Participating Media and Volume Rendering

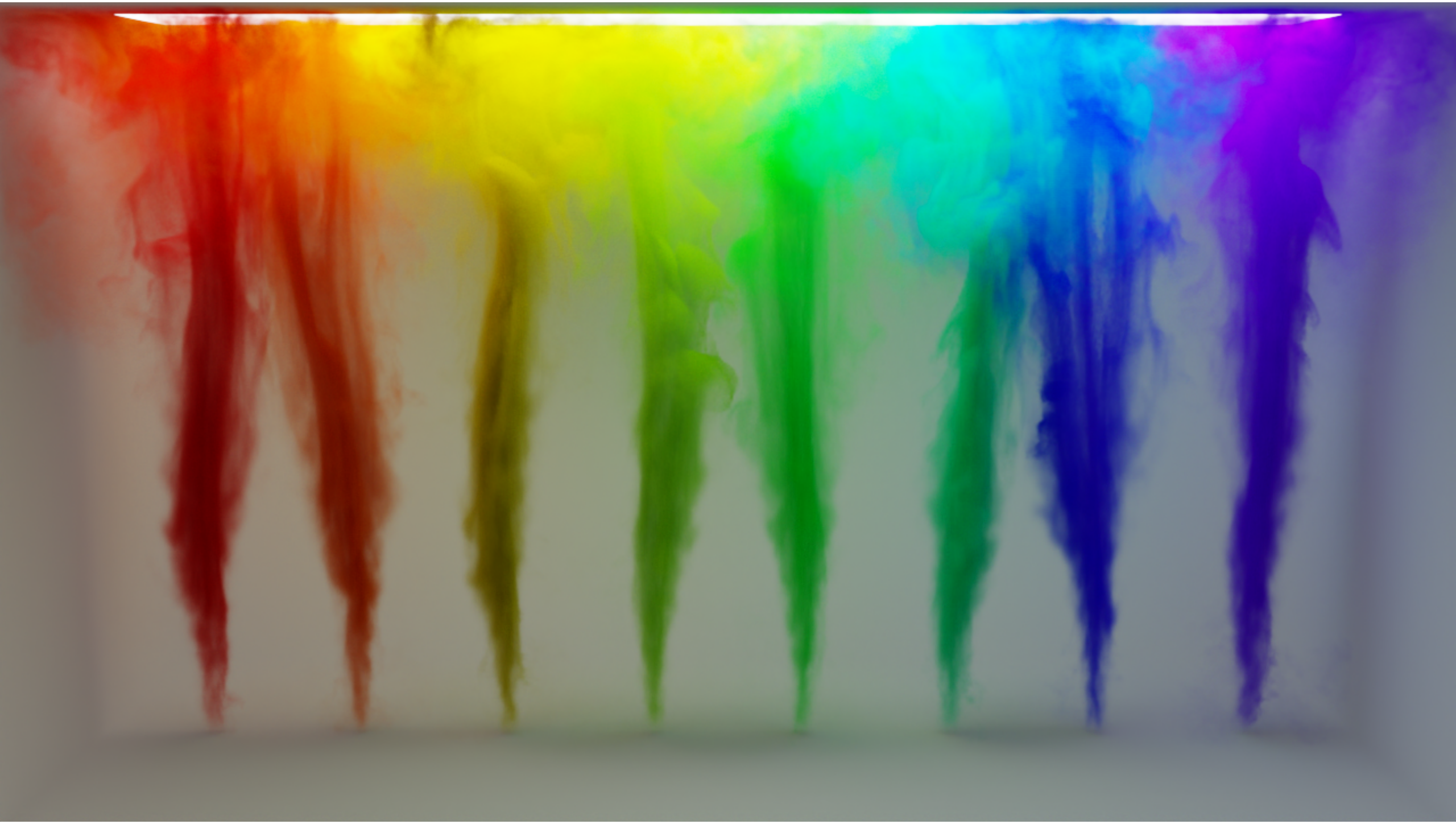
Applications:

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...

Topics:

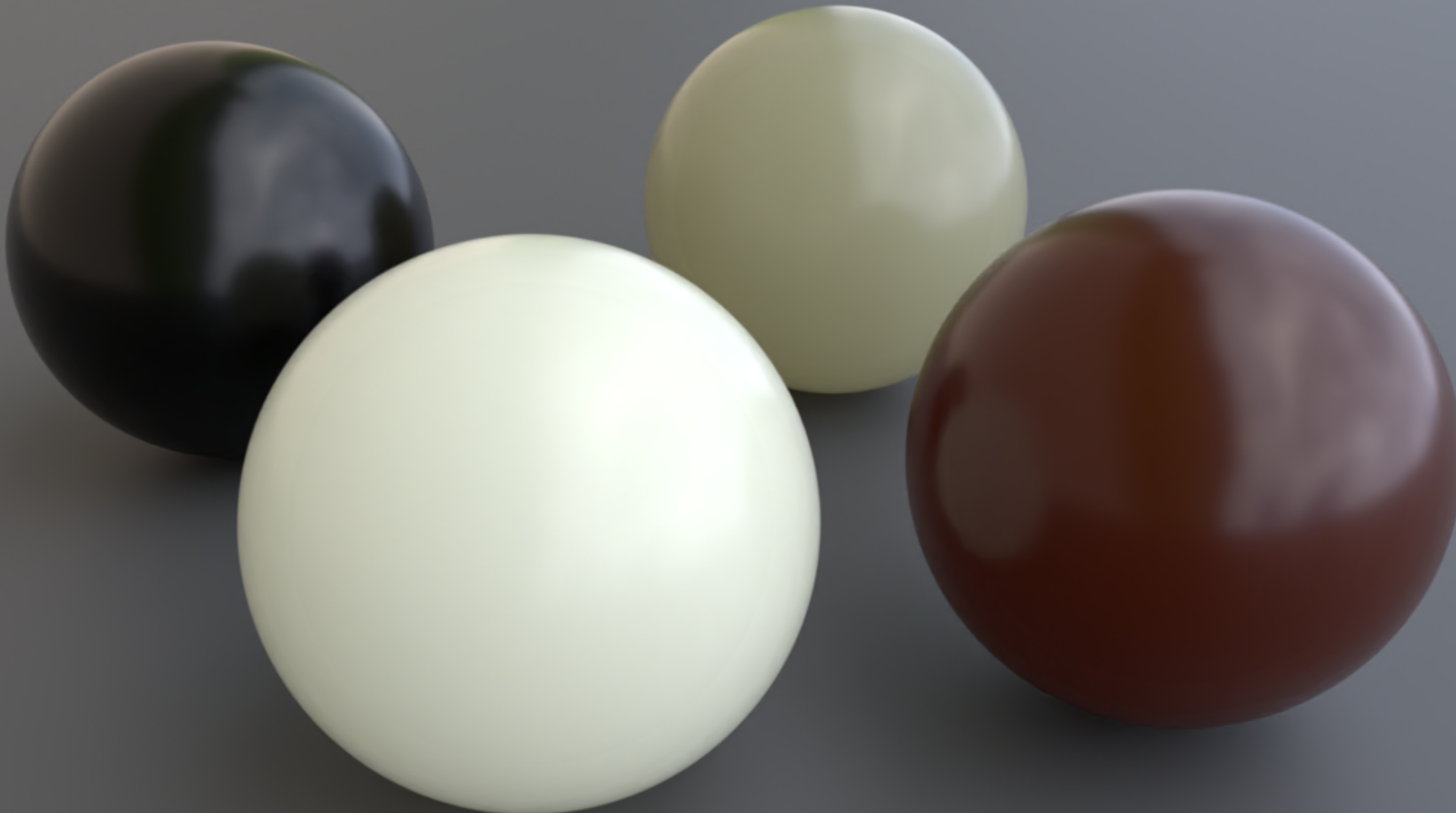
- Absorption and emission
- Scattering and phase functions
- The volume rendering equation
- Null scattering and sampling for Monte Carlo integration

Participating Media



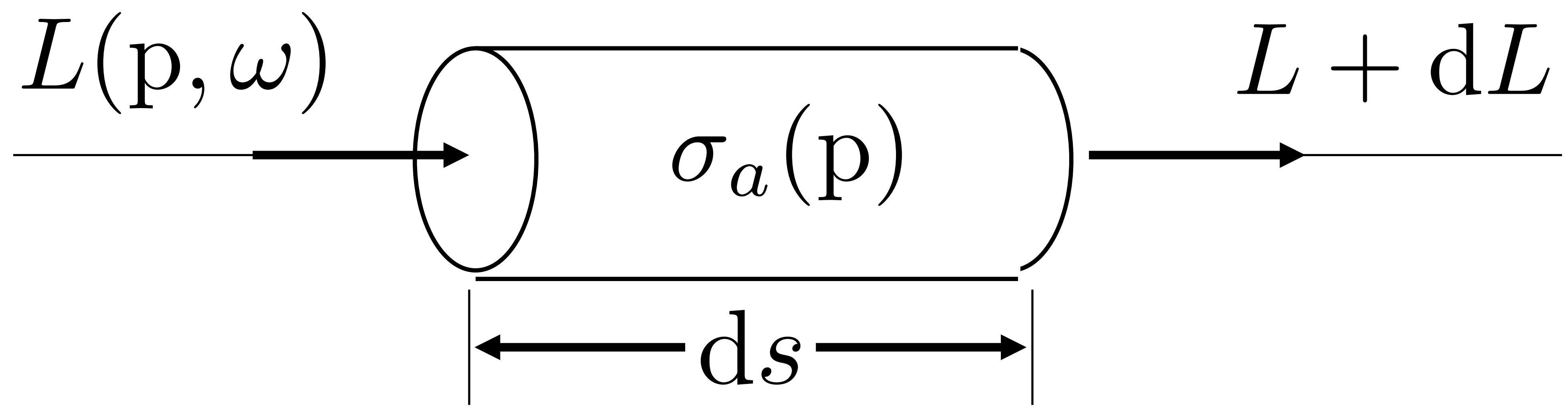
Jim Price

Participating Media



Participating Media

Absorption



$$\mathrm{d}L(\mathrm{p}, \omega) = -\sigma_a(\mathrm{p})\, L(\mathrm{p}, \omega)\, \mathrm{d}s$$

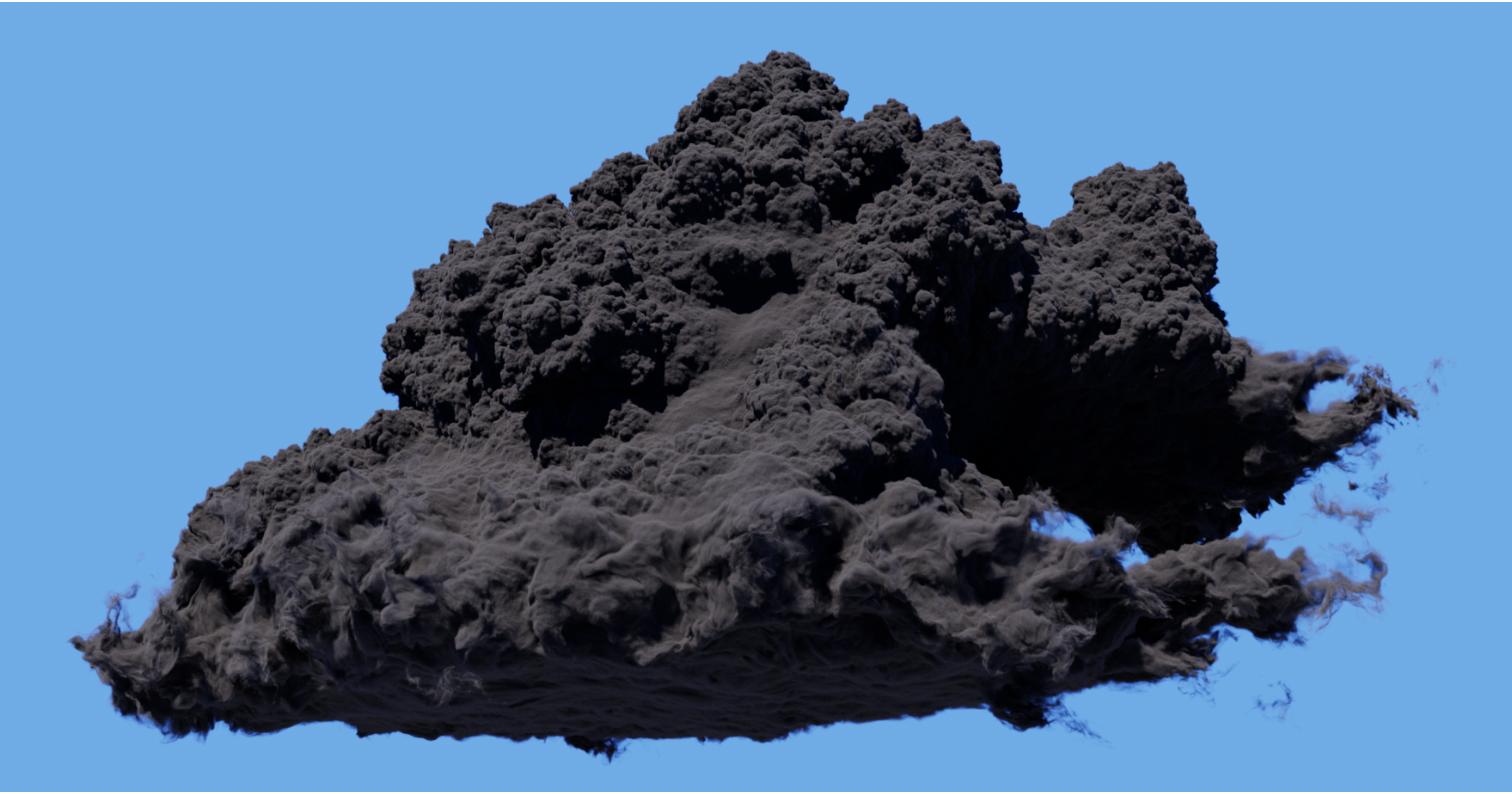
Absorption cross section: $\sigma_a(\mathrm{p})$

