Lecture 4: Efficiently Scheduling Image Processing Pipelines (in Halide)

Visual Computing Systems
Stanford CS348K, Spring 2024
Today’s themes

- Techniques for efficiently mapping image processing applications (like those we’ve discussed in the past two classes) to multi-core CPUs and GPUs

- The design of programming abstractions that facilitate efficient image processing applications
Reminder: key aspect in the design of any system
Choosing the “right” representations for the job

- Good representations are productive to use:
  - Embody the natural way of thinking about a problem

- Good representations enable the system to provide useful services:
  - Validating/providing certain guarantees (correctness, resource bounds, conversion of quantities, type checking)
  - Performance optimizations (parallelization, vectorization, use of specialized hardware)
  - Implementations of common, difficult-to-implement functionality (complex array indexing code, texture mapping in 3D graphics, auto-differentiation, etc.)
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1.f/9, 1.f/9, 1.f/9, 1.f/9, 1.f/9, 1.f/9, 1.f/9, 1.f/9, 1.f/9};

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
        output[j*WIDTH + i] = tmp;
    }
}
Consider a new task: sharpening an image

Input

Output

Question: imagine you were asked to design a system for executing sharpen as efficiently as possible on a variety of parallel processors (CPUs, GPUs, etc.)

What would the interface to your system be?
Four different representations of sharpen

1. Image input;
   Image output = sharpen(input);

2. \[
   F = \begin{bmatrix}
   0 & -1 & 0 \\
   -1 & 5 & -1 \\
   0 & -1 & 0 
   \end{bmatrix}
   \]

3. Image input;
   Image output;
   output[i][j] = F[0][0] * input[i-1][j-1] +
                  F[0][1] * input[i-1][j]  +
                  F[0][2] * input[i-1][j+1] +
                  F[1][0] * input[i][j-1]  +
                  F[1][1] * input[i][j]    +
                  ...  

4. float input[(WIDTH+2) * (HEIGHT+2)];
   float output[WIDTH * HEIGHT];

   float weights[] = {0., -1., 0.,
                      -1., 5, -1.,
                      0., -1., 0.};

   for (int j=0; j<HEIGHT; j++) {
       for (int i=0; i<WIDTH; i++) {
           float tmp = 0.f;
           for (int jj=0; jj<3; jj++)
               for (int ii=0; ii<3; ii++)
                   tmp += input[(j+jj)*(WIDTH+2) + (i+ii)]
                         * weights[jj*3 + ii];
           output[j*WIDTH + i] = tmp;
       }
   }
Diversity of tasks: image processing tasks from previous lectures

**Sobel Edge Detection**

\[
G_x = \begin{bmatrix}
-1 & 0 & 1 \\
-2 & 0 & 2 \\
-1 & 0 & 1
\end{bmatrix} \ast I
\]

\[
G_y = \begin{bmatrix}
-1 & -2 & -1 \\
0 & 0 & 0 \\
1 & 2 & 1
\end{bmatrix} \ast I
\]

\[
G = \sqrt{G_x^2 + G_y^2}
\]

**3x3 Gaussian blur**

\[
F = \begin{bmatrix}
0.075 & 0.124 & 0.075 \\
0.124 & 0.204 & 0.124 \\
0.075 & 0.124 & 0.075
\end{bmatrix}
\]

**2x2 downsample (via averaging)**

\[
output[x][y] = \frac{\text{input}[2x][2y] + \text{input}[2x+1][2y] + \text{input}[2x][2y+1] + \text{input}[2x+1][2y+1]}{4.0};
\]

**Gamma Correction**

\[
output[x][y] = \text{pow}(\text{input}[x][y], 0.5f);
\]

**LUT-based correction**

\[
output[x][y] = \text{lookup_table}[(\text{input}[x][y])];
\]

**Histogram**

\[
\text{bin}[\text{input}[x][y]]++;
\]

**Local Pixel Clamp**

```c
float f(image input) {
    float min_value = min( min(input[x-1][y], input[x+1][y]),
                          min(input[x][y-1], input[x][y+1]) );
    float max_value = max( max(input[x-1][y], input[x+1][y]),
                          max(input[x][y-1], input[x][y+1]) );
    output[x][y] = clamp(min_value, max_value, input[x][y]);
    output[x][y] = f(input);
}
```
Image processing workload characteristics

- Structure: sequences (more precisely: DAGs) of operations on images
- Natural to think about algorithms in terms of their local, per-pixel behavior: e.g., output at pixel $(x,y)$ is function of input image pixels in the neighborhood around $(x,y)$
- Common case: computing value of output pixel $(x,y)$ depends on access to a bounded local “window” of input image pixels around $(x,y)$… (e.g. convolution, but also true of median filter, bilateral filter, etc.)
- Some algorithms require data-dependent data access (e.g., data-dependent access to lookup tables)
- Upsampling/downsampling (e.g., to create image pyramids)
- Computations that reduce information across many pixels (e.g., computing maximum brightness pixel in an image, building a histogram)
- FFTs on small patches of an image (to convert from pixel domain to frequency domain)
Halide language for image processing

[Ragan-Kelley / Adams 2012]
Halide goals

- Expressive: facilitate intuitive expression of a broad class of image processing applications
  - e.g., all the components of a modern camera RAW pipeline

