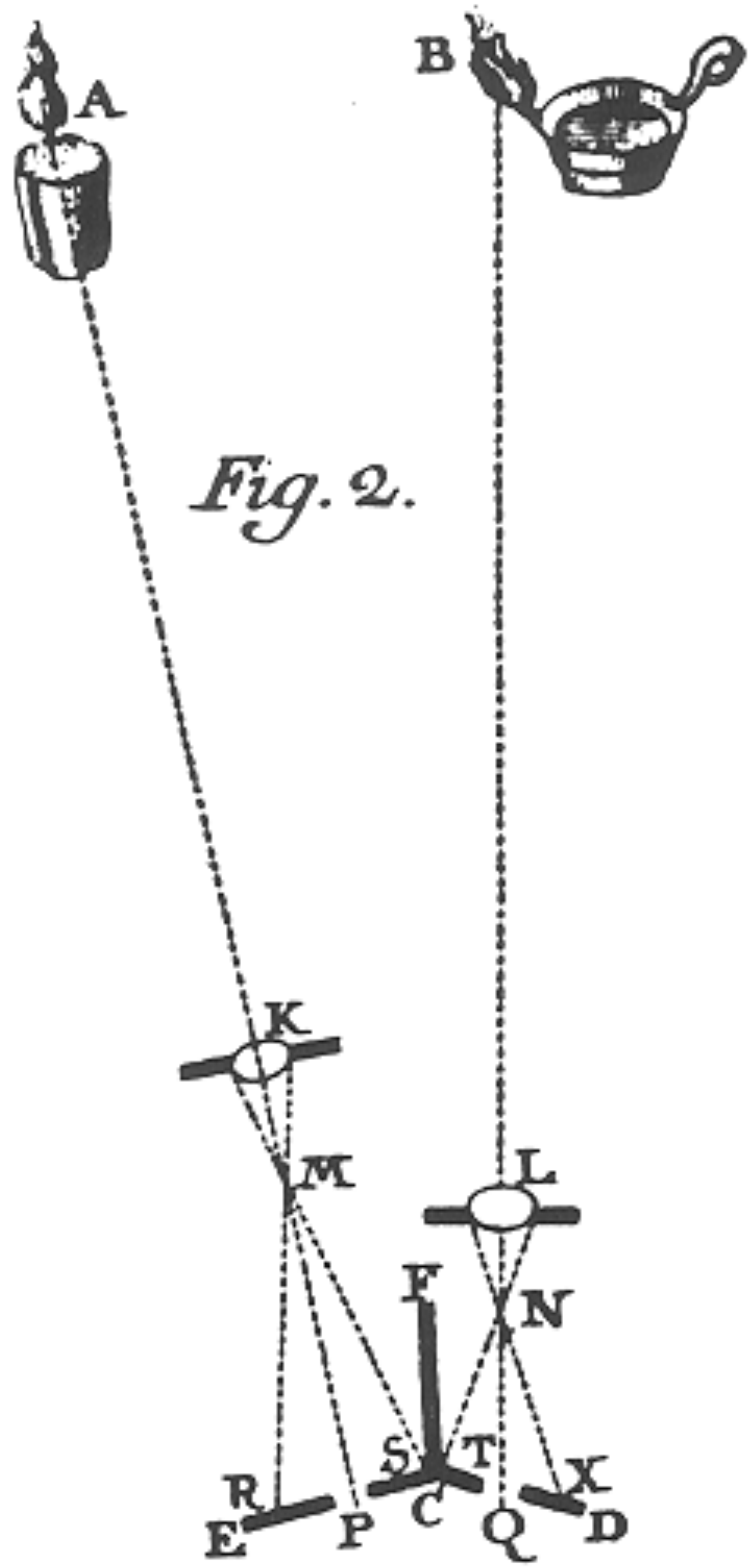
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$E = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}$$

The Invention of Photometry



Bouguer's classic experiment

- Compare a light source and a candle
- Move until they both appear equally bright
- Intensity is proportional to ratio of distances squared

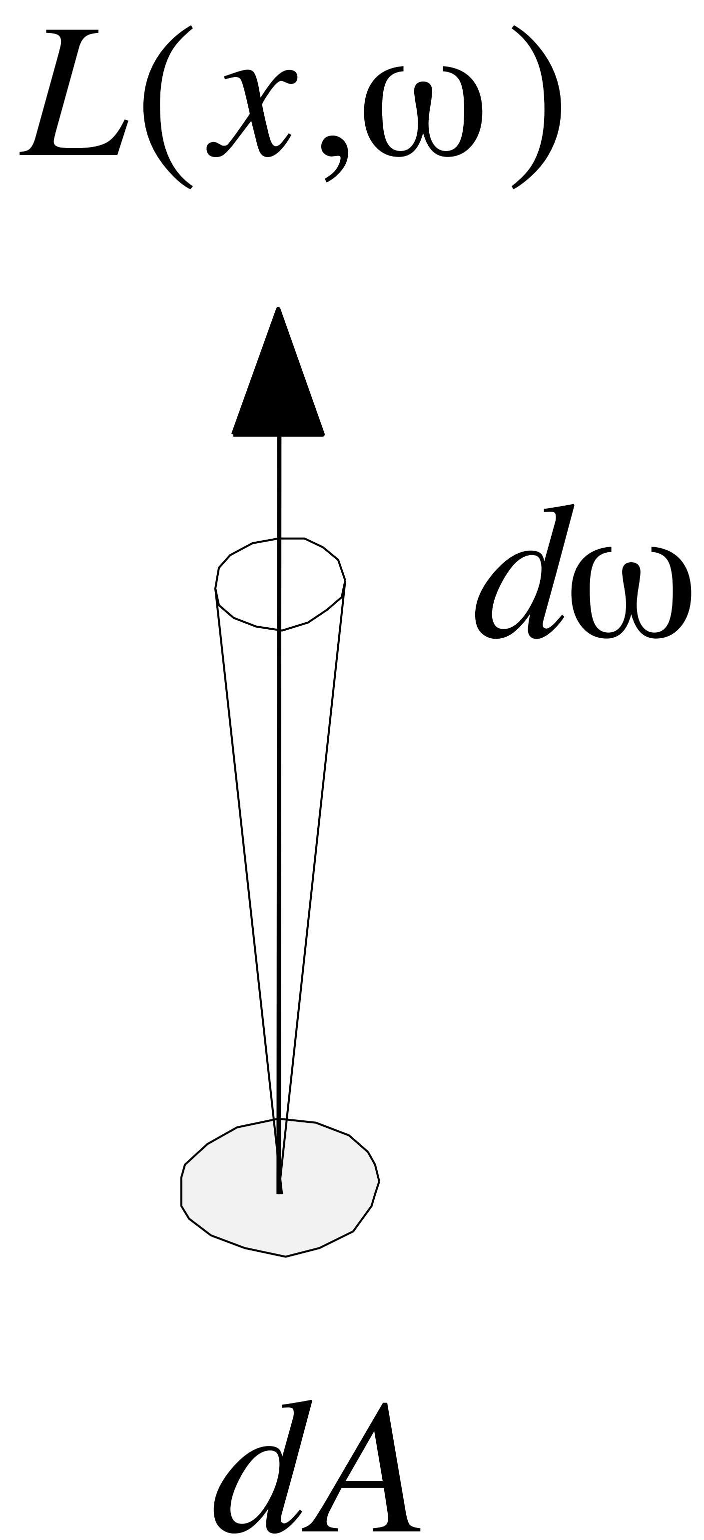
Definition of a candela

- Originally a "standard" candle
- Currently 550 nm laser with 1/683 W/sr
- 1 of 6 fundamental <u>SI units</u>

Radiance

Area Lights – Surface Radiance

Definition: The surface *radiance* (*luminance*) is the intensity per unit area leaving a surface



$$L(x,\omega) \equiv \frac{dI(x,\omega)}{dA}$$

$$= \frac{d^2\Phi(x,\omega)}{d\omega\, dA}$$

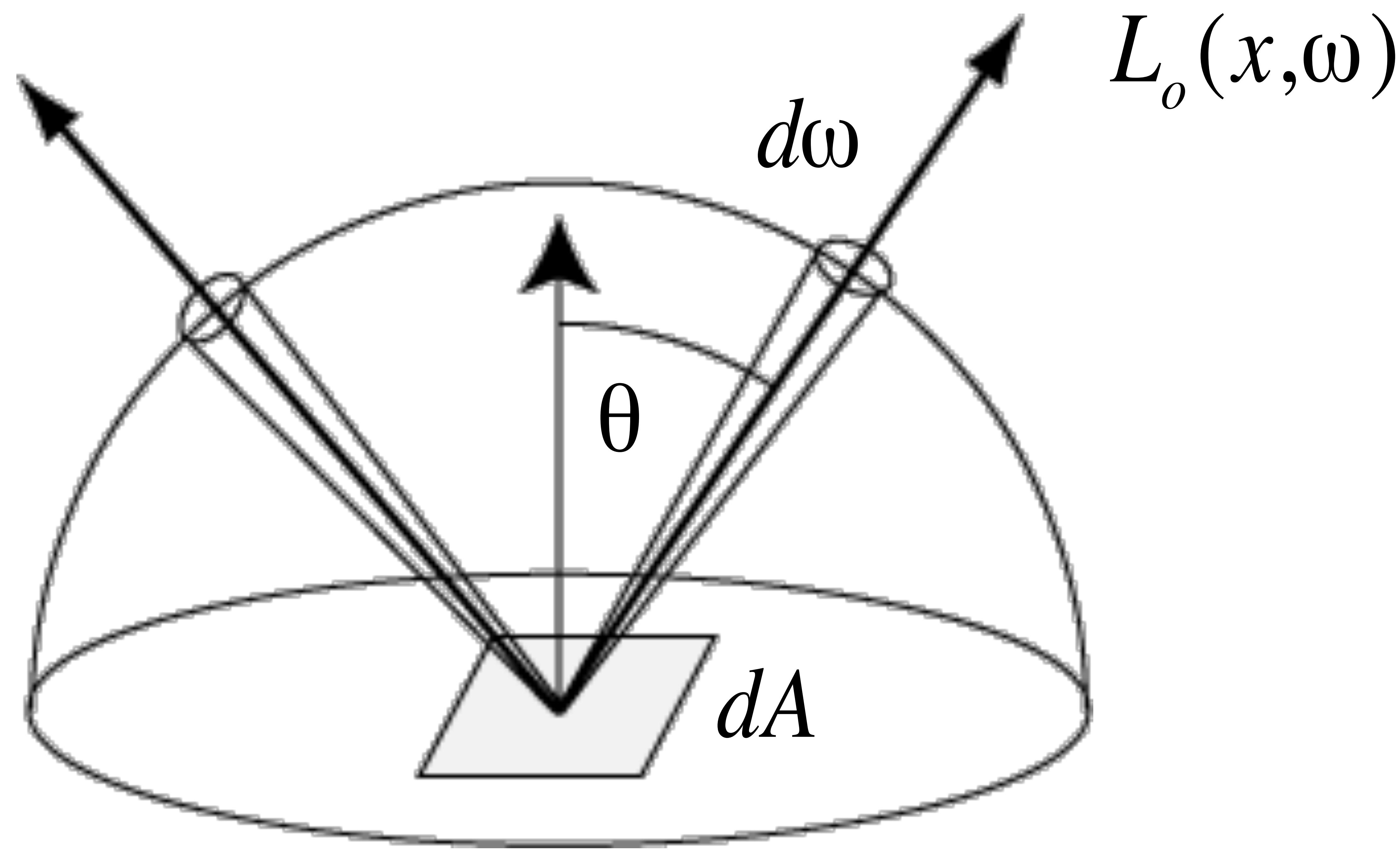
$$\left[\frac{W}{sr\, m^2}\right]\left[\frac{cd}{m^2} = \frac{lm}{sr\, m^2} = nit\right]$$

Typical Values of Luminance [cd/m²]

Surface of the sun	2,000,000,000 nit
Sunlight clouds	30,000
Clear sky	3,000
Overcast sky	300
Moon	0.03

Directional Power Leaving a Surface

$$d^2\Phi_o(x,\omega) = L_o(x,\omega)\cos\theta\, dA\, d\omega$$



Same *dA* for all directions introduces cosine

Radiant Exitance

(Radiosity)

Radiant Exitance

Definition: The *radiant* (luminous) *exitance* is the power per unit area leaving a surface.

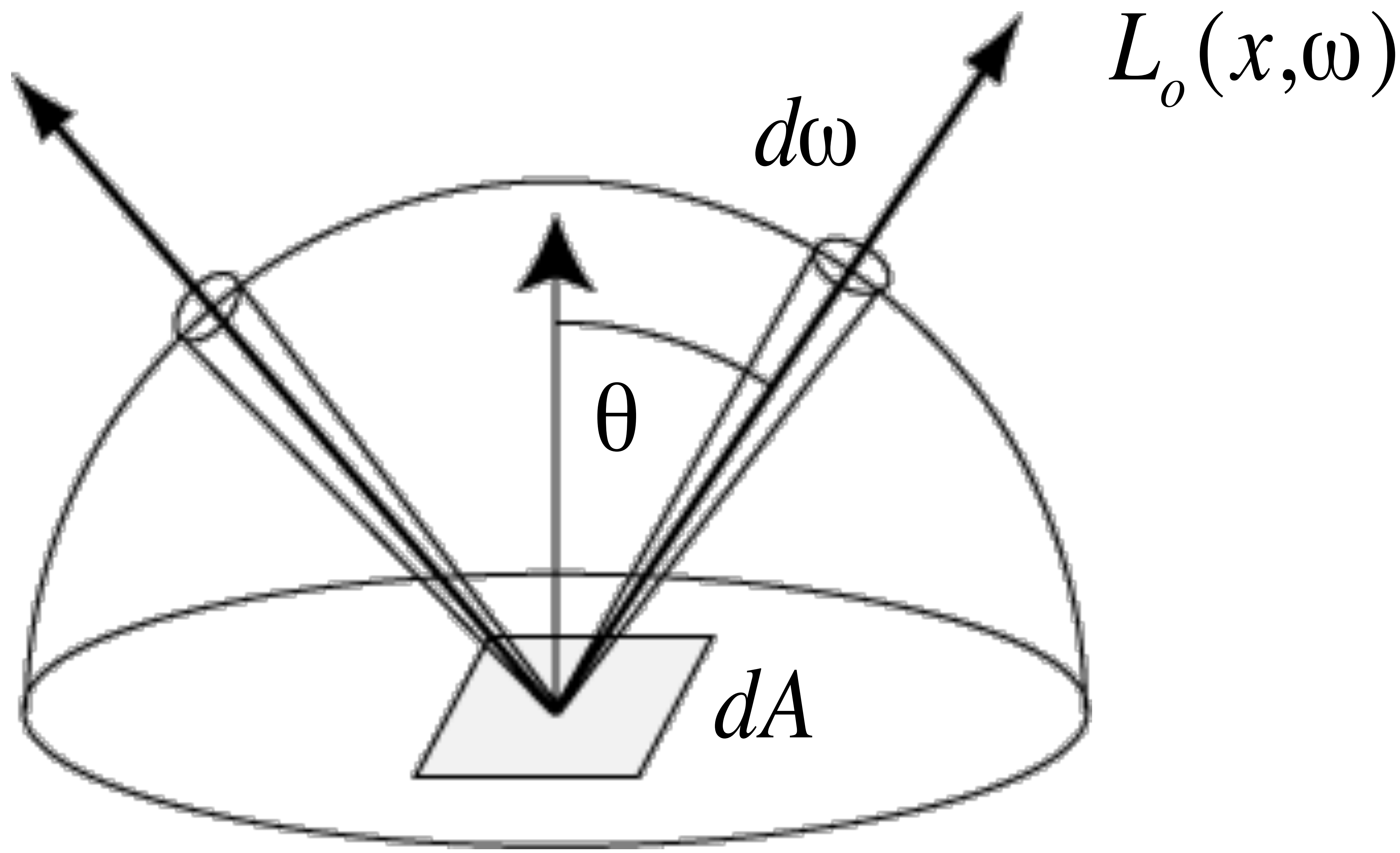
$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2}\right] \quad \left[\frac{lm}{m^2} = lux\right]$$

In computer graphics, this quantity is usually referred to as the *radiosity* (*B*)

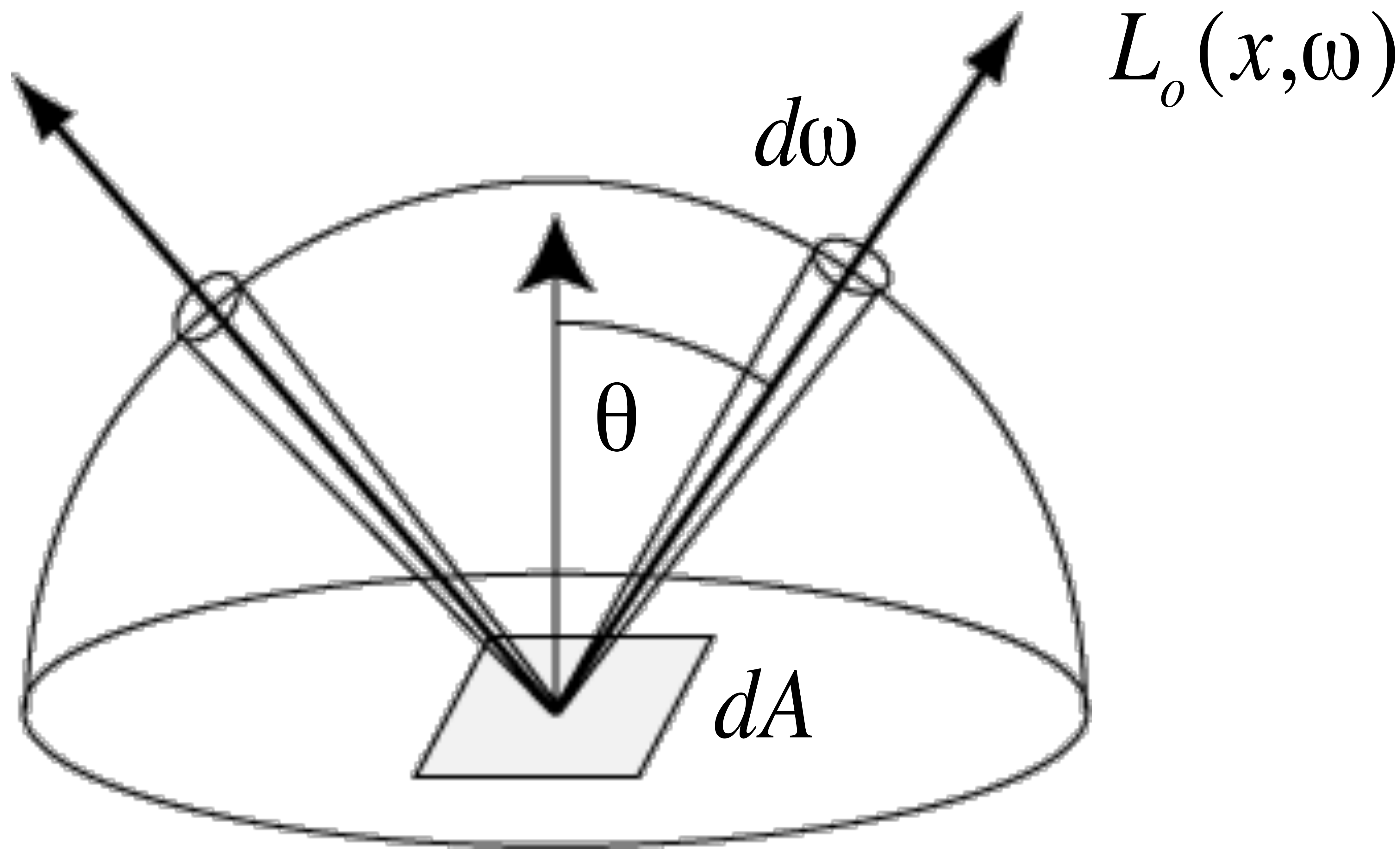
Area Light Source

$$d^2\Phi_o(x,\omega) = L_o(x,\omega)\cos\theta\, dA\, d\omega$$



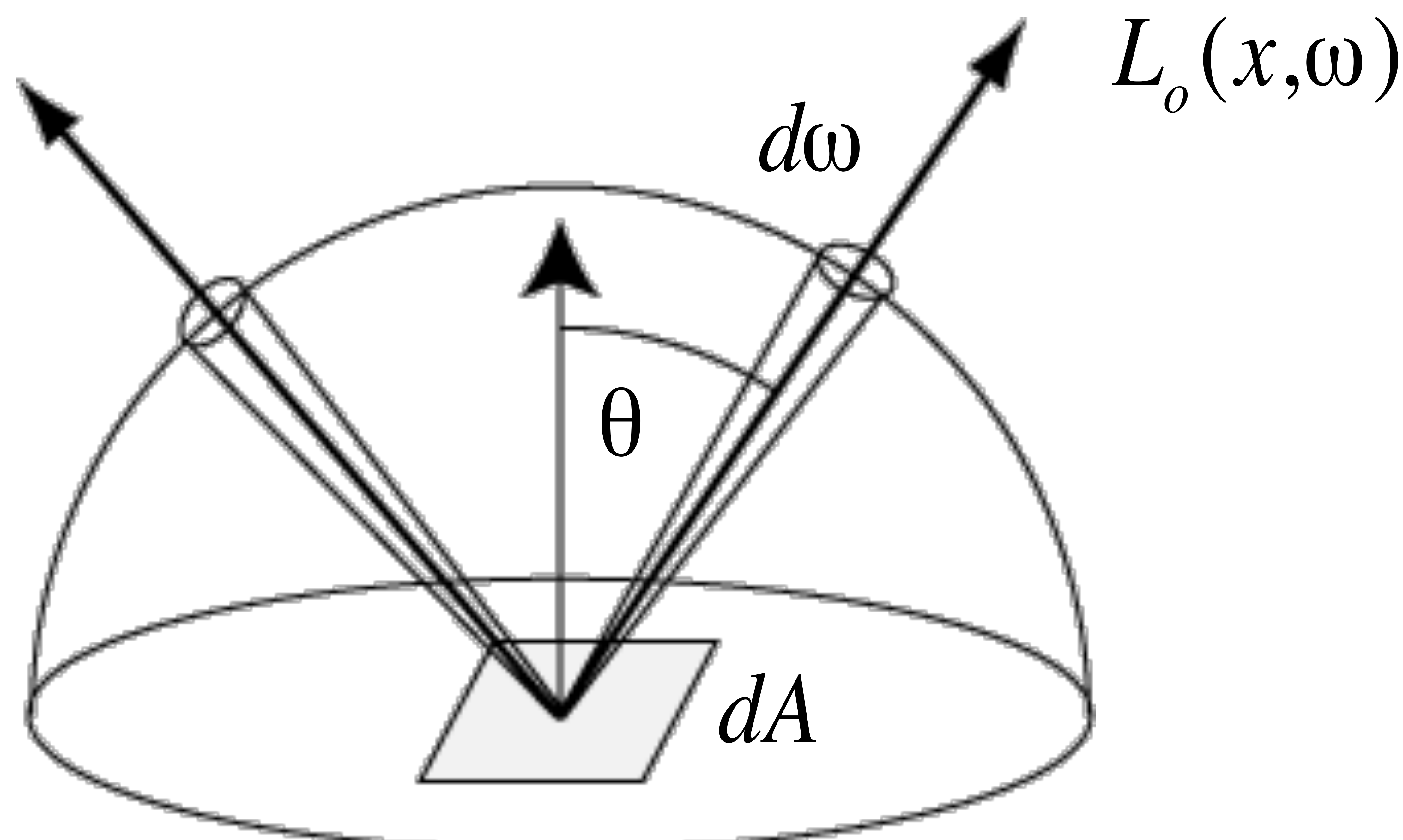
Area Light Source

$$dM(x,\omega) = \frac{d^2\Phi_o(x,\omega)}{dA} = L_o(x,\omega)\cos\theta\, d\omega$$