- **Probability of being absorbed per unit length**
- **Units: 1/distance**

Absorption: Lower Density

Walt Disney Animation Studios

Absorption: Higher Density



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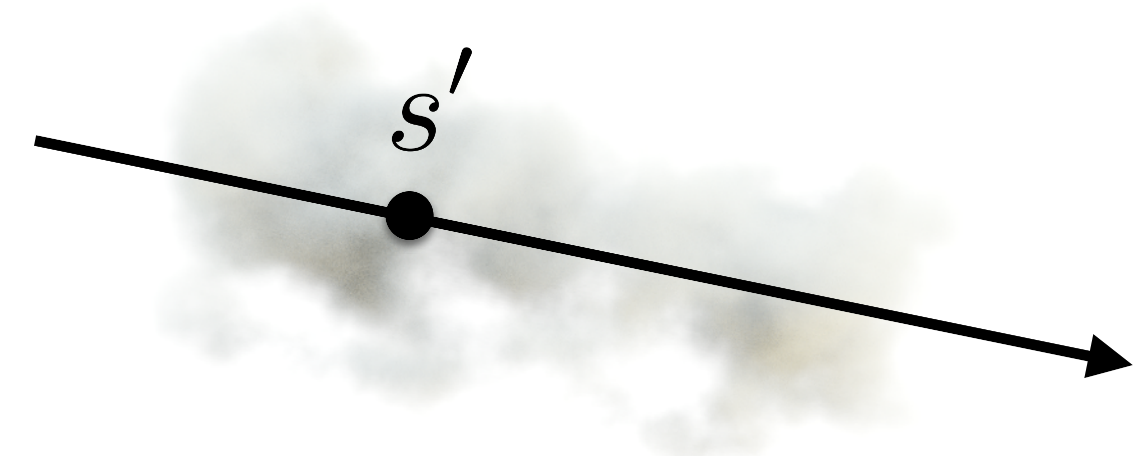
Transmittance

$$\mathrm{d}L(\mathrm{p},\omega) = -\sigma_a(\mathrm{p})\, L(\mathrm{p},\omega)\,\mathrm{d}s$$



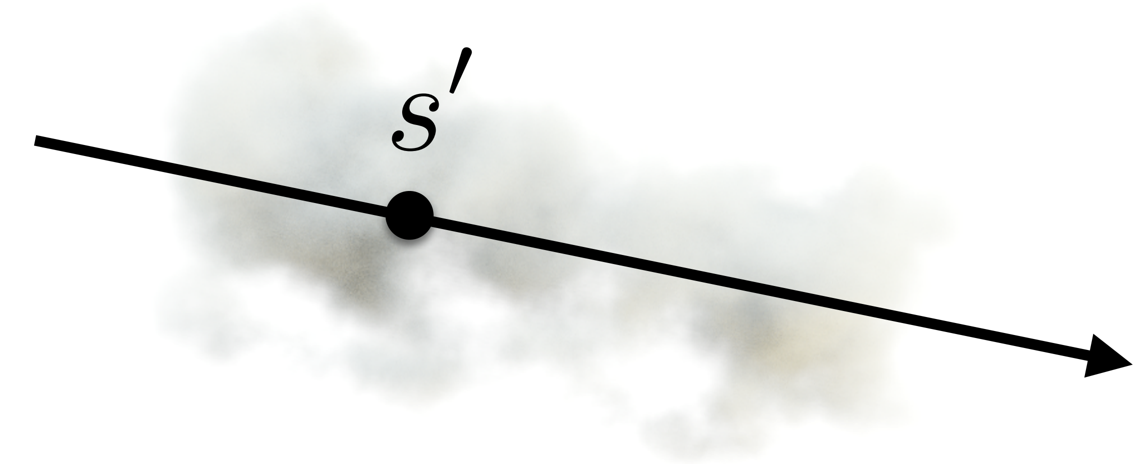
$$\frac{\mathrm{d}L(\mathrm{p},\omega)}{L(\mathrm{p},\omega)} = -\sigma_a(\mathrm{p})\,\mathrm{d}s$$

$$\log L(\mathrm{p}+s\omega,\omega) - \log L(\mathrm{p},\omega) = -\int_0^s \sigma_a(\mathrm{p}+s'\omega,\omega)\,\mathrm{d}s'$$

$$L(\mathrm{p}+s\omega,\omega) = \mathrm{e}^{-\int_0^s \sigma_\mathrm{a}(\mathrm{p}+s'\omega,\omega)\,\mathrm{d}s'} L(\mathrm{p},\omega) = T(s)\, L(\mathrm{p},\omega)$$

Transmittance: $T(s) = \mathrm{e}^{-\int_0^s \sigma_\mathrm{a}(\mathrm{p}+s'\omega,\omega)\,\mathrm{d}s'}$

Optical Thickness and Beer's Law



Optical distance (depth):

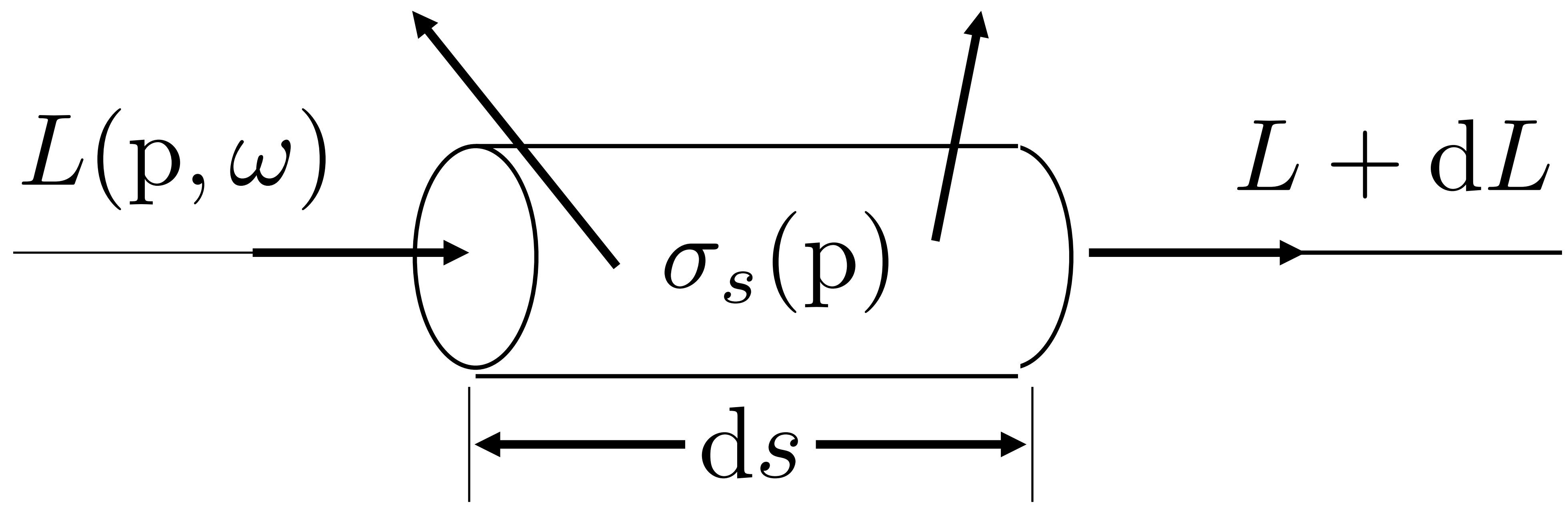
$$\tau(s) = \int_0^s \sigma_a(\mathrm{p}')\,\mathrm{d}s'$$

$$\mathrm{p}' = \mathrm{p} + s'\omega$$

Homogeneous medium–constant σ_a**:** $\tau(s) = \sigma_a s$

Beer's Law: $T(s) = \mathrm{e}^{-\tau(s)} = \mathrm{e}^{-\sigma_\mathrm{a} s}$

Out-Scattering

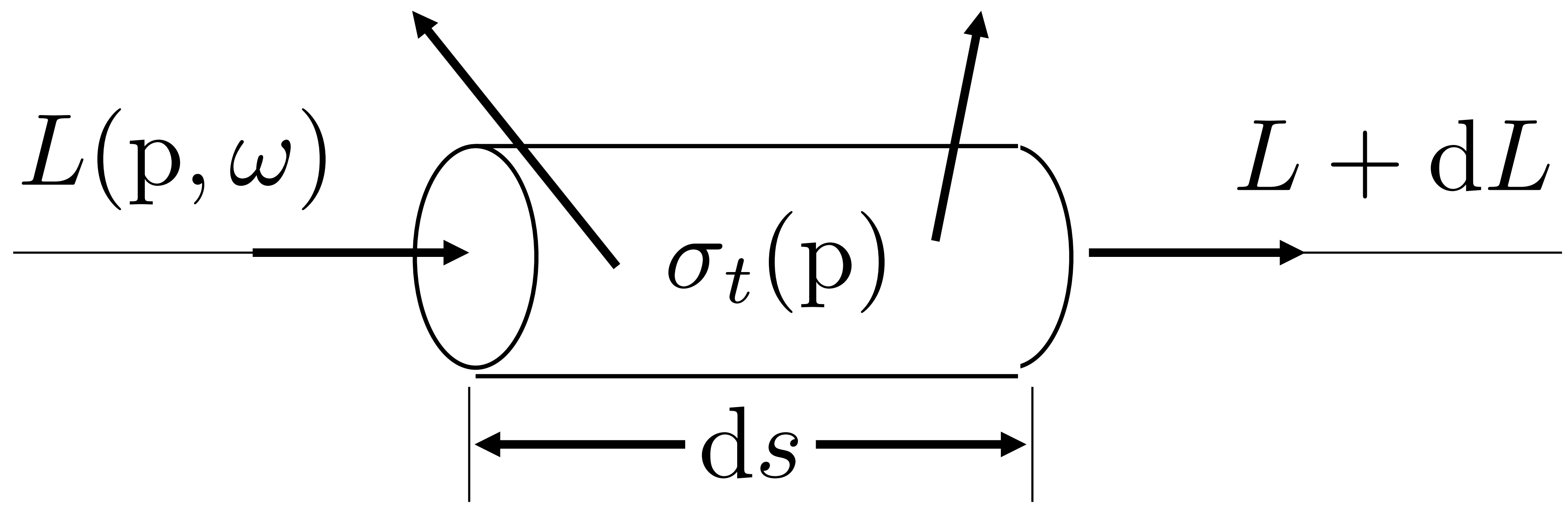


$$\mathrm{d}L(\mathrm{p}, \omega) = -\sigma_s(\mathrm{p})\, L(\mathrm{p}, \omega)\, \mathrm{d}s$$

Scattering cross-section: σ_s

- **Probability of being scattered per unit length**

Extinction



$$\mathrm{d}L(\mathrm{p}, \omega) = -\sigma_t(\mathrm{p})\, L(\mathrm{p}, \omega)\, \mathrm{d}s$$