- High performance: want to generate code that efficiently utilizes the multi-core and SIMD processing resources of modern CPUs and GPUs, and is memory bandwidth efficient
Halide used in practice

- Halide used to implement camera processing pipelines on Google phones
  - HDR+, aspects of portrait mode, etc...
- Industry usage at Instagram, Adobe, etc.
C++ code for a 3x3 “box blur”

```c
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1./9, 1./9, 1./9,
                  1./9, 1./9, 1./9,
                  1./9, 1./9, 1./9};

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
        output[j*WIDTH + i] = tmp;
    }
}
```

Total work per output image = $9 \times WIDTH \times HEIGHT$

For NxN filter: $N^2 \times WIDTH \times HEIGHT$

For now: ignore boundary pixels and assume output image is smaller than input (makes convolution loop bounds much simpler to write)
3x3 box blur in Halide

```cpp
Var x, y;
Func blurx, out;
Image<uint8_t> in = load_image("myimage.jpg");

// expression for computing convolution result for one output pixel
out(x,y) = 1/9.f * (in(x-1,y-1) + in(x,y-1) + in(x+1,y-1) +
                  in(x-1,y)   + in(x,y)   + in(x+1,y) +
                  in(x-1,y+1) + in(x,y+1) + in(x+1,y+1));

// execute pipeline on domain of size 1024x1024
Image<uint8_t> result = out.realize(1024, 1024);
```

Total work per output image =
9 x WIDTH x HEIGHT

For NxN filter: N\(^2\) x WIDTH x HEIGHT

Value of blurx at coordinate (x,y) is given by expression
that accesses three values of in

Halide function: an infinite (but discrete) set of values defined on N-D domain
Halide expression: a side-effect free expression that describes how to compute a
function's value at a point in its domain in terms of the values of other functions.
An example application: two-pass blur

A 2D separable filter (such as a box filter) can be evaluated via two 1D filtering operations.

Note: I’ve exaggerated the blur for illustration (the end result is actually a 30x30 blur, not 3x3)
Two-pass 3x3 blur in C++

```cpp
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * (HEIGHT+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1.f/3, 1.f/3, 1.f/3};

for (int j=0; j<(HEIGHT+2); j++)
for (int i=0; i<WIDTH; i++) {
    float tmp = 0.f;
    for (int ii=0; ii<3; ii++)
        tmp += input[j*(WIDTH+2) + i+ii] * weights[ii];
    tmp_buf[j*WIDTH + i] = tmp;
}

for (int j=0; j<HEIGHT; j++)
for (int i=0; i<WIDTH; i++) {
    float tmp = 0.f;
    for (int jj=0; jj<3; jj++)
        tmp += tmp_buf[(j+jj)*WIDTH + i] * weights[jj];
    output[j*WIDTH + i] = tmp;
}
```

Total work per image = 6 x WIDTH x HEIGHT
For NxN filter: 2N x WIDTH x HEIGHT
WIDTH x HEIGHT extra storage
2x lower arithmetic intensity than 2D blur. Why?
Two pass blur in Halide

Simple domain-specific language embedded in C++ for describing sequences of image processing operations

```cpp
Var x, y;
Func blurx, out;
Halide::Buffer<uint8_t> in = load_image("myimage.jpg");

// perform 3x3 box blur in two-passes
blurx(x,y) = 1/3.f * (in(x-1,y) + in(x,y) + in(x+1,y));
out(x,y) = 1/3.f * (blurx(x,y-1) + blurx(x,y) + blurx(x,y+1));

// execute pipeline on domain of size 800x600
Halide::Buffer<uint8_t> result = out.realize(800, 600);
```

Halide function: an infinite (but discrete) set of values defined on N-D domain
Halide expression: a side-effect free expression that describes how to compute a function's value at a point in its domain in terms of the values of other functions.

Functions map integer coordinates to values (e.g., colors of corresponding pixels)
Value of blurx at coordinate (x,y) is given by expression that accesses three values of in
A more complicated Halide program

Simple domain-specific language embedded in C++ for describing sequences of image processing operations

```cpp
Var x, y;
Func blurx, blury, bright, out;
Halide::Buffer<uint8_t> in = load_image("myimage.jpg");
Halide::Buffer<uint8_t> lookup = load_image("s_curve.jpg"); // 255-pixel 1D image

// perform 3x3 box blur in two-passes
blurx(x,y) = 1/3.f * (in(x-1,y) + in(x,y) + in(x+1,y));
blury(x,y) = 1/3.f * (blurx(x,y-1) + blurx(x,y) + blurx(x,y+1));

// brighten blurred result by 25%, then clamp
bright(x,y) = min(blury(x,y) * 1.25f, 255);

// access lookup table to contrast enhance
out(x,y) = lookup(bright(x,y));

// execute pipeline to materialize values of out in range (0:800,0:600)
Halide::Buffer<uint8_t> result = out.realize(800, 600);
```

Functions map integer coordinates to values (e.g., colors of corresponding pixels)

Value of \( \text{blurx} \) at coordinate \((x,y)\) is given by expression accessing three values of \( \text{in} \)

Functions map integer coordinates to values (e.g., colors of corresponding pixels)