Area Light Source

$$M = \int_{H^2} dM(x,\omega) = \int_{H^2} L_o(x,\omega)\cos\theta \, d\omega$$



H^2 **Hemisphere**

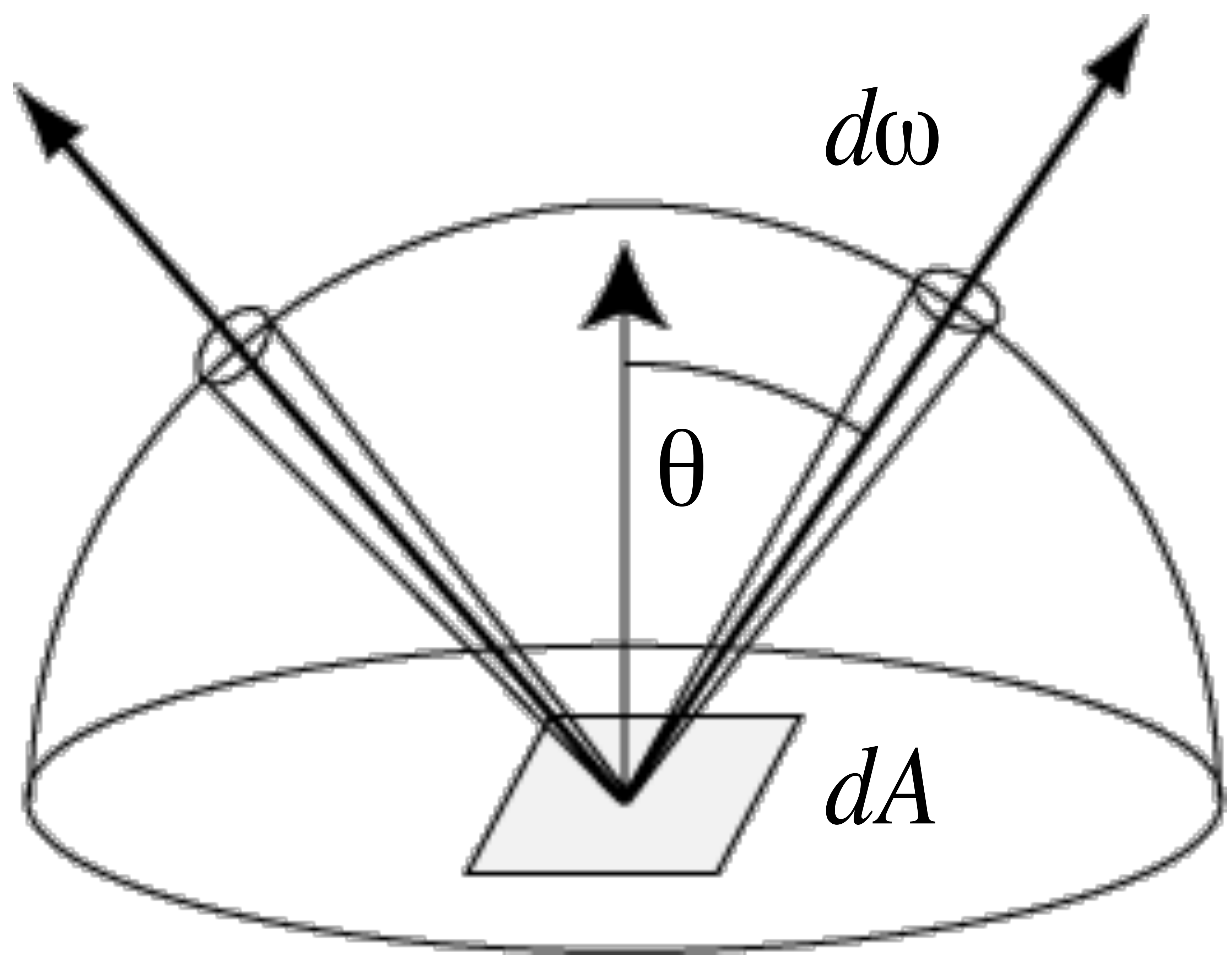
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

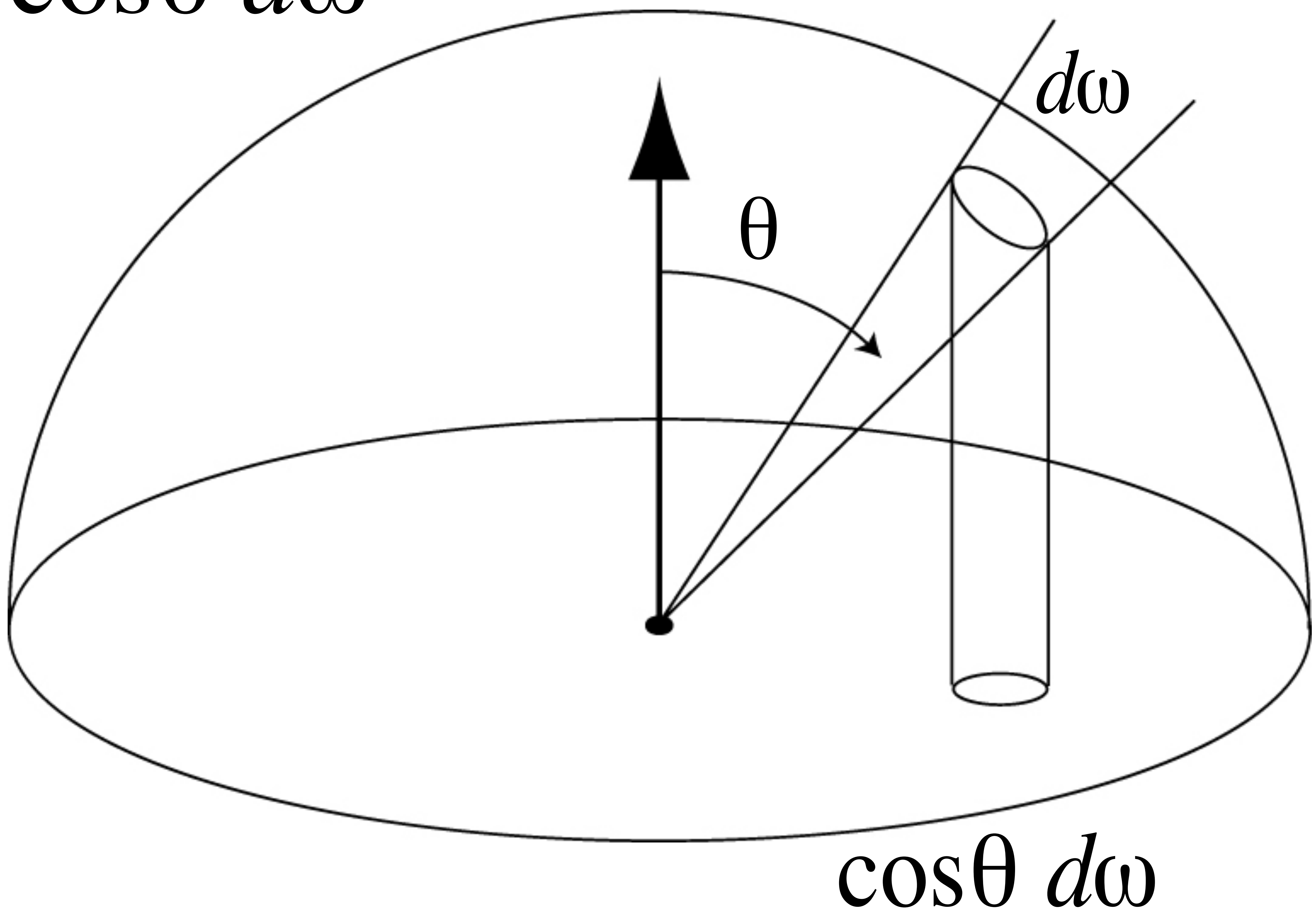
Uniform means L_o is not a function of direction

$$L_o(x,\omega) = L_o$$



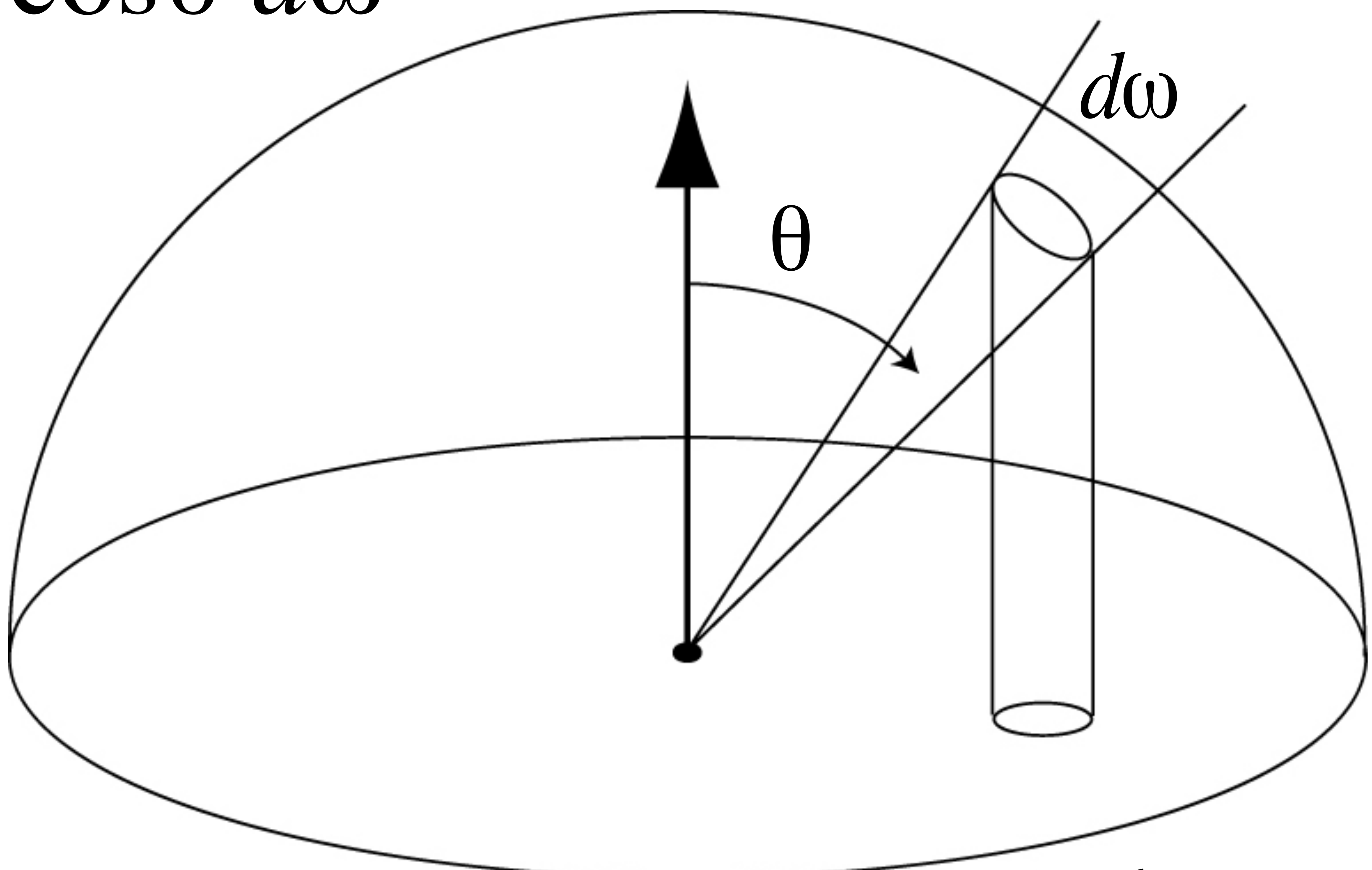
Projected Solid Angle

$$\tilde{\Omega} \equiv \int_{\Omega} \cos\theta \, d\omega$$



Projected Solid Angle

$$\tilde{\Omega} \equiv \int_{\Omega} \cos\theta \, d\omega$$



$$\tilde{\Omega} = \int_{H^2} \cos\theta \, d\omega = \pi$$

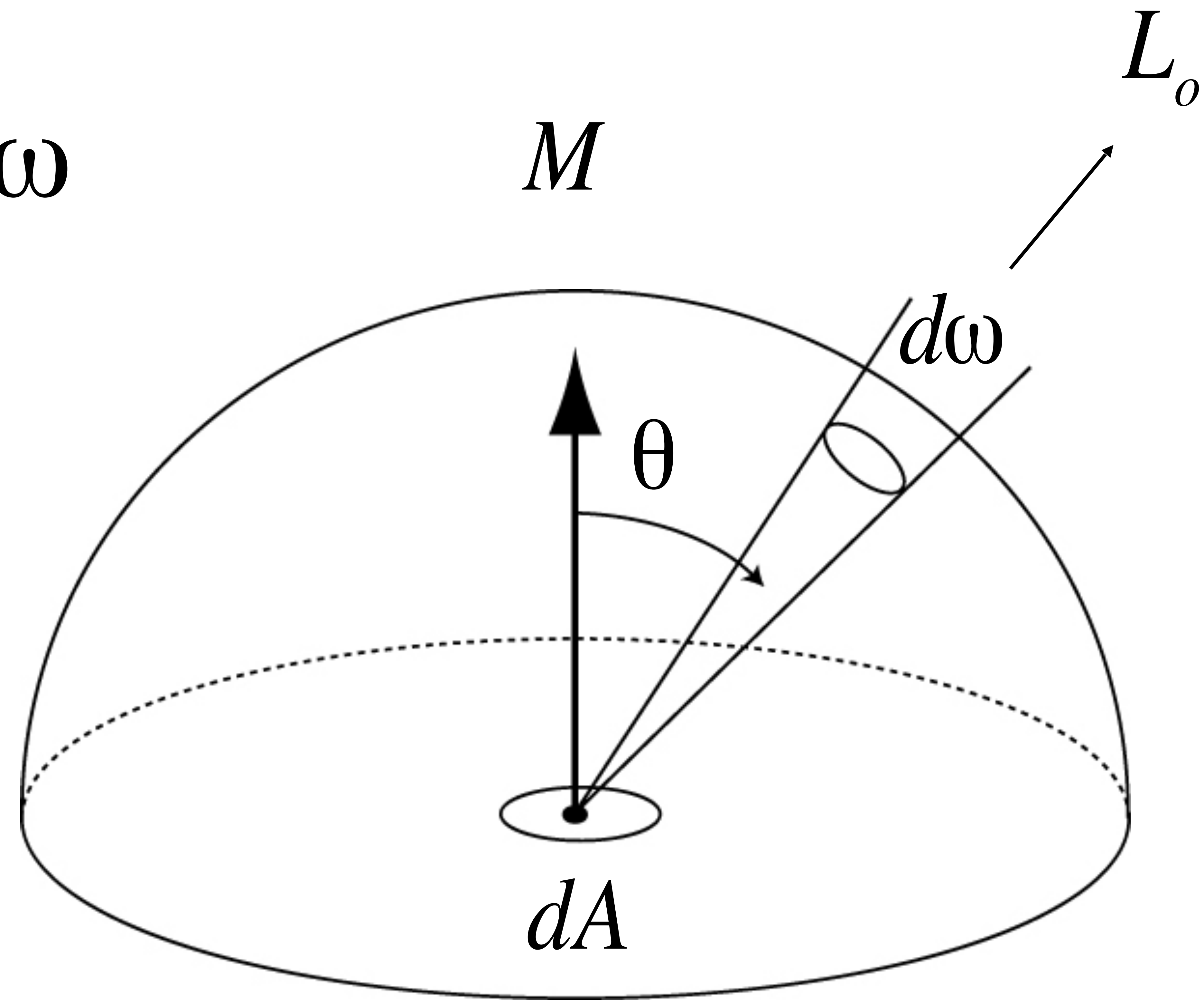
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

$$= \pi L_o$$

$$L_o = \frac{M}{\pi}$$



Radiometry and Photometry

Summary

Radiometric and Photometric Terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance Radiosity	Illuminance Luminosity
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	Units		
	MKS	CGS	British
Luminous Energy	Talbot		
Luminous Power	Lumen		
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit Apostilb, Blondel	Stilb Lambert	 Footlambert
Luminous Intensity	Candela (Candle, Candlepower, Carcel, Hefner)		

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?"

-- James Kajiya

Next up!!!

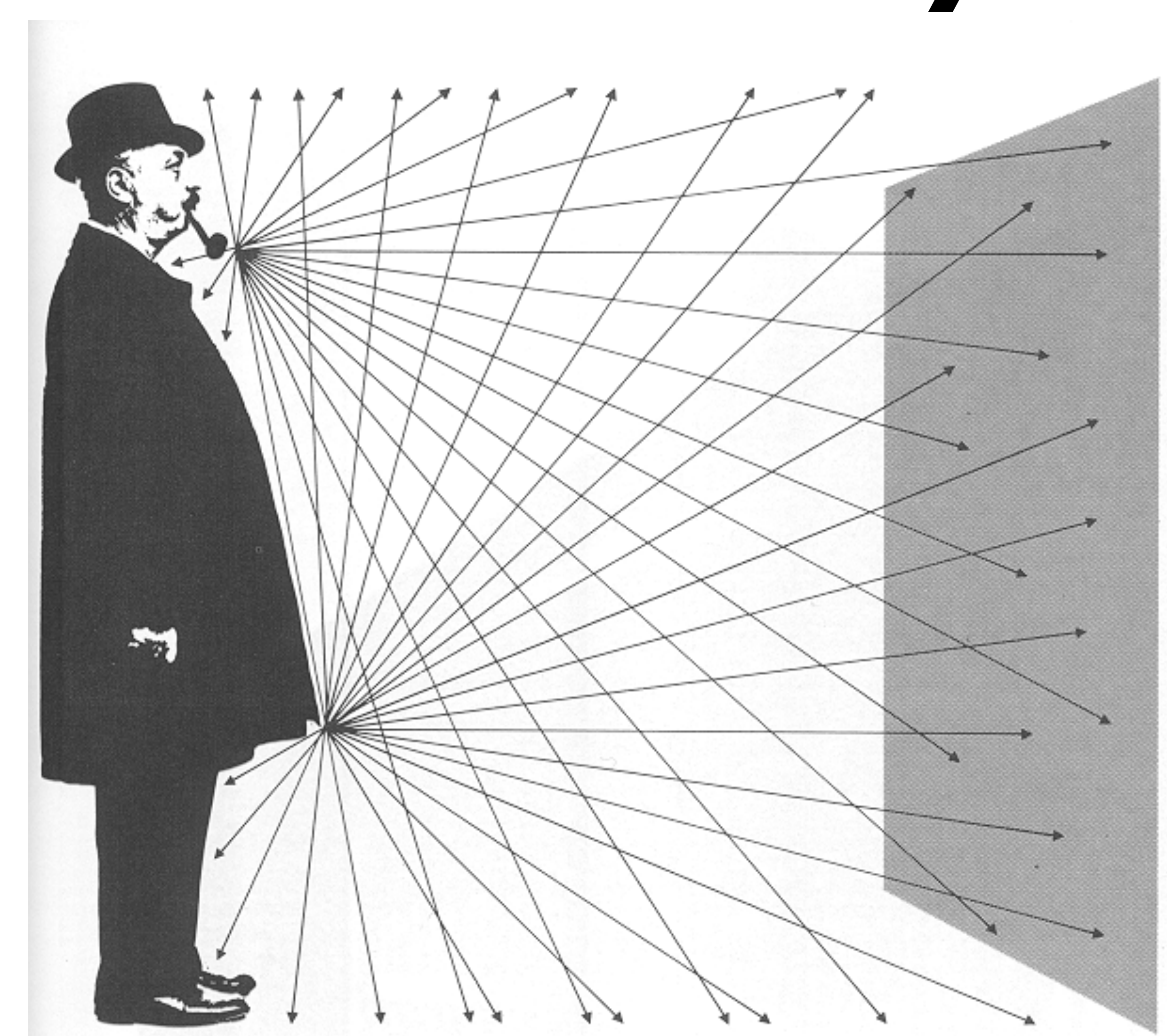
The Light Field

The Light Field

Last lecture: Radiometry and photometry

This lecture: Light field = radiance function on rays

- **Conservation of radiance**
- **Measurement equation**
- **Throughput and counting rays**
- **Irradiance calculations**



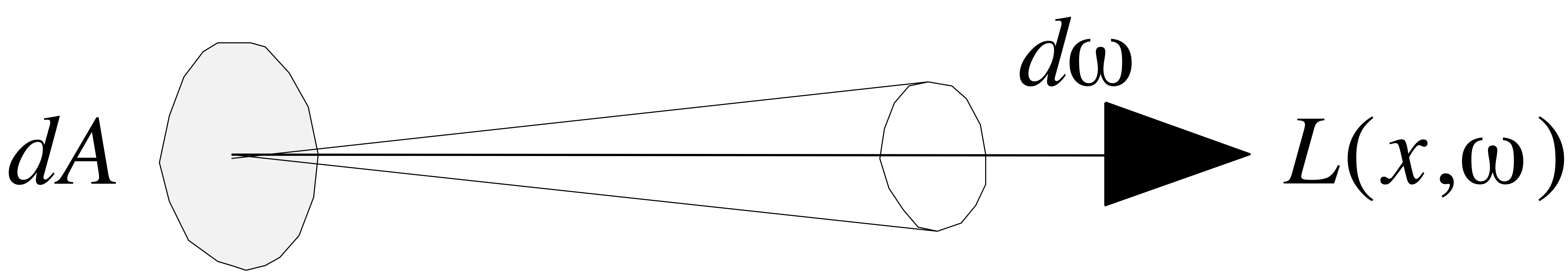
From London and Upton

Subtitle: "Lines and Light"