Total cross section: $\sigma_t = \sigma_a + \sigma_s$

Albedo: $W = \dfrac{\sigma_s}{\sigma_t} = \dfrac{\sigma_s}{\sigma_a + \sigma_s}$

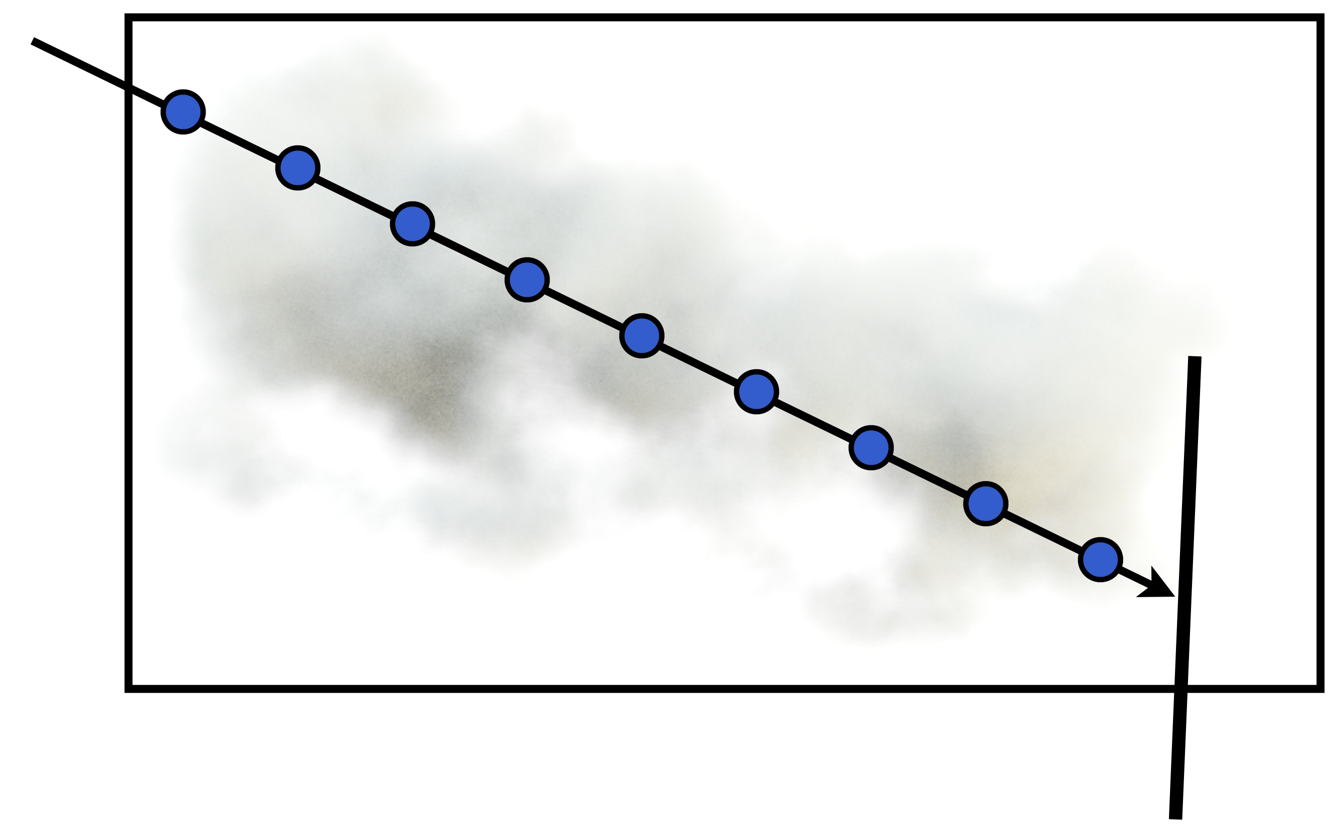
Optical distance from absorption and scattering:

$$\tau(s) = \int_0^s \sigma_t(\mathrm{p}')\, \mathrm{d}s'$$

Ray Marching to Compute Transmittance

$$\tau(s) = \int_0^s \sigma_t(\mathrm{p} + s'\omega)\,\mathrm{d}s$$

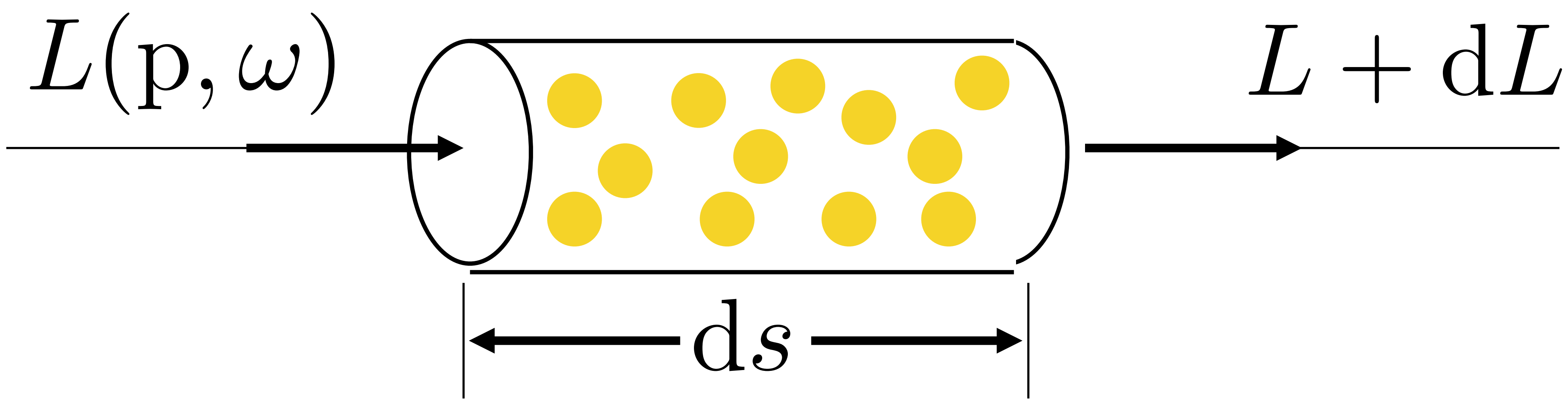
$$T(s) = \mathrm{e}^{-\tau(s)}$$



Riemann sum: $$\tau(s) \approx \frac{s}{N} \sum_i^N \sigma_t(x_i)$$

$$x_i = x + \frac{i + 0.5}{N}\omega$$

Emission

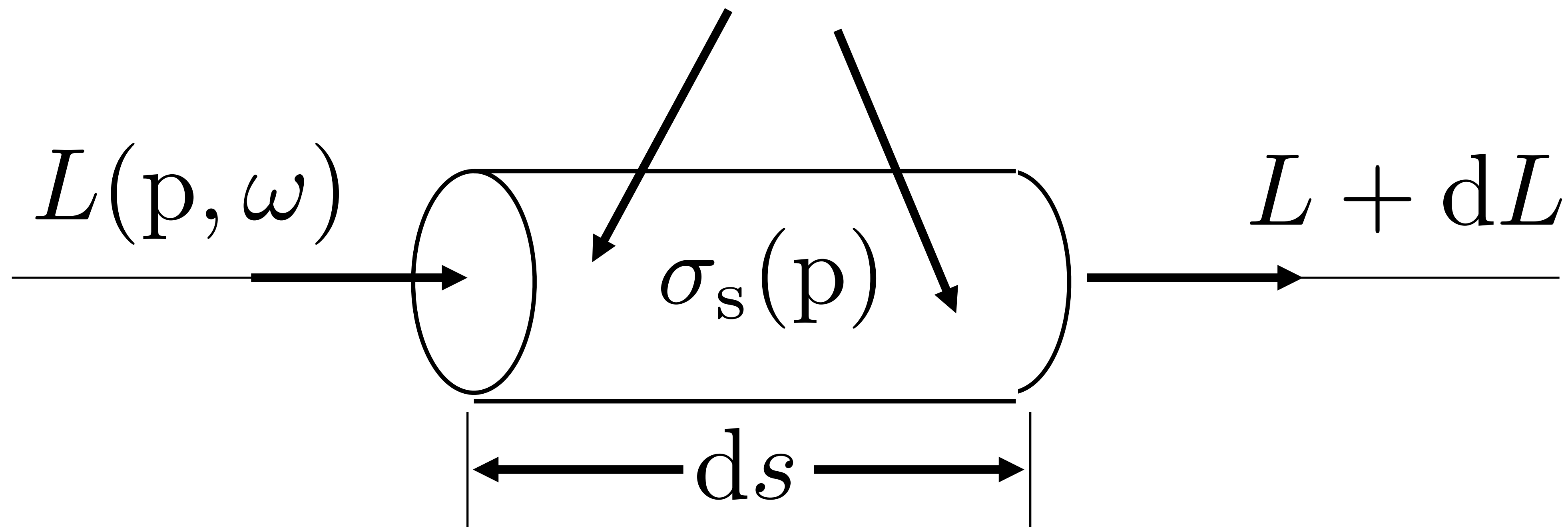


$$\mathrm{d}L(\mathrm{p}, \omega) = \sigma_a(\mathrm{p}) L_\mathrm{e}(\mathrm{p}, \omega)\, \mathrm{d}s$$

Jim Price

Jim Price

In-Scattering



$$S(\mathrm{p}, \omega) = \sigma_s(\mathrm{p}) \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p}, \omega')\, \mathrm{d}\omega'$$

Phase function: $p(\omega' \to \omega)$

Reciprocity: $p(\omega' \to \omega) = p(\omega \to \omega')$

Energy conservation: $\int_{S^2} p(\omega' \to \omega)\, \mathrm{d}\omega' = 1$

Scattering: Lower Density

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Scattering: Higher Density

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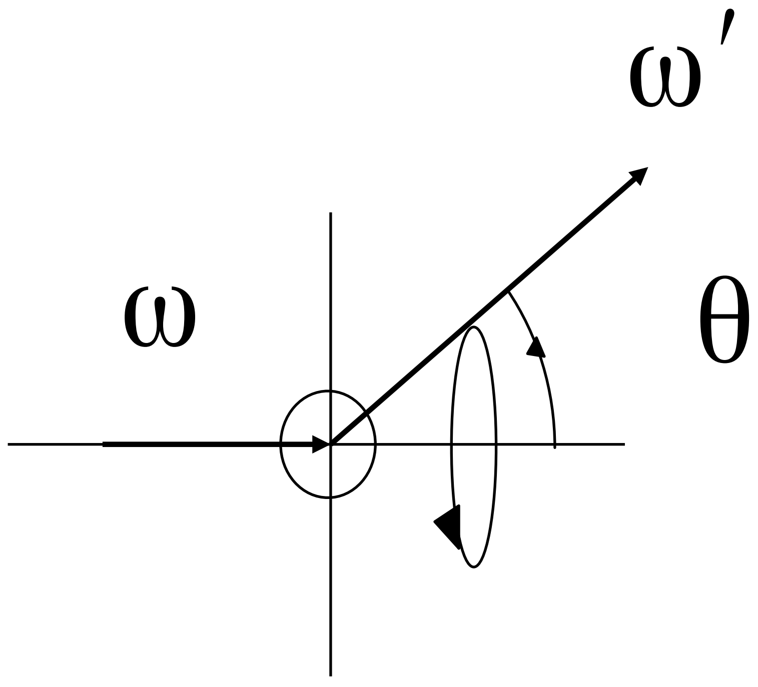
Phase Functions

Phase angle $\cos\theta = \omega \cdot \omega'$

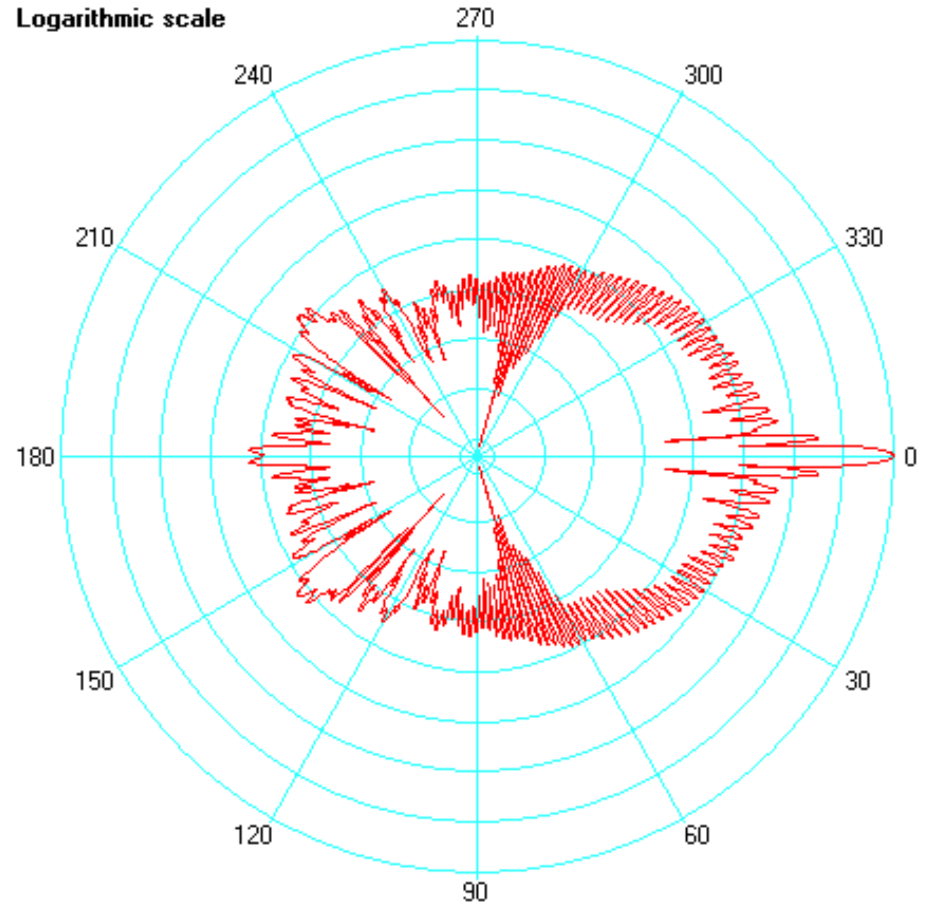


Phase functions

- **Isotropic:** $p(\cos\theta) = \frac{1}{4\pi}$

- **Rayleigh:** $p(\cos\theta) = \frac{3}{4}(1 + \cos^2\theta)$ **with** $\sigma_s \propto \frac{1}{\lambda^4}$