Value of \( \text{blurx} \) at coordinate \((x,y)\) is given by expression accessing three values of \( \text{in} \)
Image processing as a DAG

Simple domain-specific language embedded in C++ for describing sequences of image processing operations.

myimage.jpg

in

blurx

blury

brighten

out

s_curve.jpg

lookup
Image processing pipelines feature complex DAGs of functions

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Number of Halide functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-pass blur</td>
<td>2</td>
</tr>
<tr>
<td>Unsharp mask</td>
<td>9</td>
</tr>
<tr>
<td>Harris Corner detection</td>
<td>13</td>
</tr>
<tr>
<td>Camera RAW processing</td>
<td>30</td>
</tr>
<tr>
<td>Non-local means denoising</td>
<td>13</td>
</tr>
<tr>
<td>Max-brightness filter</td>
<td>9</td>
</tr>
<tr>
<td>Multi-scale interpolation</td>
<td>52</td>
</tr>
<tr>
<td>Local-laplacian filter</td>
<td>103</td>
</tr>
<tr>
<td>Synthetic depth-of-field</td>
<td>74</td>
</tr>
<tr>
<td>Bilateral filter</td>
<td>8</td>
</tr>
<tr>
<td>Histogram equalization</td>
<td>7</td>
</tr>
<tr>
<td>VGG-16 deep network eval</td>
<td>64</td>
</tr>
</tbody>
</table>

Real-world production applications may feature hundreds to thousands of functions!

Google HDR+ pipeline: over 2000 Halide functions.
Key aspects of representation

- **Intuitive expression:**
  - Adopts local “point wise” view of expressing algorithms
  - Halide language is declarative. It does not define order of iteration over elements in a domain, or even what values in domain are stored!
  - It only defines what operations are needed to compute these values.
  - Iteration over domain points is implicit (no explicit loops)

```cpp
Var x, y;
Func blurx, out;
Halide::Buffer<uint8_t> in = load_image("myimage.jpg");

// perform 3x3 box blur in two-passes
blurx(x, y) = 1/3.f * (in(x-1, y) + in(x, y) + in(x+1, y));
out(x, y) = 1/3.f * (blurx(x, y-1) + blurx(x, y) + blurx(x, y+1));

// execute pipeline on domain of size 800x600
Halide::Buffer<uint8_t> result = out.realize(800, 600);
```
Efficiently executing Halide programs
Review (I hope): what is a data cache?

- A cache is a hardware implementation detail that does not impact the output of a program, only its performance.
- Cache is on-chip storage that maintains a copy of a subset of the values in memory.
- If an address is stored “in the cache” the processor can load/store to this address more quickly than if the data resides only in DRAM.
- Caches operate at the granularity of “cache lines”.

In the figure, the cache:
- Has a capacity of 2 lines
- Each line holds 4 bytes of data

```
<table>
<thead>
<tr>
<th>Address</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x0</td>
<td>16</td>
</tr>
<tr>
<td>0x1</td>
<td>255</td>
</tr>
<tr>
<td>0x2</td>
<td>14</td>
</tr>
<tr>
<td>0x3</td>
<td>0</td>
</tr>
<tr>
<td>0x4</td>
<td>0</td>
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<tr>
<td>0x5</td>
<td>0</td>
</tr>
<tr>
<td>0x6</td>
<td>6</td>
</tr>
<tr>
<td>0x7</td>
<td>0</td>
</tr>
<tr>
<td>0x8</td>
<td>32</td>
</tr>
<tr>
<td>0x9</td>
<td>48</td>
</tr>
<tr>
<td>0xA</td>
<td>255</td>
</tr>
<tr>
<td>0xB</td>
<td>255</td>
</tr>
<tr>
<td>0xC</td>
<td>255</td>
</tr>
<tr>
<td>0xD</td>
<td>0</td>
</tr>
<tr>
<td>0xE</td>
<td>0</td>
</tr>
<tr>
<td>0xF</td>
<td>0</td>
</tr>
<tr>
<td>0x10</td>
<td>128</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>0x1F</td>
<td>0</td>
</tr>
</tbody>
</table>
```
Which program performs better?

**Program 1**

```c
void add(int n, float* A, float* B, float* C) {
    for (int i=0; i<n; i++)
        C[i] = A[i] + B[i];
}

void mul(int n, float* A, float* B, float* C) {
    for (int i=0; i<n; i++)
        C[i] = A[i] * B[i];
}

// assume arrays are allocated here
// compute E = D + ((A + B) * C)
add(n, A, B, tmp1);
mul(n, tmp1, C, tmp2);
add(n, tmp2, D, E);
```

**Program 2**

```c
void fused(int n, float* A, float* B, float* C, float* D, float* E) {
    for (int i=0; i<n; i++)
        E[i] = D[i] + (A[i] + B[i]) * C[i];
}

// compute E = D + (A + B) * C
fused(n, A, B, C, D, E);
```

Which code structuring style would you rather write?
# Two-pass 3x3 blur in C++

```cpp
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * (HEIGHT+2)];
float output[WIDTH * HEIGHT];
float weights[] = {1.f/3, 1.f/3, 1.f/3};

for (int j=0; j<(HEIGHT+2); j++)
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int ii=0; ii<3; ii++)
            tmp += input[j*(WIDTH+2) + i+ii] * weights[ii];
        tmp_buf[j*WIDTH + i] = tmp;
    }

for (int j=0; j<HEIGHT; j++)
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            tmp += tmp_buf[(j+jj)*WIDTH + i] * weights[jj];
        output[j*WIDTH + i] = tmp;
    }
```

Total work per image $= 6 \times WIDTH \times HEIGHT$

For $N \times N$ filter: $2N \times WIDTH \times HEIGHT$

$WIDTH \times HEIGHT$ extra storage

2x lower arithmetic intensity than 2D blur. Why?

