Reflection Models II

Last lecture

- The reflection equation and the BRDF
- Ideal reflection and refraction
- Diffuse surfaces and layered materials

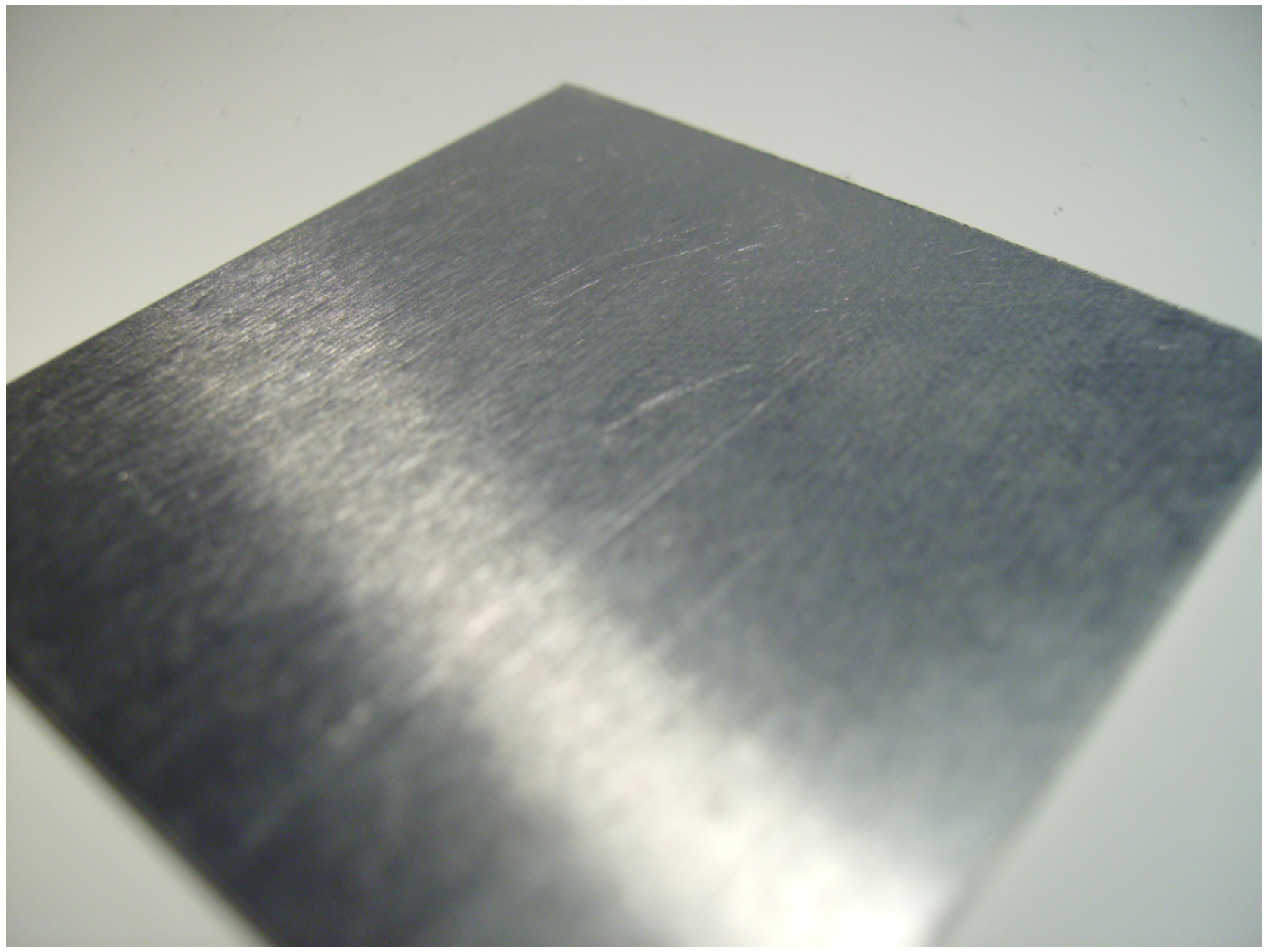
Today

- Microfacet models
  - Advanced topics: inter-reflection, wave effects, glints
- Layered reflection

Properties of BRDFs

3. Isotropic: BRDF is a 3D function

$$f_r((\theta_i, \phi_i) \rightarrow (\theta_o, \phi_o)) \Rightarrow f_r(\theta_i, \theta_o, \phi_i - \phi_o)$$

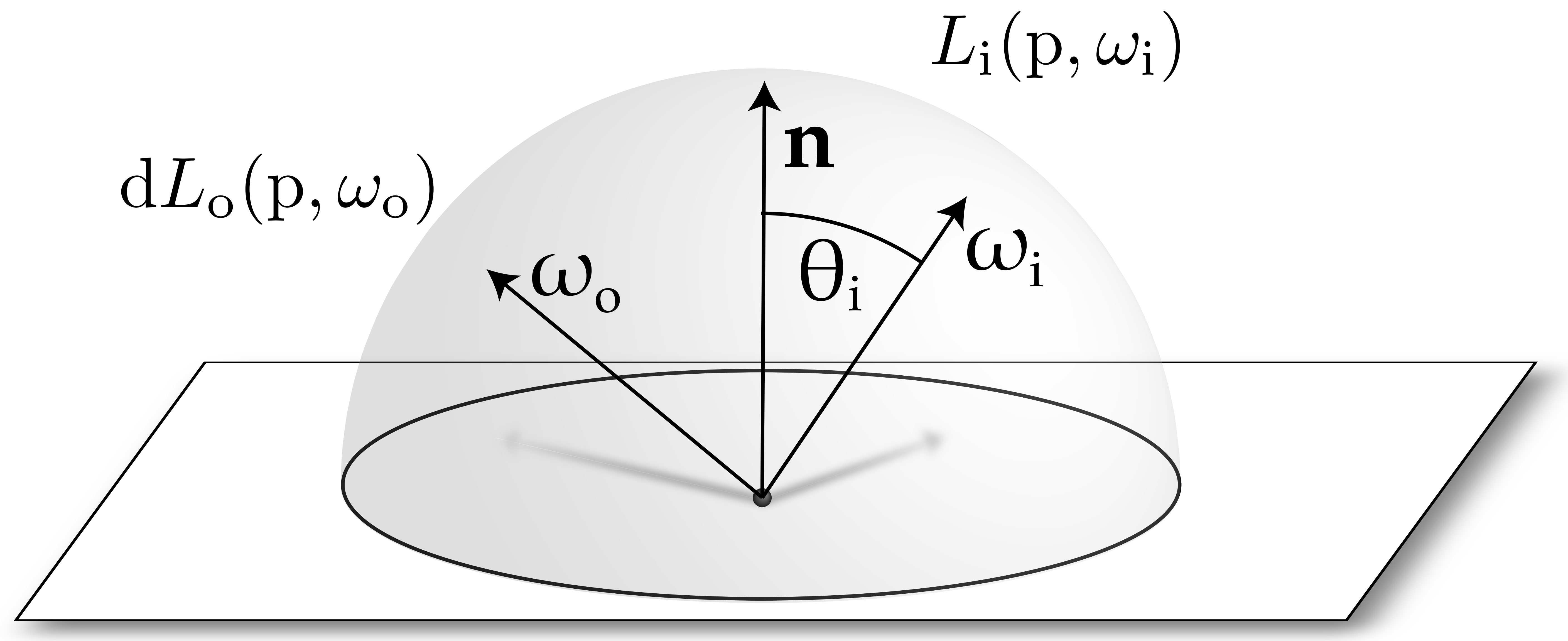


Anisotropic: e.g., brushed metal

(Wikiepedia CC-A-SA 3.0)

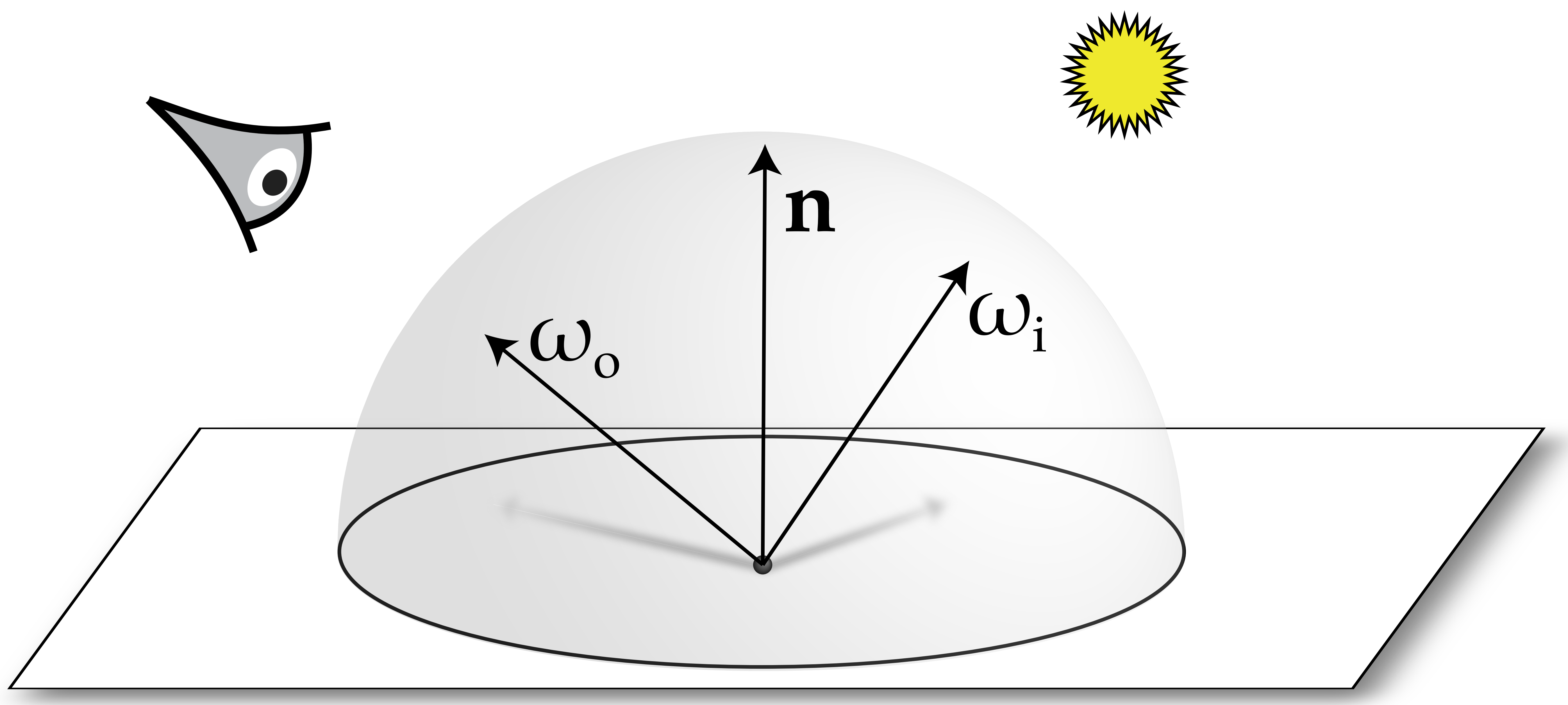
The BRDF

Bidirectional Reflectance-Distribution Function



$$f_{\mathrm{r}}(\omega_{\mathrm{i}} \rightarrow \omega_{\mathrm{o}}) \equiv \frac{\mathrm{d}L_{\mathrm{o}}(\omega_{\mathrm{o}})}{\mathrm{d}E_{\mathrm{i}}(\omega_{\mathrm{i}})} \quad \left[\frac{1}{sr}\right]$$

The Reflection Equation

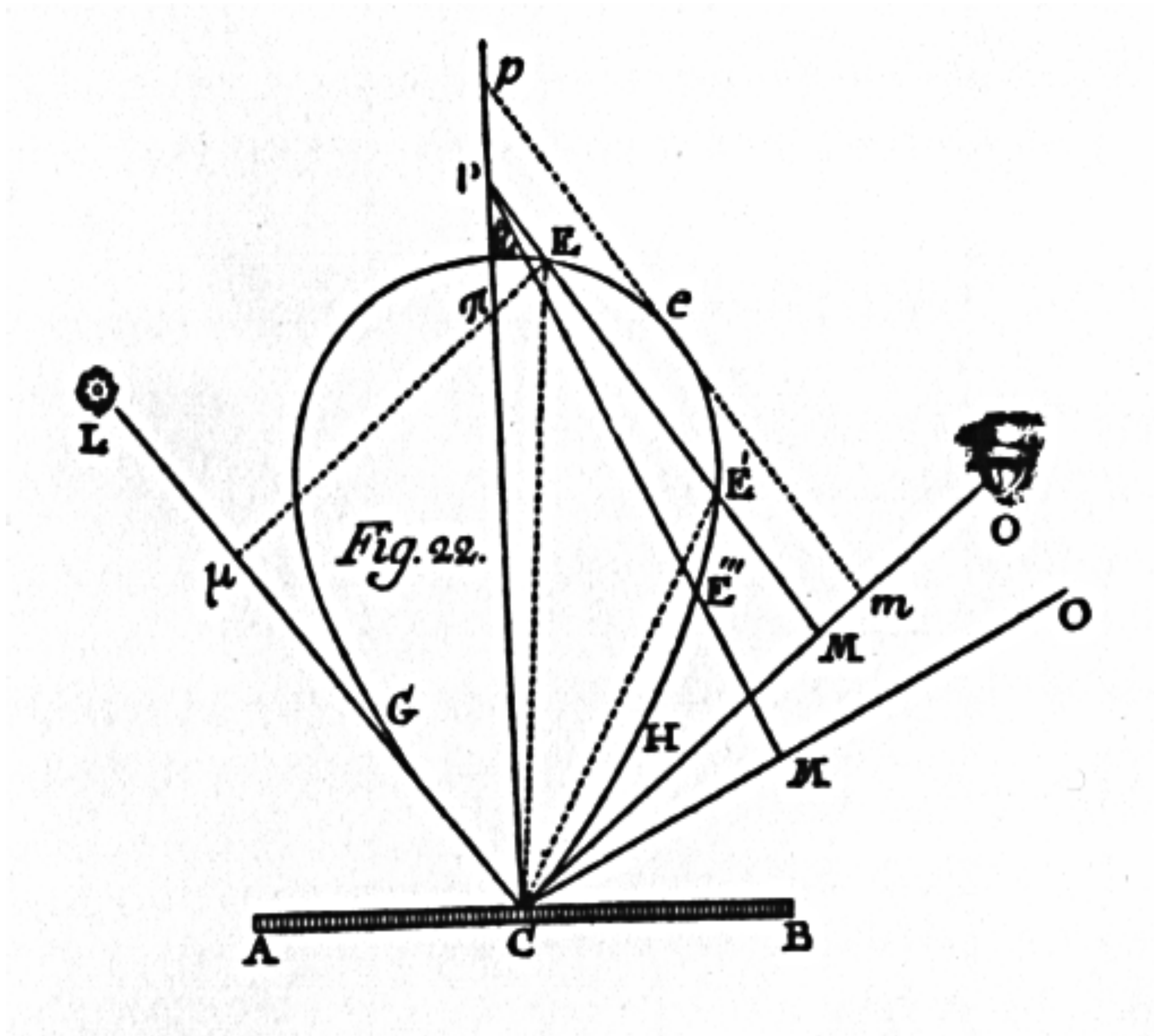


$$L_o(p, \omega_o) = \int_{\Omega^2} \underbrace{f_r(p, \omega_i \to \omega_o)}_{\textbf{BRDF}} \underbrace{L_i(p, \omega_i)\ \cos\theta_i}_{\textbf{Illumination}} \, d\omega_i$$

Microfacet Reflection

https://twitter.com/Cmdr_Hadfield/status/318986491063828480/photo/1

Bouguer's "little faces"



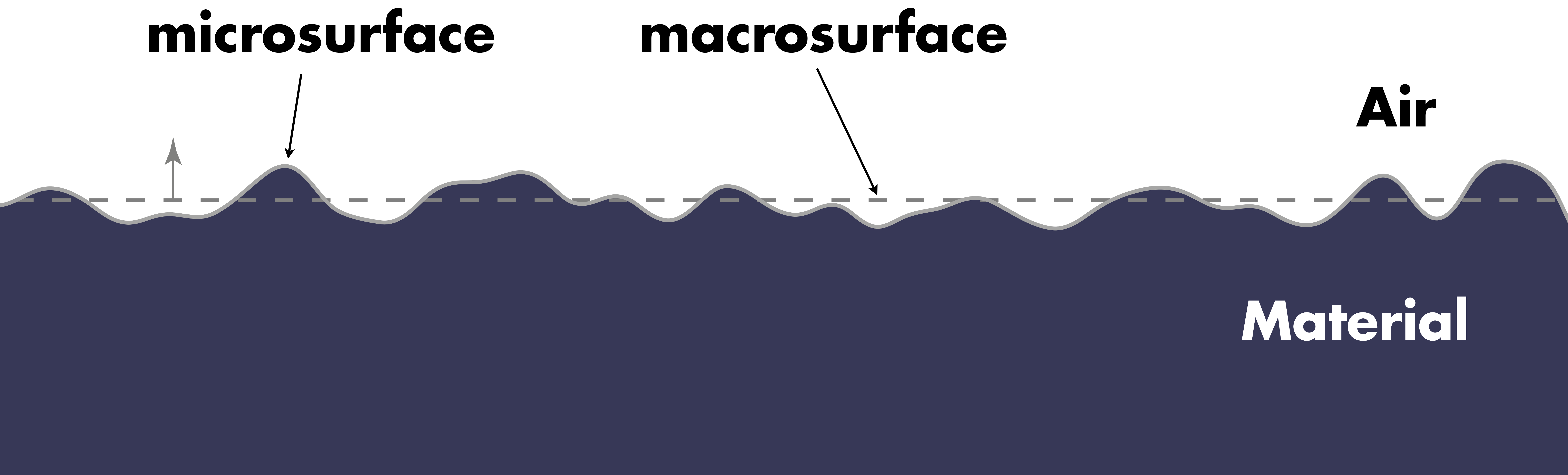
P. Bouguer, Optical Treatise on the Gradation of Light, 1760

