Reflection Models I: Ideal Materials

Today

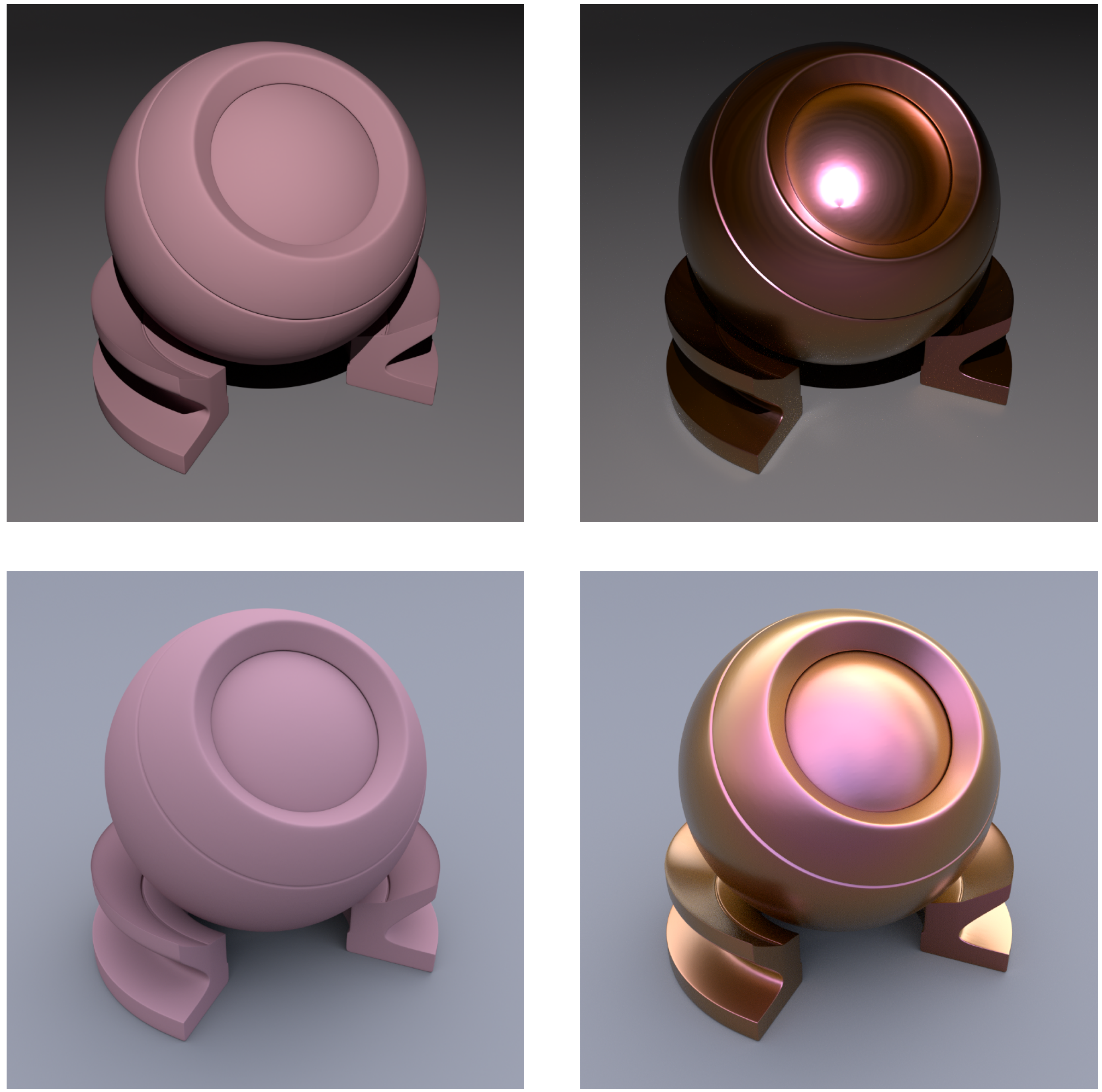
- The reflection equation
- The BRDF and reflectance
- Types of reflection models
- Ideal reflection and refraction
- Fresnel effect
- Diffuse reflection

Next lecture

- Rough surfaces and microfacet distributions

Surface Appearance

Illumination



Reflection

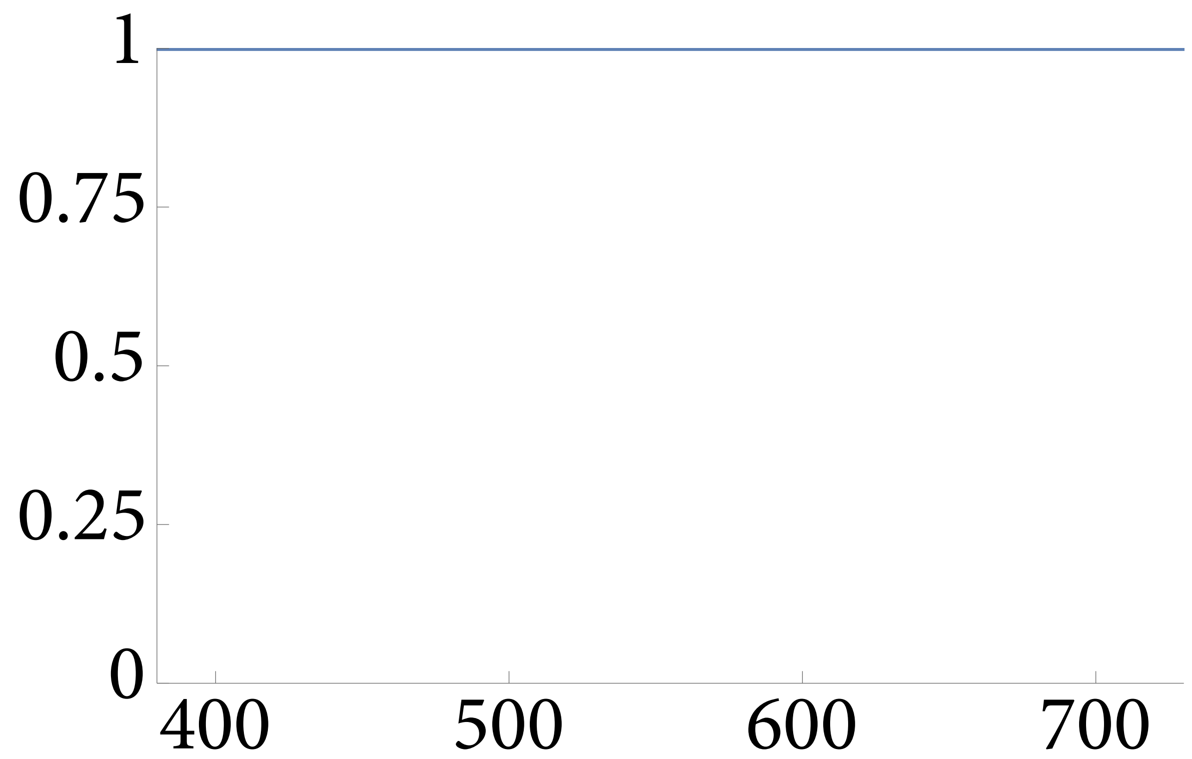
Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

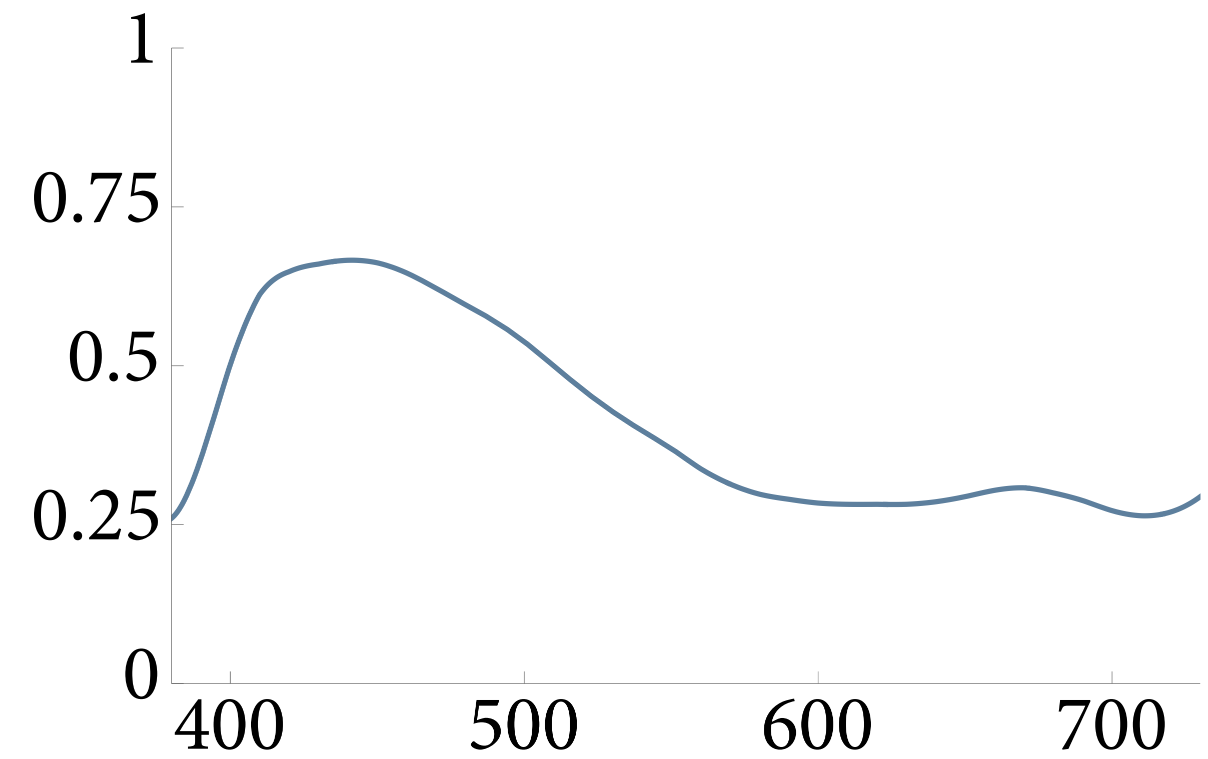
- **Spectral distribution**
- **Polarization**
- **Directional distribution**

Spectral Reflection

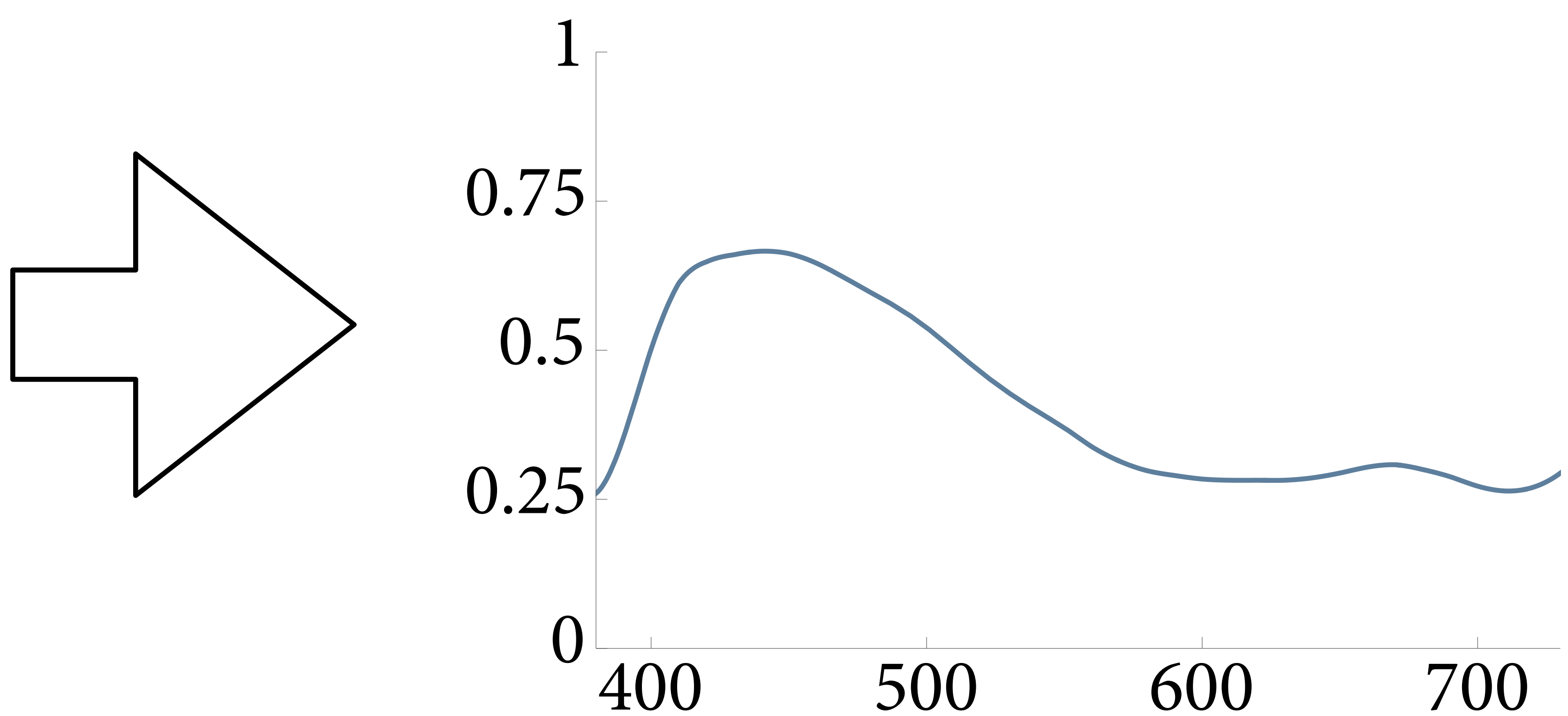


Illumination

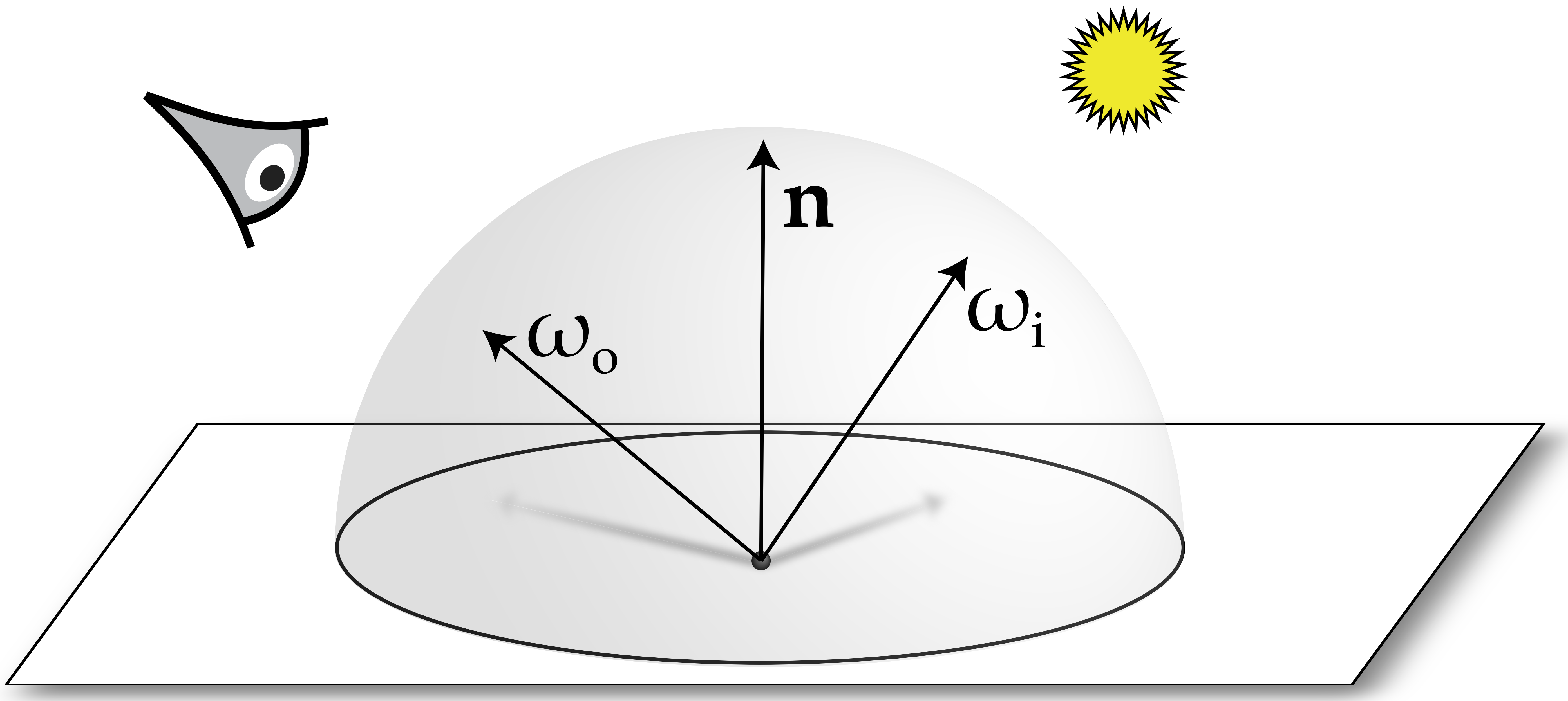
*



Reflection



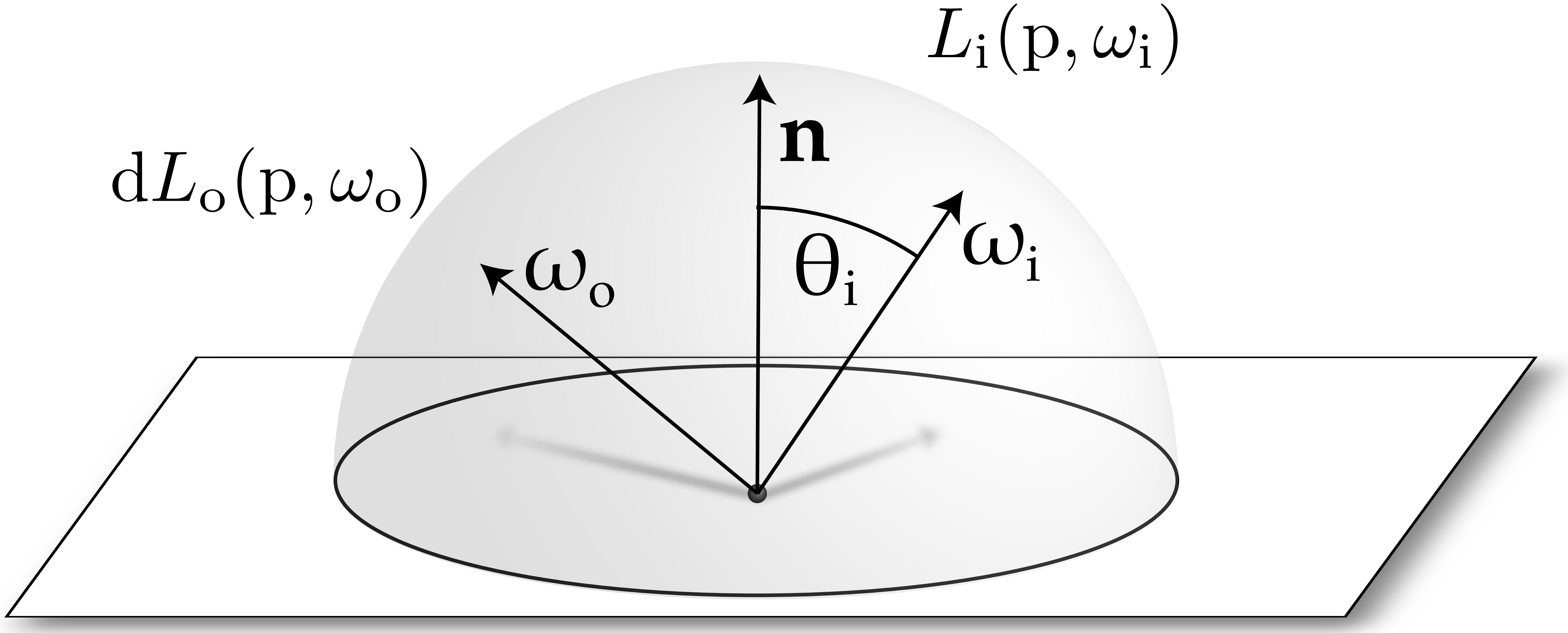
The Reflection Equation



$$L_o(p, \omega_o) = \int_{\Omega^2} \underbrace{f_r(p, \omega_i \to \omega_o)}_{\textbf{BRDF}} \underbrace{L_i(p, \omega_i) \cos\theta_i}_{\textbf{Illumination}} \, d\omega_i$$

The BRDF

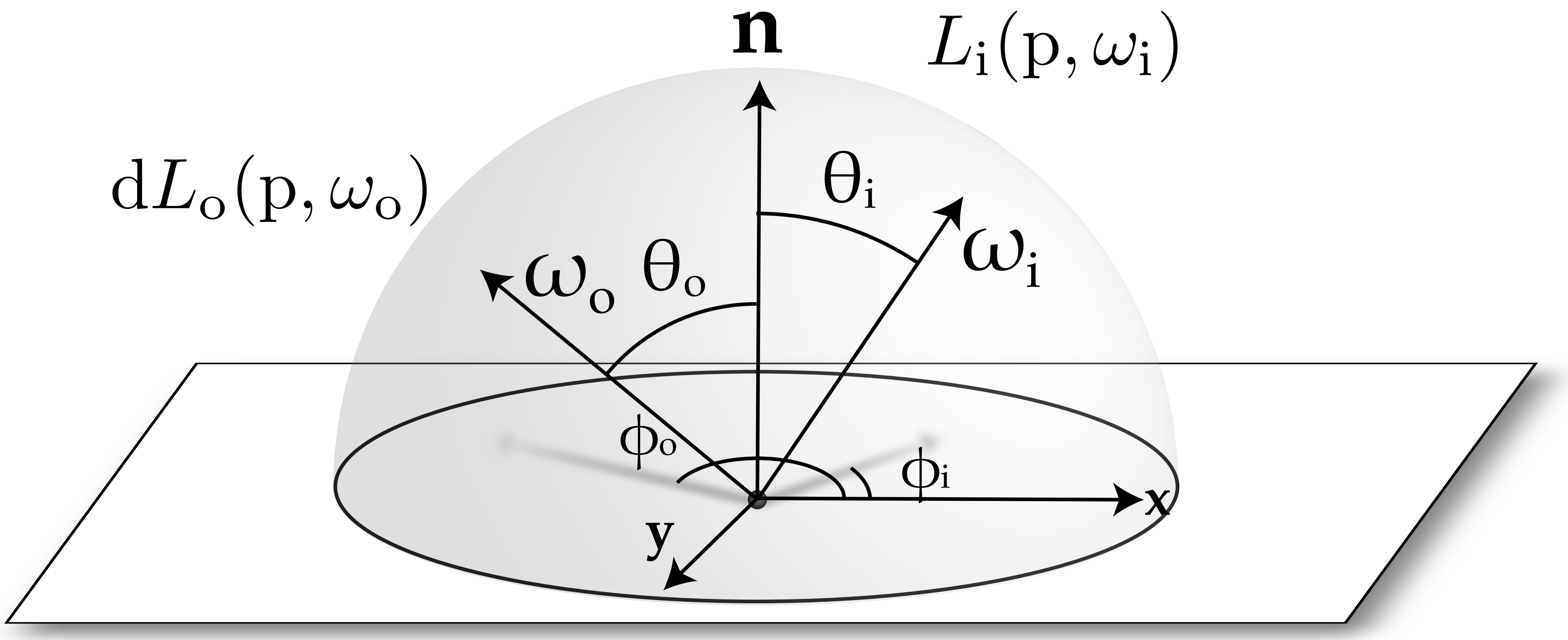
Bidirectional Reflectance-Distribution Function



$$f_{\mathrm{r}}(\omega_{\mathrm{i}} \rightarrow \omega_{\mathrm{o}}) \equiv \frac{\mathrm{d}L_{\mathrm{o}}(\omega_{\mathrm{o}})}{\mathrm{d}E_{\mathrm{i}}(\omega_{\mathrm{i}})} \quad \left[\frac{1}{sr}\right]$$