---

**1D horizontal blur**

- **Input:** $(W+2) \times (H+2)$
- **tmp buf:** $W \times (H+2)$
- **Output:** $W \times H$

---

**1D vertical blur**

- **Input:** $(W+2) \times (H+2)$
- **tmp buf:** $W \times (H+2)$
- **Output:** $W \times H$
Two-pass image blur: locality

Intrinsic bandwidth requirements of blur algorithm:
Application must read each element of input image and must write each element of output image.

Data from input reused three times. (immediately reused in next two i-loop iterations after first load, never loaded again.)
- Perfect cache behavior: never load required data more than once
- Perfect use of cache lines (don’t load unnecessary data into cache)

Data from tmp_buf reused three times (but three rows of image data are accessed in between)
- Never load required data more than once… if cache has capacity for three rows of image
- Perfect use of cache lines (don’t load unnecessary data into cache)

Two pass: loads/stores to tmp_buf are overhead (this memory traffic is an artifact of the two-pass implementation: it is not intrinsic to computation being performed)

int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * (HEIGHT+2)];
float output[WIDTH * HEIGHT];
float weights[] = {1.f/3, 1.f/3, 1.f/3};

for (int j=0; j<(HEIGHT+2); j++)
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int ii=0; ii<3; ii++)
            tmp += input[j*(WIDTH+2) + i+ii] * weights[ii];
        tmp_buf[j*WIDTH + i] = tmp;
    }

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            tmp += tmp_buf[(j+jj)*WIDTH + i] * weights[jj];
        output[j*WIDTH + i] = tmp;
    }
}
Two-pass image blur, “chunked” (version 1)

```c
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * 3];
float output[WIDTH * HEIGHT];

float weights[] = {1.f/3, 1.f/3, 1.f/3};

for (int j=0; j<HEIGHT; j++) {
    for (int j2=0; j2<3; j2++)
        for (int i=0; i<WIDTH; i++) {
            float tmp = 0.f;
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+j2)*(WIDTH+2) + i+ii] * weights[ii];
            tmp_buf[j2*WIDTH + i] = tmp;
        }
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            tmp += tmp_buf[jj*WIDTH + i] * weights[jj];
        output[j*WIDTH + i] = tmp;
    }
}
```

Only 3 rows of intermediate buffer need to be allocated

Produce 3 rows of tmp_buf (only what’s needed for one row of output)

Combine them together to get one row of output

Total work per row of output:
- step 1: 3 x 3 x WIDTH work
- step 2: 3 x WIDTH work
Total work per image = 12 x WIDTH x HEIGHT

Loads from tmp_buffer are cached (assuming tmp_buffer fits in cache)
Two-pass image blur, “chunked” (version 2)

```c
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * (CHUNK_SIZE+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1.f/3, 1.f/3, 1.f/3};
for (int j=0; j<HEIGHT; j+CHUNK_SIZE) {
    for (int j2=0; j2<CHUNK_SIZE+2; j2++)
        for (int i=0; i<WIDTH; i++) {
            float tmp = 0.f;
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+j2)*(WIDTH+2) + i+ii] * weights[ii];
            tmp_buf[j2*WIDTH + i] = tmp;
        }
    for (int j2=0; j2<CHUNK_SIZE; j2++)
        for (int i=0; i<WIDTH; i++) {
            float tmp = 0.f;
            for (int jj=0; jj<3; jj++)
                tmp += tmp_buf[(j2+jj)*WIDTH + i] * weights[jj];
            output[(j+j2)*WIDTH + i] = tmp;
        }
}
```

Produce enough rows of tmp_buf to produce a CHUNK_SIZE number of rows of output.

Sized so entire buffer fits in cache (capture all producer-consumer locality)

Produce CHUNK_SIZE rows of output

Total work per chuck of output: (assume CHUNK_SIZE = 16)
- Step 1: 18 x 3 x WIDTH work
- Step 2: 16 x 3 x WIDTH work

Total work per image: \( \frac{34}{16} \times 3 \times WIDTH \times HEIGHT \) = 6.4 x WIDTH x HEIGHT

Tends to ideal value of 6 x WIDTH x HEIGHT as CHUNK_SIZE is increased!
Still not done

- We have not parallelized loops for multi-core execution
- We have not used SIMD instructions to execute loops bodies
- Other basic optimizations: loop unrolling, etc...
Optimized implementation of 3x3 box blur in x86 SSE intrinsics