Microfacet Scattering Models

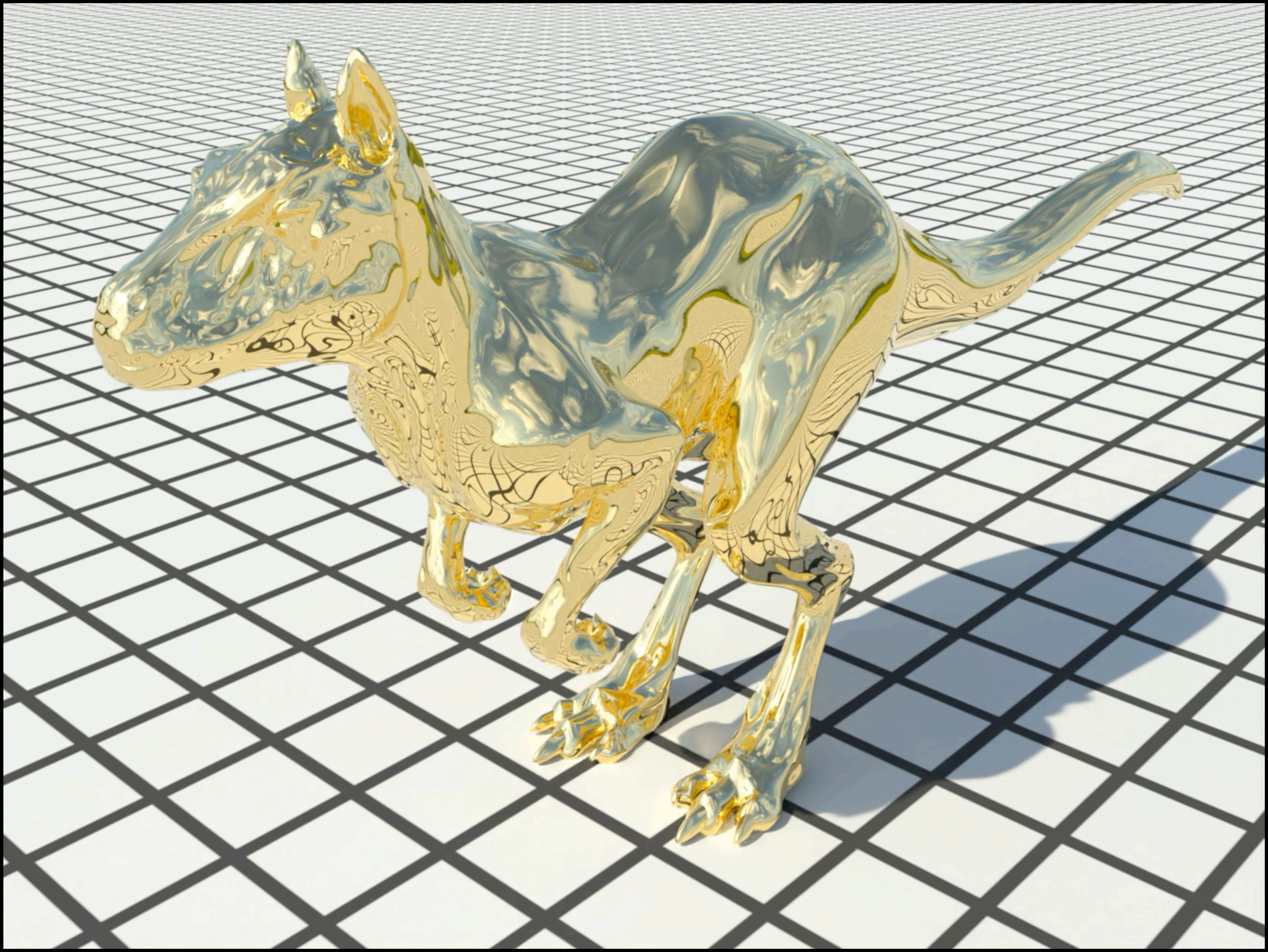
Rough surface

- Smooth at wavelength scale
- Rough at microscale
- Flat at macroscale

Individual elements of surface act like mirrors

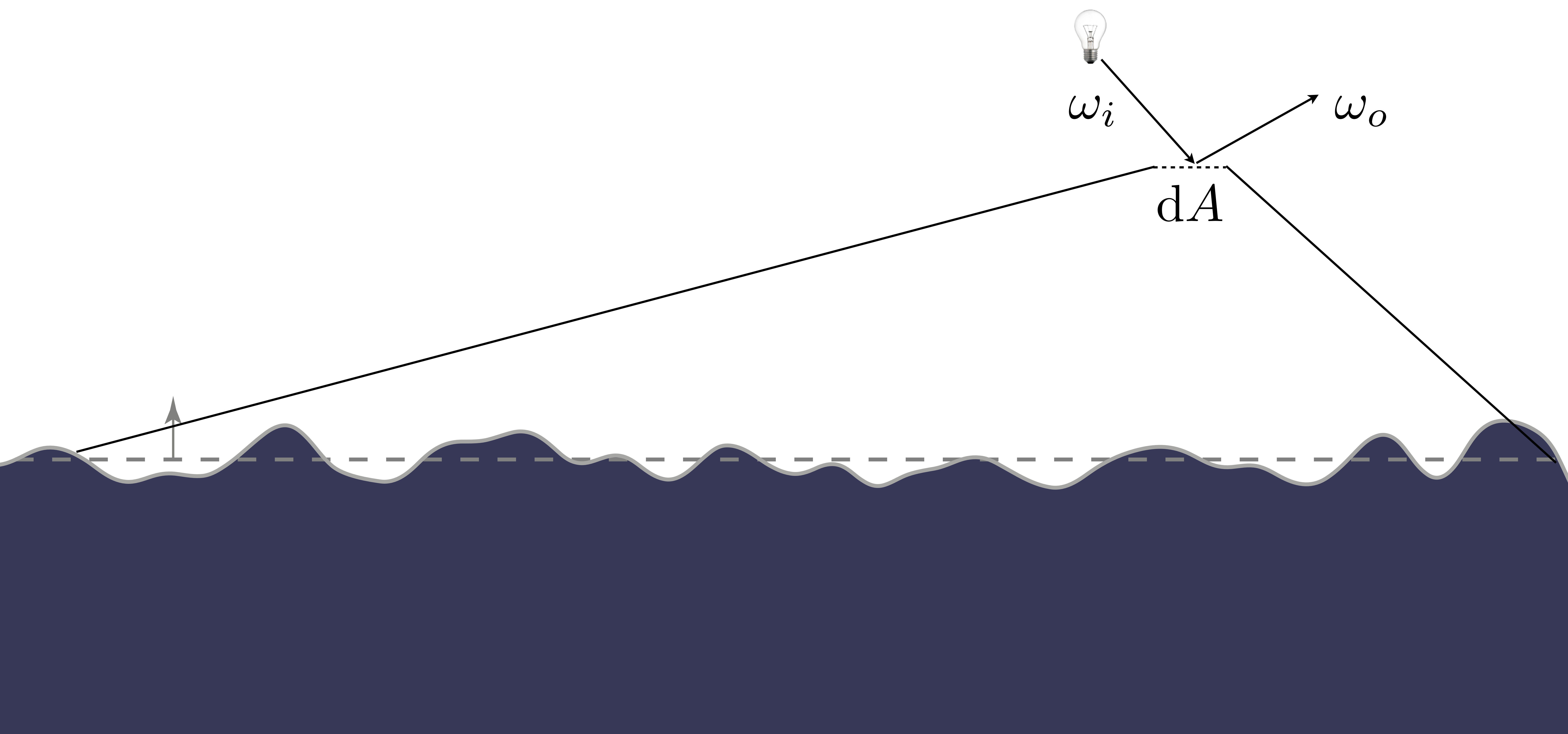


Microfacet Scattering Models



Microfacet Scattering Models

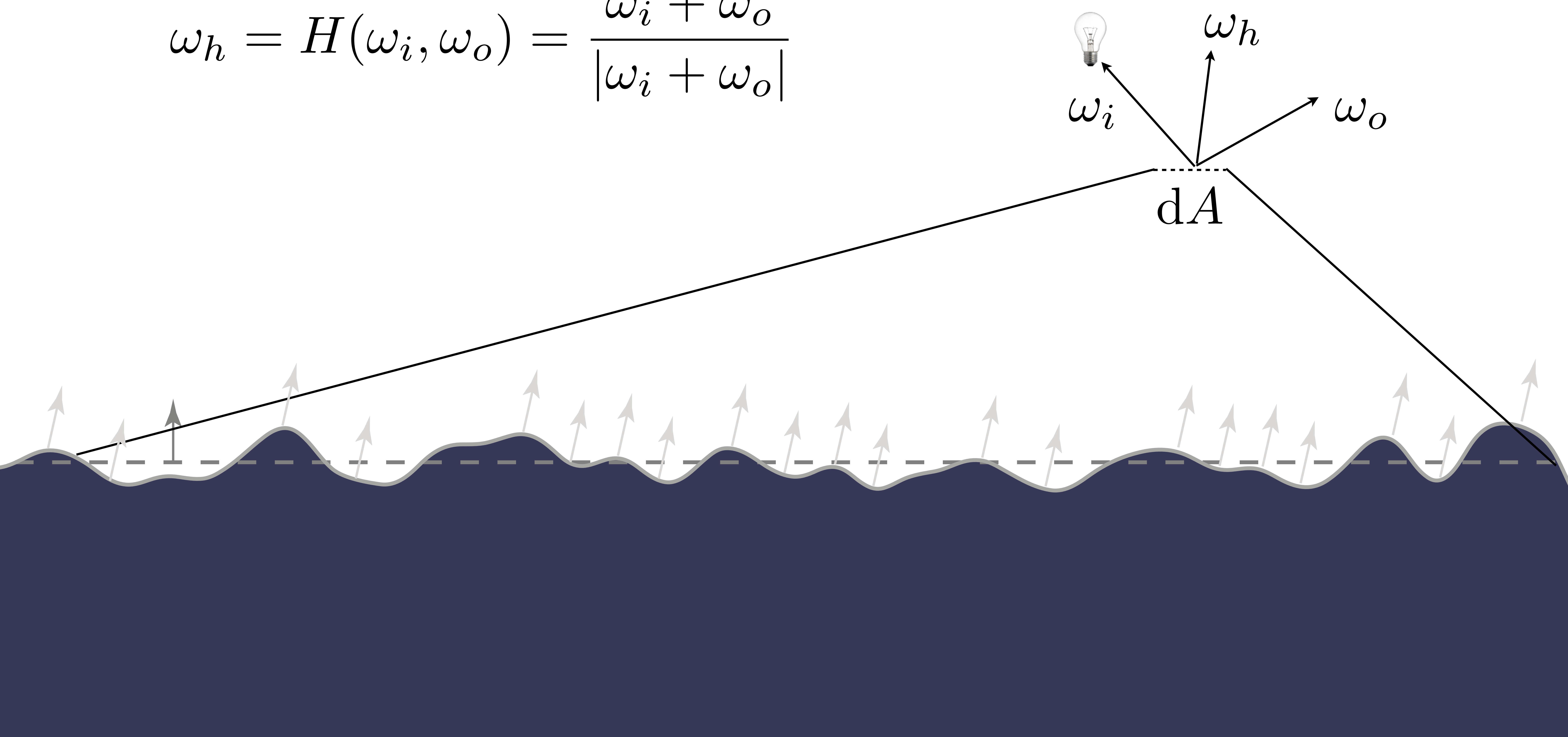
- Incident irradiance E_i illuminates macrosurface area dA from direction ω_i
- Scattered radiance L is measured in direction ω_o



"Half-Vector" Function

- **Gives the one microsurface normal ω_h that will scatter light from ω_i to ω_o**

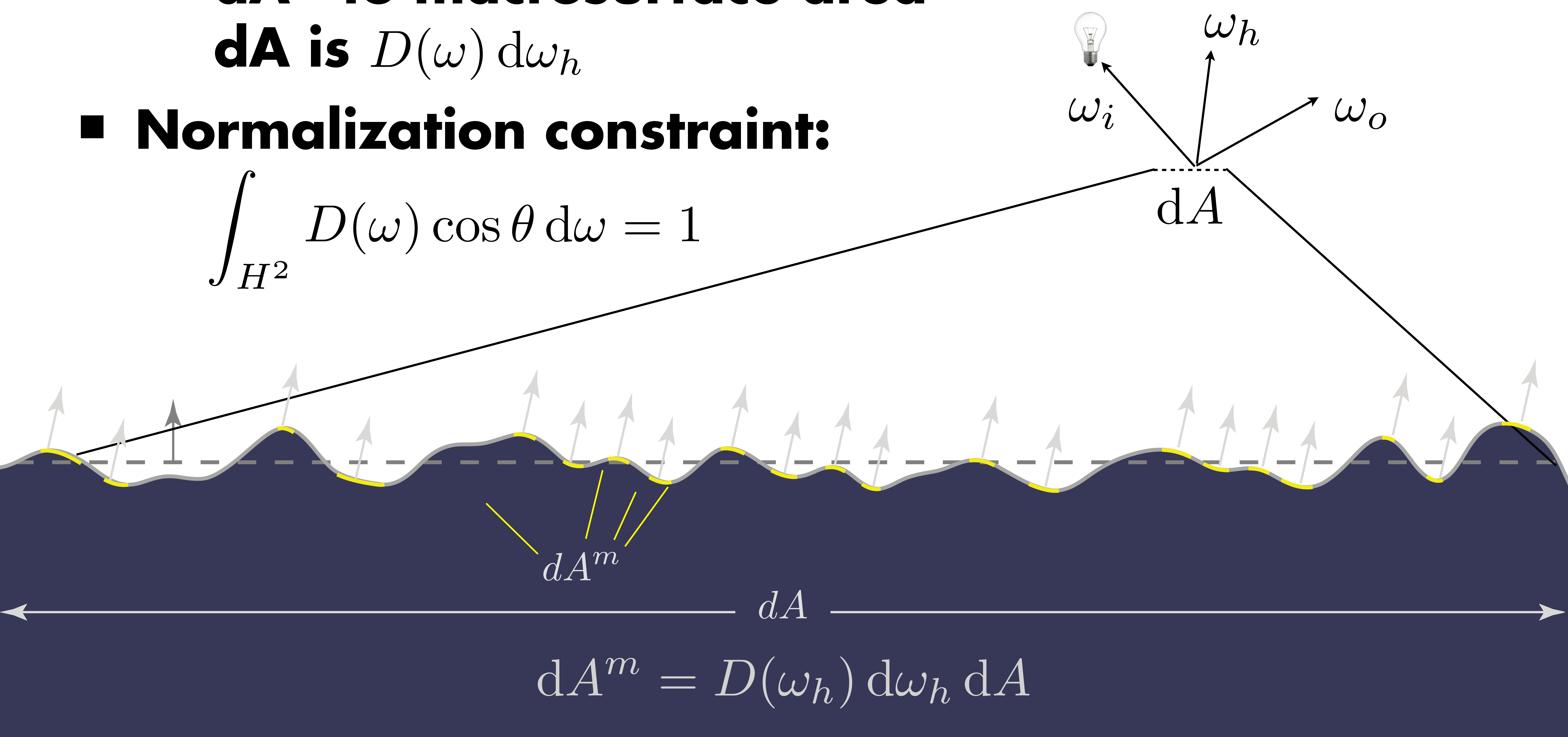
$$\omega_h = H(\omega_i, \omega_o) = \frac{\omega_i + \omega_o}{|\omega_i + \omega_o|}$$



Normal Distribution Function

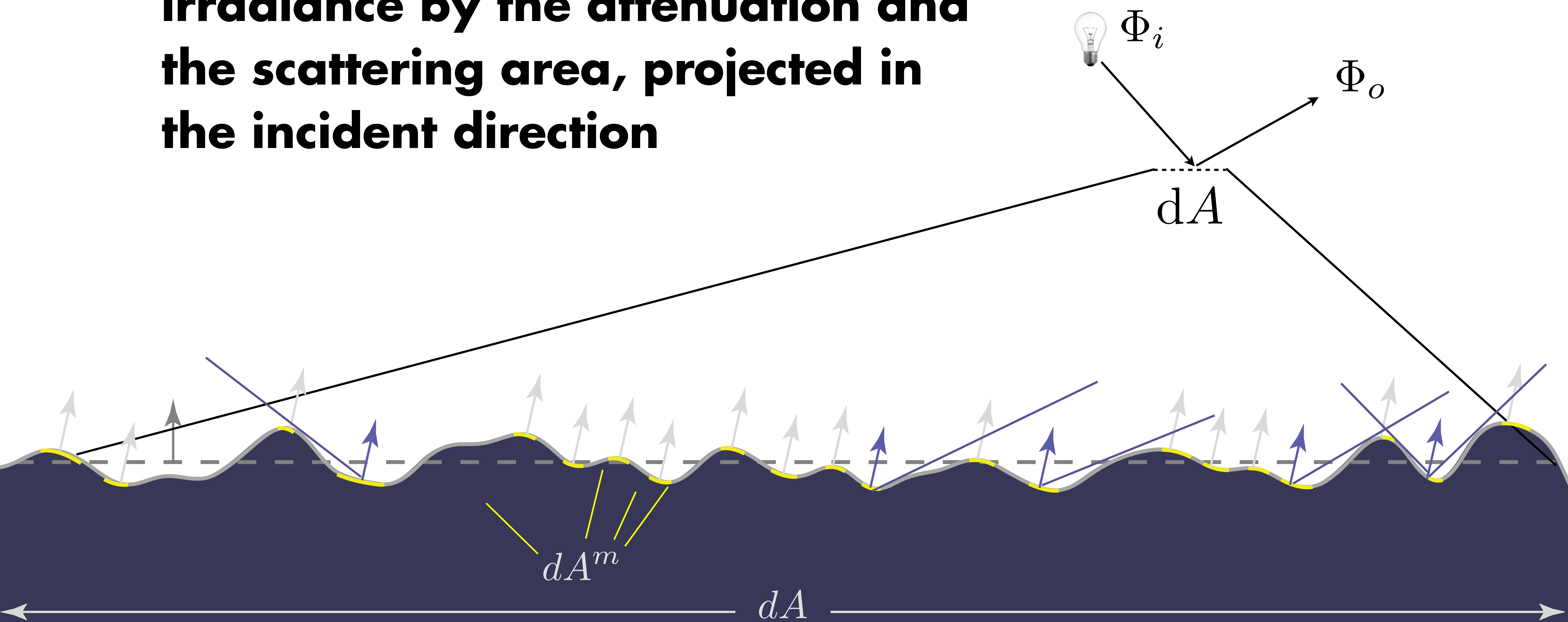
- **Measures density of microsurface area w.r.t. microsurface normal**
- **The ratio of microsurface area dA^m to macrosurface area dA is** $D(\omega)\,\mathrm{d}\omega_h$
- **Normalization constraint:**