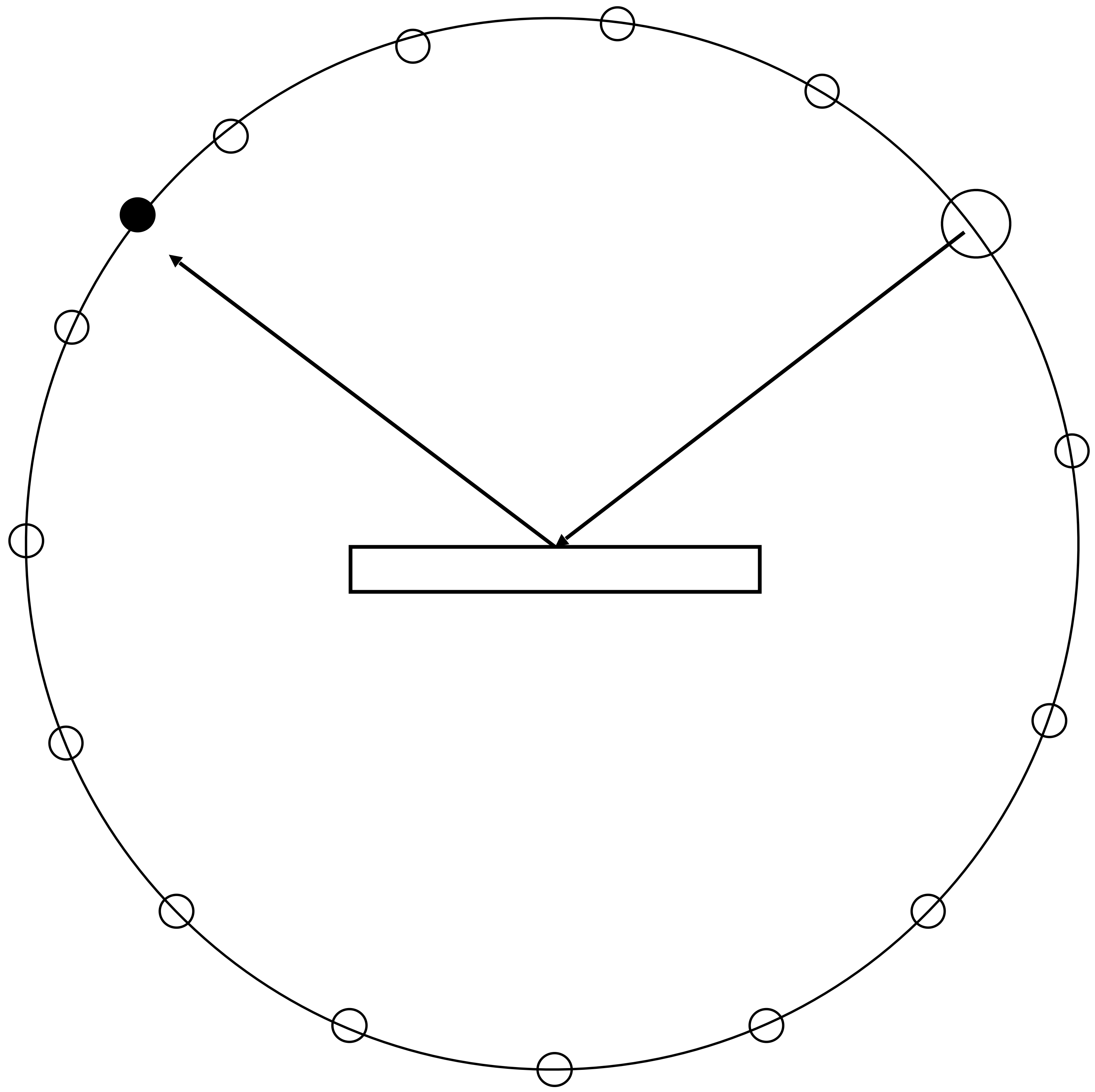
The BRDF

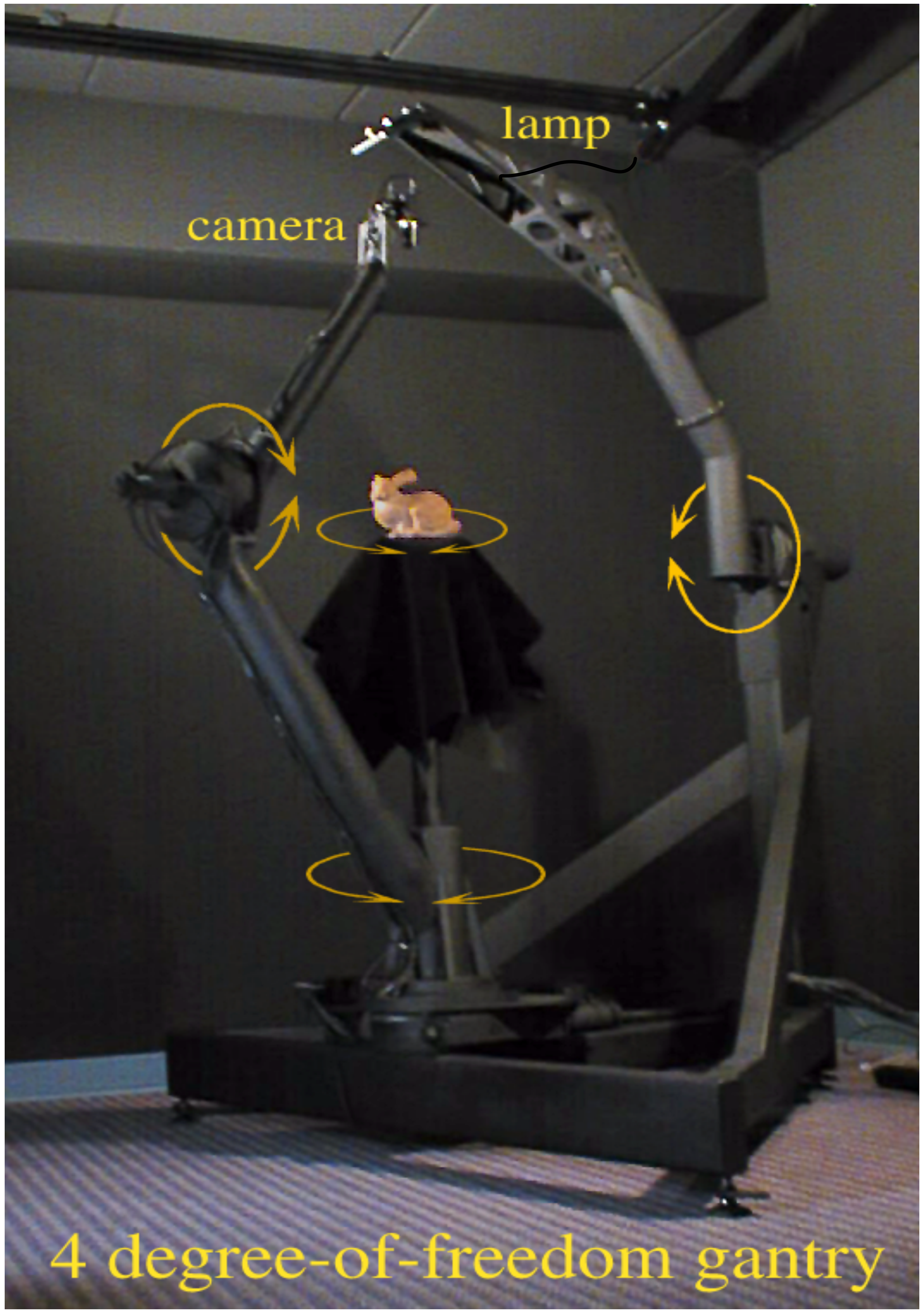
Bidirectional Reflectance-Distribution Function



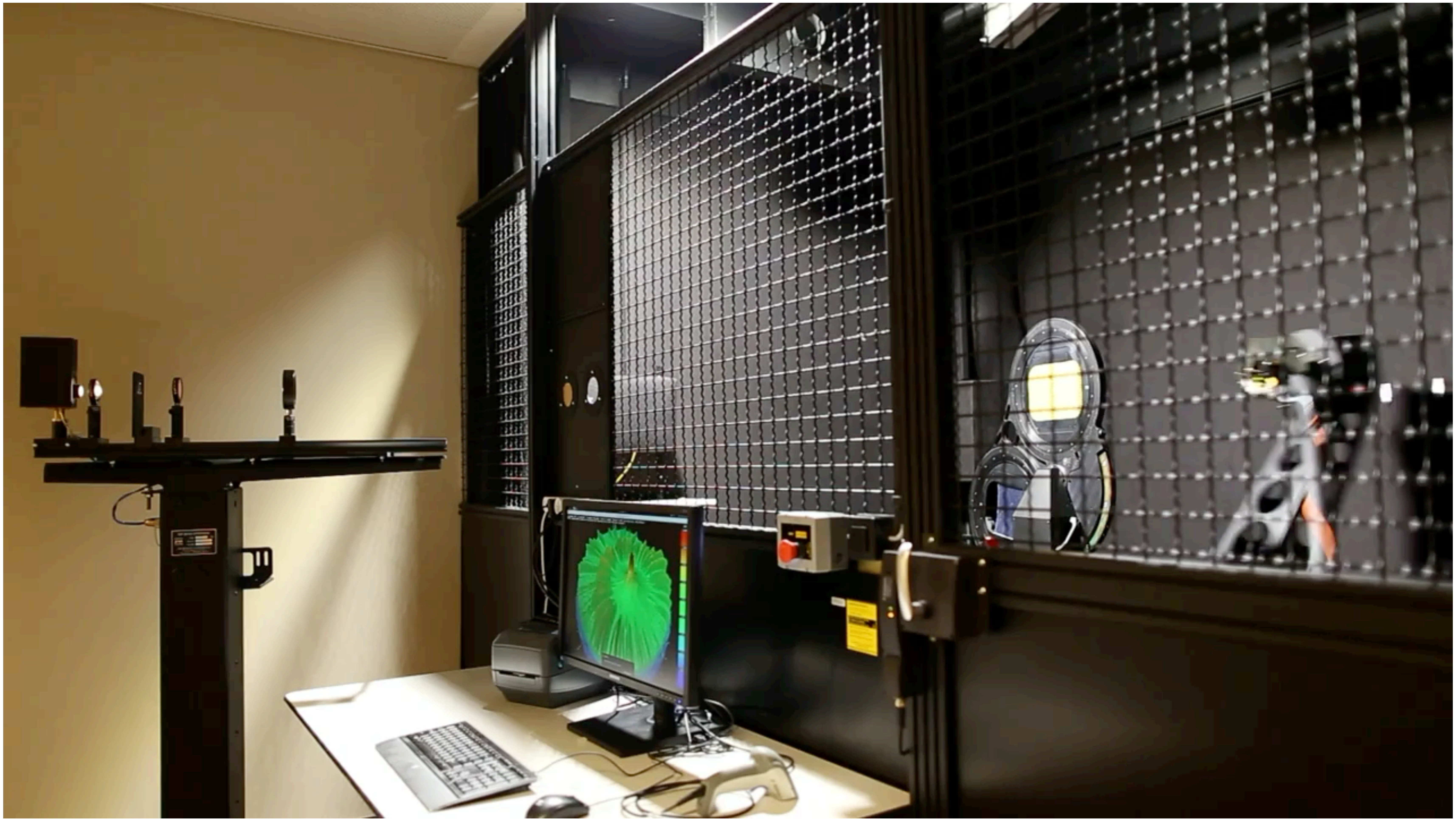
$$f_r(\omega_i \rightarrow \omega_o) \equiv \frac{dL_o(\omega_o)}{dE_i(\omega_i)} \quad \left[\frac{1}{sr}\right]$$

Stanford Gonioreflectometer



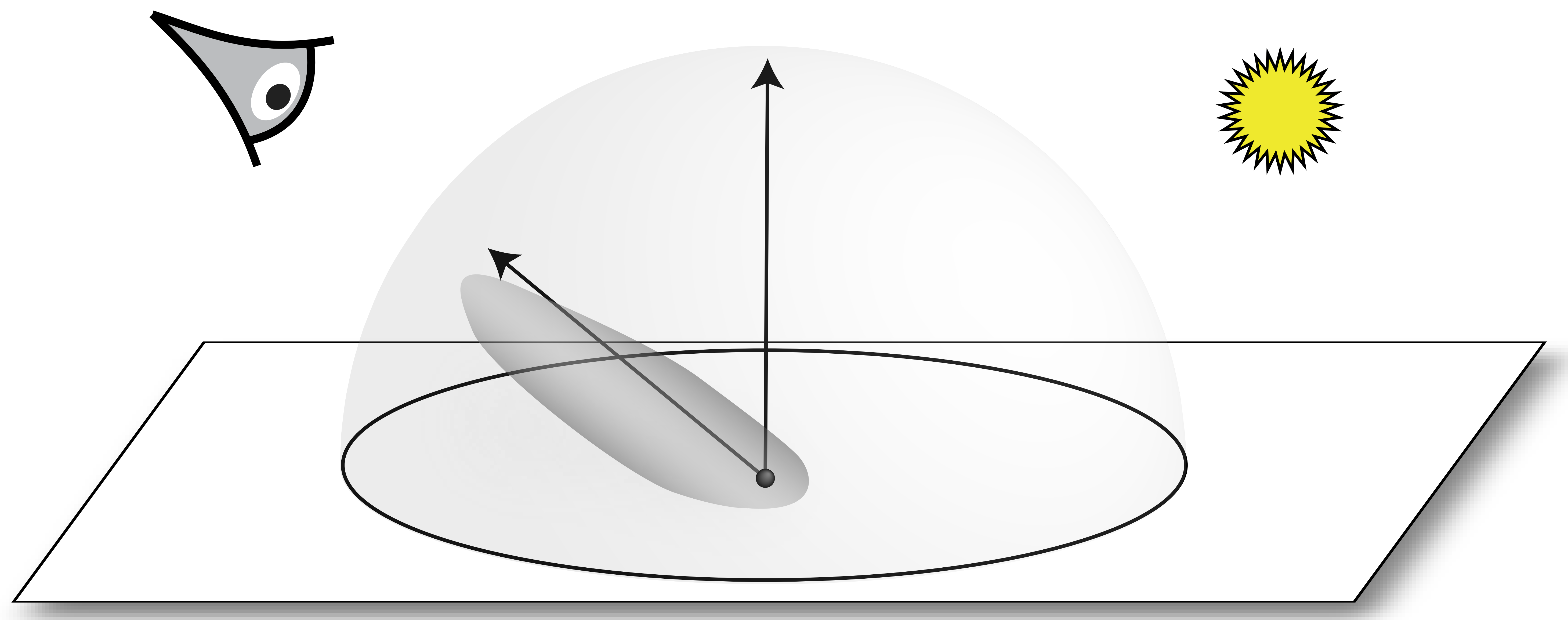


EPFL Gonioreflectometer



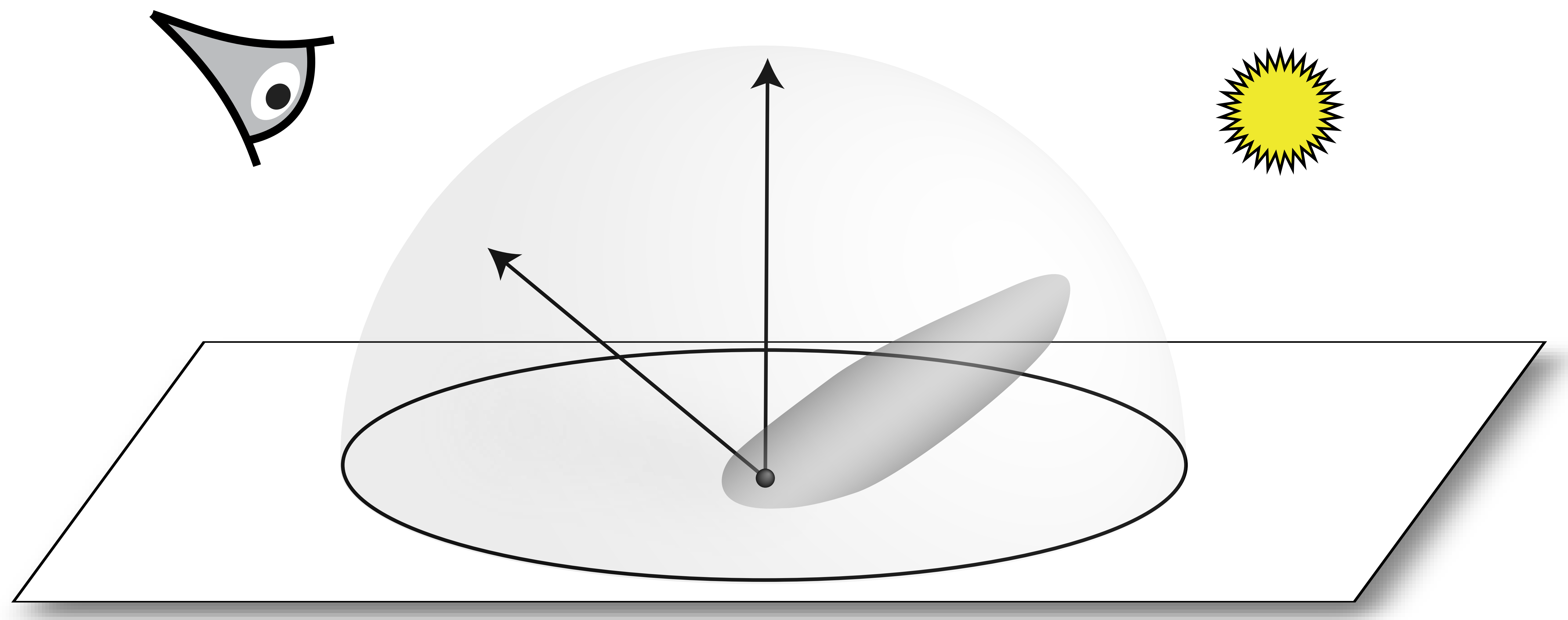
2D Slices of the BRDF: Eye

Fix lighting and vary viewing direction



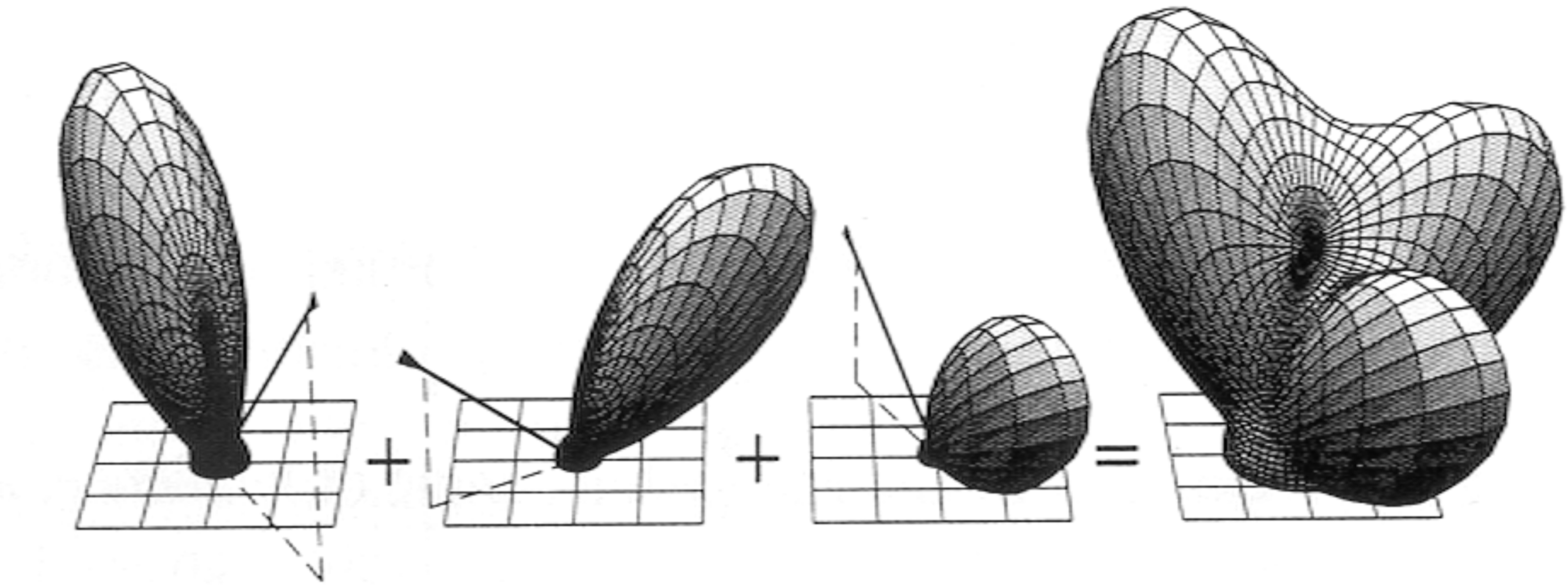
2D Slices of the BRDF: Lighting

Fix viewing direction and vary lighting



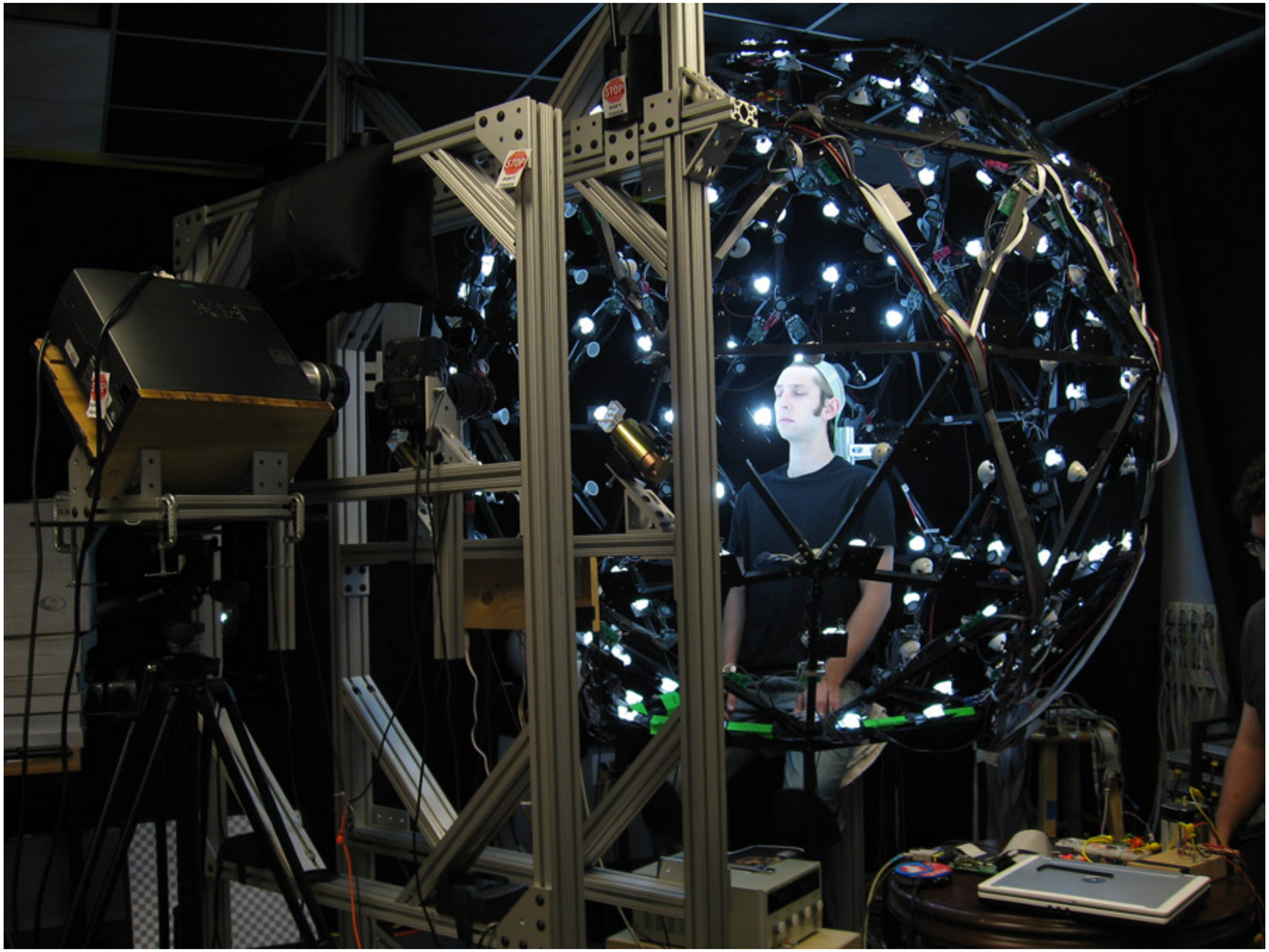
Properties of BRDFs

1. Linearity

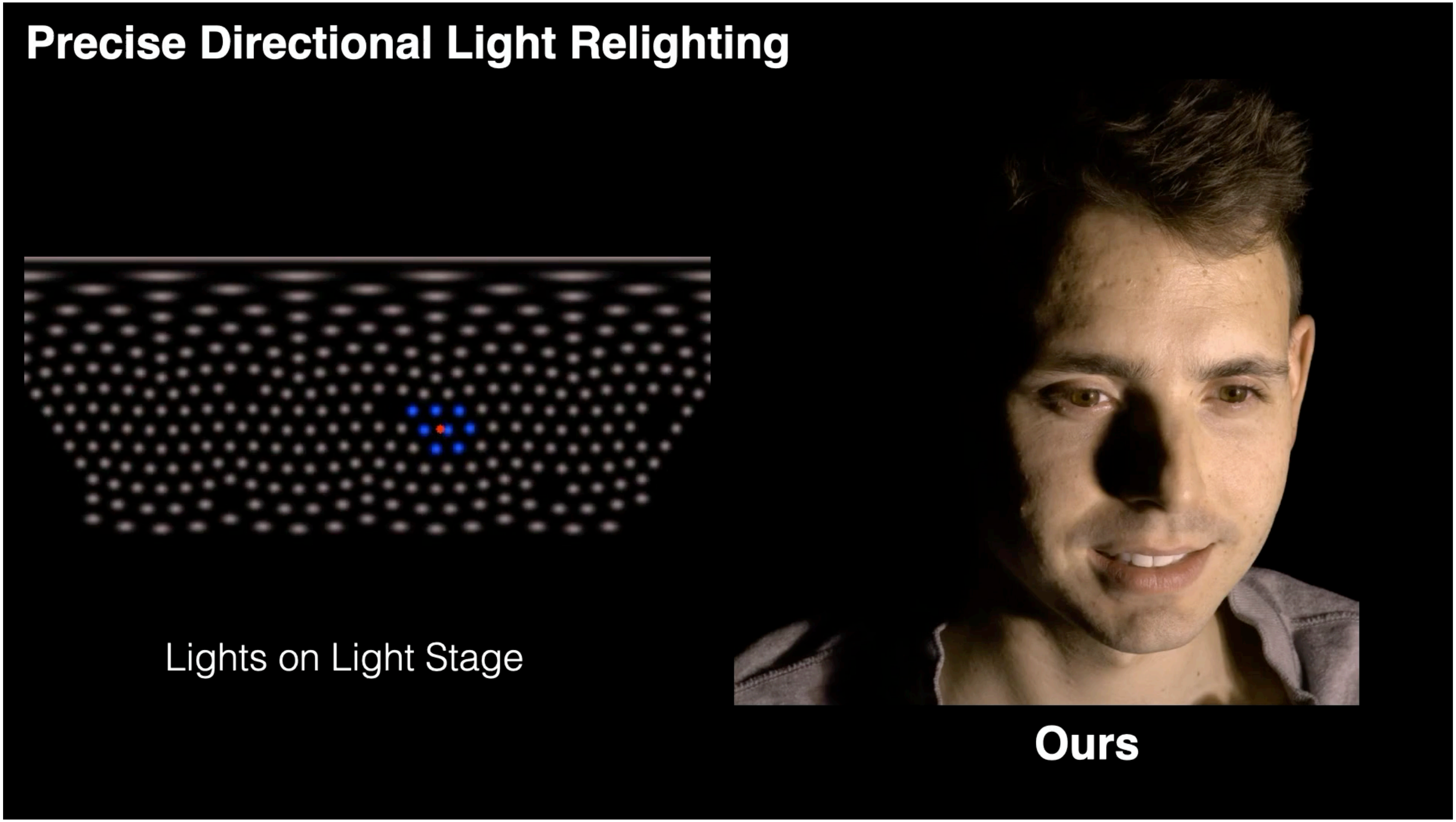


From Sillion, Arvo, Westin, Greenberg

The Light Stage



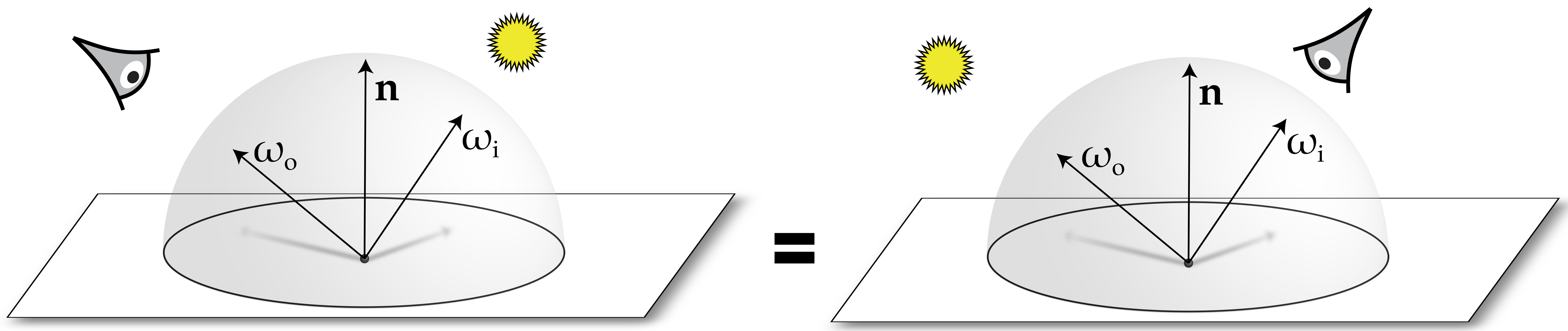
Leveraging BRDF Linearity: Relighting



[Sun et al. 2020]