Good: ~10x faster on a quad-core CPU than my original two-pass code
Bad: specific to SSE (not AVX2), CPU-code only, hard to tell what is going on at all!

```c
void fast_blur(const Image *in, Image *blurred) {
    _m128i one_third = _mm_set1_epi16(21846);
    #pragma omp parallel for
    for (int yTile = 0; yTile < in.height(); yTile += 32) {
        _m128i a, b, c, sum, avg;
        _m128i tmp[(256/8)*(32+2)];
        for (int xTile = 0; xTile < in.width(); xTile += 256) {
            _m128i *tmpPtr = tmp;
            for (int y = -1; y < 32+1; y++) {
                const uint16_t *inPtr = &(in(xTile, yTile+y));
                for (int x = 0; x < 256; x += 8) {
                    a = _mm_loadu_si128(_m128i)(inPtr-1);
                    b = _mm_loadu_si128(_m128i)(inPtr);
                    c = _mm_load_si128(_m128i)(inPtr);
                    sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
                    avg = _mm_mulhi_epi16(sum, one_third);
                    _mm_store_si128(tmpPtr++, avg);
                    inPtr += 8;
                }
                tmpPtr = tmp;
            }
            _m128i *outPtr = (_m128i *)(blurred(xTile, yTile+y));
            for (int x = 0; x < 256; x += 8) {
                a = _mm_load_si128(tmpPtr+(2*256)/8);
                b = _mm_load_si128(tmpPtr+256/8);
                c = _mm_load_si128(tmpPtr++);
                sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
                avg = _mm_mulhi_epi16(sum, one_third);
                _mm_store_si128(outPtr++, avg);
            }
        }
    }
}
```
One (serial) implementation of Halide

Func blurx, out;
Var x, y, xi, yi;
Halide::Buffer<uint8_t> in = load_image(“myimage.jpg”);

// the “algorithm description” (declaration of what to do)
blurx(x,y) = (in(x-1, y) + in(x,y) + in(x+1,y)) / 3.0f;
out(x,y) = (blurx(x,y-1) + blurx(x,y) + blurx(x,y+1)) / 3.0f;

// execute pipeline on domain of size 1024x1024
Halide::Buffer<uint8_t> result = out.realize(1024, 1024);

Equivalent “C-style” loop nest:

allocate in(1024+2, 1024+2); // (width,height)… initialize from image
allocate blurx(1024,1024+2); // (width,height)
allocate out(1024,1024); // (width,height)

for y=0 to 1024:
  for x=0 to 1024+2:
    blurx(x,y) = … compute from in

for y=0 to 1024:
  for x=0 to 1024:
    out(x,y) = … compute from blurx
Key aspect in the design of any system: Choosing the “right” representations for the job

- Good representations are productive to use:
  - Embody the natural way of thinking about a problem

- Good representations enable the system to provide the application useful services:
  - Validating/providing certain guarantees (correctness, resource bounds, type checking)
  - Performance (parallelization, vectorization, use of specialized hardware)

Now the job is not expressing an image processing computation, but generating an efficient implementation of a specific Halide program.
A second set of representations for “scheduling”

```cpp
Func blurx, out;
Var x, y, xi, yi;
Halide::Buffer<uint8_t> in = load_image("myimage.jpg");

// the “algorithm description” (declaration of what to do)
blurx(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y)) / 3.0f;
out(x, y) = (blurx(x, y-1) + blurx(x, y) + blurx(x, y+1)) / 3.0f;

// “the schedule” (how to do it)
out.tile(x, y, xi, yi, 256, 32).vectorize(xi, 8).parallel(y);
blurx.compute_at(x).vectorize(x, 8);

// execute pipeline on domain of size 1024x1024
Halide::Buffer<uint8_t> result = out.realize(1024, 1024);
```

Scheduling primitives allow the programmer to specify a high-level “sketch” of how to schedule the algorithm onto a parallel machine, but leave the details of emitting the low-level platform-specific code to the Halide compiler.
# Primitives for orderation order

Specify both order and how to parallelize (multi-thread, vectorize via SIMD instr)

2D blocked iteration order

(In diagram, numbers indicate sequential order of processing within a thread)
Ordering Halide loop nests

Halide algorithm:

\[
\begin{align*}
\text{blurx}(x,y) &= (\text{in}(x-1, y) + \text{in}(x,y) + \text{in}(x+1,y)) / 3.0f; \\
\text{out}(x,y) &= (\text{blurx}(x,y-1) + \text{blurx}(x,y) + \text{blurx}(x,y+1)) / 3.0f;
\end{align*}
\]

Halide::Buffer<uint8_t> result = out.realize(1024, 1024);

Loop nest diagram of implementation:

One possible implementation:

allocate in(1024+2, 1024+2); // (width, height) ... initialize from image
allocate blurx(1024,1024+2);  // (width, height)
allocate out(1024,1024);       // (width, height)

for y=0 to 1024:
    for x=0 to 1024+2:
        blurx(x,y) = ... compute from in

for y=0 to 1024:
    for x=0 to 1024:
        out(x,y) = ... compute from blurx

Loops for computing values of blurx
Loops for computing values of out
Ordering Halide loop nests

Halide algorithm:

\[
\begin{align*}
\text{blurx}(x,y) &= (\text{in}(x-1,y) + \text{in}(x,y) + \text{in}(x+1,y)) / 3.0f; \\
\text{out}(x,y) &= (\text{blurx}(x,y-1) + \text{blurx}(x,y) + \text{blurx}(x,y+1)) / 3.0f;
\end{align*}
\]

Halide::Buffer<uint8_t> result = out.realize(1024, 1024);

Another possible implementation:

allocate in(1024+2, 1024+2); // (width,height)... initialize from image
allocate out(1024,1024); // (width,height)

for y=0 to num_tiles_y:
    for x=0 to num_tiles_x:
        allocate blurx(258, 34)
        for yi=0 to 32+2:
            for xi=0 to 256+2:
                tmp_blurx(xi,yi) = // compute blurx from in
        for yi=0 to 32:
            for xi=0 to 256:
                idx_x = x*256+xi;
                idx_y = y*32+yi
                out(idx_x, idx_y) = ...