$$\int_{H^2} D(\omega)\cos\theta\,\mathrm{d}\omega = 1$$



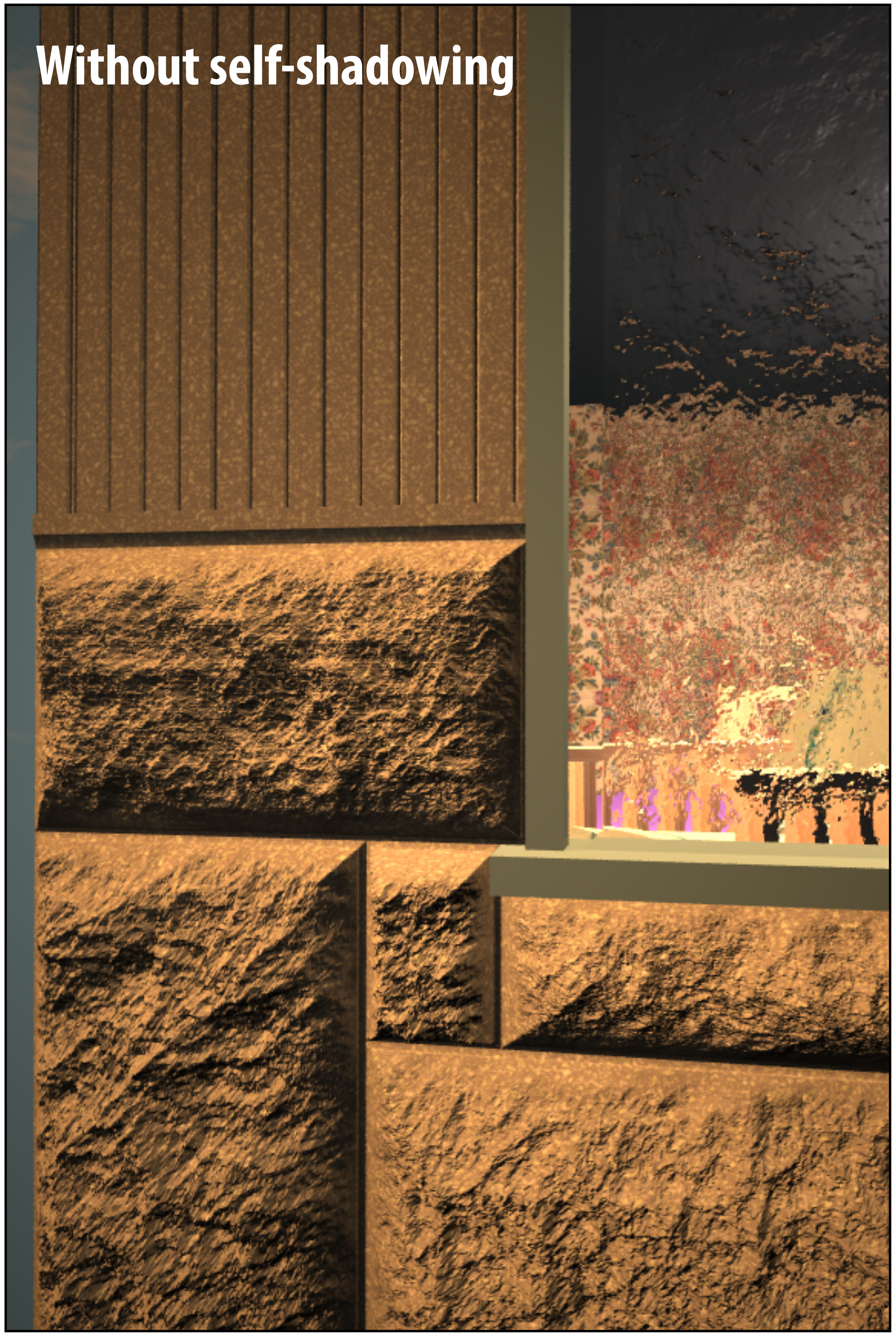
Attenuation Function

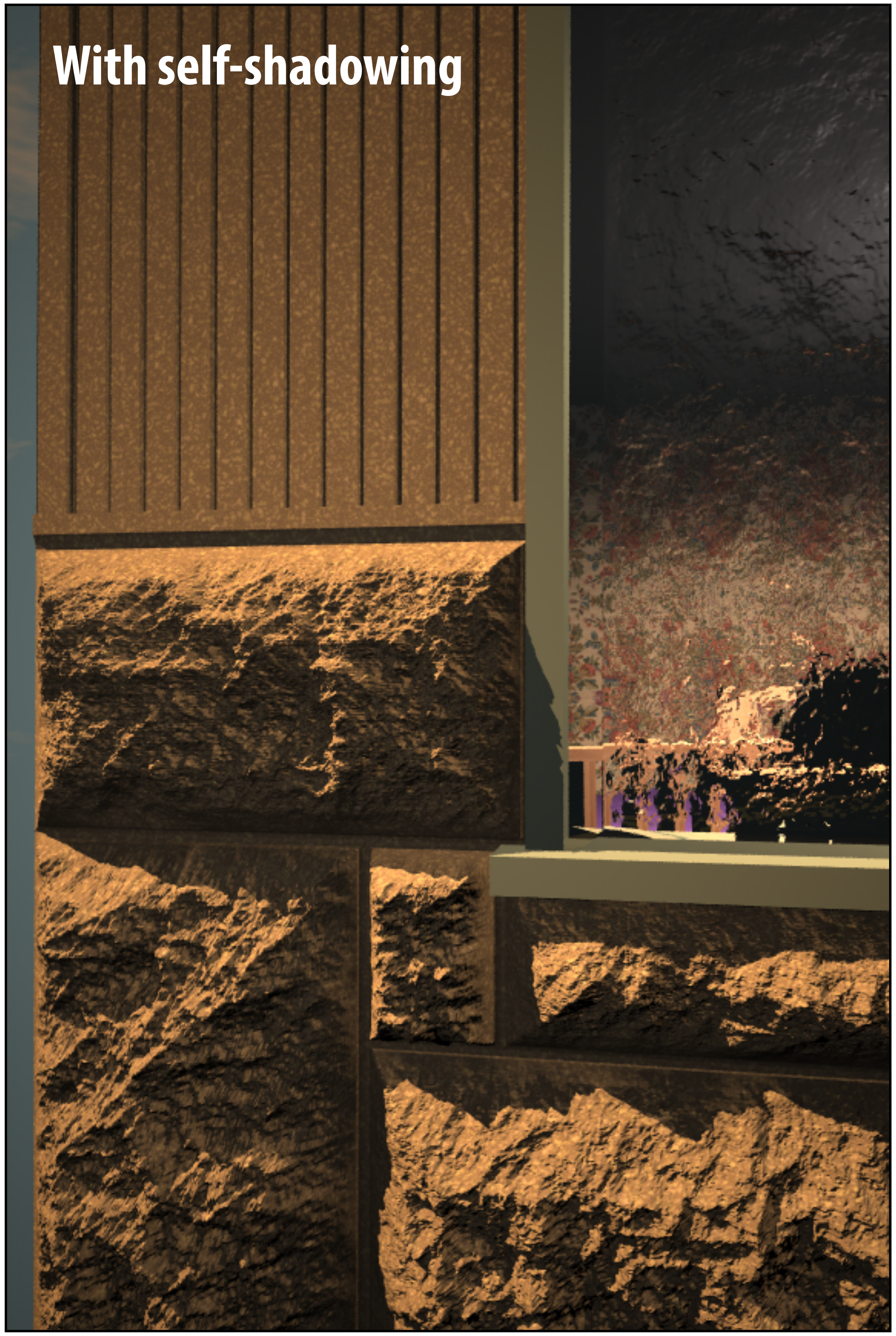
- **Gives the fraction of power incident on the scattering area $\mathrm{d}A$ that is scattered**
- **Scattered power is related to the incident irradiance by the attenuation and the scattering area, projected in the incident direction**



$$\mathrm{d}\Phi_o^m = \frac{|\omega_i \cdot \omega_h|}{\omega_i \cdot \vec{\mathrm{n}}} f_\mathrm{r}(\omega_i, \omega_o)\,\mathrm{d}A^m\,\mathrm{d}E_i$$

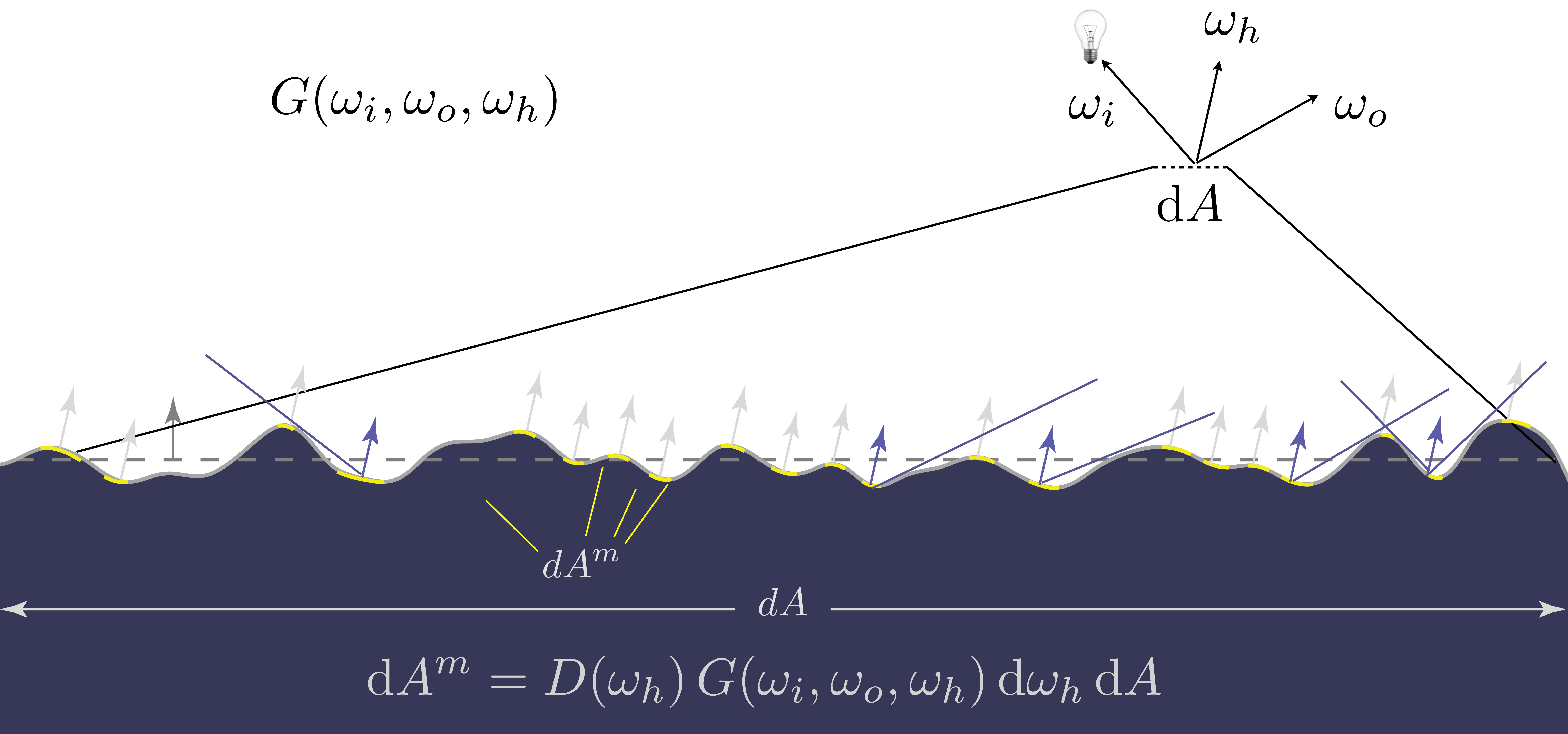
Self-Shadowing





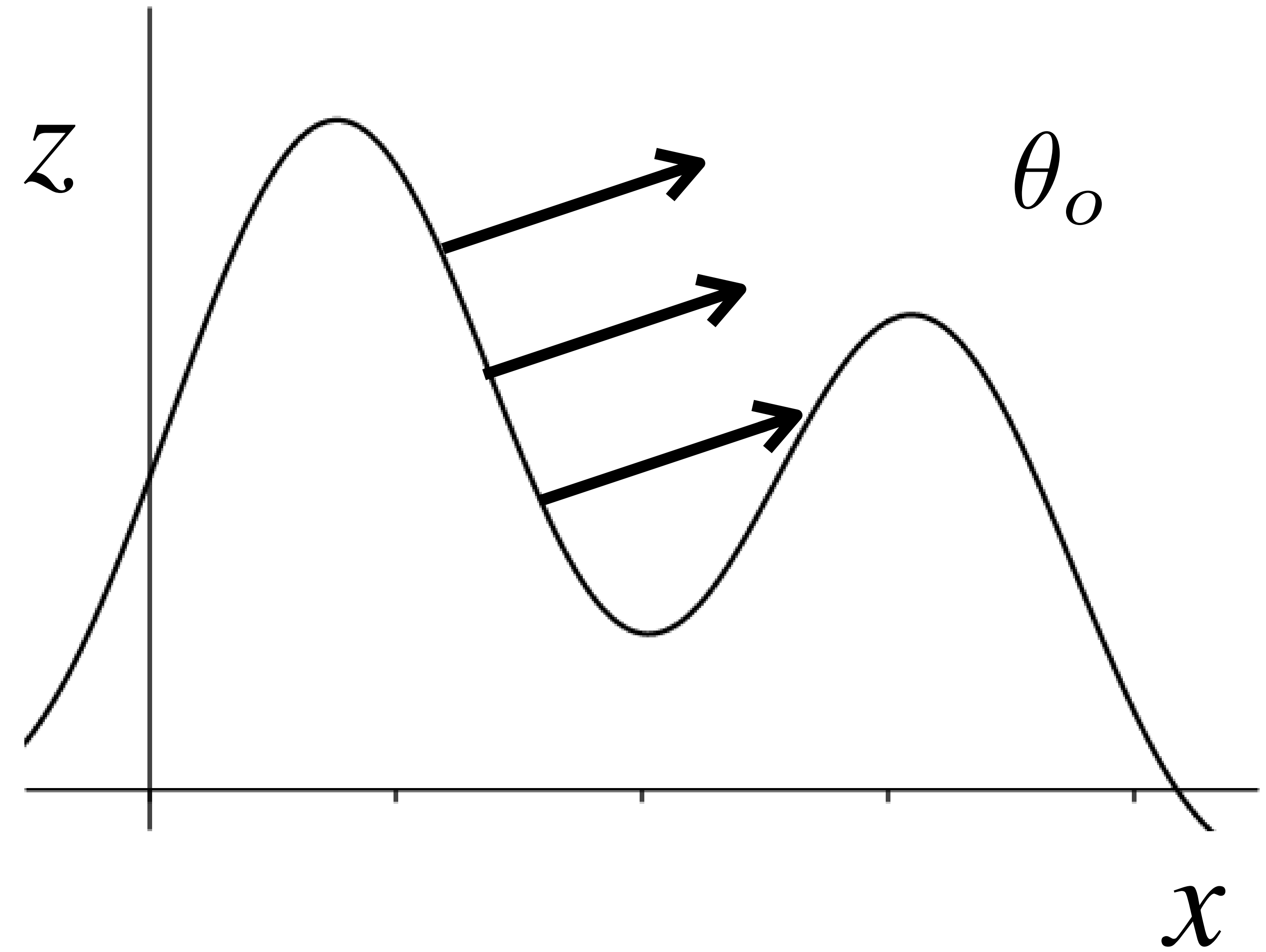
Shadowing-Masking Function

Measures fraction of microsurface points with normal ω_h that are visible in directions ω_i and ω_o



Smith Self-Shadowing Function

Assume probability of shadowing is independent of the normal



$$G(\theta_o) = \frac{1}{1 + \Lambda(\theta_o)}$$

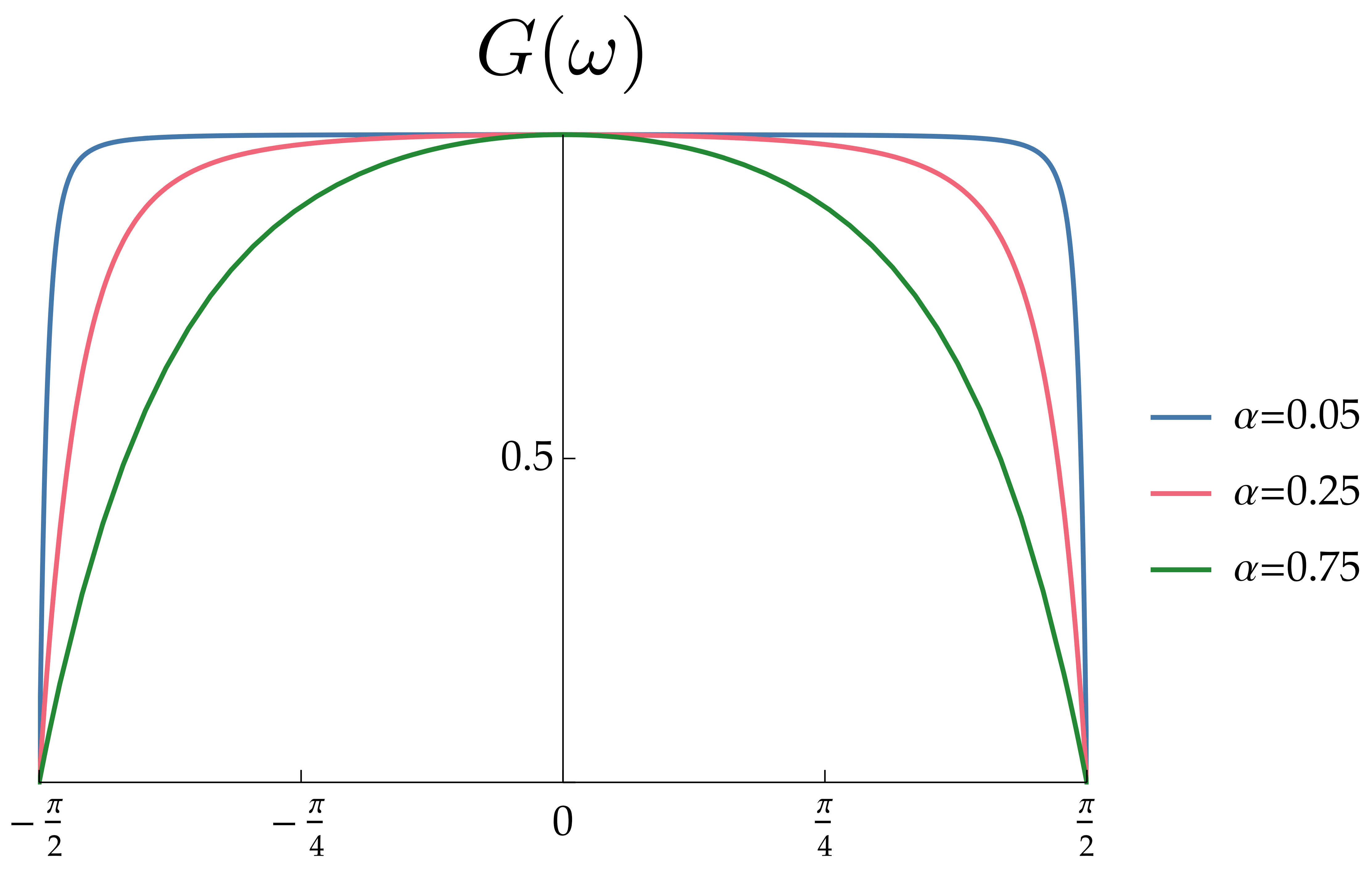
$$\Lambda(\theta_o) = \frac{\mathrm{erf}(\mathrm{a}) - 1}{2} + \frac{1}{2a\sqrt{\pi}} \exp(-a^2)$$

$$a = \frac{1}{\alpha \tan \theta_o}$$

$$\mathrm{erf}(\mathrm{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\mathrm{x}} \mathrm{e}^{-\mathrm{t}^2}\, \mathrm{dt}$$