Properties of BRDFs

2. Reciprocity principle



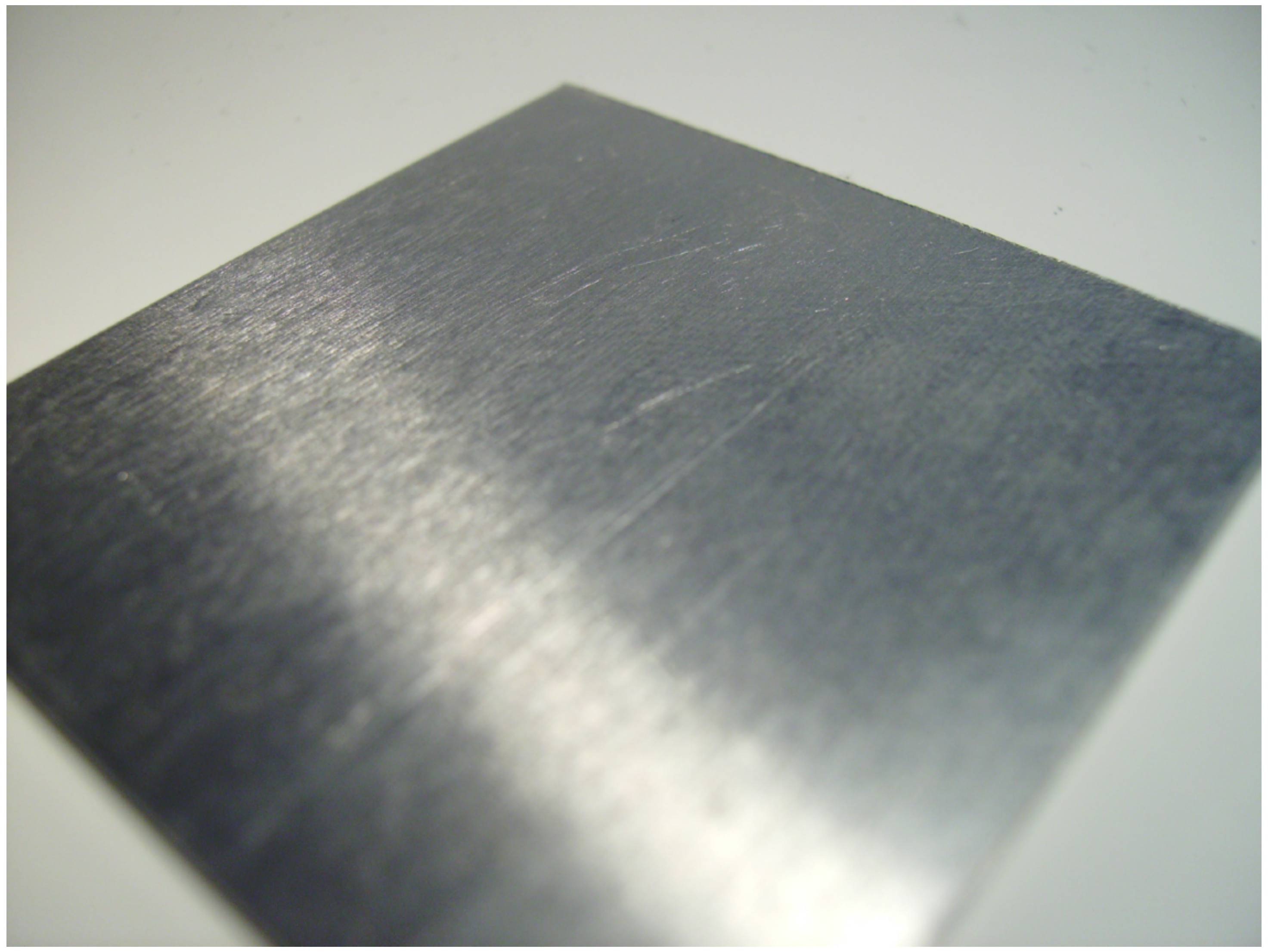
$$f(\omega_{\mathrm{i}} \to \omega_{\mathrm{o}}) = f(\omega_{\mathrm{o}} \to \omega_{\mathrm{i}})$$

Scattered radiance does not obey reciprocity—why?

Properties of BRDFs

3. Isotropic: BRDF is a 3D function

$$f_{\mathrm{r}}((\theta_{\mathrm{i}}, \phi_{\mathrm{i}}) \to (\theta_{\mathrm{o}}, \phi_{\mathrm{o}})) \Rightarrow f_{\mathrm{r}}(\theta_{\mathrm{i}}, \theta_{\mathrm{o}}, \phi_{\mathrm{i}} - \phi_{\mathrm{o}})$$



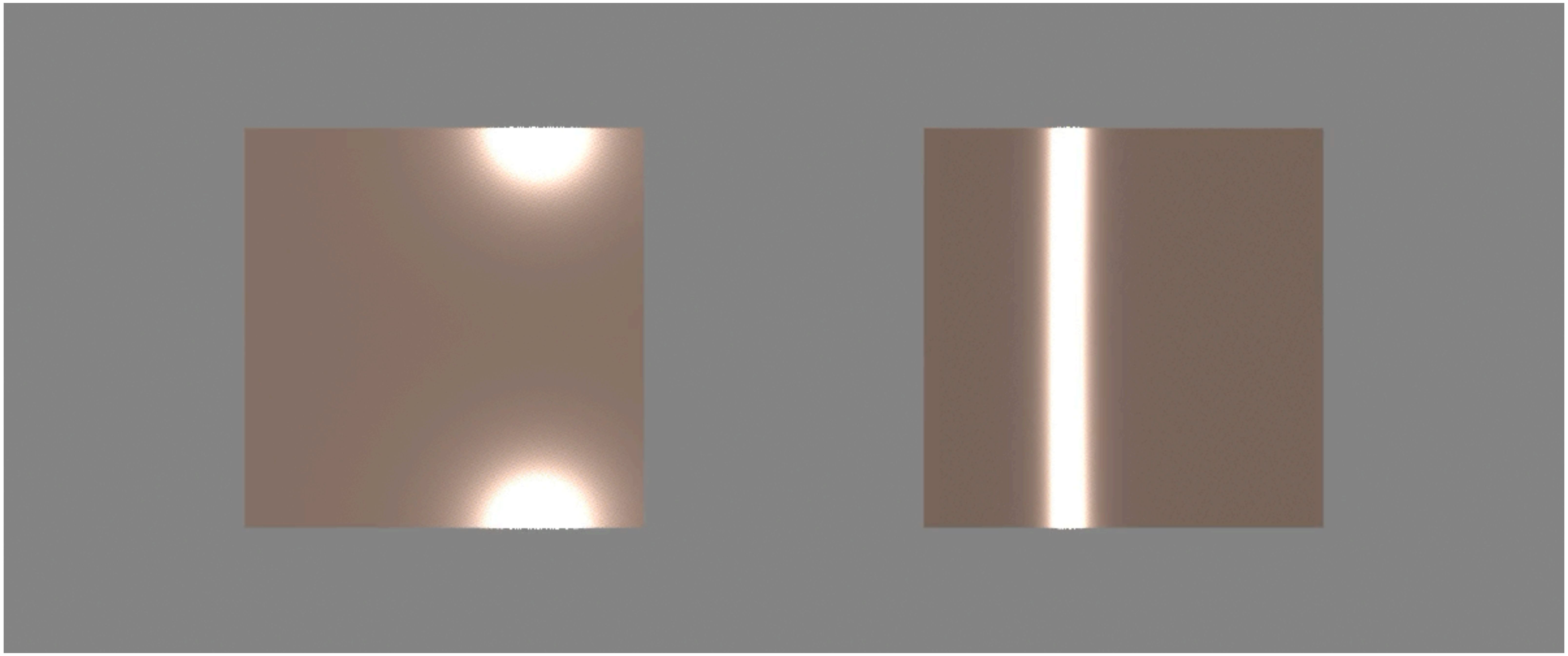
Anisotropic: e.g., brushed metal

(Wikiepedia CC-A-SA 3.0)

Properties of BRDFs

3. Isotropic: BRDF is a 3D function

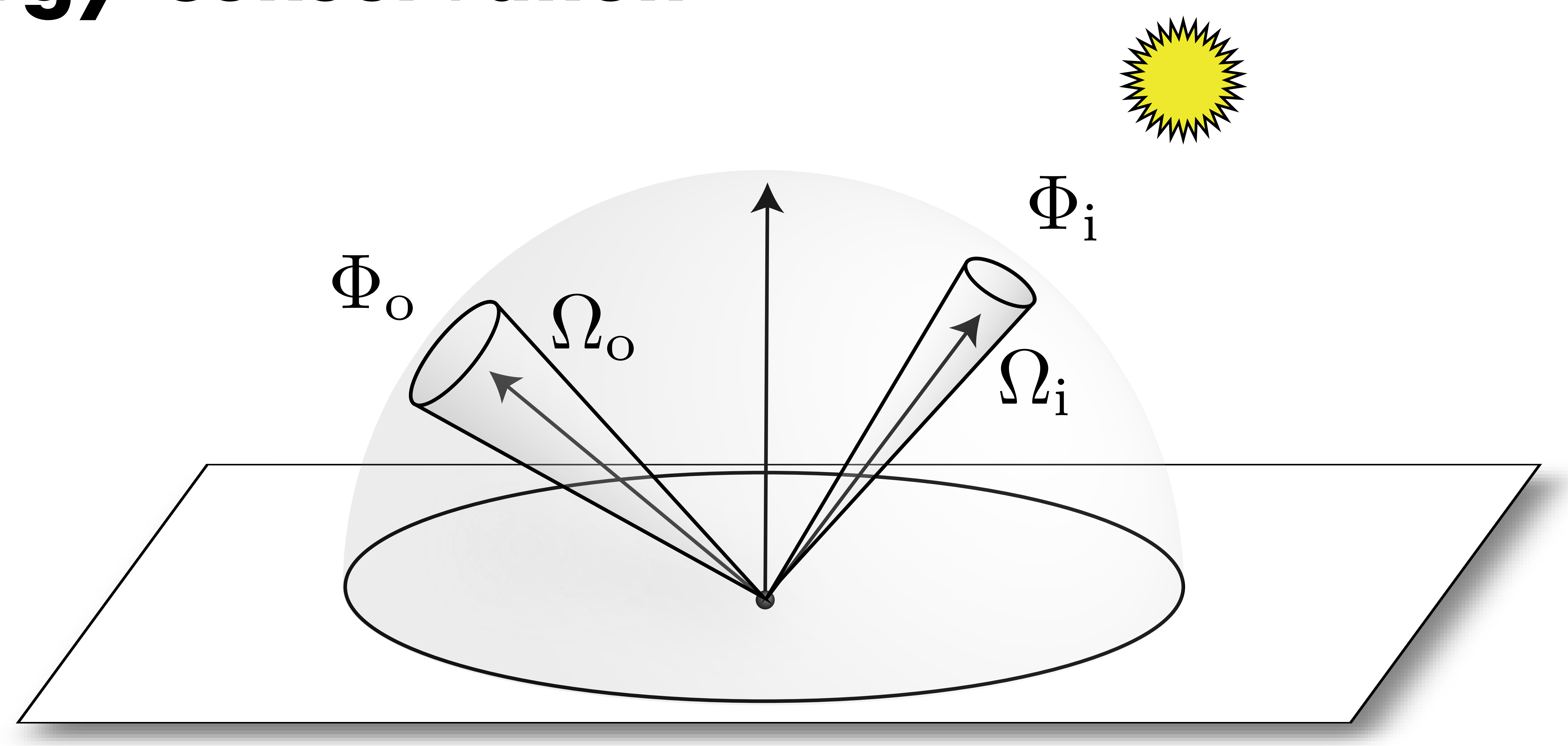
$$f_r((\theta_i, \phi_i) \to (\theta_o, \phi_o)) \Rightarrow f_r(\theta_i, \theta_o, \phi_i - \phi_o)$$



Isotropic Anisotropic

Properties of BRDFs

4. Energy Conservation



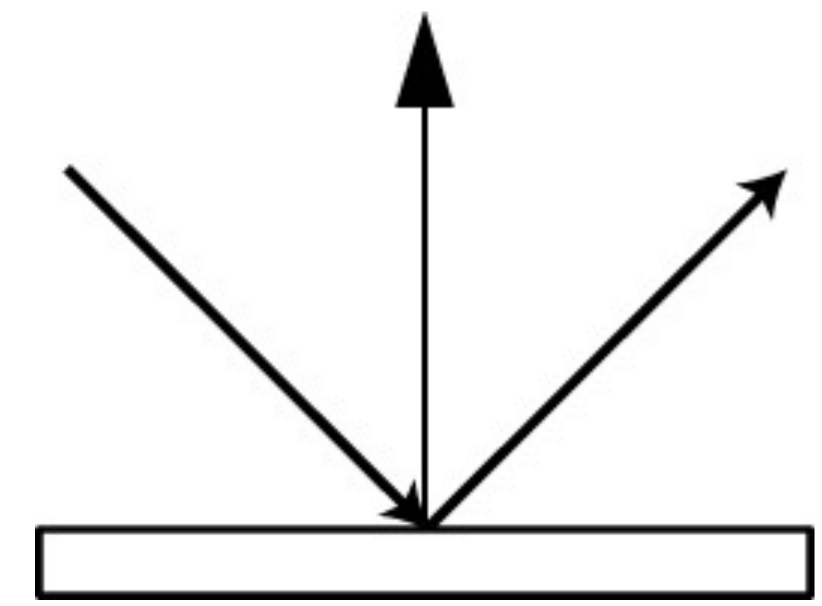
Reflectance $$\rho = \frac{\Phi_\mathrm{o}}{\Phi_\mathrm{i}} = \frac{\int_{\Omega_\mathrm{o}} L_\mathrm{o}(\omega_\mathrm{o}) \cos\theta_\mathrm{o} \, \mathrm{d}\omega_\mathrm{o}}{\int_{\Omega_\mathrm{i}} L_\mathrm{i}(\omega_\mathrm{i}) \cos\theta_\mathrm{i} \, \mathrm{d}\omega_\mathrm{i}}$$

$$0 \leq \rho \leq 1$$

Types of Reflection Functions

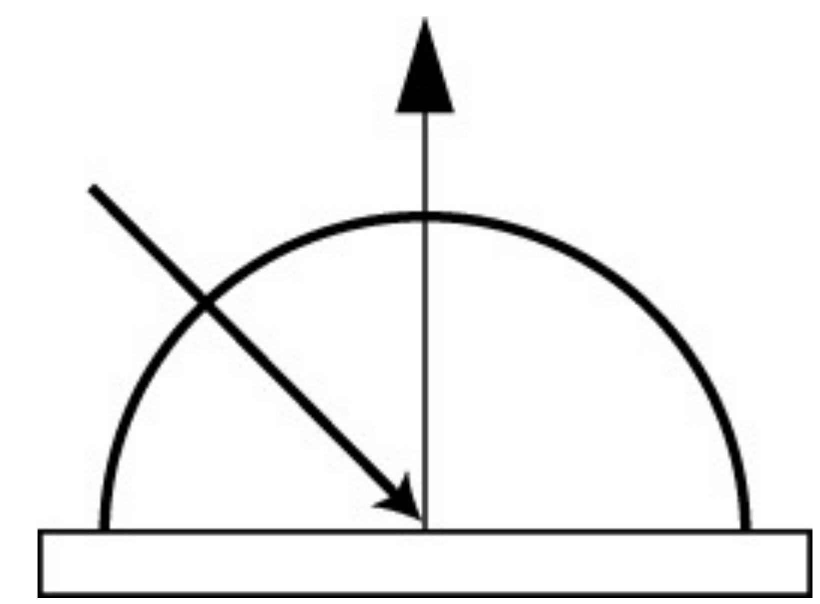
Ideal Specular

- Reflection Law
- Mirror



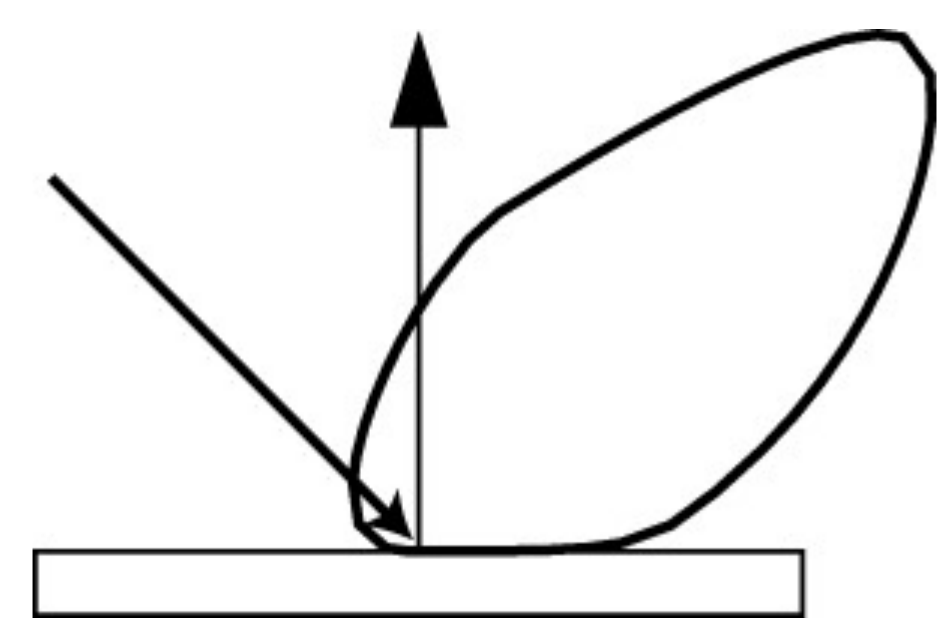
Ideal Diffuse

- Lambert's Law
- Matte

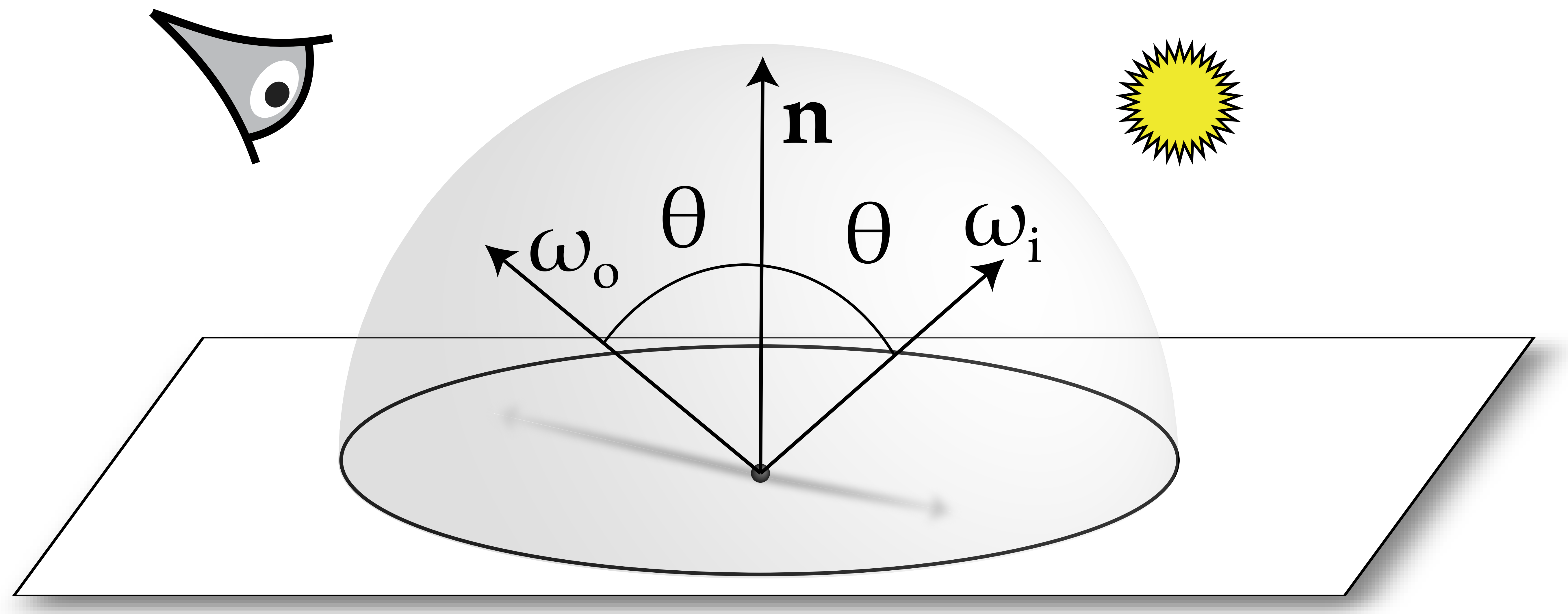


Specular

- Glossy
- Directional diffuse

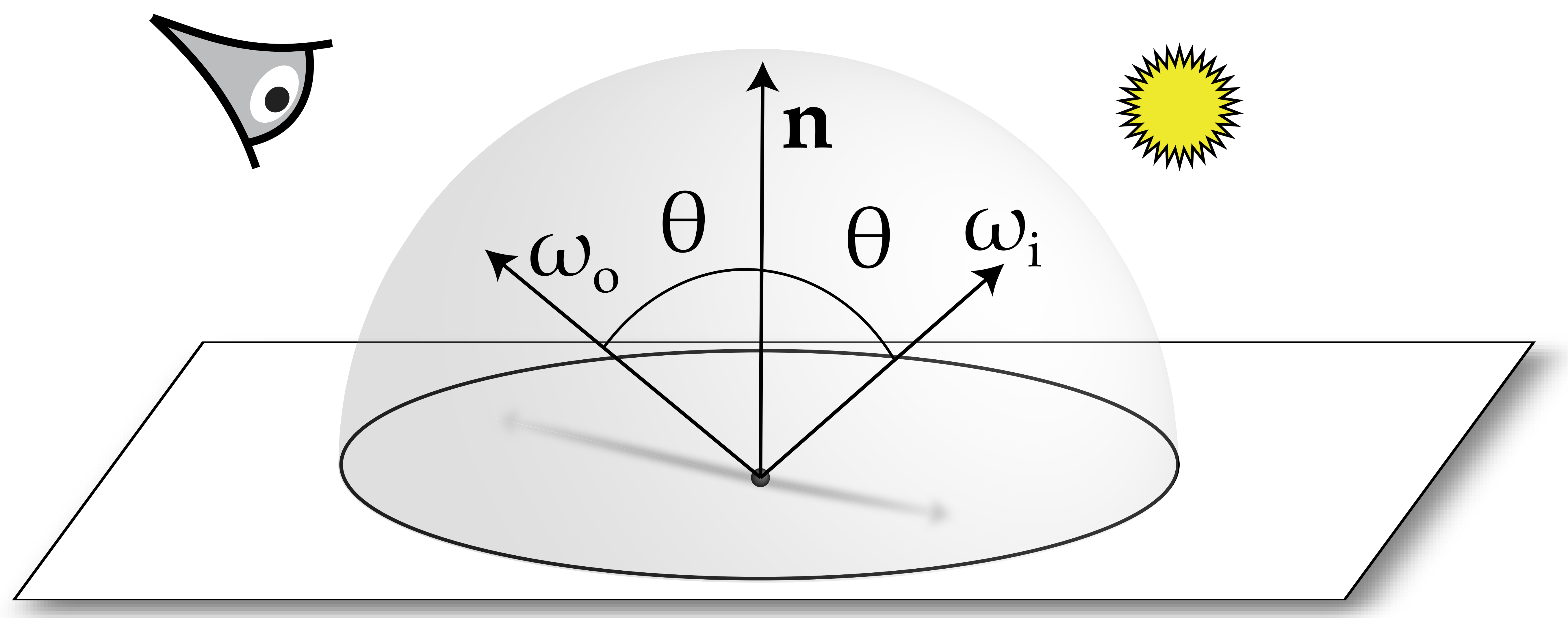


Law of Reflection



$$\omega_{\rm i} = R(\omega_{\rm o}, \mathbf{n}) = -\omega_{\rm o} + 2(\omega_{\rm o} \cdot \mathbf{n})\mathbf{n}$$

Ideal Reflection BRDF

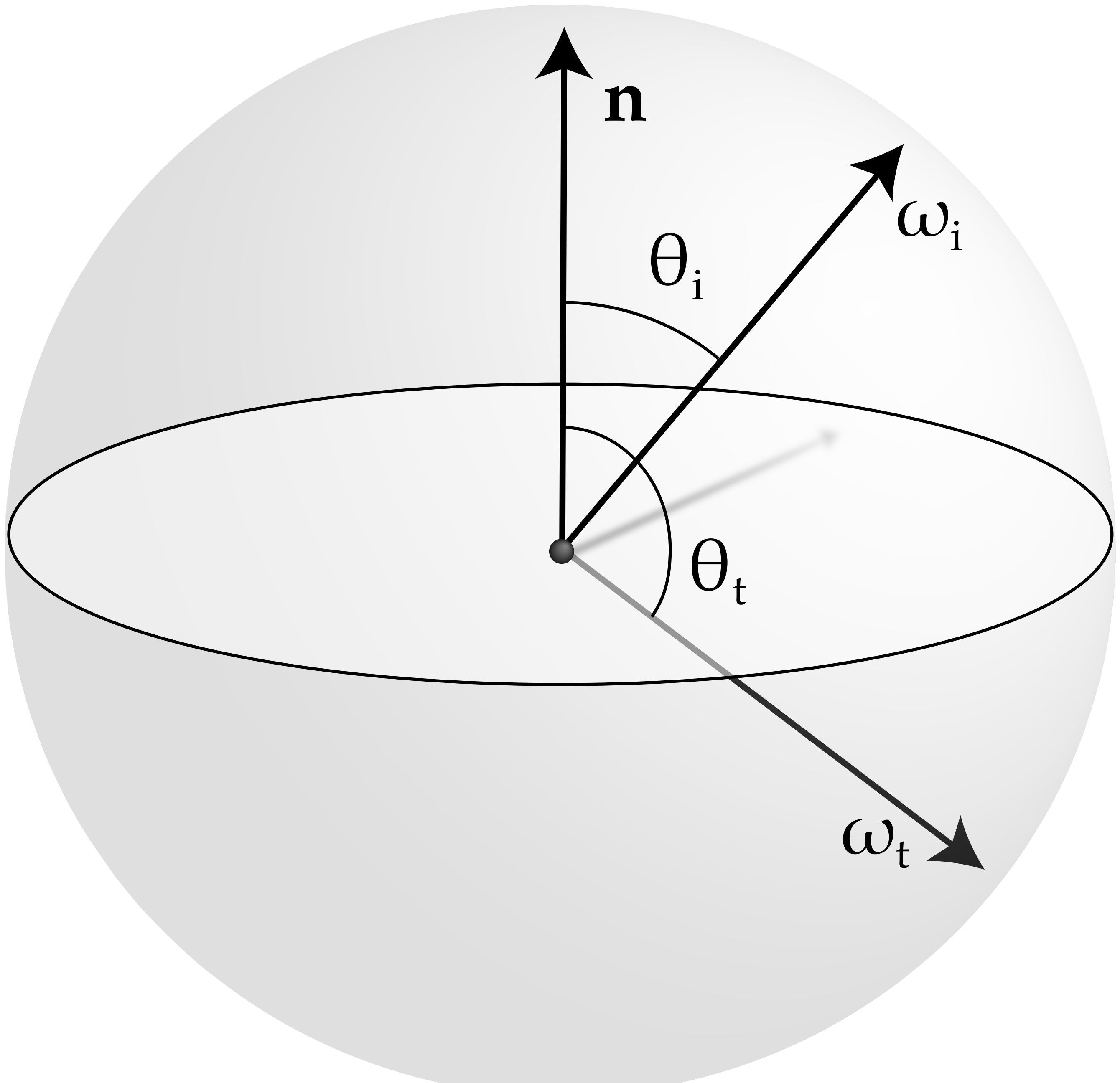


$$f_r(\omega_o \to \omega_i) = \frac{\delta(\omega_i - R(\omega_o, \mathbf{n}))}{\cos\theta_i}$$

$$\int f_r(\omega_o \to \omega_i)\, L_i(\omega_i)\, \cos\theta_i\, d\omega_i =$$

$$\int \frac{\delta(\omega_i - R(\omega_o, \mathbf{n}))}{\cos\theta_i}\, L_i(\omega_i)\, \cos\theta_i\, d\omega_i = L_i(\omega_i)$$