Acknowledgement: Pat Hanrahan

Field Radiance or Light Field

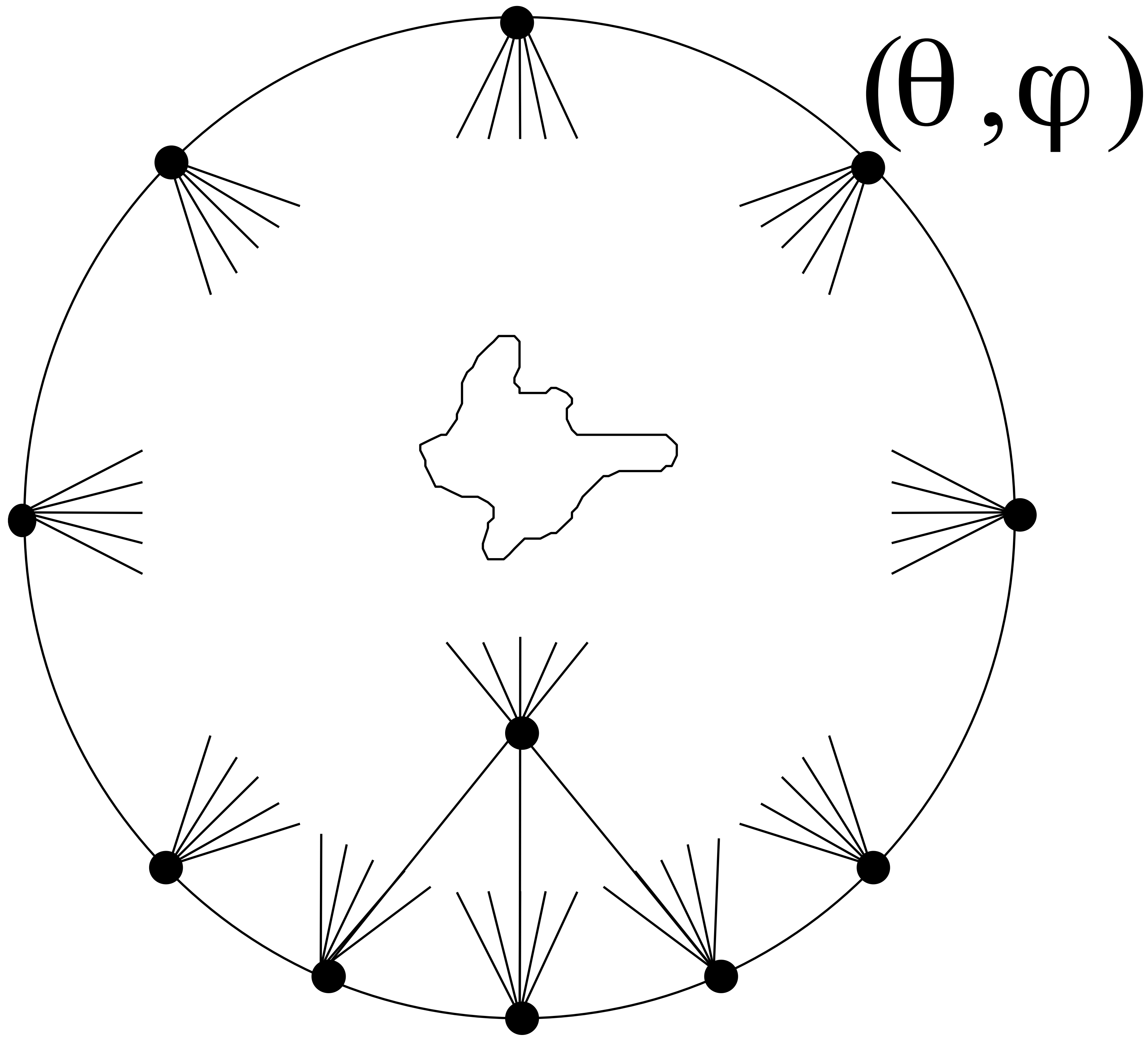
Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction



$$L(x, \omega) \equiv \frac{d^2\Phi(x, \omega)}{d\omega\, dA}$$

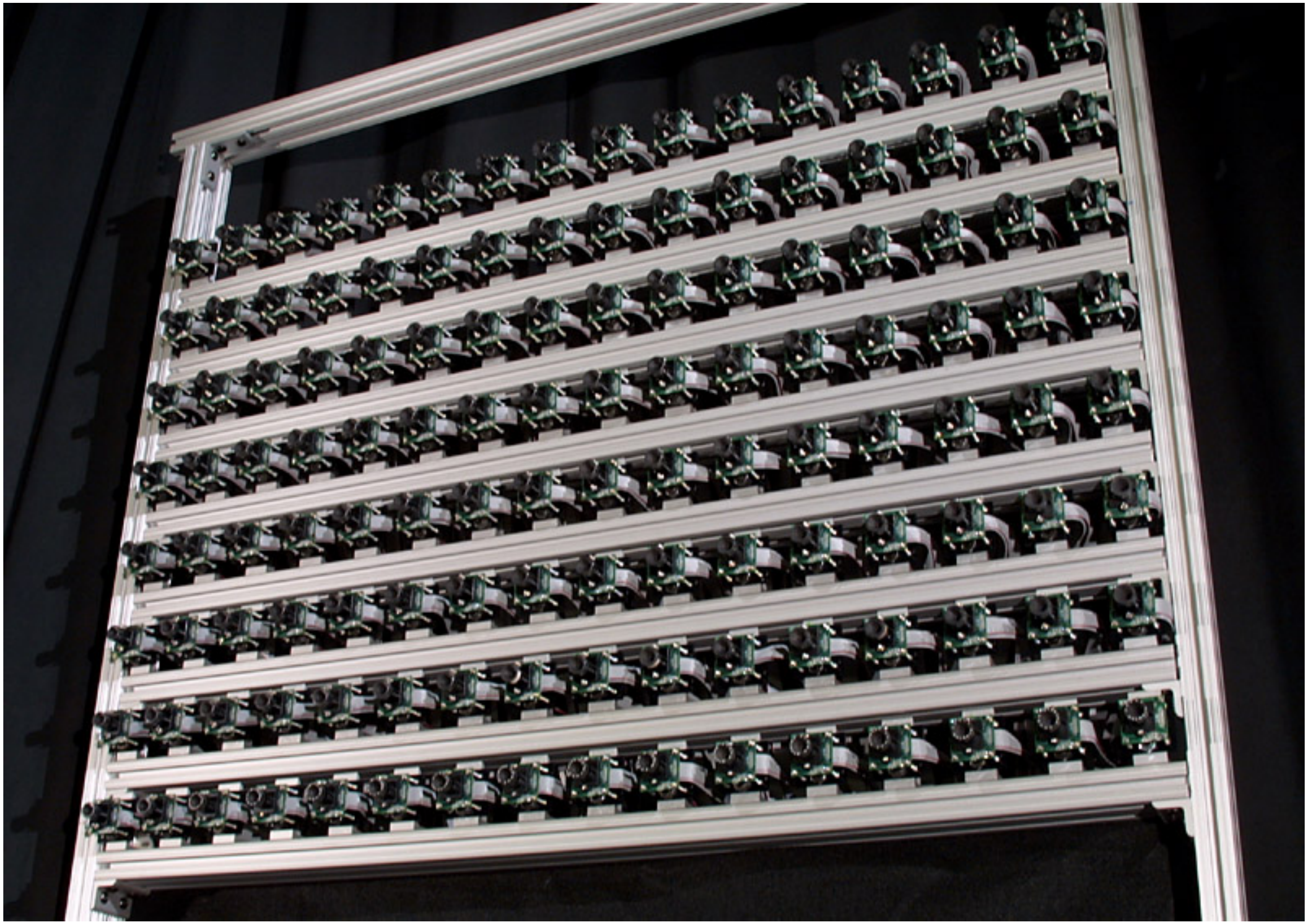
Capture Light Field

$$L(x, y, \theta, \varphi)$$

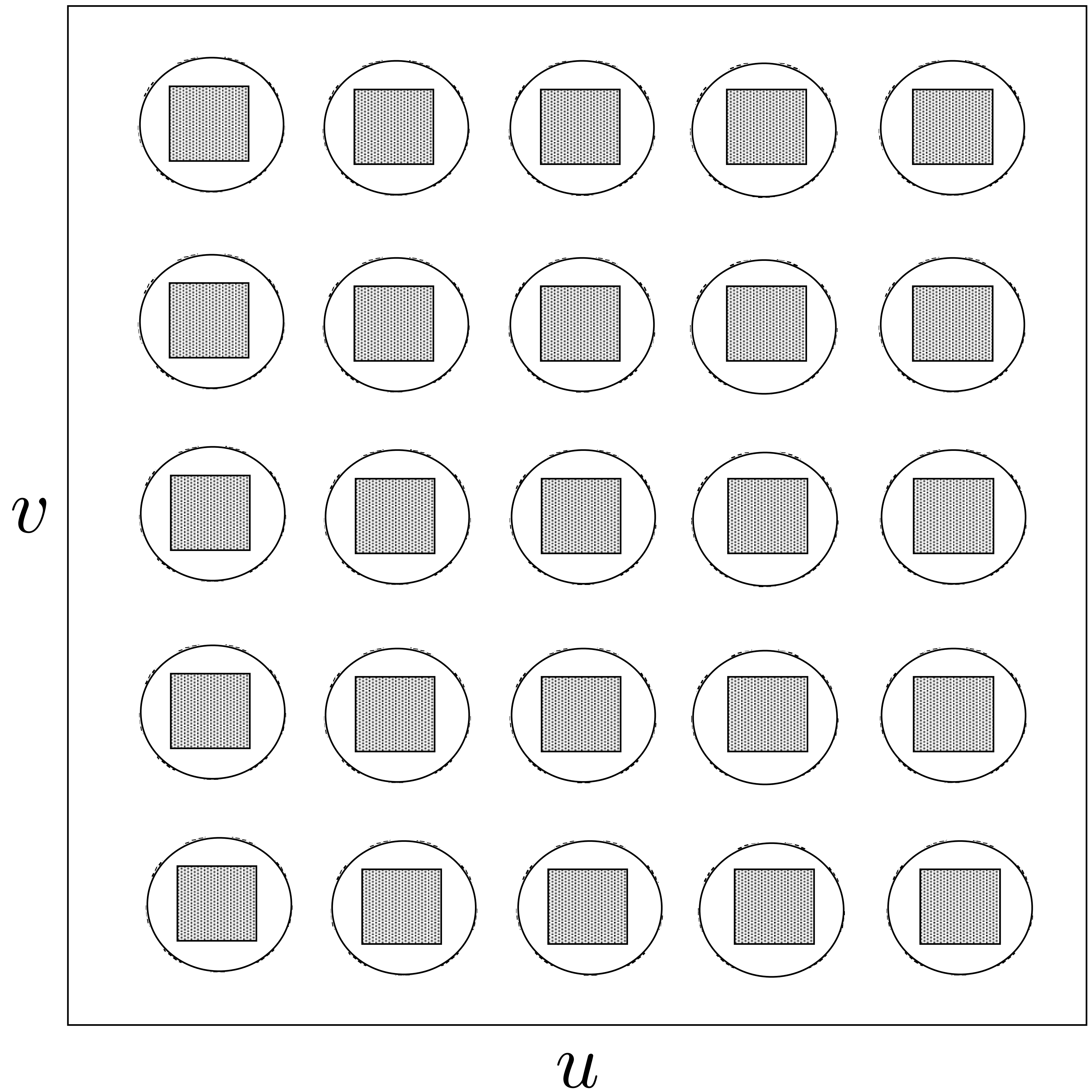


Captures all the light leaving an object - like a hologram

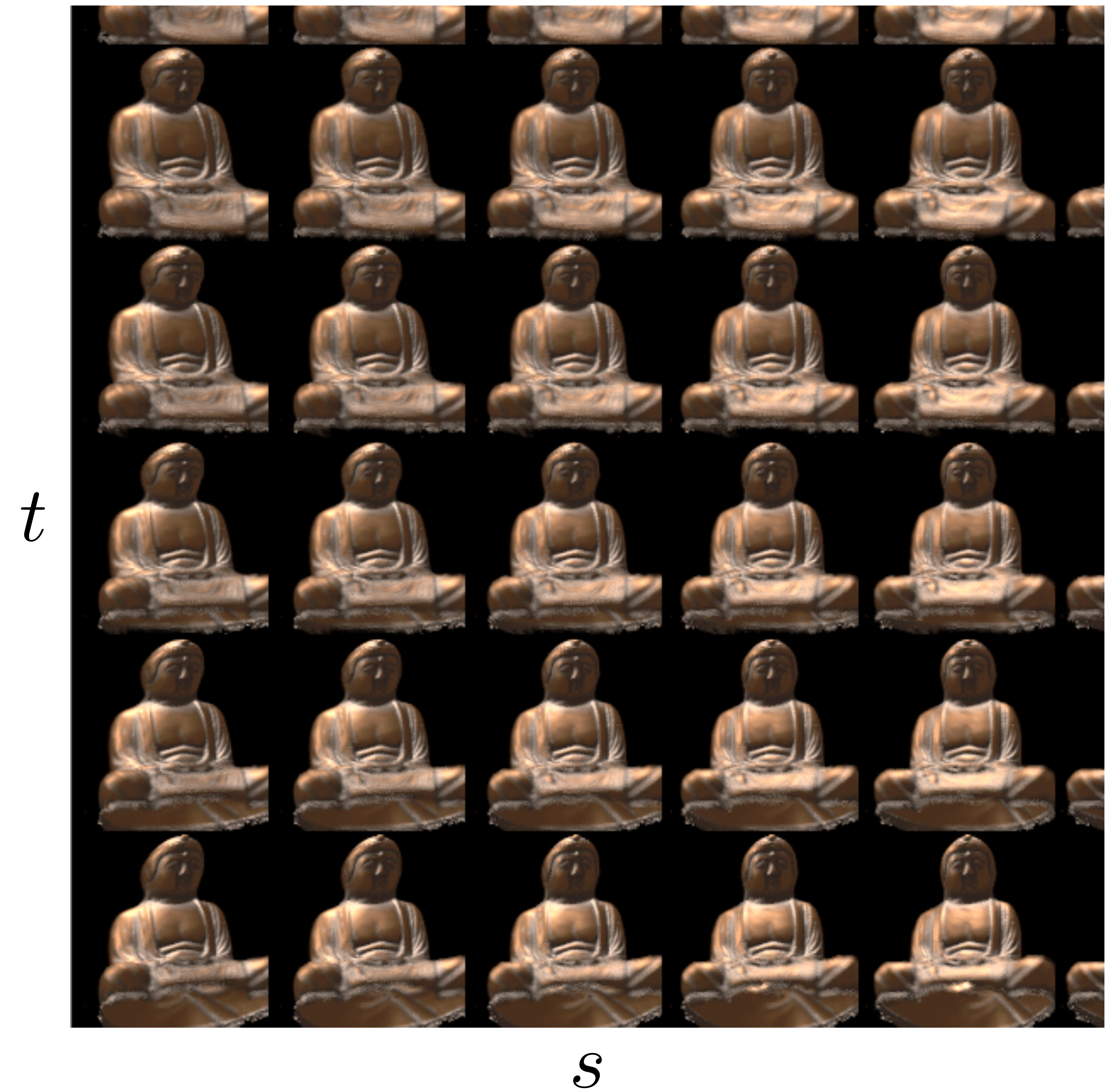
Multi-Camera Array



Two-Plane 4D Light Field



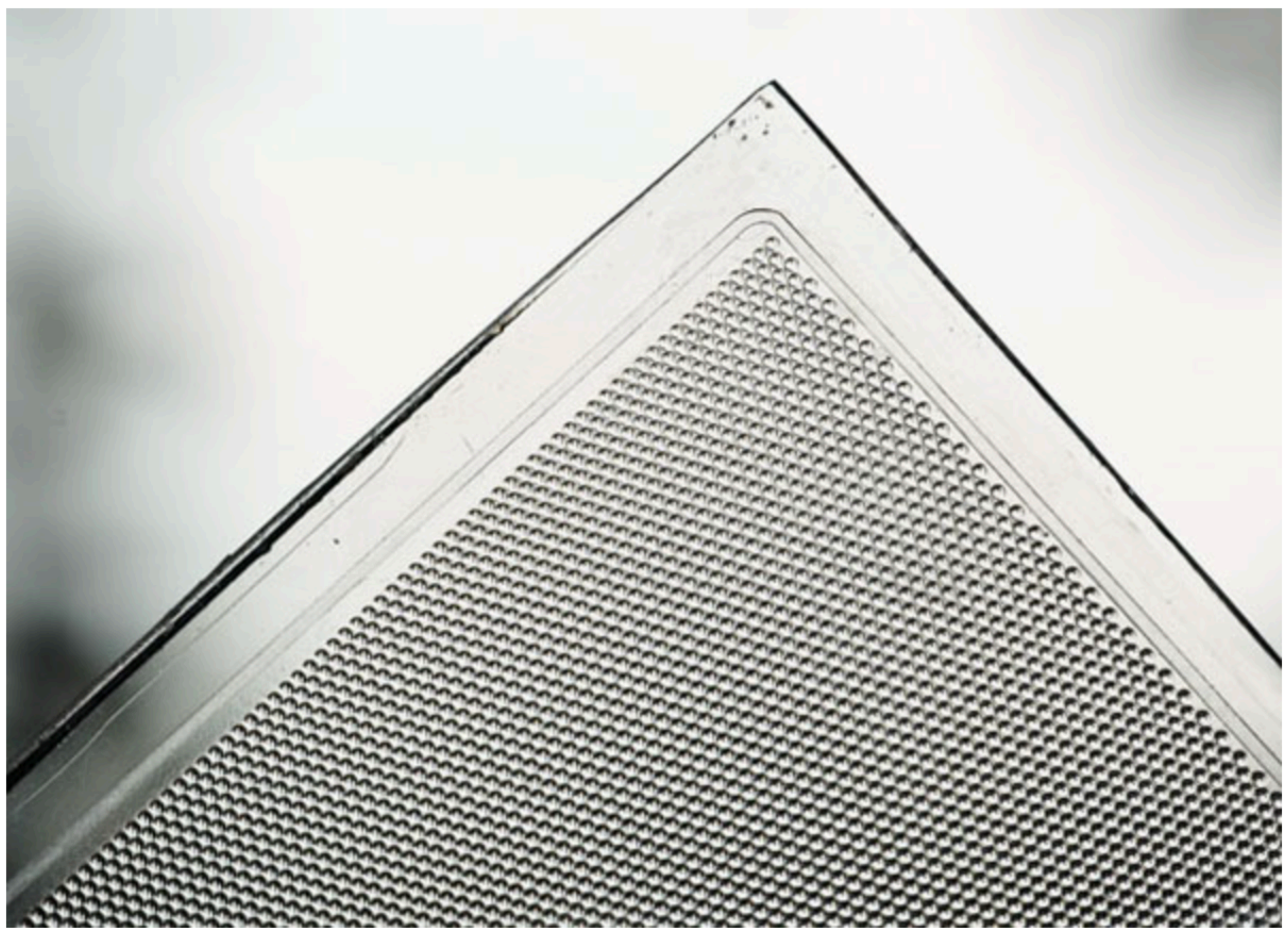
2D Array of Cameras



2D Array of Images

$$L(u,v,s,t)$$

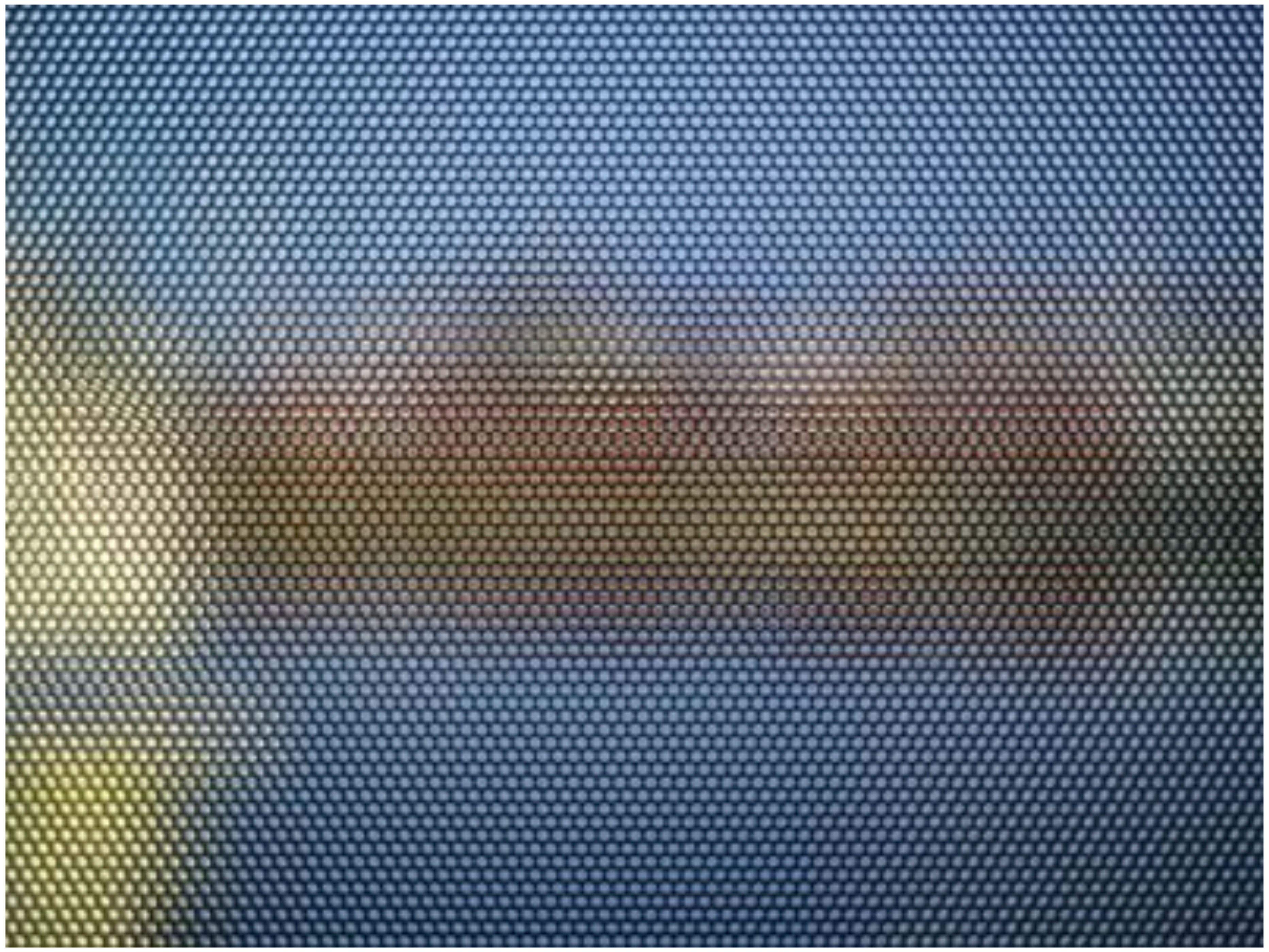
Lenslet Arrays

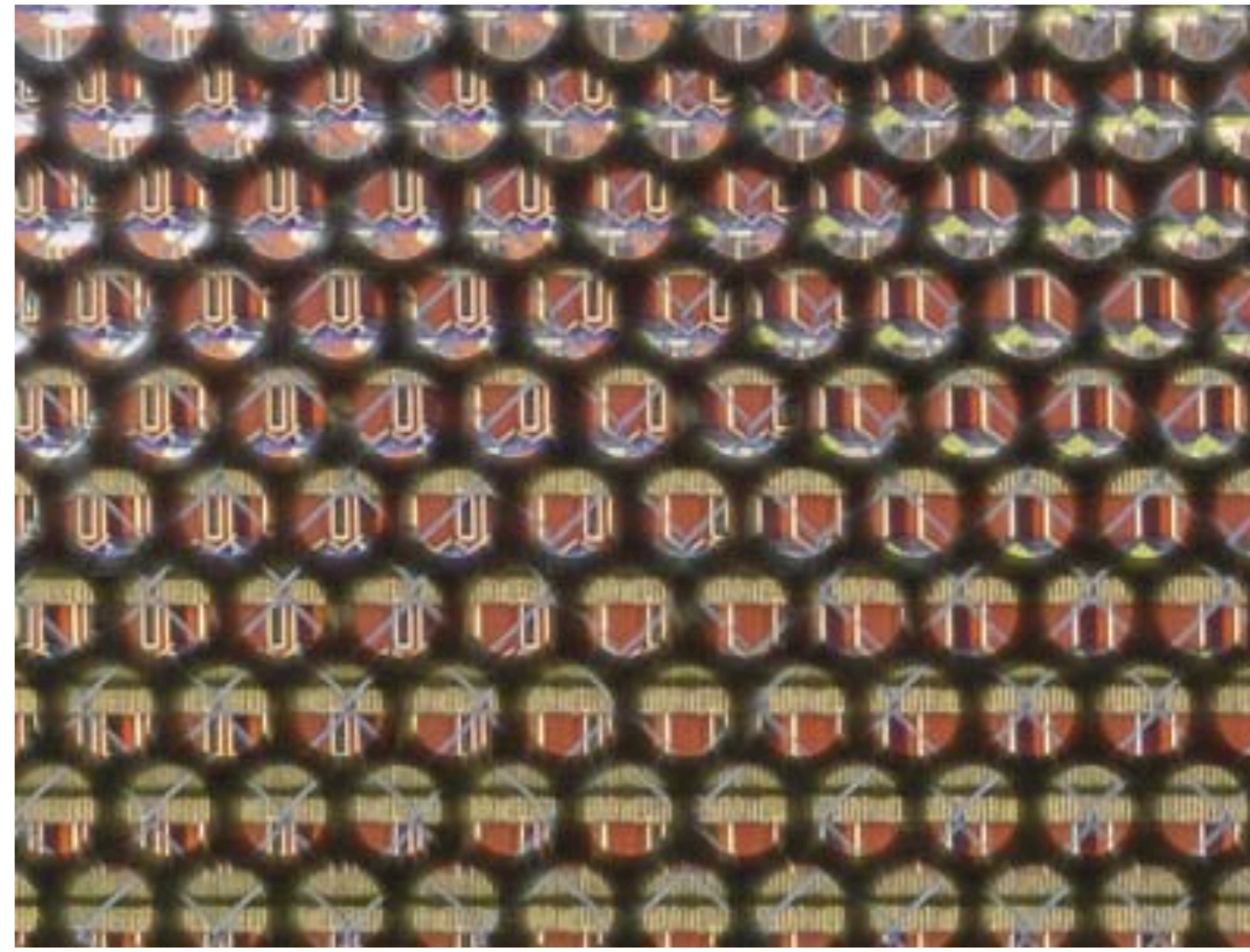


The micro lens array is placed in
front of the image sensor.