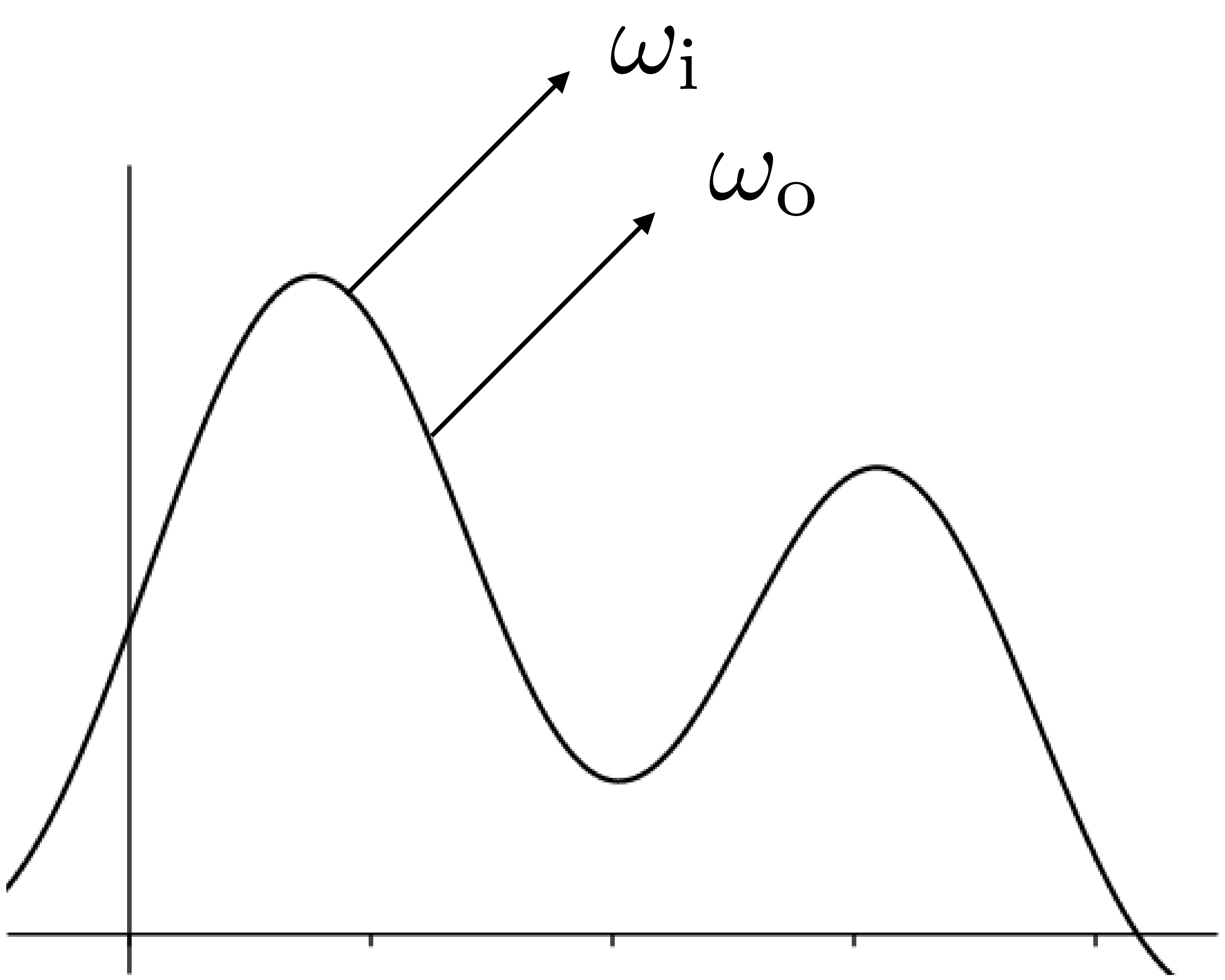
From Smith, 1967

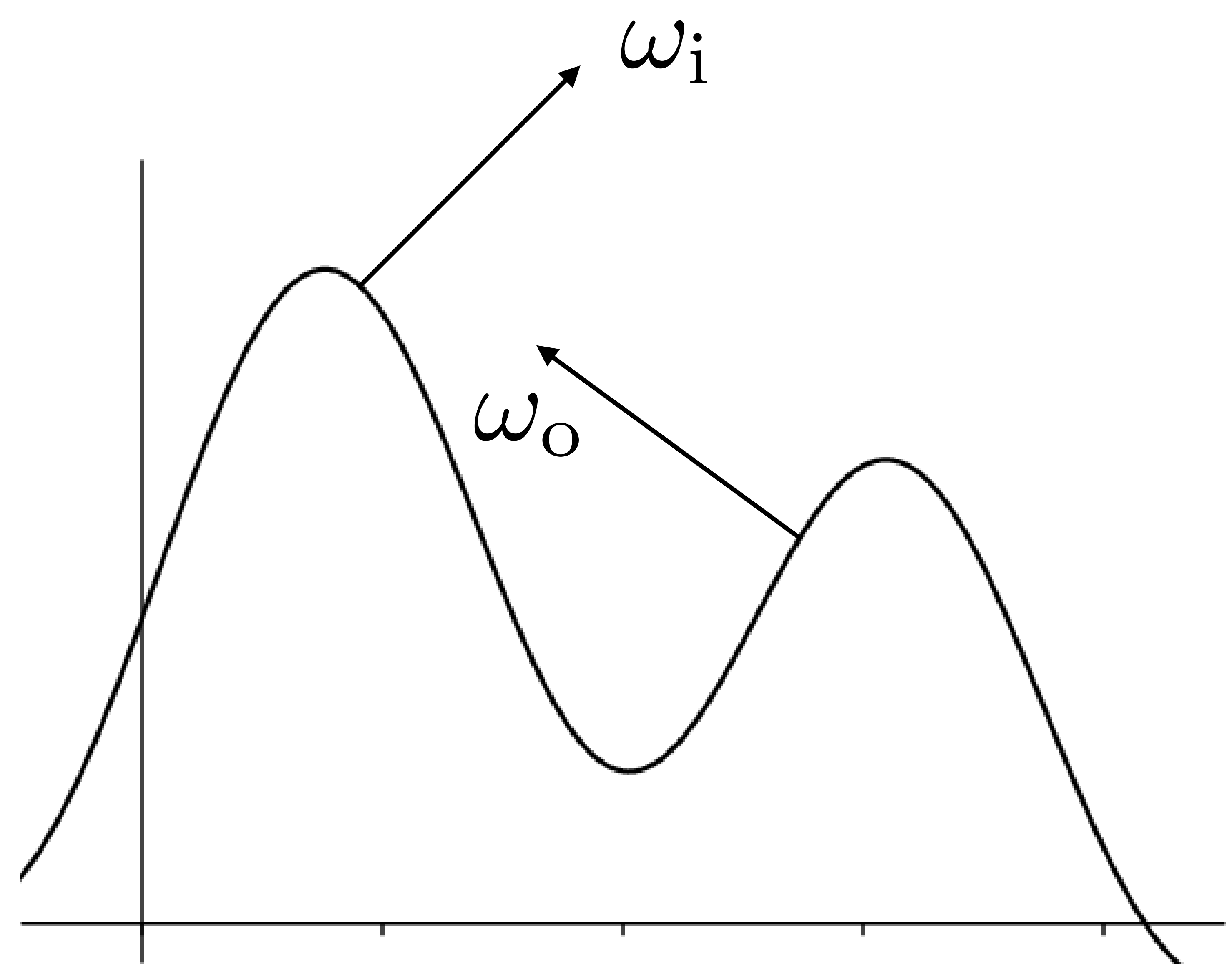
Smith Self-Shadowing Function



Roughness

Joint Masking-Shadowing





Correlated $G(\omega_\mathrm{i})$ **and** $G(\omega_\mathrm{o})$

$$G(\omega_\mathrm{i}, \omega_\mathrm{o}) \approx G(\omega_\mathrm{i})$$

Uncorrelated $G(\omega_\mathrm{i})$ **and** $G(\omega_\mathrm{o})$

$$G(\omega_\mathrm{i}, \omega_\mathrm{o}) \approx G(\omega_\mathrm{i})\, G(\omega_\mathrm{o})$$

Torrance-Sparrow BRDF

$$f_r(\omega_i \to \omega_o) = \frac{D(\omega_h)G(\omega_i, \omega_o)F_r(\omega_o)}{4\cos\theta_i \cos\theta_o}$$

Derivation is independent of a particular microfacet distribution function

Can use either conductor or dielectric Fresnel functions

See full derivation in Section 8.4.2

Microfacet Reflection

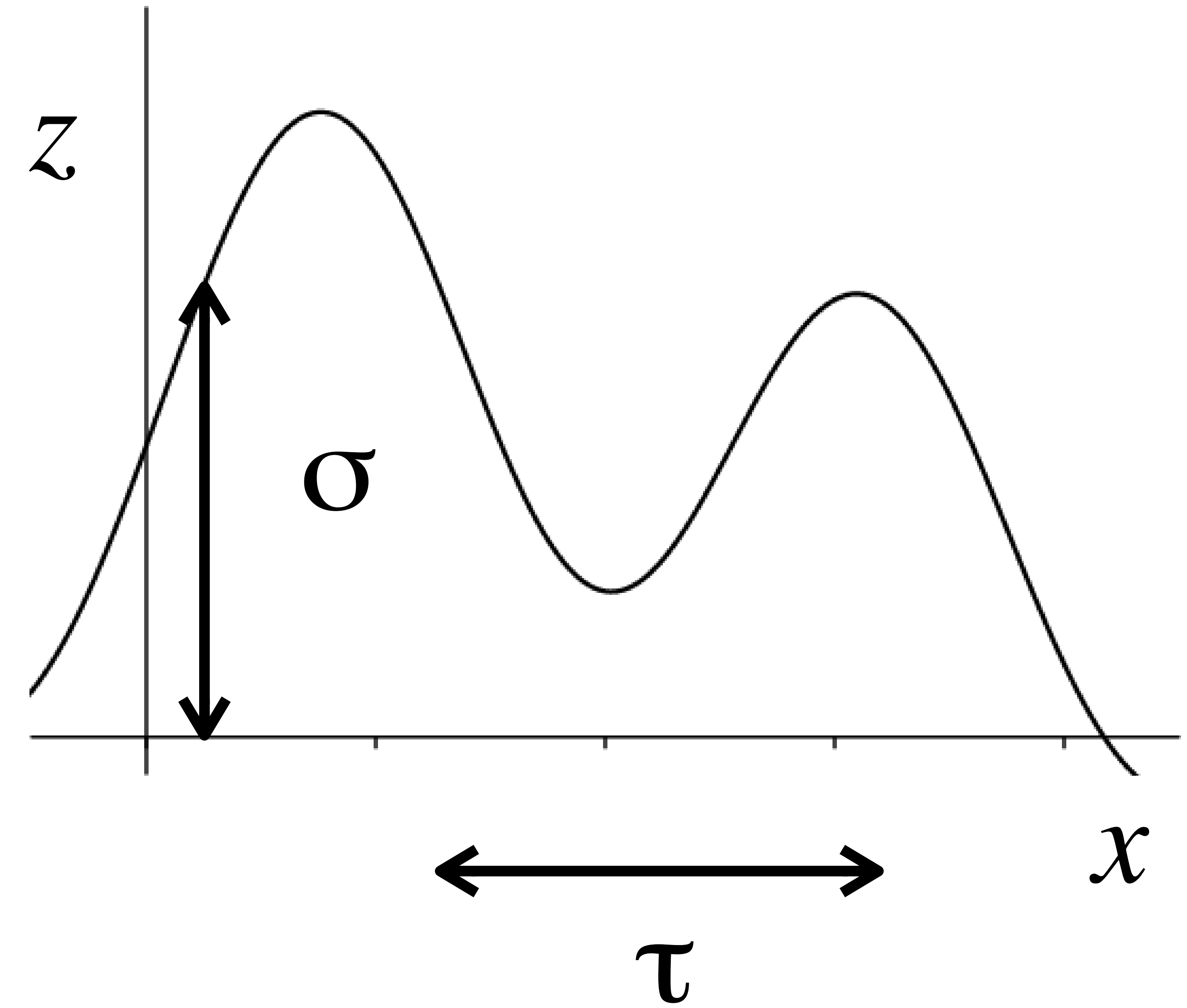
Microfacet Reflection + Transmission

Microfacet Distribution Functions

Gaussian Rough Surface

Gaussian distribution of heights

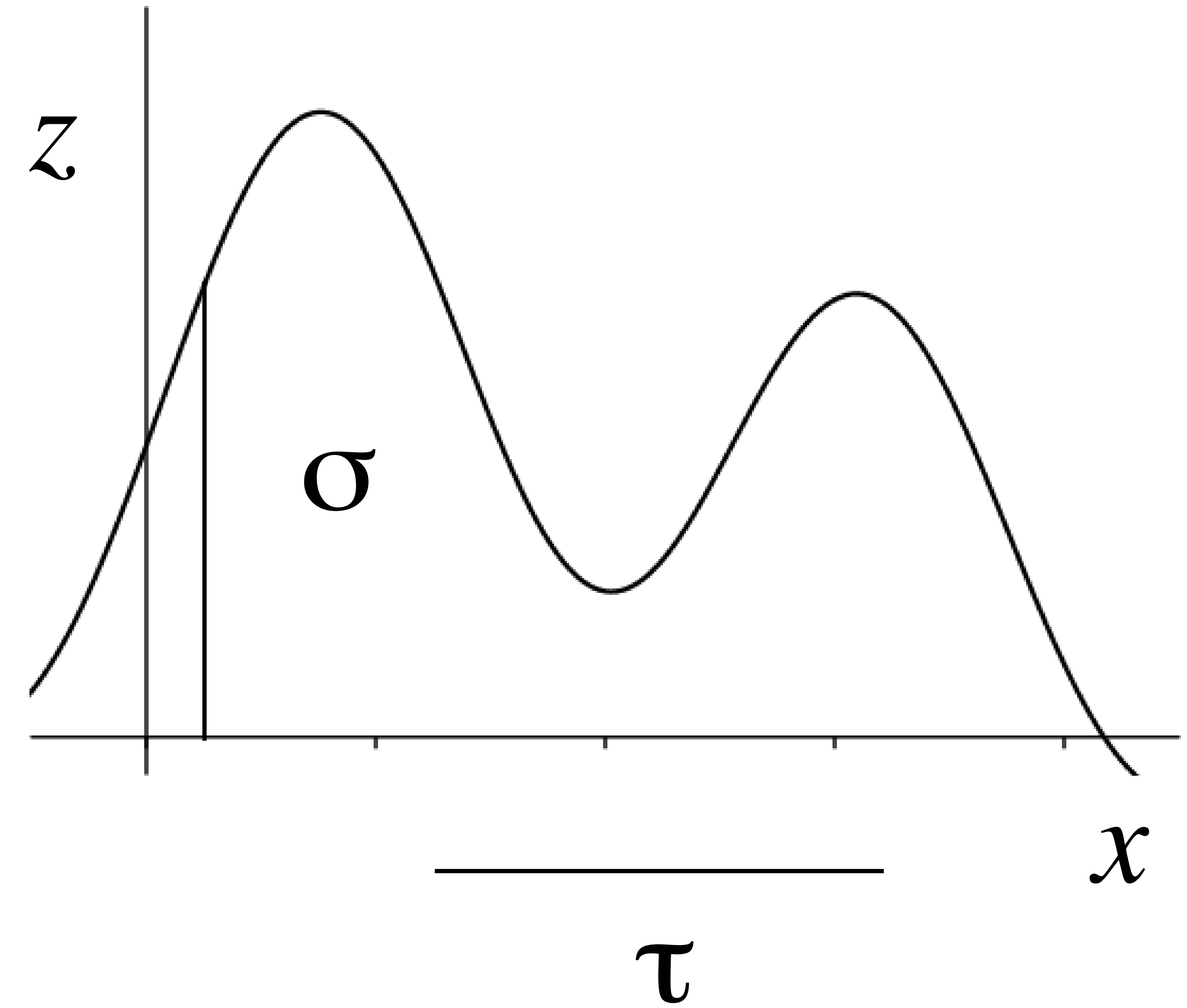
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Beckmann Distribution

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Beckmann distribution of normals

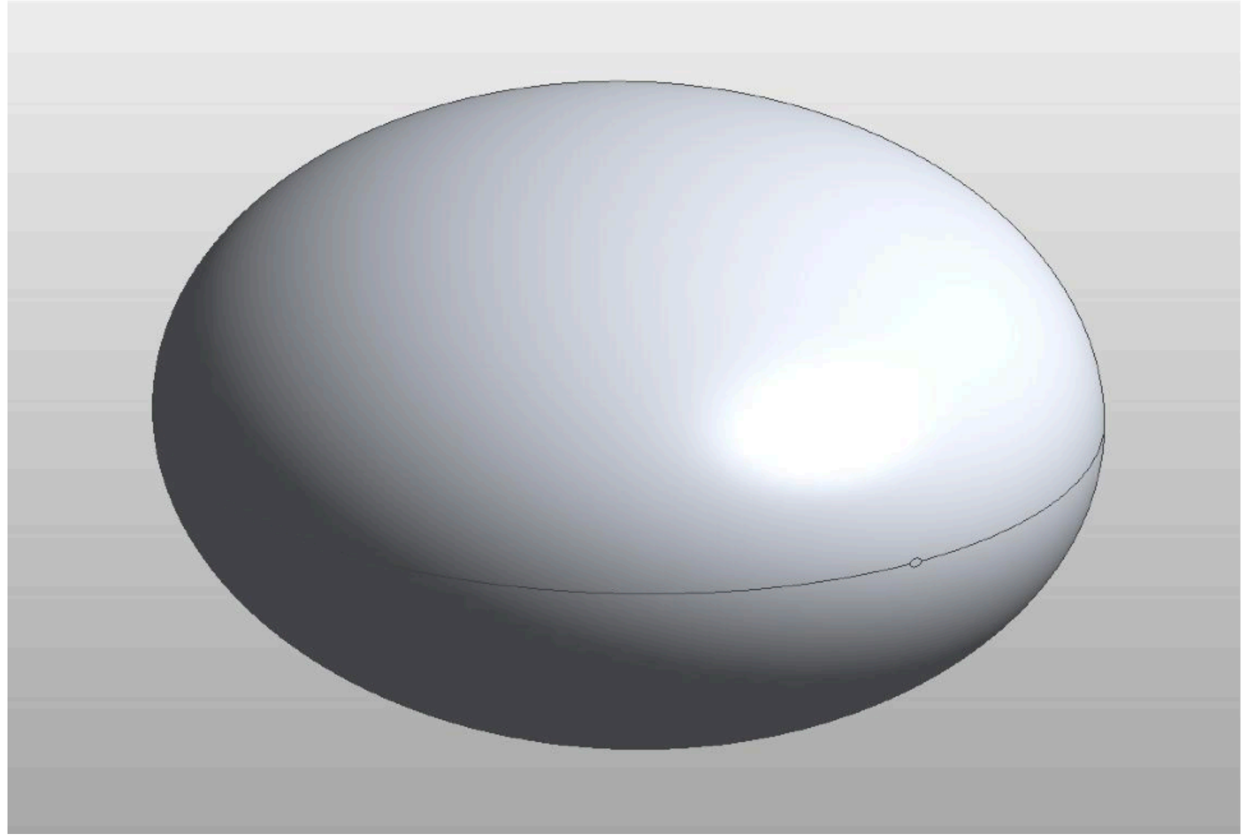
$$D(\omega_m) = \frac{e^{\frac{-\tan^2\theta_m}{\alpha^2}}}{\pi\alpha^2\cos^4\theta_m}$$

$$\alpha = \sqrt{2}\frac{\sigma}{\tau}$$

mean slope

Trowbridge-Reitz (GGX) Distribution

Ellipsoidal

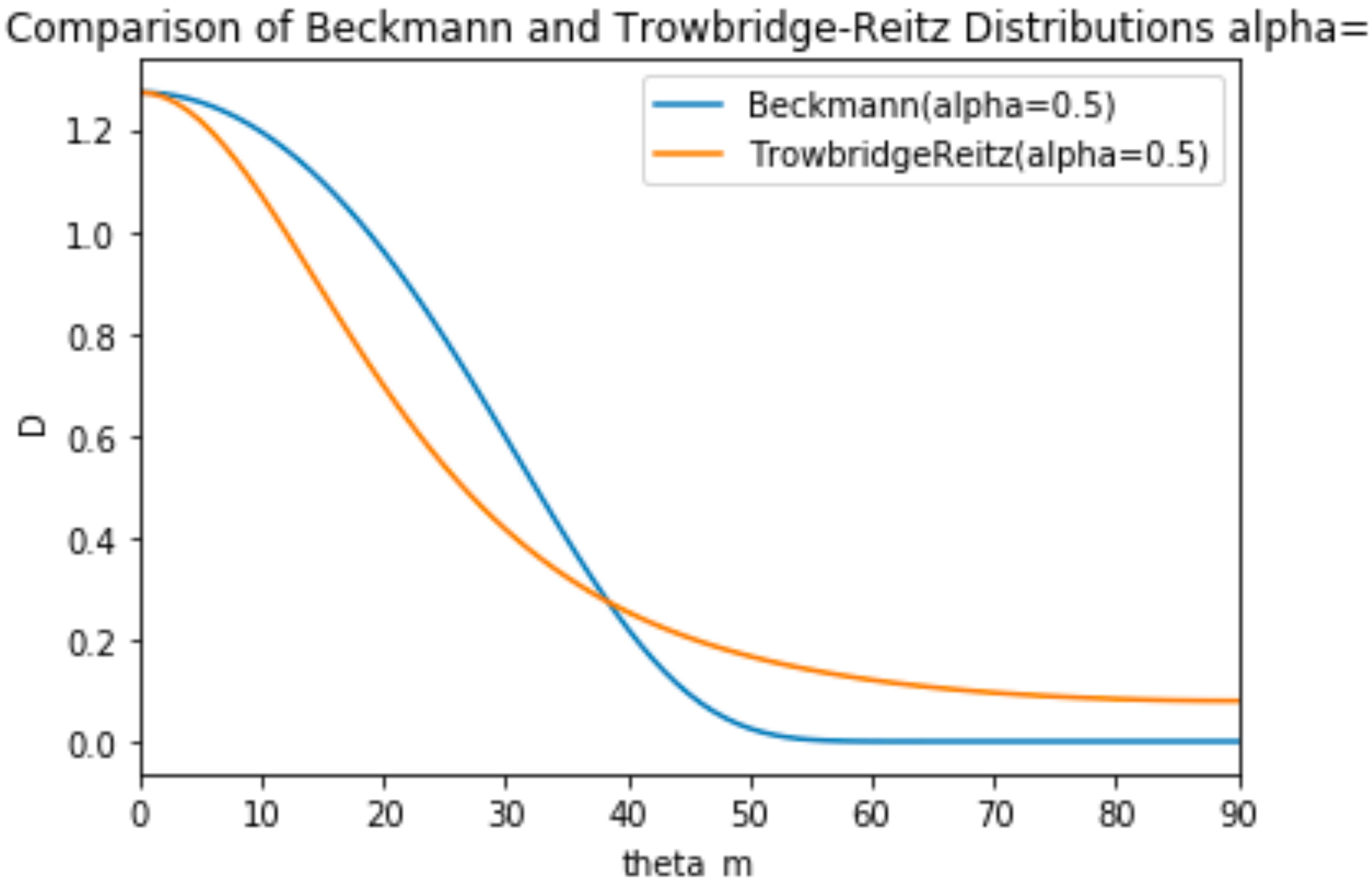


$$z = \alpha(1 - x^2 - y^2)^{(1/2)}$$

Normal distribution

$$D(\omega_m) = \frac{1}{\pi\alpha^2 \cos^4\theta_m (1 + \frac{\tan^2\theta_m}{\alpha^2})^2}$$