Ideal Specular Transmission



$$\eta = \frac{\eta_\mathrm{i}}{\eta_\mathrm{t}}$$

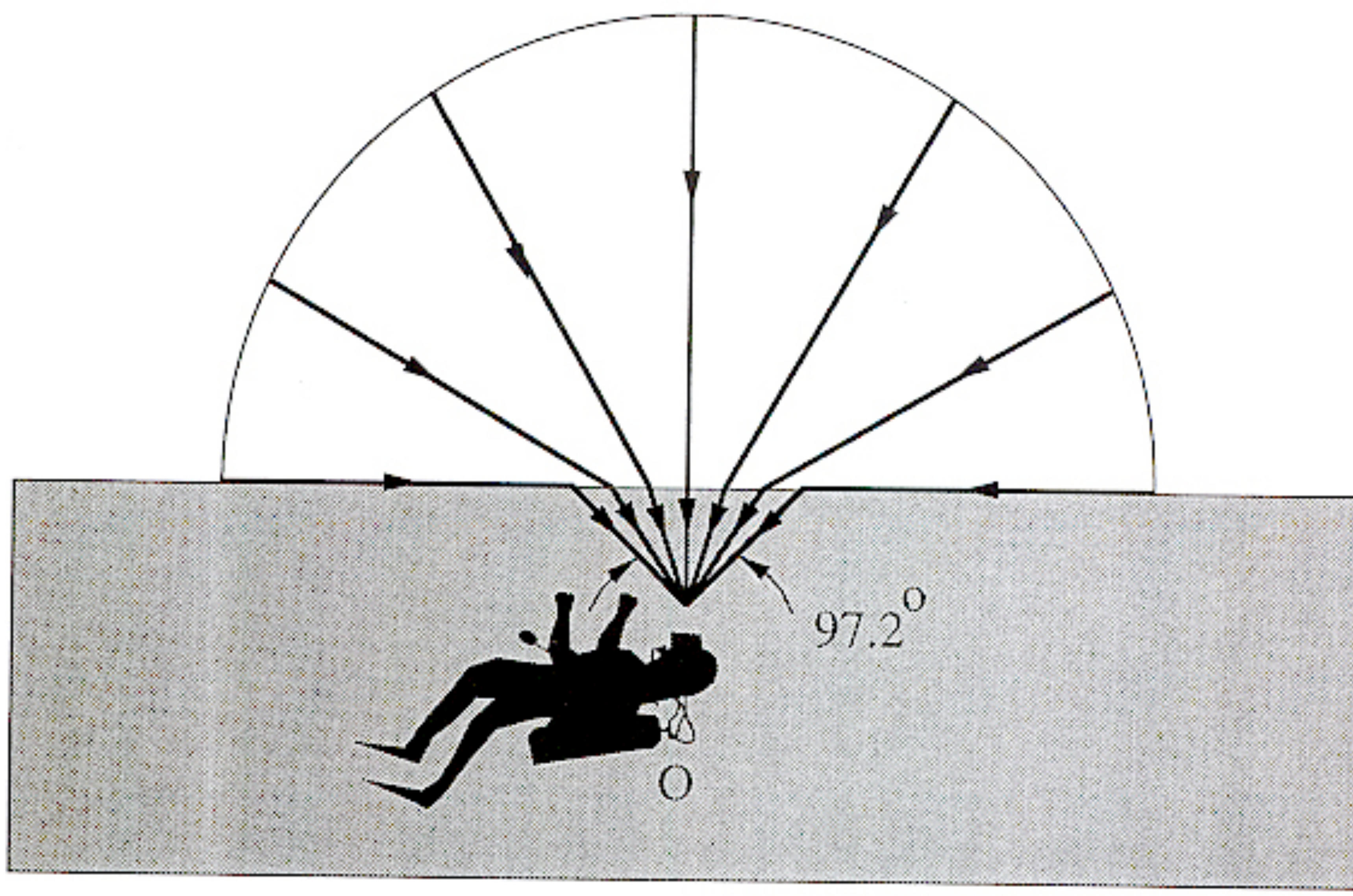
$$\omega_\mathrm{t} = \frac{-\omega_\mathrm{i}}{\eta} + \left(\frac{\cos\theta_\mathrm{i}}{\eta} - \cos\theta_\mathrm{t}\right)\mathbf{n}$$

Snell's law: $\eta_\mathrm{i}\sin\theta_\mathrm{i} = \eta_\mathrm{t}\sin\theta_\mathrm{t}$

Total internal reflection: $\sin\theta_\mathrm{t} = \frac{\eta_\mathrm{i}}{\eta_\mathrm{t}}\sin\theta_\mathrm{i} > 1$

Optical Manhole

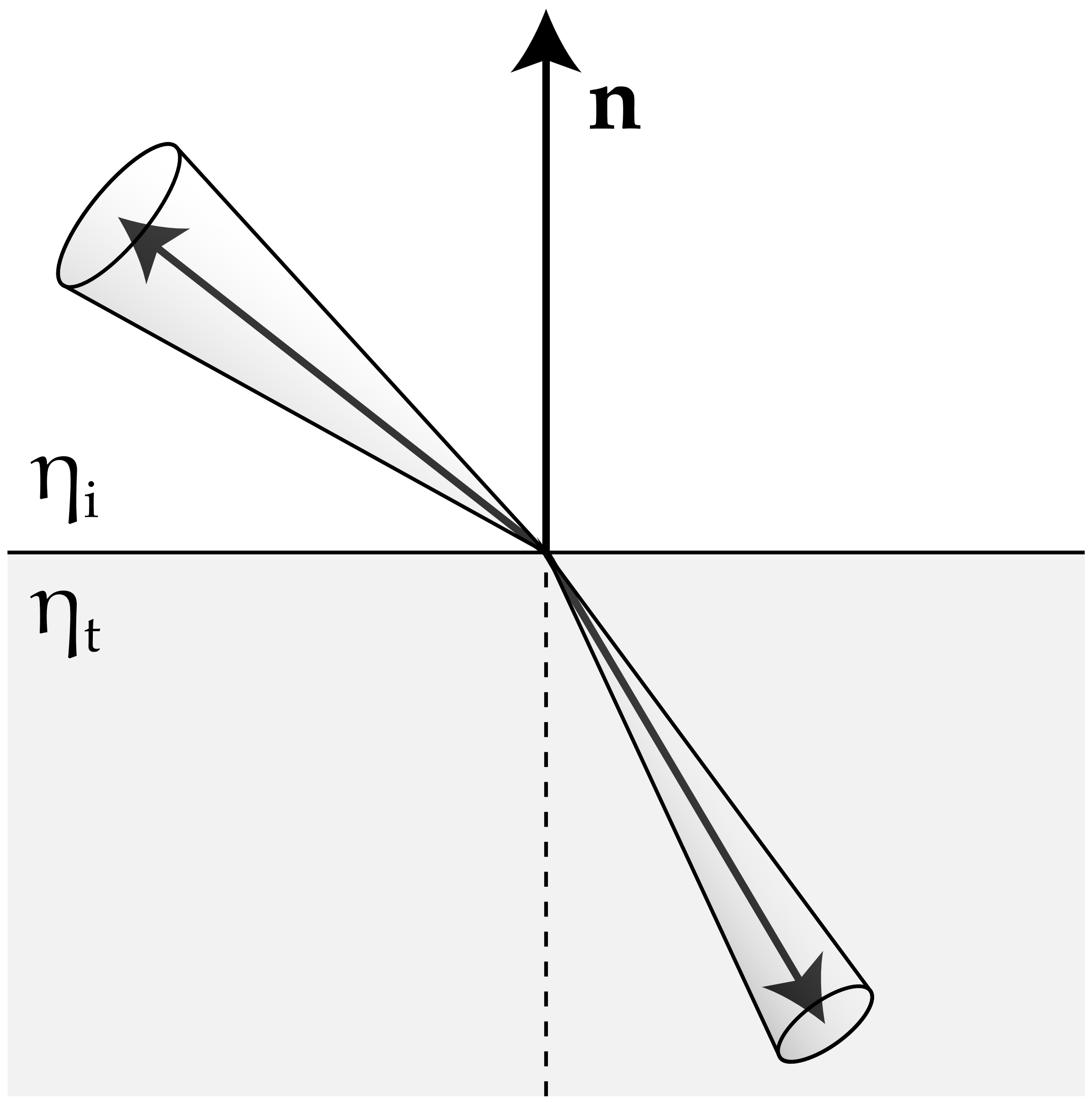
Total internal reflection



$$n_w = \frac{4}{3}$$

From Livingston and Lynch

Change in Radiance With Refraction

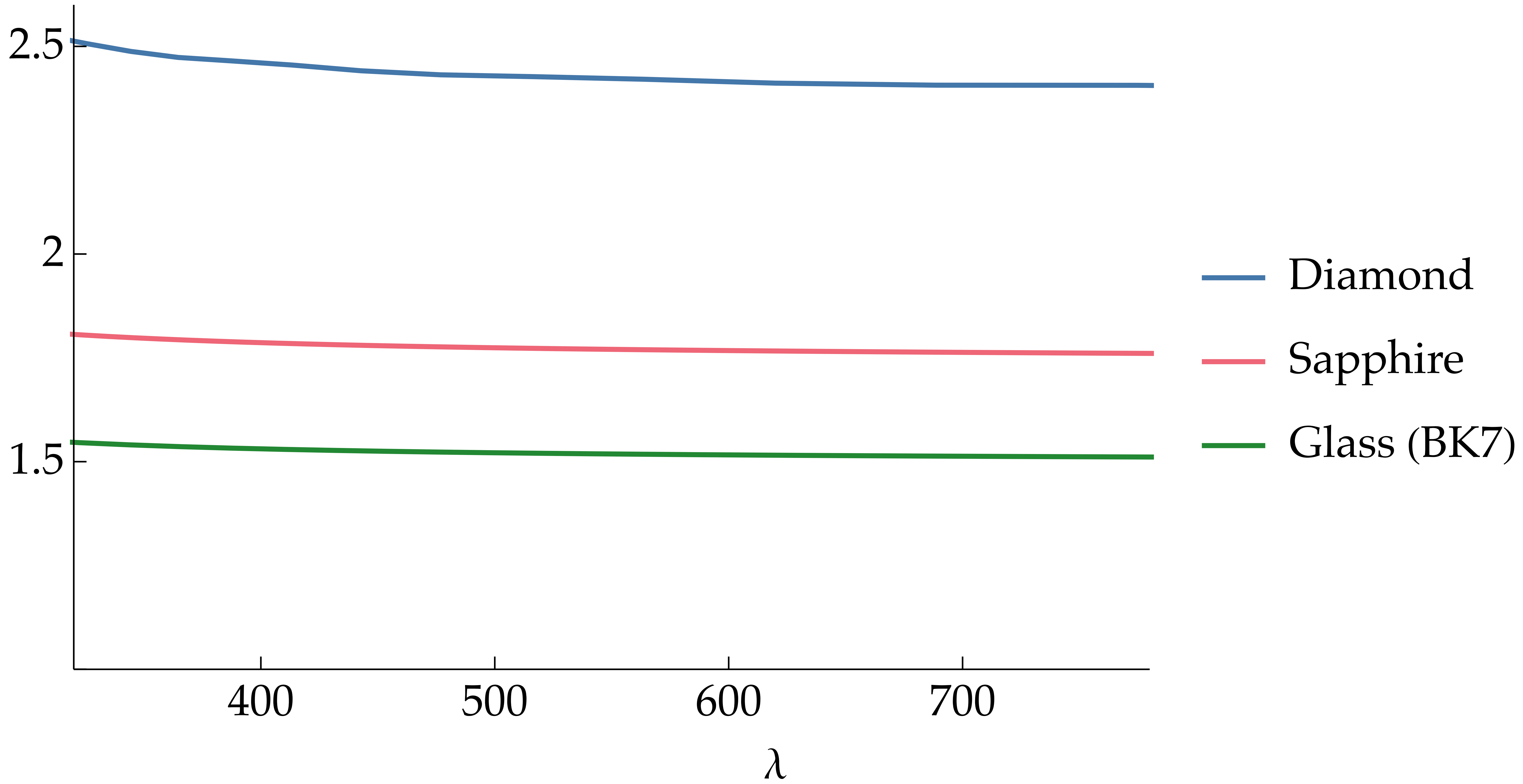


BTDFs are not reciprocal!

$$f_t(\omega_o \to \omega_i) \neq f_t(\omega_i \to \omega_o)$$

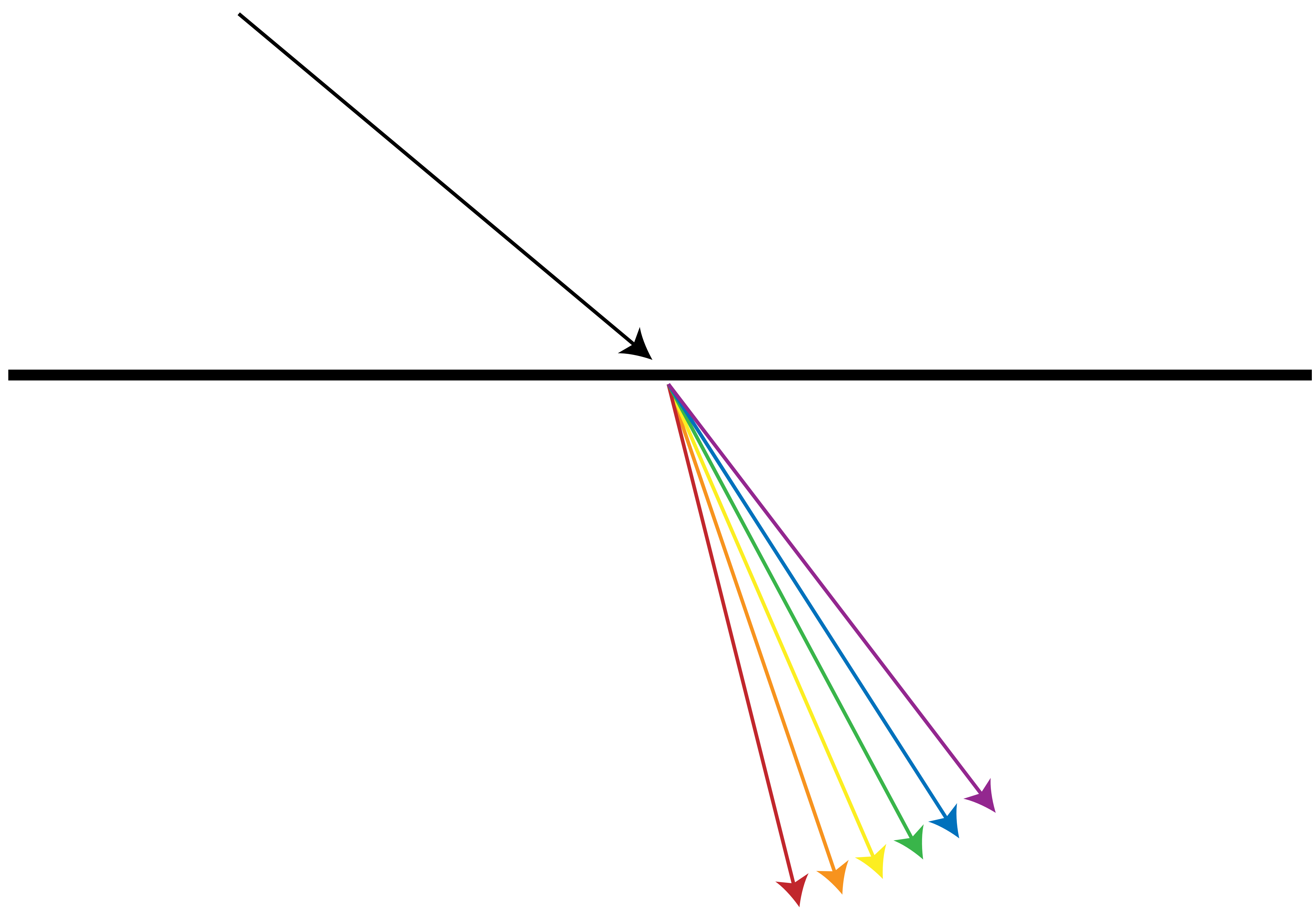
Conservation of basic radiance: $$\frac{L_i}{\eta_i^2} = \frac{L_t}{\eta_t^2}$$