Loops for computing values of out (loops over tiles)

Inner loops for computing values of out (loops over elements)

Outer loops for computing values of blurx

Only allocate a tile of blurx
Primitives for how to interleave producer/consumer processing

```
blurx(x,y) = (in(x-1, y) + in(x,y) + in(x+1,y)) / 3.0f;
out(x,y)   = (blurx(x,y-1) + blurx(x,y) + blurx(x,y+1)) / 3.0f;
```

```
out.tile(x, y, xi, yi, 256, 32);
```

```
blurx.compute_root();  // Do not compute blurx within out’s loop nest.
Compute all of blurx, then all of out
```

```
allocate blurx(1024,1024+2)
for y=0 to HEIGHT:
   for x=0 to WIDTH:
      blurx(x,y) = ...  // access values from buffer “in”

for y=0 to num_tiles_y:
   for x=0 to num_tiles_x:
      for yi=0 to 32:
         for xi=0 to 256:
            idx_x = x*256+xi;
            idx_y = y*32+yi
            out(idx_x, idx_y) = ...  // access values from buffer blurx
```

all of blurx is computed here

values of blurx consumed here
blurx(x,y) = (in(x−1, y) + in(x,y) + in(x+1,y)) / 3.0f;
out(x,y) = (blurx(x,y−1) + blurx(x,y) + blurx(x,y+1)) / 3.0f;

out.tile(x, y, xi, yi, 256, 32);

allocate blurx(1,3)

// compute 3 elements of blurx needed for out(idx_x, idx_y) here
for blurx_y=0 to 3:
    blurx(0, blurx_y) = … // access values from buffer “in”

out(idx_x, idx_y) = … // access values from buffer blurx

Compute necessary elements of blurx within out’s xi loop nest

Note: Halide compiler performs analysis that the output of each iteration of the xi loop required 3 elements of blurx
Primitives for how to interleave producer/consumer processing

\[ \text{blur}(x,y) = \frac{(\text{in}(x-1, y) + \text{in}(x,y) + \text{in}(x+1,y))}{3.0}; \]
\[ \text{out}(x,y) = \frac{(\text{blur}(x,y-1) + \text{blur}(x,y) + \text{blur}(x,y+1))}{3.0}; \]

\[ \text{out}.tile(x, y, xi, yi, 256, 32); \]

\[ \text{blur}.compute\_at(out, x); \]

\text{Compute necessary elements of blur within out's x loop nest (all necessary elements for one tile of out)}

\begin{verbatim}
for y=0 to num_tiles_y:
  for x=0 to num_tiles_x:
    allocate blur(256,34)
    for yi=0 to 32+2:
      for xi=0 to 256:
        blur(xi, yi) = // compute blur from in

    for yi=0 to 32:
      for xi=0 to 256:
        idx_x = x*256+xi;
        idx_y = y*32+yi
        out(idx_x, idx_y) = ... // access values from buffer blur

    tile of blur is computed here

    tile of blur is consumed here
\end{verbatim}
An interesting Halide schedule

```cpp
blurx(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y)) / 3.0f;
out(x, y) = (blurx(x, y-1) + blurx(x, y) + blurx(x, y+1)) / 3.0f;
out.tile(x, y, xi, yi, 256, 32);
```

```cpp
blurx.store_at(out, x)
blurx.compute_at(out, xi);
```

```cpp
for y=0 to num_tiles_y:
  for x=0 to num_tiles_x:
    allocate blurx(256,34)
    for yi=0 to 32:
      for xi=0 to 256:
        idx_x = x*256+xi;
        idx_y = y*32+yi;
        // compute 3 elements of blurx needed for out(idx_x, idx_y) here
        for blurx_y=0 to 3:
          blurx(xi, yi + blurx_y) = … // access values from buffer “in”
        out(idx_x, idx_y) = … // access values from buffer blurx
```

This recomputes values. Can compiler be smarter?
"Sliding optimization" (reduces redundant computation)

\[
\begin{align*}
    \text{blurx}(x, y) &= (\text{in}(x-1, y) + \text{in}(x, y) + \text{in}(x+1, y)) / 3.0f; \\
    \text{out}(x, y) &= (\text{blurx}(x, y-1) + \text{blurx}(x, y) + \text{blurx}(x, y+1)) / 3.0f; \\
    \text{out}.\text{tile}(x, y, x_i, y_i, 256, 32);
\end{align*}
\]

\[
\text{blurx}.\text{store}_\text{at}(\text{out}, x) \\
\text{blurx}.\text{compute}_\text{at}(\text{out}, x_i);
\]

for \(y=0\) to num\_tiles\_y:
  for \(x=0\) to num\_tiles\_x:
    allocate \text{blurx}(256x34)

for \(yi=0\) to 32:
  for \(xi=0\) to 256:
    idx\_x = x*256+\(xi\);
    idx\_y = y*32+\(yi\);

    if (\(yi=0\)) {
      // compute 3 elements of blurx needed for \(\text{out}(\text{idx}_x, \text{idx}_y)\) here
      for \(\text{blurx}_y=0\) to 3:
        \text{blurx}(\text{xi}, \text{yi} + \text{blurx}_y) = \ldots // access values from buffer "in"
    } else
      \text{blurx}(\text{xi}, \text{yi} + 2) = \ldots // only compute one additional element of blurx