Comparison

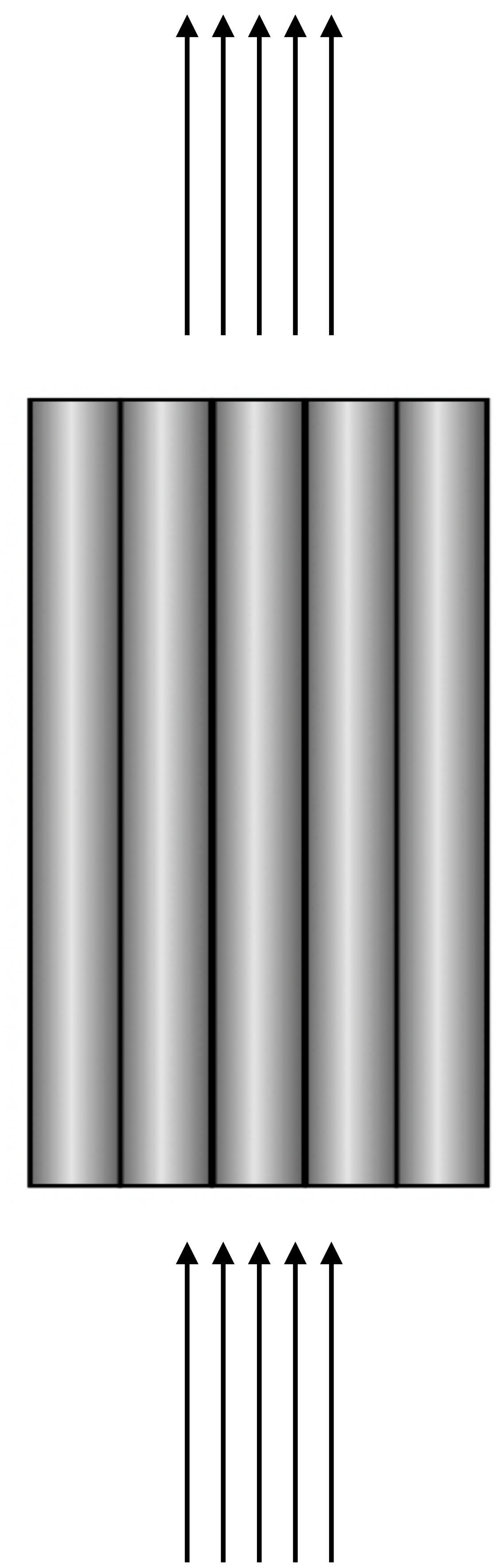


Trowbridge-Reitz has a longer tail; matches experimental data better

Anisotropy

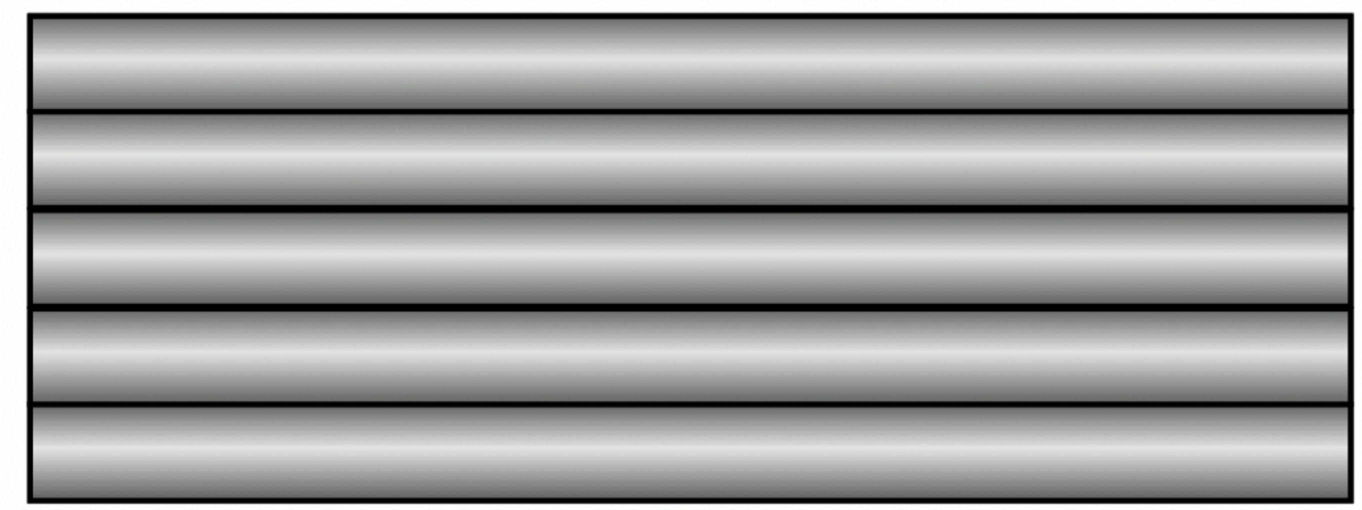
Anisotropic Microfacet Distributions

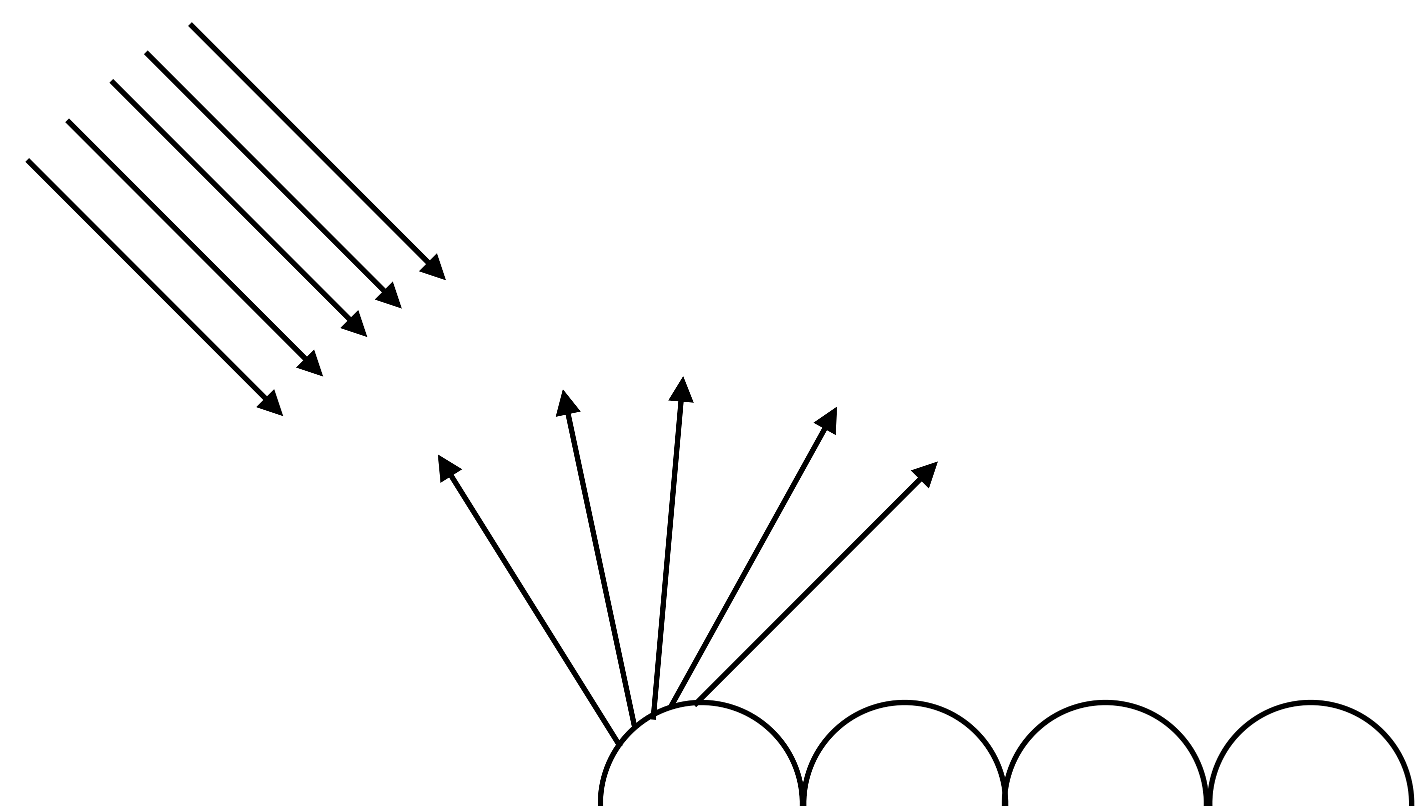
Cylindrical Microsurface



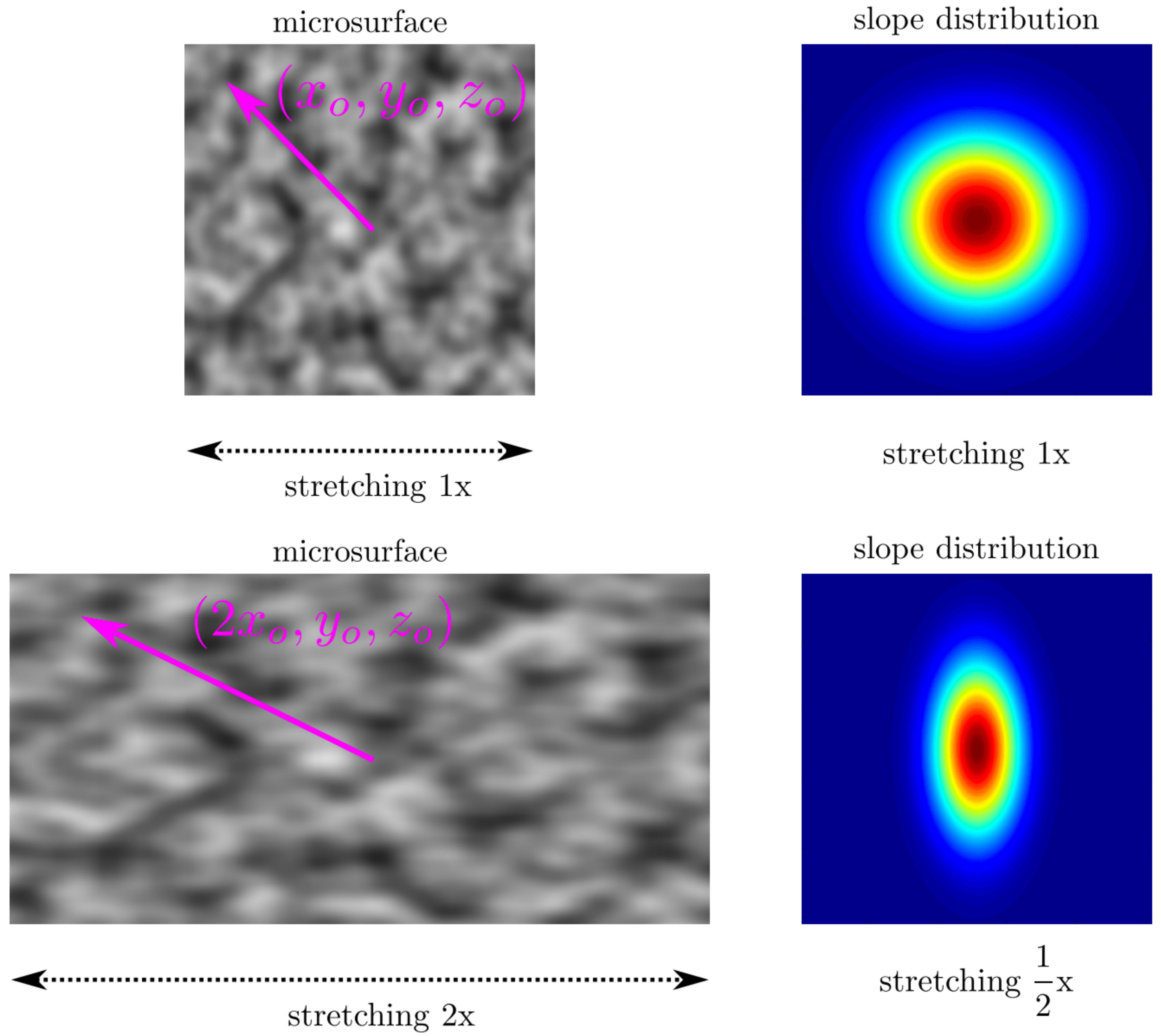
Anisotropic Microfacet Distributions

Cylindrical Microsurface





Anisotropic Distributions via Stretching



[Heitz 14]

Anisotropic Distributions via Stretching

Idea: use isotropic microfacet distribution function with $\alpha = \alpha_y$

Define x stretch factor by: $s_x = \alpha_y / \alpha_x$

Remap directions: $(x, y, z) \rightarrow (\widehat{s_x x, y}, z)$

Can use existing shadowing/masking functions, isotropic microfacet distribution functions without needing to generalize them to be anisotropic!

Sampling

Estimating The Reflection Equation

$$L_{\mathrm{o}}(\mathrm{p}, \omega_{\mathrm{o}}) = \int_{\Omega^2} f_{\mathrm{r}}(\mathrm{p}, \omega_{\mathrm{i}} \rightarrow \omega_{\mathrm{o}})\, L_{\mathrm{i}}(\mathrm{p}, \omega_{\mathrm{i}})\, \cos\theta_{\mathrm{i}}\, \mathrm{d}\omega_{\mathrm{i}}$$

Estimating The Reflection Equation

$$L_{\mathrm{o}}(\mathrm{p},\omega_{\mathrm{o}}) = \int_{\Omega^2} f_{\mathrm{r}}(\mathrm{p},\omega_{\mathrm{i}} \to \omega_{\mathrm{o}})\, L_{\mathrm{i}}(\mathrm{p},\omega_{\mathrm{i}})\, \cos\theta_{\mathrm{i}}\, \mathrm{d}\omega_{\mathrm{i}}$$

$$\approx \frac{f_{\mathrm{r}}(\mathrm{p},\omega' \to \omega_{\mathrm{o}})\, L_{\mathrm{i}}(\mathrm{p},\omega')\, \cos\theta'}{p(\omega')} \qquad \omega' \sim p$$

$$= \frac{D(\omega_h)\, G(\omega',\omega_o)\, F_r(\omega')\, L_{\mathrm{i}}(\mathrm{p},\omega')\, \cos\theta'}{4\cos\theta' \cos\theta_{\mathrm{o}}\, p(\omega')}$$

Recall the MC estimator:

$$\int f(x)\,\mathrm{d}x \approx \frac{1}{n}\sum_{i}^{n} \frac{f(x_i)}{p(x_i)} \qquad x_i \sim p$$

Estimating The Reflection Equation

$$L_\mathrm{o}(\mathrm{p},\omega_\mathrm{o}) = \int_{\Omega^2} f_\mathrm{r}(\mathrm{p},\omega_\mathrm{i}\to\omega_\mathrm{o})\, L_\mathrm{i}(\mathrm{p},\omega_\mathrm{i})\, \cos\theta_\mathrm{i}\, \mathrm{d}\omega_\mathrm{i}$$

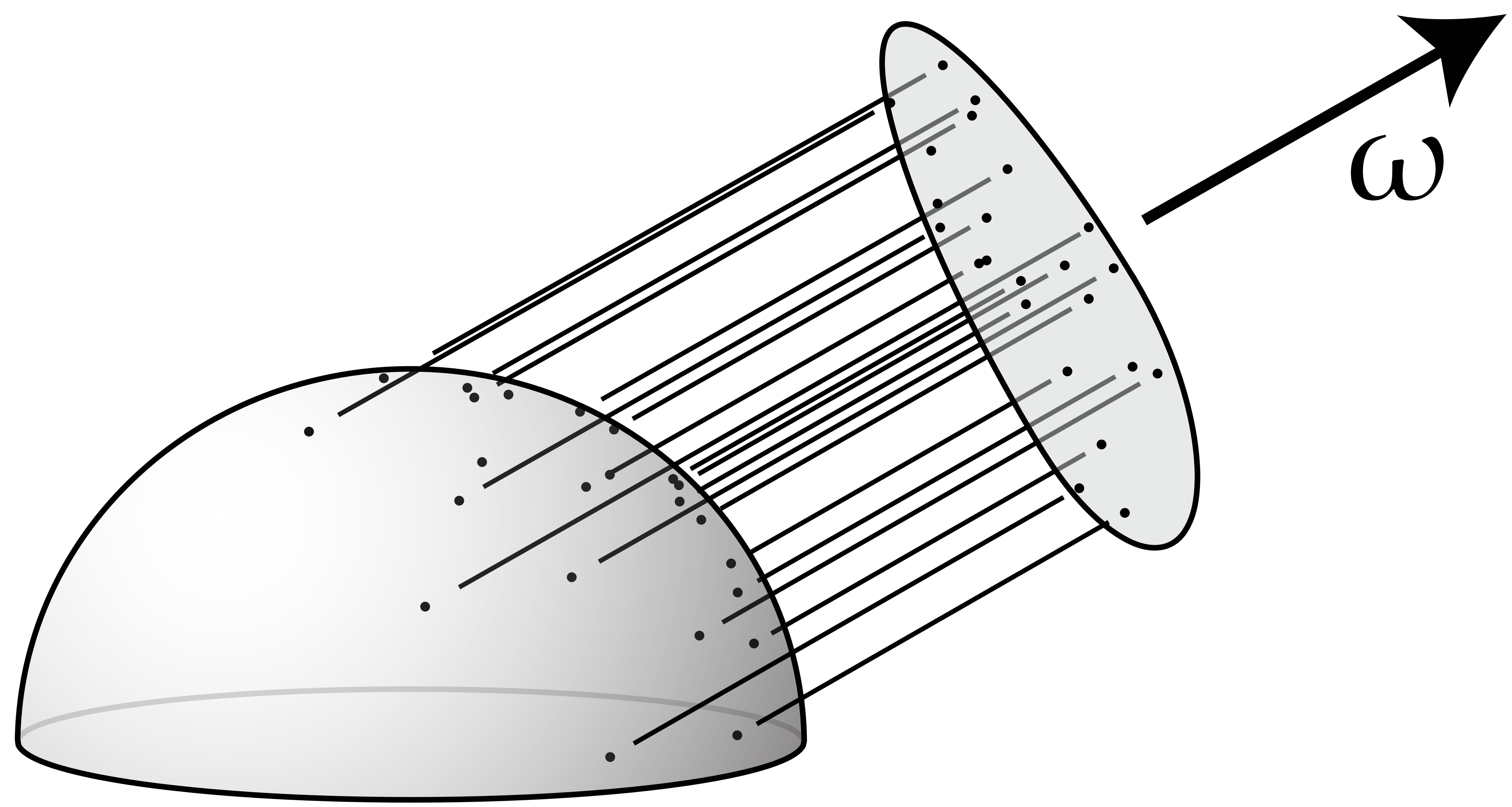
$$\approx \frac{f_\mathrm{r}(\mathrm{p},\omega'\to\omega_\mathrm{o})\, L_\mathrm{i}(\mathrm{p},\omega')\, \cos\theta'}{p(\omega')} \qquad \omega' \sim p$$

$$= \frac{D(\omega_h)\, G(\omega',\omega_o)\, F_r(\omega')\, L_\mathrm{i}(\mathrm{p},\omega')\, \cos\theta'}{4\cos\theta'\cos\theta_\mathrm{o}\, p(\omega')}$$

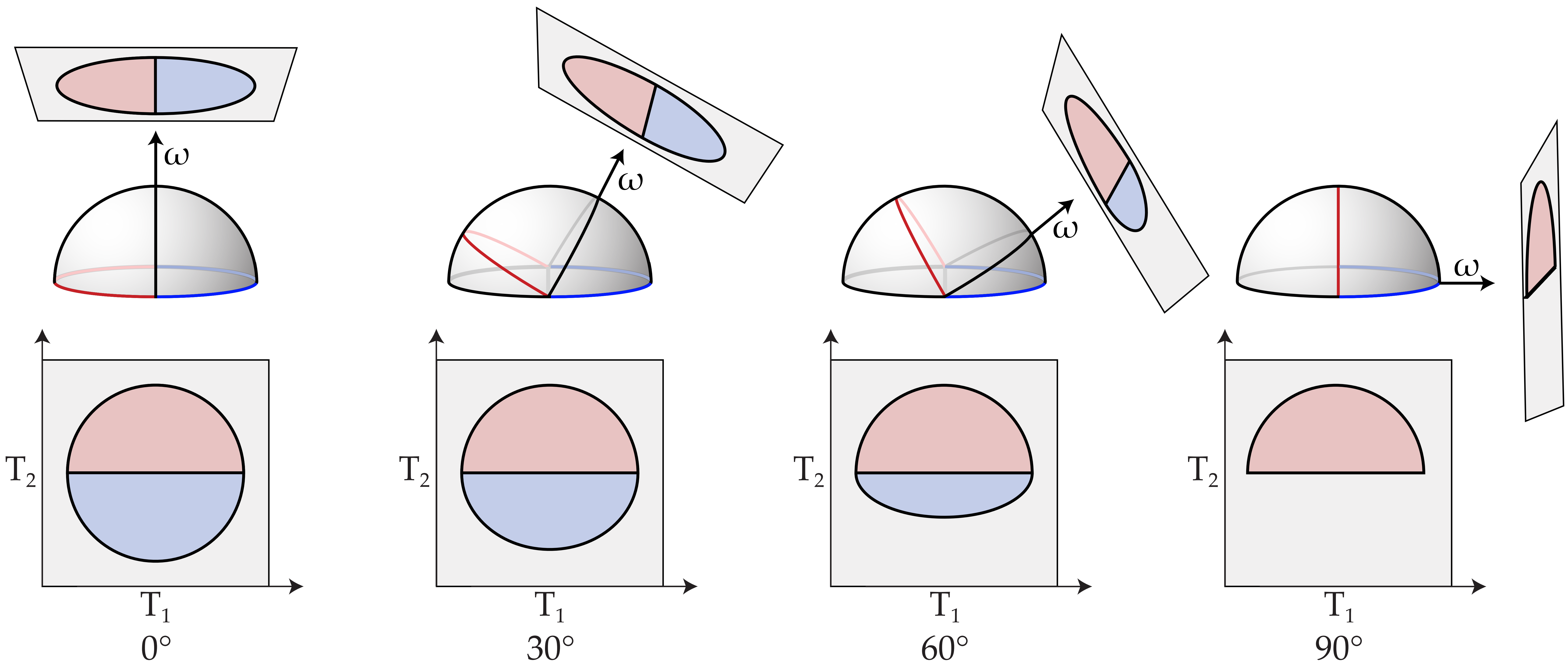
Effective approach: start by sampling $\omega_h \sim D$; ω' follows.