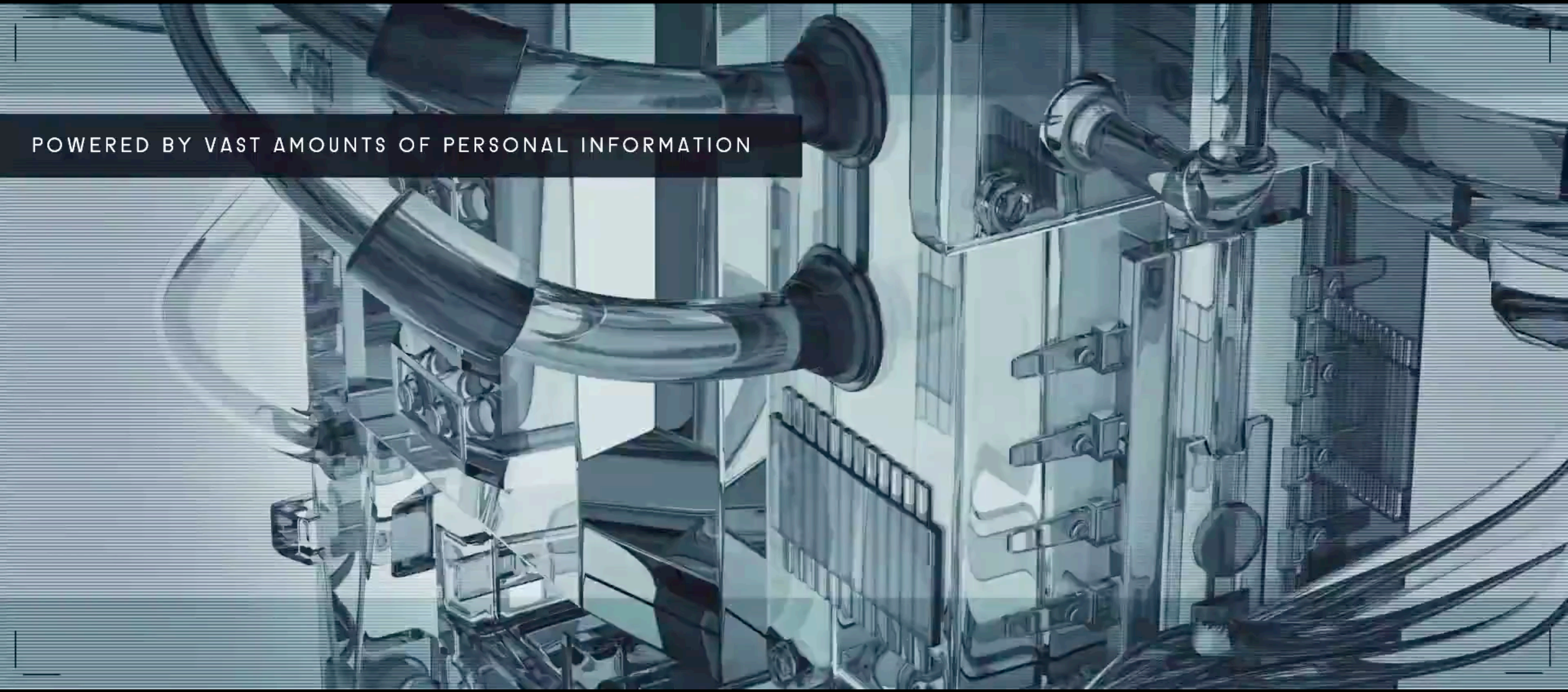
Wavelength-Dependent IOR



https://refractiveindex.info

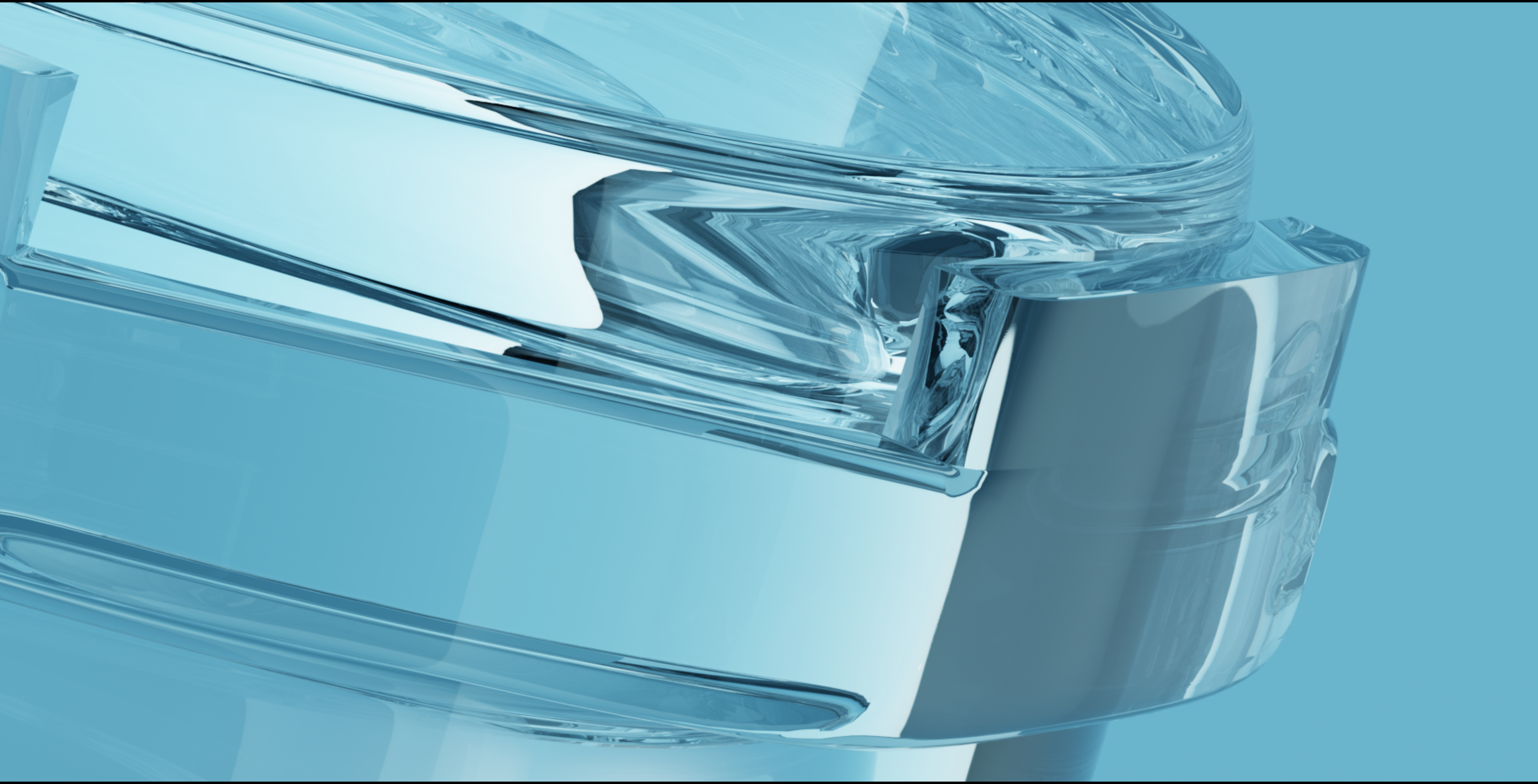
Wavelength-Dependent IOR





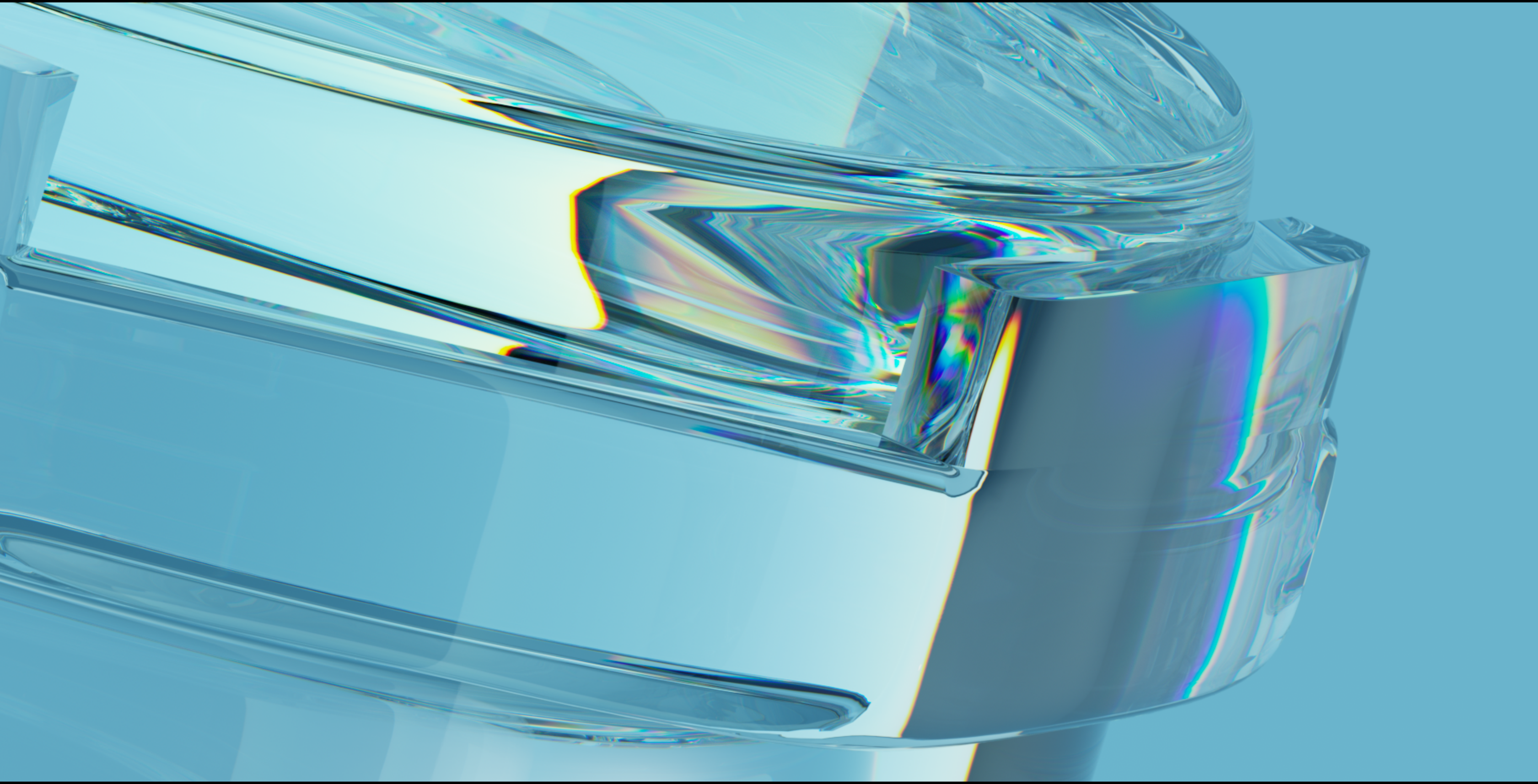
@beeple

Fixed Index of Refraction



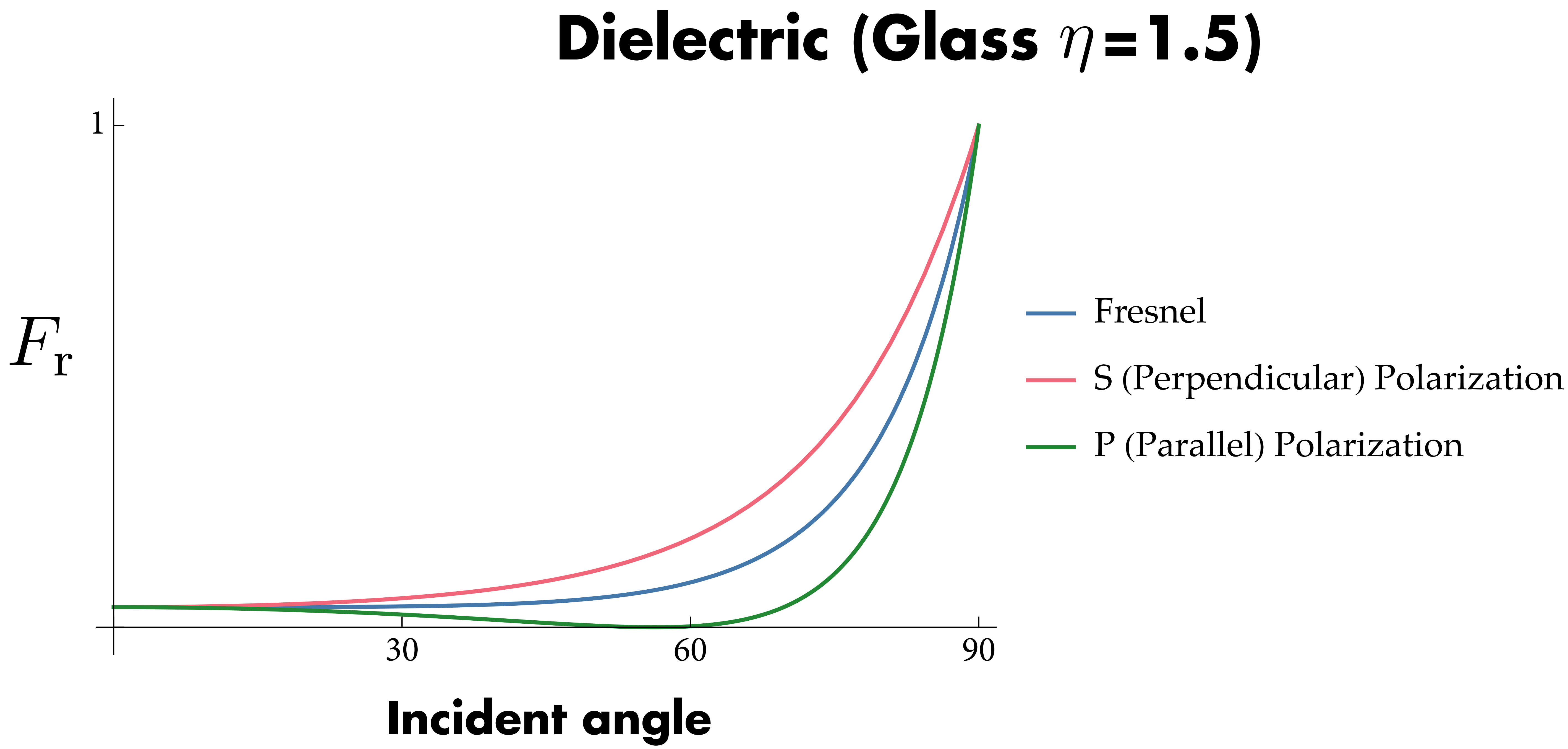
@beeple

Wavelength-Dependent IOR



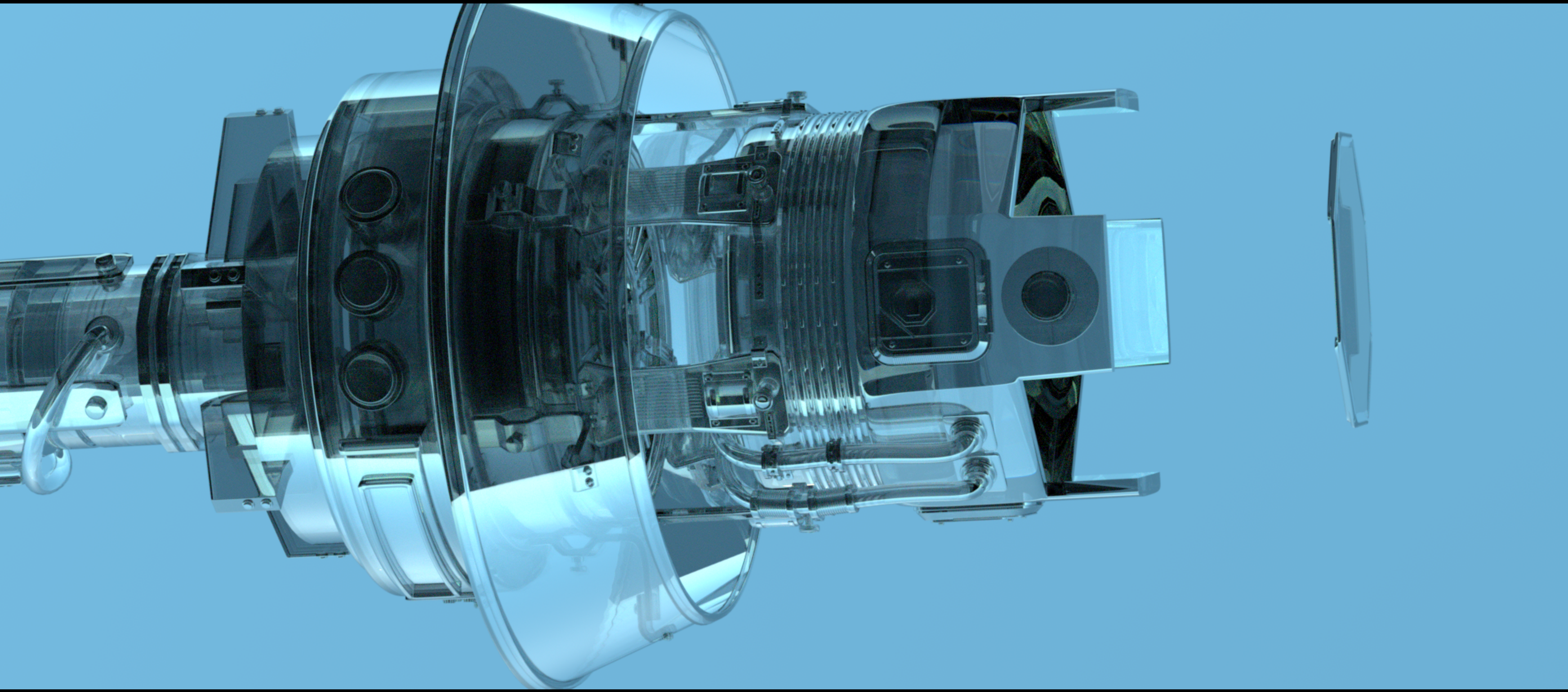
@beeple

Fresnel Reflectance



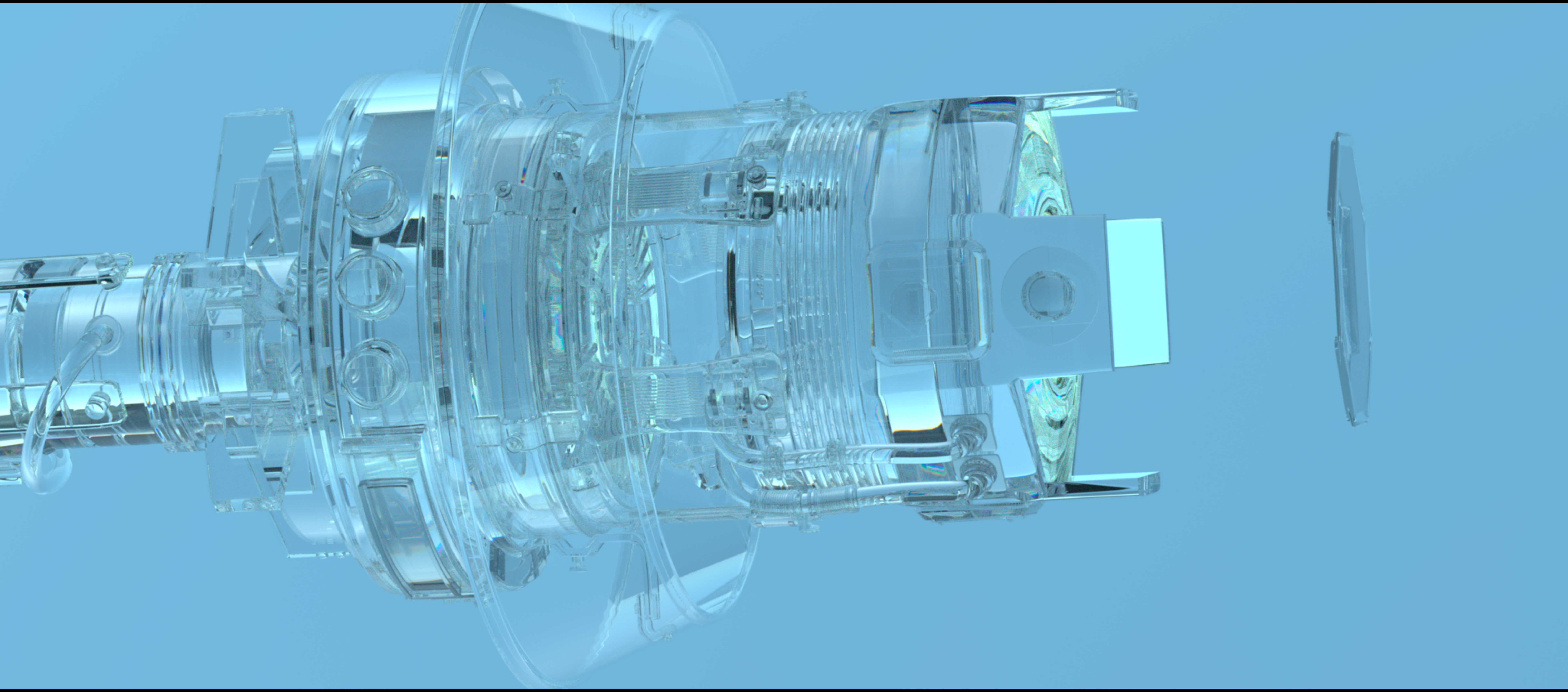
Transmission: $1 - F_r$

Reflect 0.5, Transmit 0.5



@beeple

Fresnel Reflection and Transmission



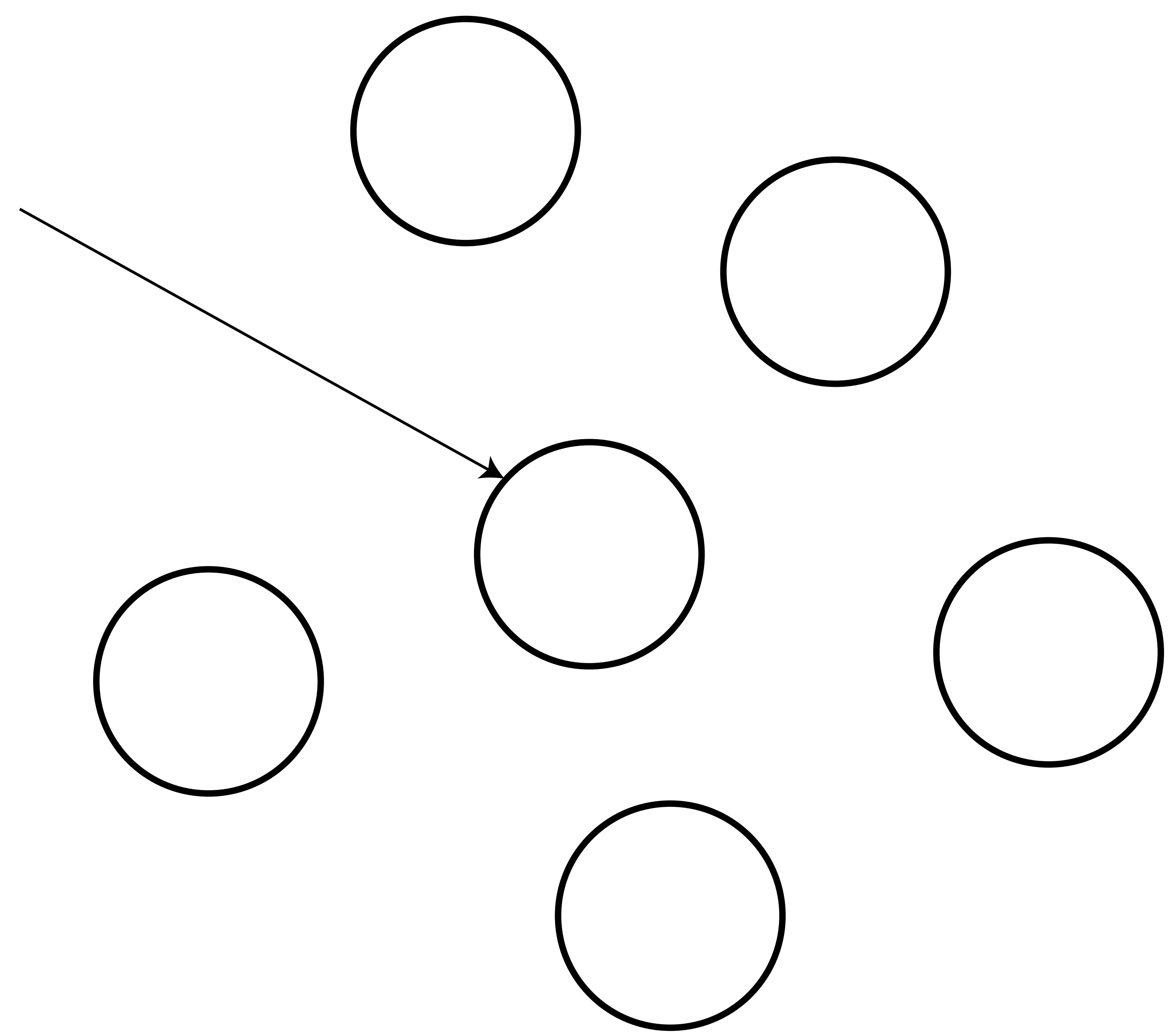
@beeple

Sampling Reflection + Transmission

$$
\begin{aligned}
L_{\mathrm{o}}(\mathrm{p},\omega) &= \int_{\mathcal{S}^2} f(\mathrm{p},\omega_{\mathrm{i}} \to \omega_{\mathrm{o}})\, L_{\mathrm{i}}(\mathrm{p},\omega_{\mathrm{i}})\, \cos\theta_{\mathrm{i}}\, \mathrm{d}\omega_{\mathrm{i}} \\
&= \int_{\mathcal{S}^2} \left(F_{\mathrm{r}}\, \frac{\delta(\omega_{\mathrm{i}} - R(\omega_{\mathrm{o}},\mathrm{n}))}{\cos\theta_{\mathrm{i}}} + (1 - F_{\mathrm{r}})\, \frac{\delta(\omega_{\mathrm{i}} - T(\omega_{\mathrm{o}},\mathrm{n}))}{\cos\theta_{\mathrm{i}}} \right) L_{\mathrm{i}}(\mathrm{p},\omega_{\mathrm{i}})\, \cos\theta_{\mathrm{i}}\, \mathrm{d}\omega_{\mathrm{i}} \\
&= F_{\mathrm{r}} L_{\mathrm{i}}(\mathrm{p}, R(\omega_{\mathrm{o}},\mathrm{n})) + (1 - F_{\mathrm{r}}) L_{\mathrm{i}}(\mathrm{p}, T(\omega_{\mathrm{o}},\mathrm{n}))
\end{aligned}
$$

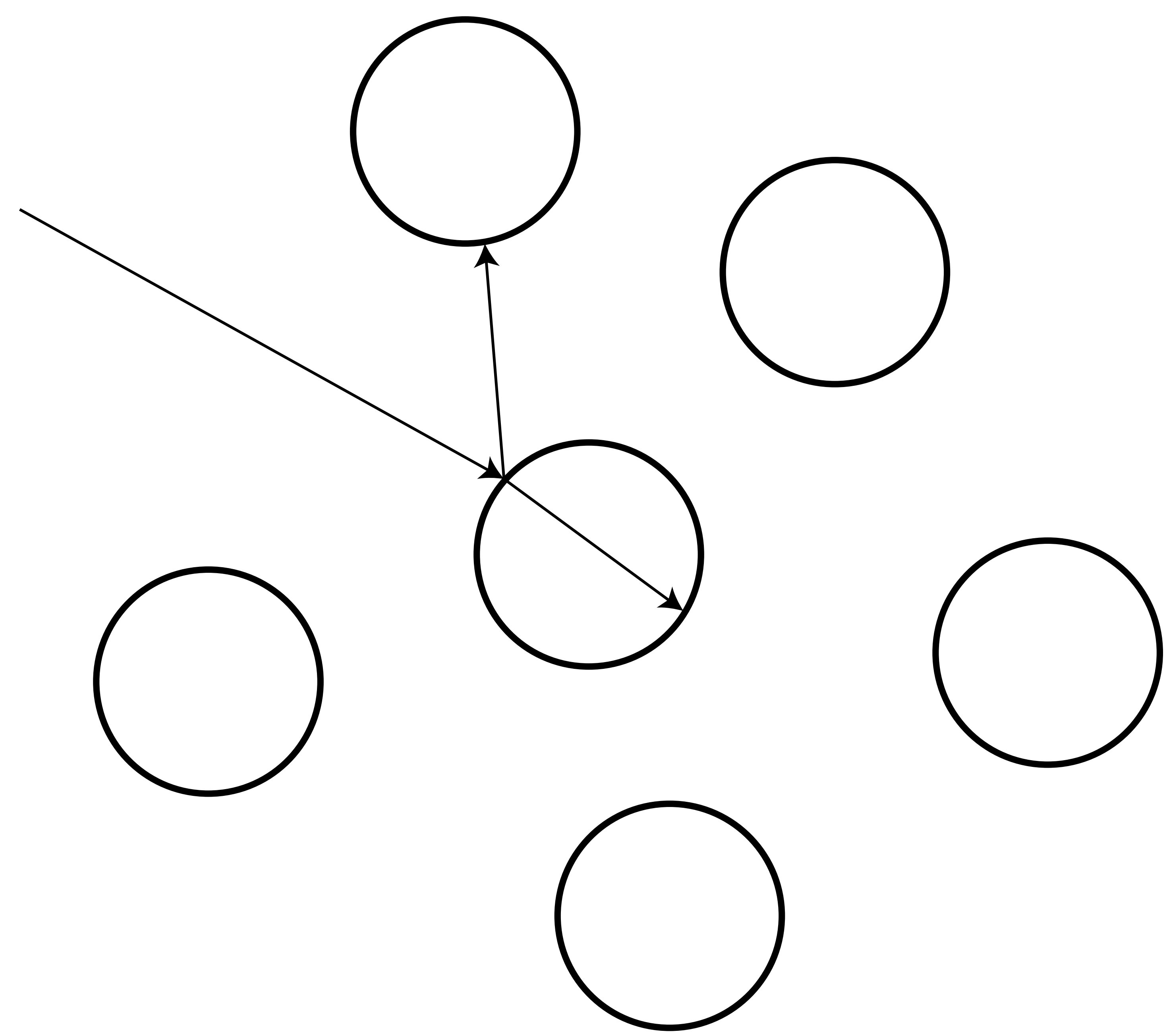
Sampling Reflection + Transmission

$$L_{\mathrm{o}}(\mathrm{p}, \omega) = F_{\mathrm{r}} L_{\mathrm{i}}(\mathrm{p}, R(\omega_{\mathrm{o}}, \mathrm{n})) + (1 - F_{\mathrm{r}}) L_{\mathrm{i}}(\mathrm{p}, T(\omega_{\mathrm{o}}, \mathrm{n}))$$



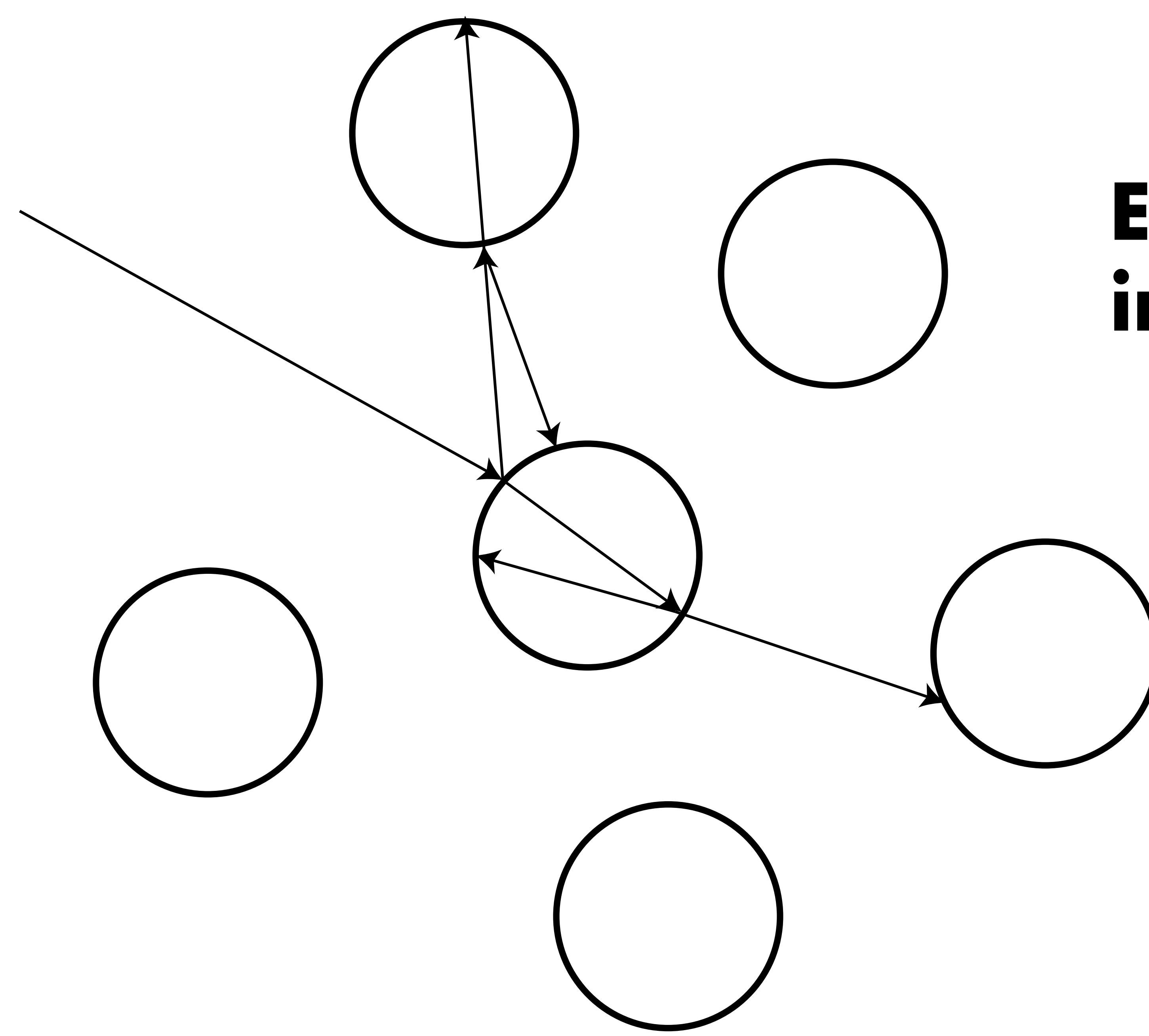
Sampling Reflection + Transmission

$$L_{\mathrm{o}}(\mathrm{p}, \omega) = F_{\mathrm{r}} L_{\mathrm{i}}(\mathrm{p}, R(\omega_{\mathrm{o}}, \mathrm{n})) + (1 - F_{\mathrm{r}}) L_{\mathrm{i}}(\mathrm{p}, T(\omega_{\mathrm{o}}, \mathrm{n}))$$



Sampling Reflection + Transmission

$$L_o(\mathrm{p},\omega) = F_r L_i(\mathrm{p}, R(\omega_o, \mathrm{n})) + (1 - F_r) L_i(\mathrm{p}, T(\omega_o, \mathrm{n}))$$



Exponential growth in number of rays!