http://cameramaker.se/Lightfield.htm

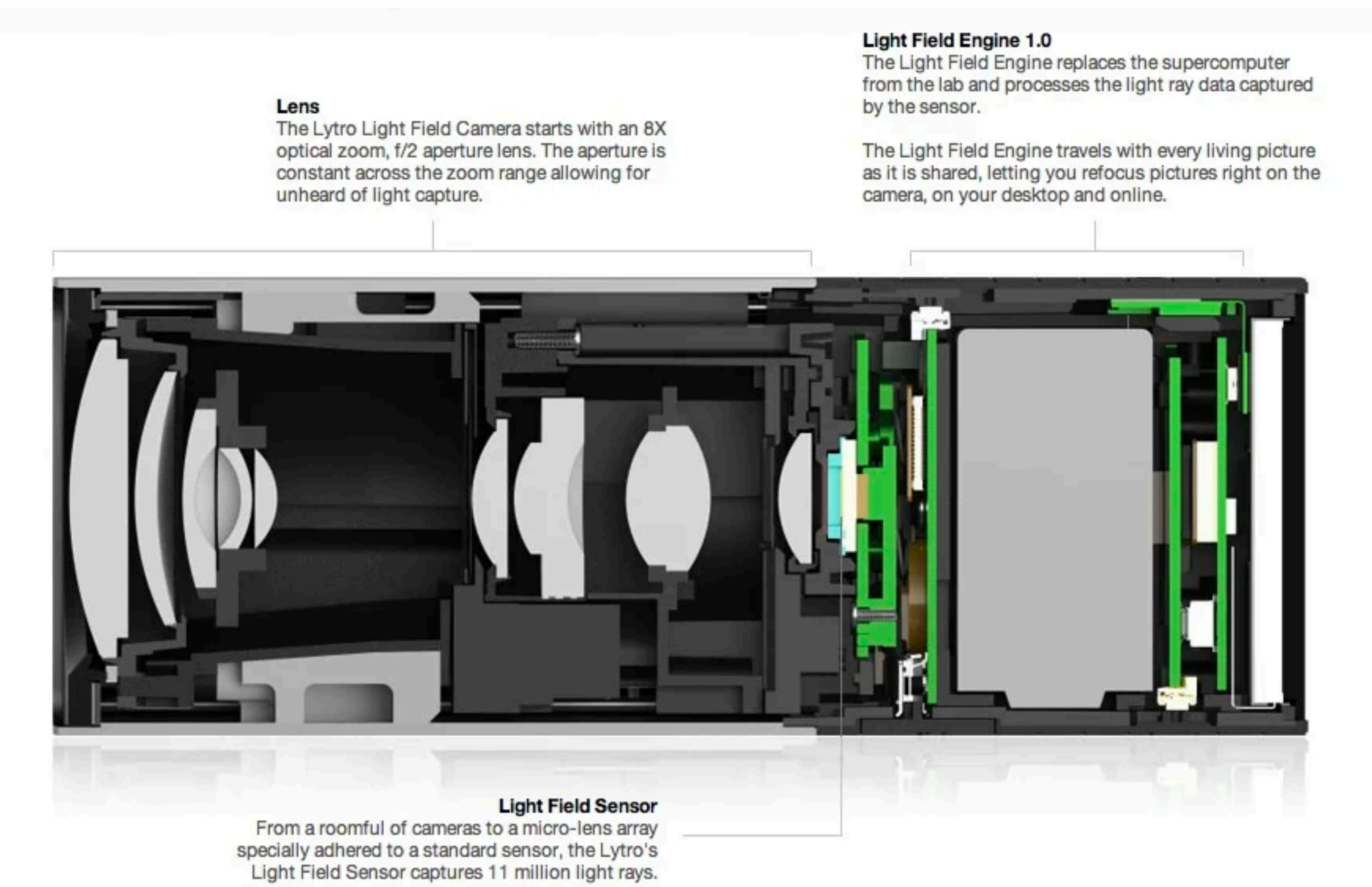
Lenslet Arrays


The unprocessed light field image


Detail of the light field image

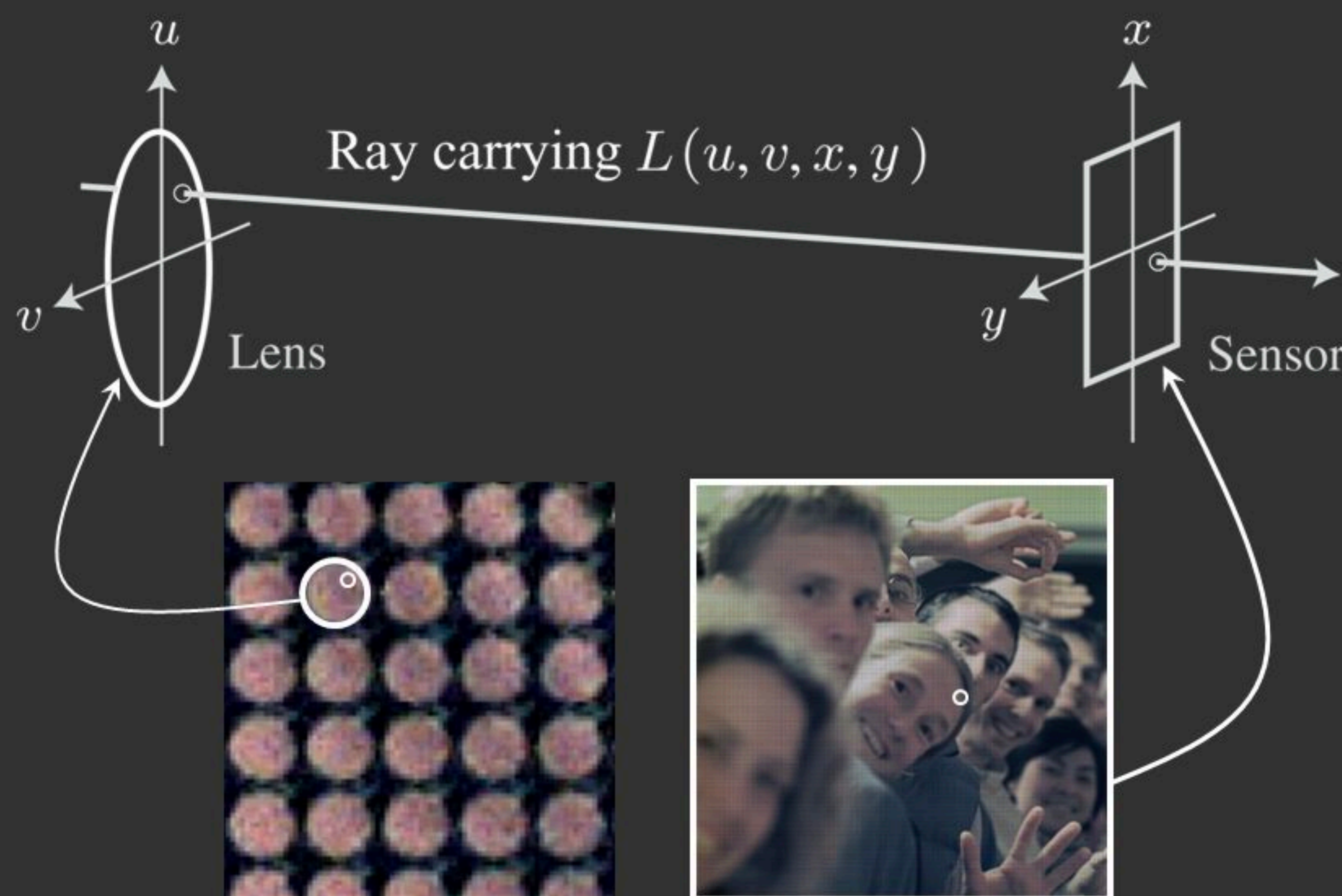
http://cameramaker.se/Lightfield.htm

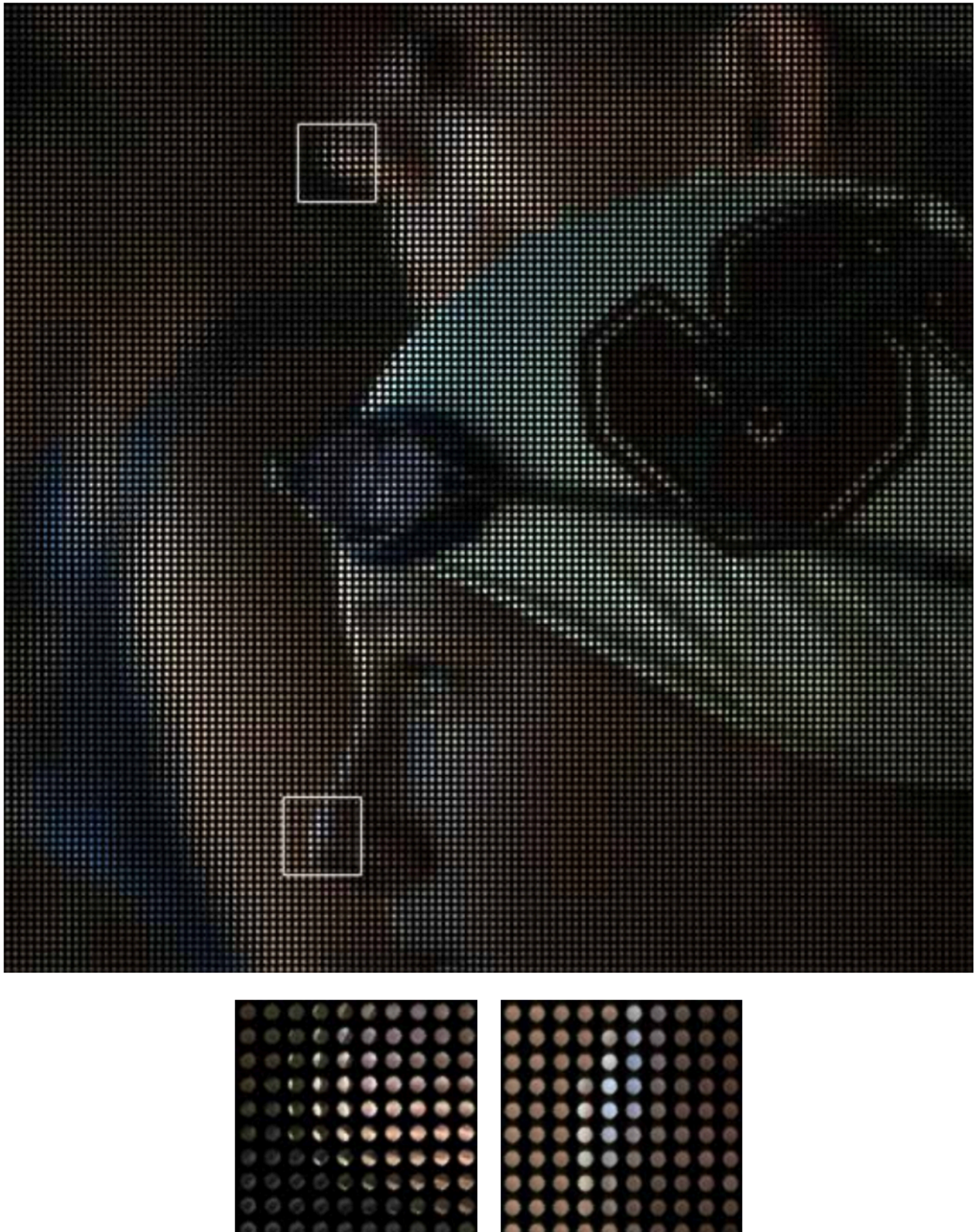
Ren Ng
Light Field Camera



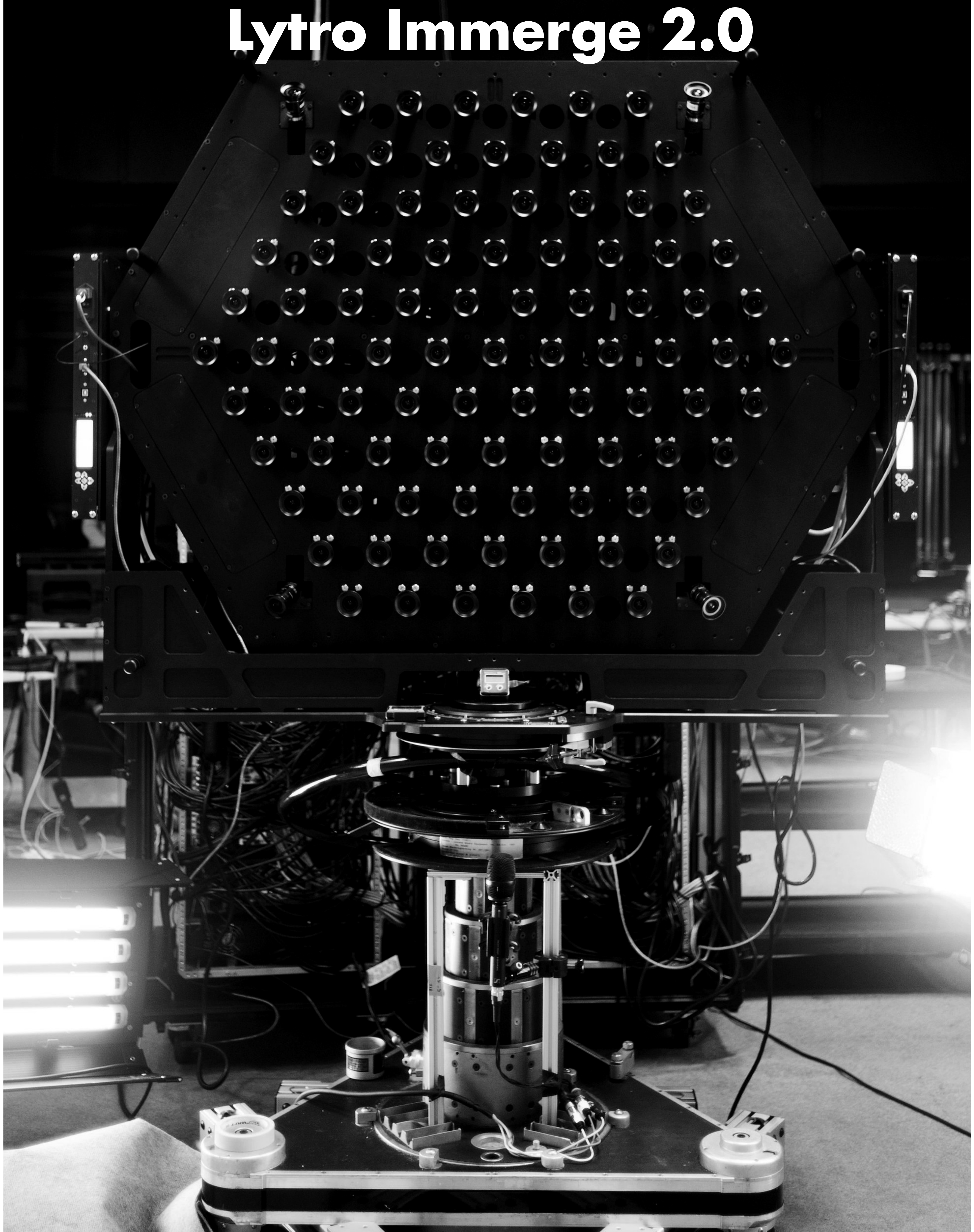
Lytro Camera

Light Field Inside the Camera Body

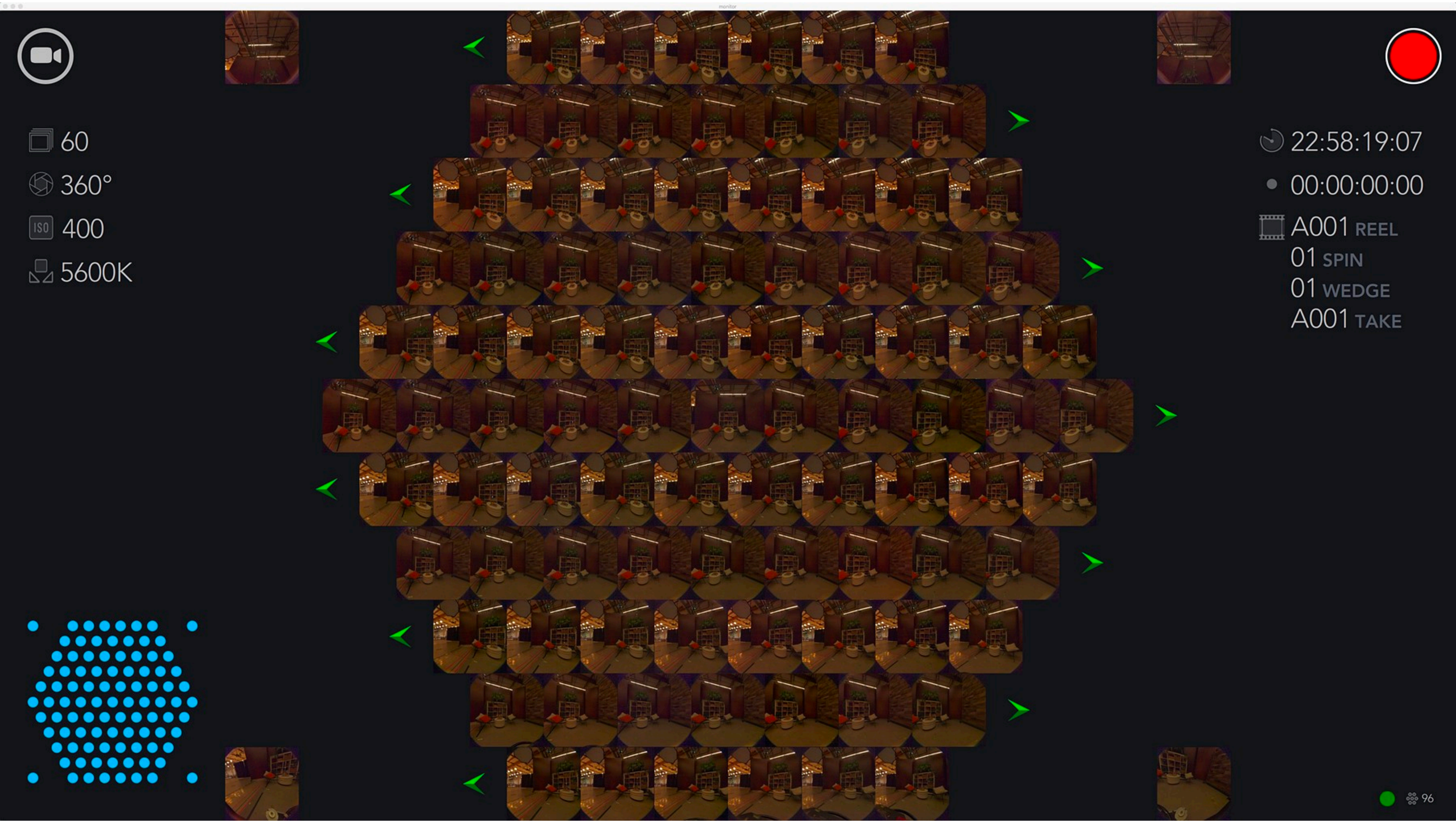




Lytro Immerge 2.0

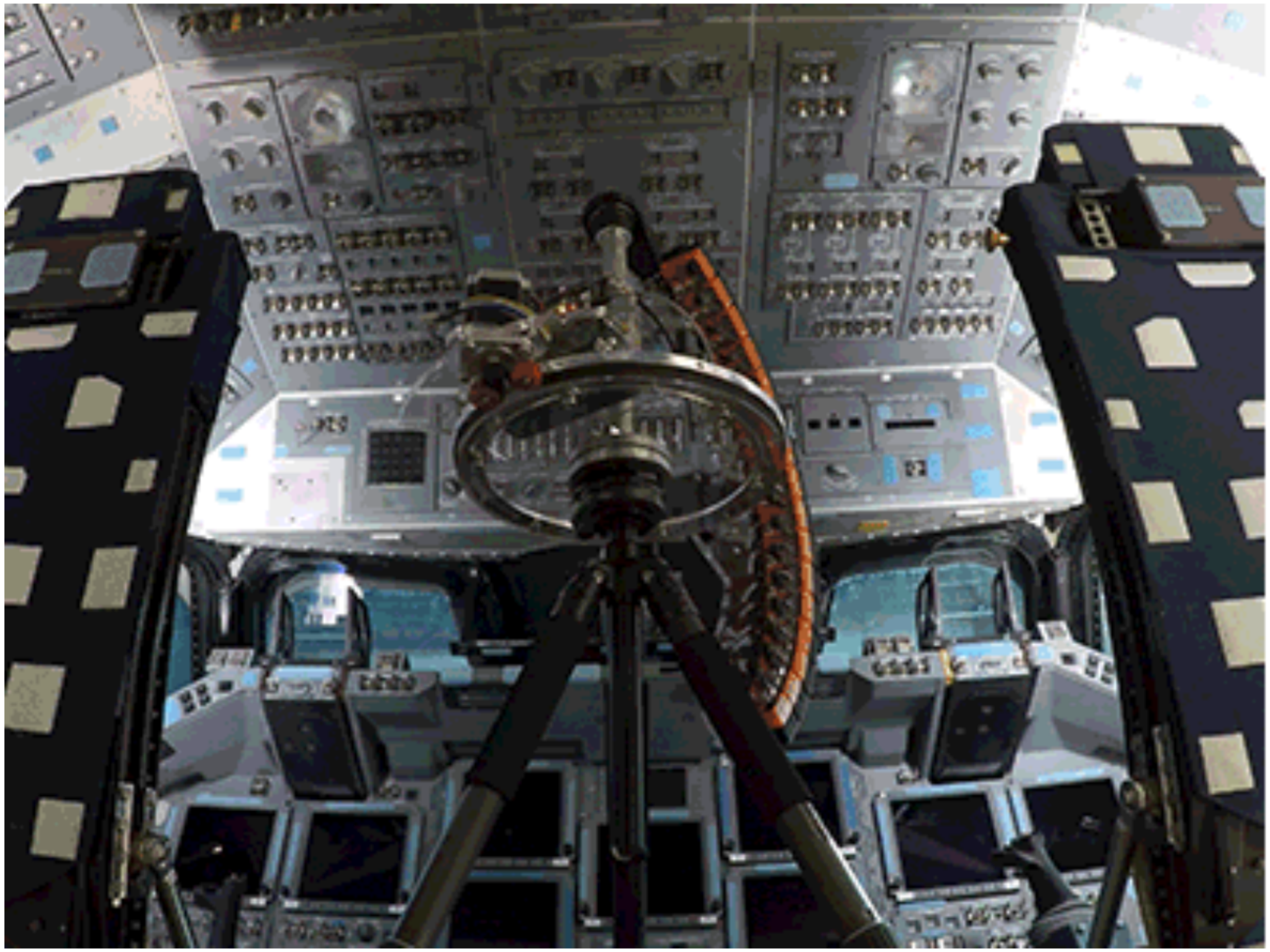


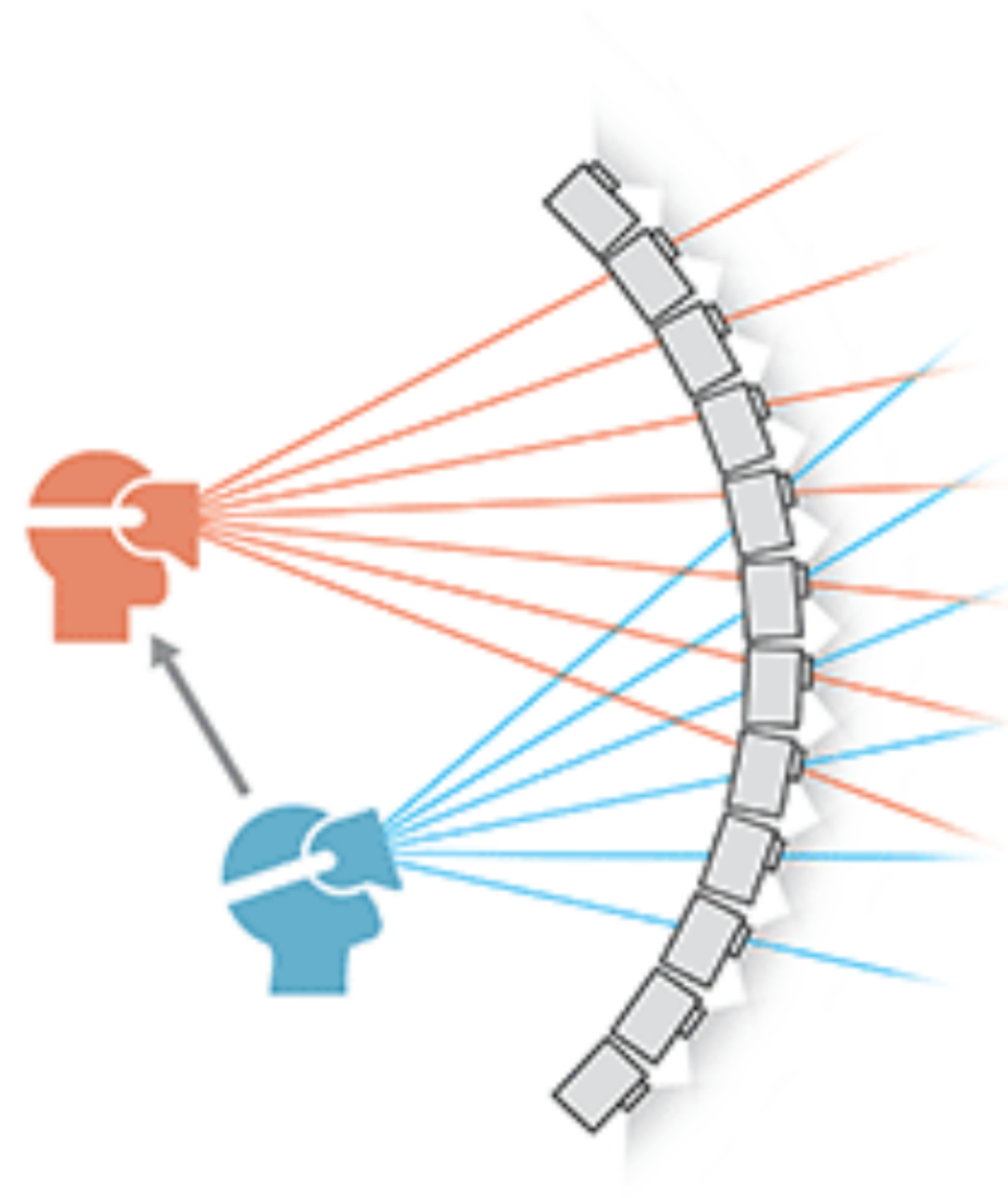
Lytro Immerge 2.0



https://www.dpreview.com/sample-galleries/1120152581/lytro-immerge-2-0/3070632200

Google VR180

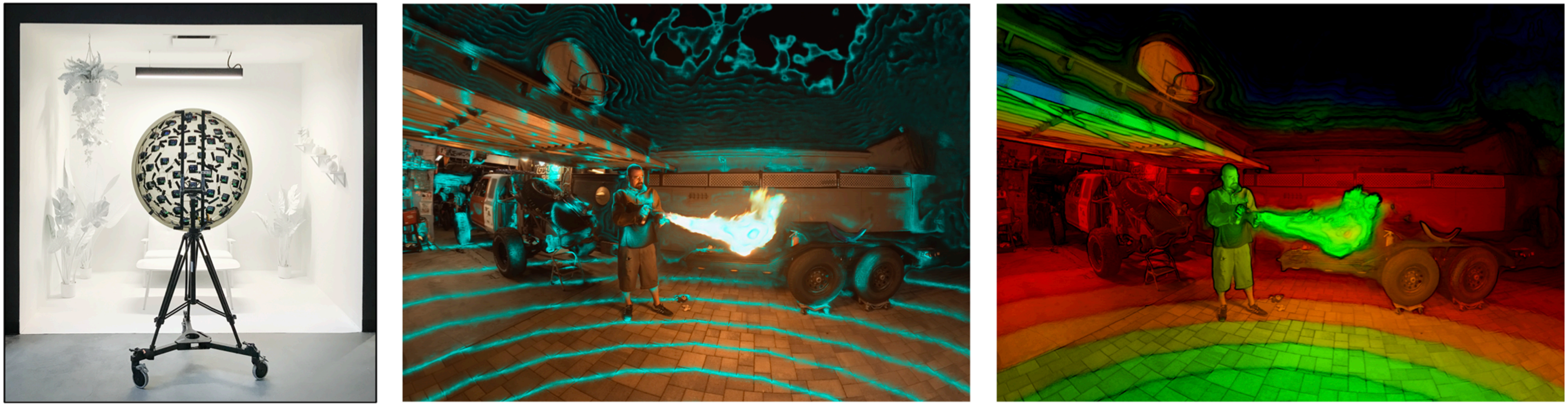




Left: A time lapse video of recording a spherical light field on the flight deck of Space Shuttle Discovery.
Right: Light field rendering allows us to synthesize new views of the scene anywhere within the spherical volume by sampling and interpolating the rays of light recorded by the cameras on the rig.

https://www.blog.google/products/google-vr/experimenting-light-fields

Light Field Video



(a) Capture Rig (b) Multi-Sphere Image (c) Layered Mesh Representation