\text{out}(\text{idx}_x, \text{idx}_y) = \ldots // access values from buffer \text{blurx}

---

**Compute necessary elements of blurx within out's xi loop nest, but fill in tile-sized buffer allocated at x loop nest.**

**Steady state:** only one new element of blurx needs to be computed per output.
“Folding optimization” (reduces intermediate storage)

\[
\text{blurx}(x,y) = \frac{\text{in}(x-1,y) + \text{in}(x,y) + \text{in}(x+1,y)}{3.0f;}
\]
\[
\text{out}(x,y) = \frac{\text{blurx}(x,y-1) + \text{blurx}(x,y) + \text{blurx}(x,y+1)}{3.0f;}
\]

\[
\text{out}.\text{tile}(x, y, xi, yi, 256, 32);
\]

\[
\text{blurx}.\text{store}\_\text{at}(\text{out}, x) \quad \text{Compute necessary elements of blurx within out's xi loop}
\]
\[
\text{blurx}.\text{compute}\_\text{at}(\text{out}, xi);
\]

\[
\text{for } y=0 \text{ to num_tiles}_y:
\]
\[
\text{for } x=0 \text{ to num_tiles}_x:
\]
\[
\quad \text{allocate tmp_blurx}(256, 3)
\]

\[
\text{for } yi=0 \text{ to 32:}
\]
\[
\text{for } xi=0 \text{ to 256:}
\]
\[
\quad \text{idx}_x = x*256+xi;
\]
\[
\quad \text{idx}_y = y*32+yi;
\]

\[
\text{if } (yi=0) \{
\]
\[
\quad \text{// compute 3 elements of blurx needed for out(idx}_x, \text{idx}_y) \text{ here}
\]
\[
\quad \text{for } \text{blurx}_{y=0} \text{ to 3:}
\]
\[
\quad \quad \text{blurx}(xi, \text{blurx}_{y}) = \ldots
\]
\[
\}
\text{else}
\]
\[
\quad \text{blurx}(xi, (yi + 2) \% 3) = \ldots \quad \text{// only compute one additional element of blurx}
\]

\[
\text{out}(\text{idx}_x, \text{idx}_y) = \ldots
\]

Circular buffer of 3 rows

Steady state: only one new element of blurx needs to be computed per output

Accesses to blurx modified to access appropriate row of circular buffer: e.g., \((\text{idx}_y+1)\%3\)
Summary of scheduling the 3x3 box blur

// the “algorithm description” (declaration of what to do)
blurx(x,y) = (in(x-1, y) + in(x,y) + in(x+1,y)) / 3.0f;
out(x,y)   = (blurx(x,y-1) + blurx(x,y) + blurx(x,y+1)) / 3.0f;

// “the schedule” (how to do it)
out.tile(x, y, xi, yi, 256, 32).vectorize(xi,8).parallel(y);
blurx.compute_at(out, x).vectorize(x, 8);

Equivalent parallel loop nest:

for y=0 to num_tiles_y:    // iters of this loop are parallelized using threads
    for x=0 to num_tiles_x:
        allocate blurx(256, 34)
        for yi=0 to 32+2:
            for xi=0 to 256+2 by 8:
                blurx(xi,yi) = ...  // compute blurx from in using 8-wide
                                  // SIMD instructions here
                                  // compiler generates boundary conditions
                                  // since 256+2 isn’t evenly divided by 8
        for yi=0 to 32:
            for xi=0 to 256 by 8:
                idx_x = x*256+xi;
                idx_y = y*32+yi
                out(idx_x, idx_y) = ...  // compute out from blurx using 8-wide
                                         // SIMD instructions here
What is the philosophy of Halide

- **Programmer** is responsible for describing an image processing algorithm
- **Programmer** has knowledge to schedule application efficiently on machine (but it’s slow and tedious), so give programmer another language to express their high-level scheduling decisions
  - Loop structure of code
  - Unrolling / vectorization / multi-core parallelization

- **The system** (Halide compiler) is not smart, it provides the service of mechanically carrying out the nitty gritty details of implementing the schedule using mechanisms available on the target machine (pthreads, AVX intrinsics, CUDA code, etc.)
  - There are deviations from this philosophy in Halide? What are they?
Constraints on language
(to enable compiler to provide desired services)

- Application domain scope: computation on regular N-D domains
- Only feed-forward pipelines (includes special support for reductions and fixed recursion depth)
- All dependencies inferable by compiler
Initial academic Halide results

- Application 1: camera RAW processing pipeline
  (Convert RAW sensor data to RGB image)
  - Original: 463 lines of hand-tuned ARM NEON assembly
  - Halide: 2.75x less code, 5% faster

- Application 2: bilateral filter
  (Common image filtering operation used in many applications)
  - Original 122 lines of C++
  - Halide: 34 lines algorithm + 6 lines schedule
    - CPU implementation: 5.9x faster
    - GPU implementation: 2x faster than hand-written CUDA

[Ragan-Kelley 2012]
Stepping back: what is Halide?