- **Mie:**



[Philip Laven]

Rayleigh Scattering: Blue Sky, Red Sunset

From Greenler: Rainbows, Halos, and Glories

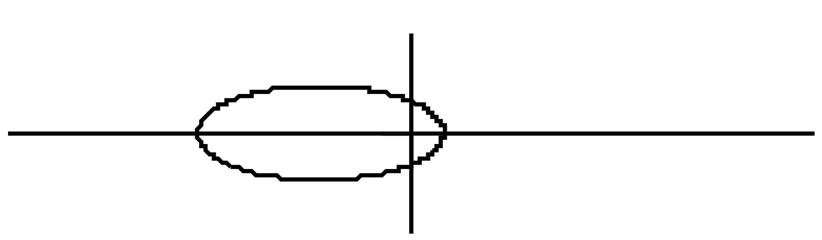
Henyey-Greenstein Phase Function

Empirical phase function

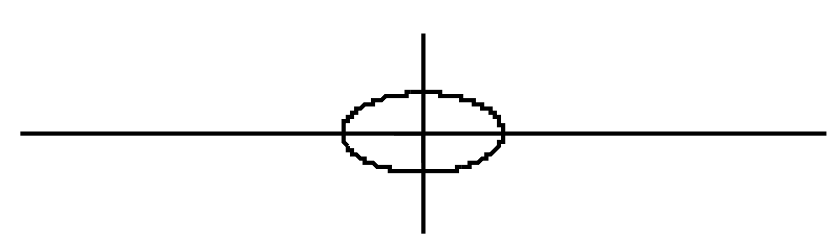
$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}$$

Average phase angle g:

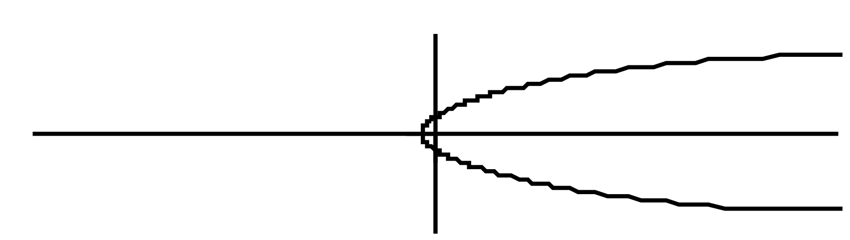
$$g = 2\pi\int_0^{2\pi} p(\cos\theta)\ \cos\theta\sin\theta\,\mathrm{d}\theta$$



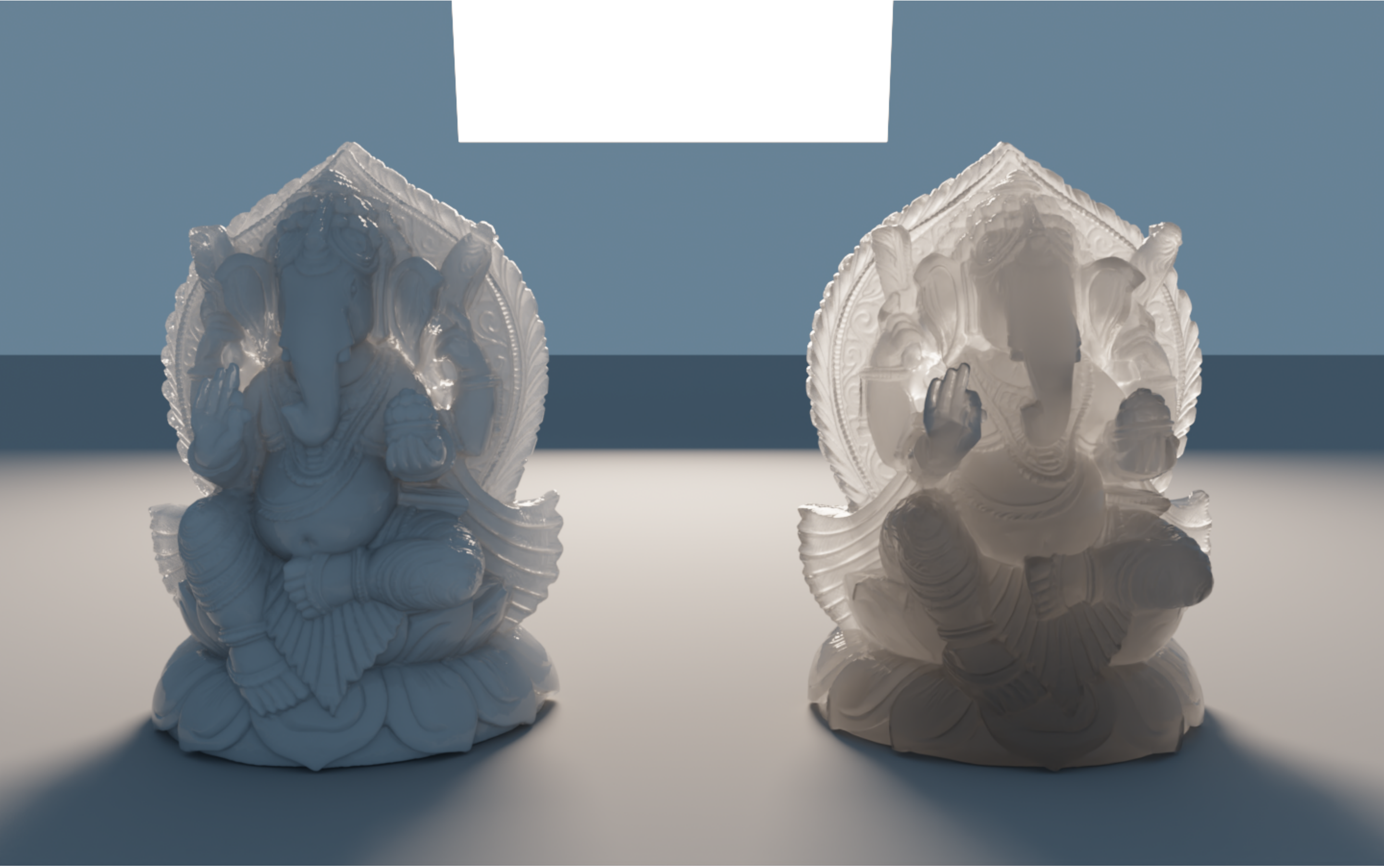
$g = -0.3$



$g = 0$



$g = 0.6$



More Backward Scattering

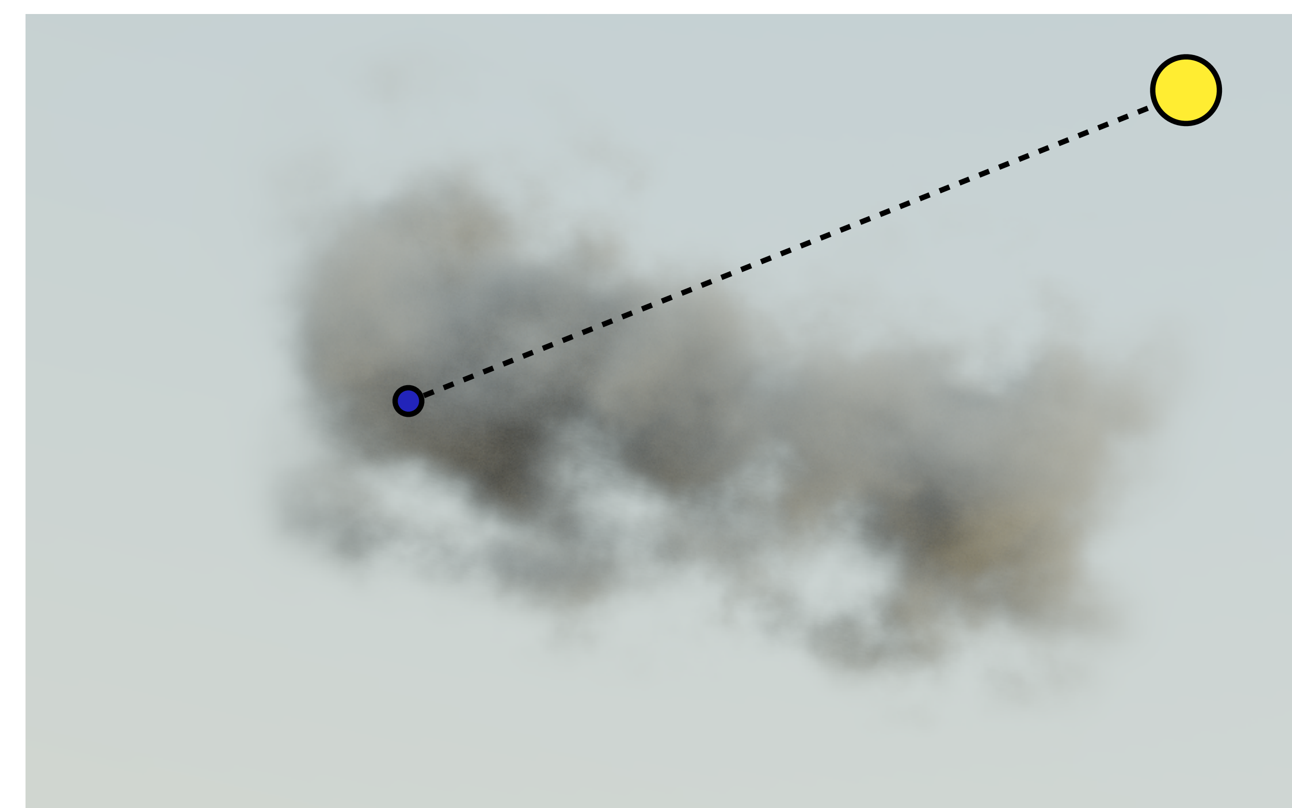
More Forward Scattering

Direct Illumination in a Volume

$$S_{\mathrm{d}}(\mathrm{p}', \omega) = \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L_{\mathrm{d}}(\mathrm{p}', \omega')\, \mathrm{d}\omega'$$

Can treat like direct illumination at a surface

- **Sample from phase function's distribution**
- **Sample from light source distributions**
- **Weight using multiple importance sampling**



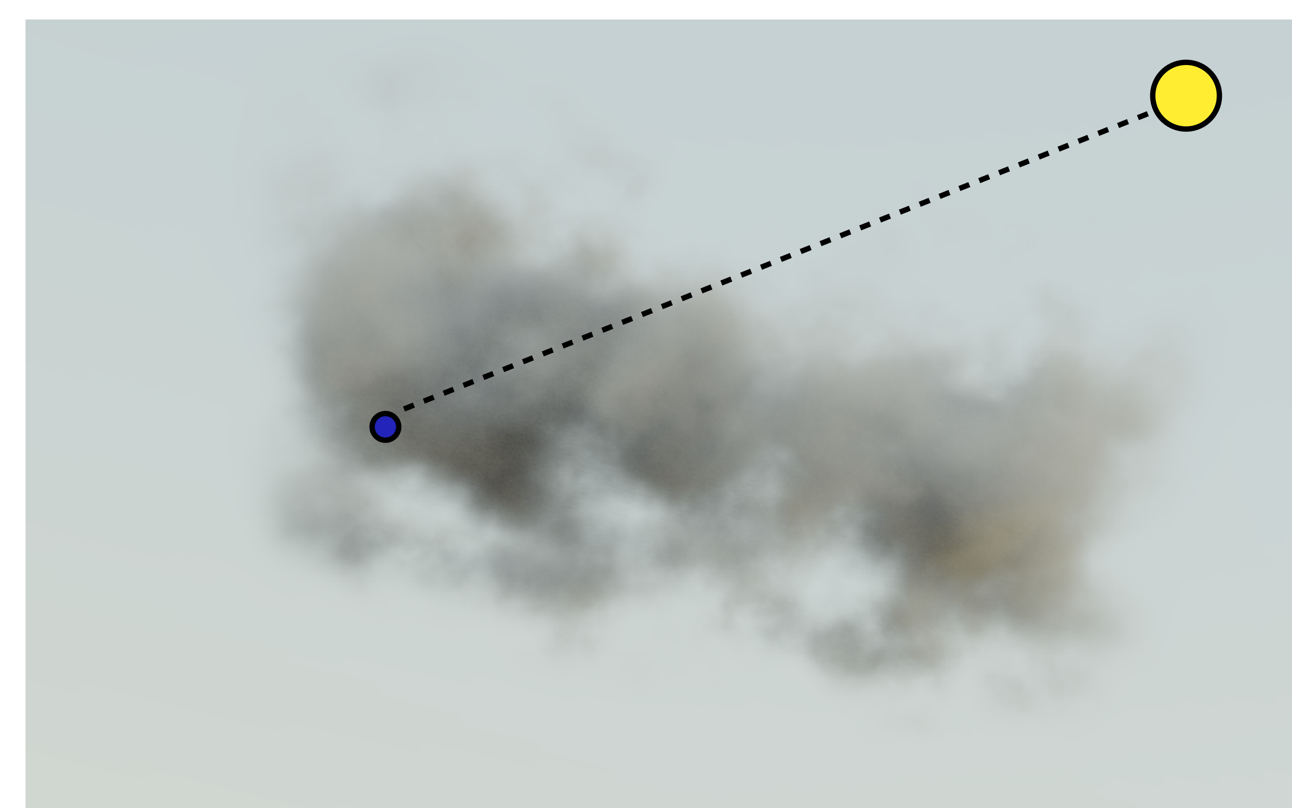
Direct Illumination in a Volume

$$S_{\mathrm{d}}(\mathrm{p}', \omega) = \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L_{\mathrm{d}}(\mathrm{p}', \omega')\, \mathrm{d}\omega'$$