Fig. 1. Our light field capture rig and scene representations. (a) We record immersive light field video using 46 action sports cameras mounted to an acrylic dome. (b) Using deep view synthesis we infer an RGBA multi-sphere image (MSI) from the light field views. Every 10th spherical shell is highlighted. (c) We convert groups of MSI layers into Layered Meshes (each shown as a different color), which are texture atlased and compressed into light field video.

https://augmentedperception.github.io/deepviewvideo

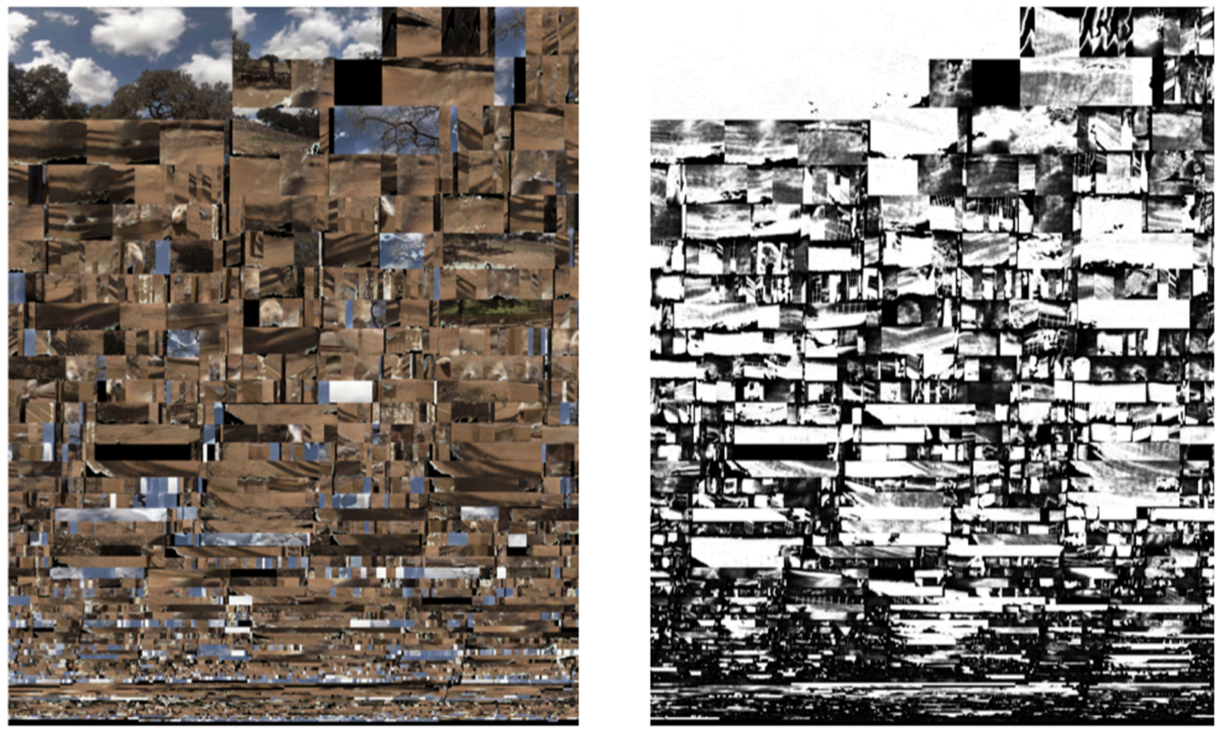


Fig. 7. An example texture atlas from the Dog scene in Figure 8.

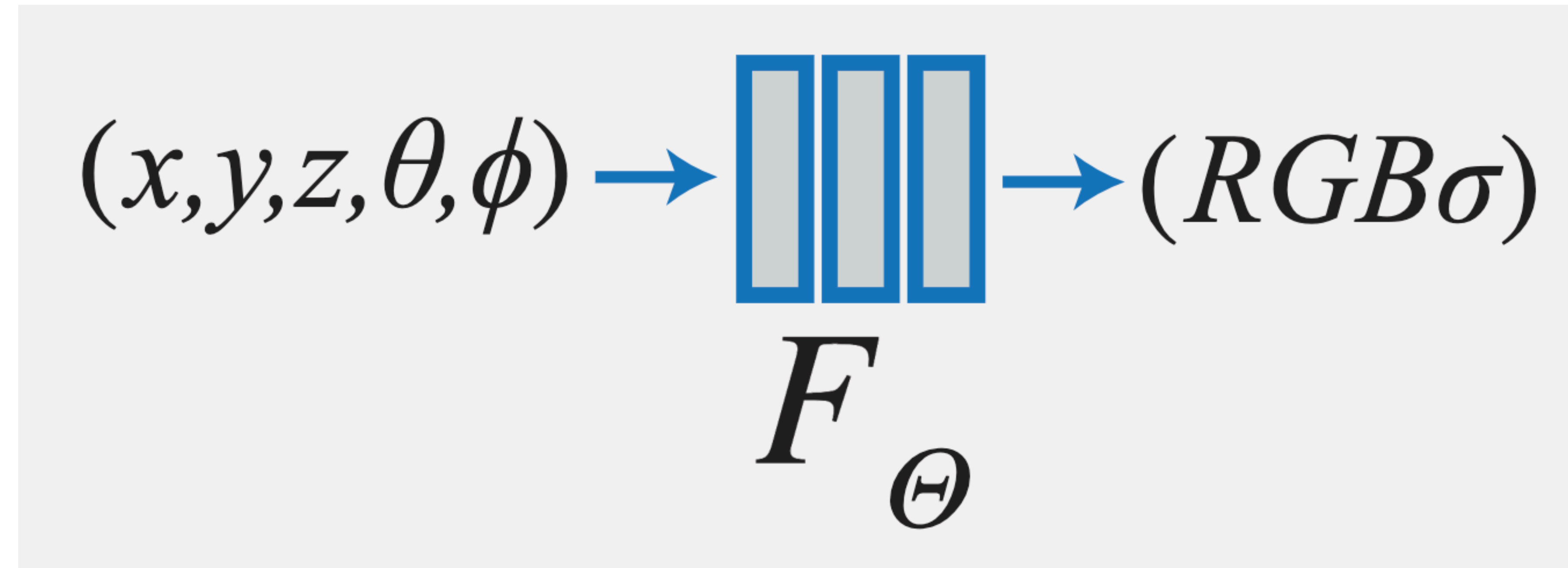
Estimating Light Fields from Images

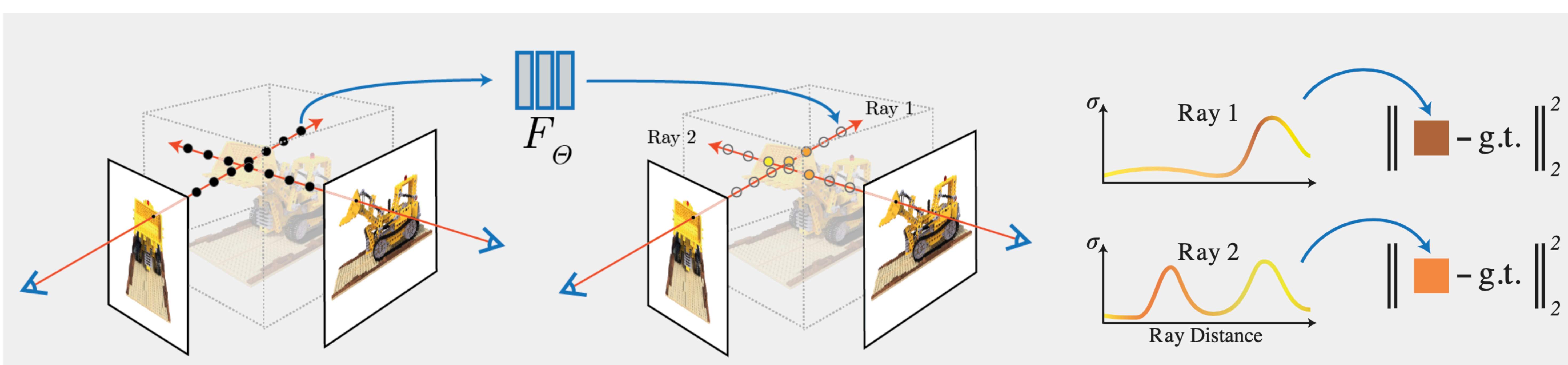
NeRF

Representing Scenes as Neural Radiance Fields for View Synthesis

ECCV 2020

https://www.matthewtancik.com/nerf





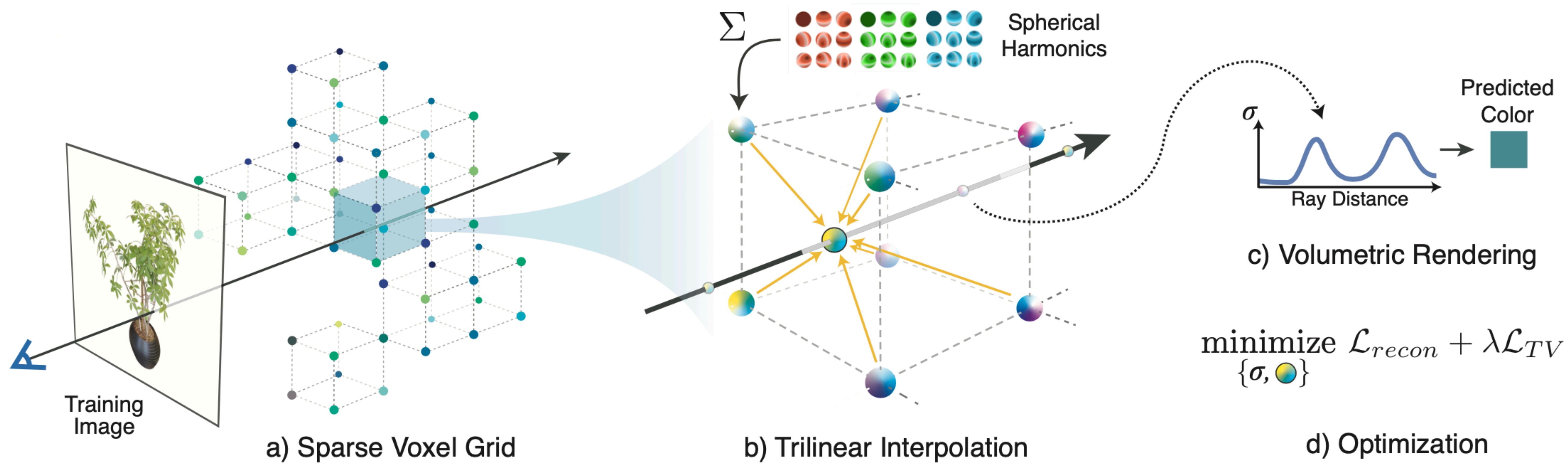
Estimating Light Fields from Images

Plenoxels

Radiance Fields without Neural Networks

CVPR 2022

https://alexyu.net/plenoxels



Properties of Radiance

Properties of Radiance

Fundamental field quantity that characterizes the distribution of light in an environment