More effective approach: sample ω_h according to *visible* microfacet distribution.

Visible Area Sampling

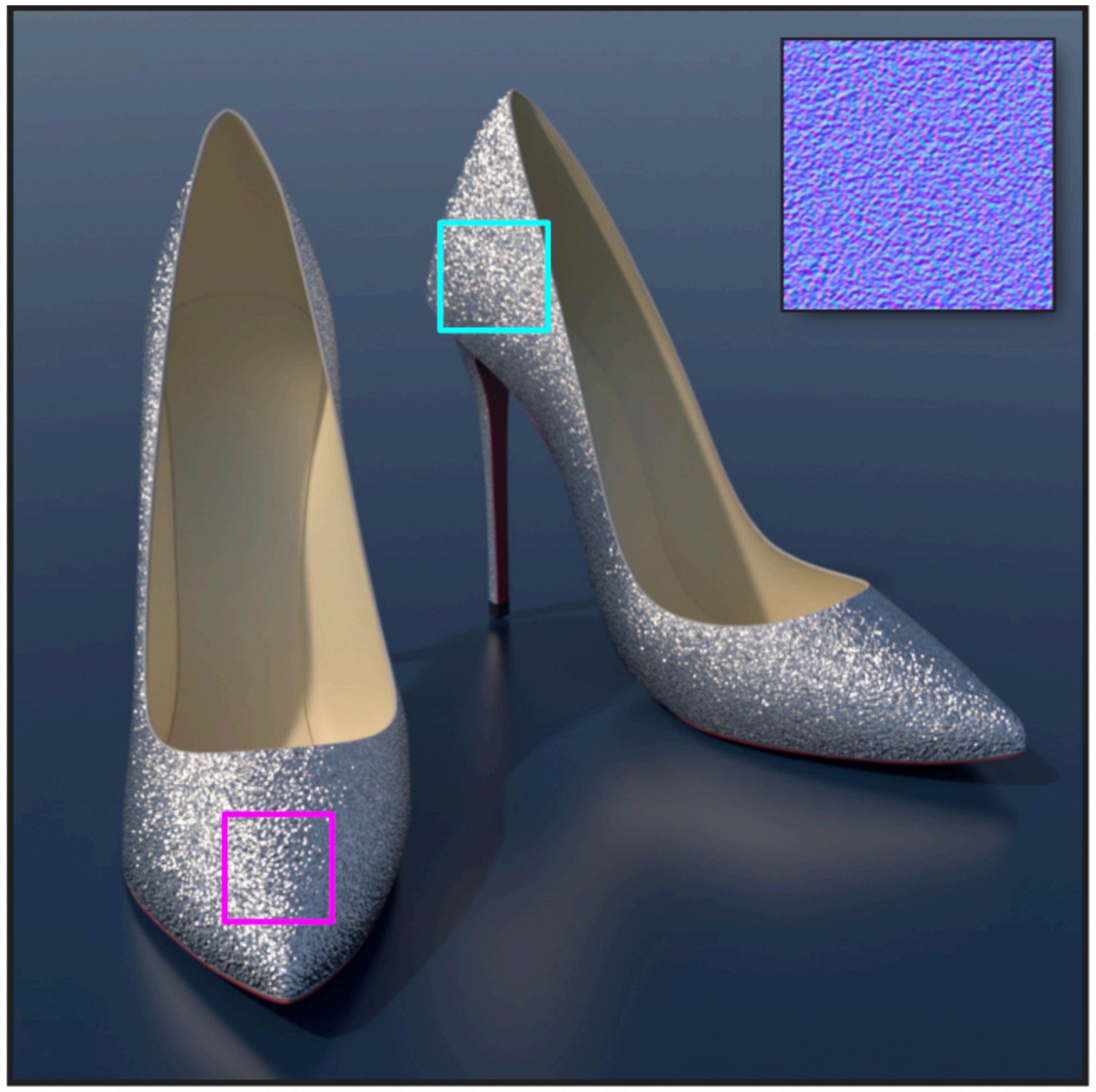


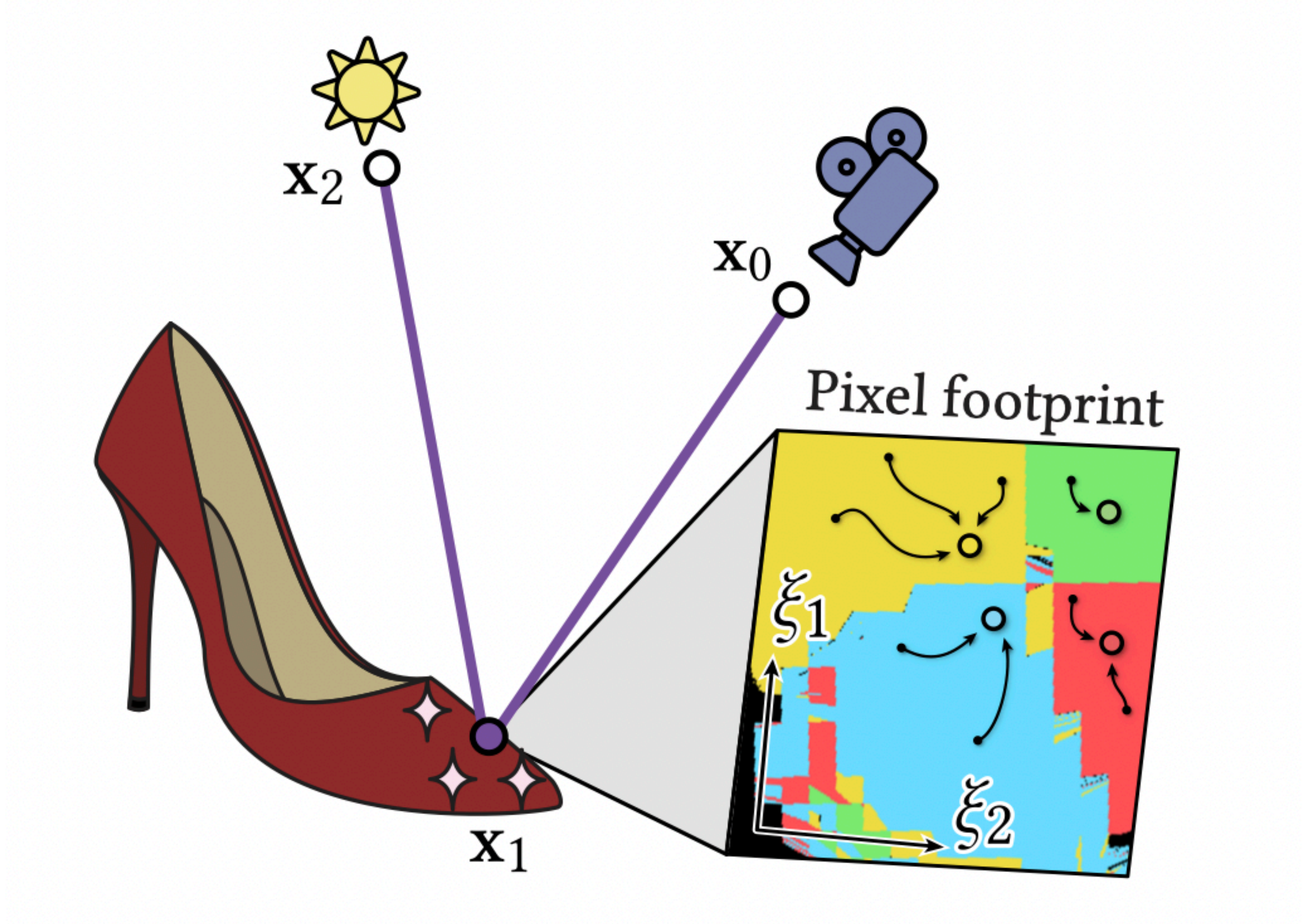
Visible Area Sampling



Advanced Topics in Microfacet Reflection

Glints



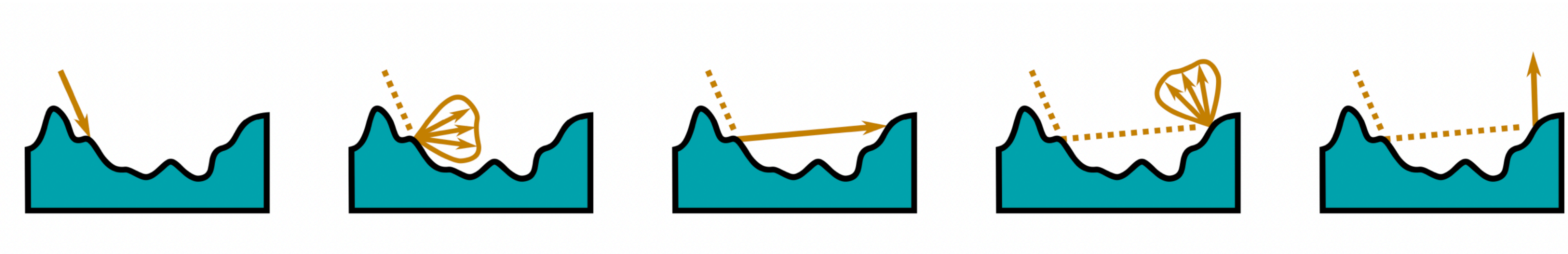


Stochastic search for connecting vertex x_1 given x_0 and x_2.

Zeltner et al. 2020. *Specular Manifold Sampling for Rendering High-Frequency Caustics and Glints.*

[Zeltner et al. 2020]

Interreflection



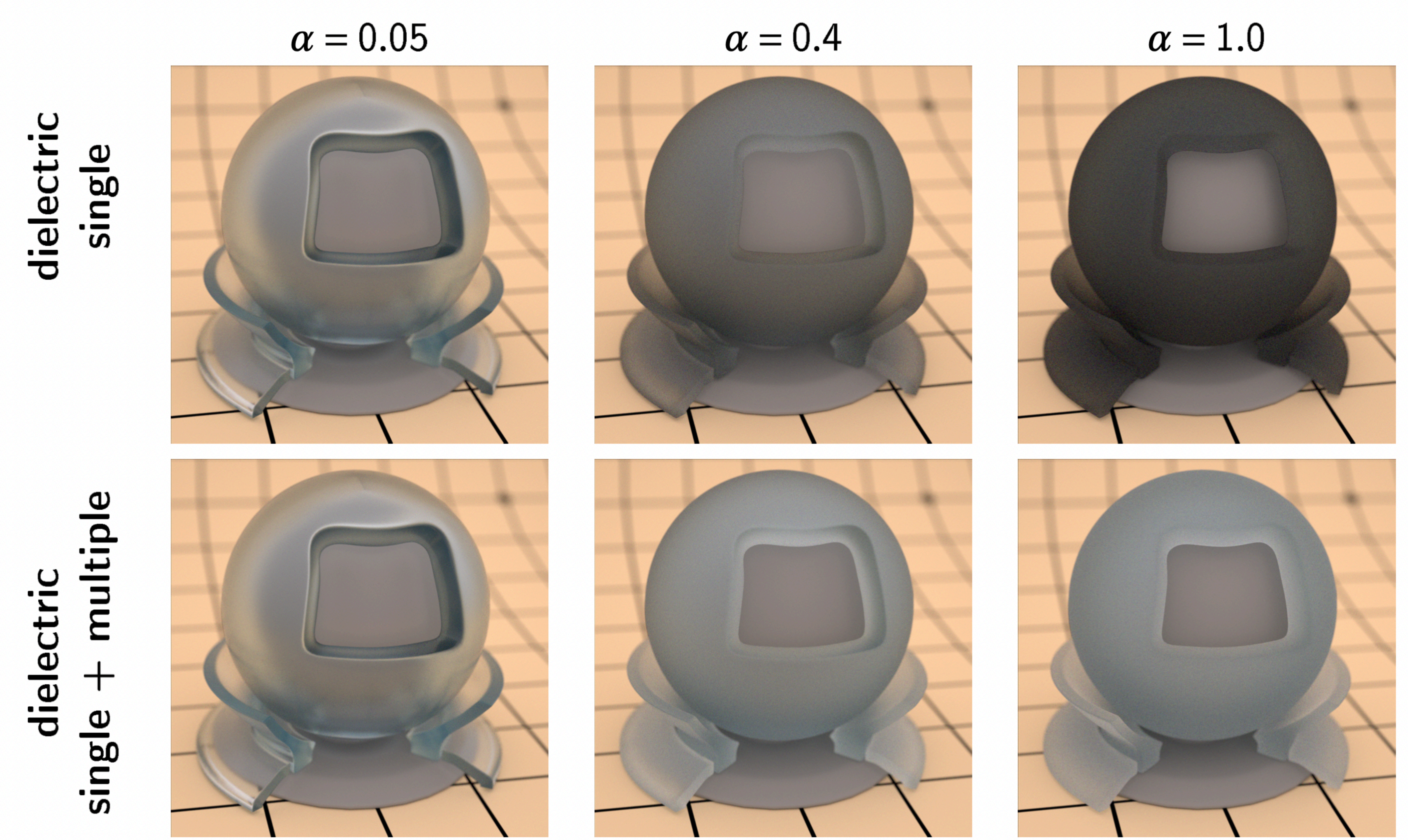
Heitz, E. et al. 2016. ***Multiple-Scattering Microfacet BSDFs with the Smith Model.***

Idea: *stochastic* evaluation of BRDF

Correct result in expectation

No explicit representation of microgeometry; instead, probabilistic model for intersections

Microfacet Interreflection

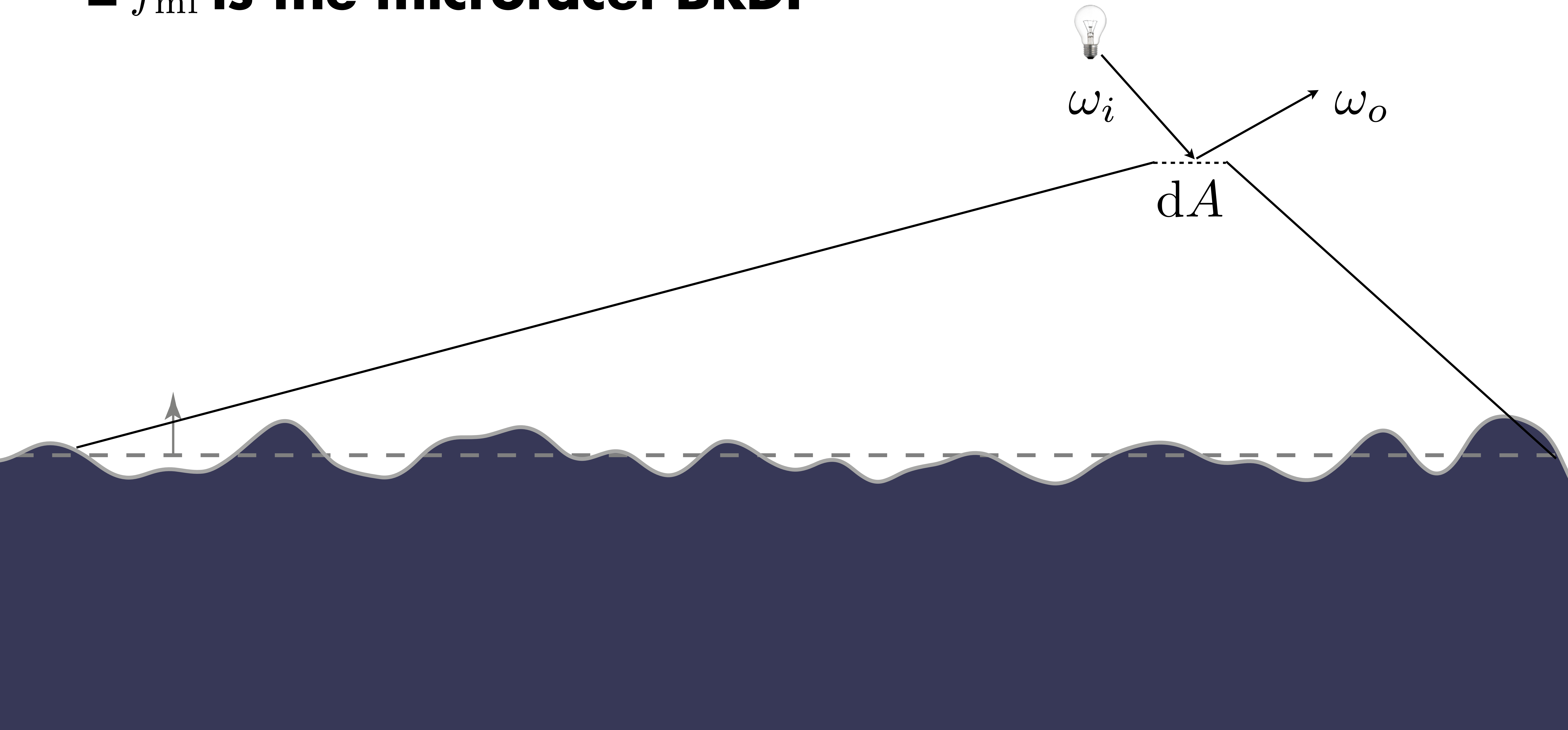


Heitz, E. et al. 2016. *Multiple-Scattering Microfacet BSDFs with the Smith Model.*

Non-Specular Microfacets

$$f_r(\omega_i \to \omega_o) = \frac{1}{\cos\theta_i \cos\theta_o} \int_\Omega D(\omega_h)\, G(\omega_i, \omega_o)\, f_{mf}(\omega_h; \omega_i \to \omega_o)\, (\omega_i \cdot \omega_h)\, (\omega_o \cdot \omega_h)\, d\omega_m$$