- Halide is a DSL for helping expert developers optimize image processing code more rapidly
  - Halide does not decide how to optimize a program for a novice programmer (ignoring the auto scheduler, see tonight’s reading)
  - Halide provides a small number of primitives for a programmer that has strong knowledge of code optimization to rapidly express what optimizations the system should apply
    - parallel, vector, unroll, split, reorder, store_at, compute_at...
  - Halide compiler carries out the mapping of that strategy to a machine
Automatically generating Halide schedules

- Problem: it turned out that very few programmers have the technical ability to write good Halide schedules
  - Circa 2017... 80+ programmers at Google write Halide
  - Very small number trusted to write schedules

- Recent work: Halide compiler analyzes the Halide program to automatically generate efficient schedules for the programmer [Mullapudi 2016, Adams 2019]
  - As of Adams 2019, you’d have to work hard to manually author a schedule that is better than the schedule generated by the Halide autoscheduler for a complex image processing pipeline
Halide extensions

[Li 2018]

Differentiable Programming for Image Processing and Deep Learning in Halide

Tru-Mao Li, Michael Gharbi, Andrew Adams, Eldad Doron, Jonathan Ragan-Kelley
MIT CSAIL, Facebook AI Research, MIT CSAIL, UC Berkeley, Google

Better GPU support

[Anderson 2021]

Efficient Automatic Scheduling of Imaging and Vision Pipelines for the GPU

LUKE ANDERSON, Massachusetts Institute of Technology, USA
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TIAN JIN, Massachusetts Institute of Technology, USA
JONATHAN RAGAN-KELLEY, Massachusetts Institute of Technology, USA

We present a new algorithm to quickly generate high-performance GPU implementations of complex imaging and vision pipelines, directly from high-level Halide algorithm code. It is fully automatic, requiring no schedule templates or hand-optimized kernels. We address the scalability challenge of extending search-based automatic scheduling to map large real-world programs to the deep hierarchies of memory and parallelism on GPU architectures in reasonable compile time. We achieve this using (1) a two-phase search algorithm that first "freezes" decisions for the lowest cost sections of a program, allowing relatively more time to be spent on the important stages, (2) a hierarchical sampling strategy that groups schedules based on their structural similarity, then samples representatives to be evaluated, allowing us to explore a large space with few samples, and (3) memoization of repeated partial schedules, amortizing their cost over all their occurrences. We guide the process with an efficient cost model combining machine learning, program analysis, and GPU architecture knowledge.

We evaluate our method’s performance on a diverse suite of real-world imaging and vision pipelines. Our scalability optimizations lead to average compile time speedups of 49x (up to 530x). We find schedules that
Influence on code generation for ML applications

Example: Apache TVM

Apache TVM
An End to End Machine Learning Compiler Framework for CPUs, GPUs and accelerators

Schedule Primitives in TVM
- split
- tile
- fuse
- reorder
- bind
- compute_at
- compute_inline
- compute_root
- Summary
- Reduction

Tuning Parameters of Thread Numbers

How to schedule the workload, say, 32x32 among the threads of one cuda block? Intuitively, it should be like:

```
num_thread_y = 8
num_thread_x = 8
thread_y = tvm.thread_axis((0, num_thread_y), "threadIdx.y")
thread_x = tvm.thread_axis((0, num_thread_x), "threadIdx.x")
ty, yi = s[Output].split(h_dim, nparts=num_thread_y)
tx, xi = s[Output].split(w_dim, nparts=num_thread_x)
s[Output].reorder(ty, tx, yi, xi)
s[Output].bind(ty, thread_y)
s[Output].bind(tx, thread_x)
```

There are two parameters in the schedule: `num_thread_y` and `num_thread_x`. How to determine the optimal values?

Below is the result with Filter = [256, 1, 3, 3] and stride = [1, 1]:

<table>
<thead>
<tr>
<th>Case</th>
<th>Input</th>
<th>num_thread_y</th>
<th>num_thread_x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[1, 256, 32, 32]</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>[1, 256, 32, 32]</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>[1, 256, 32, 32]</td>
<td>1</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>[1, 256, 32, 32]</td>
<td>32</td>
<td>1</td>
</tr>
</tbody>
</table>

Many interesting observations from above results:
Darkroom/Rigel/Aetherling

Goal: directly synthesize ASIC or FPGA implementation of image processing pipelines from a high-level algorithm description (a constrained “Halide-like” language)

### Stencil Language

```plaintext
bx = im(x,y)
   (I(x-1,y) + I(x,y) + I(x+1,y))/3
end
by = im(x,y)
   (bx(x,y-1) + bx(x,y) + bx(x,y+1))/3
end
sharpened = im(x,y)
   (I(x,y) + 0.1*)
   (I(x,y) - by(x,y))
end
```

Goal: very-high efficiency image processing