- Radiance is the quantity associated with a ray
- Ray tracers compute radiance

Radiance $L(x, \omega)$ is invariant along a ray

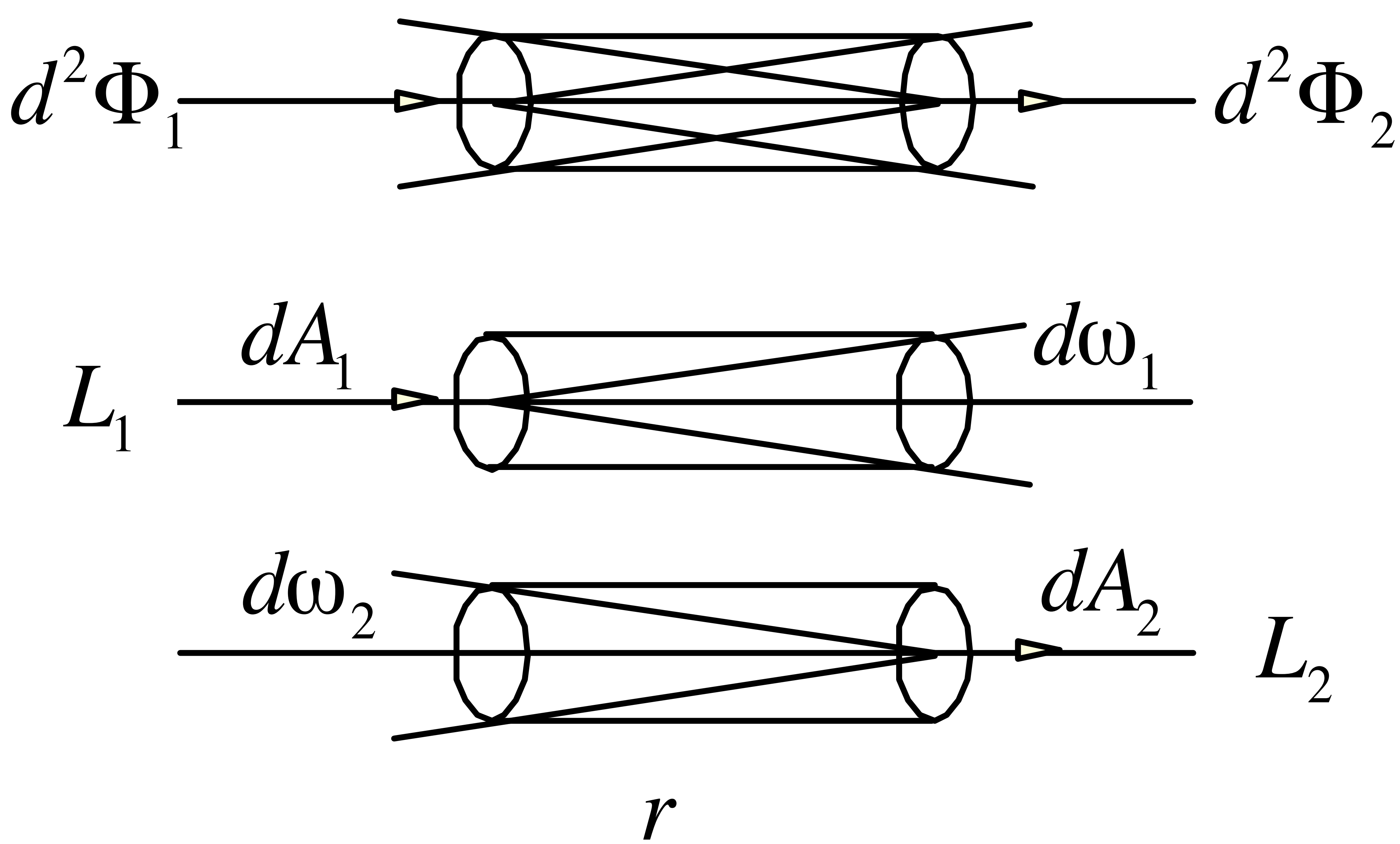
- Reduces parameters from 5D to 4D

Response of a sensor proportional to the radiance

- Cameras measure radiance

1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

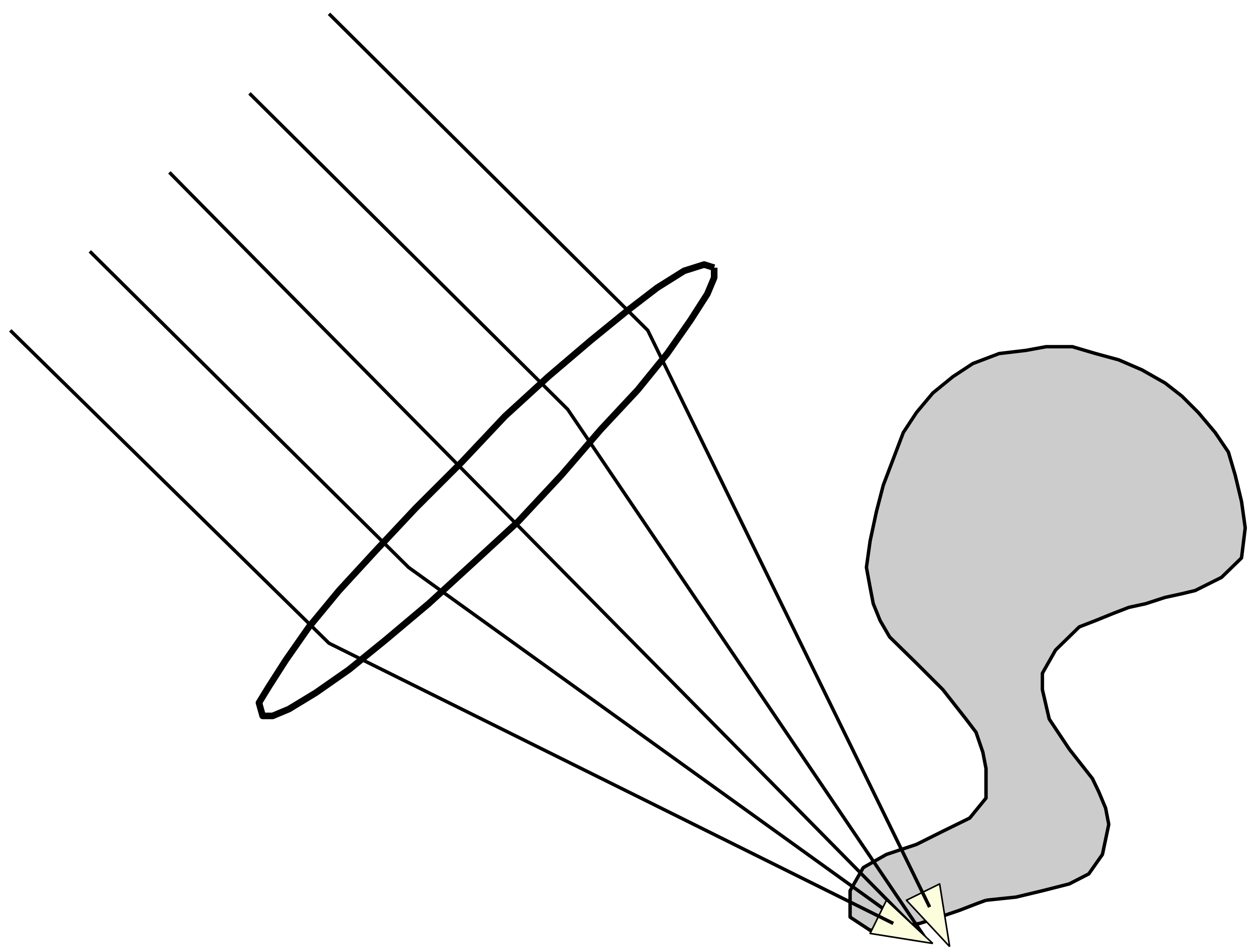
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

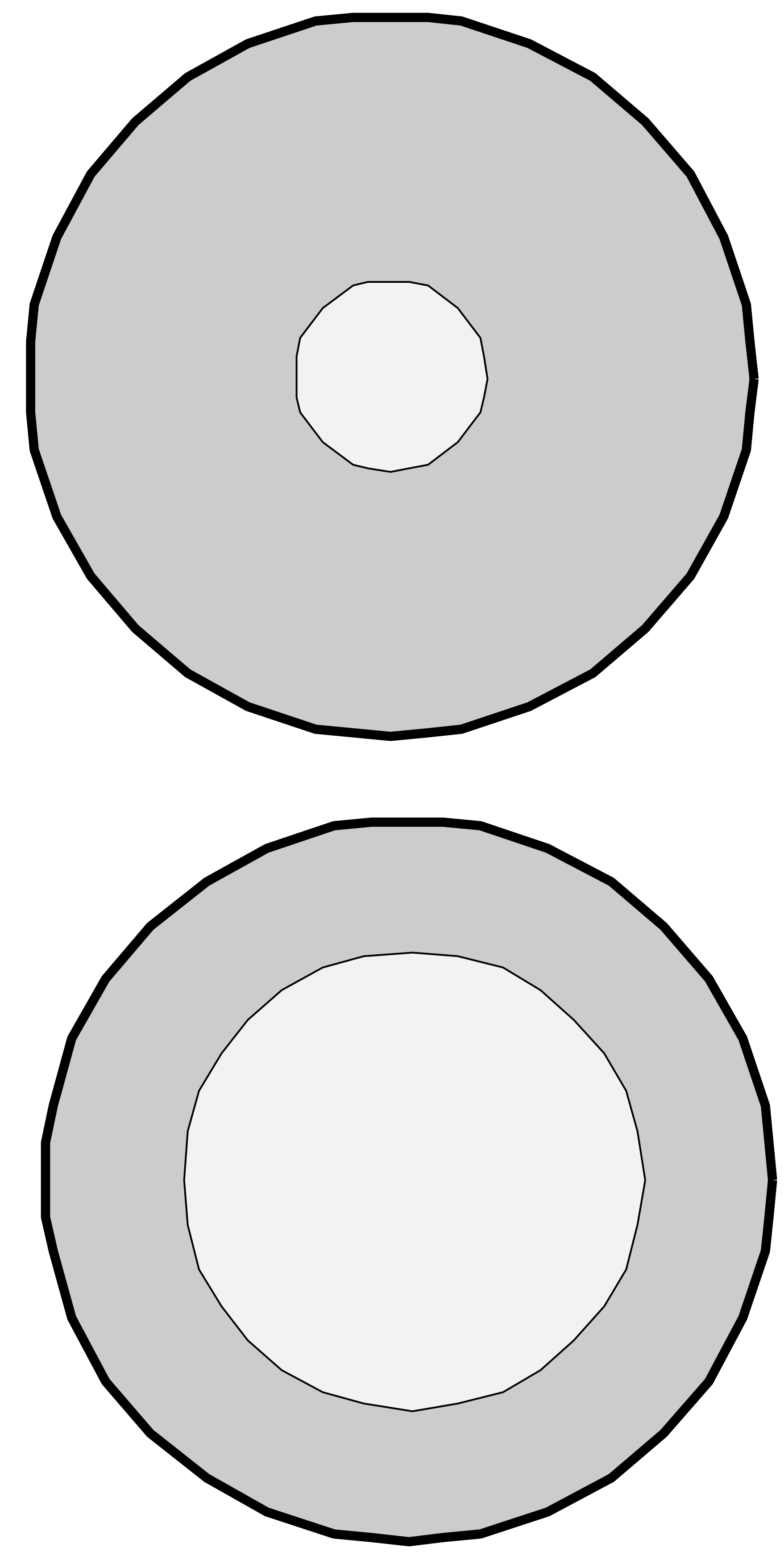
$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



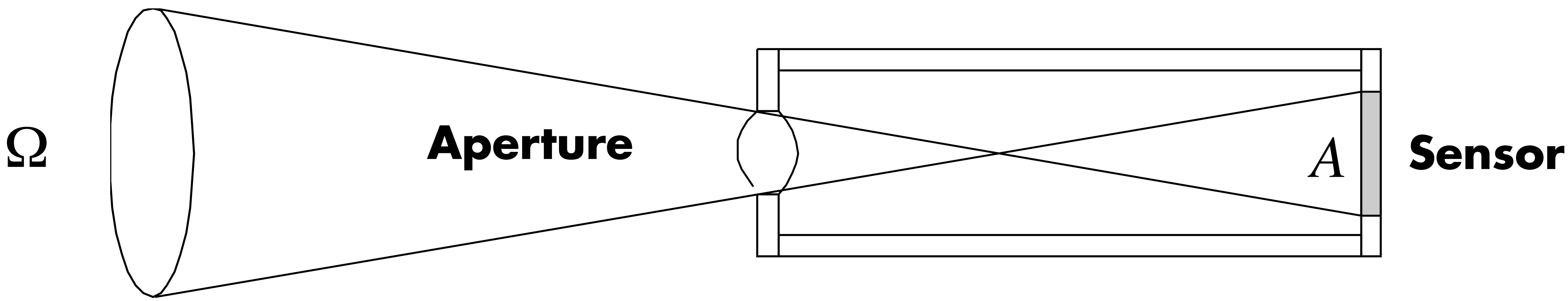
No!!



The Measurement Equation

2nd Law: Sensors measure radiance

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



$$R = \int_A \int_\Omega L \, d\omega \, dA = \bar{L} T \qquad T = \int_A \int_\Omega d\omega \, dA$$

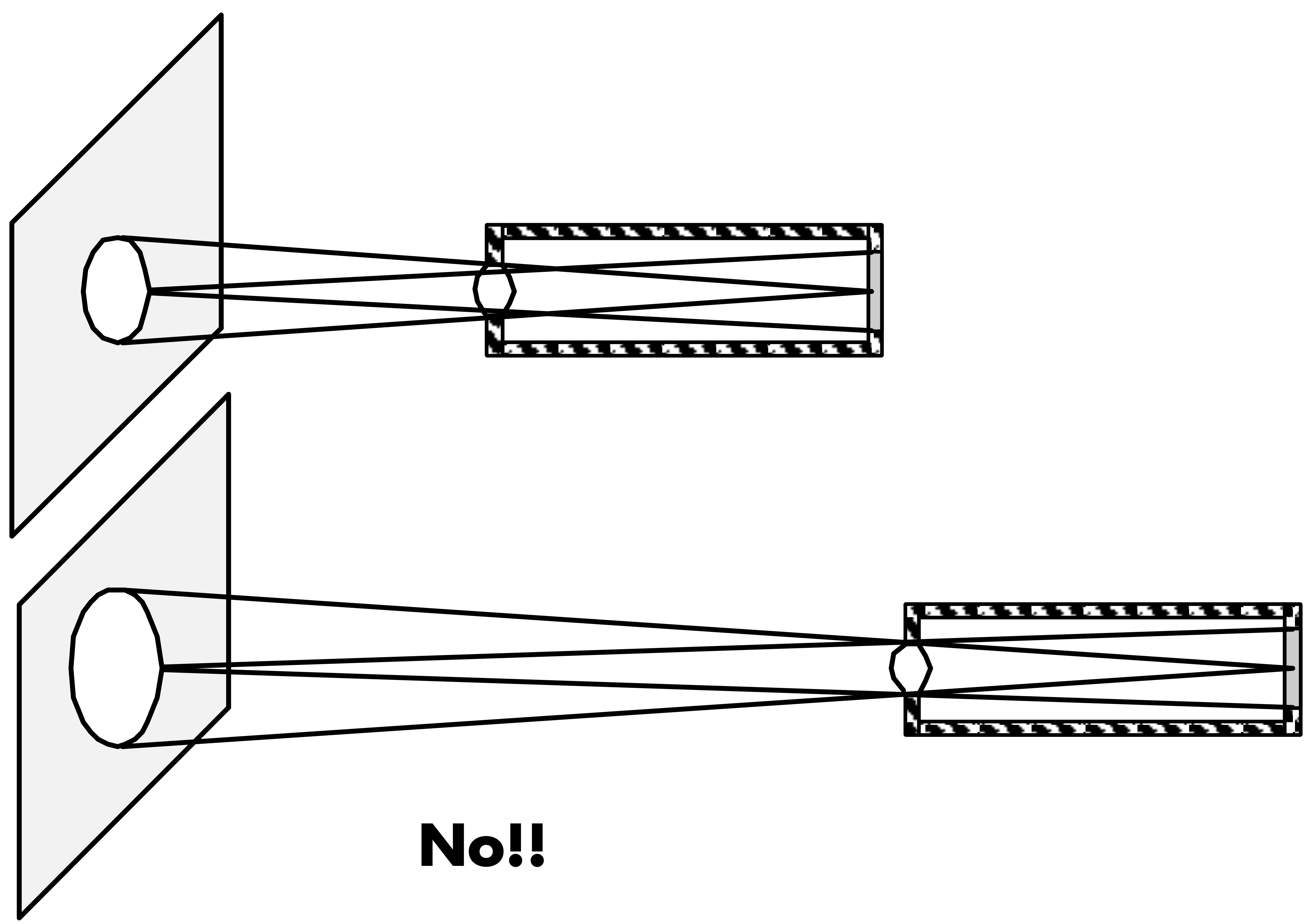
T **(throughput) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered**

T **is constant, sensor response** R **proportional to average radiance** $\bar{L}$

Quiz

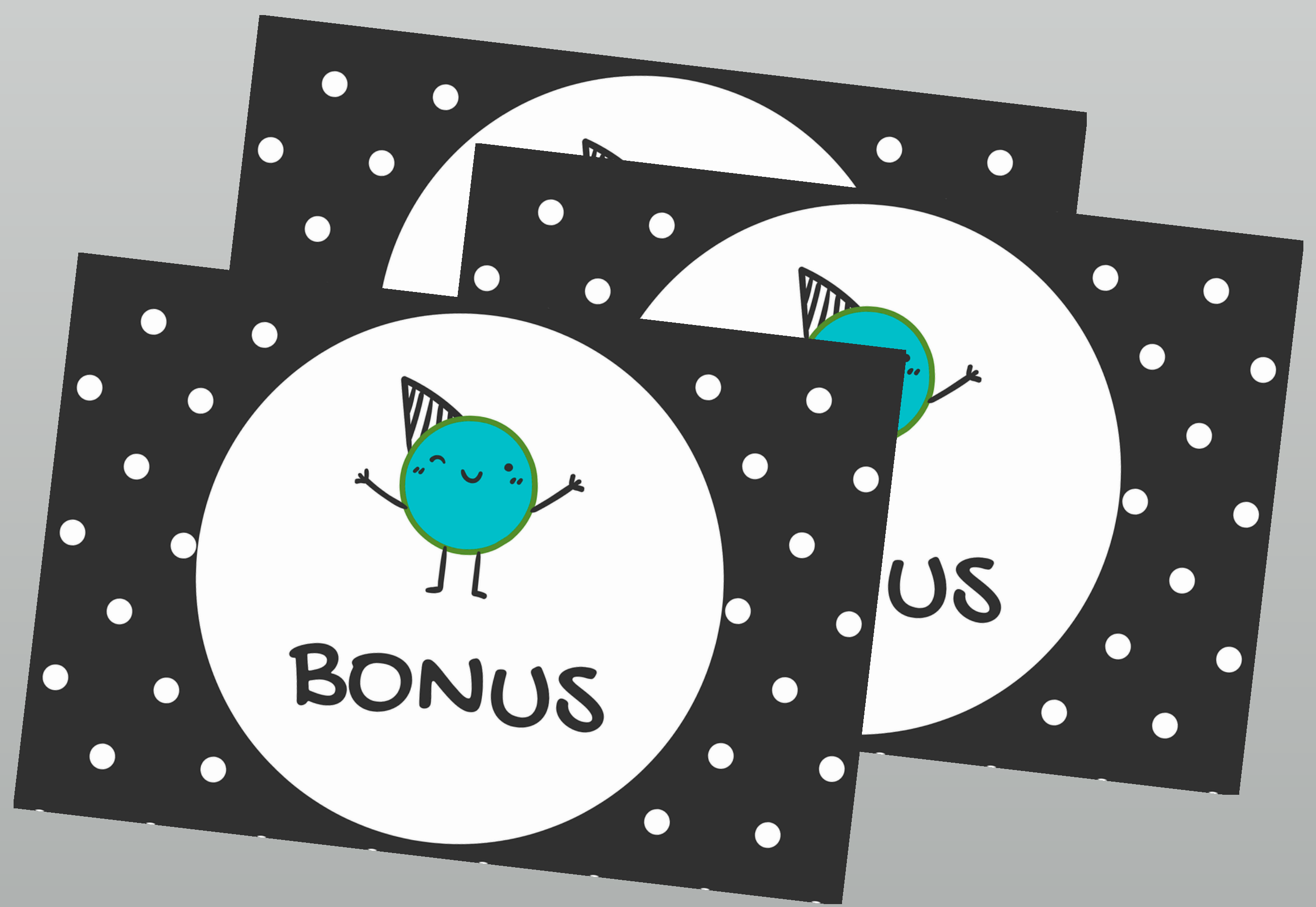
Assume a wall has constant radiance

Does the response of a sensor when pointed a wall depend on the distance to the wall?



No!!

Supplemental Material



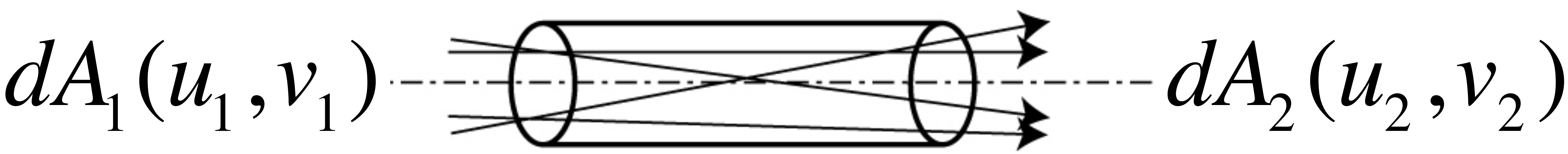
Throughput

Measuring the Number of Rays

Beam of Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$



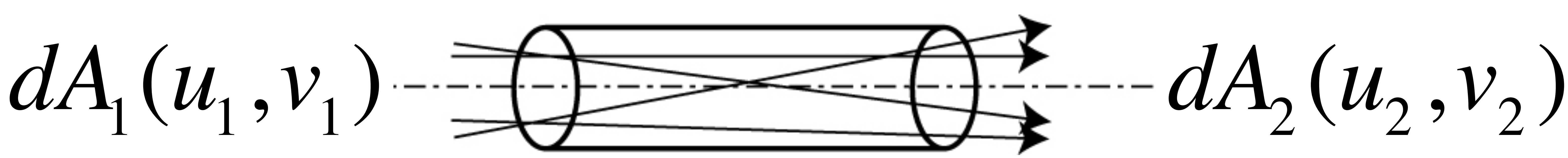
Definition:

The throughput is the number of rays in a beam.

Number of Rays in a Beam

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$

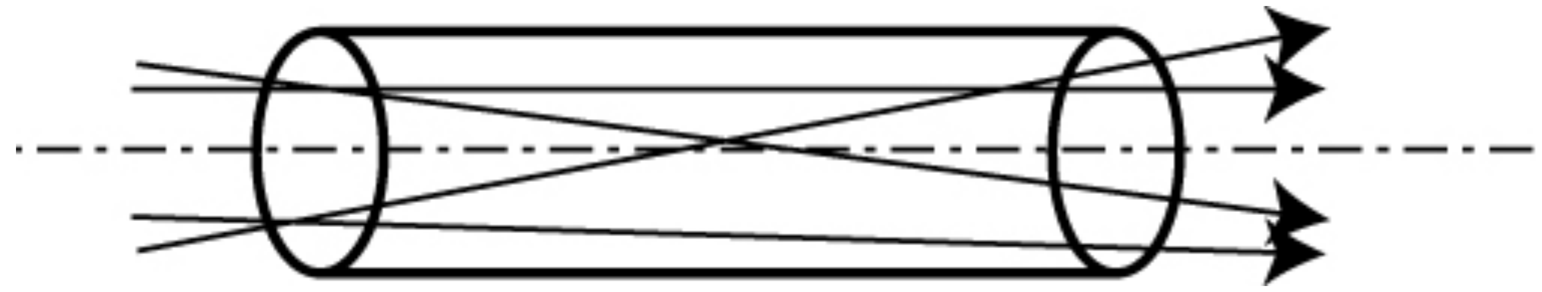


How many rays are in this beam?

This many: $d^2T = \frac{dA_1 dA_2}{\left|x_1 - x_2\right|^2}$

Why is this the Right Measure?