- f_{mf} **is the microfacet BRDF**

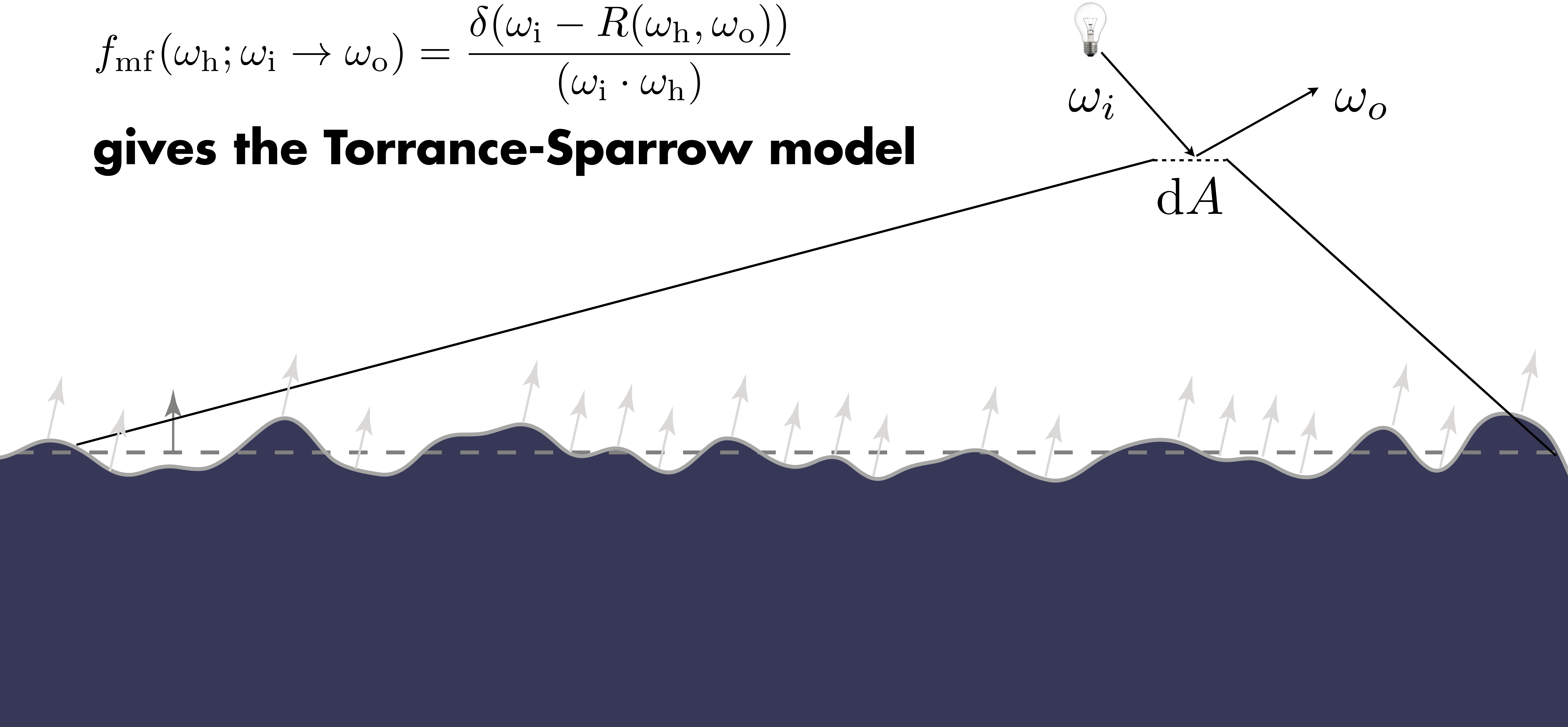


Specular Microfacets Redux

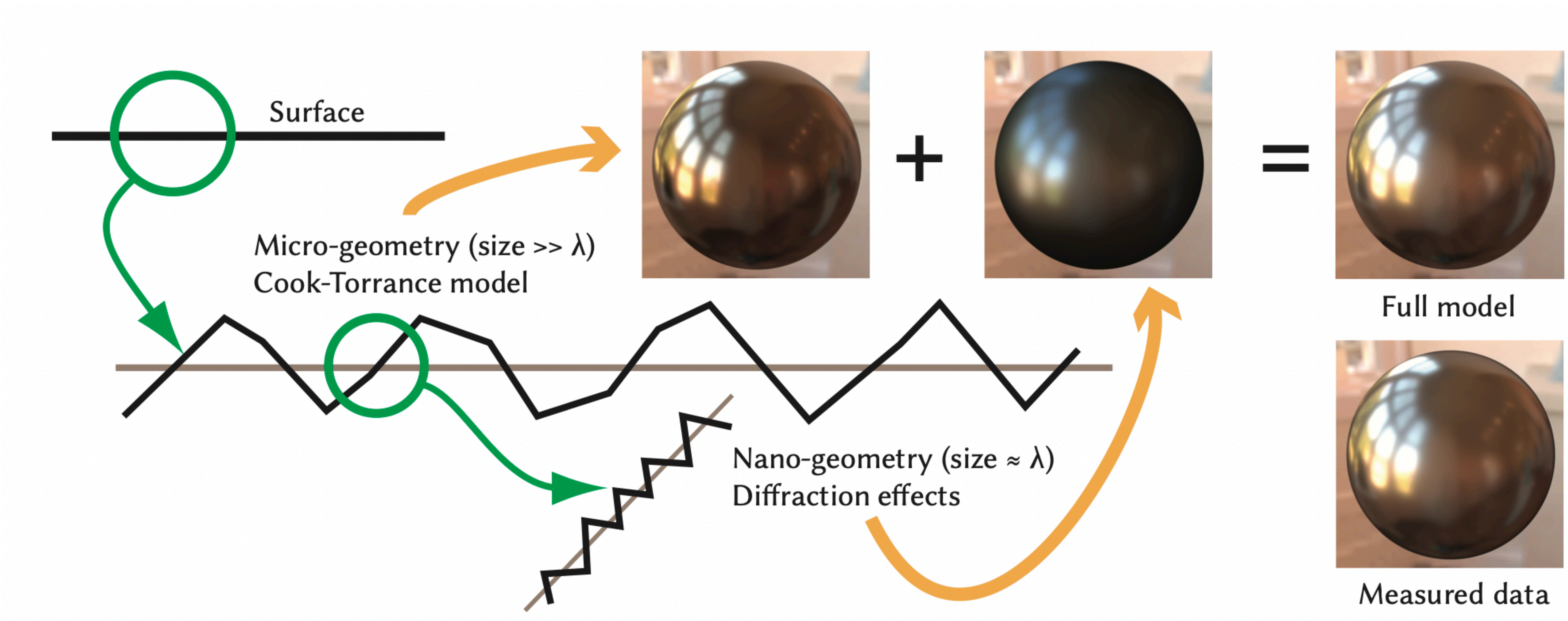
$$f_{\mathrm{r}}(\omega_{\mathrm{i}} \to \omega_{\mathrm{o}}) = \frac{1}{\cos\theta_{\mathrm{i}}\cos\theta_{\mathrm{o}}} \int_{\Omega} D(\omega_{\mathrm{h}})\, G(\omega_{\mathrm{i}}, \omega_{\mathrm{o}})\, f_{\mathrm{mf}}(\omega_{\mathrm{h}}; \omega_{\mathrm{i}} \to \omega_{\mathrm{o}})\, (\omega_{\mathrm{i}} \cdot \omega_{\mathrm{h}})\, (\omega_{\mathrm{o}} \cdot \omega_{\mathrm{h}})\, \mathrm{d}\omega_{\mathrm{m}}$$

$$f_{\mathrm{mf}}(\omega_{\mathrm{h}}; \omega_{\mathrm{i}} \to \omega_{\mathrm{o}}) = \frac{\delta(\omega_{\mathrm{i}} - R(\omega_{\mathrm{h}}, \omega_{\mathrm{o}}))}{(\omega_{\mathrm{i}} \cdot \omega_{\mathrm{h}})}$$

gives the Torrance-Sparrow model

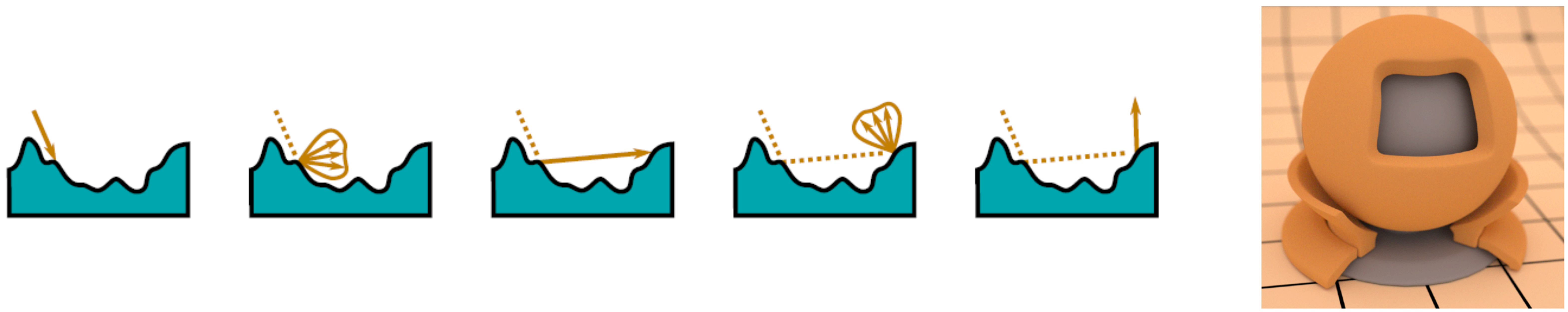


Wave Effects

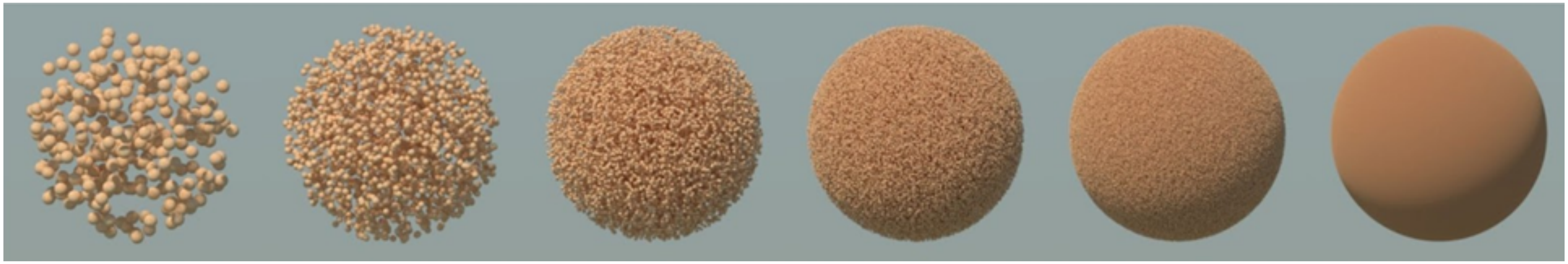


Holzschuch and Pacanowski. 2017.
A two-scale microfacet reflectance model combining reflection and diffraction.

Lambertian Generalizations



Heitz, E. et al. 2016. *Multiple-Scattering Microfacet BSDFs with the Smith Model.*



d'Eon, E. 2021. *An Analytic BRDF for Materials with Spherical Lambertian Scatterers.*

BRDF follow from inserting Equation 15 into the general solution [HC61, p.55], and we find

$$\Psi^{(0)}(\mu) = \frac{1}{384}c\left(-15(c-1)(4c+9)\mu^4 + (c(20c+281)-346)\mu^2 + 207\right),$$
$$\Psi^{(1)}(\mu) = -\frac{1}{192}c\left(\mu^2-1\right)\left(5(4c+9)\mu^2-64\right),$$
$$\Psi^{(2)}(\mu) = \frac{15}{256}c\left(\mu^2-1\right)^2.$$

We can then numerically evaluate the H functions using the Fok/Chandrasekhar equation [Foc44, Kre62]

$$H^{(i)}(\mu) = \exp\left(-\frac{\mu}{\pi}\int_0^\infty \frac{1}{1+\mu^2t^2}\log K^{(i)}(t)dt\right), \quad (22)$$

where the functions $K^{(i)}(t)$ are given by [Kre62]

$$K^{(i)}(t) = 1 - \int_1^\infty \left(\frac{1}{s-it}+\frac{1}{s+it}\right)\frac{\Psi^{(i)}\left(\frac{1}{s}\right)}{s}ds. \quad (23)$$

Working these out, we find

$$K^{(0)}(t) = 1 - \frac{c\left((256c-301)t^3 + \left((346-c(20c+281))t^2 - 15(c-1)(4c+9) + 207t^4\right)\tan^{-1}(t) + 15(c-1)(4c+9)t\right)}{192t^5}, \quad (24)$$

$$K^{(1)}(t) = 1 - \frac{c\left((40c+282)t^3 - 3\left(t^2+1\right)\left(20c+64t^2+45\right)\tan^{-1}(t) + 15(4c+9)t\right)}{288t^5}, \quad (25)$$

$$K^{(2)}(t) = 1 - \frac{5c\left(3\left(t^2+1\right)^2\tan^{-1}(t) - t\left(5t^2+3\right)\right)}{128t^5}. \quad (26)$$

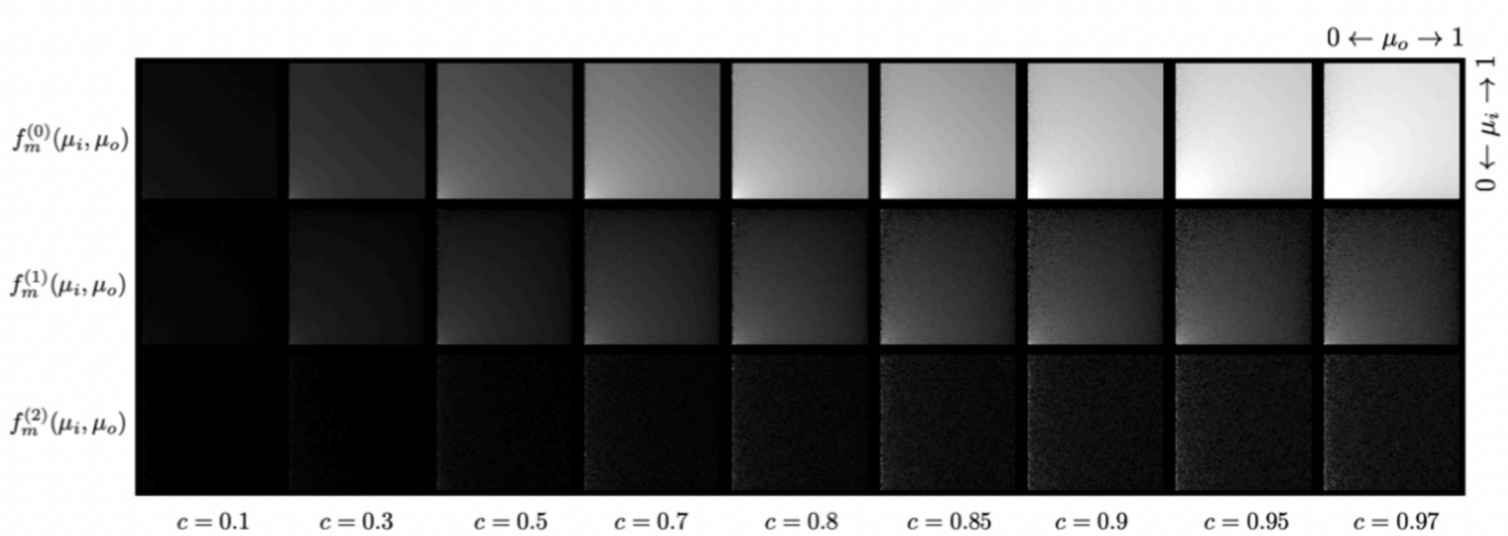


Figure 3: *Using Monte Carlo reference, we observe comparatively weak signal in the second-order mode of the multiple-scattering portion of the BRDF, $f^{(2)}(\mu_i,\mu_o)$ (bottom row).*

3.3. First-order Fourier mode

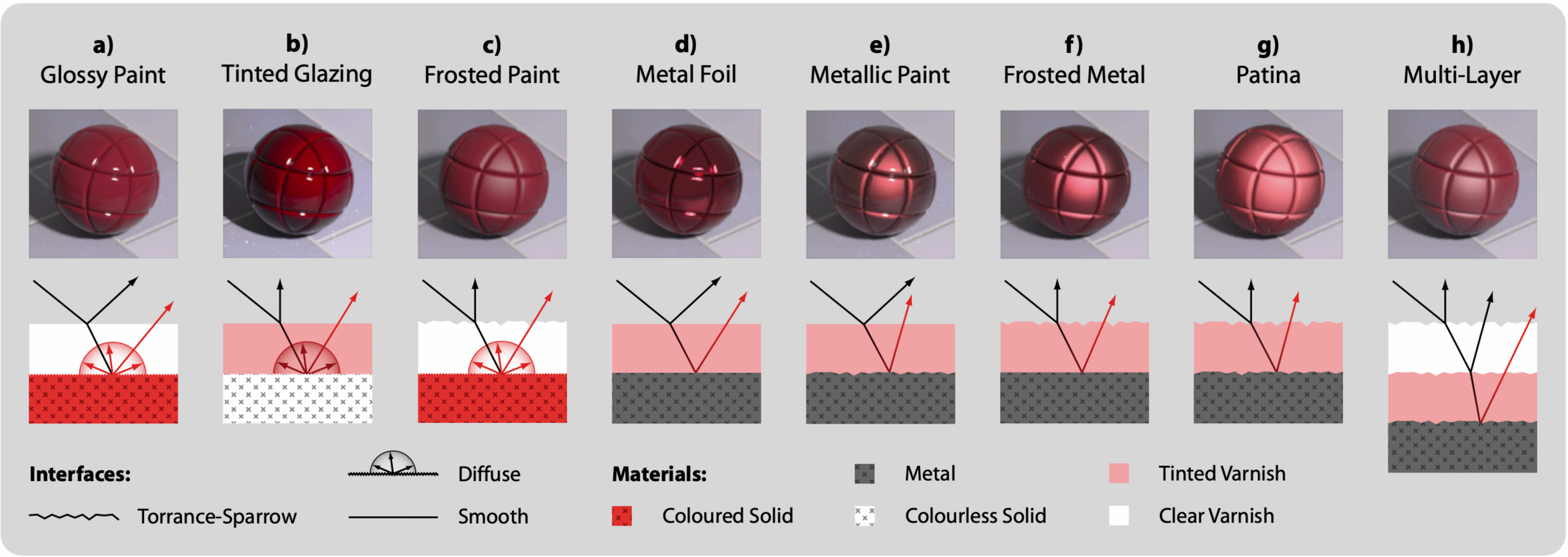
In Figure 3 we see that the first-order Fourier mode $f_m^{(1)}$ of the multiple scattering is non-negligible. This requires that f_m has a term of the form $f(\mu_i,\mu_o)\cos(\phi)$ for some function f. We approximate this term from the exact solution and this one of the key differences of our BRDF to previous approximations, which assume $f_m^{(1)} = 0$ [Hap81, Hap02].

The exact first-order mode of the BRDF is [HC61, Eq.(43)]

$$f^{(1)}(\mu_i,\mu_o) = \frac{cH^{(1)}(\mu_i)H^{(1)}(\mu_o)}{6\pi(\mu_i+\mu_o)}\sqrt{\left(1-\mu_i^2\right)\left(1-\mu_o^2\right)}\times$$
$$\times\left(1+\left(l^2+\frac{45m}{64}\right)\mu_i\mu_o + l(\mu_i+\mu_o)\right), \quad (27)$$

Layered Surfaces

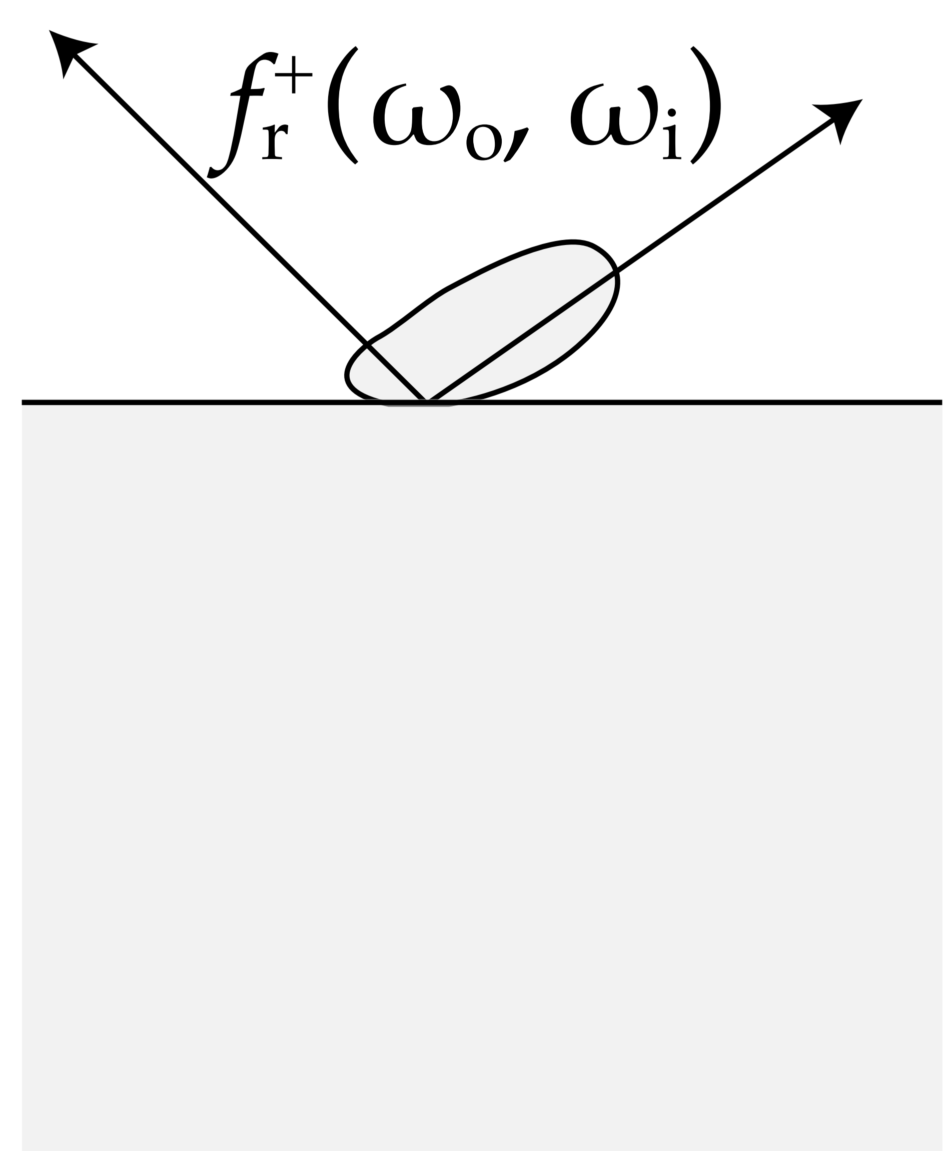
Layered Reflection Models



Weidlich and Wilkie. 2007.
Arbitrarily Layered Micro-Facet Surfaces.