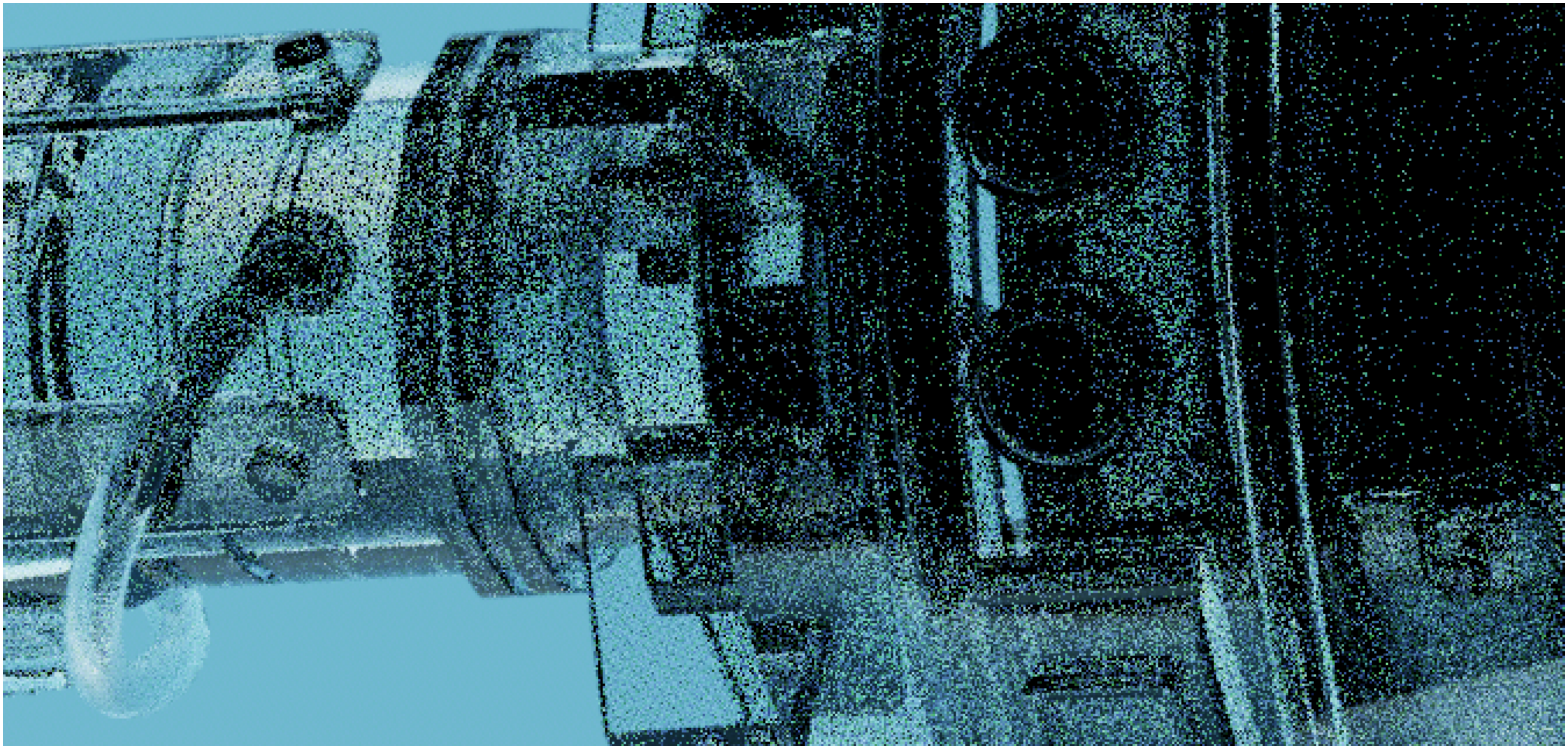
Sampling Reflection + Transmission

Monte Carlo: evaluate a single term using probability p_{r}

$$L_{\mathrm{o}}(\mathrm{p}, \omega) = F_{\mathrm{r}} L_{\mathrm{i}}(\mathrm{p}, R(\omega_{\mathrm{o}}, \mathrm{n})) + (1 - F_{\mathrm{r}}) L_{\mathrm{i}}(\mathrm{p}, T(\omega_{\mathrm{o}}, \mathrm{n}))$$

$$\approx \begin{cases} \dfrac{F_{\mathrm{r}} L_{\mathrm{i}}(\mathrm{p}, R(\omega_{\mathrm{o}}, \mathrm{n}))}{p_{\mathrm{r}}}, & \text{with probability } p_{\mathrm{r}} \\ \dfrac{(1 - F_{\mathrm{r}}) L_{\mathrm{i}}(\mathrm{p}, T(\omega_{\mathrm{o}}, \mathrm{n}))}{1 - p_{\mathrm{r}}}, & \text{otherwise} \end{cases}$$

Sampling Reflection + Transmission



$$p_{\mathrm{r}} = \frac{1}{2}$$

Sampling Reflection + Transmission

Monte Carlo: evaluate a single term using probability p_r

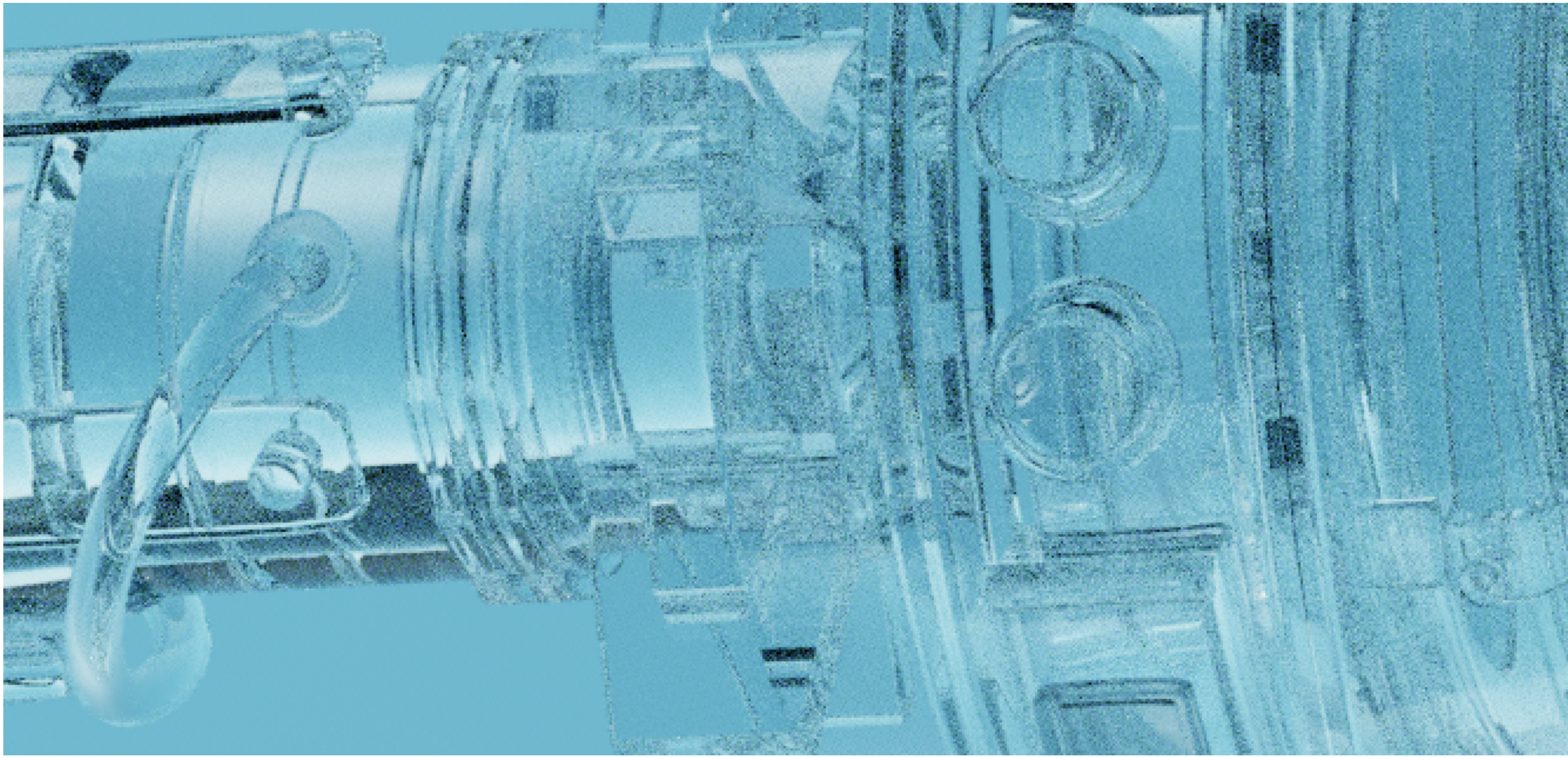
$$L_\mathrm{o}(\mathrm{p}, \omega) = F_\mathrm{r} L_\mathrm{i}(\mathrm{p}, R(\omega_\mathrm{o}, \mathrm{n})) + (1 - F_\mathrm{r}) L_\mathrm{i}(\mathrm{p}, T(\omega_\mathrm{o}, \mathrm{n}))$$

$$\approx \begin{cases} \dfrac{F_\mathrm{r} L_\mathrm{i}(\mathrm{p}, R(\omega_\mathrm{o}, \mathrm{n}))}{p_\mathrm{r}}, & \text{with probability } p_\mathrm{r} \\ \dfrac{(1 - F_\mathrm{r}) L_\mathrm{i}(\mathrm{p}, T(\omega_\mathrm{o}, \mathrm{n}))}{1 - p_\mathrm{r}}, & \text{otherwise} \end{cases}$$

Best approach: $p_\mathrm{r} = F_\mathrm{r}$

$$L_\mathrm{o}(\mathrm{p}, \omega) \approx \begin{cases} L_\mathrm{i}(\mathrm{p}, R(\omega_\mathrm{o}, \mathrm{n})), \text{ with probability } F_\mathrm{r} \\ L_\mathrm{i}(\mathrm{p}, T(\omega_\mathrm{o}, \mathrm{n})), \text{ otherwise} \end{cases}$$

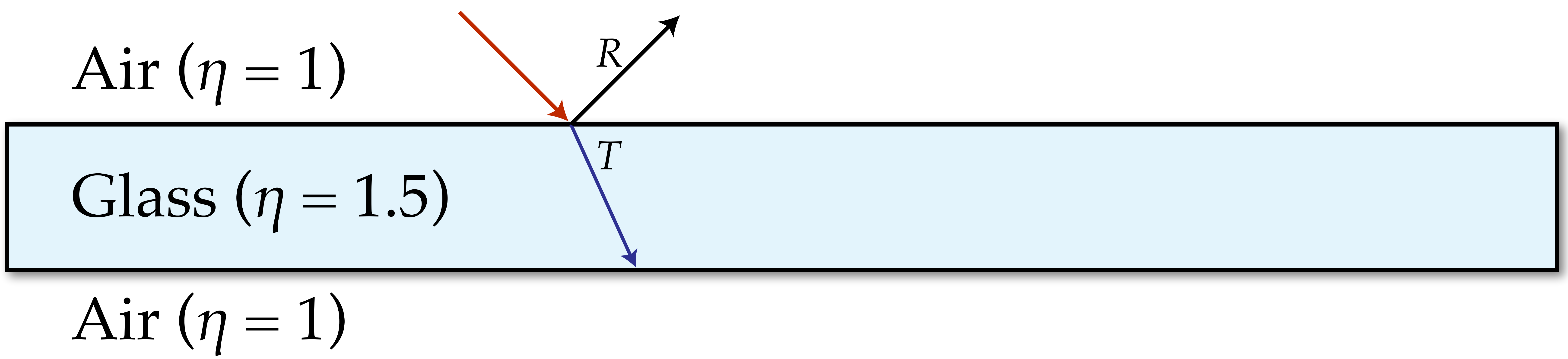
Sampling Reflection + Transmission



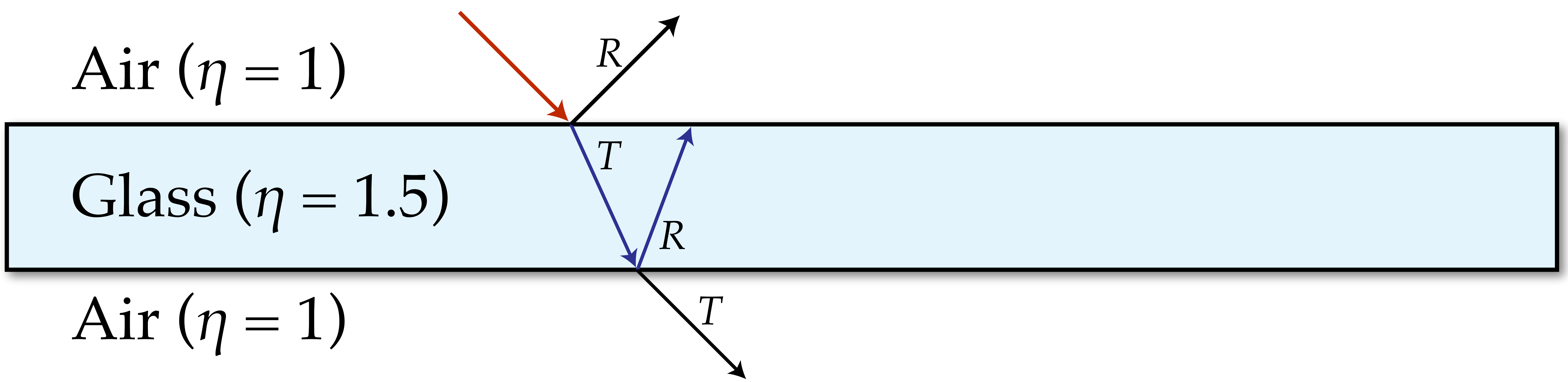
$$p_{\mathrm{r}} = F_{\mathrm{r}}$$

Thin Dielectric BSDF

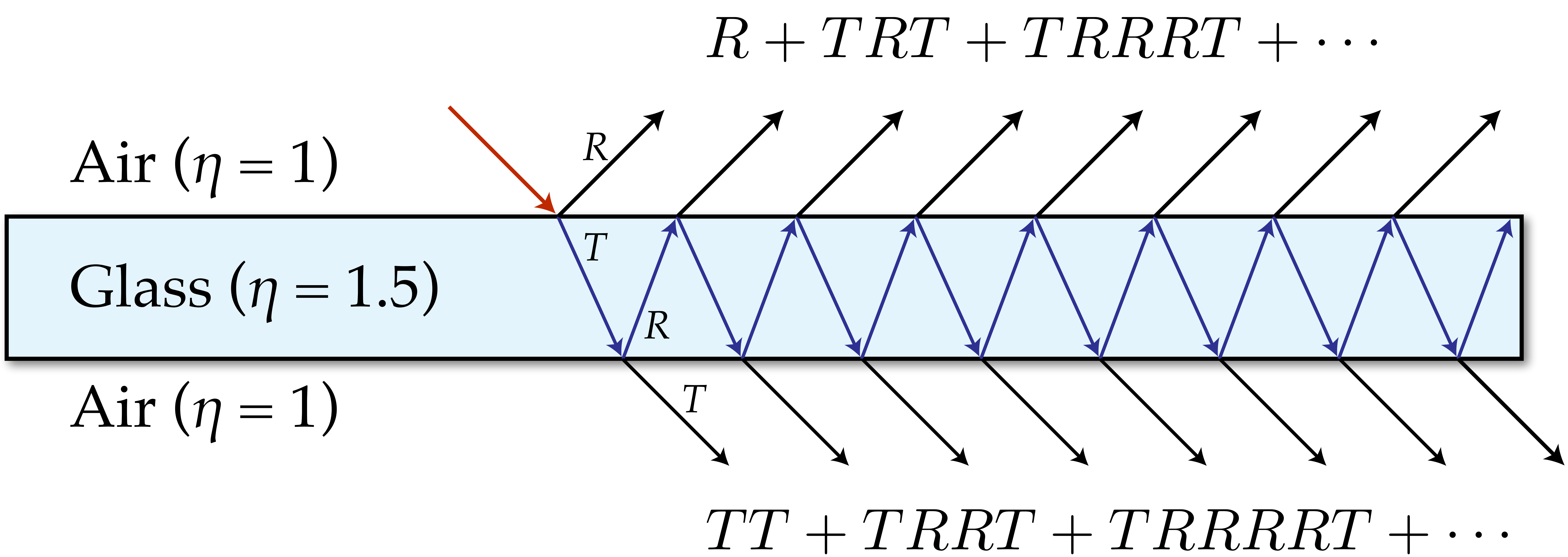
Thin Dielectric BSDF



Thin Dielectric BSDF

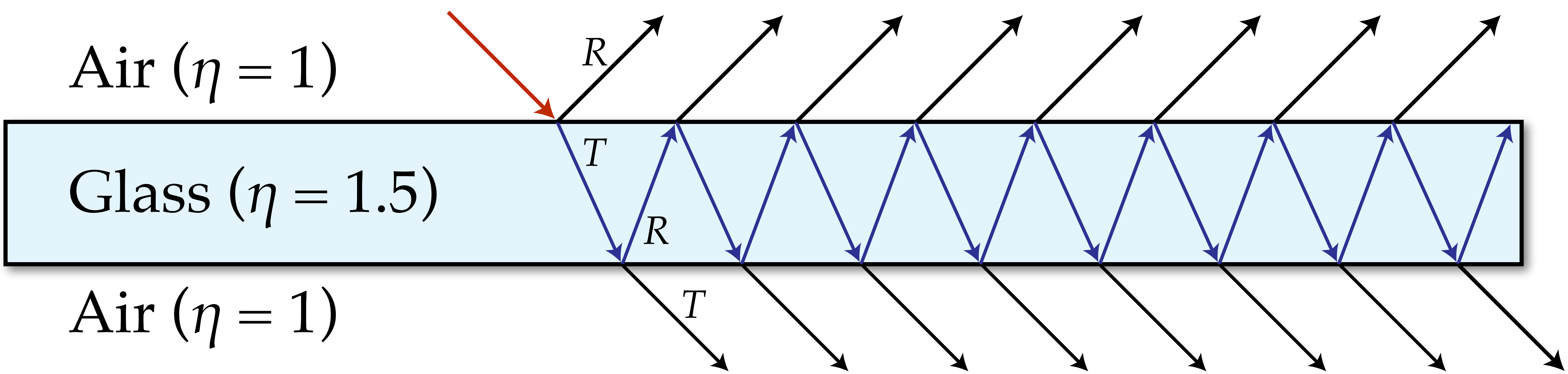


Thin Dielectric BSDF



Thin Dielectric BSDF

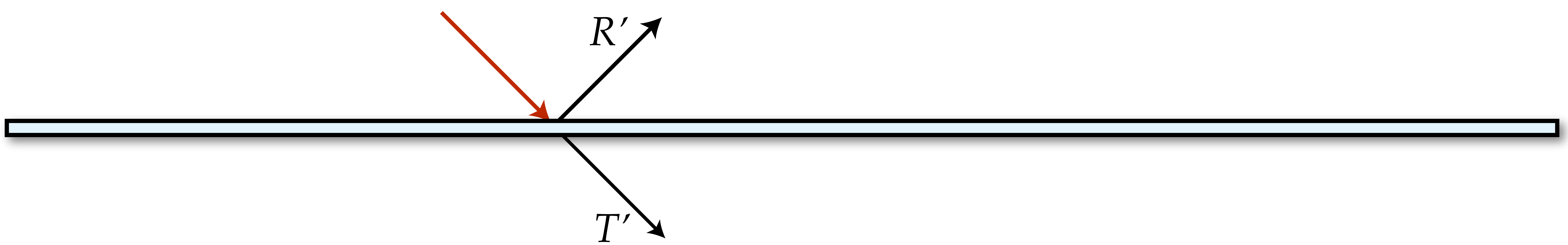
$$R' = R + TRT + TRRRT + \cdots = R + \frac{T^2 R}{1 - R^2}$$



$$T' = TT + TRRT + TRRRRT + \cdots = 1 - R'$$

Thin Dielectric BSDF

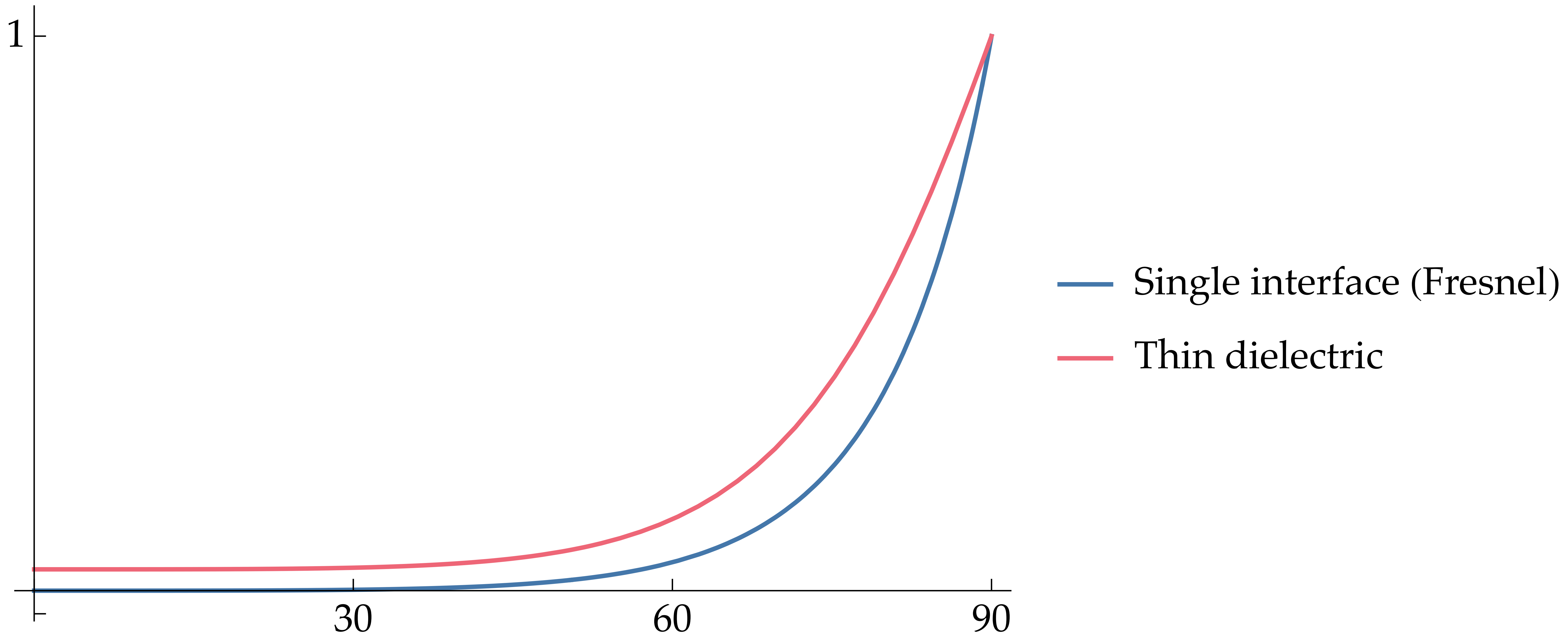
$$R' = R + TRT + TRRRT + \cdots = R + \frac{T^2 R}{1 - R^2}$$



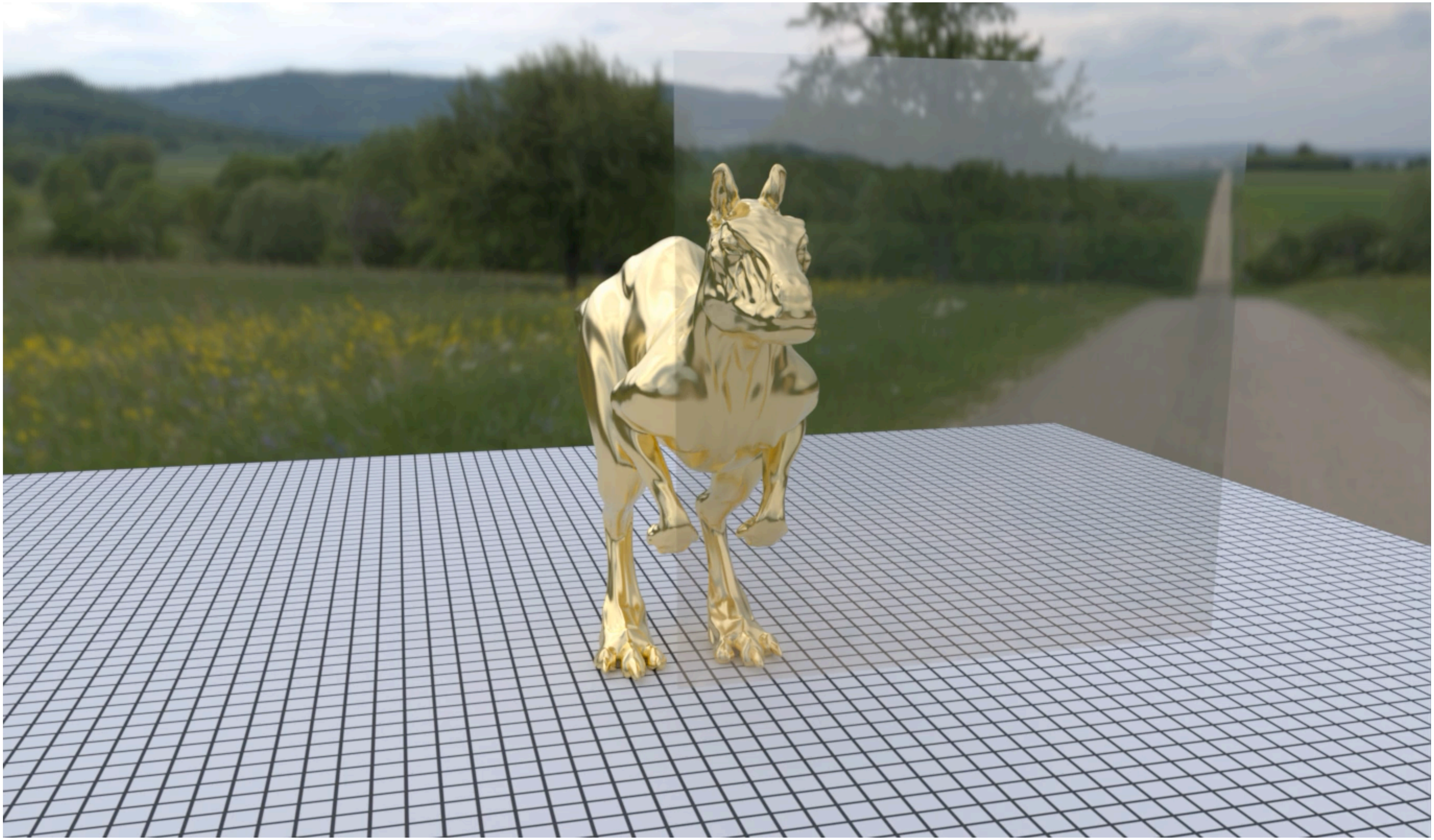
$$T' = TT + TRRT + TRRRRT + \cdots = 1 - R'$$

[Stokes 1860] ***On the intensity of the light reflected from or transmitted through a pile of plates.***