Estimator: $\sigma_s(\mathrm{p}') \frac{1}{N} \sum_{i}^{N} \frac{p(\omega_i \to \omega)\, L_d(\mathrm{p}', \omega_i)}{p(\omega_i)}$

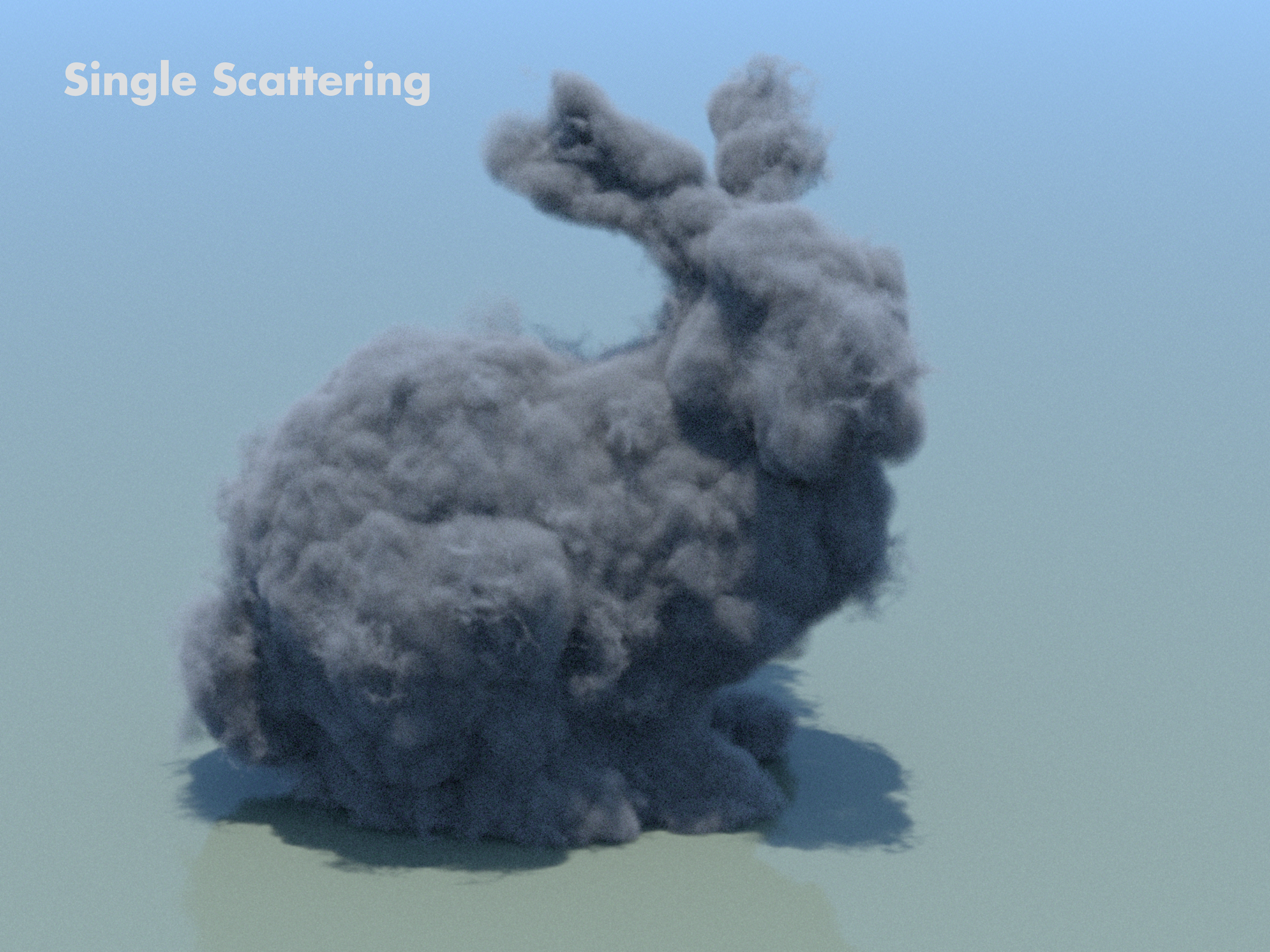
Computing direct lighting, L_d can be expensive

- **Not just a shadow ray– need to compute transmittance**



Single-Scattering

Minneart: Color and Light In The Open Air


Single Scattering

The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(\mathrm{p}, \omega)}{\partial s} = -\sigma_t L(\mathrm{p}, \omega) + S(\mathrm{p}, \omega)$$

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}', \omega)\, \mathrm{d}s'$$

Attenuation: absorption and scattering

$$\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p}'')\, \mathrm{d}s''}$$

Source: in-scattering and emission

$$L_\mathrm{e}(\mathrm{p}) + \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p}', \omega')\, \mathrm{d}\omega'$$

Volumetric Path Tracing

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}', \omega)\, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}', \omega)}{p(s')}$

Evaluating the Estimator: S

Include indirect illumination in the source term:

$$L_{\mathrm{e}}(\mathrm{p}) + \sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L(\mathrm{p}', \omega')\, \mathrm{d}\omega'$$

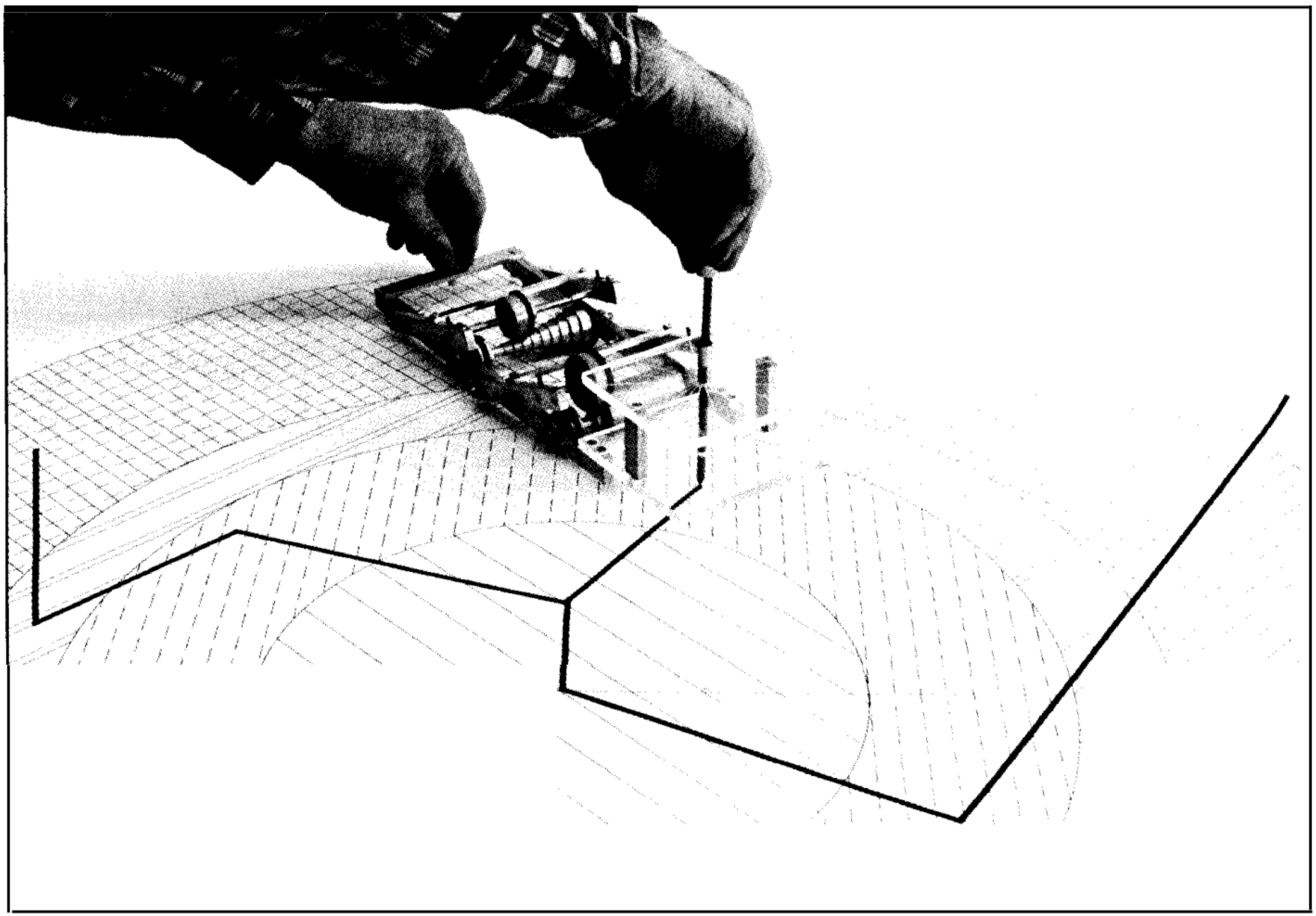
$$\Downarrow$$

$$L(\mathrm{p}, \omega') = L_{\mathrm{d}}(\mathrm{p}, \omega') + L_{\mathrm{i}}(\mathrm{p}, \omega')$$

- **Compute direct lighting as before**
- **Sample incident direction from the phase function's distribution, trace a ray recursively...**

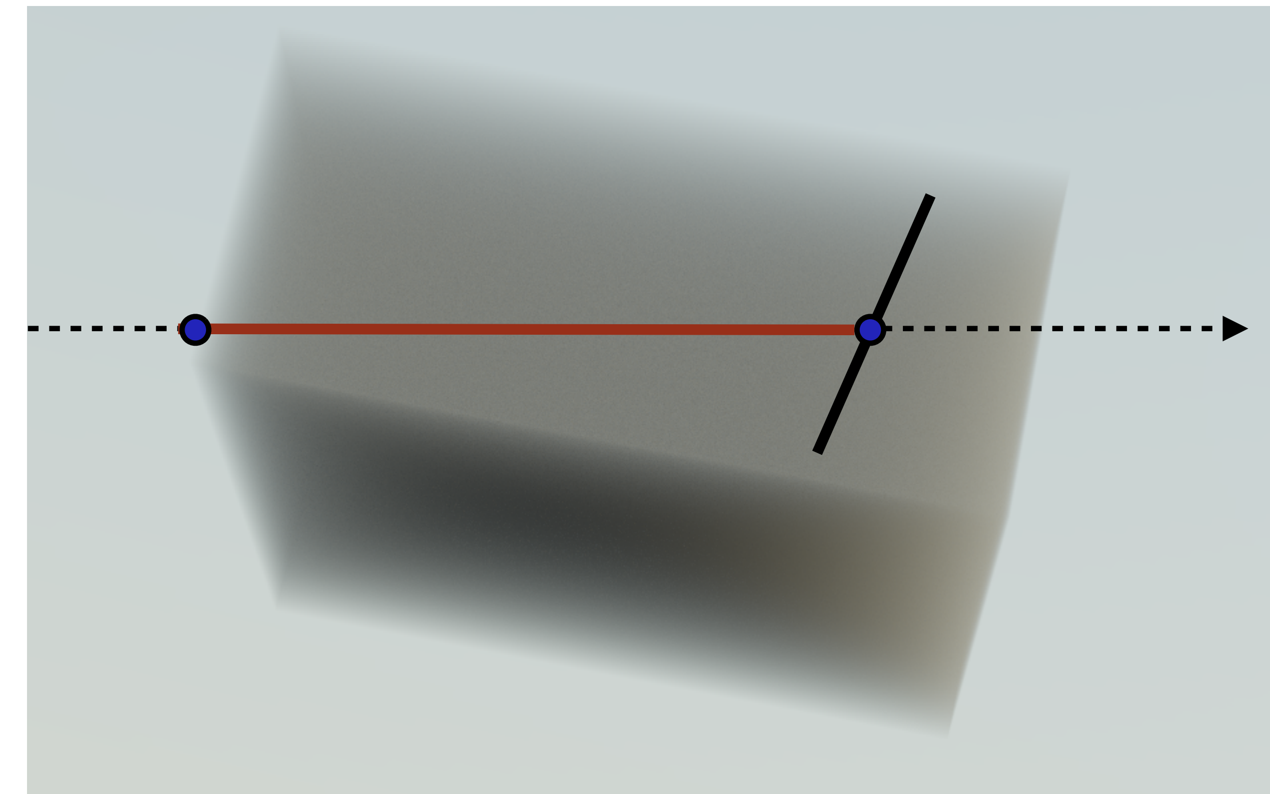
$$L_{\mathrm{i}}(\mathrm{p}, \omega') \approx \frac{p(\omega'' \to \omega')\, L(\mathrm{p}, \omega'')}{p(\omega'')}$$

The Fermiac



Linear Sampling of T

We want samples along a finite ray $[0, t_{max}]$.