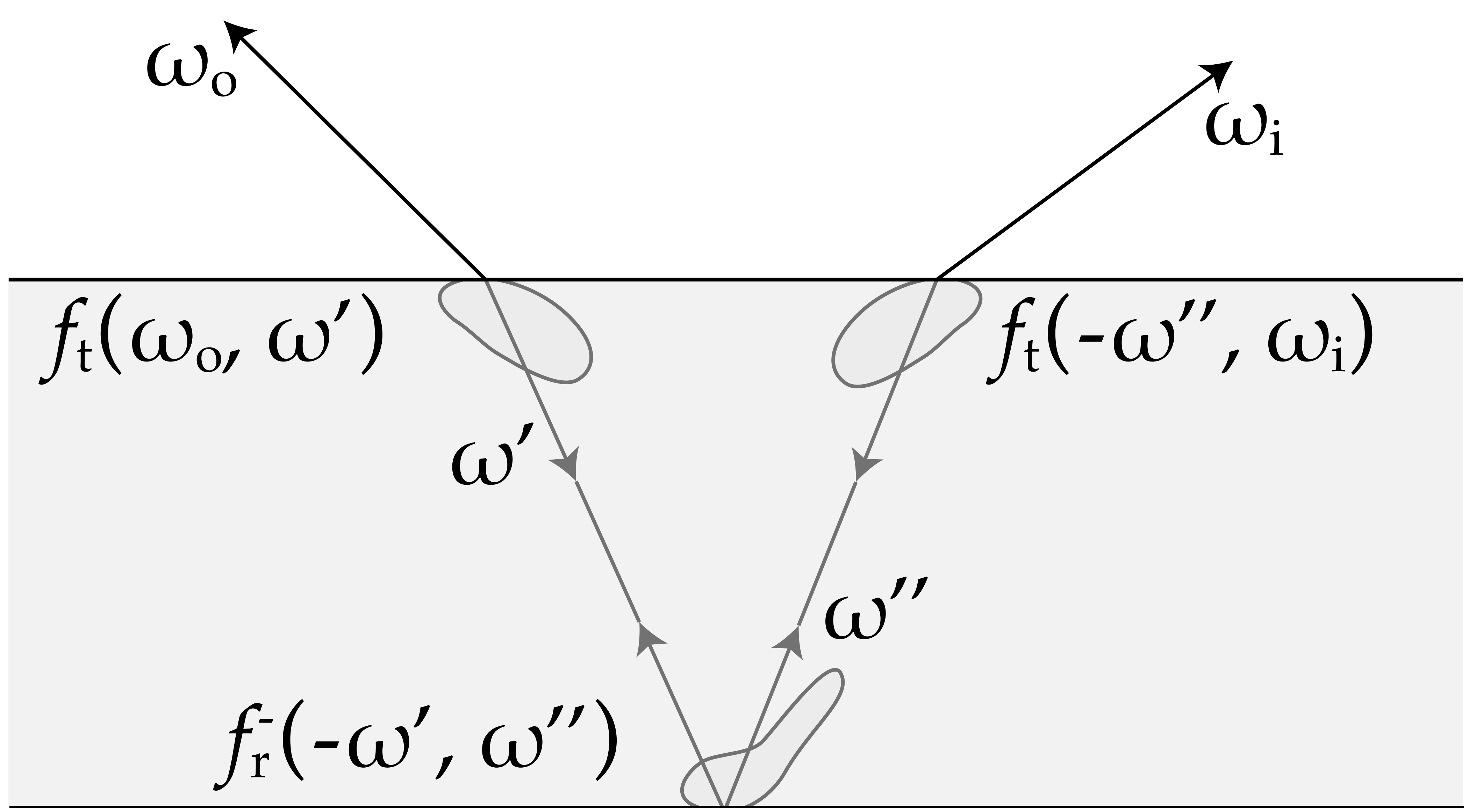
Layered Reflection Models

$$f_\mathrm{r}(\omega_\mathrm{o} \to \omega_\mathrm{i}) = f_\mathrm{r}^{+}(\omega_\mathrm{o} \to \omega_\mathrm{i}) + \cdots$$



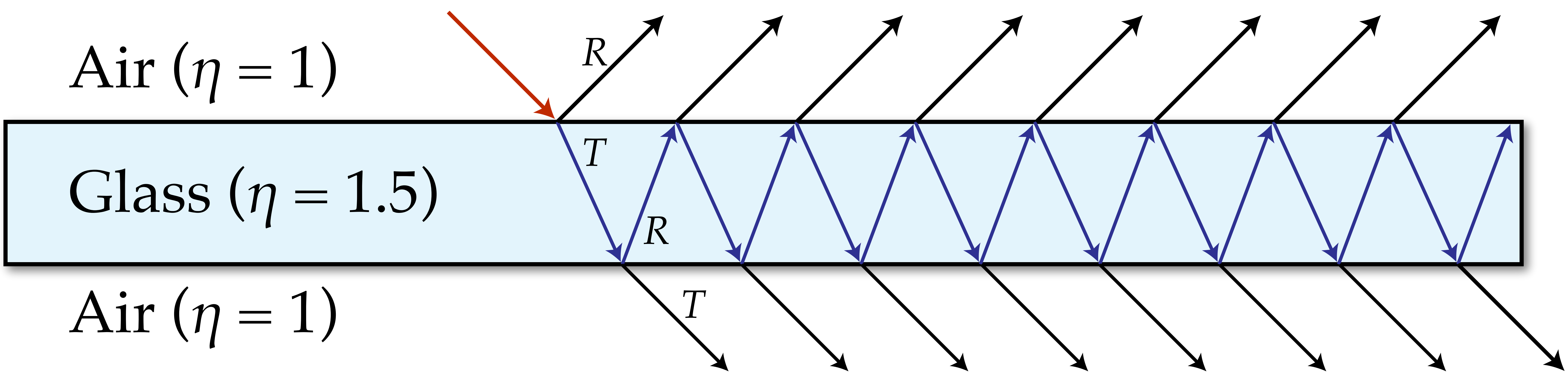
Layered Reflection Models

$$f_r(\omega_o \to \omega_i) = f_r^+(\omega_o \to \omega_i) + \int_\Omega \int_\Omega f_t(\omega_o \to \omega') \cos\theta' f_r(\omega' \to -\omega'') \cos\theta'' f_t(\omega'' \to \omega_i)\, d\omega' d\omega'' + \cdots$$



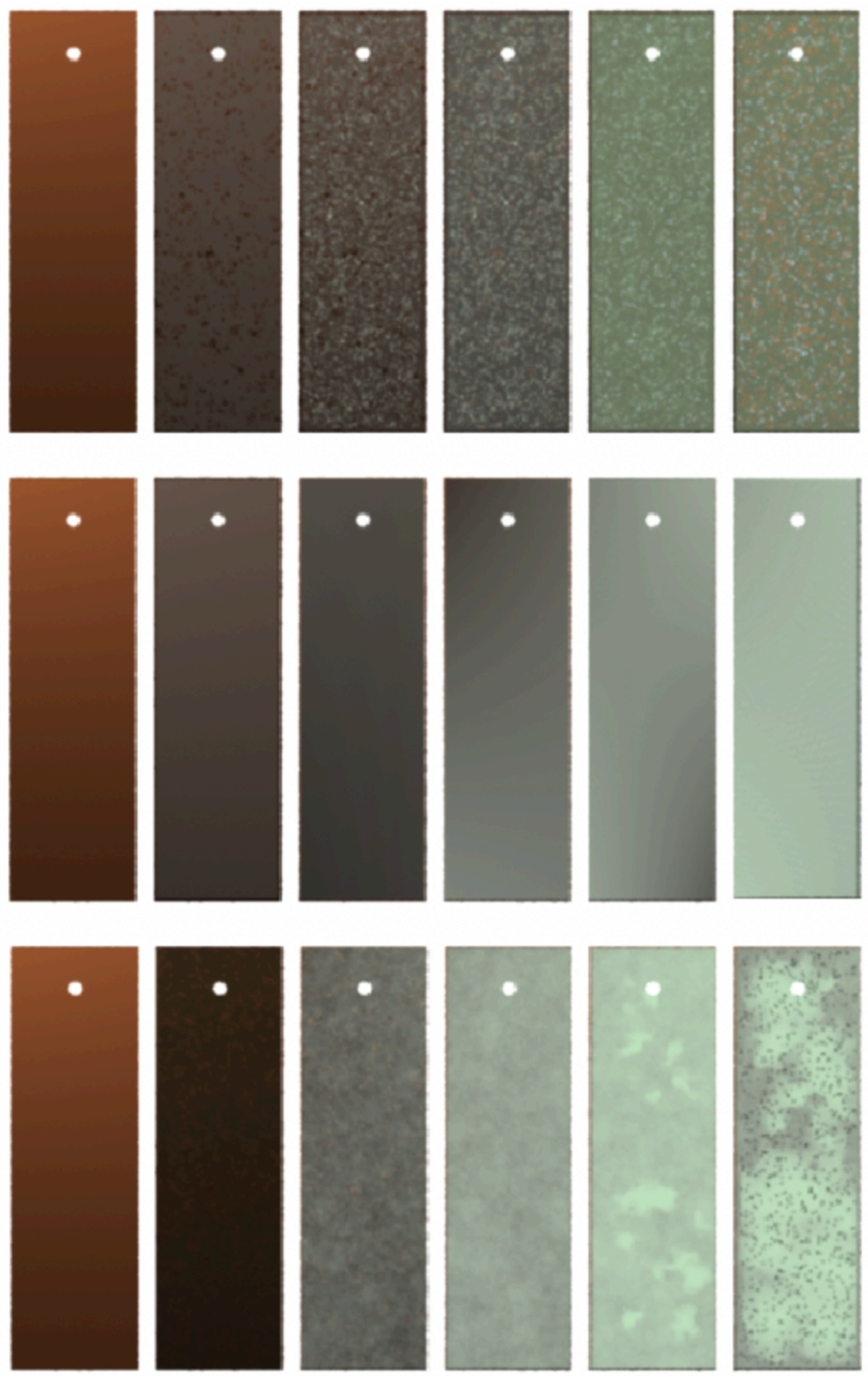
Thin Dielectric BSDF

$$R' = R + TRT + TRRRT + \cdots = R + \frac{T^2 R}{1 - R^2}$$

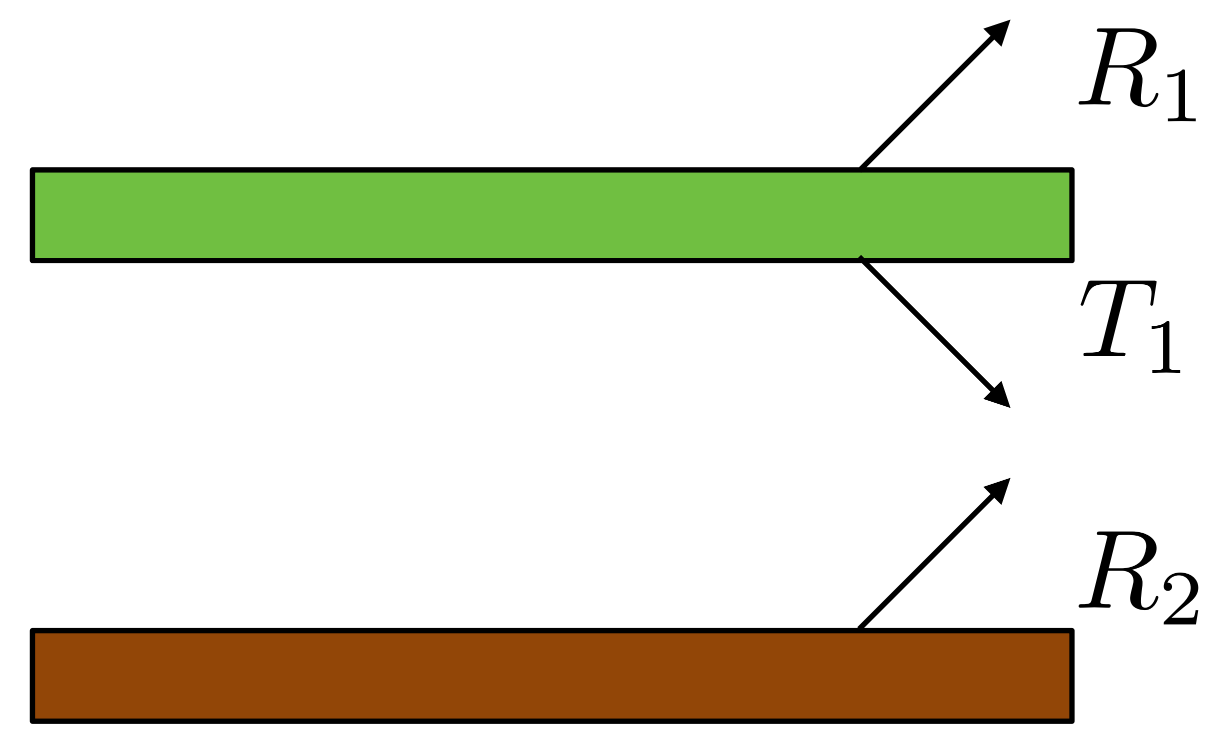


$$T' = TT + TRRT + TRRRRT + \cdots = 1 - R'$$

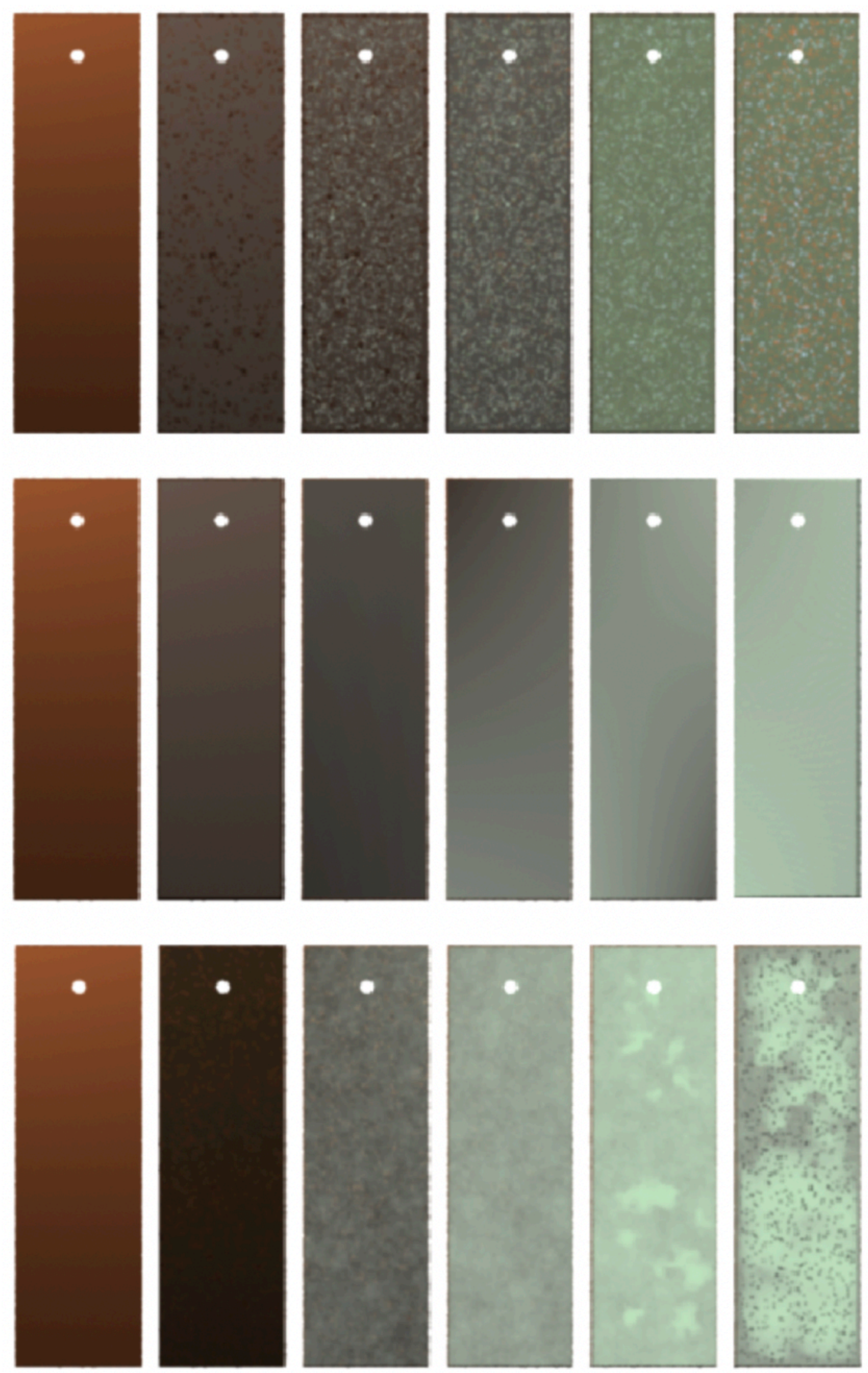
Layered Reflection Models



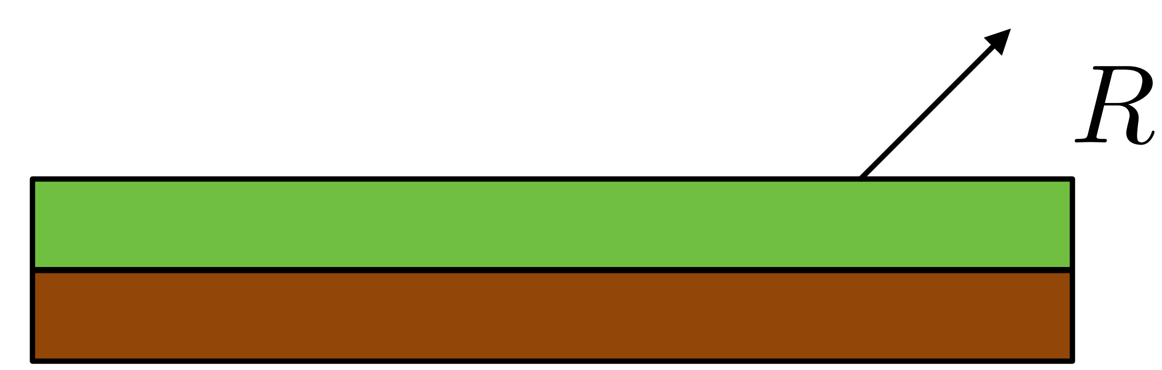
Copper patinas,
Dorsey and Hanrahan



Layered Reflection Models



Copper patinas,
Dorsey and Hanrahan



$$R = R_1 + T_1\, R_2\, T_1 + \cdots = R_1 + \frac{T_1^2\, R_2}{1 - R_1\, R_2}$$

Layered Reflection Models

Single-scattering approximation

$$f_{\mathrm{r}}(\omega_{\mathrm{o}} \to \omega_{\mathrm{i}}) \approx f_{\mathrm{r}}^{+}(\omega_{\mathrm{o}} \to \omega_{\mathrm{i}}) + \\ f_{\mathrm{t}}(\omega_{\mathrm{o}} \to \omega') \cos\theta' f_{\mathrm{r}}(\omega' \to -\omega'') \cos\theta'' f_{\mathrm{t}}(\omega'' \to \omega_{\mathrm{i}})$$

Single evaluation directions ω' and ω'' found by sampling specular transmission.

Weidlich and Wilkie. 2007.
Arbitrarily Layered Micro-Facet Surfaces.

Acknowledgement

Thanks to Steve Marschner for microsurface slides