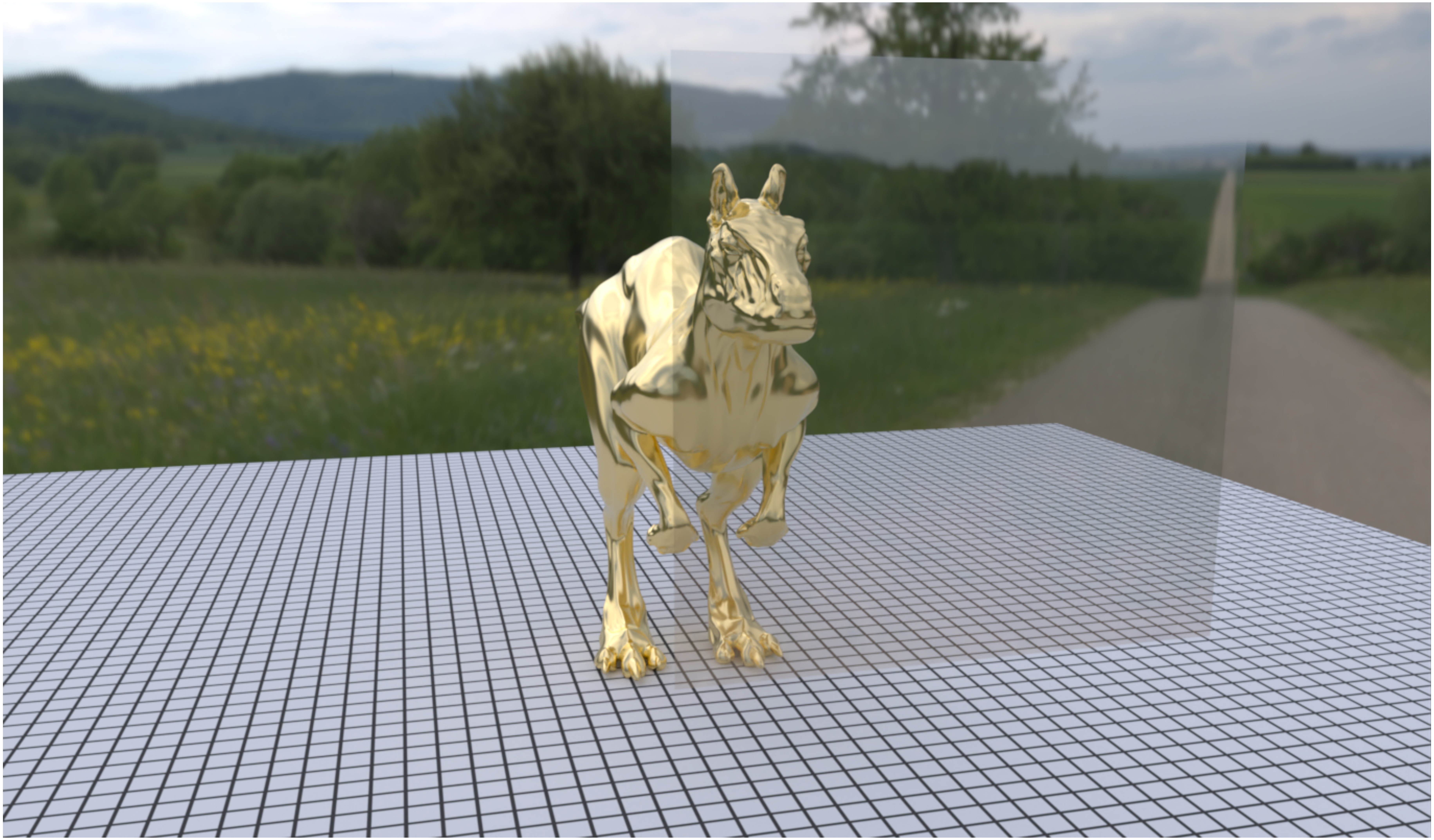
Thin Dielectric Reflectance



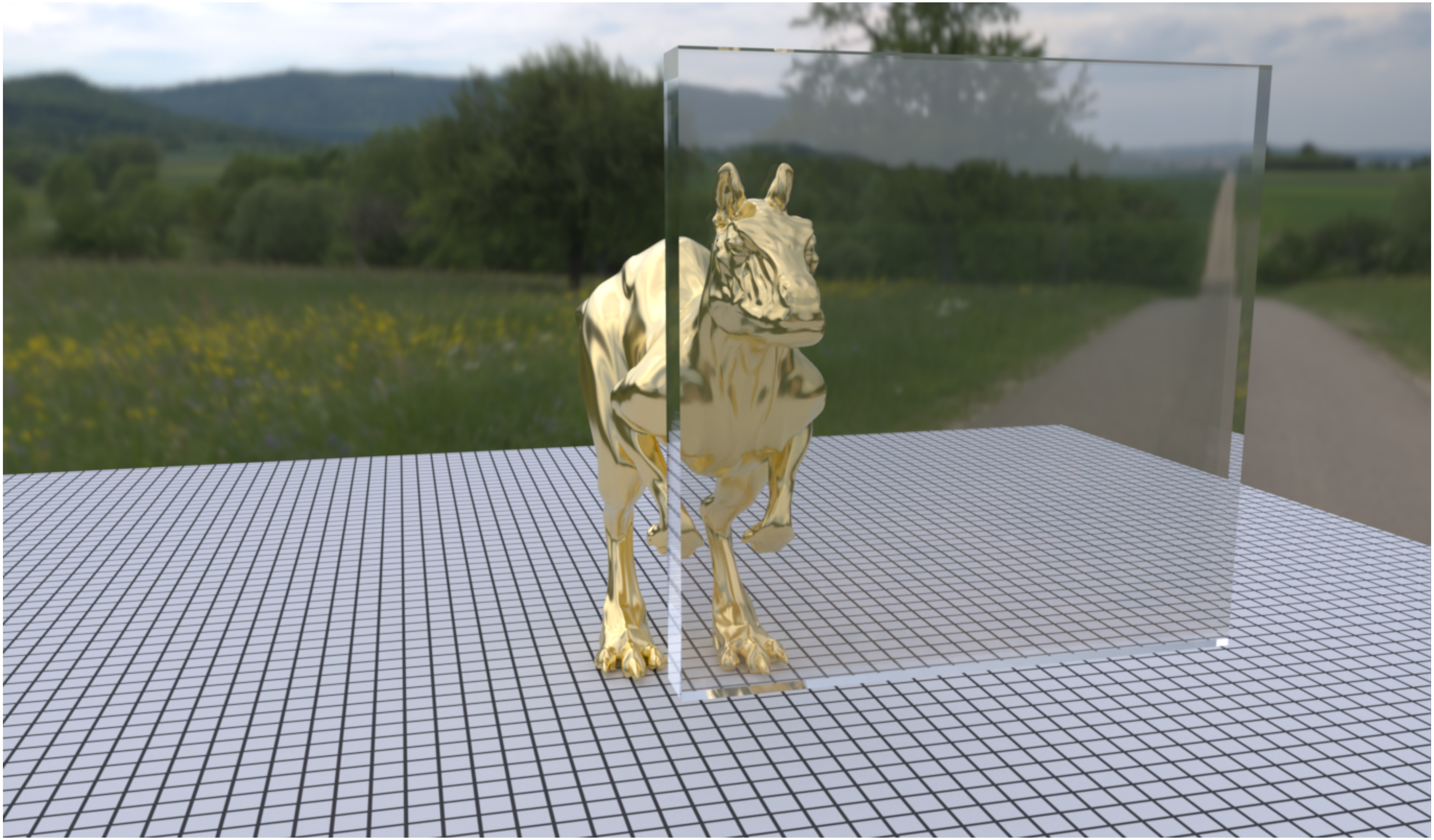
Thin vs Thick Glass



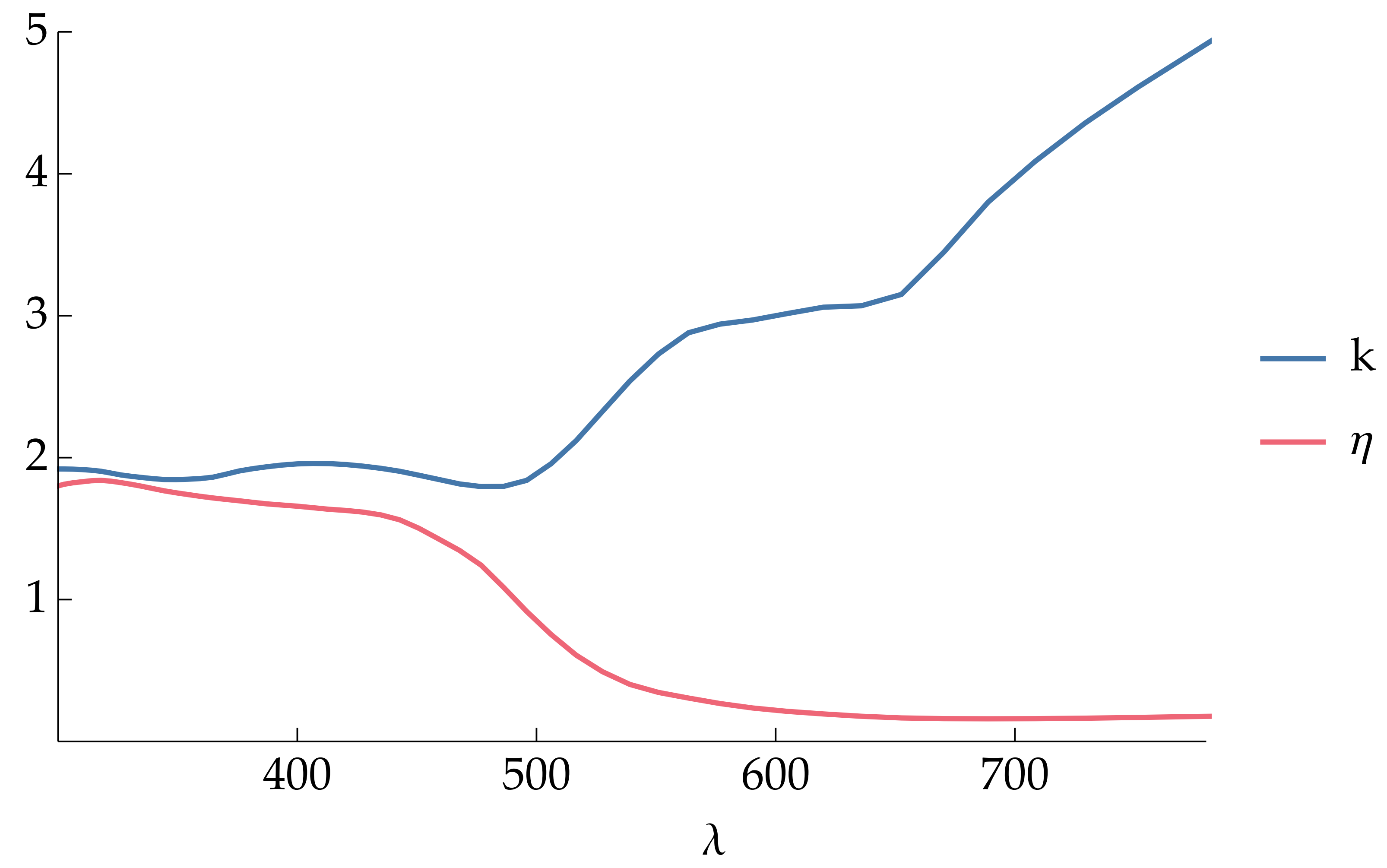
Thin Glass

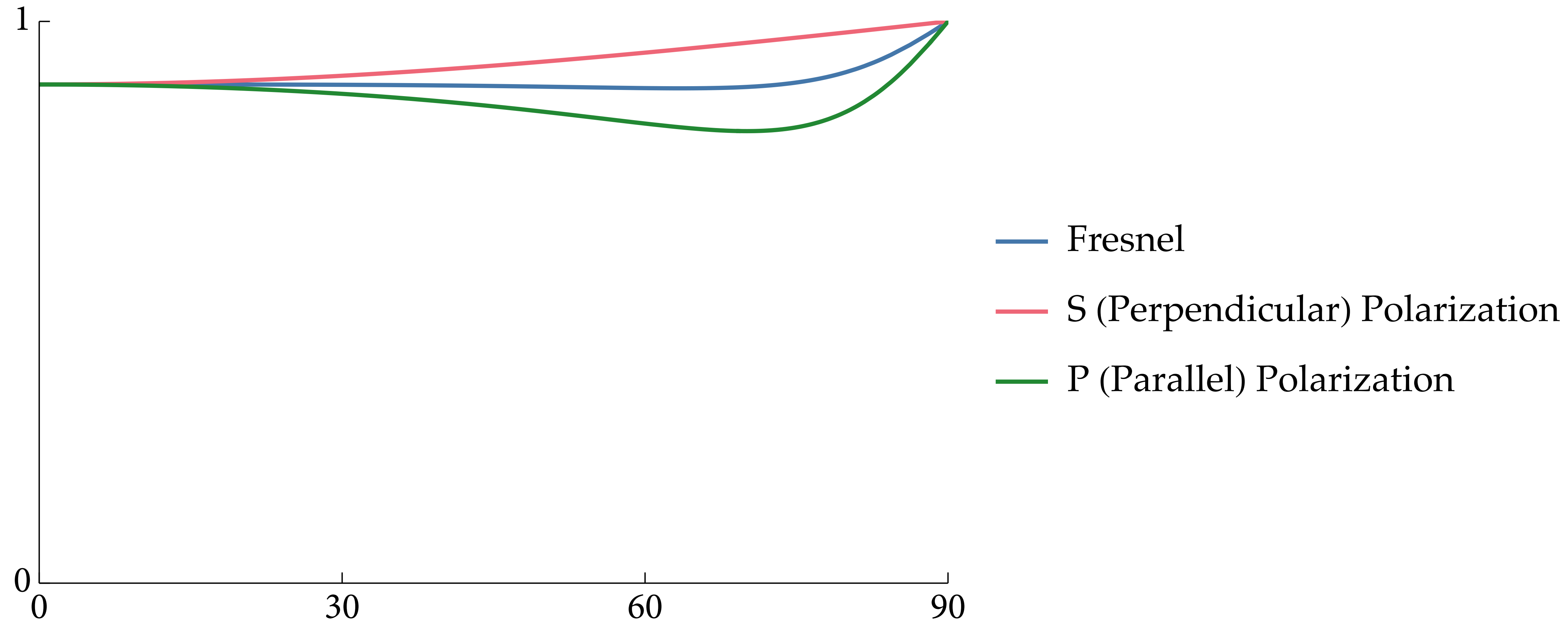


Thick Glass



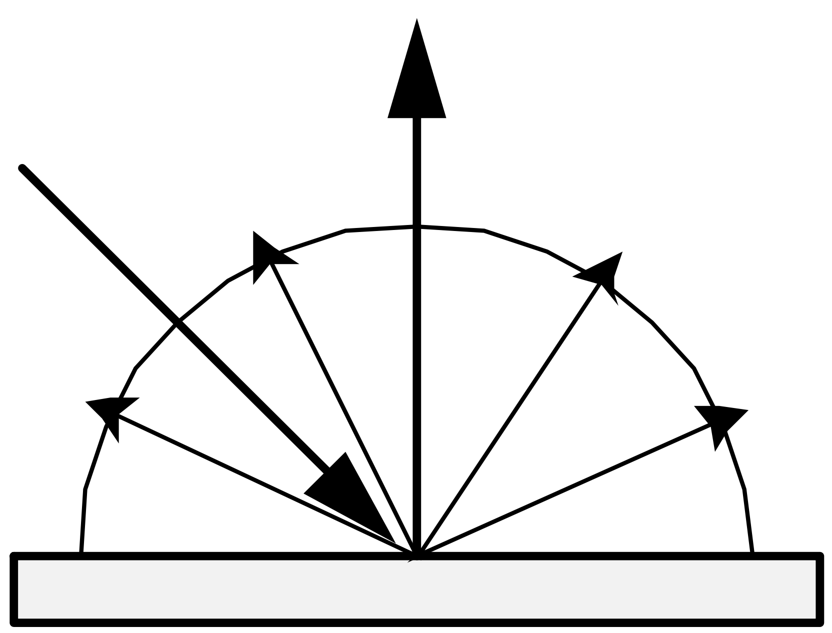
Fresnel Conductor: Gold





Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction



$$f_{r,d} = c$$

$$L_\mathrm{o}(\omega_\mathrm{o}) = \int f_\mathrm{r}\, L_\mathrm{i}(\omega_\mathrm{i})\, \cos\theta_\mathrm{i} \mathrm{d}\omega \mathrm{i}$$
$$= f_\mathrm{r} \int L_\mathrm{i}(\omega_\mathrm{i})\, \cos\theta_\mathrm{i} \mathrm{d}\omega \mathrm{i}$$
$$= f_\mathrm{r} E$$

$$\rho_\mathrm{d} = \frac{\int L_\mathrm{o}(\omega_\mathrm{o})\, \cos\theta_\mathrm{o}\, \mathrm{d}\omega_\mathrm{o}}{\int L_\mathrm{i}(\omega_\mathrm{i})\, \cos\theta_\mathrm{i}\, \mathrm{d}\omega_\mathrm{i}} = \frac{L_\mathrm{o} \int \cos\theta_\mathrm{o}\, \mathrm{d}\omega_\mathrm{o}}{E}$$
$$= \frac{L_\mathrm{o}\pi}{E} = \frac{f_\mathrm{o} E\pi}{E} = \pi f_\mathrm{r} \qquad \longrightarrow \boxed{f_\mathrm{r} = \frac{\rho_\mathrm{d}}{\pi}}$$

“Diffuse” Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufactures attempt to create ideal diffuse

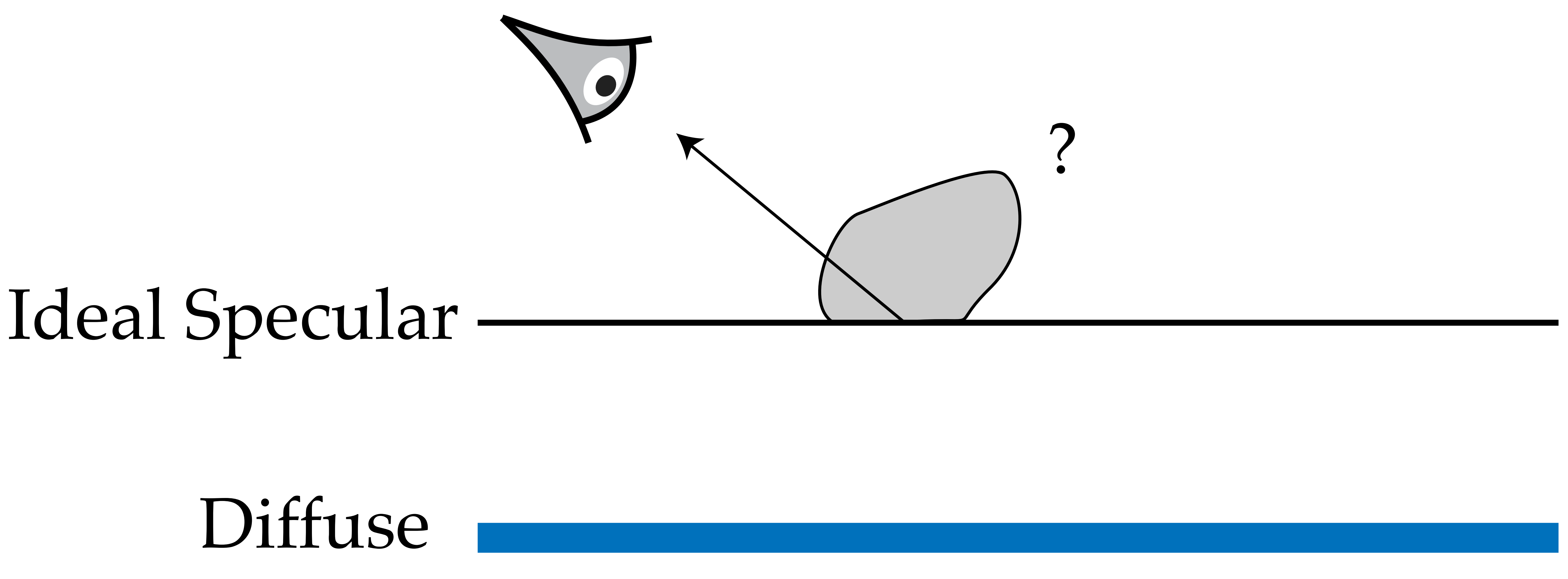
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

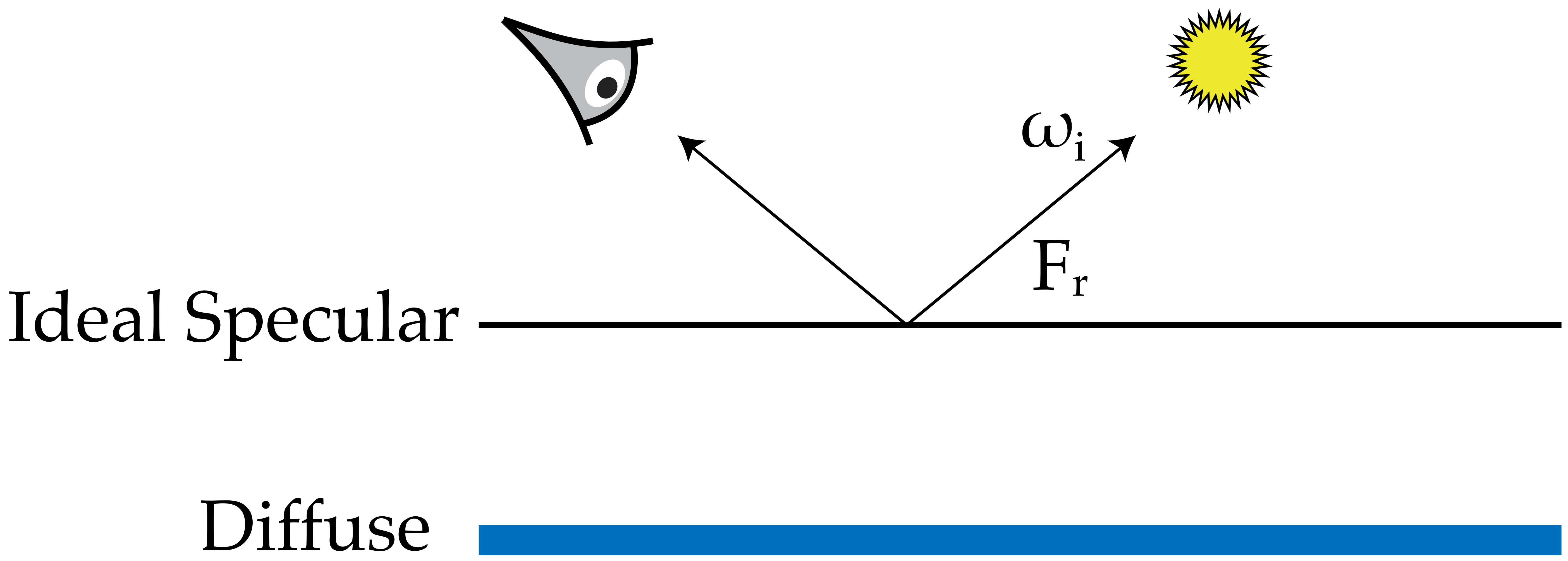
Reflection is greater at glancing angles

Coated Diffuse BRDF



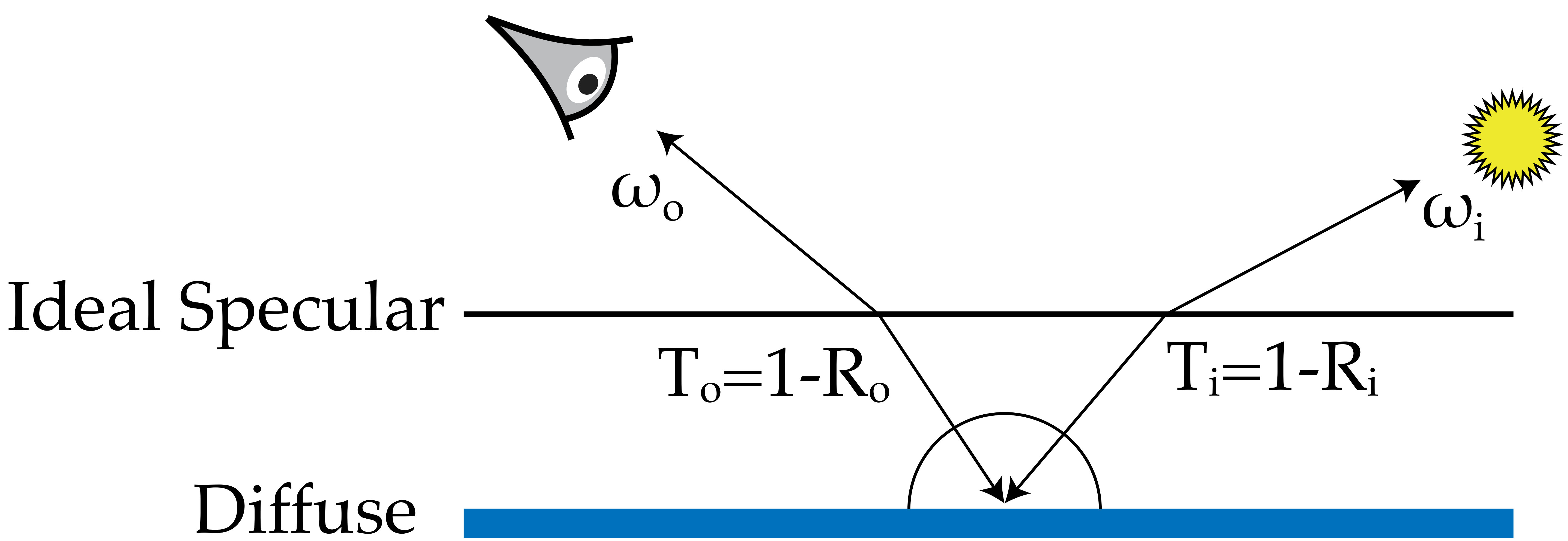
Coated Diffuse BRDF

$$f_\mathrm{r}(\omega_\mathrm{o} \to \omega_\mathrm{i}) = F_\mathrm{r}(\omega_\mathrm{i}) \frac{\delta(\omega_\mathrm{i} - R(\omega_\mathrm{o}, \mathbf{n}))}{\cos\theta_\mathrm{i}} + \cdots$$