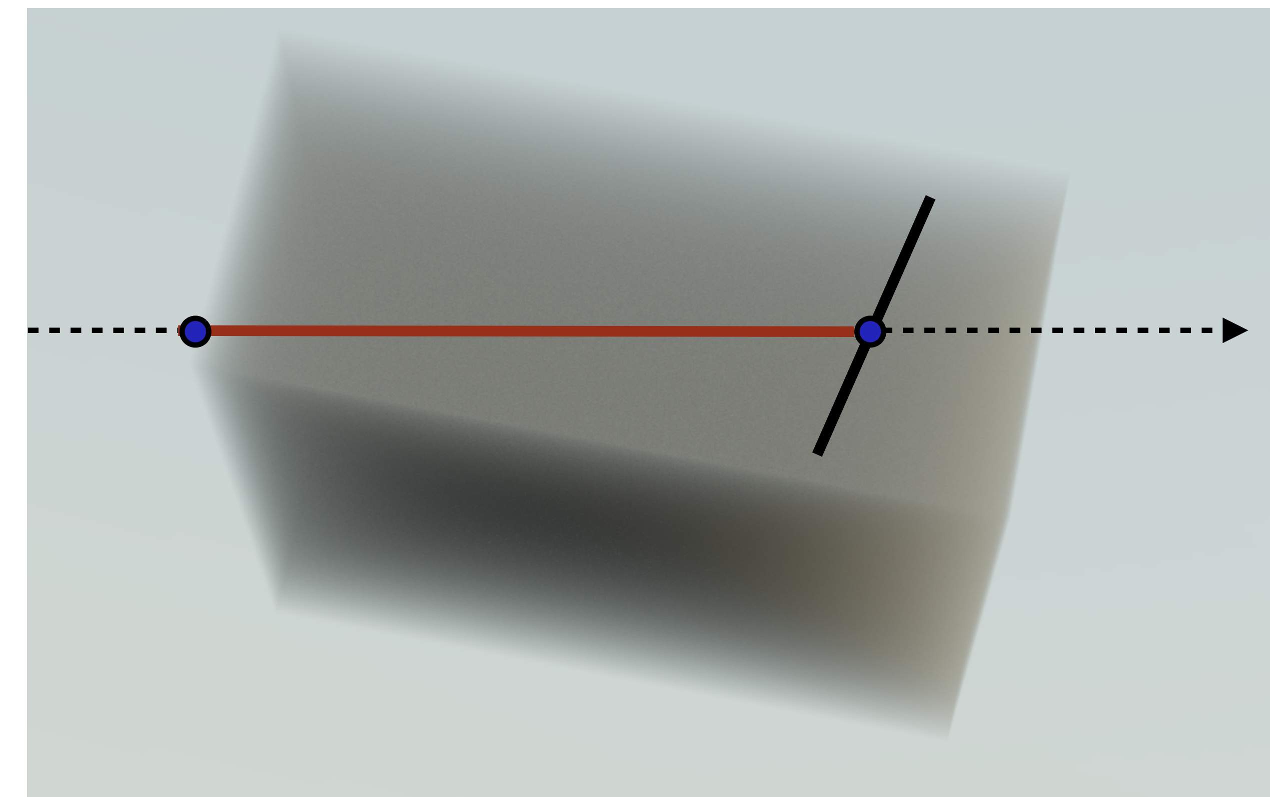
- **Uniform probability along the ray:**

$$p(t) = \frac{1}{t_{\mathrm{max}}}$$

- **Sampling recipe:**

$$\xi = \int_0^t p(t)\,\mathrm{d}t \qquad\qquad t = \xi t_{\mathrm{max}}$$

Exact Sampling of Uniform T



We want samples along a finite ray [0,t_{max}], $p(t) \propto \mathrm{e}^{-\sigma t}$

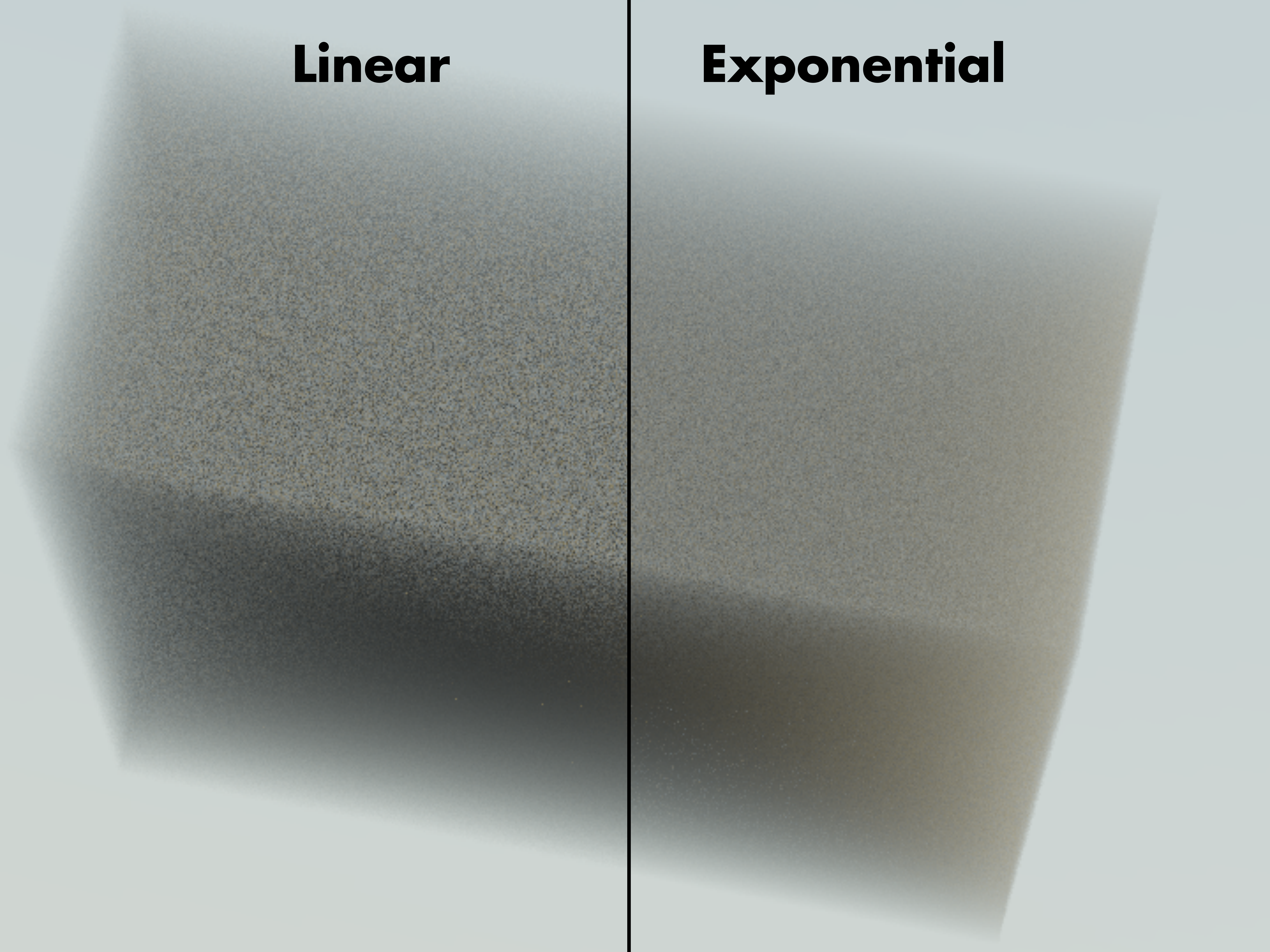
- **Normalize to find PDF:**

$$\int_0^{t_{\max}} \mathrm{e}^{-\sigma t}\,\mathrm{d}t = -\frac{1}{\sigma}(\mathrm{e}^{-\sigma t_{\max}} - 1) = c \qquad p(t) = c\mathrm{e}^{-\sigma t}$$

- **Invert to find t for a random sample:**

$$\xi = \int_0^t p(t)\,\mathrm{d}t$$

$$t = -\frac{1}{\sigma}\log(1 - \xi(1 - \mathrm{e}^{-\sigma t_{\max}}))$$


Linear
Exponential

Volumetric Path Tracing

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}', \omega)\, \mathrm{d}s'$$

Monte Carlo integration: sample $s' \sim p(s)$

Estimator: $\dfrac{T(\mathrm{p}')S(\mathrm{p}', \omega)}{p(s')}$

We "just" need to be able to evaluate $T(\mathrm{p}')$ **and to sample** $s' \sim p(s)$

Problem #1...

We would like an unbiased estimator of

$$T(s) = \mathrm{e}^{-\int_0^s \sigma_t(\mathrm{p}+s'\omega,\omega)\,\mathrm{d}s'}$$

However, using MC to estimate

$$X = \int_0^{s'} \sigma_t(\mathrm{p} + s''\omega)\,\mathrm{d}s'' \approx \frac{\sigma_t(\mathrm{p}')}{p(\mathrm{p}')}$$

and then taking e^{-X} **is biased:** $E[\mathrm{e}^X] \neq \mathrm{e}^{E[X]}$

Systemically underestimates transmittance(!)

Solution?

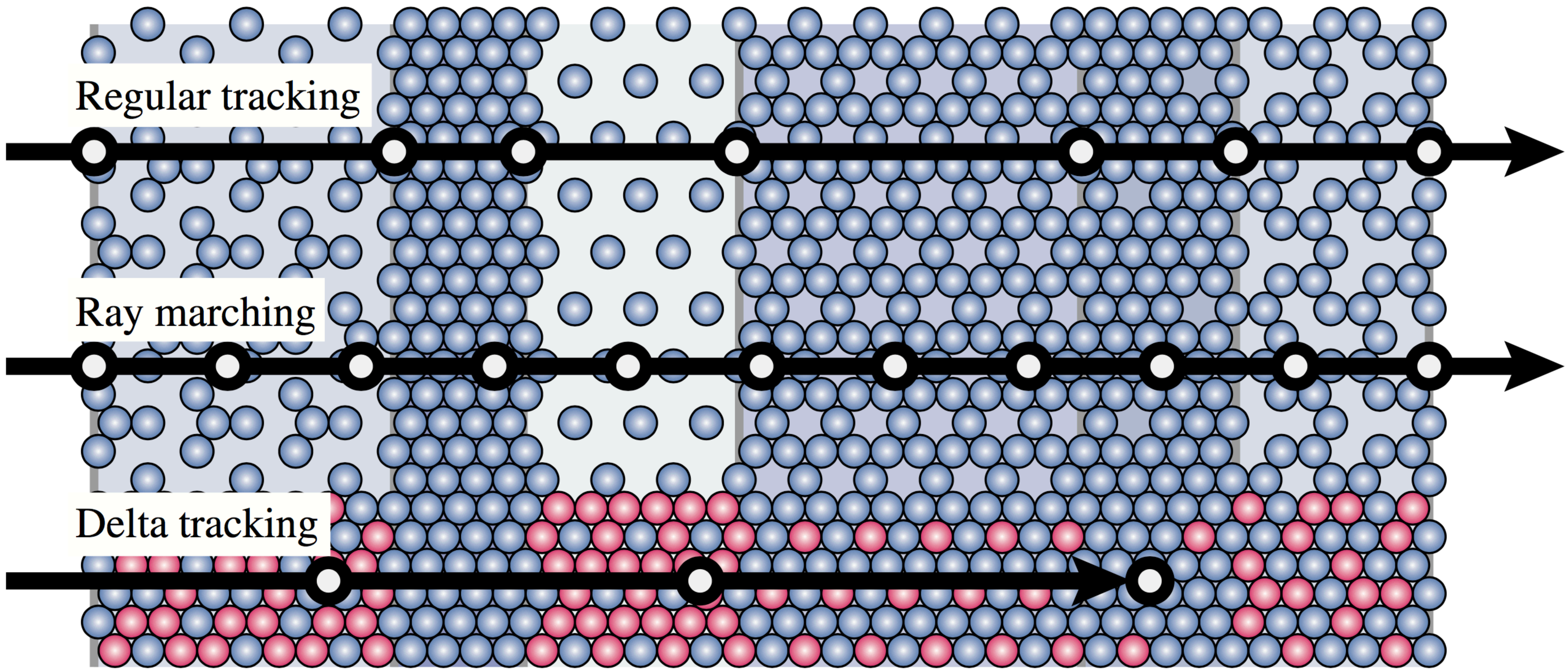
Want to integrate $L(\mathrm{p},\omega) = \int_0^\infty T(\mathrm{p}')\, S(\mathrm{p}',\omega)\, \mathrm{d}s'$

Would like to draw samples from $\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p}+s''\omega)\, \mathrm{d}s''}$ **for arbitrary** σ_t**.**

Then $T(\mathrm{p}')/p(s')$ **is a constant.**

Idea: introduce fictitious "null scattering" particles that make the density uniform.