$dA_1(u_1, v_1)$



$dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2}$$

Suppose we double the area dA_1 (or dA_2), then the number of rays in the beam should double

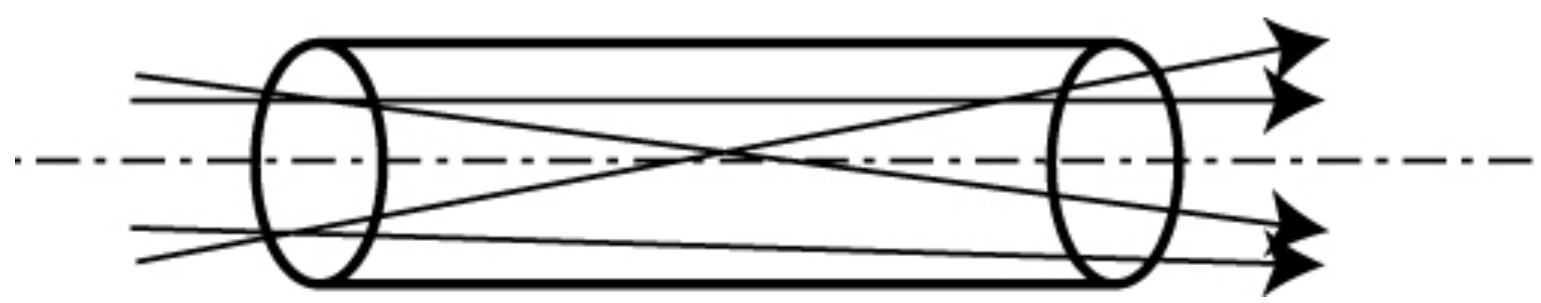
Suppose we half the distance $|x_1 - x_2|^2$, then the number of rays also doubles

Alternative Definition of Radiance

Two physical laws

- Energy is conserved $\Delta\Phi$
- The size of a beam ΔT does not change as rays propagate

Beam



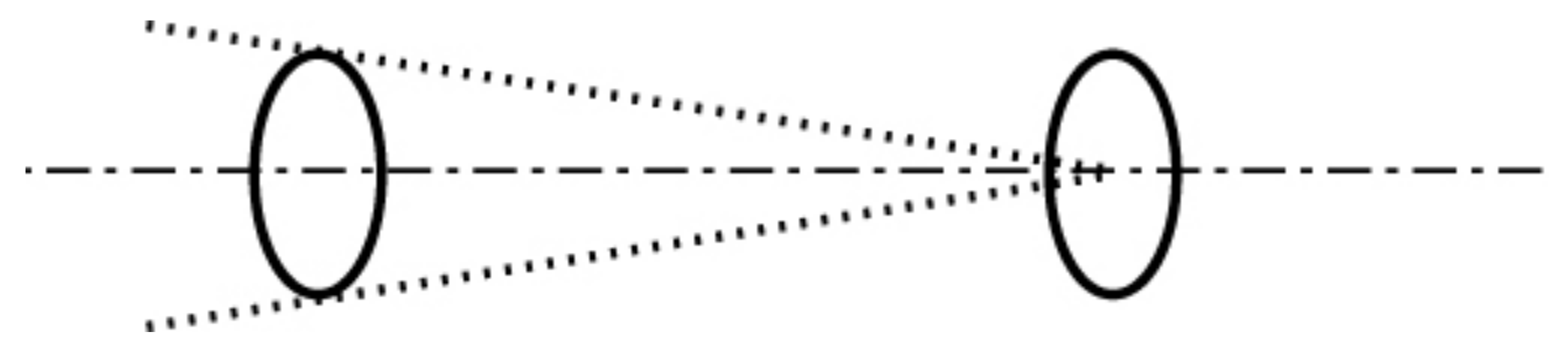
$$L(r) = \lim_{\Delta T \to r} \frac{\Delta\Phi}{\Delta T}$$

Radiance is the ratio of 2 conserved quantities, therefore, the radiance is also conserved

Changing Ray Coordinates

Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

$d\omega_2(\theta_2, \phi_2)$



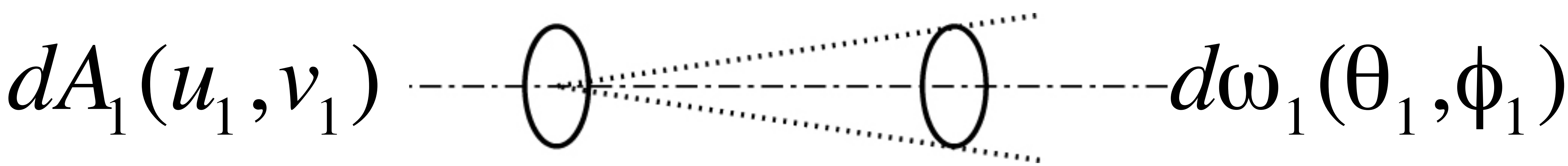
$dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1}{\left|x_1 - x_2\right|^2} dA_2 = d\omega_2 dA_2$$

Same value for the throughput, different formula

Changing Ray Coordinates

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

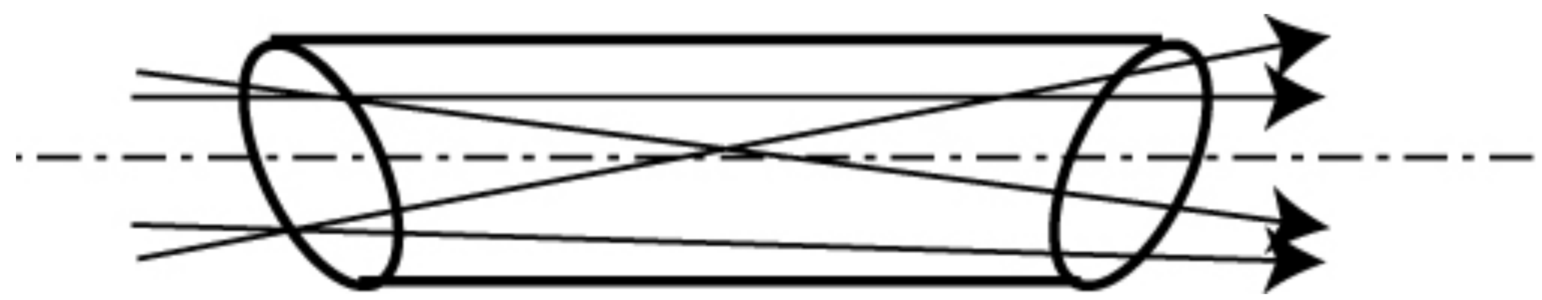


$$d^2T = dA_1 \frac{dA_2}{\left|x_1 - x_2\right|^2} = dA_1 d\omega_1$$

Changing Ray Coordinates Again

Tilting the surfaces re-parameterizes the rays!

$$r(u_1, v_1, u_2, v_2)$$

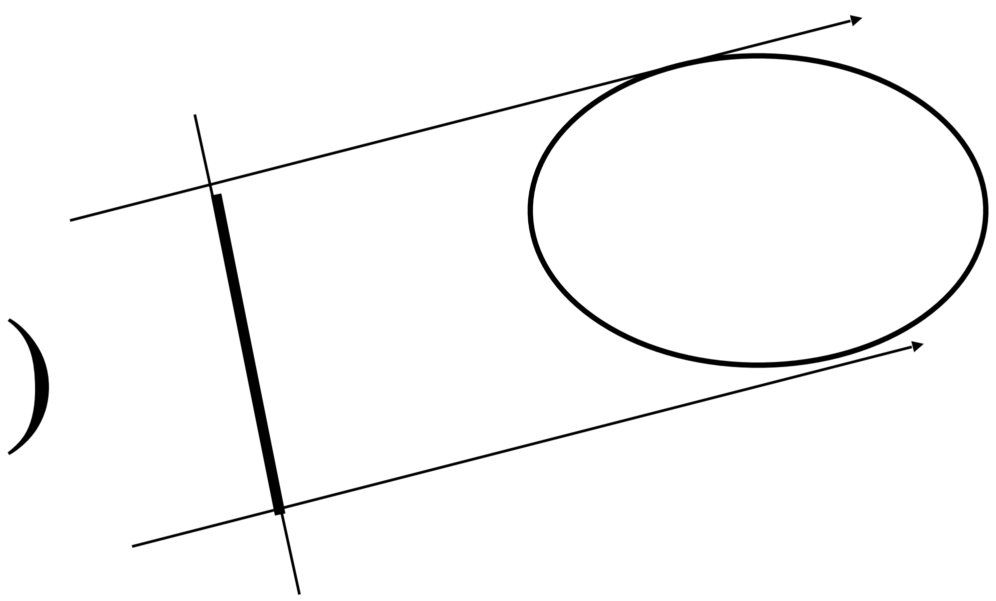
$dA_1(u_1, v_1)$  $dA_2(u_2, v_2)$

$$d^2T = \frac{\cos\theta_1 \cos\theta_2}{|x_1 - x_2|^2} dA_1 dA_2$$

Number of Rays Hitting a Shape

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\vec{\omega})$



Measuring the number or rays that hit a shape

$$T = \int_{S^2} d\omega(\theta,\varphi) dA(x,y) = \int_{S^2} d\omega(\theta,\varphi) \int_{R^2} dA(x,y)$$

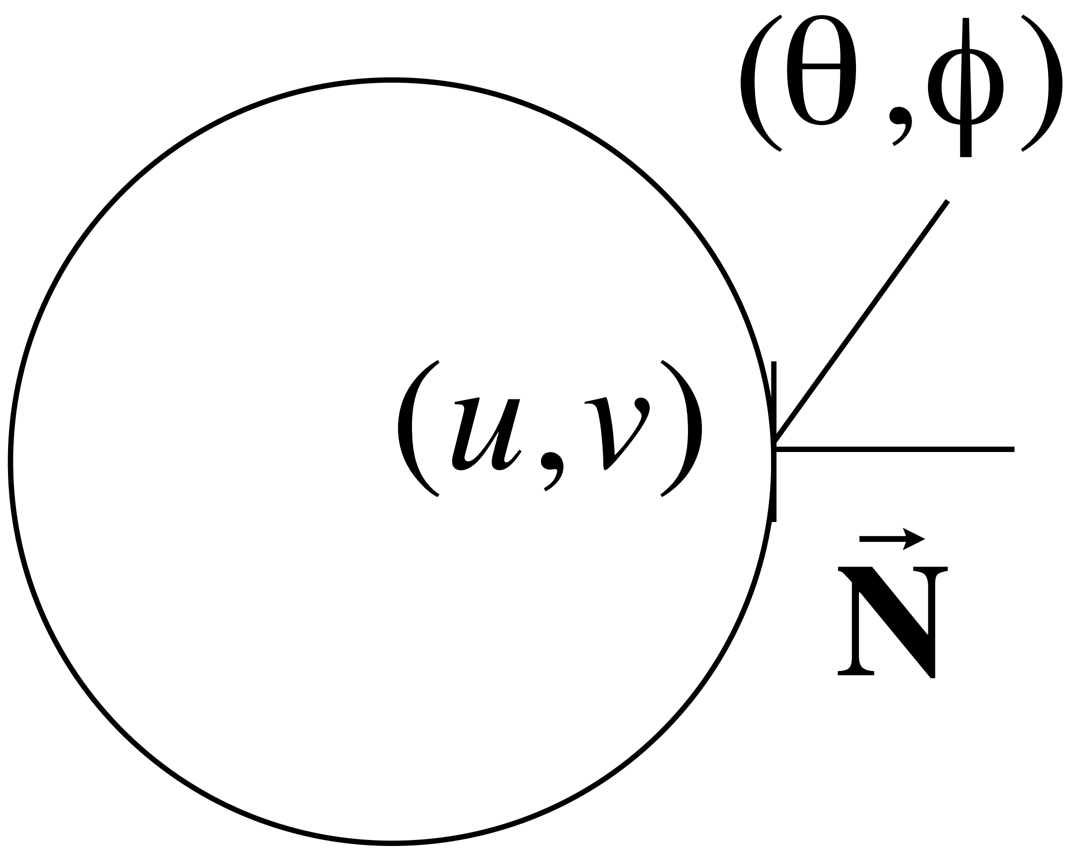
$$= \int_{S^2} \tilde{A}(\theta,\varphi)\, d\omega(\theta,\varphi)$$

$$= 4\pi \bar{\tilde{A}}$$

Sphere: $T = 4\pi \bar{\tilde{A}} = 4\pi^2 R^2$

Calculated Another Way

Parameterize rays by $r(u, v, \theta, \phi)$



$$T = \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta, \varphi) \right]}_{\pi} \underbrace{\left[\int_{M^2} dA(u, v) \right]}_{S}$$

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \frac{S}{4}$

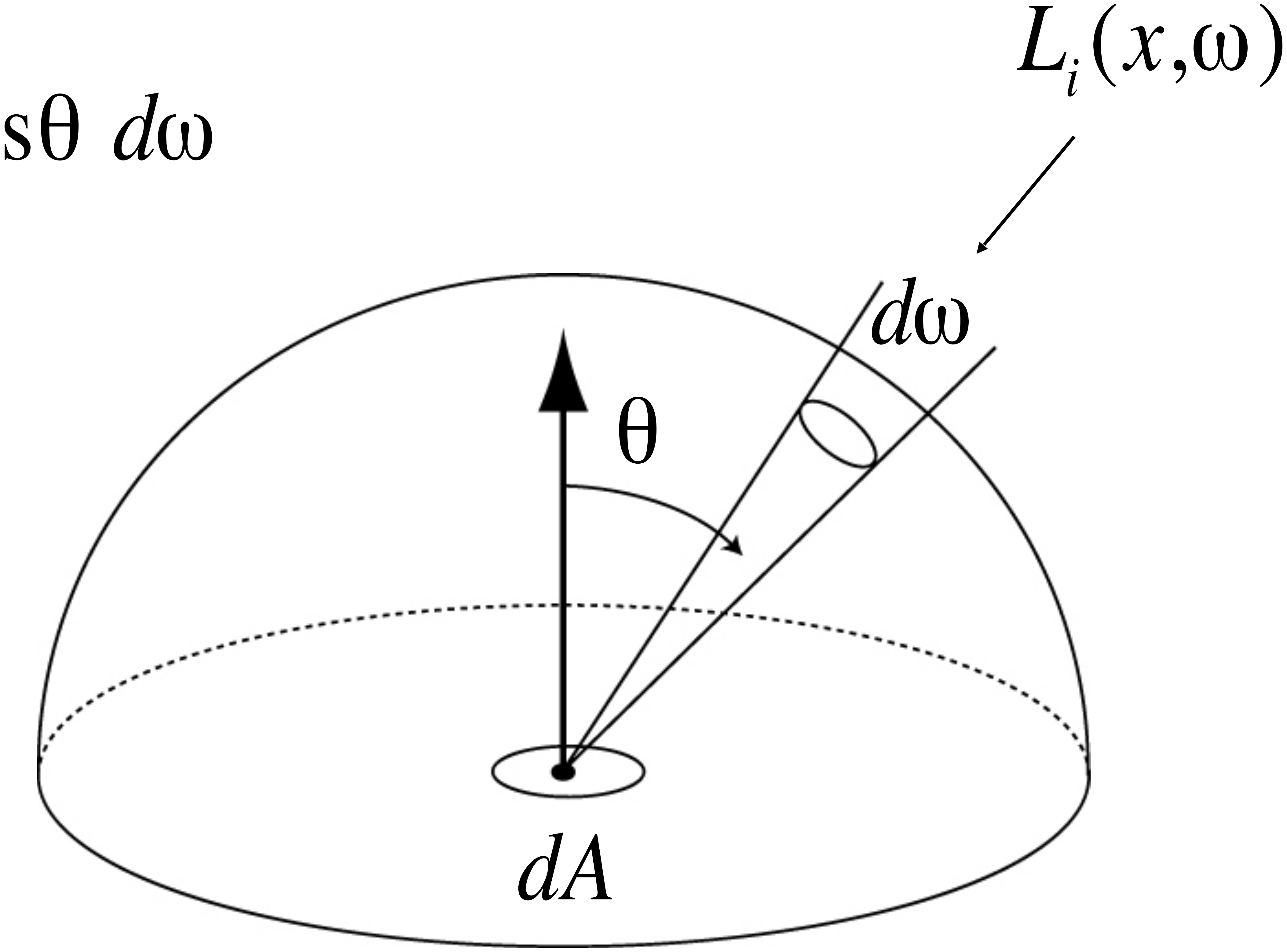
Irradiance
from a
Uniform Area Light Source

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta\ d\omega$$

Light meter



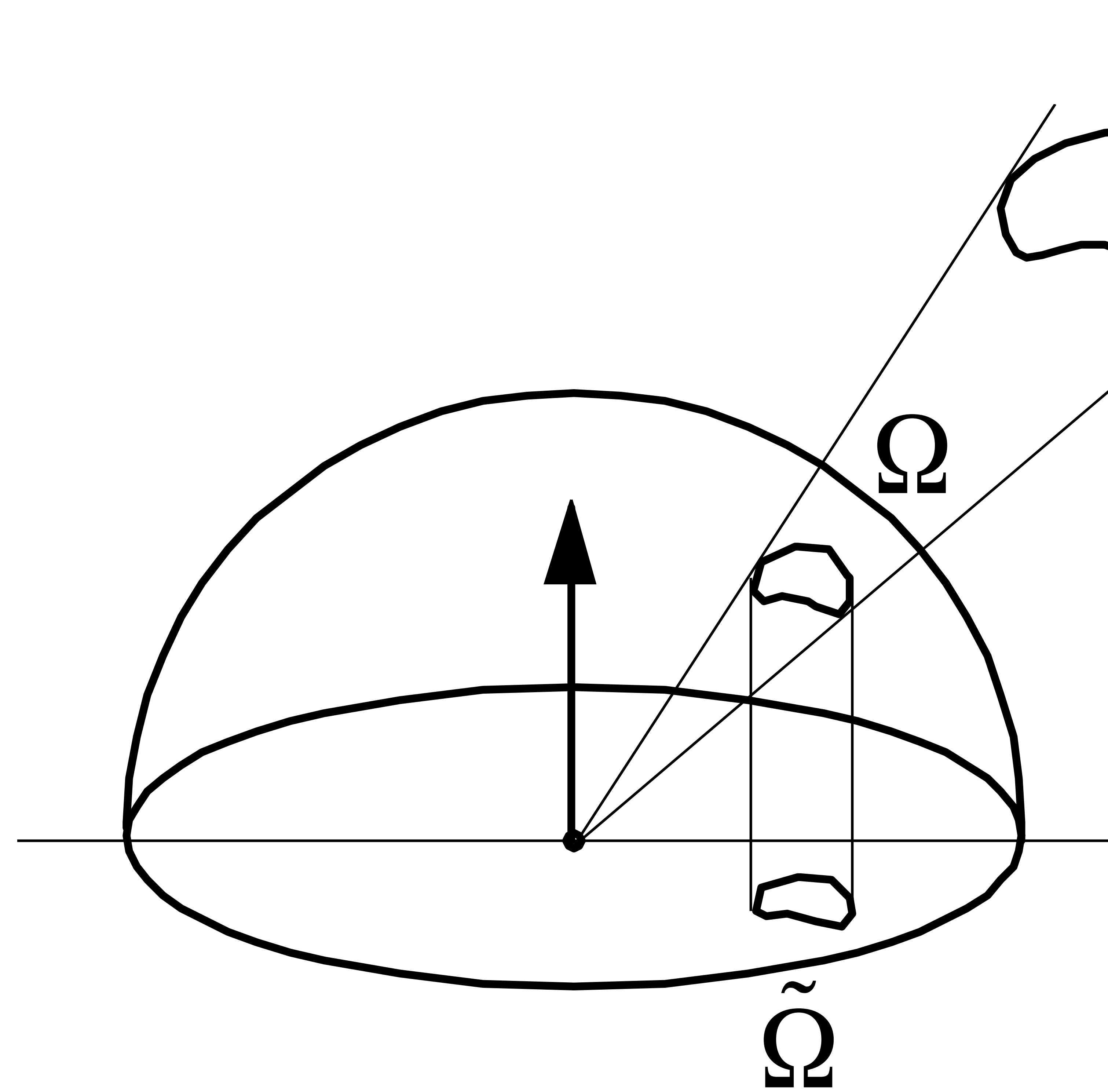
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\ d\omega$$