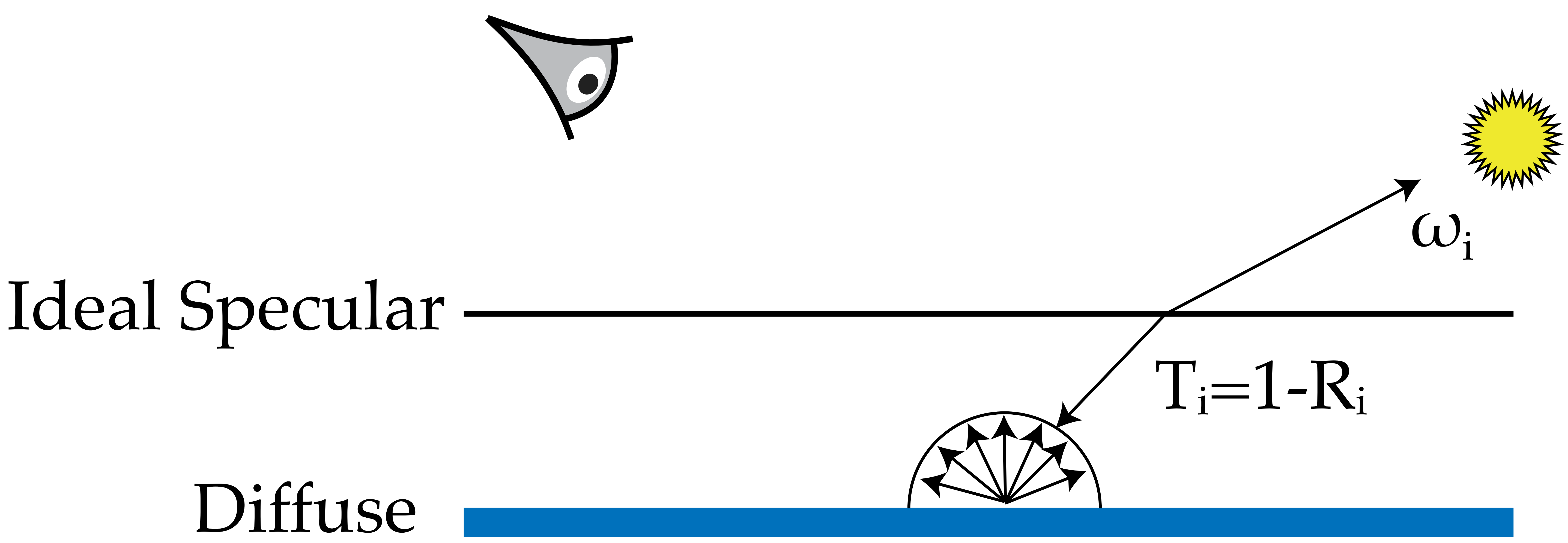


Coated Diffuse BRDF

$$f_r(\omega_o \to \omega_i) = F_r(\omega_i)\frac{\delta(\omega_i - R(\omega_o, \mathbf{n}))}{\cos\theta_i} + (1 - F_r(\omega_o))\frac{\rho_d}{\pi}(1 - F_r(\omega_i)) + \cdots$$



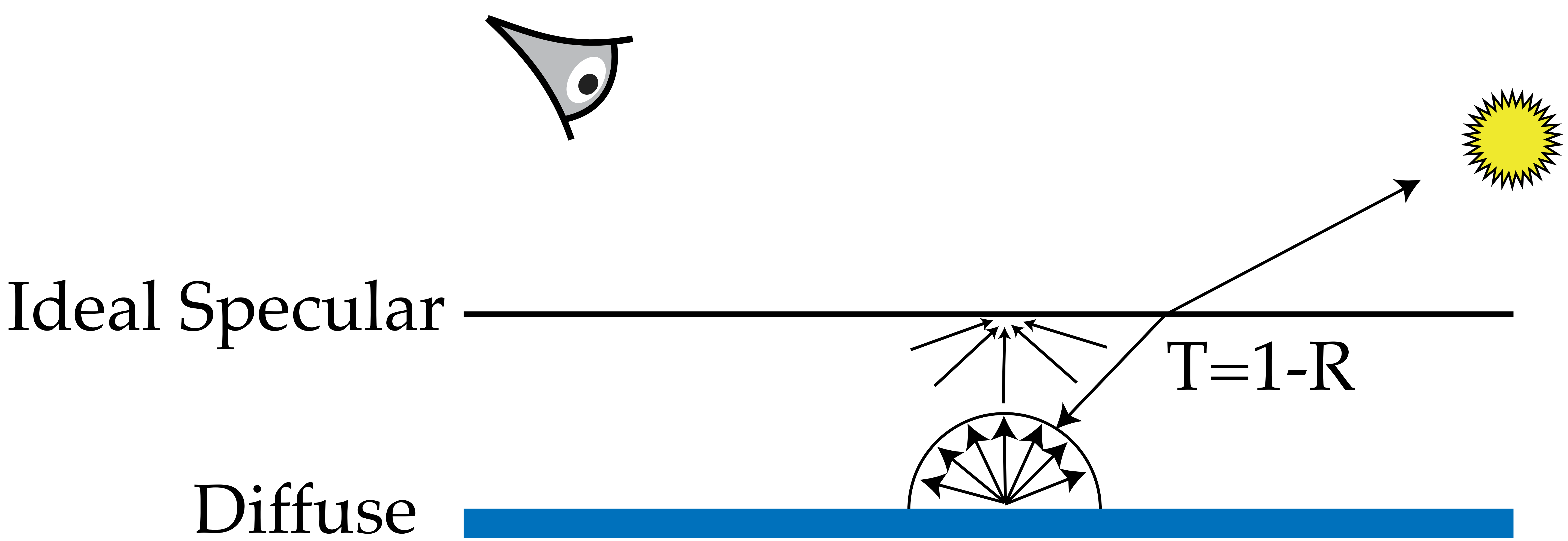
Coated Diffuse BRDF



Coated Diffuse BRDF

How much light is reflected downward at the interface?

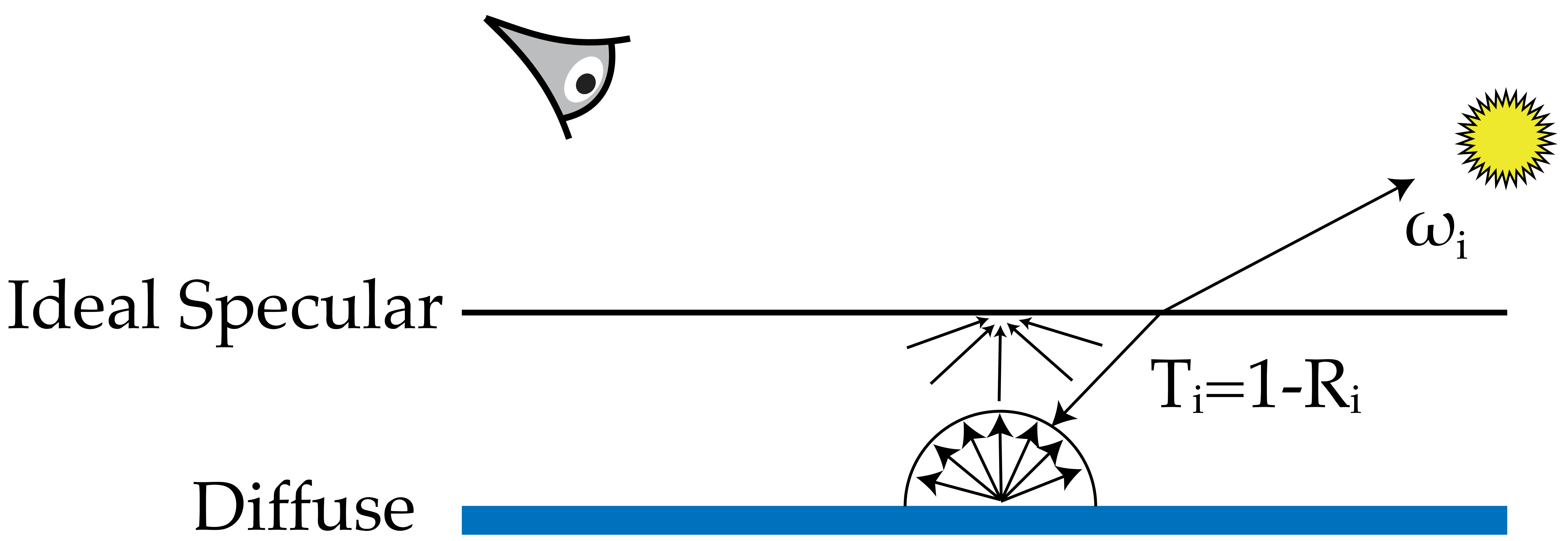
$$\tilde{r} = \int_{\Omega} F_{\mathrm{r}}(\omega, \eta)\,\mathrm{d}\omega$$



Coated Diffuse BRDF

Inter-reflection inside the layer:

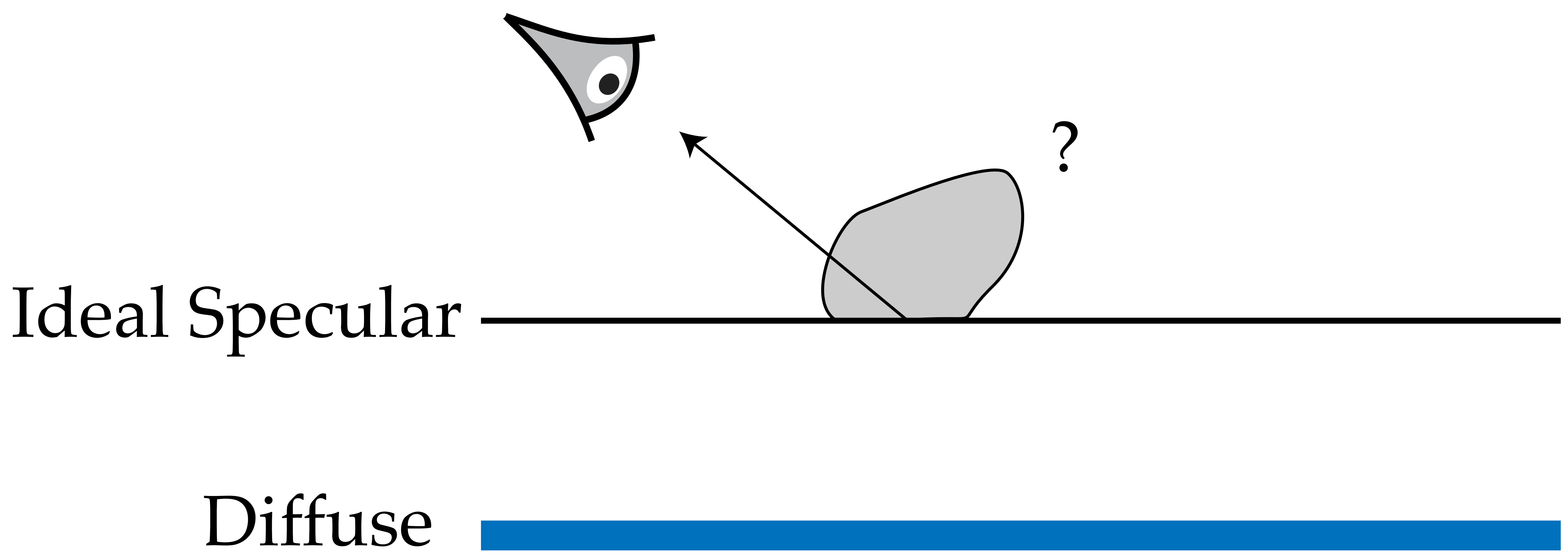
$$f_\mathrm{r} + f_\mathrm{r}\,\tilde{r}\,f_\mathrm{r} + f_\mathrm{r}\,\tilde{r}\,f_\mathrm{r}\,\tilde{r}\,f_\mathrm{r} + \cdots = \frac{f_\mathrm{r}}{1 - \tilde{r}\,f_\mathrm{r}}$$



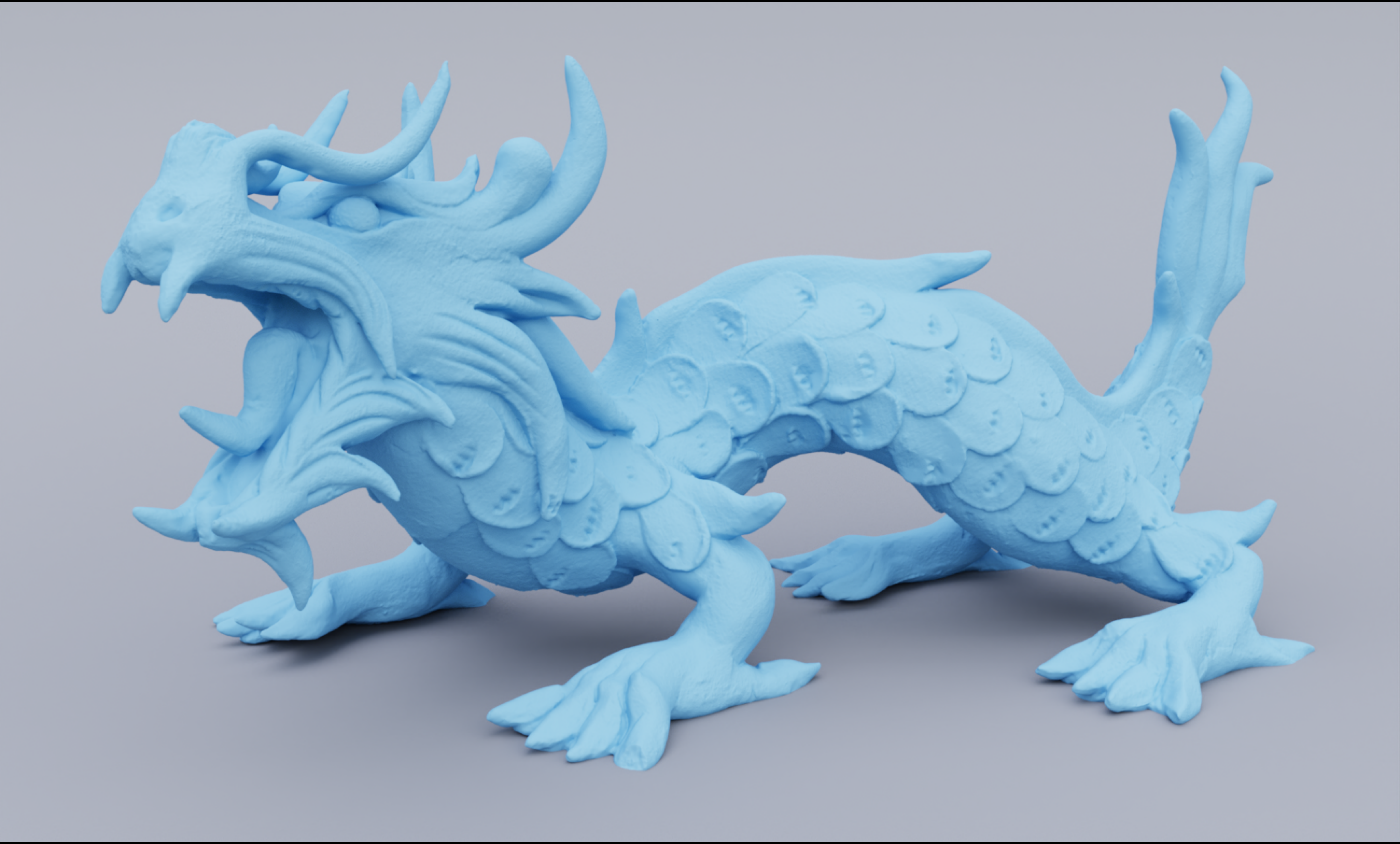
Coated Diffuse BRDF

$$f_r(\omega_o \to \omega_i) = F_r(\omega_i)\frac{\delta(\omega_i - R(\omega_o, \mathbf{n}))}{\cos\theta_i} +$$

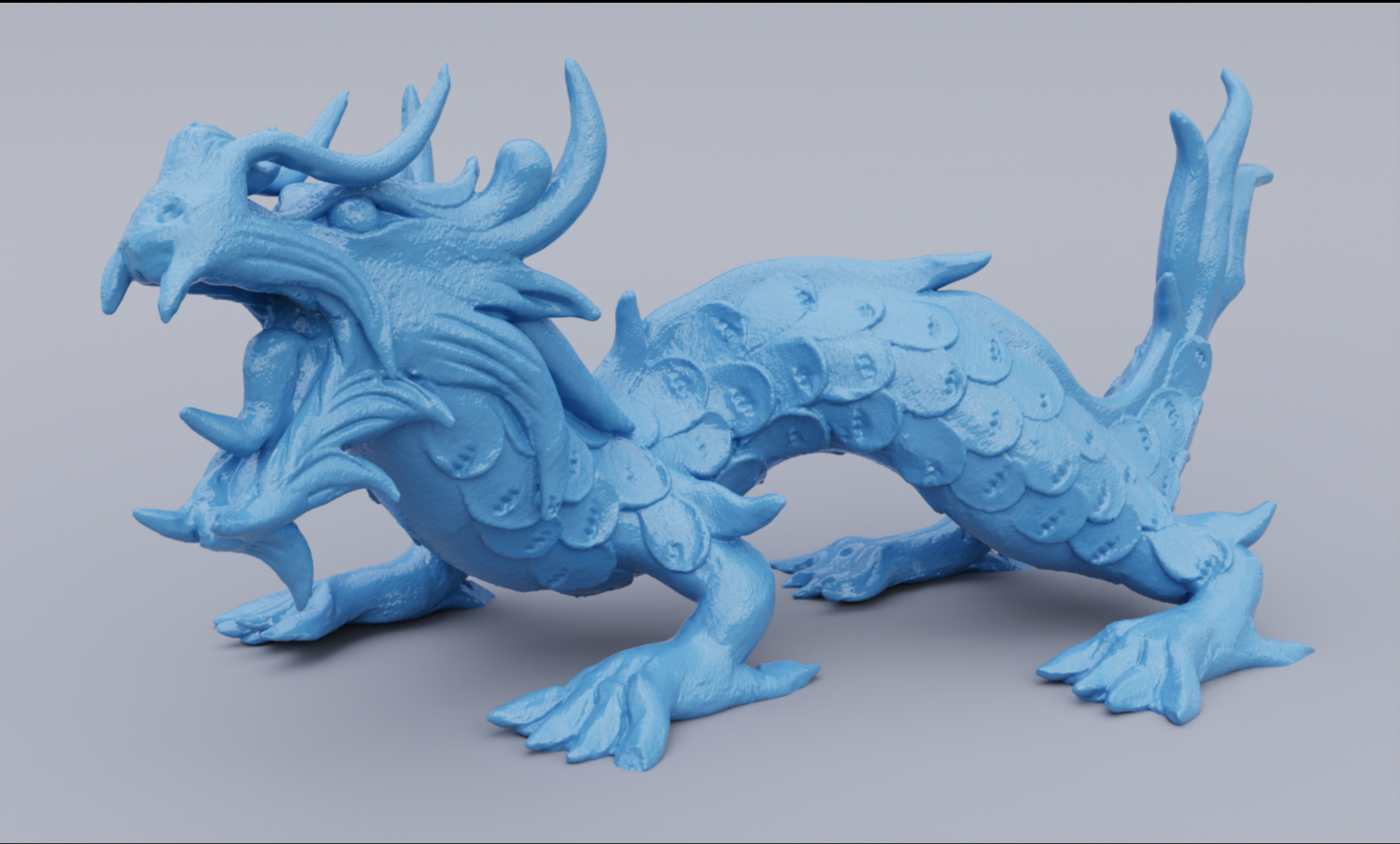
$$(1 - F_r(\omega_o))\frac{f_r}{1 - \tilde{r} f_r}(1 - F_r(\omega_i))$$



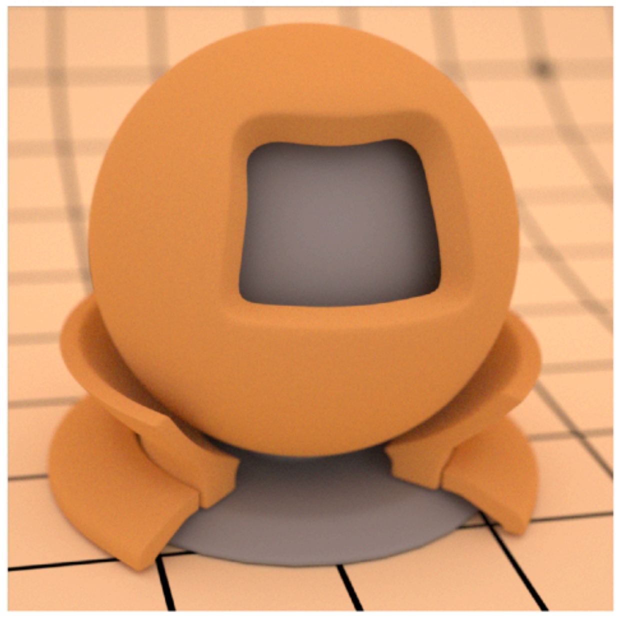
Diffuse BRDF



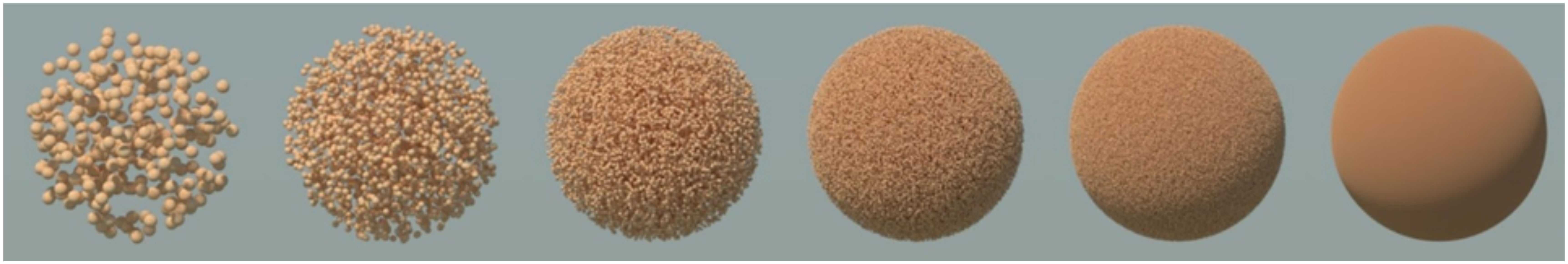
Coated Diffuse BRDF



Lambertian Generalizations



Heitz, E. et al. 2016. *Multiple-Scattering Microfacet BSDFs with the Smith Model.*



d'Eon, E. 2021. *An Analytic BRDF for Materials with Spherical Lambertian Scatterers.*

BRDF follow from inserting Equation 15 into the general solution [HC61, p.55], and we find

$$\Psi^{(0)}(\mu) = \frac{1}{384}c\left(-15(c-1)(4c+9)\mu^4 + (c(20c+281)-346)\mu^2+207\right),$$
$$\Psi^{(1)}(\mu) = -\frac{1}{192}c\left(\mu^2-1\right)\left(5(4c+9)\mu^2-64\right),$$
$$\Psi^{(2)}(\mu) = \frac{15}{256}c\left(\mu^2-1\right)^2.$$

We can then numerically evaluate the H functions using the Fok/Chandrasekhar equation [Foc44, Kre62]

$$H^{(i)}(\mu) = \exp\left(-\frac{\mu}{\pi}\int_0^\infty \frac{1}{1+\mu^2t^2}\log K^{(i)}(t)dt\right), \tag{22}$$

where the functions $K^{(i)}(t)$ are given by [Kre62]

$$K^{(i)}(t) = 1 - \int_1^\infty \left(\frac{1}{s-it}+\frac{1}{s+it}\right)\frac{\Psi^{(i)}\left(\frac{1}{s}\right)}{s}ds. \tag{23}$$

Working these out, we find

$$K^{(0)}(t) = 1 - \frac{c\left((256c-301)t^3 + \left((346-c(20c+281))t^2 - 15(c-1)(4c+9)+207t^4\right)\tan^{-1}(t) + 15(c-1)(4c+9)t\right)}{192t^5}, \tag{24}$$

$$K^{(1)}(t) = 1 - \frac{c\left((40c+282)t^3 - 3\left(t^2+1\right)\left(20c+64t^2+45\right)\tan^{-1}(t)+15(4c+9)t\right)}{288t^5}, \tag{25}$$

$$K^{(2)}(t) = 1 - \frac{5c\left(3\left(t^2+1\right)^2\tan^{-1}(t) - t\left(5t^2+3\right)\right)}{128t^5}. \tag{26}$$

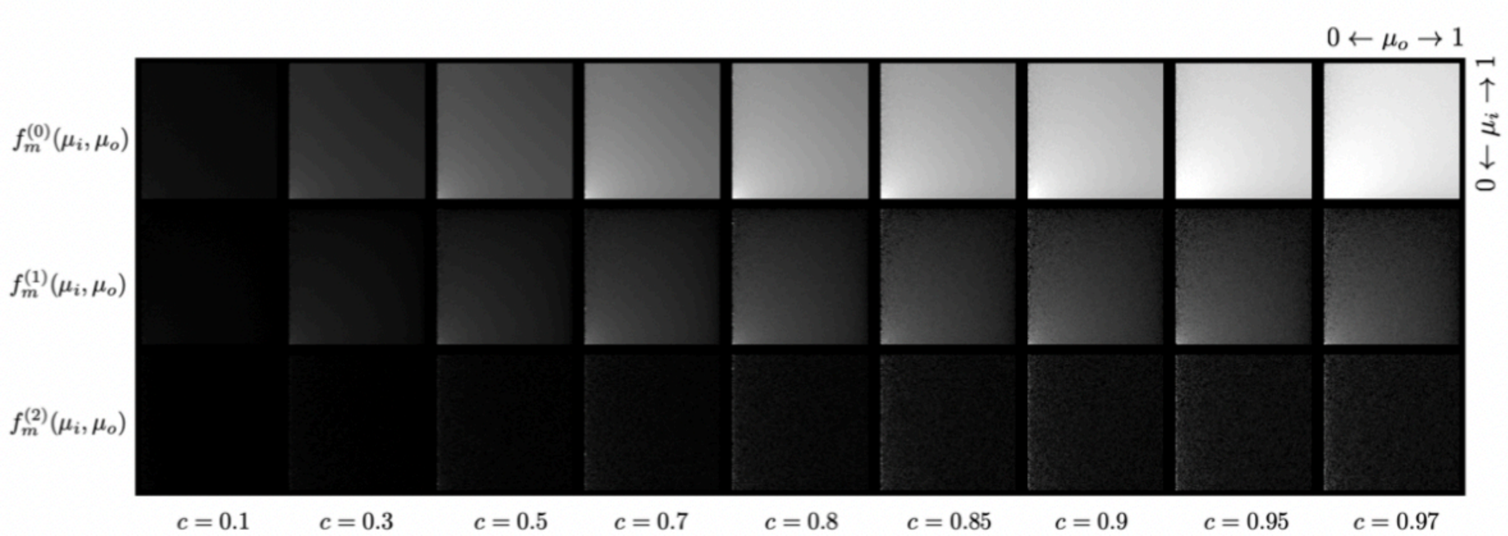


Figure 3: *Using Monte Carlo reference, we observe comparatively weak signal in the second-order mode of the multiple-scattering portion of the BRDF, $f^{(2)}(\mu_i,\mu_o)$ (bottom row).*

3.3. First-order Fourier mode

In Figure 3 we see that the first-order Fourier mode $f_m^{(1)}$ of the multiple scattering is non-negligible. This requires that f_m has a term of the form $f(\mu_i,\mu_o)\cos(\phi)$ for some function f. We approximate this term from the exact solution and this one of the key differences of our BRDF to previous approximations, which assume $f_m^{(1)} = 0$ [Hap81, Hap02].

The exact first-order mode of the BRDF is [HC61, Eq.(43)]

$$f^{(1)}(\mu_i,\mu_o) = \frac{cH^{(1)}(\mu_i)H^{(1)}(\mu_o)}{6\pi(\mu_i+\mu_o)}\sqrt{\left(1-\mu_i^2\right)\left(1-\mu_o^2\right)}\times$$
$$\times\left(1+\left(l^2+\frac{45m}{64}\right)\mu_i\mu_o + l(\mu_i+\mu_o)\right), \tag{27}$$