Delta Tracking



[Novák et al. 2014]

Delta Tracking

Compute majorant $\hat{\sigma}_t \geq \sigma_t(\mathrm{p})\ \forall \mathrm{p}$

While (still in the volume)

Sample: $s' = -\frac{1}{\hat{\sigma}_t} \log(1 - \xi)$

Accept with probability: $\frac{\sigma_t(\mathrm{p} + s'\omega)}{\hat{\sigma}_t}$

Otherwise, restart at s'

Samples exactly according to $\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p}+s''\omega)\,\mathrm{d}s''}$ **!**

Null-Collision Volume LTE

Before:

$$\frac{\partial L(\mathrm{p},\omega)}{\partial s} = -\sigma_t L(\mathrm{p},\omega) + S(\mathrm{p},\omega)$$

Using:

$$-\sigma_n(\mathrm{p})L(\mathrm{p},\omega) + \sigma_n(\mathrm{p}) \int \delta(\omega-\omega')L(\mathrm{p},\omega')\,\mathrm{d}\omega' = 0$$

Now:

$$\frac{\partial L(\mathrm{p},\omega)}{\partial s} = -\,\sigma_t(\mathrm{p})L(\mathrm{p},\omega) + -\sigma_n(\mathrm{p})L(\mathrm{p},\omega) + S(\mathrm{p},\omega) + \sigma_n(\mathrm{p}) \int \delta(\omega-\omega')L(\mathrm{p},\omega')\,\mathrm{d}\omega'$$

Null-Collision Volume LTE

$$L(\mathrm{p}, \omega) = \int_0^\infty T_n(\mathrm{p}')\left(S(\mathrm{p}', \omega) + \sigma_n(\mathrm{p}')L(\mathrm{p}', \omega)\right)\mathrm{d}s'$$

Attenuation: absorption, scattering, null-scattering

$$\mathrm{e}^{-\int_0^{t'} \sigma_t(p'')+\sigma_n(p'')\,\mathrm{d}t''}$$

Null-scattering compensation

(Same old source term)

Null-collision Volume LTE

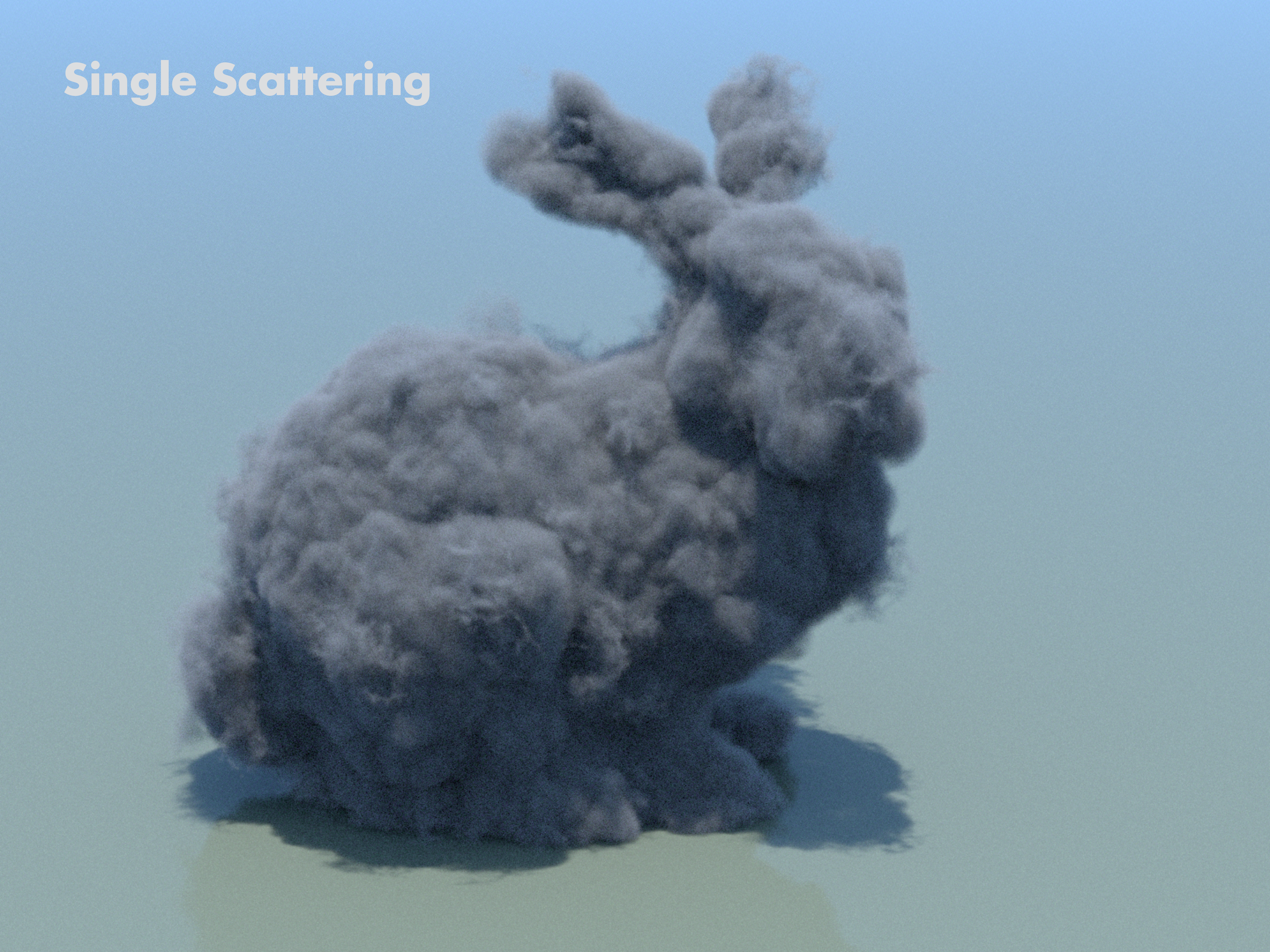
$$L(\mathrm{p},\omega) = \int_0^\infty T_n(\mathrm{p}')\,(S(\mathrm{p}',\omega) + \sigma_n(\mathrm{p}')L(\mathrm{p}',\omega))\mathrm{d}s'$$

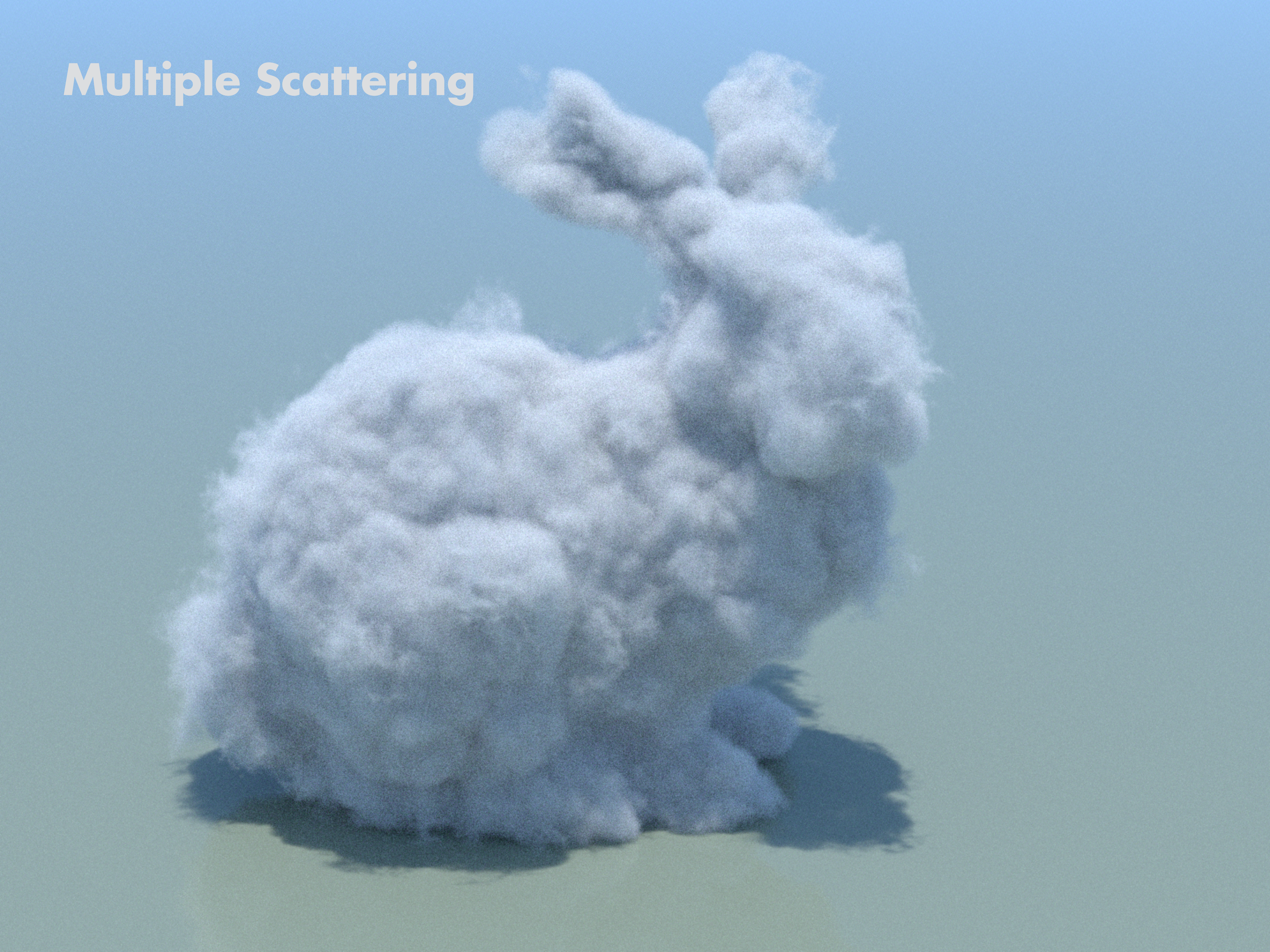
Monte Carlo integration:

Define $\sigma_n(\mathrm{p}) = \hat{\sigma}_t - \sigma_t(\mathrm{p})$

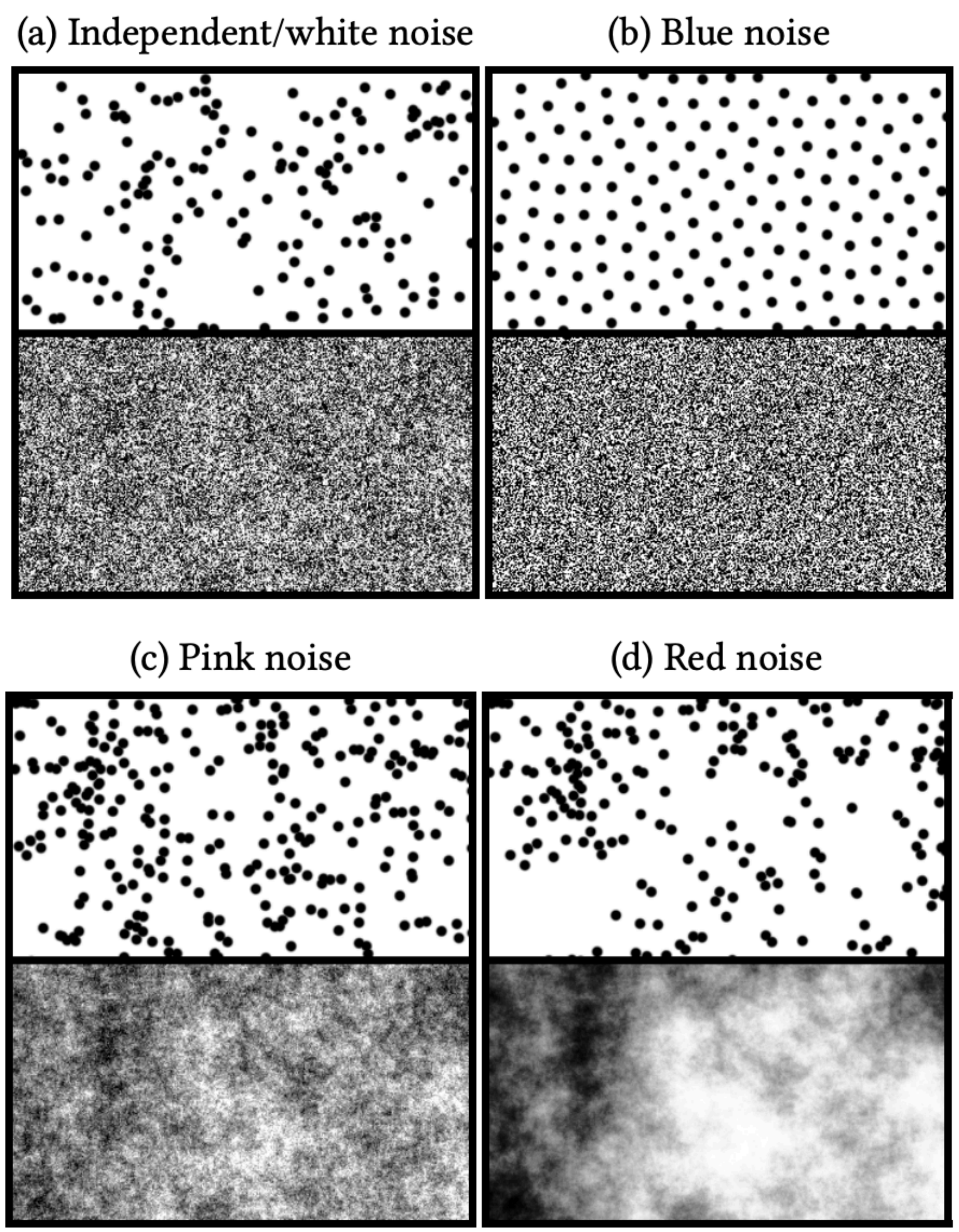
Now $T_n(\mathrm{p}') = \mathrm{e}^{-\int_0^{t'} \hat{\sigma}_t \, \mathrm{d}t''}$