Irradiance from a Uniform Area Source

$$E(x) = \int_{H^2} L \cos\theta \, d\omega$$

$$= L \int_{\Omega} \cos\theta \, d\omega$$

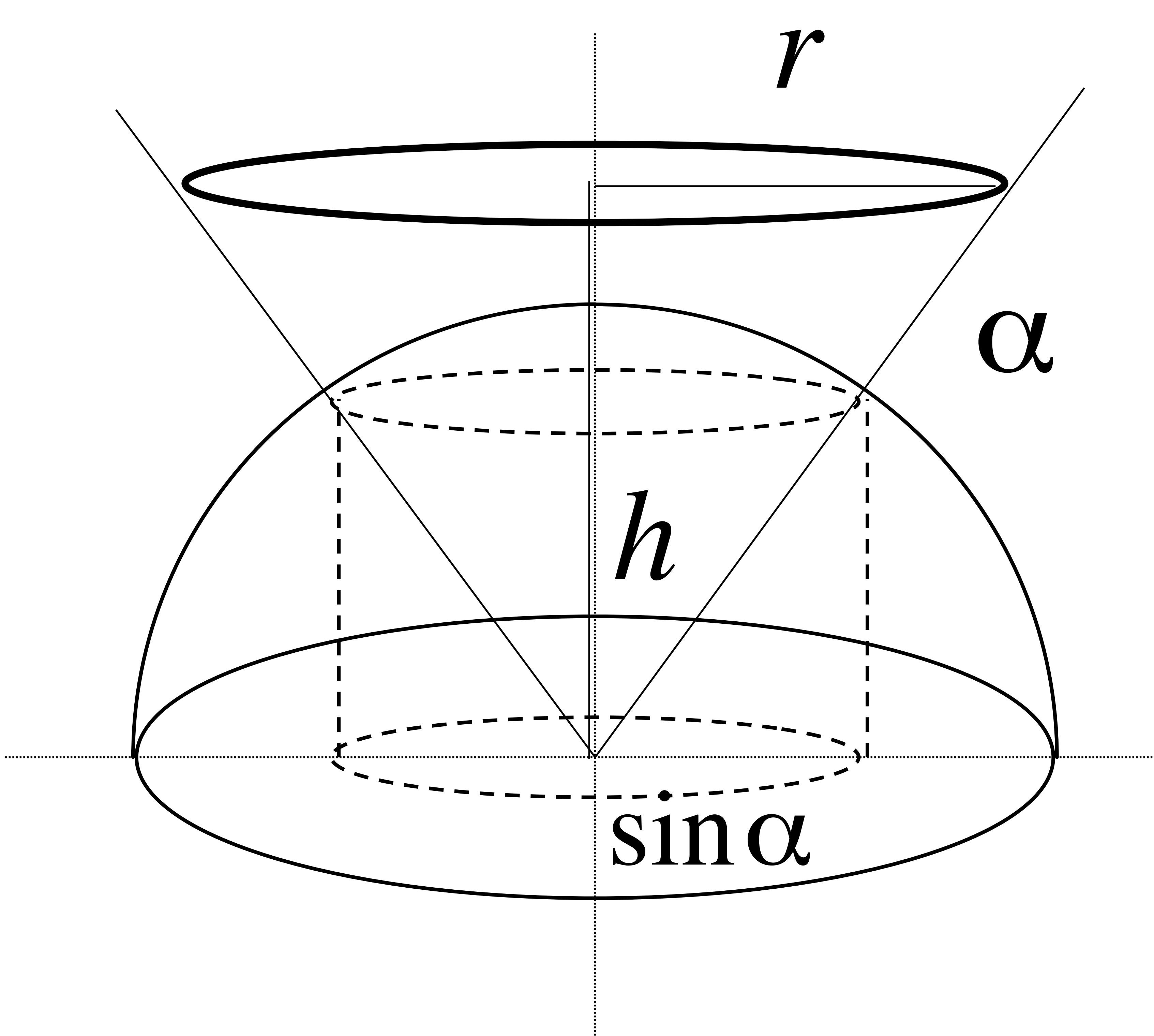
$$= L\tilde{\Omega}$$



Direct Illumination

Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \; d\phi \, d\cos\theta$$

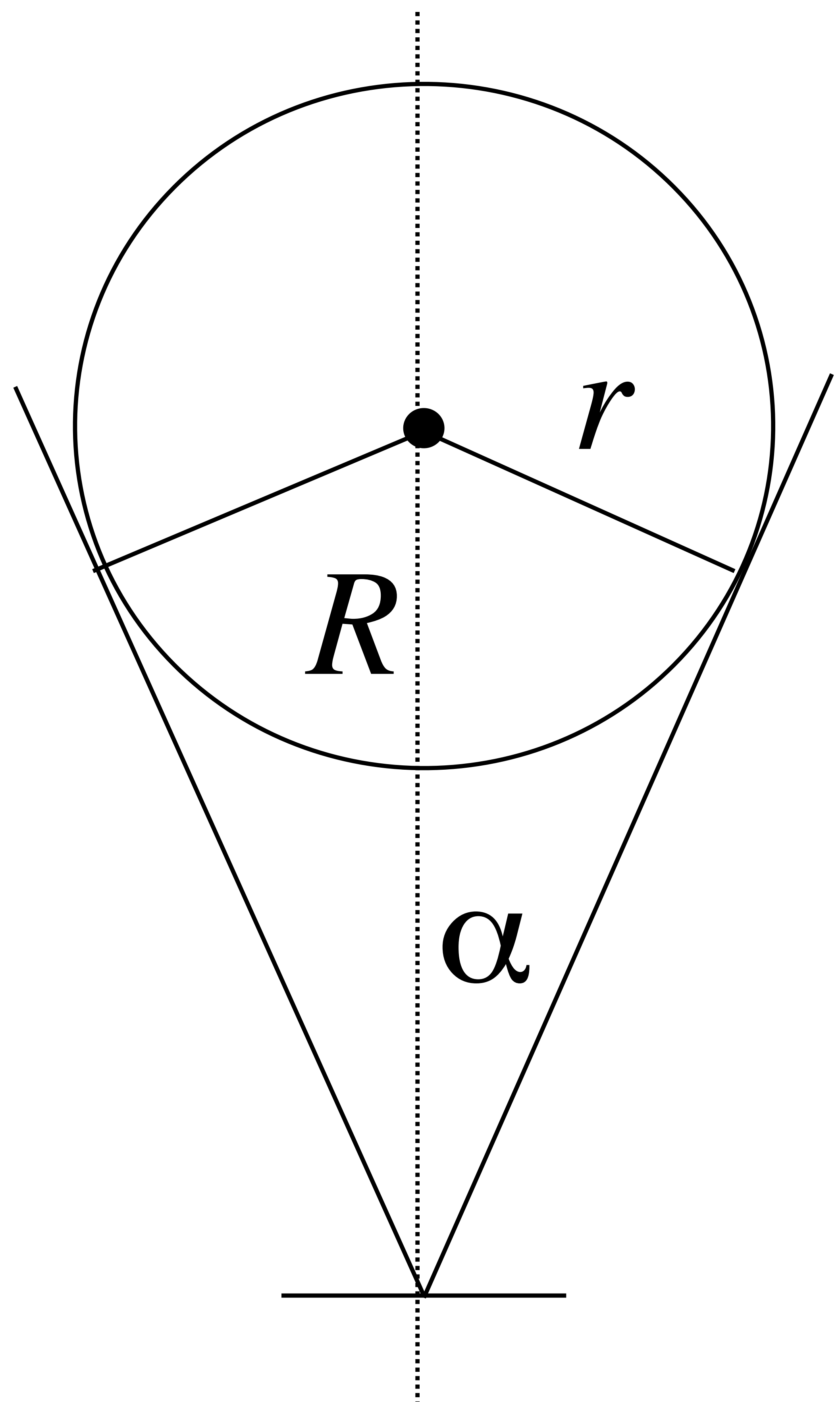
$$= 2\pi \left. \frac{\cos^2\theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{R^2}$$

The Sun

Solar constant (normal incidence at zenith)

Irradiance **1353 W/m²**

Illuminance **127,500 lm/m² = 127.5 kilolux**

Solar angle

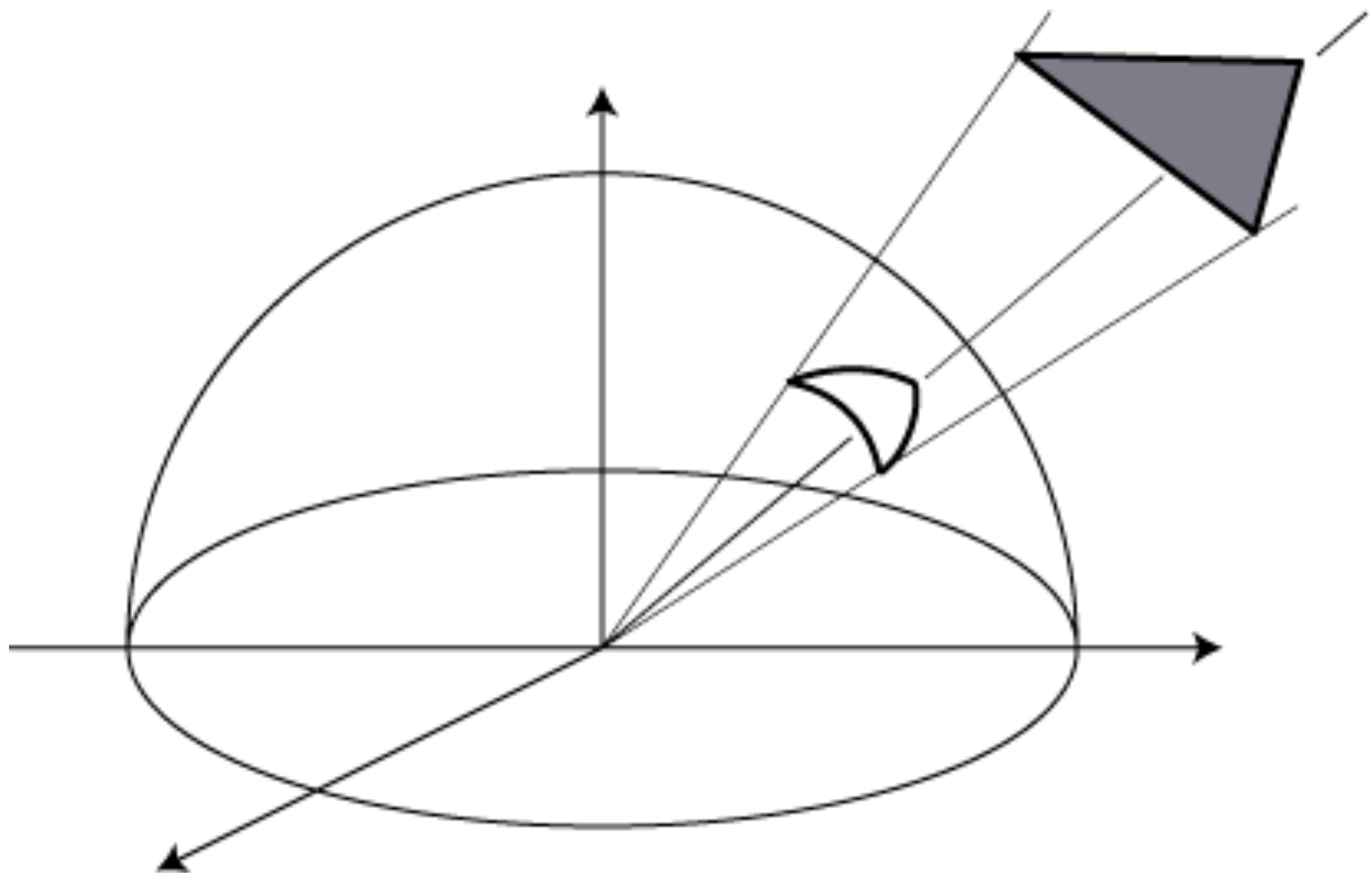
α **= .25 degrees = .004 radians (half angle)**

$\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \alpha^2$ **= 6 x 10-5 steradians**

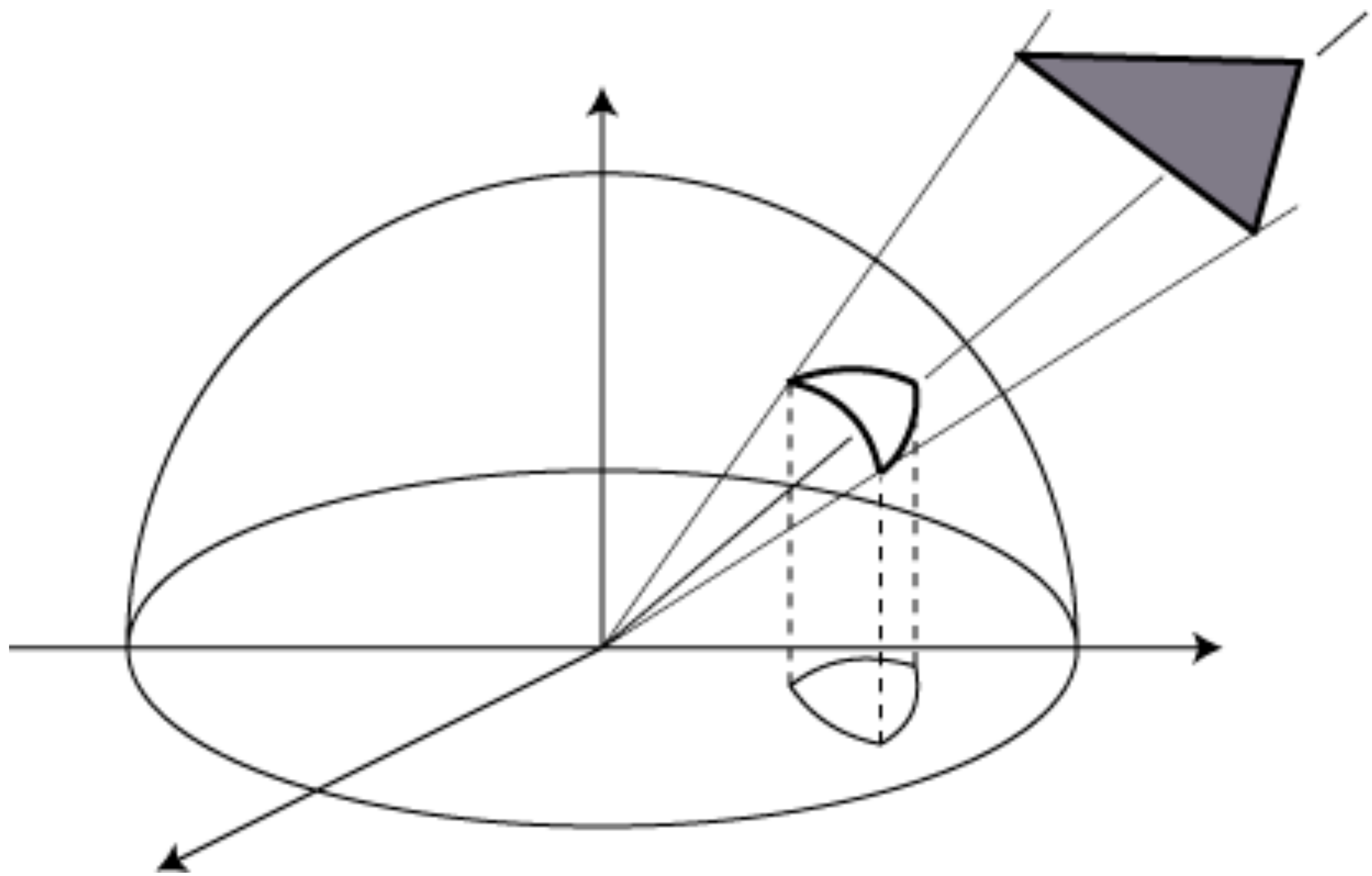
Solar radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W / m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \frac{\mathrm{W}}{m^2 \cdot sr}$$

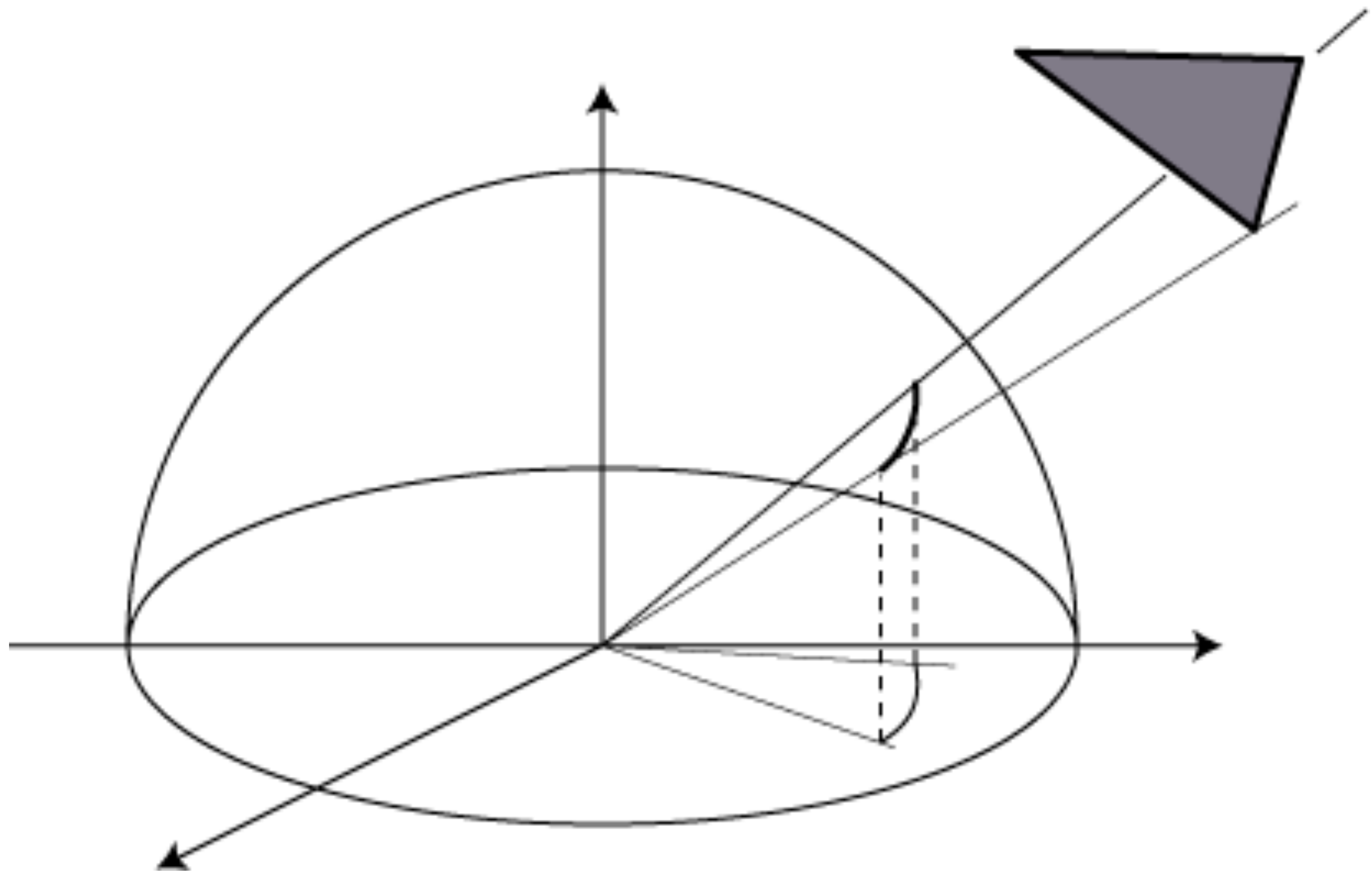
Polygonal Source



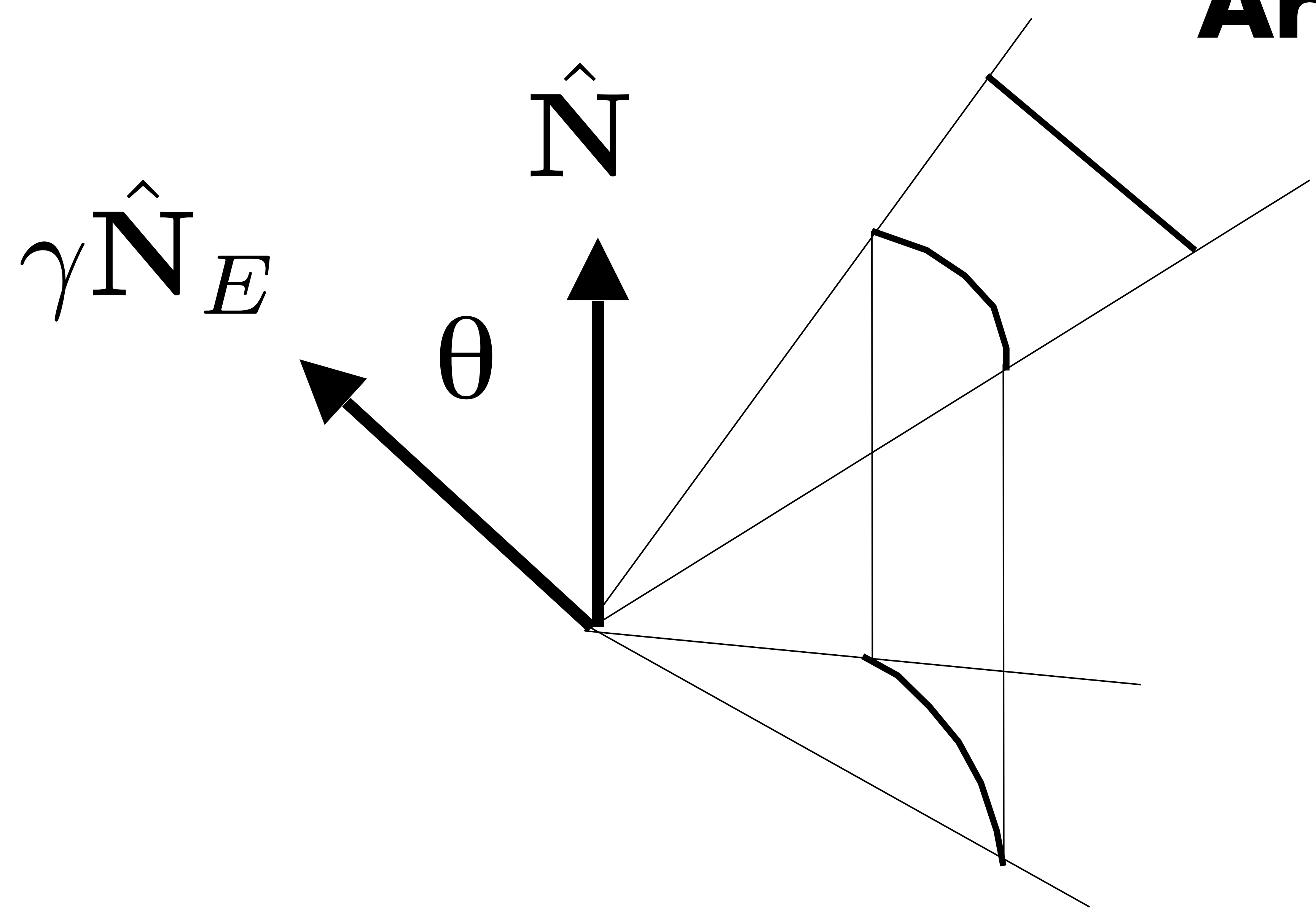
Polygonal Source



Polygonal Source



Consider 1 Edge

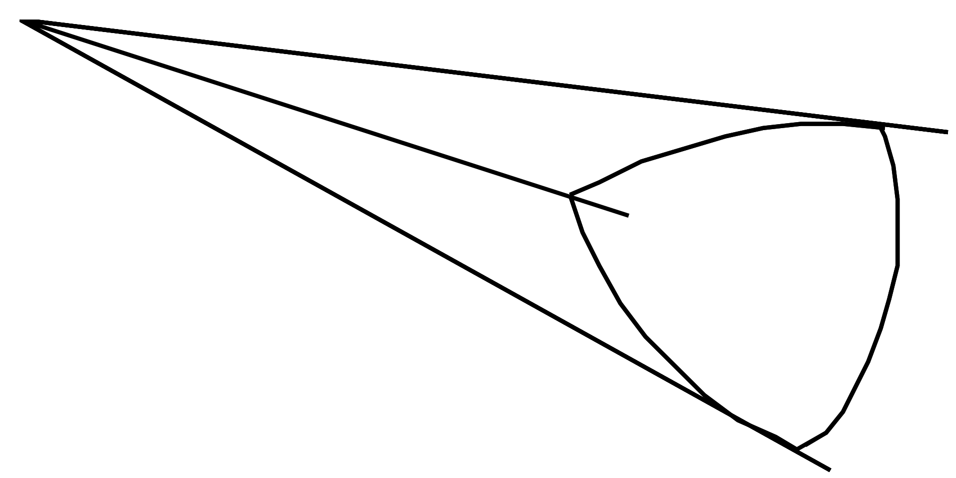


Area of sector γ

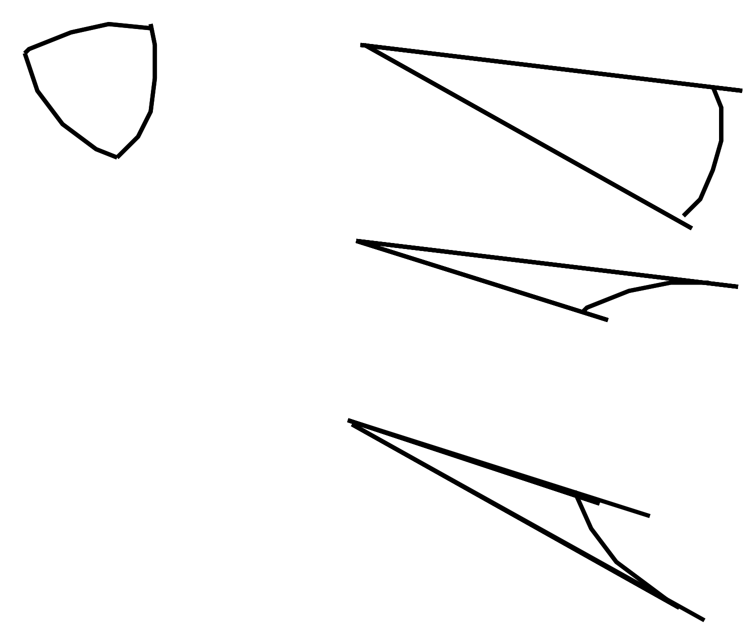
Projected area of sector

$$A = \gamma \cos\theta = \gamma \vec{\mathbf{N}}_E \bullet \vec{\mathbf{N}}$$

Lambert's Formula



$$\sum_{i=1}^{3} A_i$$



$$A_1 - A_2 - A_3$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \, \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

Penumbras and Umbras