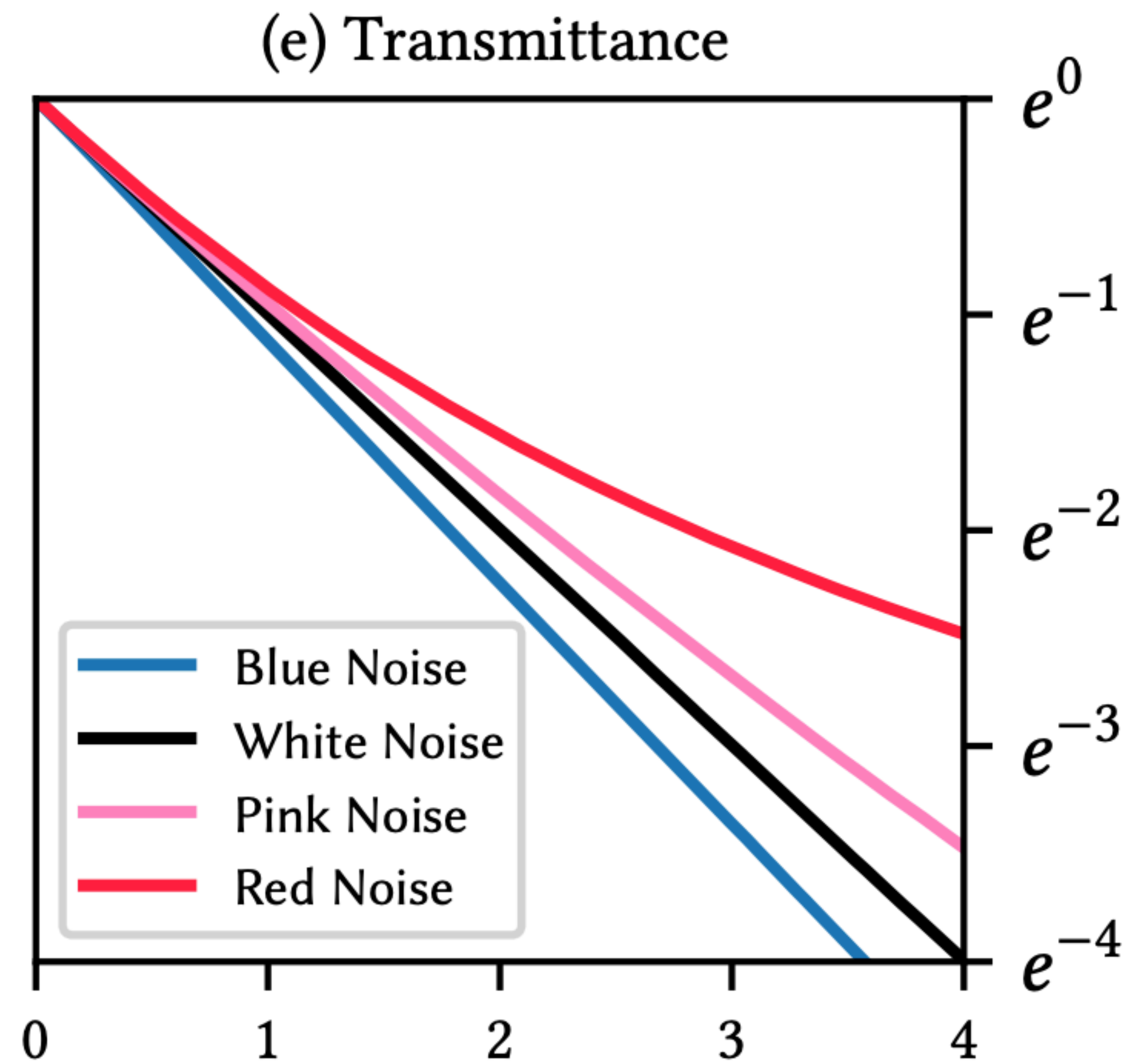
Easy to sample from & T_n is analytic!


Single Scattering


Multiple Scattering

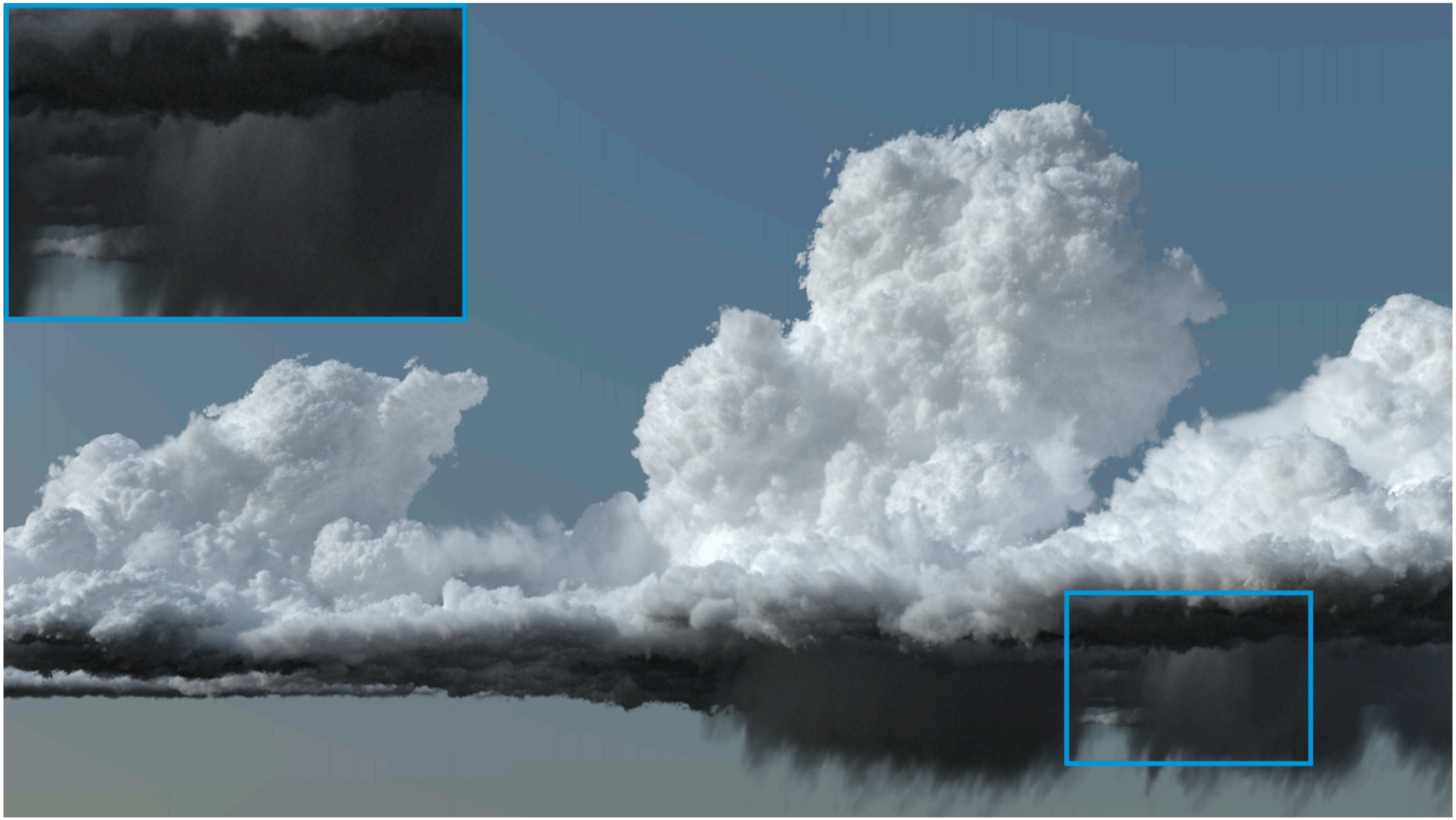
Non-Exponential Media





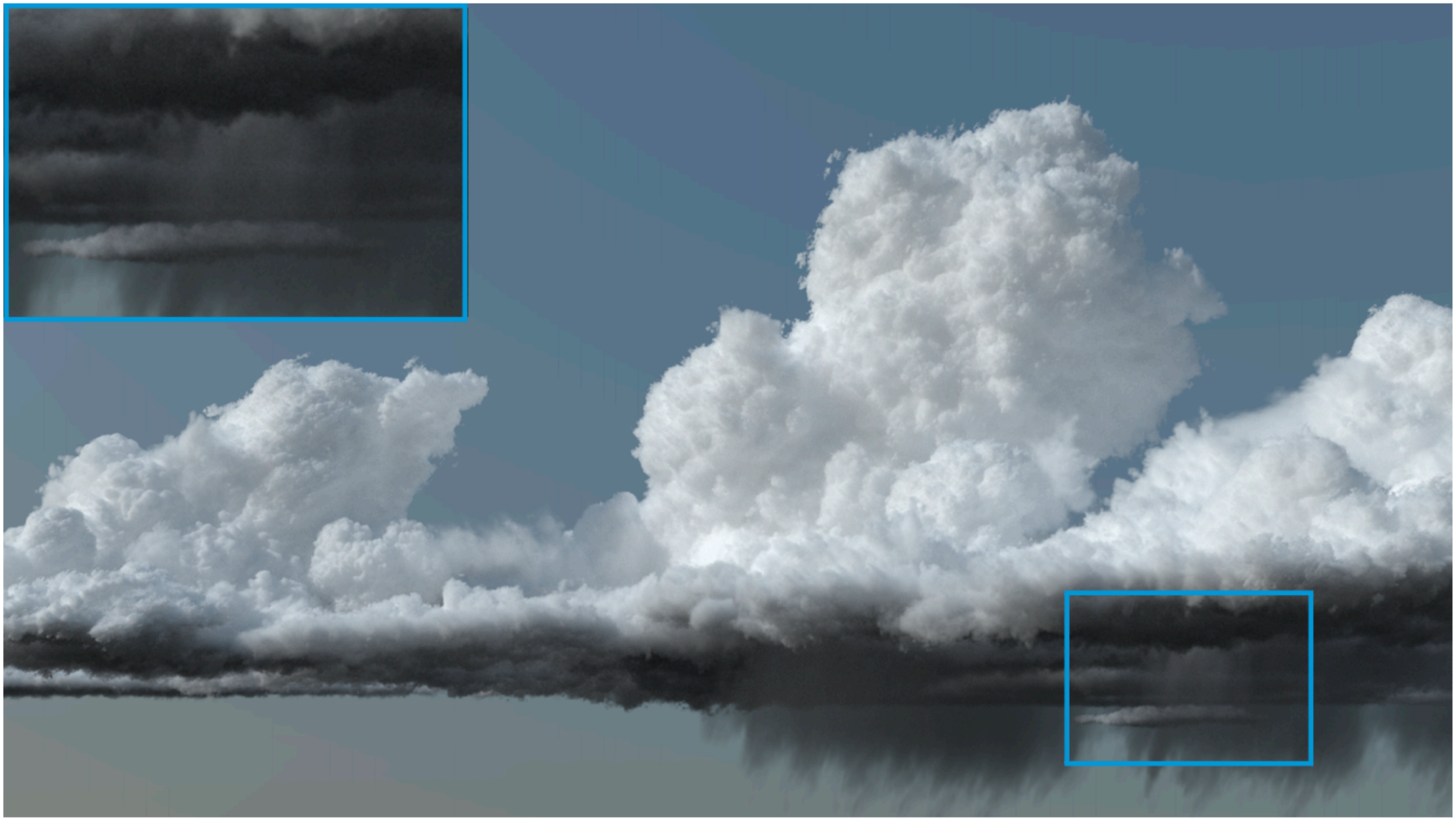
[Bitterli et al. 2018]

Exponential



[Bitterli et al. 2018]

Non-Exponential



[Bitterli et al. 2018]